

EFFECTS OF SKEWED DISTRIBUTIONS IN
INDIVIDUAL MEASUREMENT X AND
MOVING RANGE (n=2) MR
CONTROL CHARTS

BY

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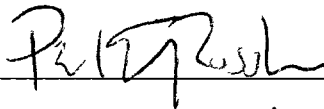
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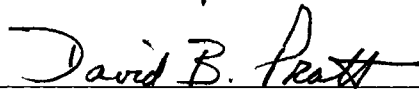
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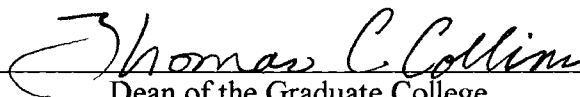
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CHAPTER I

THE PROBLEM AND ITS SETTING

INTRODUCTION

Control charts were originally developed by Dr. Walter A. Shewhart in order to determine if a sequence of data (measurements) obtained from an industrial process may be used for predictions of what this process will yield in the future. Shewhart reached the brilliant conclusion that it would be desirable and possible to set limits upon the natural variation of any process, so that fluctuations within these limits could be readily explained by chance or common causes, but any variation outside these limits would indicate the presence of special or assignable causes of variation in the process. When special causes are present, prediction is not feasible due to the presence of sources of uncontrolled variation. When only common causes are present, the process is said to be in state of statistical control (SOSC) and prediction is feasible. According to Shewhart (16, p. 6), "A phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to behave in the future."

Any quality characteristic of an industrial process can be studied by some type of control chart which has limits of variation based on the inherent or natural variability of the process. If the quality characteristic is measurable, it is common practice to organize the data in a rational manner in order to monitor the level and dispersion of the values generated by the process under study. Normally samples or subgroups of four or five

measurements are taken and the average \bar{X} and the range R of the subgroup are calculated to measure level and variability, respectively. Then, a series of such \bar{X} and R values are plotted in graphs in order to construct the \bar{X} and R control charts.

The setting of limits on the average \bar{X} control chart and range R control chart is based on the assumption of normal distribution underlying the process in study. For the average control chart the assumption is justified by the central limit theorem. The normal assumption is introduced into the \bar{X} and R control charts through the use of the control chart constants A_2 , D_3 , and D_4 . Many researches have been done to justify the robustness of the average \bar{X} and range R control charts when the underlying distribution is non-normal.

In some cases, the use of the average and range control charts is not appropriate because natural subgrouping is not possible. This might happen when the process is too slow in producing results being operationally impractical to wait until a subgroup of four or five can be formed, or when each measurement represents a given condition of a batch at a given time. Under these circumstances the logical subgroup size is $n=1$ and a combination of individual measurement X and moving range MR based on two consecutive observations can be used to monitor the level and dispersion of the process simultaneously. This is accomplished by constructing a control chart for individuals and a control chart for the moving range of two consecutive items. After all subgroups based on two consecutive observations are formed, the moving range MR for each subgroup and the average moving range \bar{MR} for all subgroups are calculated. Then, one can set the upper control limit for the moving range control chart UCL_{MR} using the equation $D_4\bar{MR}$

(the lower control limit for the moving range $n = 2$ MR control chart does not exist). If there are $MRs > UCL_{MR}$ the delete and revise procedure should be applied one time and the average moving range \overline{MR} and the UCL_{MR} recalculated. From the revised average moving range of two consecutive measurements \overline{MR} , one can estimate the standard deviation of the process σ using $\hat{\sigma} = \frac{\overline{MR}}{d_2}$. This estimate is likely to be only slightly affected by uncontrolled variation in individuals due to special causes. Then, one can set the control limits for the individual measurement control chart using $\bar{X} \pm 3\frac{\overline{MR}}{d_2}$.

The assumption of normality underlying the process is more critical for the individual measurement X control chart because the effect of the central limit theorem is not present. In addition, it is a well-known fact that many of the distributions encountered in our every-day experience cannot be described as normal distributions. Chemical, economical, biological, and physical factors have distributions which are commonly skewed in character. One of the commonest causes of non-normality is that the distribution may be unable to go beyond a certain point, such as zero. According to Burr (2, p. 67), "The Pearson type III family of distributions can be used as a second approximation of practical data when asymmetry is present." However, the individual measurement X and moving range $n = 2$ MR control charts are used widespread in industry under the blind assumption of normality. According to Duncan (1, p.400), "Control charts for individuals must be very carefully interpreted if the process shows evidence of marked departure from normality. In such cases, the multiples of σ used to set control limits might be better derived from other distributions for which the percentage points have been computed." Not too much work has been done to justify the

validity of the widespread use of the individual measurement X and moving range $n = 2$ MR control charts in industry when the underlying process distribution is non-normal.

GENERAL STATEMENT OF THE RESEARCH PROBLEM

This creative component research attempts to evaluate the effects of skewed distributions represented by the Pearson type III family of distributions with location parameter $c = 0$ (gamma distributions) on the control limits for individual measurement X and moving range $n = 2$ MR control charts.

The objectives of the study are stated as follows:

- (1) Evaluate the performance of the individual measurement X and moving range $n = 2$ MR control charts, using the control chart constants d_2 , d_3 , and D_4 under the assumption of normality, when the underlying distribution is Pearson type III family of distributions with location parameter $c = 0$ (gamma distributions) so often encountered in industry.
- (2) Determine empirical functions $f(\alpha, \beta)$ for the control chart constants d_2 , d_3 , and D_4 when the process distribution can be approximated by a Pearson type III distribution with location parameter $c = 0$, shape parameter α , and scale parameter β .
- (3) Compare the performance of the individual measurement X and moving range $n = 2$ MR control charts, using the normal control chart constants, with the performance of these control charts using the Pearson type III $c = 0$ control chart constants evaluated in objective two, when 10, 30, and 50 data values are available from an unknown process distribution.

STATEMENT OF THE HYPOTHESES

The first hypothesis is that individual measurement X and moving range $n = 2$ MR control charts, through the values of the normal curve control chart constants d_2 , d_3 , and D_4 used in the calculation of the control limits, work well for practical purposes in industry when the underlying process distribution can be approximated for a Pearson type III distribution with location parameter $c = 0$ (gamma distribution). For the X control chart, practical purpose in industry means a minimum average run length (ARL) of 100 without run rules when the process is in state of statistical control (no shifts) and maximum average run lengths (ARLs) of 43.9, 6.3, and 2 without run rules when the process is under shifts in the process average of 1, 2, and 3 process standard deviations, respectively. For the $n = 2$ MR control chart, practical purpose in industry means a minimum average run length (ARL) of 100 without run rules when the process is in control.

The second hypothesis is that the individual measurement X and moving range $n = 2$ MR control charts, using gamma control chart constants, have better performance than the control charts using the normal curve control chart constants, when 10, 30, and 50 data values are available from an unknown process distribution. Better performance means less empirical false alarm rate in control (X and MR control charts) and power of detection (X chart), for shifts in the process average of 1 and 2 sigma, at least equal to the theoretical power using the normal assumption (0.0228 and 0.1587 respectively for shifts in the process average of 1 and 2 sigma).

THE DELIMITATIONS

The study is limited to the effects of non-normality on the control limits for individual measurement X and moving range $n = 2$ MR control charts when the underlying process distribution is approximated for a Pearson type III distribution with $c = 0$.

The study will not address composite distributions (mixtures) as a severe test to evaluate the effects of non-normality because the consideration of a mixture of processes in a control chart is outside the good use of the control chart technique.

The evaluation of the type I error (α') does not include the effects of run rules. It means that only a point outside control limits is considered a signal.

The evaluation of the average run length (ARL) considers only shifts in the process average. It means that no shifts in the process standard deviation σ are considered.

In the evaluation of the performance of the individual measurement X control chart, using the Pearson type III $c = 0$ (gamma) bias correction factor d_2 , only the probability of type I error and two shifts in the process average (1 and 2 sigma) are considered.

In the evaluation of the performance of the moving range $n = 2$ MR control chart, using the gamma control chart constant D_4 , only the probability of type I error is considered.

The quality characteristic X under study cannot take negative values. This is due to the fact that the Pearson type III family of distributions with location parameter $c = 0$

(gamma distributions) does not take negative values. The range is $(0, +\infty)$.

THE DEFINITION OF TERMS

- ▶ Central limit theorem: Let X_1, X_2, \dots, X_n be a random sample from a distribution with mean " μ " and standard deviation " σ ". Then, if " n " is sufficiently large, the sample average has approximately a normal distribution with mean " μ " and standard deviation " σ/\sqrt{n} ". The larger the value of " n " the better the approximation.
- ▶ Population or Process: The totality of the set of items, units, measurements, etc., real or conceptual, that is under consideration or study.
- ▶ Sample or subgroup: A group of units, or portion of material, taken from a large collection of units, or quantity of material, which serves to provide information that can be used as a basis for judging the quality of the larger quantity (population), or as a basis for action on the larger quantity or on the production process.
- ▶ Random sample: A sample that contains independent observations selected from the same population or universe.
- ▶ Pearson type III family of distributions: This is a particular type of the Pearson system of distributions which was developed to cover different combinations of curve-shape characteristics α_3 and α_4 , where $2\alpha_4 - 3\alpha_3^2 - 6 = 0$. According to Burr (2, p. 67), the Pearson type III of distributions can be used as a second approximation to practical data when asymmetry is present. The normal curve is

considered as at least a first approximation to practical data if there is one mode and if the frequencies decrease on both sides with something like symmetry. Type III distributions with location parameter $c = 0$ are gamma distributions which go from a J-shaped curve with range $(0, +\infty)$ to a bell-shaped curve with range $(0, +\infty)$.

- ▶ Skewness (α_3): It is a measure for curve shape designed to measure the extent to which the distribution is unsymmetrical around the mode. For normal data $\alpha_3=0$. It is defined as the ratio between the third central moment and the standard deviation to the third power.
- ▶ Kurtosis (α_4): It is a second measure for curve shape, supplementing skewness, and it basically measures the relative rapidity of the frequencies to approach zero. For normal data $\alpha_4=3$. It is defined as the ratio between the fourth central moment and the standard deviation to the fourth power.
- ▶ Range: It is an order statistic; that is, its measure depends on the ordering of the individual values in a sample or subgroup, which serves as a measure of dispersion mainly in the quality control field. It is the difference between the largest and smallest values of the sample or subgroup.
- ▶ Moving range $n = 2$ MR: It is the successive differences between the individual values. It represents the short-term variation for a sequence of individual values.
- ▶ Control chart: A graphical chart with control limits and plotted values of some statistical measure for a series of samples or individual values. It is a tool created by Walter Shewhart in order to detect the presence of uncontrolled variation. It helps people to understand when they can make predictions regarding future

observations safely and when they cannot do so.

- ▶ Individual measurement \bar{X} control chart: It is a control chart for evaluating the process level in terms of a single observation per sample used when rational subgrouping is not appropriate.
- ▶ Moving range $n = 2$ MR control chart: It is a control chart for evaluating the variability within a process in terms of the range of the latest two observations in which the current observation has replaced the oldest of the previous two observations.
- ▶ Control limits: Limits on a control chart based on the data or standards given which are used as criteria for action or for judging the significance of variations between samples or individual values.
- ▶ d_2 : A bias correction factor, varying with the sample size n , used to convert ranges into σ .
- ▶ d_3 : A bias correction factor, varying with the sample size n , used to transform ranges into σ_R .
- ▶ D_4 : Control chart constant, varying with the sample size n , used to calculate the upper control limit for range R and moving range MR control charts.
- ▶ E_2 : Control chart constant, varying with the sample size n , used to set control limits for the individual measurement \bar{X} control chart.
- ▶ Robustness: Even though the data used to compute the control limits come from a non-normal distribution, the individual measurement \bar{X} and moving range $n = 2$ MR control charts based on the normal theory are capable of identifying non-

random variation with not too much type I error (false alarm rate).

- ▶ Run length: The run length of any control procedure is the number of sampling periods or subgroups before an out-of-control signal is given on the control chart.
- ▶ Average run length (ARL): It is the average number of subgroups or samples taken before an out-of-control condition is given on the control chart. It is a measure of control chart performance. A large ARL is desired when the process is stable or in control, and a small ARL (quick detection) otherwise.
- ▶ Average run length curve: It is a curve that shows the performance of a particular control chart under different shifts in the process average.
- ▶ Type I error (false alarm rate): It is the probability of thinking that the process is unstable when it really is stable. In other words, it is the probability of looking for problems when nothing has happened

ABBREVIATIONS AND NOTATION

Symbol	Term	Definition
μ	Population or process mean	
σ or σ_X	Population or process standard deviation	
X	Observed value or observation	
\bar{X}	Sample or subgroup average	$\sum X_i/n$
s	Sample or subgroup standard deviation	$\sum (X_i - \bar{X})^2 / (n-1)$
R	Sample or subgroup range	$X_{\max} - X_{\min}$
σ_R	Standard deviation of the theoretical distribution of ranges	$d_3\sigma$

\bar{R}	Average range	$\sum R_i / N$
MR	Moving range n = 2	$ X_{i+1} - X_i $
\overline{MR}	Average moving range	$\sum MR / (N-1)$
n	Sample or subgroup size	
N	Number of samples or subgroups	
k	Number of individual observations	
k'	Number of new individual observations	
α_3	Skewness parameter	$\int [(X - \mu)^3 / \sigma^3] f(x) dx$
α_4	Kurtosis parameter	$\int [(X - \mu)^4 / \sigma^4] f(x) dx$
a_3	Skewness statistic	$\sum (X_i - \bar{X})^3 / ns^3$
a_4	Kurtosis statistic	$\sum (X_i - \bar{X})^4 / ns^4$
α	Shape parameter gamma distribution	
β	Scale parameter gamma distribution	
c	Location parameter Pearson type III distribution	
α'	Type I error (false alarm rate)	
ARL	Average Run Length	$1 / P$ or $1 / P'$
P	Probability of detection X chart	Prob. (UCL < X < LCL)
P'	Probability of detection MR chart	Prob. (MR > UCL _{MR})
IID	Independent and identical distributed	
U (0, 1)	Uniform distribution between 0 and 1	
X (α , β)	Gamma distribution with parameters α and β	
R ²	Coefficient of determination	$1 - [\sum (y_i - \hat{y}_i)^2 / \sum (y_i - \bar{y})^2]$

X chart	Individual measurement control chart	
MR chart	Moving range of n = 2 control chart	
SOSC	State of statistical control	
UCL _X	Upper control limit for individuals	$\bar{X} + 3\overline{MR}/d_2$
LCL _X	Lower control limit for individuals	$\bar{X} - 3\overline{MR}/d_2$
CL _X	Center line for individuals	\bar{X}
UCL _{MR}	Upper control limit for moving ranges	$D_4 \overline{MR}$
CL _{MR}	Center line for moving ranges	\overline{MR}
d ₂	Bias correction factor	\bar{R}/σ or \overline{MR}/σ
d ₃	Bias correction factor	σ_R / σ
D ₄	Control chart constant	$1 + 3d_3/d_2$
E ₂	Control chart constant	$3 / d_2$

ASSUMPTIONS

- ▶ Average run lengths of a minimum of 100 for both control charts (X and MR), when the process is in a state of statistical control, are quite acceptable in industry for practical purposes. It means that people in industry are willing to accept a maximum risk of 1 % (1 / 100).
- ▶ There is not correlation between individual measurements (batches).
- ▶ The widespread use in industry of individual measurement X and moving range n = 2 MR control charts will continue in the future.

THE IMPORTANCE OF THE STUDY

The importance of this thesis research is to create the quantitative foundations to support the widespread use in industry of the individual measurement \bar{X} and moving range $n = 2$ MR control charts.

CHAPTER II

REVIEW OF THE LITERATURE

An overview of the literature relevant to the research objectives is presented in this chapter. Also, related research efforts are reviewed. This chapter is divided into seven sections. They are:

2.1 An historical overview of quality control and the control chart technique.

2.2 A discussion of the basis of control charts.

2.3 A description of the main types of variables control charts, their control limits, and related formulas.

2.4 A discussion about the estimation of the process standard deviation σ required to set control limits for the \bar{X} chart and X chart.

2.5 A discussion of the estimation of the range standard deviation σ_R required to set control limits for the R chart and MR ($n=2$) charts.

2.6 Estimation of the bias correction factors d_2 and d_3 used to evaluate the control chart constants A_2 , D_3 , D_4 , and E_2 .

2.7 Identification and discussion of previous researches related to the effects of non-normality in control charts.

2.1 Quality Control and Control Charts - An Historical Overview

Quality Control includes efforts to manage quality and maintain assurance of continued high quality of products and services. The word quality here refers to customer

satisfaction through the fitness of a product or service for its intended use. It means customer satisfaction by providing products or services which meet customer needs (product features) and are free from deficiencies (conformance to requirements). This definition of quality and others are discussed in detail in Juran (10), Crosby (11), ANSI/ASQC Standard A3 (12), and Juran and Gryna (13).

Even though most of the developments in quality control are fairly recent, the need for it has existed for a long time. The need for inspection of manufactured products has been around since processes and factories that produce these products have existed. Before the industrial revolution, workers and artisans inspected their own work. However, as work became more complicated and specialized, inspectors were hired to inspect the work of others. In this way, production workers left the responsibility for quality up to the inspector. As industries became larger and larger, the inspection job became too much for inspectors. Therefore, the technology of Quality Control was developed to design tools and techniques which have helped inspectors and quality control personnel to do their job.

According to Wadsworth, et al. (14, p. 6), "From 1925 to 1941, the development of quality control and quality assurance methodology was remarkable. In the December issue of *The Journal of the American Statistical Association*, Shewhart set the tone with his paper: *The Application of Statistics as an Aid in Maintaining Quality of a Manufactured Product*." In this paper, Shewhart (15) introduced the control chart concept.

The control chart is one of the earliest techniques developed in the quality control

field. When control charts are employed in their original form, they are called Shewhart charts. Dr. Walter Shewhart developed control charts in order to determine if a sequence of data (measurements) obtained from an industrial process may be used for predictions of what this process will yield in the future. Shewhart reached the brilliant conclusion that it would be desirable and possible to set limits based upon the natural variation of any process, so that fluctuations within these limits could be readily explained by chance or common causes, but any variation outside these limits would indicate the presence of special or assignable causes of variation. When special causes are present, prediction is not feasible due to the presence of sources of uncontrolled variation. When only common causes are present, the process is said to be in state of statistical control (SOSC) and prediction is feasible. According to Shewhart (16, p. 6), "A phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to behave in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that the observed phenomenon will fall within the given limits."

2.2 The Basis of Control Charts

As stated in 2.1, control charts were developed by Dr. Walter Shewhart in 1925 to determine if a sequence of observations obtained from an industrial process may be used for prediction of what this process will yield in the future. It means that the control chart technique represents a scientific basis for prediction.

The basis of all control charts is the following: Any varying quantity (whether an

individual measurement X , an average \bar{X} , a sample standard deviation s , a range R , a moving range MR , a fraction defective p , or a count on the number of defects c) forms a distribution if chance causes alone are present. Any such distribution has a mean and a standard deviation. According to Burr (2, p. 92), "Quite regardless of the shape of the distribution (unless extremely badly behaved), there will be, by chance causes only, very few points outside of the band between the mean minus three standard deviations and the mean plus three standard deviations. Hence, having set such limits, we have a band of normal variability for the statistical measure in question." Then, when a point falls outside this three sigma band, it is a much better bet to think that the point is due to the presence of special causes of variation than to think that the point is due to chance (common) causes alone. On the other hand, when a point falls inside the 3σ band, one can safely say that there is not evidence that an assignable (special) cause of variation is present and the process is said to be in state of statistical control (SOSC).

The primary concern of statistical control is predictability. According to Wheeler and Chambers (17, p. 38). "A process is predictable when it is in a state of statistical control, and it is unpredictable when it is not in state of statistical control ... Since the decision is to be based on past experiences, it follows that one will need to begin with data generated by the phenomenon in question. When a reasonable amount of these data have been accumulated, they are used to calculate appropriate limits. If the historical data fall within these limits, and if data collected after the limits have been calculated also stay within these historical limits, then it becomes reasonable to make a prediction regarding future observations."

The control chart technique is a powerful tool for making more fruitful people's efforts in stabilizing and controlling industrial processes at desired levels of performance.

2.2.1 The basis for establishing control limits

Any quality characteristic of an industrial process can be represented by a distribution function $f(x)$. Therefore, according to Shewhart (16, p. 275), "Knowing the distribution function $f(x)$, it is possible, in general, to find a distribution function $f_{\theta}(\theta, n)$ for a given statistic θ calculated from samples of size n such that the integral

$$P = \int_{\theta_1}^{\theta_2} f_{\theta}(\theta, n) d\theta$$

gives the probability that the statistic θ will have a value lying within the limits θ_1 and θ_2 ." However, people normally do not know $f(x)$ and $f_{\theta}(\theta, n)$ in sufficient detail to set up such control limits.

As a consequence, Shewhart says (16, p. 276) that the basis for establishing such allowable limits on the variability of sample statistics must be empirical. Under such conditions, Shewhart states (16, p. 276) that it seems reasonable to choose limits θ_1 and θ_2 on some statistic θ such that the associated probability P is economic. It means, according to Shewhart (16, p. 276), that "if more than one statistic is used, then the limits on all the statistics should be chosen so that the probability of looking for trouble when any one of the chosen statistics fall outside its own limits is economic."

Shewhart stresses (16, p. 276) that one must find a balance between the advantages of increasing the value of P through reduction in the probability of looking for trouble when none exist (type I error) and the disadvantages occasioned by failing to detect troubles when they really exist (type II error).

For all these reasons, Shewhart recommends (16, p. 277) a symmetrical range characterized by limits

$$\bar{\theta} \pm t\sigma_{\theta}$$

symmetrically spaced in reference to $\bar{\theta}$. Experience indicates that $t = 3$ seems to be an acceptable economic value.

According to Wheeler and Chambers (17, p. 60), "The strongest justification of three sigma limits is the empirical evidence that three sigma limits work well in practice ... that they provide effective action limits when applied to real world data."

Wheeler and Chambers (17) use part three of the Empirical Rule to empirically justify the selection of $t = 3$. Part three of the Empirical Rule states that given an homogeneous set of data, approximately 99% to 100% of the data will be located within three sigma units on either side of the average, regardless of the distribution function of the data. In order to evaluate the robustness of part three of the Empirical Rule, Wheeler and Chambers (17) use six different probability models: uniform, triangular, normal, the Burr distribution, chi-square, and exponential. The results showed that only for the chi-square distribution (98.6%) and exponential distribution (98.2%), the percentage of the data within three sigma units on either side of the average is scarcely less than the 99% to 100% stated in part three of the Empirical Rule. Therefore, Wheeler and Chambers (17, p. 64) conclude that "No matter how skewed, no matter how heavy-tailed, virtually all of the distribution will fall within three standard deviations of the mean ... Since data which display statistical control are, by definition, reasonably homogeneous, part three of the Empirical Rule provides an explanation why the control chart technique with $t = 3$

will yield very few false alarms. At the same time, when a point falls outside 3σ limits it is very likely to be due to the presence of an assignable cause of variation."

2.2.1.1 Standard given

According to ANSI/ASQC Standard A1 1978 (18, p. 1), "Control chart with standard given is a control chart whose control limits are based on adopted standard values applicable to the statistical measure plotted on the chart." This type of control chart is used to discover whether observed values of X , \bar{X} , p , c , etc., for samples differ from standard values X_0 , \bar{X}_0 , p_0 , c_0 , etc., by an amount greater than should be attributed to chance alone.

According to the ASTM special technical publication 15D (19, p. 75), "The standard value X_0 , \bar{X}_0 , p_0 , c_0 , etc., may be an experienced value based on representative prior data, or an economic value established on consideration of needs of service and costs of production, or a desired or aimed-at value designated by specifications." The problem with this method of setting control limits at $\bar{\theta}_0 \pm 3\sigma_\theta$ is that people almost never know the standard value of the mean $\bar{\theta}_0$ and standard deviation σ_θ of the statistic θ . Therefore, this method is rarely used in industry to set control limits for the statistical measure plotted on the chart.

2.2.1.2 No standard given

According to ANSI/ASQC Standard A1 1978 (18, p. 1), "Control chart with no standard given is a control chart whose control limits are based on the sample or subgroup data plotted on the chart." These control charts are based entirely on the data from the samples being evaluated. This type of control chart is used to determine whether

observed values of X , \bar{X} , R , MR , etc., for a series of samples or subgroups vary among themselves by an amount greater than should be attributed to chance alone.

In this thesis research, control charts with no standard given are the ones to be considered because, normally in industry, control limits are based on data rather than on standards.

2.3 Shewhart Control Charts based on Variables Data

Any quality characteristic of an industrial process can be studied and analyzed by some type of Shewhart control chart which has limits of variation based on the inherent or natural variability of the process. If the quality characteristic is measurable (variable), it is common practice to organize the data in a rational manner (rational subgroups) in order to monitor the level and dispersion of the values of the quality characteristic generated by the process in study. Usually, subgroups of n observations are taken and the average \bar{X} and the range R of the subgroup are calculated to measure level and variability, respectively. Then, a series of such \bar{X} and R values are plotted in a time series with control limits to construct the average \bar{X} and range R control charts.

The idea behind the use of subgroups to monitor the process in study is to minimize the sources of variation within a subgroup in order to make the control charts more sensitive to changes in the level and dispersion of the process (tighter control limits). According to the ASTM special technical publication 15D (19, p. 76), "One of the essential features of the control chart method is what is referred to as breaking up the data into rationally chosen subgroups called rational subgroups; that is, classifying the

observations under consideration into subgroups or samples, within which the variation may be considered on engineering grounds to be due to nonassignable chance causes only, but between which the differences may be due to assignable causes whose presence is suspected or considered possible."

2.3.1 \bar{X} and R control charts

The variable control charts that are most commonly used are average, or \bar{X} charts, and range, or R charts. According to Duncan (1, p. 381), "When control is undertaken by using variables instead of attributes, it usually takes the form of employing an \bar{X} chart to control the average of the process and an R chart to control the general variability of the process. The two taken together will give reasonably good control of the whole process."

Normally, samples or subgroups of four or five measurements are taken and the average \bar{X} and the range R of the subgroups are calculated. In fact, these two statistics (\bar{X} and R) are the ones actually used to monitor the process. Individual measurements X are primarily used to get values for the subgroup average and the subgroup range.

2.3.1.1 \bar{X} charts

An \bar{X} chart is a control chart for evaluating the process level or subgroup differences in terms of the subgroup average \bar{X} . According to ANSI/ASQC Standard A1-1978 (18, p. 3), "Averages are generally used for the purpose of determining whether there are differences between subgroup levels."

The setting of limits on the average \bar{X} control chart is based on the assumption of normality justified by the central limit theorem. The normal assumption is introduced into the \bar{X} charts through the use of the control chart constants d_2 and A_2 . The central line is

set at

$$CL_{\bar{X}} = \bar{\bar{X}}$$

where $\bar{\bar{X}}$ is the average of all the data.

The control limits $UCL_{\bar{X}}$ and $LCL_{\bar{X}}$ are set at $\pm 3 \hat{\sigma}_{\bar{X}}$ from $\bar{\bar{X}}$

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3 \hat{\sigma}_{\bar{X}}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - 3 \hat{\sigma}_{\bar{X}}$$

where $\hat{\sigma}_{\bar{X}}$ is an estimate of $\sigma_{\bar{X}}$ derived from the data.

Due to the central limit theorem

$$\hat{\sigma}_{\bar{X}} = \hat{\sigma}_X / \sqrt{n}$$

where $\hat{\sigma}_X$ is an estimate of the process standard deviation σ_X derived from the data. It is shown in section 2.4 below that $\hat{\sigma}_X$ is equal to \bar{R} / d_2 . Factor d_2 is the bias correction factor used in estimating the process standard deviation using the average sample range as a measure of dispersion.

Then,

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3 (\hat{\sigma}_X / \sqrt{n}) = \bar{\bar{X}} + 3 (\bar{R} / d_2 \sqrt{n})$$

where $3 / d_2 \sqrt{n}$ is known as the control chart constant A_2 .

Therefore,

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R}$$

In the same way

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$$

2.3.1.2 R charts

An R chart is a control chart for evaluating the variability within a process in

terms of the subgroup range R . According to ANSI/ASQC Standard A1-1978 (18, p. 3), "Ranges of the individual observations within the subgroup or sample are used to estimate the variability from chance causes within short time intervals and ordinarily should not include assignable causes. These ranges serve to estimate the inherent variability within an essentially unchanging process." Ranges are usually easier to compute than sample standard deviations but they are not recommended for $n > 10$.

The setting of limits on the R chart is based on the assumption of normality underlying the process in study. The normal assumption is introduced into the R charts through the use of the control chart constants d_2 , d_3 , and D_4 . The central line is set at

$$CL_R = \bar{R}$$

where \bar{R} is the average of the subgroup ranges.

The control limits UCL_R and LCL_R are set at $\pm 3 \hat{\sigma}_R$ from \bar{R}

$$UCL_R = \bar{R} + 3 \hat{\sigma}_R$$

$$LCL_R = \bar{R} - 3 \hat{\sigma}_R$$

where $\hat{\sigma}_R$ is an estimate of the range standard deviation σ_R derived from the data. It is shown in section 2.5 below that $\hat{\sigma}_R = d_3 \hat{\sigma}_X = (d_3 / d_2) \bar{R}$. Factor d_3 is the bias correction factor used in estimating the range standard deviation.

Then,

$$UCL_R = \bar{R} + 3 \hat{\sigma}_R = \bar{R} + 3 (d_3 / d_2) \bar{R} = \bar{R} [1 + 3 (d_3 / d_2)]$$

where $1 + 3 (d_3 / d_2)$ is known as the control chart constant D_4 .

Therefore,

$$UCL_R = D_4 \bar{R}$$

In the same way

$$LCL_R = \bar{R} - 3 \hat{\sigma}_R = \bar{R} - 3 (d_3 / d_2) \bar{R} = \bar{R} [1 - 3 (d_3 / d_2)] = D_3 \bar{R}$$

where $1 - 3 (d_3 / d_2)$ is known as the control chart constant D_3 .

For $n \leq 6$ the lower control limit for the range LCL_R does not exist.

2.3.2 Individual measurement X and moving range n = 2 MR control charts

In some cases, the use of the average and range control charts is not appropriate because natural subgrouping is not possible. This might happen when the process is too slow in producing results, being operationally impractical to wait until a subgroup of four or five can be formed, or when each measurement represents a given condition of a batch at a given time. Under these circumstances the logical subgroup size is $n=1$ and a combination of individual measurements X and moving ranges MR based on two consecutive observations can be used to monitor the level and dispersion of the process, respectively.

2.3.2.1 Control chart for individual observations (X chart)

An X chart is a control chart for evaluating the process level in terms of a single observation per visit to the process. According to Wadsworth, et al. (14, p. 143), "Their use is generally reserved for process and product characteristics for which it is impractical or unreasonable to replicate observations and to form subgroups of observations to aid the study of process variation."

The setting of limits on the individual measurement X control chart is based on the assumption of normality underlying the process under study. The normal assumption is introduced into the X chart through the use of the control chart constants d_2 and E_2 . The

central line is set at

$$CL_X = \bar{X}$$

where \bar{X} is the average of all the data.

The control limits UCL_X and LCL_X are set at $\pm 3 \hat{\sigma}_X$ from \bar{X}

$$UCL_X = \bar{X} + 3 \hat{\sigma}_X$$

$$LCL_X = \bar{X} - 3 \hat{\sigma}_X$$

where $\hat{\sigma}_X$ is an estimate of the process standard deviation σ_X derived from the data. It is shown in section 2.4 below that $\hat{\sigma}_X$ is equal to \bar{MR} / d_2 . Factor d_2 is the bias correction factor used in estimating the process standard deviation using the moving range ($n=2$) as a measure of dispersion.

Then,

$$UCL_X = \bar{X} + 3 (\bar{MR} / d_2)$$

where $3 / d_2$ is known as the control chart constant E_2 .

Therefore,

$$UCL_X = \bar{X} + E_2 \bar{MR}$$

In the same way

$$LCL_X = \bar{X} - 3 (\bar{MR} / d_2) = \bar{X} - E_2 \bar{MR}$$

2.3.2.2 Moving range $n = 2$ MR control charts

A moving range $n = 2$ MR control chart is a chart for evaluating the variability within a process in terms of the range of the latest two observations in which the current observation has replaced the older of the previous two observations.

The setting of limits on the MR chart is based on the assumption of normality

underlying the process under study. The normal assumption is introduced into the MR chart through the use of the control chart constants d_2 , d_3 , and D_4 . The central line is set at

$$CL_{MR} = \bar{MR}$$

where \bar{MR} is the average of the $k - 1$ moving ranges formed of $n = 2$.

The upper control limit UCL_{MR} is set at $3 \hat{\sigma}_{MR}$ above the central line \bar{MR}

$$UCL_{MR} = \bar{MR} + 3 \hat{\sigma}_{MR}$$

where $\hat{\sigma}_{MR}$ is an estimate of the moving range standard deviation for $n = 2$ (σ_{MR}) derived from the data. It is shown in section 2.5 below that $\hat{\sigma}_{MR} = d_3 \hat{\sigma}_X = (d_3 / d_2) \bar{MR}$. Factor d_3 is the bias correction factor used in estimating the moving range standard deviation for $n = 2$.

Then,

$$UCL_{MR} = \bar{MR} + 3 \hat{\sigma}_{MR} = \bar{MR} + 3 (d_3 / d_2) \bar{MR} = \bar{MR} [1 + 3 (d_3 / d_2)]$$

where $1 + 3 (d_3 / d_2)$ is known as the control chart constant D_4 .

Therefore,

$$UCL_{MR} = D_4 \bar{MR}$$

Due to the fact that the lower control limit for the range does not exist for $n \leq 6$, the lower control limit for the moving range with $n = 2$ does not exist.

The values of the bias correction factors d_2 and d_3 and the values of the control chart constants A_2 , D_3 , D_4 , and E_2 , when the underlying process distribution is assumed normal, are given in Appendix A. This table is reproduced from ANSI/ASQC Standard A1-1978 (18, p. 13).

In section 2.6 below, the meaning of the bias correction factors d_2 and d_3 is

explained in detail. The other control chart constants used in this creative component research (A_2 , D_3 , D_4 , and E_2) derive from the two bias correction factors d_2 and d_3 .

2.4 Estimation of the Process Standard Deviation required to set control limits for the \bar{X} and X charts.

In small samples or subgroups, the standard deviation and the range are likely to vary together. Thus, if the sample standard deviation is large, the sample range is also likely to be large. If the sample standard deviation is small, the sample range is likely to be small, too. However, in large subgroups, the occurrence of one extreme value will cause the sample range to be large, but it may have much less effect on the sample standard deviation. Therefore, if one is interested in finding an estimate of the process standard deviation, the sample range may often be employed as a substitute for the standard deviation with little loss in efficiency for small subgroup sizes. Grubbs and Weaver (21, p. 224-225) state that "The range should not be used when the sample size is greater than about 15 or 20. The reason for this is due to the practical loss in the efficiency of the range as compared to the standard deviation for sample sizes greater than about 10." Thus, the sample range is not recommended to estimate the process standard deviation for subgroup size $n > 10$.

A table which provides the efficiency of the estimator $\hat{\sigma}$ based on the sample range statistic was published by David (20, p. 185). From the table, David states that "the efficiency of $\hat{\sigma}$ seems to be adequate for $n \leq 12$ and very good for the small sample sizes (typically $n = 5$) generally used in quality control work." Mathematical statisticians

define the most efficient estimator $\hat{\theta}$ of a parameter θ as the estimator that has the minimum variance. The efficiency of any other unbiased estimator is defined as the ratio of the variance of the sampling distribution of the efficient estimator to the variance of the sampling distribution of the other estimator. David (20, p. 185) defines the efficiency of $\hat{\sigma}$ based on the sample range statistic as:

$$\text{eff}(\hat{\sigma}) = \text{Var } S' / \text{Var } \hat{\sigma}$$

where S' is the unbiased root mean square estimator of σ .

The sample range, in addition to being nearly as efficient as the sample standard deviation for small samples ($n \leq 10$), is easy to calculate. For that reason, it is usually preferred to the standard deviation in quality control analysis.

In statistical quality control, the average subgroup range is used to estimate the process standard deviation. However, the average range is a biased estimator of σ . Therefore, a bias correction factor (d_2), as a function of the subgroup size n , is required to get an unbiased estimator of the process standard deviation σ based on the average subgroup range.

$$\hat{\sigma} = \bar{R} / d_2$$

According to Grubbs and Weaver (21, p. 224), "A measure of the true variation in a lot could be estimated by simply taken the difference between the largest and smallest observations in a sample of n items and dividing the range obtained by a factor which depends on the sample size. Although the sample standard deviation is a more efficient estimate of dispersion, it may be desirable from a practical standpoint to use the range in view of its simplicity and since only a slight loss in efficiency is suffered in the case of

small samples."

This estimator $\hat{\sigma} = \bar{R} / d_2$ of the process standard deviation σ based on the average subgroup range has been also emphasized in the quality control literature by Shone (22), Nelson (23), and Masuyama (24).

In cases where natural subgrouping is not possible, because the process is too slow in producing results being operationally impractical to wait until a subgroup of four or five can be formed or because each measurement represents a given condition of a batch at a given time, the logical subgroup size is $n=1$. In this situation, a combination of individual measurement X and moving range MR based on two consecutive observations can be used to monitor the level and dispersion of the process simultaneously. When subgroup size $n = 1$, the subgroup range is impossible to be evaluated. Therefore, the process standard deviation σ required to set control limits for the X chart cannot be estimated using the equation

$$\hat{\sigma} = \bar{R} / d_2$$

According to Cryer and Ryan (25, p. 187), "The standard approach to estimating sigma for an individual observation control chart is to use moving ranges of two consecutive observations." From the average moving range of two consecutive measurements \bar{MR} , one can easily estimate the process standard deviation σ by using the equation

$$\hat{\sigma} = \bar{MR} / d_2$$

This estimate is likely to be slightly affected by uncontrolled variation in individuals due to special causes. In this regard, Keen and Page (26, p. 13) state that "a rapid estimate of

the standard deviation of a distribution that is nearly normal can be obtained by regarding the mean successive differences between adjacent readings, arranged in order of collection, as an approximation of the mean range of two. This method was suggested by Mr. W. J. Jennett for control chart work carried out in the MO Valve Company Ltd. in 1942, and has since been used extensively in The General Electric Company Ltd. The calculation of the standard deviation is made in three steps: (1) take the differences between the consecutive values, (2) average the differences (ignoring sign), and (3) regarding them as ranges in random samples of two, divide the mean difference by the bias correction factor $d_2 (n = 2) = 1.128$."

2.5 Estimation of the Range Standard Deviation σ_R required to set control limits for the R chart and MR (n = 2) charts.

Considering moving ranges of two consecutive measurements MR as ranges R in random samples of two ($n = 2$), the setting of control limits for the MR ($n = 2$) chart can be seen as a particular case of the R chart with $n = 2$. Therefore, in this section, only the estimation of the range standard deviation σ_R is discussed.

As described in sections 2.3.1.2 (p. 23) and 2.3.2.2 (p. 26), the setting of limits on the R chart and MR chart is based on the general Shewhart equations:

$$UCL_R = \bar{R} + 3 \hat{\sigma}_R$$

$$LCL_R = \bar{R} - 3 \hat{\sigma}_R$$

where $\hat{\sigma}_R$ is an estimate of the range standard deviation σ_R derived from the data.

The estimator $\hat{\sigma} = \bar{R} / d_2$ is a biased estimator of the range standard deviation.

Therefore, a bias correction factor (d_3), as a function of the subgroup size n , is required to get an unbiased estimator of the range standard deviation σ_R based on the process standard deviation.

$$\hat{\sigma}_R = d_3 \hat{\sigma} = (d_3 / d_2) \bar{R}$$

In the same way

$$\hat{\sigma}_{MR} = d_3 \hat{\sigma} = (d_3 / d_2) \overline{MR}$$

2.6 Estimation of the Bias Correction Factors d_2 and d_3 used to evaluate the control chart constants A_2 , D_3 , D_4 , and E_2 .

The difficulty in the estimation of the bias correction factors d_2 and d_3 is that the distribution of the sample range does not have a particular probability distribution which represents it. According to Burr (28, p. 636-637) "Formidable difficulties of approximation are usually involved in finding the density function and the first four moments of the range. This is especially true when the population distribution function, $F(x)$ can only be expressed in terms of an integral." In this regard, Grant (41, p. 86) states that for the distribution of the range no simple formula gives either \bar{R} or σ_R . However, theory fully defines the expected distribution of R when the parent population is normal. According to Duncan (1, p. 126) "Fortunately, tables of the distribution of the relative range $W = \bar{R} / \sigma$ have been worked out for a normal universe." In addition, the sample range distribution is greatly affected by the shape of the parent distribution. According to Hartley (32, p. 334), "extensive investigations have shown that its random sampling distribution is markedly dependent on the parental population." In the same way,

Shewhart (16, p. 202) states that "We do not know the distribution function of the range ... there is a marked influence of the functional form of the universe upon the distribution function of the ranges."

As mentioned in chapter I (in the definition of terms), the range is an order statistic which serves as a measure of dispersion mainly in the quality control field. It is the difference between the largest and smallest values of the sample or subgroup. Because of its simplicity, the sample range is often used instead of the sample standard deviation as an estimator of the process standard deviation. David (20), Grubbs and Weaver (21), Shone (22), Nelson (23), Masuyama (24), Cryer and Ryan (25), Keen and Page (26), and others have used the sample range, the sample mean range, the sample moving range of two consecutive observations, and the average sample moving range of two consecutive measurements as estimators of the process standard deviation in normal theory tests.

Since the early paper by Tippett (1925), interest in the distribution of sample ranges and associated applications has steadily grown. In his paper, Tippett (27) states that "It has not been found possible to write down the distribution of ranges in any useful form, so the procedure has been to find those constants involving the first four moments." Values of the bias correction factors $d_2 = \bar{R} / \sigma$ and $d_3 = \sigma_R / \sigma$ were found by Tippett (27) for all sample sizes from two to one thousand when the underlying process distribution is normal.

According to Burr (28, p. 636), "Since the first paper by Tippett (27), the distribution of ranges from independent observations from a normal distribution has been

well covered." Pearson (29, 1926), Pearson (30, 1932), Pearson (31, 1942), Hartley (32, 1942), Harter (33, 1960), and others have showed interest in the range and its properties including quality control applications. All this work has generated tables giving the standardized mean values (d_2) and the standardized standard deviation (d_3) of the range from random samples drawn from the normal distribution as well as tables of the probability integral of the range.

The effect of non-normality on the mean range has been experimentally studied by Pearson (34, 1928) and Shone (22, 1949). Pearson (35, 1950) has summarized earlier empirical studies on the effect of non-normality on the range and concluded that the range may be used with the appropriate adjustment (d_2) for a normal population as an estimator of the process standard deviation, provided that samples of not more than 10 observations are taken. David (36, 1954) derived some exact results regarding the mean and probability integral of the range in samples from a number of non-normal populations such as the rectangular, exponential and others. Cox (37, 1954) got a general picture of the effect of process kurtosis on the mean and coefficient of variation of the range in small samples and assessed the effect of non-normality on the common applications of the range.

As mentioned before, Burr (28, p. 636-637) states that "Formidable difficulties of approximation are usually involved in finding the density function and the first four moments of the range. This is especially true when the process distribution function, $F(x)$, can only be expressed in terms of an integral. When, however, $F(x)$ can be given in closed form, the calculations become more feasible." Burr (28), using 81 distributions

from the Burr system of distributions $F(x) = 1 - (1 + x^c)^{-k}$ for $x \geq 0$ and $c, k > 0$, evaluated the four moments ($\mu_R, \sigma_R, \alpha_{3:R}, \alpha_{4:R}$) for the corresponding 81 range distributions generated from the 81 parent distributions.

Due to the complexity in finding the density function and the four moments of the range mentioned by Burr (28), and due to the fact that d_2 and d_3 have been tabulated when the parent population is normal, the quality control field has centered its attention only in the use of the normal d_2 and d_3 bias correction factors. These normal bias correction factors are used in the process of setting control limits for the \bar{X} chart, R chart, X chart, and MR ($n = 2$) control charts.

It is one of the objectives of this thesis research to evaluate the performance of these normal d_2 and d_3 bias correction factors when the underlying process distribution is a Pearson type III with location parameter $c = 0$. In addition, another objective of this study is to find the empirical (approximate) values of d_2 and d_3 when the process distribution is approximated by a Pearson type III with $c = 0$.

2.7 Previous researches - Effects of non-normality in control charts.

Researches in this field have been mainly concentrated on the effects of non-normality in the average \bar{X} and range R control charts where the central limit theorem plays an important role in the assumption of normality.

2.7.1 Niemann's experimental thesis

Niemann (38) in an unpublished master thesis studied the effect of lack of symmetry of the population upon the control chart constants. The population sampled is

approximately a Pearson type III curve. In his experiment, 4000 samples of $n = 4$ each were drawn with replacement. The values of the mean, standard deviation, and range were calculated for each subgroup and the results tabulated. Then, the values of the control chart constants were approximated and compared with the corresponding values for the normal curve control chart constants for $n = 4$. Comparison of the two set of limits indicates similarity, although the subgroup size is small ($n = 4$) and the process distribution is quite skewed.

Niemann's general conclusion is that if control chart constants for the normal curve are used, when the process distribution is non-normal through lack of symmetry, tighter control limits should be observed. Thus, it is slightly more likely for assignable causes to be indicated when none exist (type I error). Therefore, it is also a bit more difficult to get a process into a perfect state of statistical control than if the unknown correct constants are used.

This experimental study is described in Burr (2, p. 163-165). According to Burr (2, p. 163), "This experimental thesis has given us good grounds for confidently using the normal curve constants even for moderately non-normal populations. For J-shaped or very strongly skewed populations, the normal curve constants probably cannot be much relied upon."

2.7.2 Irving Burr's research

Irving Burr (39, 1967) published a paper in which he discusses the effect of non-normality on the control chart constants. He presents a set of tables where the values of eight control chart constants (d_2 , d_3 , A_2 , E_2 , and D_1 - D_4) are exactly calculated for 28

different non-normal distributions given by different combinations of skewness and kurtosis. Burr states (39, p.563) that "a literature search, involving about 100 papers, revealed that the sampling distribution for ranges is known for only a very few populations, of which only the normal distribution is of any appreciable practical importance ... the present article provides information on the effects of various practical degrees of non-normality upon the control chart constants for \bar{X} and R charts."

From the 81 non-normal distributions considered by Burr (28), 28 distributions are selected by Burr (39) in this study to cover to some extent a wide band of the plane α_3 (skewness) - α_4 (kurtosis). In reference (28), Burr calculates $E(R)$, σ_R , $\alpha_{3,R}$, and $\alpha_{4,R}$ for the 81 sampling distributions of ranges corresponding to the 81 non-normal parent populations chosen. This allows Burr to obtain the control chart constants required for three standard deviation control charts for measurements. The first two constants d_2 and d_3 (bias correction factors) were evaluated using the equations:

$$d_2 = E(R) / \sigma = \bar{R} / \sigma$$

$$d_3 = \sigma_R / \sigma$$

The last six constants (A_2 , E_2 , and D_1 - D_4) were all determined from d_2 and d_3 using the equations:

$$A_2 = 3 / d_2 \sqrt{n}$$

$$E_2 = 3 / d_2$$

$$D_1 = d_2 - 3d_3$$

$$D_2 = d_2 + 3d_3$$

$$D_3 = 1 - 3 (d_3 / d_2)$$

$$D_4 = 1 + 3 (d_3 / d_2)$$

From the values of the control chart constants obtained, Burr (39, p. 567) concludes that "study of constants given in Tables I to IV reveals that they are quite stable, in some cases surprisingly so, as for example A_2 . Thus we can use the ordinary normal curve control chart constants unless the population is markedly non-normal. When it is, the tables provide guidance on what constants to use."

2.7.3 Schilling's and Nelson's research

As mentioned before, the setting of limits on \bar{X} control charts are based on the assumption of normality justified by the central limit theorem. This theorem states that the distribution of sample means will approach the normal distribution for large sample sizes. Schilling and Nelson (40) address the questions, "How large?" and "To what approximation?"

According to Schilling and Nelson (40, p. 183), "By numerically inverting the appropriate characteristic functions, tables are provided which show the manner of approach to normality for various underlying distributions and sample sizes. Also, sample sizes are given such that, at selected points, the sum of the tail areas of the distribution of sample means will be within given values." They investigated nine different populations such as rectangular, right triangular, gammas with $\beta=1$ and $\alpha=1/2, 1, 2, 3,$ and $4,$ symmetric bimodal, and asymmetrical bimodal.

The results of this study indicate that the approach to normality of the distribution of sample means is slower than is often indicated in the literature, especially for the exponential and mixed distributions. In relation with applications to control charts for the

mean \bar{X} , Schilling and Nelson (40, p. 187) state that "If the risk of a false signal is held to 0.3% for a three sigma control chart, samples of considerable size would be required to be plotted for the exponential ($n = 166$) and asymmetric bimodal distributions ($n = 47$) ... The risk of 0.003 for three sigma control charts is often quoted on the basis of normal theory; 0.3% should however not be considered sacrosanct. When used with non-normal distributions, reasonably small probabilities of a type I error should suffice for construction of the chart. Experience over the years has shown 3σ limits with small sample sizes to be of utmost practical value. Examination of Table 1 (p. 186-187) indicates that such charts assure the risk of a false signal to be 1.4% or less when plotting subgroups of size 4 or more, over all the distributions studied. A maximum risk of 1 in 72 for the most extreme case studied (gamma with $\alpha=1/2$) seems to imply that samples of this size are sufficient for most practical applications."

2.7.4 Individual measurement X and moving range $n = 2$ MR control charts

For the individual measurement X and moving range $n = 2$ MR control charts, no work was found that addresses the effects of non-normality on these control charts.

CHAPTER III

GENERAL PROCEDURE

3.1 Objective one. The first objective is to evaluate the performance of the individual measurement and moving range $n=2$ MR control charts, using the control chart constants d_2 , d_3 , and D_4 under the assumption of normality, when the underlying distribution is the Pearson type III family of distributions with location parameter $c=0$ (gamma distributions) so often encountered in industry.

PROCEDURE I

The detailed procedure designed to accomplish objective one is the following:

1. Forty gamma distributions are selected using different combinations of shape parameter α and scale parameter β in order to have a representative set of gamma distributions which go from J-shaped curves to bell-shaped curves approximating the normal distribution as α increases.

Ten values of the shape parameter α are selected. They are:

$$\alpha = 0.5, 1, 1.5, 2, 3, 4, 5, 10, 50, \text{ and } 100$$

Four values of the scale parameter β are selected. They are:

$$\beta = 0.5, 1, 5, \text{ and } 10$$

2. Generate ten thousand (10,000) random variates for each gamma distribution indicated in step one of this procedure.

General gamma random variates are more complicated to generate than other continuous distributions because the distribution function has no simple closed form for

which one can find an inverse in order to use the inverse transform method. However, given $Y \sim$ gamma distribution $(\alpha, \beta=1)$, any gamma distribution (α, β) can be easily obtained by letting $X = \beta Y$. Therefore, if one can generate a gamma distribution $(\alpha, \beta=1)$, more general gamma distributions (α, β) can be easily generated using the equation $X = \beta Y$ indicated above.

The algorithm to be used in the generation of the 10,000 random variates for each gamma distribution depends upon the value of the shape parameter α .

2.1 Case $0 < \alpha < 1$. This algorithm is due to Ahrens and Dieter (3). Atkinson and Pearce (4) tested three algorithms for the generation of gamma variates and recommend Ahrens and Dieter algorithm for a "one-time" simulation which is the case in this thesis research. In addition, Law and Kelton (5) recommend the use of this algorithm to generate general gamma random variates with shape parameter α less than one.

Algorithm:

2.1.1 Generate U_1 as an independent and identical distributed uniform $U(0, 1)$, and let $P = bU_1$ and $b = (e + \alpha) / e$

where

$$e = \text{constant } 2.718281828$$

If $P > 1$, go to step 2.1.3. Otherwise, proceed to step 2.1.2.

2.1.2 Let $Z = P^{\frac{1}{\alpha}}$, and generate $U_2 \sim U(0, 1)$. If $U_2 \leq e^{-Z}$, return $Y = Z$ and

$X = \beta Y$ where

β is the scale parameter of the general gamma distribution to be generated.

Otherwise, go back to step 2.1.1.

2.1.3 Let $Z = -\ln\left[\frac{(b-P)}{\alpha}\right]$, and generate $U_2 \sim U(0, 1)$. If $U_2 \leq Z^{\alpha-1}$, return $Y = Z$ and $X = \beta Y$. Otherwise, go back to step 2.1.1.

2.2 Case $\alpha = 1$. A gamma distribution with shape parameter $\alpha = 1$ and scale parameter β is an exponential distribution with mean β . The algorithm to be used in order to generate gamma variates ($\alpha = 1, \beta$) is based on the inverse transform method.

Algorithm:

2.2.1 Generate $U \sim U(0, 1)$

2.2.2 Return $X = -\beta \ln(U)$

2.3 Case $\alpha > 1$. According to Law and Kelton (5, p. 489), "There are several good algorithms for the case $\alpha > 1$." However, they recommend a method due to Cheng (6), who called this the GB algorithm.

Algorithm:

2.3.1 Generate U_1 and U_2 as IID $U(0, 1)$ and let $a = 1 / (2\alpha - 1)^{1/2}$, $b = \alpha - \ln(4)$, $q = (\alpha + 1) / a$, $\theta = 4.5$, and $d = 1 + \ln(\theta)$.

2.3.2 Let $V = a \ln[U_1 / (1 - U_1)]$, $Z = \alpha(e)^V$, $T = U_1^2 U_2$, and $W = b + qV - Z$

2.3.3 If $W + d - \theta T \geq 0$, return $Y = Z$ and $X = \beta Y$. Otherwise, proceed to the next step (2.3.4.).

2.3.4 If $W \geq \ln(T)$, return $Y = Z$ and $X = \beta Y$. Otherwise, go back to step 2.3.1.

In order to generate correct gamma variates, it is essential that a statistically reliable $U(0, 1)$ random-number generator be available. In this thesis research the random-number generator used is the one included in Minitab for windows release 10. This random-number generator generates U_i 's which resemble values of true independent and

identically distributed uniform $U(0, 1)$ random variables. This generator was tested statistically to see how closely the U_i 's generated resemble IID $U(0, 1)$ random variables. The test used is a special case of the chi-square test with all parameters known. This test is recommended by Law and Kelton (5, p. 437). The results are shown in Appendix B and they indicate that the generator is good enough to generate reliable gamma variates.

3. Calculate the average \bar{X} of the 10,000 gamma variates for each of the forty gamma distributions using the equation $(\sum X_i) / 10,000$.

4. Group the 10,000 gamma variates $X_1, X_2, X_3, \dots, X_{5000}, X_{5001}, \dots, X_{9999}, X_{10000}$ for each gamma distribution generated in step 2 in subgroups of two consecutive measurements in the following manner:

$$(X_1, X_2), (X_2, X_3), \dots, (X_{5000}, X_{5001}), \dots, (X_{9999}, X_{10000}).$$

At the end there will be 9,999 (N-1) subgroups of $n = 2$ consecutive measurements for each of the forty gamma distributions generated in step 2.

Then, for each subgroup, calculate the moving range $n = 2$ MR using the equation:

$$MR = \text{Absolute value } (X_{i+1} - X_i)$$

In this way, the empirical moving range $n = 2$ MR distributions are obtained from the 9,999 moving ranges already calculated for each gamma distribution.

5. Evaluate the expected value of each of the forty empirical moving range $n = 2$ MR distributions. This evaluation is done using the equation:

$$E(MR) = \overline{MR} = (\sum MR_i) / 9,999$$

6. With the average moving range \overline{MR} for each empirical moving range $n = 2$ MR

distribution and the value of the normal control chart constants d_2 and D_4 , control limits are set for the individual measurement and moving range $n = 2$ MR control charts for each of the forty gamma distributions using the following equations:

$$UCL_X = \bar{X} + 3 \overline{MR} / d_2 = \bar{X} + 2.6596 \overline{MR}$$

$$LCL_X = \bar{X} - 3 \overline{MR} / d_2 = \bar{X} - 2.6596 \overline{MR}$$

$$UCL_{MR} = D_4 \overline{MR} = 3.268 \overline{MR}$$

where

d_2 is the bias correction factor which has a value of 1.128 for $n = 2$ under the assumption of normality.

D_4 is the control chart constant which has a value of 3.268 for $n = 2$ under the assumption of normality.

7. Evaluate the performance of the individual measurement and moving range $n = 2$ MR control charts for each of the forty gamma distributions defined in step 1 of this Procedure I.

For the individual measurement control charts, the performance will be measured through the average run length (ARL) for several shifts in the process average $k\sigma$ where $k = 0 (0.2) 3$ using the equation:

$$ARL = 1 / P$$

where

P is the probability of detection evaluated by the equation:

$$P = \text{Probability}(X > UCL_X) + \text{Probability}(X < LCL_X)$$

These probabilities will be evaluated using the gamma cumulative distribution

function routine contained in Minitab for windows release 10.

Then, empirical ARL curves will be drawn for each of the forty X charts in order to show the performance of the individual measurement X control charts, using the control chart constants d_2 and D_4 under the assumption of normality, when the underlying process distribution is Pearson type III with location parameter $c = 0$ (gamma distributions).

For the moving range $n = 2$ MR control charts, the performance will be measured through the probability of type I error since only shifts in the process average are considered in this research.

Since the moving range $n = 2$ MR distribution is theoretically unknown when the parent distribution is Pearson type III with $c = 0$, the probability of a type I error and empirical ARL in control must be evaluated empirically using the equations:

$$\text{Empirical probability (type I error)}_{\text{MR}} = P' = B / 9,999$$

$$\text{Empirical ARL in control} = \text{ARL}' = 1 / P'$$

where

B is the number of moving ranges greater than the UCL_{MR} for each of the forty moving range ($n=2$) control charts corresponding to the forty gamma distributions generated in step 2.

3.2 Objective two. The second objective is to determine the empirical formulas $f(\alpha, \beta)$ for the control chart constants d_2 , d_3 , and D_4 when the process distribution can be approximated by a Pearson type III distribution with location parameter $c = 0$, shape parameter α , and scale parameter β .

PROCEDURE II

The detailed procedure designed to accomplish objective two is the following:

1. Forty gamma distributions are selected using different combinations of shape parameter α and scale parameter β in order to have a representative set of gamma distributions which go from J-shaped curves to bell-shaped curves approximating the normal distribution as α increases.

Ten values of the shape parameter α are selected. They are:

$$\alpha = 0.5, 1, 1.5, 2, 3, 4, 5, 10, 50, \text{ and } 100$$

Four values of the scale parameter β are selected. They are:

$$\beta = 0.5, 1, 5, \text{ and } 10$$

2. Generate ten thousand (10,000) random variates for each gamma distribution indicated in step 1 of this Procedure II.

The algorithm used in the generation of the 10,000 random variates for each gamma distribution depends upon the value of the shape parameter α .

- 2.1 Case $0 < \alpha < 1$.

Algorithm:

2.1.1 Generate U_1 as an independent and identical distributed uniform $U(0, 1)$,

and let $P = bU_1$ and $b = (e + \alpha) / e$

where

$$e = \text{constant } 2.718281828$$

If $P > 1$, go to step 2.1.3. Otherwise, proceed to step 2.1.2.

2.1.2 Let $Z = P^{\frac{1}{\alpha}}$, and generate $U_2 \sim U(0, 1)$. If $U_2 \leq e^{-Z}$, return $Y = Z$ and

$X = \beta Y$ where

β is the scale parameter of the general gamma distribution to be generated.

Otherwise, go back to step 2.1.1.

2.1.3 Let $Z = -\ln\left[\frac{(b-P)}{\alpha}\right]$, and generate $U_2 \sim U(0, 1)$. If $U_2 \leq Z^{\alpha-1}$, return $Y = Z$ and $X = \beta Y$. Otherwise, go back to step 2.1.1.

2.2 Case $\alpha = 1$.

Algorithm:

2.2.1 Generate $U \sim U(0, 1)$

2.2.2 Return $X = -\beta \ln(U)$

2.3 Case $\alpha > 1$.

Algorithm:

2.3.1 Generate U_1 and U_2 as IID $U(0, 1)$ and let $a = 1 / (2\alpha - 1)^{1/2}$, $b = \alpha - \ln(4)$, $q = (\alpha + 1) / a$, $\theta = 4.5$, and $d = 1 + \ln(\theta)$.

2.3.2 Let $V = a \ln[U_1 / (1 - U_1)]$, $Z = \alpha(e)^V$, $T = U_1^2 U_2$, and $W = b + qV - Z$

2.3.3 If $W + d - \theta T \geq 0$, return $Y = Z$ and $X = \beta Y$. Otherwise, proceed to the next step (2.3.4.).

2.3.4 If $W \geq \ln(T)$, return $Y = Z$ and $X = \beta Y$. Otherwise, go back to 2.3.1.

3. Calculate the average \bar{X} of the 10,000 gamma variates for each of the forty gamma distributions using the equation $(\sum X_i) / 10,000$.

4. Group the 10,000 gamma variates $X_1, X_2, X_3, \dots, X_{5000}, X_{5001}, \dots, X_{9999}, X_{10000}$ for each gamma distribution generated in step 2 in subgroups of two consecutive measurements in the following manner:

$$(X_1, X_2), (X_2, X_3), \dots, (X_{5000}, X_{5001}), \dots, (X_{9999}, X_{10000}).$$

At the end there will be 9,999 (N-1) subgroups of n = 2 consecutive measurements for each of the forty gamma distributions generated in step 2 of this Procedure II.

Then, for each subgroup, calculate the moving range n = 2 MR using the equation:

$$MR = \text{Absolute value } (X_{i+1} - X_i)$$

In this way, the empirical moving range n = 2 MR distributions are obtained from the 9,999 moving ranges already calculated for each gamma distribution.

5. Evaluate the expected value of each of the forty empirical moving range n = 2 MR distribution. This evaluation can be done using the equation:

$$E(MR) = \overline{MR} = (\sum MR_i) / 9,999$$

6. Evaluate the standard deviation σ_{MR} of each of the forty empirical moving range n = 2 MR distributions obtained from the 9,999 moving ranges already calculated for each of the forty gamma distributions. The evaluation of σ_{MR} can be done using the equation:

$$\sigma_{MR} = [\sum (MR_i - \overline{MR})^2 / 9,998]^{1/2}$$

7. With the average moving range \overline{MR} and standard deviation σ_{MR} for each empirical moving range n = 2 MR distribution and the theoretical process standard deviation $\sigma(\alpha, \beta)$ of each gamma distribution, approximate control chart constants d_2 , d_3 , and D_4 can be calculated using the equations:

$$d_2 = \overline{MR} / \sigma(\alpha, \beta)$$

$$d_3 = \sigma_{MR} / \sigma (\alpha, \beta)$$

$$D_4 = 1 + 3d_3 / d_2$$

where

$$\sigma (\alpha, \beta) = (\beta^2 \alpha)^{1/2}$$

8. Find models in which the expected value of the control chart constants d_2 , d_3 , and D_4 can be written as functions of two independent variables α , β . It means

$$d_2 = f_1 (\alpha, \beta)$$

$$d_3 = f_2 (\alpha, \beta)$$

$$D_4 = f_3 (\alpha, \beta)$$

The method to be used in order to build the models for d_2 , d_3 , and D_4 is the method of fitting multiple regression models using the least-squares approach.

A statistical program package (Minitab for windows release 10), which features regression software, will be used to test different multiple regression models. The model which maximizes the multiple coefficient of determination R^2 will be the one selected to predict the mean value of the control chart constants d_2 , d_3 , and D_4 .

The multiple coefficient of determination R^2 is a sample statistic that tells us how well the model fits the data, and thereby represents a measure of adequacy of the model.

R^2 is defined as:

$$R^2 = 1 - [\Sigma (y_i - \hat{y}_i)^2 / \Sigma (y_i - \bar{y})^2] = 1 - SSE / SS_{yy}$$

The global F test will be used in order to test the validity of the multiple regression model selected. In this F test the null hypothesis

$$H_0 : \lambda_1 = \lambda_2 = \dots = \lambda_k = 0$$

is tested against the alternative hypothesis

H_a : at least one of the λ parameters does not equal zero

The test statistic is defined by

$$F = (R^2 / k) / \{ (1 - R^2) / [n - (k + 1)] \}$$

and the rejection region by

$$F > F_{\alpha'}(k, n - (k + 1))$$

where

n is the number of data points

k is the number of λ parameters in the model excluding the constant term λ_0 .

λ_i 's are the parameters of the multiple regression model, and

α' is the significance level.

3.3 Objective three. The third objective is to compare the performance of the individual measurement X and moving range $n = 2$ MR control charts, using the normal control chart constants, with the performance of these control charts using the Pearson type III $c = 0$ (gamma) control chart constants calculated in objective two, when 10, 30, and 50 data values are available from an unknown process distribution. The performance of the control charts will be evaluated using the proportion of control charts which indicate at least one signal when the process is in control and for shifts in the process average of 1 and 2 process standard deviations.

The motivation of this objective is due to the fact that normally people in industry use the normal control chart constants to set individual measurement and moving range control charts only having 10, 30, or 50 pieces of data from an unknown process

distribution.

The idea is to evaluate the performance of the \bar{X} and MR control charts under the blind assumption of normality and then compare this performance with the performance of these control charts using the Pearson type III (gamma) constants which cover a bigger range of underlying process distribution than the normal constants.

PROCEDURE III

The detailed procedure designed to achieve objective three is the following:

1. Five different process distributions are selected to represent unknown parent distributions. Thus, the performance of the individual measurement \bar{X} and moving range $n = 2$ MR control charts based on normal control chart constants can be compared with those control charts based on gamma control chart constants under different process distributions generally unknown by people in industry.

The five process distributions selected are:

- Lognormal ($0, 1^2$)
- Normal ($40, 10^2$)
- Exponential ($\theta = 1$)
- Gamma ($\alpha = 1.5, \beta = 1$)
- Chi-square ($df = 4$)

2. Generate one thousand (1,000) sets of $k = 10, 30,$ and 50 observations for each of the five distributions selected in step 1 of Procedure III.

2.1 Lognormal ($0, 1^2$) algorithm

Law and Kelton (5, p. 492) recommend the following algorithm to generate

lognormal random variates LN (μ, σ^2).

2.1.1 Generate $Y \sim \text{Normal}(\mu, \sigma^2)$. See algorithm in 2.2 of this Procedure

III.

2.1.2 Return $X = e^Y$

2.2 Normal ($40, 10^2$) algorithm

Given that $Y \sim N(0, 1)$, $X \sim N(\mu, \sigma^2)$ can be obtained by using $X = \mu + \sigma Y$.

Therefore, if one generates standard normal random variates, normal distributions $N(\mu, \sigma^2)$ can be easily generated.

The algorithm to be used in order to generate standard normal random variates is known as the polar method. This method is recommended by several authors including Law and Kelton (5, p. 491).

Algorithm:

2.2.1 Generate U_1 and U_2 as IID $U(0, 1)$, let $V_i = 2U_{i-1}$ for $i = 1, 2, \dots, k$ and let $W = V_1^2 + V_2^2$.

2.2.2 If $W > 1$, go back to step 2.2.1. Otherwise, let $Z = [(-2 \ln W) / W]^{1/2}$, $Y_1 = V_1 Z$, and $Y_2 = V_2 Z$. Y_1 and Y_2 are IID $N \sim (0, 1)$ random variates.

2.2.3 Return $X_1 = \mu + \sigma Y_1$ and $X_2 = \mu + \sigma Y_2$. X_1 and X_2 are IID $N(\mu, \sigma^2)$ random variates.

2.3 Exponential ($\theta = 1$) algorithm

The algorithm to be used to generate exponential random variates with mean $\theta = 1$ is described in 2.2 (Case $\alpha = 1$), Procedure I (p. 42).

2.4 Gamma ($\alpha = 1.5$, $\beta = 1$) algorithm

The algorithm to be used to generate gamma random variates with shape parameter $\alpha = 1.5$ and scale parameter $\beta = 1$ is described in 2.3 (Case $\alpha > 1$), Procedure I (p. 42).

2.5 Chi- square with 4 degrees of freedom algorithm

Knowing that a chi-square distribution is a gamma distribution with shape parameter $\alpha = \text{d.o.f.} / 2$ and scale parameter $\beta = 2$, the algorithm used to generate chi-square (d.o.f. = 4) random variates is the same algorithm as the one to generate gamma $\alpha = 2$ and $\beta = 2$ random variates. This algorithm is described in 2.3 (Case $\alpha > 1$), Procedure I (p. 42).

3. Calculate the average \bar{X} for each run of $k = 10, 30,$ and 50 observations generated from the five distributions selected in step 1 of Procedure III using the equation:

$$\bar{X} = (\sum X_j) / k$$

4. Group the runs generated in subgroups of two consecutive measurements and calculate the moving range for each subgroup using the equation:

$$MR_j = \text{Absolute value} (X_{i+1} - X_i)$$

The manner how to form the subgroups of two consecutive observations is explained in step 4, Procedure I (p. 43).

5. Evaluate the average moving range for each run using the equation:

$$\overline{MR} = (\sum MR_j) / (k - 1)$$

6. In order to get the Pearson type III with $c = 0$ (gamma) control chart constants $d_2,$ $d_3,$ and D_4 to be used in setting control limits for each run of k observations, Pearson type

III parameters α and β have to be estimated from the k observations generated.

Since the process distribution is supposedly unknown, the idea is to fit the data with a Pearson type III with $c = 0$ distribution by estimating the parameters α and β from the k data values (gamma distribution assumption as the underlying process distribution). According to Fisher (7, p. 332), the method of moments is inefficient to estimate parameters of a gamma distribution, except for a distribution closely resembling the normal distribution. Kendall and Stuart (8, p. 38) show that the efficiency of the estimated shape parameter α of a gamma distribution by the method of moments may be as low as 22 percent. Therefore, Fisher (7, p. 332) and Law and Kelton (5, p. 331) recommend the method of maximum likelihood estimation (MLE) in order to estimate the parameters α and β of type III from the data.

The difficulty in applying the method of maximum likelihood estimation to estimate the parameters α and β of a gamma distribution is that closed expressions for the maximum likelihood estimators $\hat{\alpha}$ and $\hat{\beta}$ cannot be obtained analytically. Therefore, numerical methods must be used to estimate the parameters α and β of a gamma distribution.

Choi and Wette (9, p. 683) developed a numerical technique of the maximum likelihood method to estimate the parameters of a gamma distribution. This method is recommended by Law and Kelton (5, p. 331) to estimate α and β . Therefore, this method is the one to be used in this Procedure III to estimate α and β from the data in order to fit a Pearson type III distribution with location parameter $c = 0$ (gamma distribution).

In this numerical technique of the maximum likelihood method, the statistic T is

evaluated using the equation:

$$T = [\ln \bar{X} - \sum \ln X_i / k]^{-1}$$

Then, using Table 6.19 in Law and Kelton (5, p. 411) the estimator $\hat{\alpha}$ is obtained as a function of the statistic T. This table is reproduced in Appendix J of this thesis research.

With the estimator $\hat{\alpha}$ and using the equation:

$$\hat{\beta} = \bar{X} / \hat{\alpha}$$

the scale parameter β is estimated.

Using the estimators $\hat{\alpha}$ and $\hat{\beta}$ and the functions (models) generated in Procedure II by multiple regression analysis

$$d_2 = f_1 (\alpha, \beta)$$

$$d_3 = f_2 (\alpha, \beta)$$

$$D_4 = f_3 (\alpha, \beta) = 1 + 3d_3 / d_2$$

one can obtain the approximate or empirical mean value of the Pearson type III with $c = 0$ (gamma) control chart constants d_2 , d_3 , and D_4 in order to set control limits for the individual measurement X and moving range $n = 2$ MR control charts.

7. Given the gamma control chart constants and using the control chart constants under the assumption of normality, two sets of control limits are obtained for each simulation run of $k = 10, 30,$ and 50 observations. One set of control limits is obtained using the normal constants and the other one is obtained using the Pearson type III (gamma) constants.

The equations to be used to set control limits for X and MR control charts are:

$$UCL_X = \bar{X} + 3 \overline{MR} / d_2$$

$$LCL_X = \bar{X} - 3 \overline{MR} / d_2$$

$$UCL_{MR} = D_4 \overline{MR}$$

8. Generate three sets of 1,000 new samples of $k' = 30$ observations for each of the five process distributions using the same algorithms described in step 2 of this Procedure III. The first set of 1,000 new samples is left as it is (no shift in the process average), the second set of 1,000 new samples is shifted up by one process standard deviation, and the third set of 1,000 new samples is shifted up by two process standard deviations. The shifts of 1 and 2 standard deviations in the process average are considered only up since the quality characteristic cannot take negative values (see delimitations of this research).

9. Determine if the first observation ($k'=1$), the first ten observations ($k'=10$), and the thirty observations ($k'=30$) of the first new sample of $k' = 30$ data values fall outside normal and gamma control limits for the first X control chart. This step is executed repeatedly for the five process distributions and for the three sets of 1,000 new samples ($k'=30$). Each time it is compared the n^{th} new sample of $k' = 30$ with the normal and gamma control limits corresponding to the n^{th} X control chart calculated from the n^{th} old sample of $k = 10, 30, \text{ and } 50$ observations.

10. Group the first set of 1,000 new samples of $k' = 30$ data values (no shift in the process average) in subgroups of two consecutive measurements and calculate the moving range for each subgroup using the equation:

$$MR_j' = \text{Absolute value } (X'_{i+1} - X'_i)$$

At the end, there are 30 moving ranges for each of the 1,000 new samples of $k'=30$. This is different to the logical 29 moving ranges ($N-1$) because an additional

subgroup is formed using the last observation of the 1,000 old samples of $k = 10, 30,$ and 50 observations with the first observation of the new 1,000 samples of $k' = 30$ observations (no shift).

11. Determine if the first moving range (MR_1'), the first ten moving ranges ($MR_{1...10}'$), and the thirty moving ranges ($MR_{1...30}'$) fall above the normal and gamma upper control limits corresponding to the first MR control chart. This step is done repeatedly for the five process distributions and for the first set (no shift) of 1,000 new samples of $k' = 30$. Each time it is compared the n^{th} new sample of $k' = 30$ with the normal and gamma limits corresponding to the n^{th} MR control chart calculated from the n^{th} old sample of $k = 10, 30,$ and 50 observations.

12. Calculate for each of the five distributions and for $k = 10, 30,$ and 50 the proportion of individual measurement X and moving range $n = 2$ MR control charts which show at least one out-of-control signal. This is done comparing the first observation ($k'=1$), the first ten observations ($k'=10$), and the thirty observations ($k'=30$) of the three sets of 1,000 new samples generated with the two sets of control limits calculated using the 1,000 old samples of $k = 10, 30,$ and 50 data values.

This step is done separately for the X and the MR control charts, set with the normal control chart constants $d_2, d_3,$ and D_4 , and for those X and MR control charts set with the gamma control chart constants $d_2, d_3,$ and D_4 . The equation to be used to evaluate the proportion is:

$$P = N / 1000$$

where

- P is the proportion of X and MR control charts which show at least one out-of-control signal for both sets of control limits (normal and gamma limits).
- N is the number of control charts which show at least one out-of-control signal.

The proportion P is evaluated separately for each of the five distributions, for each k (10, 30, and 50), for each of the three sets (no shift, 1 sigma shift, and 2 sigma shift) of k' (1, 10, and 30) and for each set of control limits (normal and gamma).

CHAPTER IV

RESULTS AND ANALYSIS

The results of this thesis research are presented following the three objectives described in chapter I.

4.1 Objective one.

The first objective is to evaluate the performance of the individual measurement X and moving range $n = 2$ MR control charts, using the control chart constants d_2 , d_3 , and D_4 under the assumption of normality, when the underlying process distribution is the Pearson type III family of distributions with location parameter $c = 0$ (gamma distributions) so often encountered in industry.

Following Procedure I, described in detail in chapter III (p. 40), forty gamma distributions are generated using ten values of the shape parameter α (0.5, 1, 1.5, 2, 3, 4, 5, 10, 50, and 100) and four values of the scale parameter β (0.5, 1, 5, and 10).

For each gamma distribution, ten thousand (10,000) gamma random variates are generated in a spreadsheet designed in Quattro Pro for Windows using the random number generator included in Minitab for Windows release 10 and following the algorithm described in Procedure I. Portions of the gamma distributions generated can be observed in Appendix D. The algorithm used to generate gamma variates is tested statistically to see how closely the values generated resemble IID gamma distributions (α, β) . The test is a goodness of fit test for 100 and 1,000 observations for the gamma distribution (1, 1). The results of the test are shown in Appendix C and they indicate that

the algorithm is good enough to generate reliable gamma variates. Then, for each gamma distribution the following values are calculated:

- Average \bar{X} of the 10,000 gamma variates.
- Average moving range \overline{MR} of the 9,999 subgroups of two consecutive measurements.
- Control limits for the individual measurement X control charts (UCL_X and LCL_X) using the normal bias correction factor $d_2 = 1.128$.
- Upper control limits for the moving range $n = 2$ MR control charts (UCL_{MR}) using the normal control chart constant $D_4 = 3.268$.

These values are summarized in Table 4.1.

TABLE 4.1

SUMMARY TABLE - VALUES CALCULATED USING PROCEDURE I.

Alpha α	Beta β	Avg. \bar{X}	Avg. \overline{MR}	UCL_X	LCL_X	UCL_{MR}
0.5	0.5	0.2476	0.3149	1.0851	-0.5899	1.0291
	1.0	0.5045	0.6374	2.1997	-1.1908	2.0831
	5.0	2.4525	3.0912	10.6737	-5.7688	10.1020
	10.0	4.9689	6.3167	21.7687	-11.8308	20.6430
1.0	0.5	0.4966	0.5006	1.8280	-0.8348	1.6360
	1.0	1.0207	0.9976	3.6738	-1.6323	3.2600
	5.0	4.9572	4.9450	18.1089	-8.1945	16.1604
	10.0	9.9641	10.1105	36.8536	-16.9254	33.0410
1.5	0.5	0.7451	0.6300	2.4205	-0.9303	2.0587
	1.0	1.4920	1.2539	4.8267	-1.8427	4.0976
	5.0	7.5528	6.3645	24.4797	-9.3741	20.7992

Alpha α	Beta β	Avg. \bar{X}	Avg. \overline{MR}	UCL_x	LCL_x	UCL_{MR}
1.5	10.0	15.0286	12.7060	48.8212	-18.7641	41.5234
2.0	0.5	0.9971	0.7352	2.9524	-0.9582	2.4026
	1.0	1.9625	1.4976	5.9454	-2.0204	4.8940
	5.0	9.9023	7.3454	29.4380	-9.6335	24.0049
	10.0	20.0860	15.1945	60.4969	-20.3249	49.6556
3.0	0.5	1.5018	0.9459	4.0174	-1.0139	3.0911
	1.0	3.0280	1.8759	8.0171	-1.9611	6.1304
	5.0	15.1172	9.4211	40.1733	-9.9389	30.7881
	10.0	30.1224	18.7847	80.0816	-19.8368	61.3882
4.0	0.5	1.9969	1.0826	4.8761	-0.8823	3.5379
	1.0	4.0032	2.1963	9.8443	-1.8379	7.1774
	5.0	19.9459	11.0745	49.3993	-9.5074	36.1913
	10.0	40.2152	22.0412	98.8354	-18.4051	72.0307
5.0	0.5	2.5080	1.2346	5.7916	-0.7756	4.0347
	1.0	5.0065	2.4833	11.6111	-1.5982	8.1156
	5.0	24.9952	12.3467	57.8322	-7.8419	40.3492
	10.0	49.8423	24.3149	114.5095	-14.8250	79.4611
10.0	0.5	4.9852	1.7319	9.5913	0.3791	5.6598
	1.0	9.9423	3.5097	19.2765	0.6081	11.4696
	5.0	50.1309	17.4940	96.06574	3.6044	57.1703
	10.0	100.6372	34.9837	193.6788	7.5955	114.3266
50.0	0.5	24.9684	3.9957	35.5952	14.3416	13.0579
	1.0	49.9587	7.9154	71.0104	28.9071	25.8676
	5.0	250.2511	39.4540	355.1820	145.3202	128.9358
	10.0	500.8203	80.2515	714.2551	287.3855	262.2619

Alpha α	Beta β	Avg. \bar{X}	Avg. \bar{MR}	UCL_x	LCL_x	UCL_{MR}
100.0	0.5	49.9843	5.6459	65.0001	34.9686	18.4509
	1.0	100.1233	11.2389	130.0140	70.2325	36.7288
	5.0	499.9215	55.5181	647.5760	352.2670	181.4331
	10.0	1001.3230	112.9637	1301.7580	700.8875	369.1654

More details about the forty gamma distributions and their values can be observed in the spreadsheets shown in Appendix D.

Then, using the values of the individual measurement X control limits (UCL_x and LCL_x) and the gamma cumulative distribution function routine contained in Minitab for Windows release 10, the average run lengths (ARLs) for several shifts in the process average ($k\sigma$, where $k = 0 (0.2) 3$) are computed using the equation:

$$ARL = 1 / P$$

where

P is the probability of detection evaluated by the equation:

$$P = \text{Probability}(x > UCL_x) + \text{Probability}(x < LCL_x)$$

The probabilities of detection (P) calculated in Minitab and the average run lengths (ARLs) are summarized in Tables 4.2 and 4.3 respectively for the forty gamma distributions generated in Procedure I.

TABLE 4.2 a

PROBABILITIES OF DETECTION (P) FOR DIFFERENT SHIFTS IN THE PROCESS

AVERAGE $k\sigma$ WHERE $k = 0 (0.2) 1.4$

α	β	Shift in the process average $k\sigma$							
		0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
0.5	0.5	0.0372	0.0440	0.0520	0.0617	0.0732	0.0872	0.1040	0.1244
	1.0	0.0360	0.0425	0.0502	0.0595	0.0706	0.0840	0.1002	0.1198
	5.0	0.0388	0.0459	0.0543	0.0644	0.0765	0.0911	0.1087	0.1302
	10	0.0369	0.0436	0.0516	0.0612	0.0726	0.0864	0.1031	0.1234
1.0	0.5	0.0258	0.0316	0.0385	0.0471	0.0575	0.0702	0.0858	0.1048
	1.0	0.0254	0.0310	0.0379	0.0462	0.0565	0.0690	0.0843	0.1029
	5.0	0.0267	0.0327	0.0399	0.0487	0.0595	0.0727	0.0888	0.1084
	10	0.0251	0.0306	0.0374	0.0457	0.0558	0.0682	0.0833	0.1017
1.5	0.5	0.0215	0.0268	0.0335	0.0418	0.0521	0.0648	0.0806	0.0999
	1.0	0.0218	0.0272	0.0340	0.0424	0.0528	0.0657	0.0816	0.1012
	5.0	0.0204	0.0255	0.0319	0.0398	0.0496	0.0617	0.0768	0.0952
	10	0.0207	0.0259	0.0323	0.0403	0.0502	0.0625	0.0777	0.0964
2.0	0.5	0.0188	0.0240	0.0304	0.0386	0.0488	0.0616	0.0775	0.0972
	1.0	0.0182	0.0231	0.0294	0.0373	0.0472	0.0596	0.0750	0.0941
	5.0	0.0191	0.0243	0.0309	0.0391	0.0495	0.0624	0.0786	0.0986
	10	0.0166	0.0212	0.0269	0.0342	0.0433	0.0547	0.0689	0.0866
3.0	0.5	0.0134	0.0175	0.0229	0.0297	0.0385	0.0497	0.0639	0.0818
	1.0	0.0136	0.0178	0.0232	0.0301	0.0390	0.0504	0.0648	0.0828
	5.0	0.0134	0.0175	0.0229	0.0297	0.0385	0.0498	0.0640	0.0818
	10	0.0137	0.0179	0.0233	0.0303	0.0393	0.0507	0.0652	0.0834
4.0	0.5	0.0124	0.0165	0.0220	0.0290	0.0382	0.0500	0.0652	0.0843

α	β	Shift in the process average $k\sigma$							
		0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
4.0	1.0	0.0116	0.0155	0.0206	0.0272	0.0359	0.0471	0.0613	0.0795
	5.0	0.0113	0.0151	0.0201	0.0266	0.0350	0.0460	0.0599	0.0777
	10	0.0113	0.0150	0.0200	0.0265	0.0349	0.0458	0.0598	0.0775
5.0	0.5	0.0101	0.0138	0.0186	0.0250	0.0334	0.0443	0.0584	0.0765
	1.0	0.0099	0.0135	0.0183	0.0245	0.0328	0.0436	0.0574	0.0752
	5.0	0.0103	0.0139	0.0188	0.0253	0.0338	0.0448	0.0590	0.0772
	10	0.0111	0.0151	0.0203	0.0273	0.0363	0.0481	0.0633	0.0827
10	0.5	0.0079	0.0114	0.0161	0.0225	0.0312	0.0429	0.0582	0.0782
	1.0	0.0076	0.0108	0.0153	0.0214	0.0298	0.0409	0.0557	0.0749
	5.0	0.0073	0.0105	0.0148	0.0208	0.0289	0.0398	0.0542	0.0730
	10	0.0072	0.0103	0.0146	0.0204	0.0284	0.0391	0.0533	0.0718
50	0.5	0.0037	0.0056	0.0087	0.0134	0.0202	0.0299	0.0436	0.0622
	1.0	0.0039	0.0059	0.0092	0.0141	0.0212	0.0315	0.0457	0.0650
	5.0	0.0039	0.0059	0.0091	0.0140	0.0211	0.0313	0.0454	0.0646
	10	0.0034	0.0052	0.0081	0.0125	0.0189	0.0281	0.0411	0.0587
100	0.5	0.0032	0.0047	0.0075	0.0118	0.0183	0.0279	0.0414	0.0600
	1.0	0.0032	0.0047	0.0075	0.0118	0.0183	0.0278	0.0413	0.0599
	5.0	0.0037	0.0054	0.0084	0.0132	0.0203	0.0307	0.0454	0.0654
	10	0.0031	0.0045	0.0072	0.0114	0.0177	0.0269	0.0400	0.0581

TABLE 4.2 b

PROBABILITIES OF DETECTION (P) FOR DIFFERENT SHIFTS IN THE PROCESS

AVERAGE $k\sigma$ WHERE $k = 1.6 (0.2) 3.0$

α	β	Shift in the process average $k\sigma$							
		1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
0.5	0.5	0.1495	0.1803	0.2188	0.2676	0.3306	0.4153	0.5372	0.7544
	1.0	0.1438	0.1733	0.2101	0.2564	0.3160	0.3953	0.5073	0.6921
	5.0	0.1566	0.1892	0.2299	0.2818	0.3495	0.4414	0.5779	0.8698
	10	0.1482	0.1787	0.2168	0.2650	0.3273	0.4107	0.5302	0.7389
1.0	0.5	0.1280	0.1563	0.1909	0.2332	0.2848	0.3479	0.4249	0.5189
	1.0	0.1257	0.1535	0.1875	0.2290	0.2798	0.3417	0.4173	0.5097
	5.0	0.1324	0.1617	0.1975	0.2413	0.2947	0.3599	0.4397	0.5370
	10	0.1243	0.1518	0.1854	0.2264	0.2765	0.3378	0.4126	0.5039
1.5	0.5	0.1237	0.1529	0.1884	0.2315	0.2835	0.3458	0.4197	0.5061
	1.0	0.1253	0.1548	0.1907	0.2343	0.2868	0.3498	0.4243	0.5116
	5.0	0.1180	0.1458	0.1798	0.2211	0.2710	0.3309	0.4020	0.4856
	10	0.1194	0.1476	0.1819	0.2237	0.2741	0.3346	0.4064	0.4907
2.0	0.5	0.1216	0.1515	0.1880	0.2322	0.2851	0.3478	0.4211	0.5051
	1.0	0.1178	0.1469	0.1823	0.2253	0.2769	0.3382	0.4099	0.4924
	5.0	0.1233	0.1536	0.1905	0.2351	0.2886	0.3520	0.4259	0.5105
	10	0.1085	0.1355	0.1684	0.2085	0.2568	0.3144	0.3822	0.4607
3.0	0.5	0.1041	0.1318	0.1658	0.2070	0.2565	0.3151	0.3830	0.4603
	1.0	0.1054	0.1334	0.1677	0.2094	0.2593	0.3183	0.3868	0.4645
	5.0	0.1042	0.1318	0.1658	0.2071	0.2566	0.3151	0.3831	0.4603
	10	0.1061	0.1342	0.1686	0.2105	0.2607	0.3199	0.3887	0.4666
4.0	0.5	0.1083	0.1381	0.1747	0.2190	0.2718	0.3336	0.4043	0.4833

α	β	Shift in the process average $k\sigma$							
		1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
4.0	1.0	0.1023	0.1307	0.1657	0.2081	0.2589	0.3185	0.3873	0.4645
	5.0	0.1001	0.1279	0.1623	0.2040	0.2540	0.3129	0.3808	0.4573
	10	0.0998	0.1276	0.1619	0.2036	0.2535	0.3123	0.3801	0.4566
5.0	0.5	0.0993	0.1279	0.1631	0.2059	0.2571	0.3171	0.3859	0.4627
	1.0	0.0977	0.1259	0.1607	0.2030	0.2537	0.3131	0.3813	0.4577
	5.0	0.1003	0.1291	0.1646	0.2077	0.2592	0.3195	0.3886	0.4657
	10	0.1071	0.1376	0.1749	0.2202	0.2739	0.3365	0.4078	0.4867
10	0.5	0.1037	0.1358	0.1754	0.2233	0.2798	0.3448	0.4176	0.4965
	1.0	0.0995	0.1306	0.1690	0.2156	0.2708	0.3346	0.4064	0.4845
	5.0	0.0971	0.1276	0.1654	0.2112	0.2657	0.3287	0.3998	0.4775
	10	0.0956	0.1257	0.1630	0.2083	0.2623	0.3249	0.3956	0.4729
50	0.5	0.0868	0.1187	0.1586	0.2074	0.2651	0.3312	0.4044	0.4825
	1.0	0.0905	0.1233	0.1644	0.2143	0.2731	0.3402	0.4141	0.4926
	5.0	0.0899	0.1226	0.1635	0.2133	0.2719	0.3389	0.4127	0.4912
	10	0.0823	0.1128	0.1514	0.1987	0.2549	0.3197	0.3918	0.4693
100	0.5	0.0850	0.1174	0.1583	0.2082	0.2671	0.3343	0.4083	0.4867
	1.0	0.0848	0.1171	0.1580	0.2078	0.2667	0.3338	0.4078	0.4862
	5.0	0.0921	0.1265	0.1695	0.2217	0.2827	0.3517	0.4270	0.5061
	10	0.0825	0.1142	0.1543	0.2034	0.2616	0.3281	0.4016	0.4797

TABLE 4.3 a

AVERAGE RUN LENGTHS (ARLs) FOR DIFFERENT SHIFTS IN THE PROCESS

AVERAGE $k\sigma$ WHERE $k = 0 (0.2) 1.4$

α	β	Shift in the process average $k\sigma$							
		0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
0.5	0.5	26.9	22.7	19.2	16.2	13.7	11.5	9.6	8.0
	1.0	27.8	23.5	19.9	16.8	14.2	11.9	9.9	8.3
	5.0	25.8	21.8	18.4	15.5	13.1	10.9	9.2	7.7
	10	27.1	22.9	19.4	16.3	13.8	11.6	9.7	8.1
1.0	0.5	38.7	31.7	25.9	21.2	17.4	14.2	11.7	9.5
	1.0	39.4	32.3	26.4	21.6	17.7	14.5	11.9	9.7
	5.0	37.4	30.6	25.1	20.5	16.8	13.8	11.3	9.2
	10	39.9	32.6	26.7	21.9	17.9	14.7	12.0	9.8
1.5	0.5	46.6	37.3	29.8	23.9	19.2	15.4	12.4	10.0
	1.0	45.9	36.8	29.4	23.6	18.9	15.2	12.3	9.9
	5.0	48.9	39.2	31.4	25.1	20.2	16.2	13.0	10.5
	10	48.4	38.7	30.9	24.8	19.9	15.9	12.9	10.4
2.0	0.5	53.1	41.7	32.9	25.9	20.5	16.2	12.9	10.3
	1.0	54.9	43.2	34.0	26.8	21.2	16.8	13.3	10.6
	5.0	52.3	41.1	32.4	25.6	20.2	16.0	12.7	10.1
	10	60.1	47.2	37.1	29.3	23.1	18.3	14.5	11.5
3.0	0.5	74.7	57.1	43.7	33.6	25.9	20.1	15.6	12.2
	1.0	73.7	56.3	43.2	33.2	25.6	19.8	15.4	12.1
	5.0	74.7	57.1	43.7	33.6	25.9	20.1	15.6	12.2
	10	73.2	55.9	42.9	32.9	25.4	19.7	15.3	11.9
4.0	0.5	80.8	60.5	45.5	34.4	26.2	19.9	15.3	11.9

α	β	Shift in the process average $k\sigma$							
		0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
4.0	1.0	86.3	64.7	48.6	36.7	27.9	21.2	16.3	12.6
	5.0	88.6	66.3	49.8	37.6	28.5	21.8	16.7	12.9
	10	88.9	66.5	49.9	37.7	28.6	21.8	16.7	12.9
5.0	0.5	98.5	72.6	53.7	39.9	29.9	22.6	17.1	13.1
	1.0	100.4	73.9	54.7	40.7	30.5	22.9	17.4	13.3
	5.0	97.3	71.8	53.1	39.6	29.6	22.3	16.9	12.9
	10	90.0	66.4	49.2	36.7	27.5	20.8	15.8	12.1
10	0.5	125.2	87.9	62.2	44.4	32.0	23.3	17.2	12.8
	1.0	132.0	92.6	65.4	46.7	33.6	24.4	17.9	13.4
	5.0	136.2	95.4	67.4	48.0	34.6	25.1	18.4	13.7
	10	139.0	97.4	68.7	49.0	35.2	25.6	18.8	13.9
50	0.5	273.4	178.9	115.1	74.8	49.5	33.3	22.9	16.1
	1.0	255.7	168.5	108.8	70.9	47.0	31.8	21.9	15.4
	5.0	255.8	169.6	109.6	71.4	47.3	31.9	22.0	15.5
	10	293.9	192.8	123.8	80.3	52.9	35.6	24.4	17.0
100	0.5	315.1	211.4	133.7	84.6	54.5	35.9	24.2	16.7
	1.0	310.2	210.9	133.9	84.9	54.7	35.9	24.2	16.7
	5.0	274.1	186.7	119.1	75.9	49.1	32.5	22.0	15.3
	10	324.7	219.9	139.3	88.1	56.6	37.2	25.0	17.2

TABLE 4.3 b

AVERAGE RUN LENGTHS (ARL_s) FOR DIFFERENT SHIFTS IN THE PROCESS

AVERAGE $k\sigma$ WHERE $k = 1.6 (0.2) 3.0$

α	β	Shift in the process average $k\sigma$							
		1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
0.5	0.5	6.7	5.5	4.6	3.7	3.0	2.4	1.9	1.3
	1.0	6.9	5.8	4.8	3.9	3.2	2.5	1.9	1.4
	5.0	6.4	5.3	4.3	3.5	2.9	2.3	1.7	1.2
	10	6.7	5.6	4.6	3.8	3.1	2.4	1.9	1.4
1.0	0.5	7.8	6.4	5.2	4.3	3.5	2.9	2.4	1.9
	1.0	7.9	6.5	5.3	4.4	3.6	2.9	2.4	1.9
	5.0	7.6	6.2	5.1	4.1	3.4	2.8	2.3	1.9
	10	8.0	6.6	5.4	4.4	3.6	2.9	2.4	1.9
1.5	0.5	8.1	6.5	5.3	4.3	3.5	2.9	2.4	1.9
	1.0	8.0	6.5	5.2	4.3	3.5	2.9	2.4	1.9
	5.0	8.5	6.9	5.6	4.5	3.7	3.0	2.5	2.0
	10	8.4	6.8	5.5	4.5	3.6	3.0	2.5	2.0
2.0	0.5	8.2	6.6	5.3	4.3	3.5	2.9	2.4	1.9
	1.0	8.5	6.8	5.5	4.4	3.6	3.0	2.4	2.0
	5.0	8.1	6.5	5.2	4.3	3.5	2.8	2.3	1.9
	10	9.2	7.4	5.9	4.8	3.9	3.2	2.6	2.1
3.0	0.5	9.6	7.6	6.0	4.8	3.9	3.2	2.6	2.1
	1.0	9.5	7.5	6.0	4.8	3.9	3.1	2.6	2.1
	5.0	9.6	7.6	6.0	4.8	3.9	3.2	2.6	2.1
	10	9.4	7.5	5.9	4.7	3.8	3.1	2.6	2.1
4.0	0.5	9.2	7.2	5.7	4.6	3.7	3.0	2.5	2.0

α	β	Shift in the process average $k\sigma$							
		1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
4.0	1.0	9.8	7.7	6.0	4.8	3.9	3.1	2.6	2.1
	5.0	9.9	7.8	6.2	4.9	3.9	3.2	2.6	2.1
	10	10.0	7.8	6.2	4.9	3.9	3.2	2.6	2.2
5.0	0.5	10.1	7.8	6.1	4.9	3.9	3.2	2.6	2.1
	1.0	10.2	7.9	6.2	4.9	3.9	3.2	2.6	2.1
	5.0	10.0	7.7	6.1	4.8	3.9	3.1	2.6	2.1
	10	9.3	7.3	5.7	4.5	3.7	3.0	2.5	2.0
10	0.5	9.6	7.4	5.7	4.5	3.6	2.9	2.4	2.0
	1.0	10.0	7.7	5.9	4.6	3.7	3.0	2.5	2.0
	5.0	10.3	7.8	6.0	4.7	3.8	3.0	2.5	2.1
	10	10.5	7.9	6.1	4.8	3.8	3.1	2.5	2.1
50	0.5	11.5	8.4	6.3	4.8	3.8	3.0	2.5	2.0
	1.0	11.1	8.1	6.1	4.7	3.7	2.9	2.4	2.0
	5.0	11.1	8.2	6.1	4.7	3.7	2.9	2.4	2.0
	10	12.2	8.9	6.6	5.0	3.9	3.1	2.6	2.1
100	0.5	11.8	8.5	6.3	4.8	3.7	3.0	2.4	2.0
	1.0	11.8	8.5	6.3	4.8	3.7	3.0	2.5	2.0
	5.0	10.9	7.9	5.9	4.5	3.5	2.8	2.3	1.9
	10	12.1	8.8	6.5	4.9	3.8	3.0	2.5	2.0

More details about the calculations of the probabilities of detection (P) and average run lengths (ARLs) are shown in Appendix E.

From Table 4.3 (a and b), empirical average run length (ARL) curves are drawn for each of the forty X control charts corresponding to each of the forty gamma

distributions generated. These ARL curves show the overall performance of the individual measurement \bar{X} control charts, using the normal bias correction factor $d_2 = 1.128$, when the underlying process distribution is Pearson type III with location parameter $c = 0$ (gamma distribution).

The empirical average run length (ARL) curves for the different combinations between shape parameters α and scale parameters β can be observed from Figure 4.1 to Figure 4.10.

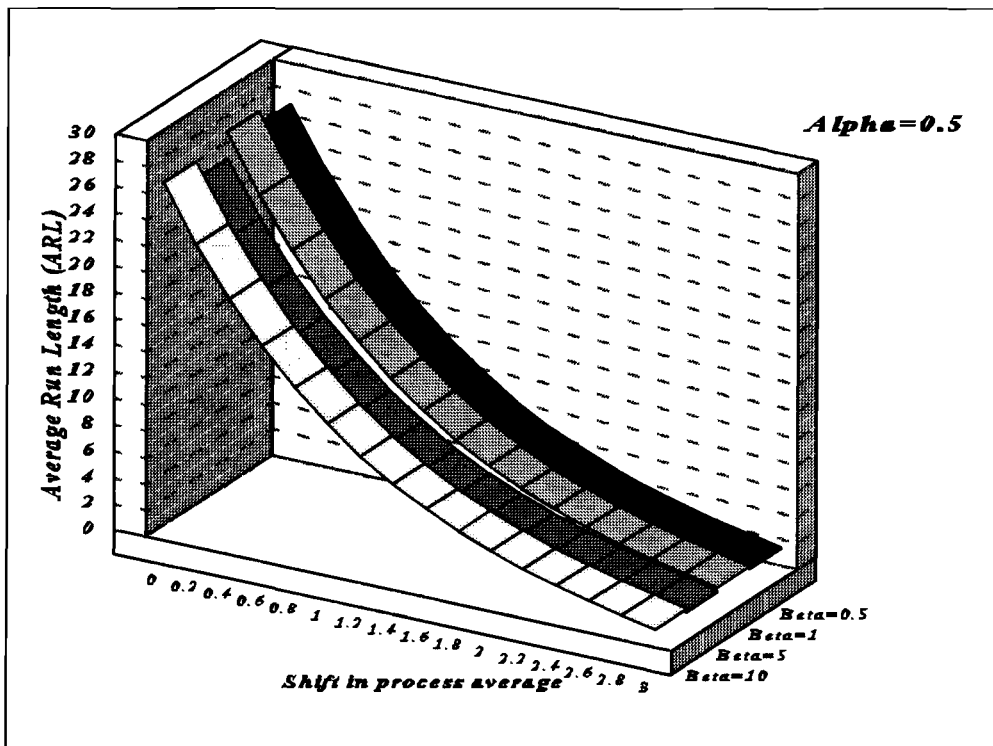
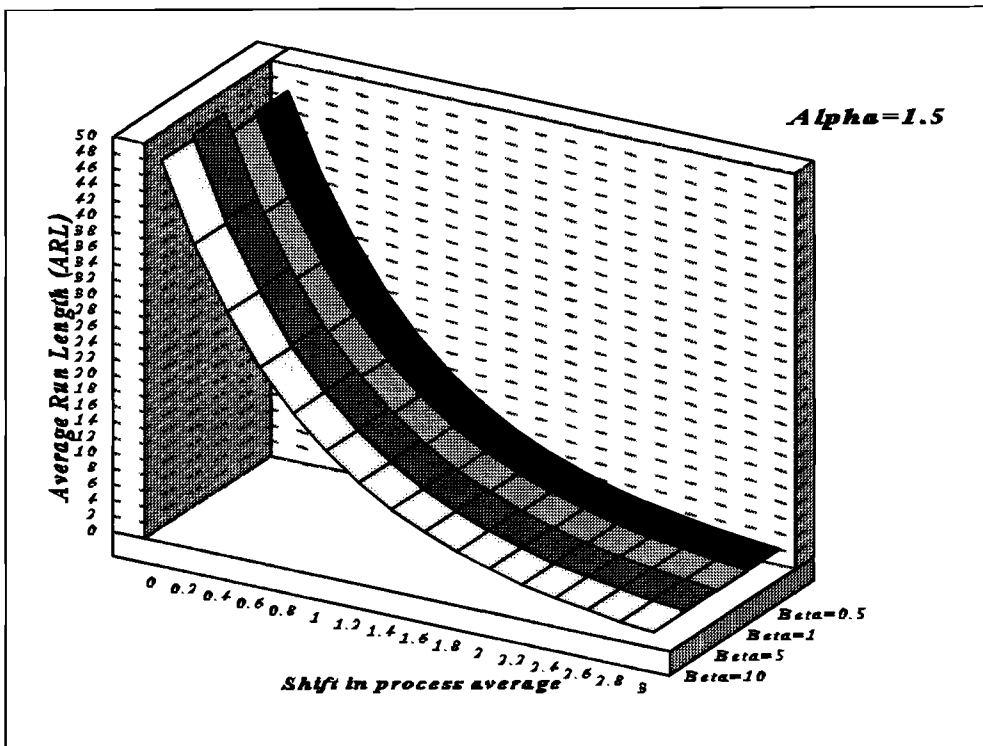
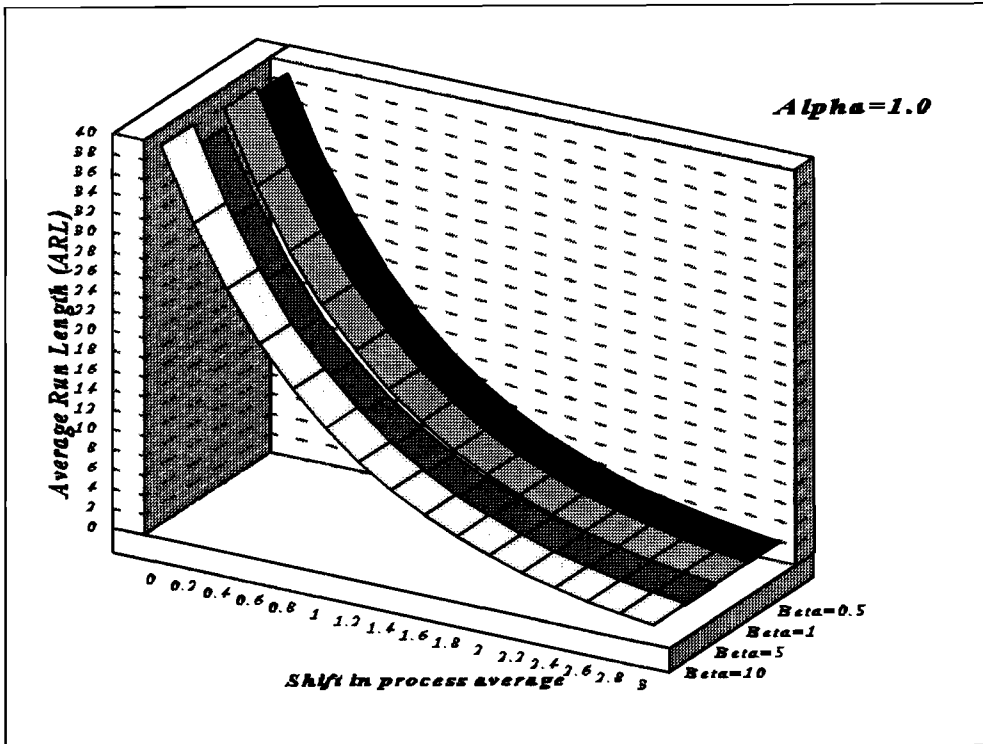
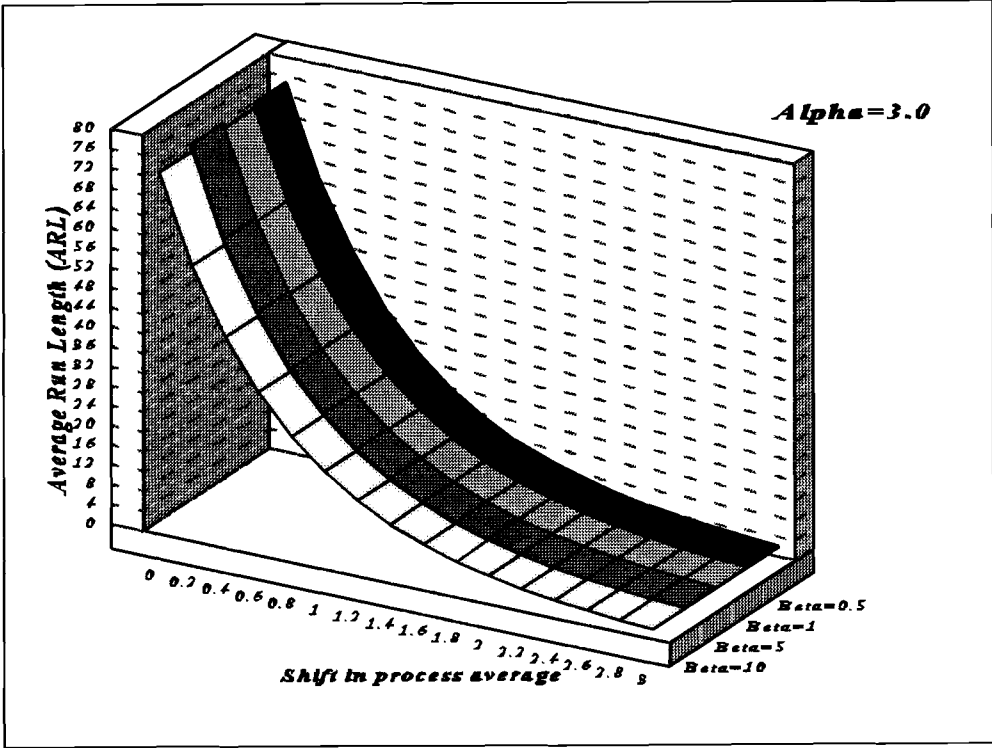
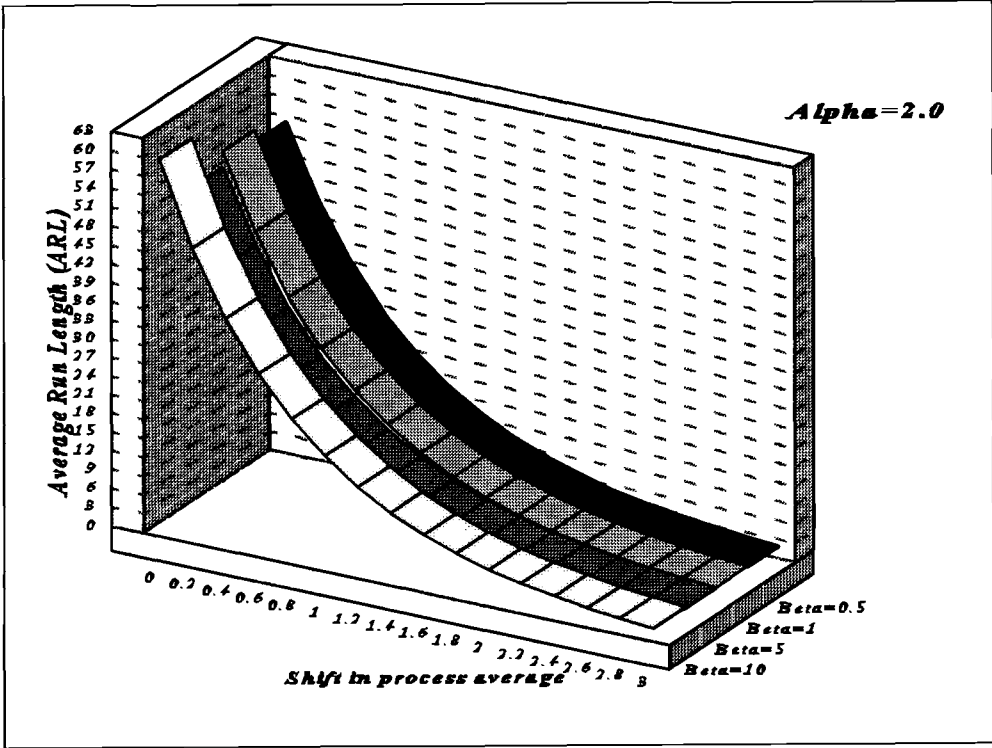


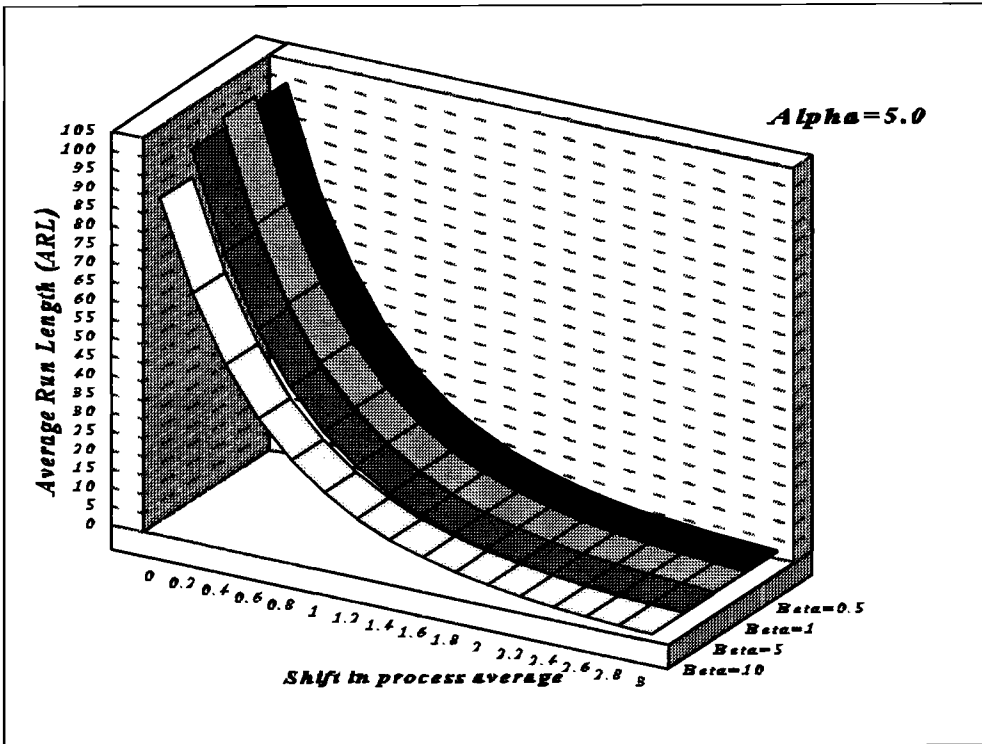
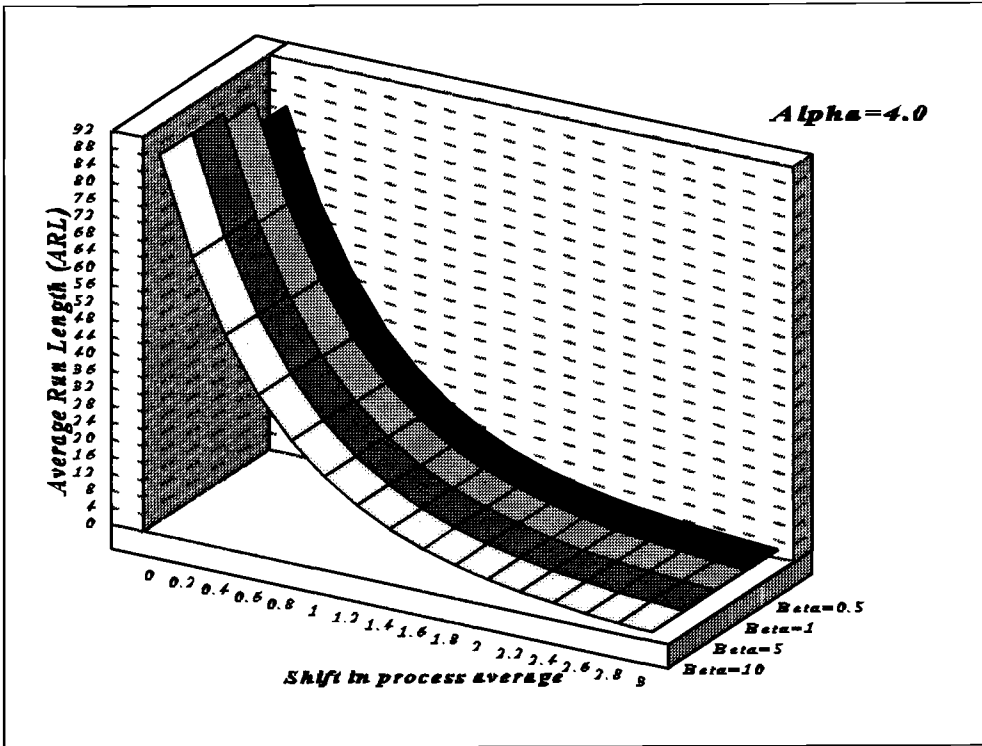
Figure 4.1 Empirical ARL curve for shape parameter $\alpha = 0.5$.



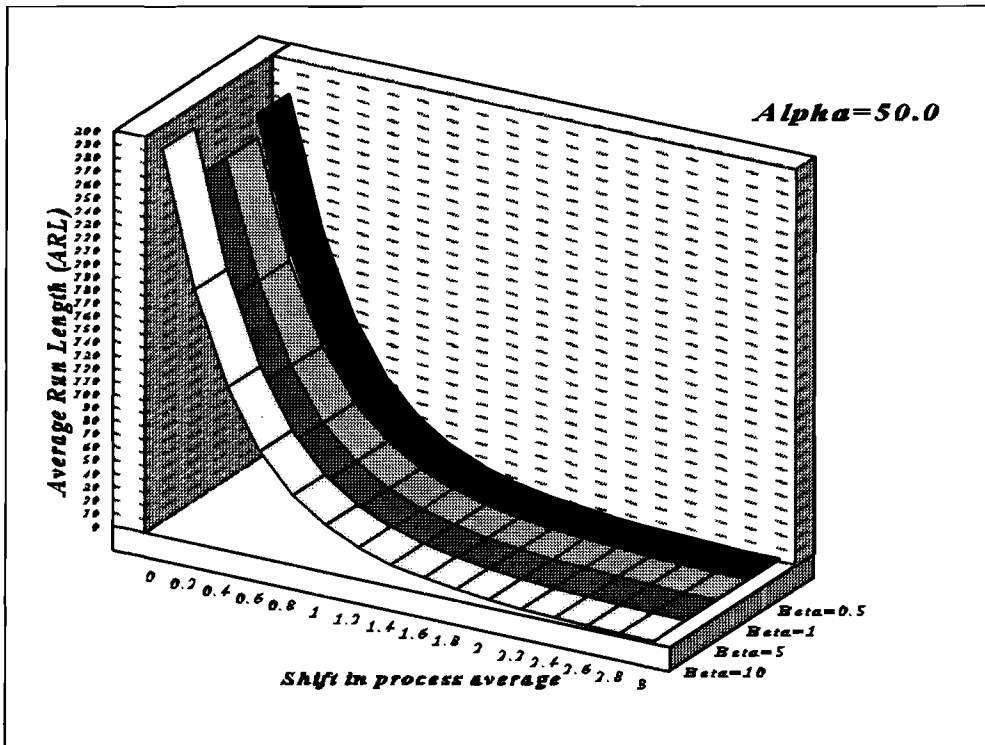
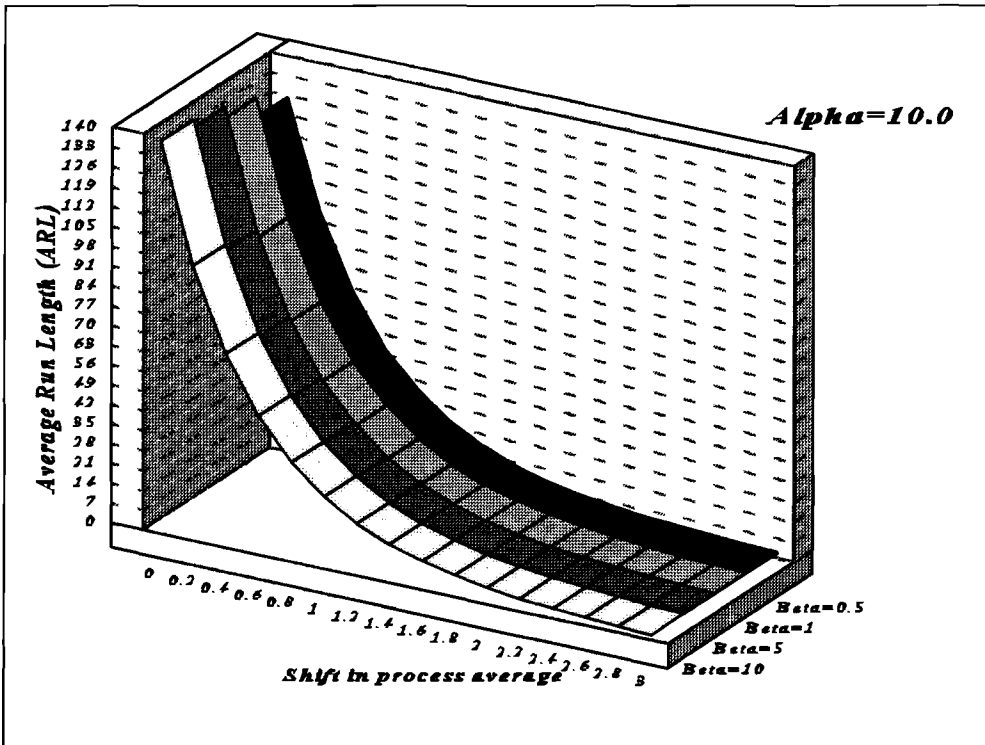
Figures 4.2 and 4.3. Empirical ARL curves for shape parameters $\alpha = 1.0$ and 1.5 .



Figures 4.4 and 4.5. Empirical ARL curves for shape parameters $\alpha = 2.0$ and 3.0 .



Figures 4.6 and 4.7. Empirical ARL curves for shape parameters $\alpha = 4.0$ and 5.0



Figures 4.8 and 4.9. Empirical ARL curves for shape parameters $\alpha = 10$ and 50.

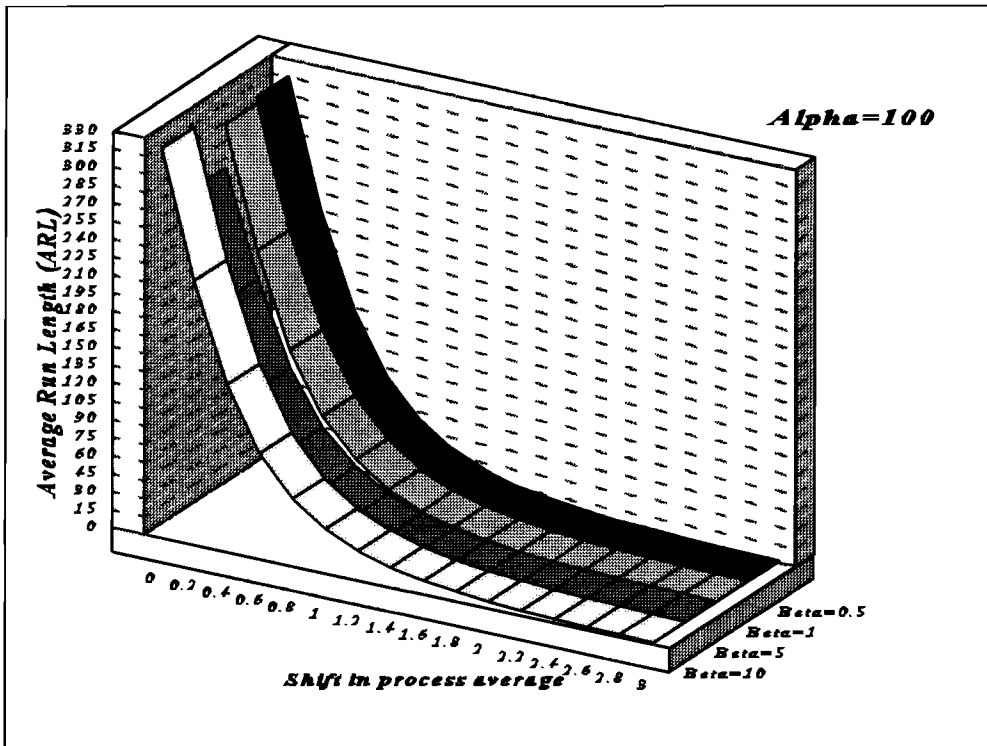


Figure 4.10. Empirical ARL curve for shape parameter $\alpha = 100$

From the empirical average run length (ARL) curves, it is observed that for shape parameters $\alpha < 5$ the ARLs in control (no shift) are substantially less (from 15% to 74%) than the 100 stated in the first hypothesis as the minimum accepted in industry for practical purposes. However, the average run lengths (ARLs) for shifts in the process average of $k\sigma = 1, 2$ and 3 process standard deviations are smaller, for most of the shape parameter α values, than the 43.9, 6.3, and 2 stated in the first hypothesis for shifts in the process average of $k = 1, 2,$ and 3 sigma, respectively. Only for a few combinations of shape parameter α and scale parameter β (for example $\alpha = 50$ and $\beta = 10$), the ARLs for shifts in the process average of $k = 2$ and 3 process standard deviations are a little bit higher (approximately 4.5%) than the theoretical ARL for normal distributions. It is the

author's belief that these tiny differences (approximately 4.5%) in some cases are due to the presence of random error in the simulation process.

Additionally, it can be seen from the empirical ARL curves that the ARL values do not vary with the scale parameter β . This is due to the fact that the scale parameter β does not affect the shape of the gamma distribution.

For the moving range $n = 2$ MR control charts, using the values of the upper control limits (UCL_{MR}) and the moving range values (MR_i), the empirical probabilities of type I error and empirical ARLs in control are calculated for each of the forty gamma distributions generated. The equations used are:

$$\text{Empirical probability (type I error)} = P' = B / 9,999$$

$$\text{Empirical ARL} = ARL' = 1 / P'$$

where

B is the number of moving ranges (MR_i) above the UCL_{MR} for each of the forty gamma distributions.

The empirical probabilities of type I error (P') and the empirical ARL' in control are shown in Table 4.4.

TABLE 4.4

EMPIRICAL PROBABILITIES OF TYPE I ERROR (P') AND EMPIRICAL ARL' IN CONTROL FOR THE MR ($n=2$) CONTROL CHARTS

α	β	Prob. P'	ARL'	α	β	Prob. P'	ARL'
0.5	0.5	0.0536	18.7	4.0	0.5	0.0176	56.8
	1.0	0.0597	16.8		1.0	0.0177	56.5

α	β	Prob. P'	ARL'	α	β	Prob. P'	ARL'
0.5	5.0	0.0558	17.9	4.0	5.0	0.0151	66.2
	10	0.0571	17.5		10	0.0187	53.5
1.0	0.5	0.0396	25.3	5.0	0.5	0.0162	61.7
	1.0	0.0386	25.9		1.0	0.0163	61.3
	5.0	0.0415	24.1		5.0	0.0183	54.6
	10	0.0367	27.3		10	0.0155	64.5
1.5	0.5	0.0304	32.9	10	0.5	0.0129	77.5
	1.0	0.0301	33.2		1.0	0.0129	77.5
	5.0	0.0288	34.7		5.0	0.0143	69.9
	10	0.0301	33.2		10	0.0156	64.1
2.0	0.5	0.0220	45.5	50	0.5	0.0100	100.0
	1.0	0.0271	36.9		1.0	0.0112	89.3
	5.0	0.0252	39.7		5.0	0.0104	96.2
	10	0.0248	40.3		10	0.0089	112.3
3.0	0.5	0.0218	45.9	100	0.5	0.0107	93.5
	1.0	0.0220	45.5		1.0	0.0087	114.9
	5.0	0.0194	51.5		5.0	0.0099	101.0
	10	0.0204	49.0		10	0.0096	104.2

More details about the calculations of the empirical probabilities of type I error P' and the empirical ARL' in control for MR (n=2) control charts can be seen in the spreadsheets shown in Appendix D.

From table 4.4, it is observed that for shape parameters $\alpha \leq 10$ the empirical ARL' in control (no shift) are substantially less (from 30% to 83%) than the 100 value stated in the first hypothesis as a minimum accepted value in industry for practical matters. It is

not only until the shape parameter α reaches a value of 50 when the empirical ARL' in control begin to approach the practical 100 value. In normal theory the ARL in control has a value of 107.14.

For all of the reasons mentioned above, the first hypothesis is rejected. It means that for practical purposes in industry the individual measurement X control chart, using the normal bias correction factor d_2 , does not work well when the underlying process distribution is skewed (asymmetric) represented by gamma distributions with shape parameter $\alpha < 5$. In relation with the moving range $n = 2$ MR control chart, it means that for practical matters this control chart does not perform well either when the underlying process distribution is asymmetric represented by gamma distributions with shape parameters $\alpha \leq 10$.

As a consequence, individual measurement X and moving range $n = 2$ MR control charts must be very carefully interpreted if the underlying process distribution is skewed (asymmetric). According to Duncan (1, p. 400), "In such cases, the multiple of σ used to set control limits might be better derived from other distributions for which the percentage points have been computed." This is exactly what this thesis research is trying to accomplish with objective three using the Pearson type III family of distributions with $c = 0$ (gamma distributions) as the assumption to set control limits for X and MR ($n=2$) control charts.

4.2 Objective two.

The second objective is to determine empirical functions $f(\alpha, \beta)$ for the control chart constants d_2 , d_3 , and D_4 when the process distribution can be approximated by a

Pearson type III distribution with location parameter $c = 0$, shape parameter α , and scale parameter β .

Using the same forty gamma distributions generated in Procedure I and based on Procedure II described in page 45, the following values are computed for each distribution:

- Average \bar{X} of the 10,000 gamma variates.
- Average moving range \overline{MR} of the 9,999 subgroups of two consecutive measurements.
- Moving range standard deviation σ_{MR} obtained from the 9,999 MR_i .
- Approximate values for the bias correction factors d_2 and d_3 and the control chart constant D_4 .

These values are summarized in Table 4.5.

TABLE 4.5

SUMMARY TABLE - VALUES CALCULATED USING PROCEDURE II

α	β	\bar{X}	\overline{MR}	$\sigma(\alpha, \beta)$	σ_{MR}	d_2	d_3	D_4
0.5	0.5	0.2476	0.3149	0.3536	0.3726	0.8907	1.0538	4.5493
	1.0	0.5045	0.6374	0.7071	0.7858	0.9014	1.1112	4.6982
	5.0	2.4525	3.0912	3.5355	3.7717	0.8743	1.0668	4.6604
	10	4.9689	6.3167	7.0711	7.6504	0.8933	1.0819	4.6334
1.0	0.5	0.4966	0.5006	0.5000	0.4979	1.0012	0.9958	3.9838
	1.0	1.0207	0.9976	1.0000	1.0008	0.9976	1.0008	4.0098
	5.0	4.9572	4.9450	5.0000	5.0334	0.9890	1.0067	4.0536
	10	9.9641	10.1105	10.0000	10.0519	1.0110	1.0052	3.9826

α	β	\bar{X}	MR	$\sigma(\alpha, \beta)$	σ_{MR}	d_2	d_3	D_4
1.5	0.5	0.7451	0.6300	0.6124	0.5831	1.0287	0.9522	3.7768
	1.0	1.4920	1.2539	1.2247	1.1480	1.0238	0.9374	3.7468
	5.0	7.5528	6.3645	6.1237	5.8146	1.0393	0.9495	3.7408
	10	15.0286	12.7060	12.2475	11.8153	1.0374	0.9647	3.7897
2.0	0.5	0.9971	0.7352	0.7071	0.6406	1.0397	0.9060	3.6141
	1.0	1.9625	1.4976	1.4142	1.3258	1.0589	0.9375	3.6559
	5.0	9.9023	7.3454	7.0711	6.4374	1.0388	0.9104	3.6291
	10	20.0860	15.1945	14.1421	13.1893	1.0744	0.9326	3.6041
3.0	0.5	1.5018	0.9459	0.8660	0.8067	1.0922	0.9315	3.5587
	1.0	3.0280	1.8759	1.7321	1.5935	1.0830	0.9200	3.5485
	5.0	15.1172	9.4211	8.6603	7.8210	1.0879	0.9031	3.4905
	10	30.1224	18.7847	17.3205	15.7486	1.0845	0.9093	3.5151
4.0	0.5	1.9969	1.0826	1.0000	0.8836	1.0826	0.8836	3.4487
	1.0	4.0032	2.1963	2.0000	1.7963	1.0981	0.8982	3.4537
	5.0	19.9459	11.0745	10.0000	8.9646	1.1075	0.8965	3.4285
	10	40.2152	22.0412	20.0000	18.1502	1.1021	0.9075	3.4704
5.0	0.5	2.5080	1.2346	1.1180	0.9972	1.1043	0.8919	3.4230
	1.0	5.0065	2.4833	2.2361	2.0015	1.1106	0.8951	3.4179
	5.0	24.9952	12.3467	11.1803	10.0514	1.1043	0.8990	3.4423
	10	49.8423	24.3149	22.3607	19.5458	1.0874	0.8741	3.4116
10.0	0.5	4.9852	1.7319	1.5811	1.3572	1.0953	0.8583	3.3509
	1.0	9.9423	3.5097	3.1623	2.7479	1.1099	0.8690	3.3489
	5.0	50.1309	17.4940	15.8114	13.7233	1.1064	0.8679	3.3534
	10	100.637	34.9837	31.6228	27.5975	1.1063	0.8727	3.3666
50.0	0.5	24.9684	3.9957	3.5355	3.0206	1.1302	0.8544	3.2679

α	β	\bar{X}	\overline{MR}	$\sigma(\alpha, \beta)$	σ_{MR}	d_2	d_3	D_4
50.0	1.0	49.9587	7.9154	7.0711	6.0473	1.1194	0.8552	3.2920
	5.0	250.251	39.4540	35.3553	29.9551	1.1159	0.8473	3.2777
	10	500.820	80.2515	70.7107	60.6941	1.1349	0.8583	3.2689
100	0.5	49.9843	5.6459	5.0000	4.2898	1.1292	0.8580	3.2794
	1.0	100.123	11.2389	10.0000	8.4914	1.1239	0.8491	3.2666
	5.0	499.921	55.5181	50.0000	42.1747	1.1104	0.8435	3.2790
	10	1001.32	112.964	100.000	85.6926	1.1296	0.8569	3.2758

The details about the calculations performed to set Table 4.5 are shown in the spreadsheets contained in Appendix D.

In order to find models in which the expected values of the control chart constants d_2 , d_3 , and D_4 are written as functions of two independent variables α and β , the empirical values of d_2 , d_3 , and D_4 shown in Table 4.5 are plotted against α and β . These graphs can be observed in Figures 4.11, 4.12, and 4.13

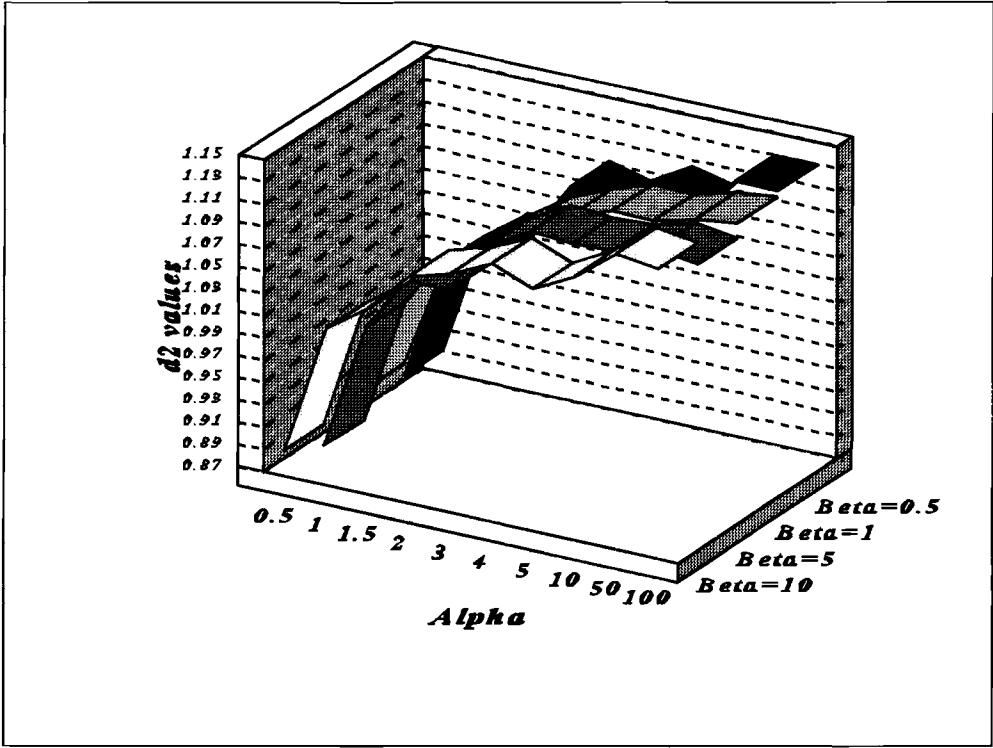
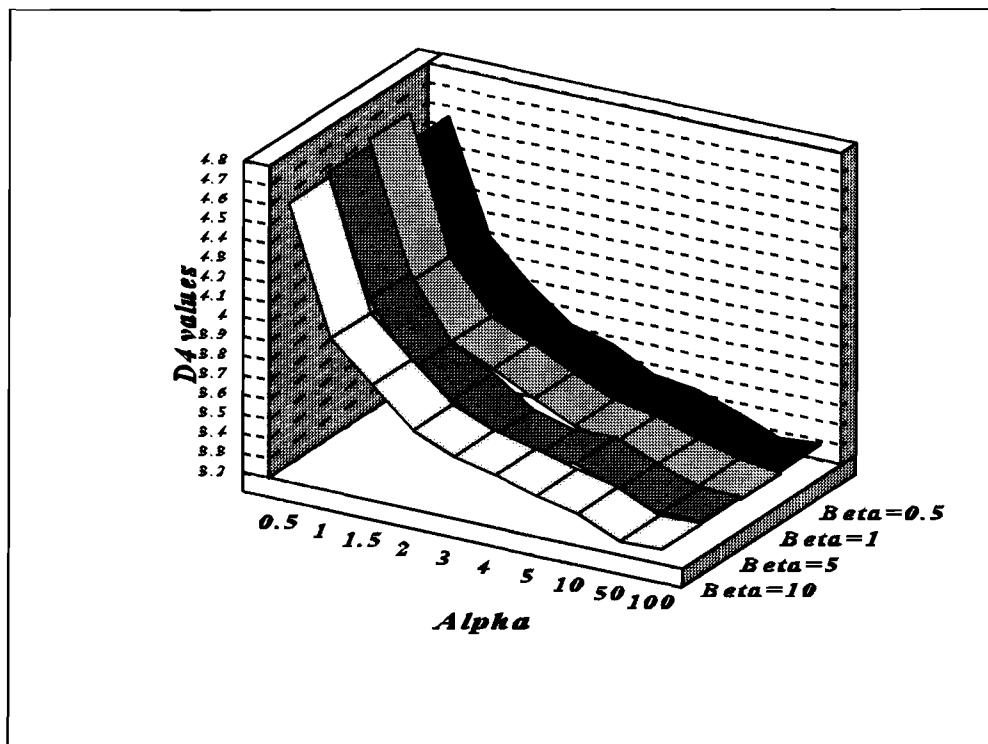
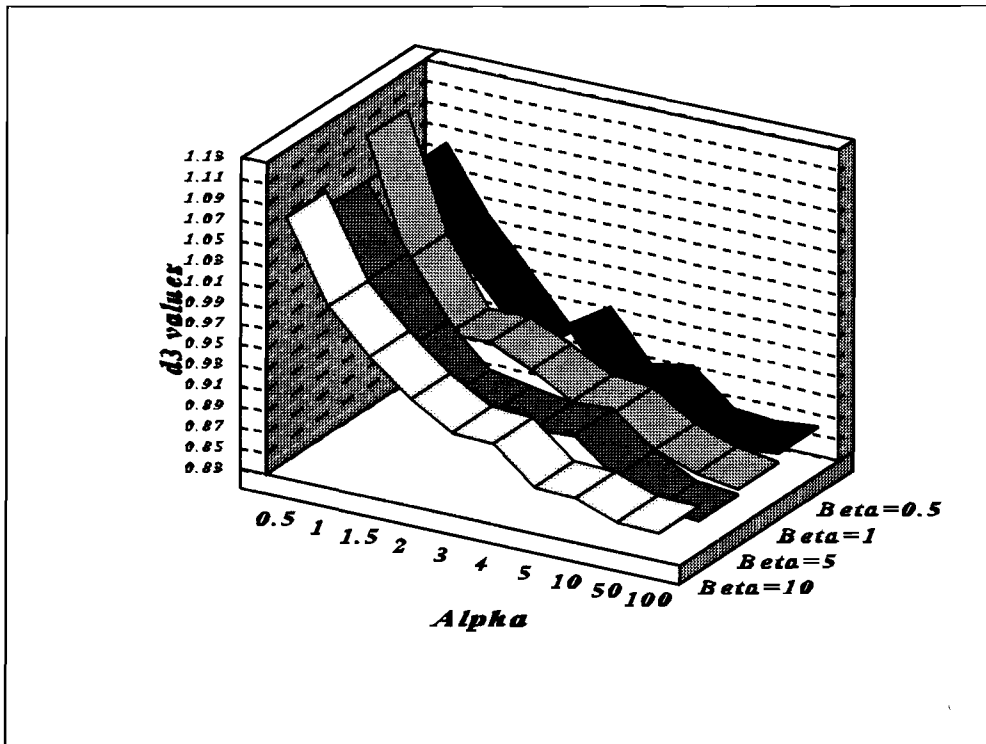


Figure 4.11. Bias correction factor d_2 as a function of α and β .



Figures 4.12 and 4.13. Empirical d_3 and D_4 values as functions of α and β .

It is obvious from the graphs (Figures 4.11, 4.12, and 4.13) that there are non-linear patterns in the three control chart constants d_2 , d_3 , and D_4 as functions of the shape parameter α . Also, it seems that the control chart constants d_2 , d_3 , and D_4 do not vary with the scale parameter β .

Based on this information, different quadratic, cubic, and exponential models are generated in Minitab for Windows release 10 using the method of fitting multiple regression models with the least-squares approach. The Minitab outputs for the different models tried can be seen in Appendix F.

Combinations of exponential functions for the shape parameter α are found to be the ones which better fit the data for the three control chart constants d_2 , d_3 , and D_4 . The scale parameter β , as suspected, is found not significant for all the models generated.

4.2.1 Bias correction factor d_2 .

For the bias correction factor d_2 , the combination of exponential functions which best fits the data is:

$$d_2 = 0.64282 + 0.09775 (1 - e^{-0.5\alpha}) + 0.35736 (1 - e^{-2\alpha}) + 0.02483 (1 - e^{-0.1\alpha})$$

This model has a multiple coefficient of determination R^2 of 97.9%. This value of R^2 indicates that the model is useful for predicting the population of d_2 .

The global F test is used as indicated in Procedure II to test the validity of the multiple regression model selected. From the Minitab output (Appendix G - Models selected), the value of the test statistic F is:

$$F = 566.07$$

Using a significance level $\alpha' = 0.01$, the rejection region for the test is defined by

the value of $F_{\alpha'(k, n-(k+1))}$. From an F table, this critical value is:

$$F_{0.01(3, 36)} = 4.39$$

Clearly the null hypothesis $H_0: \lambda_1 = \lambda_2 = \lambda_3 = 0$ is rejected since the value of the F statistic is greater than the critical value F_c :

$$F > F_{\alpha'(k, n-(k+1))}$$

$$566.07 > 4.39$$

Therefore, it is concluded that one can be very confident that this model is useful for predicting d_2 .

Graphs are built to compare the simulated values of d_2 (empirical) with the regressed values of d_2 . These graphs are shown in Appendix H.

In addition, a residual model diagnostic is performed to validate the assumption of normality for the residuals. Figure 4.14 shows the residual model diagnostic generated by Minitab. From this Figure 4.14, it can be concluded that the residuals (ϵ_i) follow a normal distribution with mean zero and variance σ^2 ($\epsilon_i \sim N(0, \sigma^2)$).

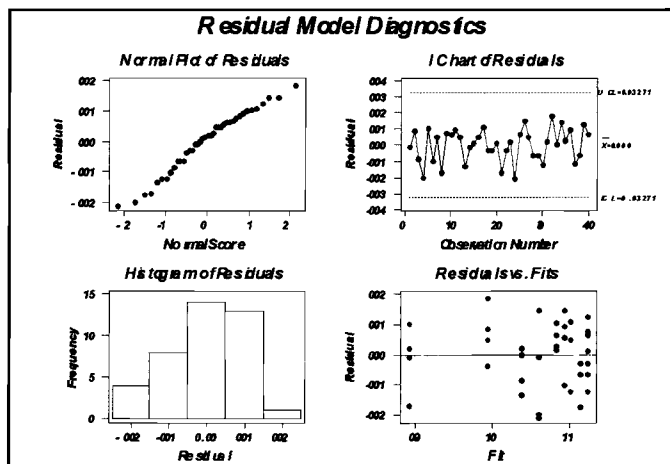


Figure 4.14. Residual model diagnostic for the bias correction factor d_2 .

4.2.2 Bias correction factor d_3 .

For the bias correction factor d_3 , the combination of exponential functions which best fits the data is:

$$d_3 = 0.859457 - 0.2964 e^{-\alpha} + 0.29099 e^{-0.5\alpha} + 0.4758 e^{-2\alpha}$$

This model has a multiple coefficient of determination R^2 of 96.5%. This value of R^2 indicates that the model is useful for predicting the population of d_3 .

The global F test is used as indicated in Procedure II to test the validity of the multiple regression model selected. From the Minitab output (Appendix G - Models selected), the value of the test statistic F is:

$$F = 327.34$$

Using a significance level $\alpha' = 0.01$, the rejection region for the test is defined by the value of $F_{\alpha' (k, n-(k+1))}$. From an F table, this critical value is:

$$F_{0.01 (3, 36)} = 4.39$$

Clearly the null hypothesis $H_0: \lambda_1 = \lambda_2 = \lambda_3 = 0$ is rejected since the value of the F statistic is greater than the critical value F_c :

$$F > F_{\alpha' (k, n-(k+1))}$$

$$327.34 > 4.39$$

Therefore, it is concluded that one can be very confident that this model is useful for predicting d_3 .

Graphs are built to compare the simulated values of d_3 (empirical) with the regressed values of d_3 . These graphs are shown in Appendix H.

In addition, a residual model diagnostic is performed to validate the assumption of

normality for the residuals. Figure 4.15 shows the residual model diagnostic generated by Minitab. From this Figure 4.15, it can be concluded that the residuals (ϵ_i) follow a normal distribution with mean zero and variance σ^2 ($\epsilon_i \sim N(0, \sigma^2)$).

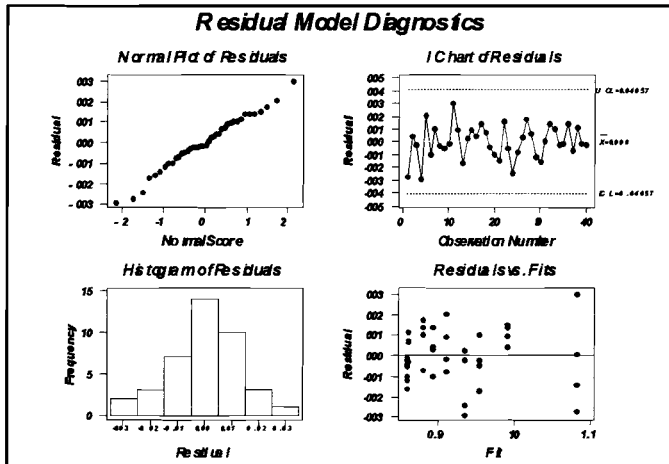


Figure 4.15. Residual model diagnostic for the bias correction factor d_3 .

4.2.3 Control chart constant D_4 .

For the control chart constant D_4 , the combination of exponential functions which best fits the data is:

$$D_4 = 3.28976 + 1.87067 e^{-\alpha} + 0.13663 e^{-0.1\alpha}$$

This model has a multiple coefficient of determination R^2 of 98.0%. This value of R^2 indicates that the model is useful for predicting the population of D_4 .

The global F test is used as indicated in Procedure II to test the validity of the multiple regression model selected. From the Minitab output (Appendix G - Models selected), the value of the test statistic F is:

$$F = 925.12$$

Using a significance level $\alpha' = 0.01$, the rejection region for the test is defined by

the value of $F_{\alpha'(k, n-(k+1))}$. From an F table, this critical value is:

$$F_{0.01(2, 37)} = 5.243$$

Clearly the null hypothesis $H_0: \lambda_1 = \lambda_2 = \lambda_3 = 0$ is rejected since the value of the F statistic is greater than the critical value F_c :

$$925.12 > 5.243$$

Therefore, it is concluded that one can be very confident that this model is useful for predicting D_4 .

Graphs are built to compare the simulated values of D_4 (empirical) with the regressed values of D_4 . These graphs are shown in Appendix H.

In addition, a residual model diagnostic is performed to validate the assumption of normality for the residuals. Figure 4.16 shows the residual model diagnostic generated by Minitab. From this Figure 4.16, it can be concluded that the residuals (ϵ_i) follow a normal distribution with mean zero and variance σ^2 ($\epsilon_i \sim N(0, \sigma^2)$).

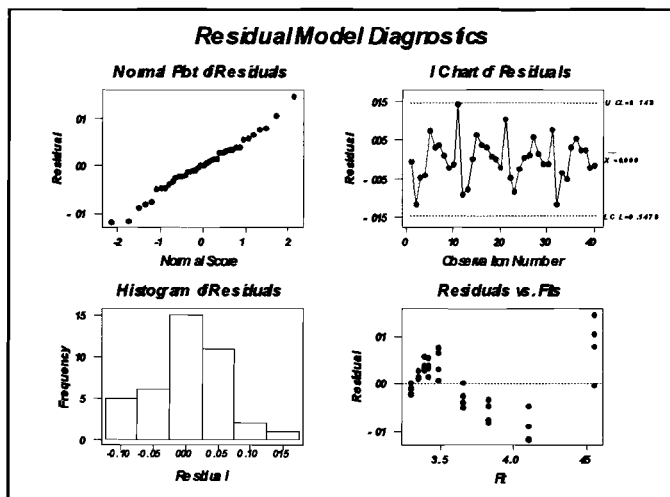


Figure 4.16. Residual model diagnostic for the control chart constant D_4 .

4.3 Objective three.

The third objective of this thesis research is to compare the performance of the individual measurement X and moving range $n = 2$ MR control charts, using the normal control chart constants, with the performance of these control charts using the Pearson type III $c = 0$ (gamma) control chart constants computed in objective two, when 10, 30, and 50 data values are available from an unknown process distribution. The performance of the control charts will be evaluated using the proportion of control charts which indicate at least one signal when the process is in control and for shifts in the process average of 1 and 2 process standard deviations σ .

The results of Objective three are presented separately for each of the five unknown process distributions. Then, an overall analysis is performed over the results of the five distributions to test the second hypothesis stated in Chapter I (p. 5).

According to Procedure III, one thousand samples of $k = 10, 30,$ and 50 observations are generated in Quattro Pro for Windows using the random number generator included in Minitab and following the algorithms described for each of the five process distributions. The algorithms used to generate gamma and normal variates are tested statistically to see how closely the values generated resemble IID gamma (α, β) and normal (μ, σ^2) distributions, respectively. The tests are goodness of fit tests for 100 and 1,000 observations for the gamma distribution (2, 1) and for the normal (40, 10^2). The results of the tests are shown in Appendix C and they indicate that both algorithms are good enough to generate reliable gamma and normal variates respectively. The other algorithms are not tested statistically because they are particular cases of either the

gamma algorithm or the normal algorithm.

Spreadsheets which place 250 samples of $k = 10, 30,$ and 50 observations are designed to perform the whole analysis contained in objective three. Therefore, for each distribution and for $k = 10, 30,$ and 50 observations, the spreadsheets are run four times to complete the 1,000 samples required in Procedure III. A portion of the spreadsheets and a subset of the results for each of the five process distributions and for $k = 10, 30,$ and 50 observations can be seen in Appendix I.

Then, for each of the five distributions and for each sample generated of $k = 10, 30,$ and 50 observations, the following values are calculated:

- Sample average \bar{X} .
- Sample average moving range \overline{MR} from the 9, 29, and 49 subgroups of two consecutive measurements formed respectively for $k = 10, 30,$ and 50 .
- Sample shape parameter α and sample scale parameter β . In order to estimate these parameters α and β , the numerical technique developed by Choi and Wette (9, p. 683) and described in Procedure III is used. The statistic T is evaluated for each sample. Then, using the table reproduced in Appendix J and included in the spreadsheets designed (Appendix I), the values of α and β are estimated.
- Gamma bias correction factors d_2 and d_3 and gamma control chart constant D_4 using the exponential models selected in objective two

$$d_2 = f_1(\alpha)$$

$$d_3 = f_2(\alpha)$$

$$D_4 = f_3(\alpha)$$

- Two sets of control limits for the X and MR ($n=2$) control charts are calculated for each simulation run of $k = 10, 30,$ and 50 observations. One set of control limits is obtained using the normal control chart constants and the other one is obtained using the gamma constants computed before.

A subset of the details of these calculations performed for each of the five distributions and for each simulation run of $k = 10, 30,$ and 50 observations can be seen partially in the spreadsheets included in Appendix I.

Next, as indicated in Procedure III, three sets of 1,000 new samples of $k' = 30$ observations are generated for each of the five process distributions. The first set is left as it is (no shift), the second set is shifted up one process standard deviation, and the third set is shifted up two process standard deviations. Only for the first set of 1,000 new samples (no shift in the process average), the observations are grouped in subgroups of two consecutive measurements and the moving range values are calculated. The first subgroup is formed using the last observation of the old samples of $k = 10, 30,$ and 50 observations with the first observation of the new 1000 samples of $k' = 30$. Therefore, there are thirty moving ranges for each new sample of $k' = 30$ data values.

Then, the first observation ($k'=1$) and first moving range, the first ten observations ($k'=10$) and first ten moving ranges, and the thirty observations ($k'=30$) and thirty moving ranges of the first new sample of $k' = 30$ data values, are compared with the two sets of control limits calculated before for the first sample of $k = 10, 30,$ and 50 measurements and for each of the five process distributions. This step is executed repeatedly for each of the 1,000 samples contained in the three sets of new samples ($k'=30$) and for each of the

five distributions. Each time the comparison is between the n^{th} new sample of $k' = 30$ with the normal and gamma control limits corresponding to the n^{th} old samples of $k = 10, 30,$ and 50 observations, respectively. It is important to clarify that for the moving range the step is repeated only for the first set of 1,000 new samples because only shifts in the process average are considered in this research.

Thus, for each of the five process distributions and for $k = 10, 30,$ and 50 data values, the proportion of individual measurement X and moving range $n = 2$ MR control charts which indicate at least one signal are recorded. This is done for the three comparisons described before ($k'=1, 10,$ and 30 new observations) and for the two set of control limits (normal and gamma). For the moving range, the proportions are computed only for the first set of 1,000 new samples of $k' = 30$.

A subset of the details about the calculations of the number of X and MR control charts which indicate at least one signal for each of the five distributions, for $k = 10, 30,$ and $50,$ for $k' = 1, 10,$ and $30,$ and for both sets of control limits (normal and gamma), can be observed partially in the spreadsheets included in Appendix I.

4.3.1 Lognormal $(0, 1^2)$

The number of normal and gamma individual measurement X and moving range $n = 2$ MR control charts which indicate at least one signal are summarized in Table 4.6 for the lognormal distribution. This table indicates the number of normal and gamma control charts out of the 1,000 generated which show at least one false alarm (no shift in the process average) and signal at least one out-of-control condition (shift in the process average of 1 and 2 sigma) for values of $k = 10, 30,$ and 50 and $k' = 1, 10,$ and 30 .

TABLE 4.6

NUMBER OF X AND MR (n=2) CONTROL CHARTS WHICH INDICATE AT LEAST ONE SIGNAL OUT OF THE 1,000 GENERATED FOR THE LOGNORMAL

Shift in sigma	X control charts (k=10)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	62	453	714	55	413	667
1	216	673	834	197	628	799
2	558	844	927	512	815	904
Shift in sigma	X control charts (k=30)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	45	367	678	39	323	622
1	129	622	858	106	566	814
2	449	874	964	367	814	934
Shift in sigma	X control charts (k=50)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	33	328	664	28	283	605
1	116	621	864	94	537	796
2	395	866	974	303	806	944
Value of k (obsv.)	MR control charts (no shift in the process average)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
10	83	469	718	56	381	623
30	60	379	669	45	283	559
50	60	343	656	40	242	545

From the values in Table 4.6, the reductions in false alarm rate and power of detection in percentage due to the use of gamma control limits rather than normal control limits are computed. These results are presented in Table 4.7.

TABLE 4.7

REDUCTION IN SIGNALS FROM NORMAL TO GAMMA CONTROL CHARTS
FOR THE LOGNORMAL DISTRIBUTION (IN PERCENT)

X control chart									
Shift in σ	k = 10			k = 30			k = 50		
	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30
0	11.29	8.83	6.58	13.33	11.99	8.26	15.15	13.72	8.89
1	8.79	6.69	4.20	17.83	9.00	5.13	18.97	13.53	7.87
2	8.24	3.44	2.48	18.26	6.86	3.11	23.29	6.93	3.08
MR control chart (no shift in the process average)									
k	k' = 1			k' = 10			k' = 30		
10	32.53			18.76			13.23		
30	25.00			25.33			16.44		
50	33.33			29.45			16.92		

Note: Values marked are the biggest reduction (in percent) in the k' column.

In order to test the second part of the second hypothesis (power of detection at least equal to the theoretical power using the normal assumption for shifts of 1 and 2 process standard deviations), the performance of the gamma X control chart for k' = 1 is compared with the performance of the theoretical normal X control chart. This comparison is shown in Table 4.8. Table 4.8 indicates percentages of signals for shifts in the process average of 1 and 2 sigma for the lognormal distribution.

TABLE 4.8

PERCENTAGE OF SIGNALS (%) - GAMMA X CONTROL CHARTS vs.
 THEORETICAL NORMAL X CONTROL CHARTS FOR SHIFTS OF 1 AND 2
 SIGMA AND $k' = 1$

Shift in σ	k = 10		k = 30		k = 50	
	Gamma	NT	Gamma	NT	Gamma	NT
1	19.7	2.28	10.6	2.28	9.4	2.28
2	51.2	15.87	36.7	15.87	30.3	15.87

NT: Normal theory.

In order to evaluate the effect of the number of observations used to set control limits ($k = 10, 30,$ and 50), the performance of only the normal X control charts for $k = 10$ are compared with the performance of those normal X control charts for $k = 30$. In the same way, the performance of only the normal X control chart constants for $k = 30$ are compared with those normal X control charts for $k = 50$. The values used to do the comparison are taken from Table 4.6. The comparison is shown in Table 4.9. Table 4.9 indicates the differences in percentage of signals between $k = 10$ and 30 and between $k = 30$ and 50 for the normal X control chart.

TABLE 4.9

DIFFERENCES IN PERCENTAGE OF SIGNALS (%) DUE TO THE VALUE OF k
FOR THE NORMAL \bar{X} CONTROL CHART - LOGNORMAL DISTRIBUTION

Shift in σ	$k = 10$ vs. $k = 30$			$k = 30$ vs. $k = 50$		
	$k' = 1$	$k' = 10$	$k' = 30$	$k' = 1$	$k' = 10$	$k' = 30$
0	(27.42)	(18.98)	(5.04)	(26.67)	(10.63)	(2.06)
1	(40.28)	(7.58)	2.88	(10.08)	(0.16)	0.70
2	(19.53)	3.55	3.99	(12.03)	(0.92)	1.04

Note: Values in parenthesis are negative percentages (reduction in percentage of signals).

4.3.2 Normal ($40, 10^2$)

The number of normal and gamma individual measurement \bar{X} and moving range $n = 2$ MR control charts which indicate at least one signal are summarized in Table 4.10 for the normal distribution. This table indicates the number of normal and gamma control charts out of the 1,000 generated which show at least one false alarm (no shift in the process average) and signal at least one out-of-control condition (shift in the process average of 1 and 2 sigma) for values of $k = 10, 30, \text{ and } 50$ and $k' = 1, 10, \text{ and } 30$.

TABLE 4.10

NUMBER OF X AND MR (n=2) CONTROL CHARTS WHICH INDICATE AT LEAST ONE SIGNAL OUT OF THE 1,000 GENERATED FOR THE NORMAL

Shift in sigma	X control charts (k=10)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	30	150	301	27	144	290
1	67	337	536	67	327	523
2	235	711	843	229	696	833
Shift in sigma	X control charts (k=30)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	10	72	165	10	66	154
1	35	276	532	33	260	509
2	167	742	912	164	734	906
Shift in sigma	X control charts (k=50)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	4	54	143	4	53	136
1	31	263	535	26	252	514
2	169	786	948	161	774	942
Value of k (obsv.)	MR control charts (no shift in the process average)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
10	31	194	350	28	188	336
30	15	119	276	13	108	253
50	11	115	256	9	99	238

From the values in Table 4.10, the reductions in false alarm rate and power of detection in percentage due to the use of gamma control limits rather than normal control limits are computed. These results are presented in Table 4.11.

TABLE 4.11

REDUCTION IN SIGNALS FROM NORMAL TO GAMMA CONTROL CHARTS
FOR THE NORMAL DISTRIBUTION (IN PERCENT)

X control chart									
Shift in σ	k = 10			k = 30			k = 50		
	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30
0	10.00	4.00	3.65	0.00	8.33	6.67	0.00	1.85	4.90
1	0.00	2.97	2.43	5.71	5.80	4.32	16.13	4.18	3.93
2	2.55	2.11	1.19	2.19	1.08	0.66	4.73	1.53	0.63
MR control chart (no shift in the process average)									
k	k' = 1			k' = 10			k' = 30		
10	9.68			3.09			4.0		
30	13.33			9.24			8.33		
50	18.18			13.91			7.03		

Note: Values marked are the biggest reduction (in percent) in the k' column.

In order to test the second part of the second hypothesis (power of detection at least equal to the theoretical power using the normal assumption for shifts of 1 and 2 process standard deviations), the performance of the gamma X control chart for k' = 1 is compared with the performance of the theoretical normal X control chart. This comparison is shown in Table 4.12. Table 4.12 indicates percentages of signals for shifts in the process average of 1 and 2 sigma for the normal distribution.

TABLE 4.12

PERCENTAGE OF SIGNALS (%) - GAMMA X CONTROL CHARTS vs.
 THEORETICAL NORMAL X CONTROL CHARTS FOR SHIFTS OF 1 AND 2
 SIGMA AND $k' = 1$

Shift in σ	k = 10		k = 30		k = 50	
	Gamma	NT	Gamma	NT	Gamma	NT
1	6.70	2.28	3.3	2.28	2.6	2.28
2	22.9	15.87	16.4	15.87	16.1	15.87

NT: Normal theory.

In order to evaluate the effect of the number of observations used to set control limits ($k = 10, 30,$ and 50), the performance of only the normal X control charts for $k = 10$ are compared with the performance of those normal X control charts for $k = 30$. In the same way, the performance of only the normal X control chart constants for $k = 30$ are compared with those normal X control charts for $k = 50$. The values used to do the comparison are taken from Table 4.10. The comparison is shown in Table 4.13. Table 4.13 indicates the differences in percentage of signals between $k = 10$ and 30 and between $k = 30$ and 50 for the normal X control chart.

TABLE 4.13

DIFFERENCES IN PERCENTAGE OF SIGNALS (%) DUE TO THE VALUE OF k
FOR THE NORMAL \bar{X} CONTROL CHART - NORMAL DISTRIBUTION

Shift in σ	$k = 10$ vs. $k = 30$			$k = 30$ vs. $k = 50$		
	$k' = 1$	$k' = 10$	$k' = 30$	$k' = 1$	$k' = 10$	$k' = 30$
0	(66.67)	(52.00)	(45.18)	(60.00)	(25.00)	(13.33)
1	(47.76)	(18.10)	(0.75)	(11.43)	(4.71)	0.56
2	(28.94)	4.36	8.19	1.20	5.93	3.95

Note: Values in parenthesis are negative percentages (reduction in percentage of signals).

4.3.3 Exponential (mean=1)

The number of normal and gamma individual measurement X and moving range $n = 2$ MR control charts which indicate at least one signal are summarized in Table 4.14 for the exponential distribution. This table indicates the number of normal and gamma control charts out of the 1,000 generated which show at least one false alarm (no shift in the process average) and signal at least one out-of-control condition (shift in the process average of 1 and 2 sigma) for values of $k = 10, 30,$ and 50 and $k' = 1, 10,$ and 30 .

TABLE 4.14

NUMBER OF X AND MR (n=2) CONTROL CHARTS WHICH INDICATE AT LEAST ONE SIGNAL OUT OF THE 1,000 GENERATED FOR THE EXPONENTIAL

Shift in sigma	X control charts (k=10)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	47	342	608	38	300	533
1	138	596	817	115	526	742
2	347	792	912	288	729	859
Shift in sigma	X control charts (k=30)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	34	280	548	21	208	451
1	89	548	825	65	444	737
2	231	820	950	172	728	902
Shift in sigma	X control charts (k=50)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	28	246	555	22	181	448
1	86	537	863	57	443	776
2	221	858	983	159	757	956
Value of k (obsv.)	MR control charts (no shift in the process average)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
10	58	353	614	32	254	468
30	55	278	552	20	153	360
50	44	248	552	23	133	330

From the values in Table 4.14, the reductions in false alarm rate and power of detection in percentage due to the use of gamma control limits rather than normal control limits are computed. These results are presented in Table 4.15.

TABLE 4.15

REDUCTION IN SIGNALS FROM NORMAL TO GAMMA CONTROL CHARTS
FOR THE EXPONENTIAL DISTRIBUTION (IN PERCENT)

X control chart									
Shift in σ	k = 10			k = 30			k = 50		
	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30
0	19.15	12.28	12.34	38.24	25.71	17.70	21.43	26.42	19.28
1	16.67	11.74	9.18	26.97	18.98	10.67	33.72	17.50	10.08
2	17.00	7.95	5.81	25.54	11.22	5.05	28.05	11.77	2.75
MR control chart (no shift in the process average)									
k	k' = 1			k' = 10			k' = 30		
10	44.83			28.05			23.78		
30	63.64			44.96			34.78		
50	47.73			46.37			40.22		

Note: Values marked are the biggest reduction (in percent) in the k' column.

In order to test the second part of the second hypothesis (power of detection at least equal to the theoretical power using the normal assumption for shifts of 1 and 2 process standard deviations), the performance of the gamma X control chart for k' = 1 is compared with the performance of the theoretical normal X control chart. This comparison is shown in Table 4.16. Table 4.16 indicates percentages of signals for shifts in the process average of 1 and 2 sigma for the exponential distribution.

TABLE 4.16

PERCENTAGE OF SIGNALS (%) - GAMMA X CONTROL CHARTS vs.
THEORETICAL NORMAL X CONTROL CHARTS FOR SHIFTS OF 1 AND 2
SIGMA AND $k' = 1$

Shift in σ	k = 10		k = 30		k = 50	
	Gamma	NT	Gamma	NT	Gamma	NT
1	11.5	2.28	6.5	2.28	5.7	2.28
2	28.8	15.87	17.2	15.87	15.90	15.87

NT: Normal theory.

In order to evaluate the effect of the number of observations used to set control limits ($k = 10, 30,$ and 50), the performance of only the normal X control charts for $k = 10$ are compared with the performance of those normal X control charts for $k = 30$. In the same way, the performance of only the normal X control chart constants for $k = 30$ are compared with those normal X control charts for $k = 50$. The values used to do the comparison are taken from Table 4.14. The comparison is shown in Table 4.17. Table 4.17 indicates the differences in percentage of signals between $k = 10$ and 30 and between $k = 30$ and 50 for the normal X control chart.

TABLE 4.17

DIFFERENCES IN PERCENTAGE OF SIGNALS (%) DUE TO THE VALUE OF k
FOR THE NORMAL \bar{X} CONTROL CHART - EXPONENTIAL DISTRIBUTION

Shift in σ	$k = 10$ vs. $k = 30$			$k = 30$ vs. $k = 50$		
	$k' = 1$	$k' = 10$	$k' = 30$	$k' = 1$	$k' = 10$	$k' = 30$
0	(27.66)	(18.13)	(9.87)	(17.65)	(12.14)	1.28
1	(35.51)	(8.05)	0.98	(3.37)	(2.01)	4.61
2	(33.43)	3.54	4.17	(4.33)	4.63	3.47

Note: Values in parenthesis are negative percentages (reduction in percentage of signals).

4.3.4 Gamma ($\alpha=1.5, \beta=1.0$)

The number of normal and gamma individual measurement X and moving range $n = 2$ MR control charts which indicate at least one signal are summarized in Table 4.18 for the gamma distribution. This table indicates the number of normal and gamma control charts out of the 1,000 generated which show at least one false alarm (no shift in the process average) and signal at least one out-of-control condition (shift in the process average of 1 and 2 sigma) for values of $k = 10, 30,$ and 50 and $k' = 1, 10,$ and 30 .

TABLE 4.18

NUMBER OF X AND MR (n=2) CONTROL CHARTS WHICH INDICATE AT LEAST ONE SIGNAL OUT OF THE 1,000 GENERATED FOR THE GAMMA

Shift in sigma	X control charts (k=10)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	36	287	539	33	252	474
1	111	506	755	99	456	692
2	304	743	896	257	691	856
Shift in sigma	X control charts (k=30)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	28	238	524	20	195	433
1	66	489	809	58	416	742
2	232	807	958	190	733	925
Shift in sigma	X control charts (k=50)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	19	216	497	14	170	418
1	57	483	811	48	412	752
2	207	827	977	162	744	948
Value of k (obsv.)	MR control charts (no shift in the process average)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
10	55	297	534	36	230	433
30	32	247	528	20	164	370
50	26	227	511	11	139	354

From the values in Table 4.18, the reductions in false alarm rate and power of detection in percentage due to the use of gamma control limits rather than normal control limits are computed. These results are presented in Table 4.19.

TABLE 4.19

REDUCTION IN SIGNALS FROM NORMAL TO GAMMA CONTROL CHARTS
FOR THE GAMMA DISTRIBUTION (IN PERCENT)

X control chart									
Shift in σ	k = 10			k = 30			k = 50		
	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30
0	8.33	12.20	12.06	28.57	18.07	17.37	26.32	21.30	15.90
1	10.81	9.88	8.34	12.12	14.93	8.28	15.79	14.70	7.27
2	15.46	7.00	4.46	18.10	9.17	3.44	21.74	10.04	2.97
MR control chart (no shift in the process average)									
k	k' = 1			k' = 10			k' = 30		
10	34.55			22.56			18.91		
30	37.50			33.60			29.92		
50	57.69			38.77			30.72		

Note: Values marked are the biggest reduction (in percent) in the k' column.

In order to test the second part of the second hypothesis (power of detection at least equal to the theoretical power using the normal assumption for shifts of 1 and 2 process standard deviations), the performance of the gamma X control chart for k' = 1 is compared with the performance of the theoretical normal X control chart. This comparison is shown in Table 4.20. Table 4.20 indicates percentages of signals for shifts in the process average of 1 and 2 sigma for the gamma distribution.

TABLE 4.20

PERCENTAGE OF SIGNALS (%) - GAMMA X CONTROL CHARTS vs.
 THEORETICAL NORMAL X CONTROL CHARTS FOR SHIFTS OF 1 AND 2
 SIGMA AND $k' = 1$

Shift in σ	k = 10		k = 30		k = 50	
	Gamma	NT	Gamma	NT	Gamma	NT
1	9.90	2.28	5.80	2.28	4.80	2.28
2	25.70	15.87	19.00	15.87	16.20	15.87

NT: Normal theory.

In order to evaluate the effect of the number of observations used to set control limits ($k = 10, 30,$ and 50), the performance of only the normal X control charts for $k = 10$ are compared with the performance of those normal X control charts for $k = 30$. In the same way, the performance of only the normal X control chart constants for $k = 30$ are compared with those normal X control charts for $k = 50$. The values used to do the comparison are taken from Table 4.18. The comparison is shown in Table 4.21. Table 4.21 indicates the differences in percentage of signals between $k = 10$ and 30 and between $k = 30$ and 50 for the normal X control chart.

TABLE 4.21

DIFFERENCES IN PERCENTAGE OF SIGNALS (%) DUE TO THE VALUE OF k
FOR THE NORMAL \bar{X} CONTROL CHART - GAMMA DISTRIBUTION

Shift in σ	$k = 10$ vs. $k = 30$			$k = 30$ vs. $k = 50$		
	$k' = 1$	$k' = 10$	$k' = 30$	$k' = 1$	$k' = 10$	$k' = 30$
0	(22.22)	(17.07)	(2.78)	(32.14)	(9.24)	(5.15)
1	(40.54)	(3.36)	7.15	(13.64)	(1.23)	0.25
2	(23.68)	8.61	6.92	(10.78)	2.48	1.98

Note: Values in parenthesis are negative percentages (reduction in percentage of signals).

4.3.5 Chi-Square χ^2 (df=4)

The number of normal and gamma individual measurement \bar{X} and moving range $n = 2$ MR control charts which indicate at least one signal are summarized in Table 4.22 for the chi-square distribution. This table indicates the number of normal and gamma control charts out of the 1,000 generated which show at least one false alarm (no shift in the process average) and signal at least one out-of-control condition (shift in the process average of 1 and 2 sigma) for values of $k = 10, 30,$ and 50 and $k' = 1, 10,$ and 30 .

TABLE 4.22

NUMBER OF X AND MR (n=2) CONTROL CHARTS WHICH INDICATE AT LEAST ONE SIGNAL OUT OF THE 1,000 GENERATED FOR THE CHI-SQUARE

Shift in sigma	X control charts (k=10)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	48	266	490	43	246	454
1	108	509	702	93	475	662
2	291	729	881	263	691	846
Shift in sigma	X control charts (k=30)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	29	194	449	23	161	397
1	80	504	773	70	437	718
2	219	790	955	195	736	931
Shift in sigma	X control charts (k=50)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
0	23	165	422	19	134	364
1	72	496	799	23	156	363
2	201	804	975	175	737	944
Value of k (obsv.)	MR control charts (no shift in the process average)					
	Normal limits			Gamma limits		
	k' = 1	k' = 10	k' = 30	k' = 1	k' = 10	k' = 30
10	56	282	500	44	243	414
30	35	213	474	23	156	363
50	34	181	444	22	124	335

From the values in Table 4.22, the reductions in false alarm rate and power of detection in percentage due to the use of gamma control limits rather than normal control limits are computed. These results are presented in Table 4.23.

TABLE 4.23

REDUCTION IN SIGNALS FROM NORMAL TO GAMMA CONTROL CHARTS
FOR THE CHI-SQUARE DISTRIBUTION (IN PERCENT)

X control chart									
Shift in σ	k = 10			k = 30			k = 50		
	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30	k'=1	k'=10	k'=30
0	10.42	7.52	7.35	20.69	17.01	11.58	17.39	18.79	13.74
1	13.84	6.68	5.70	12.50	13.29	7.12	9.72	13.51	9.01
2	9.62	5.21	3.97	10.96	6.84	2.51	12.94	8.33	3.18
MR control chart (no shift in the process average)									
k	k' = 1			k' = 10			k' = 30		
10	21.43			13.83			17.20		
30	34.29			26.76			23.42		
50	35.29			31.49			24.55		

Note: Values marked are the biggest reduction (in percent) in the k' column.

In order to test the second part of the second hypothesis (power of detection at least equal to the theoretical power using the normal assumption for shifts of 1 and 2 process standard deviations), the performance of the gamma X control chart for k' = 1 is compared with the performance of the theoretical normal X control chart. This comparison is shown in Table 4.24. Table 4.24 indicates percentages of signals for shifts in the process average of 1 and 2 sigma for the chi-square distribution.

TABLE 4.24

PERCENTAGE OF SIGNALS (%) - GAMMA X CONTROL CHARTS vs.
 THEORETICAL NORMAL X CONTROL CHARTS FOR SHIFTS OF 1 AND 2
 SIGMA AND $k' = 1$

Shift in σ	k = 10		k = 30		k = 50	
	Gamma	NT	Gamma	NT	Gamma	NT
1	9.30	2.28	7.00	2.28	6.50	2.28
2	26.30	15.87	19.50	15.87	17.50	15.87

NT: Normal theory.

In order to evaluate the effect of the number of observations used to set control limits ($k = 10, 30, \text{ and } 50$), the performance of only the normal X control charts for $k = 10$ are compared with the performance of those normal X control charts for $k = 30$. In the same way, the performance of only the normal X control chart constants for $k = 30$ are compared with those normal X control charts for $k = 50$. The values used to do the comparison are taken from Table 4.22. The comparison is shown in Table 4.25. Table 4.25 indicates the differences in percentage of signals between $k = 10$ and 30 and between $k = 30$ and 50 for the normal X control chart.

TABLE 4.25

DIFFERENCES IN PERCENTAGE OF SIGNALS (%) DUE TO THE VALUE OF k
FOR THE NORMAL \bar{X} CONTROL CHART - CHI-SQUARE DISTRIBUTION

Shift in σ	$k = 10$ vs. $k = 30$			$k = 30$ vs. $k = 50$		
	$k' = 1$	$k' = 10$	$k' = 30$	$k' = 1$	$k' = 10$	$k' = 30$
0	(39.58)	(27.07)	(8.37)	(20.69)	(14.95)	(6.01)
1	(25.93)	(0.98)	10.11	(10.00)	(1.59)	3.36
2	(24.74)	8.37	8.40	(8.22)	1.77	2.09

Note: Values in parenthesis are negative percentages (reduction in percentage of signals).

4.3.6 Overall Analysis of the Results

The overall analysis of the results described from section 4.3.1 to section 4.3.5 is based on the second hypothesis stated in Chapter I.

The second hypothesis of this thesis research states that the individual measurement X and moving range $n = 2$ MR control charts, using gamma control chart constants, have better performance than the same control charts using the normal constants, when 10, 30, and 50 data values are available from an unknown process distribution. Better performance means less false alarm rate in control and power of detection (\bar{X} chart), for shifts in the process average of 1 and 2 sigma, at least equal to the theoretical power using the normal assumption (0.0228 and 0.1587 respectively for shifts in the process average of 1 and 2 sigma).

From Tables 4.6, 4.10, 4.14, 4.18, and 4.22, it is clearly seen that the gamma control charts (\bar{X} and MR) always yield less false alarm rate than the normal control charts (\bar{X} and MR) because the gamma limits are always wider than the normal limits.

However, this reduction in false alarm rate leads to reductions in the power of detection too. This is the statistical price that the gamma control charts have to pay in order to reduce the number of false signals when the underlying process distribution is asymmetric (skewed distribution).

The higher power of detection in the normal \bar{X} control charts is unquestionable. However, in general, when the power increases the probabilities of a type I error or alpha risk also increases. According to Walker, et al. (42, p. 248), "Occasions arise in practice when a signal from the chart is automatically interpreted as an indication of an out-of-control process. Operators may be instructed to stop the process in order to search out and remove assignable causes. Under such circumstances, spurious signals can lead to counterproductive changes."

In this regard, it is important to have a criterion that does not indicate trouble too often when such trouble is not present. This statement has been stressed by Shewhart and many other quality experts. This criterion is better accomplished by the Pearson type III with $c = 0$ (gamma) control charts (less false alarm rate at the expense of reductions in the power of detection when shifts are present).

In addition, from Tables 4.7, 4.11, 4.15, 4.19, and 4.23, it is observed that in almost all of the cases for the \bar{X} chart, the reductions in false alarm rate (%) are greater than the reductions in power of detection (%) for shifts of 1 and 2 process standard deviations when gamma control limits are used instead of the normal control limits. For the MR charts, the reductions in false alarm rate are always substantially big (from 14% to 63%). Therefore, based on the empirical evidence, it seems that the gamma control

charts have better performance than the normal control charts.

Now, even though the empirical power of detection for the gamma X control charts is always less than the empirical power of detection for the normal X control charts, the empirical power of detection for the gamma X control charts is always greater than the theoretical power of detection for the normal control charts (0.0228 and 0.1587 for shifts in the process average of 1 and 2 sigma respectively). It is clearly seen in Tables 4.8, 4.12, 4.16, 4.20, and 4.24.

In conclusion, based on the empirical results, it could be said that there is not evidence to reject the statement of the second hypothesis. It means that, based on the empirical evidence, the gamma control charts (X and MR) seem to have better performance than the same control charts set with the normal constants d_2 , d_3 , and D_4 . However, even though the performance of the gamma control charts is better than the performance of the normal control charts when the process distribution is non-normal, the number of empirical false alarms (no shift) is still too big (greater than the 1% accepted for practical purposes and greater than the theoretical value 0.27%). For instance, it is observed in Table 4.14 (exponential distribution) that the false alarm rates for $k' = 1$ are 3.8%, 2.1%, and 2.2% for $k = 10, 30,$ and 50 data values, respectively. These false alarm rate values are far away the theoretical normal curve false alarm rate (0.27%) and the practical false alarm rate used in industry (1.0%). It is the author's belief that these high false alarm rates using the gamma control charts are due to the following three factors:

- The empirical nature of the study (# of observations).
- The skewness of the process distribution which affects the number of data values above

the three sigma upper control limit (tail area).

- The use of symmetric control limits (\bar{X} chart) for asymmetric process distributions.

In relation to the effects of the number of observations used to set control limits ($k = 10, 30,$ and 50) in the performance of the individual measurement \bar{X} and moving range $n = 2$ MR control charts there are not conclusive results. However, from Tables 4.9, 4.13, 4.17, 4.21, and 4.25, it seems that the differences in performance $k = 10$ versus $k = 30$ are more inconsistent than the differences in performance $k = 30$ versus $k = 50$. In addition, the number of false alarms (\bar{X} and MR), when $k = 10$ is used to set control limits, is always much greater (from 28% to 65% for the asymmetric distributions and from 150% to 200% for the normal distribution) than the number of false alarms when $k = 30$ and 50 data values are used to set control limits.

Therefore, it is the author's belief that $k = 10$ observations is not a good sample size to set reliable control limits (\bar{X} and MR) for future predictions. This goes against some authors who believe that control charts always perform well no matter what number of observations are available. Additionally, from the empirical evidence, it seems that there is not too much difference in performance between $k = 30$ and $k = 50$ data values used to set control limits. As a consequence, it seems to be safe to say that $k = 30$ measurements is a good sample size to set control limits (\bar{X} and MR) for future predictions.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions and recommendations

- The empirical ARL curves (from Figure 4.1 to Figure 4.10) indicate that the individual measurement X control chart, using the normal curve bias correction factor $d_2 = 1.128$, does not work well when the underlying process distribution shows a marked departure from normality (highly skewed distributions). This departure from normality is represented in this thesis research by a Pearson type III with $c = 0$ family of distributions (gamma distributions). For gamma distributions with shape parameter $\alpha < 5$, the ARLs in control (no shift) are substantially less (from 15% to 74%) than the 100 accepted in industry as a minimum for practical purposes and much less than the 370 stated in the normal theory ($1 / 0.0027$). Therefore, if the process is suspected to have a skewed (asymmetric) distribution, the individual measurement X control chart, using the normal bias correction factor d_2 , must be very carefully used and interpreted.

In general, it is recommended not to use the X chart, under the assumption of normality, when it is suspected that the process distribution is markedly non-normal (represented in this research by asymmetric distributions). According to Duncan (1, p. 400), "In such cases, the multiple of σ used to set control limits might be better derived from other distributions for which the percentage points have been computed."

- The empirical ARL values in control (Table 4.4) indicate that the moving range $n = 2$ MR control chart, using the normal curve control chart constant $D_4 = 3.268$, does

not work well when the underlying process distribution is skewed . From Table 4.4, it is observed that for gamma distributions with shape parameter $\alpha \leq 10$ the empirical ARLs in control are substantially less (from 30% to 83%) than the 100 accepted in industry as a minimum for practical purposes and much less than the 107.14 given by the normal theory ($1 / 0.0093$).

Therefore, if the process has a skewed (asymmetric) distribution, the moving range $n = 2$ MR control chart, using the normal D_4 , must be very carefully used and interpreted.

In general, it is recommended not to use the MR ($n=2$) chart, under the assumption of normality, when it is suspected that the process distribution is markedly non-normal (represented in this research by skewed distributions). Instead, the control limits must be based on the percentiles of a probability distribution which better fits the moving range distribution. Another approach could be to transform the original data to a new variable that can be approximated by a normal distribution. In the latter case, the control charts under the assumption of normality can be used safely on the transformed data.

- The bias correction factors d_2 and d_3 and the control chart constant D_4 show an exponential behavior approaching the normal curve values as the shape parameter α increases when the underlying process distributions are represented by gamma distributions. This behavior can be observed from Figure 4.11 to Figure 4.13 for the three constants, respectively.

The apparent exponential behavior is corroborated by the regression model selected to fit the values of d_2 , d_3 , and D_4 (Appendix G) when the underlying process

distribution is gamma with shape parameter α and scale parameter β . The regression models, using the least-squares approach, which best fit the data for the three constants are combinations of exponential functions for the shape parameter α . The scale parameter β , as suspected, is found not to be significant in all the models generated.

- For four out of the five distributions selected to represent unknown process distributions (lognormal, exponential, gamma, and chi-square), the individual measurement X and moving range $n = 2$ MR control charts, set with the gamma control chart constants d_2 , d_3 , and D_4 , have better performance than the same control charts set with the normal curve constants d_2 , d_3 , and D_4 . Better performance means less false alarm rate in control, for the X and MR control charts, and power of detection at least equal to the theoretical power using the normal assumption for the X chart. This can be observed in Tables 4.6 and 4.8, 4.14 and 4.16, 4.18 and 4.20, and 4.22 and 4.24 for the four process distributions, respectively.

For the fifth distribution (normal), the individual measurement X and moving range $n = 2$ MR control charts, set with the gamma constants, have approximately the same performance as those control charts set with the normal constants. This can be seen in Table 4.10 for the normal curve. The similar performance is due to the fact that the gamma constants d_2 , d_3 , and D_4 approach the normal values when the gamma distribution approaches the normal curve (α increases).

In addition, for all cases (five distributions), the reductions in false alarm rates are bigger than the reductions in power of detection for shifts of 1 and 2 sigma when the gamma control limits are used instead of the normal limits. This can be observed in

Tables 4.7, 4.11, 4.15, 4.19, and 4.23.

Therefore, based on the empirical evidence, it is recommended that the gamma control chart constants d_2 , d_3 , and D_4 be used to set control limits for \bar{X} and MR ($n=2$) control charts when the underlying process distribution is suspected to be a skewed distribution (asymmetric distribution).

However, even though the performance of the gamma control charts is better than the performance of the normal curve control charts when the process distribution is skewed, the number of empirical false alarms (no shift) is still too big. These high false alarm rates are far away the theoretical normal curve false alarm rate (0.27%) and the practical false alarm rate used in industry (1.0%). It is the author's belief that these high false alarm rates, using the gamma control charts, are due to the following three factors:

1. The empirical nature of the study (# of observations).
2. The skewness of the process distribution which affects the number of data values above the three sigma upper control limit (tail area).
3. The use of symmetric control limits (\bar{X} chart) for asymmetric process distributions.

As a consequence, more research is needed in this area to improve the performance of the \bar{X} and MR ($n=2$) gamma control charts.

- The number of empirical false alarms, when $k = 10$ observations are used to set control limits, is always much greater (from 28% to 65% for the asymmetric distributions and from 150% to 200% for the normal distribution) than the number of false alarms when $k = 30$ and 50 data values are used to set control limits. In addition, it is observed from Tables 4.9, 4.13, 4.17, 4.21, and 4.25 that the differences in performance $k=10$

versus $k=30$ have more variation than the differences in performance $k=30$ versus $k=50$. It means that the performance of \bar{X} and MR ($n=2$) control charts are more consistent when $k=30$ and 50 data values are used to set control limits.

Therefore, based on the empirical evidence, it is not recommended to use $k = 10$ observations to set reliable control limits (\bar{X} and MR control charts) for future predictions. Instead, it seems safe to say that $k = 30$ measurements is a good sample size to set these control limits for future predictions.

5.2 Research contributions

- This thesis research supplies the empirical evidence required to avoid using the individual measurement \bar{X} and moving range $n = 2$ MR control charts when the underlying process distribution is suspected to be a skewed (asymmetric) distribution. This is on behalf of people in industry who use this pair of control charts (\bar{X} and MR), under the assumption of normality, indiscriminately without knowing the approximate shape of the underlying process distribution.
- This research provides empirical equations to calculate approximately the correct control chart constants d_2 , d_3 , and D_4 when the underlying process distribution is a gamma distribution with shape parameter α and scale parameter β .
- This thesis provides empirical evidence that supports the hypothesis that the gamma control charts (\bar{X} and MR) perform better than the normal curve control charts (\bar{X} and MR) when the process distribution has a marked departure from normality (represented in this research by skewed distributions). However, more research is needed in this area since the number of empirical false alarms in control for the gamma control

charts is still unacceptable.

In this regard, this research opens ways for future research providing a new methodology for setting control limits (\bar{X} and MR) under skewed (asymmetric) circumstances.

- This thesis research sets the empirical foundations that support the fact that $k = 10$ observations is not a good sample size to calculate reliable control limits for future predictions (\bar{X} and MR). This goes against some authors who believe that \bar{X} and MR ($n=2$) control charts always perform well no matter what number of observations are available. In addition, this research shows that there is not too much difference in performance between $k = 30$ and $k = 50$ data values used to set control limits. As a consequence, it seems to suggest that $k = 30$ observations is a good sample size to set those control limits for future predictions.

5.3 For future research

The fact that the gamma control charts (\bar{X} and MR) still signal a large number of false alarms when the process is in control suggests that more research is needed in this area. It is the author's belief that these higher false alarm rates, using the gamma control charts, are due to the following three factors:

1. The empirical nature of the study (# of observations).
2. The skewness of the process distribution which affects the number of data values above the three sigma upper control limit (tail area).
3. The use of symmetric control limits (\bar{X} chart) for asymmetric process distributions.

In Figure 5.1, the effects of the skewness and symmetrical control limits can be observed.

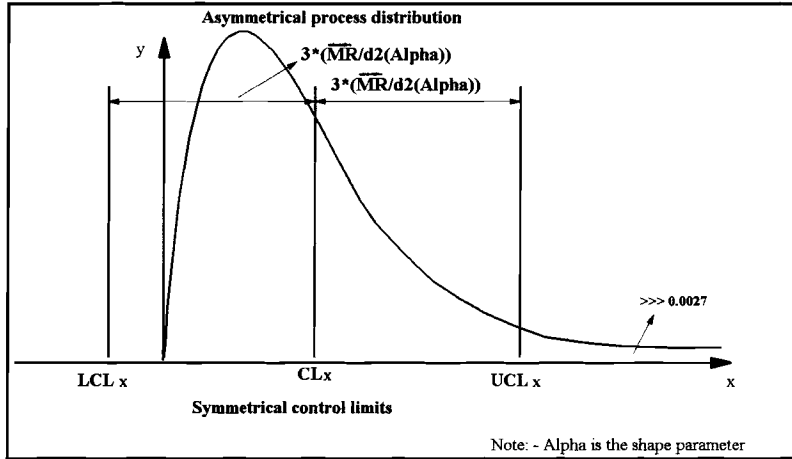


Figure 5.1. Effects of skewness and symmetrical control limits (X chart).

In order to get rid off these three factors, additional research is recommended in the field of setting asymmetric control limits (X and MR) based on the number of observations available, k , and the shape parameter, α . Following the methodology developed in this research, gamma control limits for the individual measurement X control chart are set at:

$$UCL_X = \bar{X} + J_u (\overline{MR} / d_2 (\alpha)) = \bar{X} + E_{2u} (k, \alpha) \overline{MR}$$

$$LCL_X = \bar{X} - J_l (\overline{MR} / d_2 (\alpha)) = \bar{X} - E_{2l} (k, \alpha) \overline{MR}$$

$$E_{2u} (k, \alpha) = J_u (k, \alpha) / d_2 (\alpha)$$

$$E_{2l} (k, \alpha) = J_l (k, \alpha) / d_2 (\alpha)$$

where:

- $d_2 (\alpha)$ is the gamma bias correction factor evaluated in this research.

- $E_{2u} (k, \alpha)$ is the gamma control chart constant used to set the individual measurement upper control limit. It is a function of the number of observations available k and the shape parameter α .

- $E_{2l}(k, \alpha)$ is the gamma control chart constant used to set the individual measurement lower control limit. It is a function of the number of observations available k and the shape parameter α .

- $J_u(k, \alpha)$ is a value which yields an upper tail area similar to the normal theory in control (0.00135). This value depends upon the sample size k used to set control limits and the shape parameter α .

- $J_l(k, \alpha)$ is a value which yields a lower tail area similar to the normal theory in control (0.00135). This value depends upon the sample size k used to set control limits and the shape parameter α .

The proposed idea is seen in Figure 5.2.

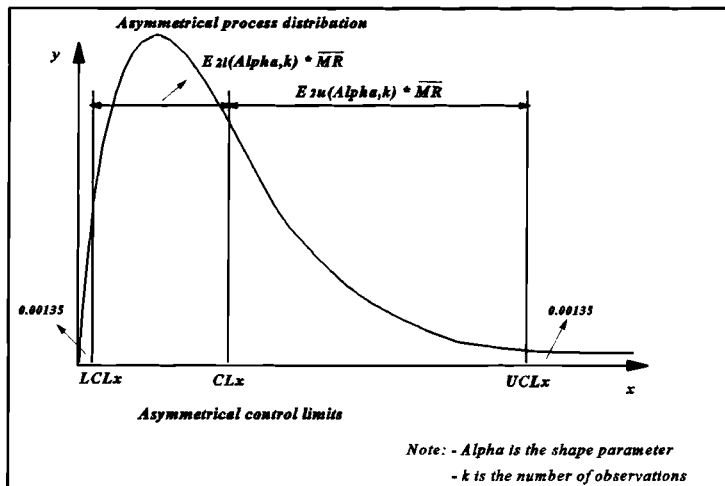


Figure 5.2. Asymmetric gamma control limits (X chart) for asymmetric process distributions.

For the MR ($n=2$) control charts, the same logic is used.

$$UCL_{MR} = \overline{MR} + L_u * \sigma_{MR} = \overline{MR} + L_u(k, \alpha) * (d_3(\alpha)/d_2(\alpha)) * \overline{MR}$$

$$UCL_{MR} = (1 + L_u(k, \alpha) * (d_3(\alpha)/d_2(\alpha))) * \overline{MR}$$

where:

- $d_2(\alpha)$ is the gamma bias correction factor evaluated in this research.
- $d_3(\alpha)$ is the gamma bias correction factor evaluated in this research.
- $L_u(k, \alpha)$ is a value which yields an upper tail area similar to the normal theory in control for the range distribution (0.0093). This value depends upon the sample size used to set control limits k and the shape parameter α .

The proposed idea for the moving range $n = 2$ MR control chart is seen in Figure 5.3.

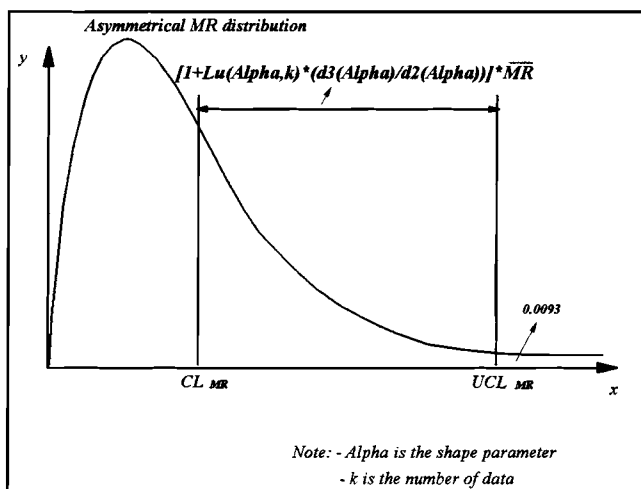


Figure 5.3. Gamma upper control limit (MR chart) for asymmetric distributions.

Future research can also be done in the following areas:

- Evaluate the performance of the gamma control charts (X and MR) under shifts in the process standard deviation (variability).
- Use a more generic family of distributions (for instance, the Burr family of distributions) to fit any set of data available to set control limits. Then, using the same methodology developed in this research, appropriate control chart constants and asymmetric control limits can be calculated.

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APPENDICES

APPENDIX A

**VALUES OF THE BIAS CORRECTION FACTORS d_2 AND d_3 AND CONTROL
CHART CONSTANTS A_2 , D_3 , D_4 , AND E_2 - NORMAL ASSUMPTION**

ANSI/ASQC Standard A1-1978

IV. FACTORS AND FORMULAS FOR CONTROL CHARTS FOR VARIABLES

Table 1
Factors for Computing Central Lines and 3-sigma Control Limits for \bar{X} , s, and R Charts

Observations in Sample, n	Chart for Averages			Chart for Standard Deviations						Chart for Ranges						
	Factors for Control Limits			Factors for Central Line		Factors for Control Limits				Factors for Central Line		Factors for Control Limits				
	A	A ₂	A ₃	c ₄	1/c ₄	B ₁	B ₂	B ₃	B ₄	d ₂	1/d ₂	d ₃	D ₁	D ₂	D ₃	D ₄
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.155	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

APPENDIX B

CHI-SQUARE TEST - RANDOM NUMBER GENERATOR IN MINITAB

This particular case of the chi-square χ^2 test is designed to check whether the U_i 's generated appear to be uniformly distributed between 0 and 1. The idea is to evaluate how well the generated U_i 's resemble values of true IID $U(0, 1)$ random variates.

Test procedure:

1. Divide the interval $[0, 1]$ into k subintervals of equal length and generate U_1, U_2, \dots, U_n .

Rules of thumb:

- k should be at least 100.
- n / k should be at least 5.

2. Calculate the statistic χ^2 using the equation:

$$\chi^2 = (n/k) \sum [f_j - (n/k)]^2$$

Where

f_j is the number of U_i 's that are in the j^{th} subinterval.

3. The null hypothesis is:

H_0 : U_i 's are IID $U(0, 1)$ random variables

4. The rejection region is defined by:

$$\chi^2_{k-1, 1-\alpha'}$$

Where

α' is the confidence level.

$k-1$ is the degrees of freedom.

5. Reject the null hypothesis if the χ^2 statistic is greater than $\chi^2_{k-1, 1-\alpha'}$.

Results of the test:

The test is performed three times with different samples generated and different values of k and n .

First test: $n = 5000$ observations.

$k = 100$ subintervals.

$\alpha' = 0.01$

1. $n/k = 5000/100 = 50$ which is greater than the value 5 recommended in the rules of thumb.

2. The value of the χ^2 statistic is:

$$\chi^2 = 118.88$$

3. The null hypothesis is:

Ho: U_i 's are IID $U(0, 1)$ random variables

4. The rejection region is defined by:

$$\chi^2_{99, 0.99} = 134.6392$$

5. There is not reason to reject the null hypothesis since $\chi^2 < \chi^2_{99, 0.99}$.

Second test: $n = 1000$ observations.

$k = 100$ subintervals.

$\alpha' = 0.01$

1. $n/k = 1000/100 = 10$ which is greater than the value 5 recommended in the rules of thumb.

2. The value of the χ^2 statistic is:

$$\chi^2 = 92.0$$

3. The null hypothesis is:

Ho: U_i 's are IID $U(0, 1)$ random variables

4. The rejection region is defined by:

$$\chi^2_{99, 0.99} = 134.6392$$

5. There is not reason to reject the null hypothesis since $\chi^2 < \chi^2_{99, 0.99}$.

Third test: $n = 5000$ observations.

$k = 200$ subintervals.

$$\alpha' = 0.01$$

1. $n/k = 5000/200 = 25$ which is greater than the value 5 recommended in the rules of thumb.

2. The value of the χ^2 statistic is:

$$\chi^2 = 168.16$$

3. The null hypothesis is:

Ho: U_i 's are IID $U(0, 1)$ random variables

4. The rejection region is defined by:

$$\chi^2_{199, 0.99} = 248.3332$$

5. There is not reason to reject the null hypothesis since $\chi^2 < \chi^2_{199, 0.99}$.

Therefore, based on the results of the three tests, it could be said that this random number generator does not behave in a way that is significantly different from that would be expected from truly IID $U(0, 1)$ random variables.

n=5000 k=100	k	Interval	f _j	n/k	$[(f_j - n/k)]^2$
	1	0.01	53	50	9
	2	0.02	59	50	81
	3	0.03	46	50	18
	4	0.04	33	50	289
	5	0.05	53	50	9
	6	0.06	46	50	16
	7	0.07	56	50	36
	8	0.08	47	50	9
	9	0.09	54	50	16
	10	0.1	51	50	1
	11	0.11	50	50	0
	12	0.12	89	50	361
	13	0.13	49	50	1
	14	0.14	50	50	0
	15	0.15	59	50	81
	16	0.16	58	50	64
	17	0.17	51	50	1
	18	0.18	56	50	36
	19	0.19	50	50	0
	20	0.2	42	50	64
	21	0.21	90	50	100
	22	0.22	44	50	36
	23	0.23	50	50	0
	24	0.24	59	50	81
	25	0.25	58	50	64
	26	0.26	42	50	64
	27	0.27	48	50	4
	28	0.28	53	50	9
	29	0.29	33	50	289
	30	0.3	57	50	49
	31	0.31	93	50	169
	32	0.32	46	50	16
	33	0.33	45	50	25
	34	0.34	59	50	81
	35	0.35	48	50	4
	36	0.36	45	50	25
	37	0.37	80	50	100
	38	0.38	63	50	169
	39	0.39	43	50	49
	40	0.4	53	50	9
	41	0.41	33	50	289
	42	0.42	56	50	36
	43	0.43	43	50	49
	44	0.44	49	50	4
	45	0.45	50	50	0
	46	0.46	43	50	49
	47	0.47	46	50	25
	48	0.48	56	50	25
	49	0.49	46	50	16
	50	0.5	53	50	9
	51	0.51	52	50	4
	52	0.52	43	50	49
	53	0.53	51	50	1
	54	0.54	54	50	16
	55	0.55	54	50	16
	56	0.56	86	50	256
	57	0.57	42	50	64
	58	0.58	51	50	1
	59	0.59	50	50	0
	60	0.6	46	50	16
	61	0.61	39	50	121
	62	0.62	49	50	1
	63	0.63	51	50	1
	64	0.64	42	50	64
	65	0.65	49	50	1
	66	0.66	57	50	49
	67	0.67	63	50	169
	68	0.68	56	50	36
	69	0.69	59	50	81
	70	0.7	49	50	1
	71	0.71	43	50	49
	72	0.72	53	50	9
	73	0.73	48	50	4
	74	0.74	39	50	121
	75	0.75	51	50	1
	76	0.76	57	50	49
	77	0.77	57	50	49
	78	0.78	39	50	121
	79	0.79	44	50	36
	80	0.8	80	50	100
	81	0.81	59	50	81
	82	0.82	44	50	36
	83	0.83	54	50	16
	84	0.84	51	50	1
	85	0.85	52	50	4
	86	0.86	44	50	36
	87	0.87	45	50	25
	88	0.88	44	50	36
	89	0.89	86	50	256
	90	0.9	39	50	121
	91	0.91	40	50	100
	92	0.92	56	50	25
	93	0.93	44	50	36
	94	0.94	60	50	100
	95	0.95	38	50	196
	96	0.96	38	50	144
	97	0.97	58	50	64
	98	0.98	39	50	121
	99	0.99	46	50	25
	100	1	40	50	100

Total: 5000 5000 5944

Chi-square value = (kn) Sum $[(f_j - n/k)]^2$
 Chi-square value = 118.88

n=1000 k=100	k	Interval	f _j	n/k	$[(f_j - n/k)]^2$
	1	0.01	14	10	16
	2	0.02	9	10	1
	3	0.03	11	10	1
	4	0.04	9	10	1
	5	0.05	9	10	1
	6	0.06	9	10	1
	7	0.07	8	10	4
	8	0.08	14	10	16
	9	0.09	10	10	0
	10	0.1	10	10	0
	11	0.11	10	10	0
	12	0.12	9	10	4
	13	0.13	4	10	36
	14	0.14	11	10	1
	15	0.15	11	10	1
	16	0.16	12	10	4
	17	0.17	10	10	0
	18	0.18	10	10	0
	19	0.19	9	10	1
	20	0.2	12	10	4
	21	0.21	7	10	9
	22	0.22	12	10	4
	23	0.23	14	10	16
	24	0.24	9	10	1
	25	0.25	10	10	0
	26	0.26	8	10	4
	27	0.27	8	10	4
	28	0.28	14	10	16
	29	0.29	10	10	0
	30	0.3	17	10	49
	31	0.31	4	10	36
	32	0.32	12	10	4
	33	0.33	10	10	0
	34	0.34	8	10	4
	35	0.35	9	10	1
	36	0.36	9	10	1
	37	0.37	10	10	0
	38	0.38	11	10	1
	39	0.39	10	10	0
	40	0.4	19	10	81
	41	0.41	9	10	1
	42	0.42	7	10	9
	43	0.43	7	10	9
	44	0.44	10	10	0
	45	0.45	18	10	64
	46	0.46	3	10	49
	47	0.47	12	10	4
	48	0.48	7	10	9
	49	0.49	5	10	25
	50	0.5	9	10	1
	51	0.51	13	10	9
	52	0.52	9	10	1
	53	0.53	11	10	1
	54	0.54	11	10	1
	55	0.55	9	10	4
	56	0.56	19	10	81
	57	0.57	11	10	1
	58	0.58	12	10	4
	59	0.59	8	10	4
	60	0.6	7	10	9
	61	0.61	9	10	1
	62	0.62	10	10	0
	63	0.63	9	10	1
	64	0.64	6	10	16
	65	0.65	8	10	4
	66	0.66	15	10	25
	67	0.67	12	10	4
	68	0.68	14	10	16
	69	0.69	14	10	16
	70	0.7	9	10	1
	71	0.71	6	10	16
	72	0.72	9	10	1
	73	0.73	12	10	4
	74	0.74	10	10	0
	75	0.75	10	10	0
	76	0.76	15	10	25
	77	0.77	12	10	4
	78	0.78	9	10	1
	79	0.79	13	10	9
	80	0.8	10	10	0
	81	0.81	9	10	1
	82	0.82	11	10	1
	83	0.83	6	10	16
	84	0.84	6	10	16
	85	0.85	17	10	49
	86	0.86	13	10	9
	87	0.87	9	10	1
	88	0.88	6	10	16
	89	0.89	10	10	0
	90	0.9	9	10	1
	91	0.91	9	10	1
	92	0.92	9	10	1
	93	0.93	4	10	36
	94	0.94	8	10	4
	95	0.95	11	10	1
	96	0.96	8	10	4
	97	0.97	9	10	1
	98	0.98	10	10	0
	99	0.99	7	10	9
	100	1	10	10	0

Total: 1000 1000 920

Chi-square value = (kn) Sum $[(f_j - n/k)]^2$
 Chi-square value = 92

n=5000 k=200									
k	Interval	fj	nk	$[f_j - (n/k)]^2$	k	Interval	fj	nk	$[f_j - (n/k)]^2$
1	0.005	34	25	81	101	0.505	24	25	1
2	0.01	30	25	25	102	0.51	19	25	36
3	0.015	30	25	25	103	0.515	33	25	64
4	0.02	24	25	1	104	0.52	21	25	16
5	0.025	22	25	9	105	0.525	27	25	4
6	0.03	22	25	9	106	0.53	21	25	16
7	0.035	18	25	49	107	0.535	30	25	25
8	0.04	24	25	1	108	0.54	25	25	0
9	0.045	27	25	4	109	0.545	26	25	1
10	0.05	21	25	16	110	0.55	26	25	0
11	0.055	36	25	121	111	0.555	33	25	64
12	0.06	33	25	64	112	0.56	22	25	9
13	0.065	29	25	16	113	0.565	26	25	1
14	0.07	24	25	1	114	0.57	28	25	9
15	0.075	19	25	36	115	0.575	24	25	1
16	0.08	29	25	16	116	0.58	21	25	16
17	0.085	21	25	16	117	0.585	26	25	1
18	0.09	21	25	16	118	0.59	23	25	4
19	0.095	24	25	1	119	0.595	28	25	9
20	0.1	28	25	9	120	0.6	19	25	36
21	0.105	30	25	25	121	0.605	25	25	0
22	0.11	29	25	16	122	0.61	24	25	1
23	0.115	30	25	25	123	0.615	28	25	9
24	0.12	18	25	49	124	0.62	21	25	16
25	0.125	27	25	4	125	0.625	24	25	1
26	0.13	35	25	100	126	0.63	25	25	0
27	0.135	29	25	16	127	0.635	22	25	9
28	0.14	24	25	1	128	0.64	29	25	16
29	0.145	33	25	64	129	0.645	18	25	49
30	0.15	22	25	9	130	0.65	27	25	4
31	0.155	24	25	1	131	0.655	30	25	25
32	0.16	23	25	4	132	0.66	24	25	1
33	0.165	20	25	25	133	0.665	18	25	49
34	0.17	21	25	16	134	0.67	22	25	9
35	0.175	27	25	4	135	0.675	28	25	9
36	0.18	28	25	9	136	0.68	34	25	81
37	0.185	26	25	1	137	0.685	29	25	16
38	0.19	25	25	0	138	0.69	21	25	16
39	0.195	16	25	81	139	0.695	23	25	4
40	0.2	29	25	16	140	0.7	15	25	100
41	0.205	32	25	49	141	0.705	21	25	16
42	0.21	28	25	9	142	0.71	22	25	9
43	0.215	24	25	1	143	0.715	15	25	100
44	0.22	28	25	9	144	0.72	28	25	9
45	0.225	21	25	16	145	0.725	33	25	64
46	0.23	17	25	64	146	0.73	26	25	1
47	0.235	28	25	9	147	0.735	20	25	25
48	0.24	24	25	1	148	0.74	27	25	4
49	0.245	15	25	100	149	0.745	24	25	1
50	0.25	17	25	64	150	0.75	18	25	49
51	0.255	33	25	64	151	0.755	19	25	36
52	0.26	28	25	9	152	0.76	27	25	4
53	0.265	20	25	25	153	0.765	25	25	0
54	0.27	32	25	49	154	0.77	19	25	36
55	0.275	23	25	4	155	0.775	25	25	0
56	0.28	27	25	4	156	0.78	25	25	0
57	0.285	26	25	1	157	0.785	28	25	9
58	0.29	30	25	25	158	0.79	23	25	4
59	0.295	27	25	4	159	0.795	25	25	0
60	0.3	25	25	0	160	0.8	26	25	1
61	0.305	16	25	81	161	0.805	26	25	1
62	0.31	30	25	25	162	0.81	22	25	9
63	0.315	30	25	25	163	0.815	25	25	0
64	0.32	28	25	9	164	0.82	17	25	64
65	0.325	31	25	36	165	0.825	22	25	9
66	0.33	21	25	16	166	0.83	32	25	49
67	0.335	20	25	25	167	0.835	24	25	1
68	0.34	27	25	4	168	0.84	29	25	16
69	0.345	21	25	16	169	0.845	30	25	25
70	0.35	28	25	9	170	0.85	21	25	16
71	0.355	23	25	4	171	0.855	29	25	16
72	0.36	21	25	16	172	0.86	18	25	81
73	0.365	30	25	25	173	0.865	27	25	4
74	0.37	25	25	0	174	0.87	27	25	4
75	0.375	34	25	81	175	0.875	15	25	100
76	0.38	33	25	64	176	0.88	24	25	1
77	0.385	23	25	4	177	0.885	26	25	1
78	0.39	23	25	4	178	0.89	25	25	0
79	0.395	27	25	4	179	0.895	25	25	0
80	0.4	27	25	4	180	0.9	21	25	16
81	0.405	26	25	1	181	0.905	24	25	1
82	0.41	17	25	64	182	0.91	23	25	4
83	0.415	20	25	25	183	0.915	29	25	16
84	0.42	26	25	1	184	0.92	17	25	64
85	0.425	29	25	16	185	0.925	26	25	1
86	0.43	28	25	16	186	0.93	18	25	49
87	0.435	27	25	4	187	0.935	25	25	0
88	0.44	32	25	49	188	0.94	29	25	16
89	0.445	18	25	49	189	0.945	25	25	0
90	0.45	33	25	64	190	0.95	29	25	16
91	0.455	20	25	25	191	0.955	31	25	36
92	0.46	24	25	1	192	0.96	16	25	81
93	0.465	28	25	9	193	0.965	25	25	0
94	0.47	27	25	4	194	0.97	26	25	1
95	0.475	31	25	36	195	0.975	25	25	0
96	0.48	20	25	25	196	0.98	30	25	25
97	0.485	21	25	16	197	0.985	23	25	4
98	0.49	24	25	1	198	0.99	25	25	0
99	0.495	22	25	9	199	0.995	30	25	25
100	0.5	27	25	4	200	1	23	25	4

Total: 5000 5000 4204

Chi-square value = (kh) Sum $[f_j - (n/k)]^2$
 Chi-square value = 168.16

APPENDIX C

GOODNESS OF FIT TESTS - GENERATION GAMMA AND NORMAL RANDOM VARIATES

The generation of gamma and normal random variables is tested statistically to check whether the values generated, in fact, agree with the underlying probabilistic model assumed for these data. The test used is the χ^2 goodness-of-fit test.

Only the generation of gamma and normal random variates is tested statistically because the other distributions generated in this research are either particular cases or derivations from these two distributions. The exponential and chi-square distributions are particular cases of the gamma distribution. The lognormal distribution is derived from the normal distribution.

For the gamma distribution, four tests are performed using different parameters and number of observations. For the normal distribution, two tests are run using different number of observations.

1. Gamma distribution

1.1 First test

Gamma distribution with shape parameter $\alpha = 1$ and scale parameter $\beta = 1$. This is equivalent to an exponential distribution with mean 1. The number of observations used to perform the test is 1000.

The null hypothesis H_0 to be tested is:

H_0 : The data generated can be regarded as a gamma (1, 1) distribution.

From the spreadsheet designed in Quattro Pro, the χ^2 chi-square statistic is:

$$\chi^2 = 7.20$$

Using a significance level of $\alpha' = 0.01$, the rejection region is defined by the following critical value:

$$\chi^2_{\alpha', k-1} = \chi^2_{0.99, 12} = 26.217$$

Since the χ^2 chi-square statistic is not greater than the critical value $\chi^2_{0.99, 12}$, there is not statistical reasons to reject the null hypothesis that the 1000 data values generated come from a gamma (1, 1) distribution.

1.2 Second test

Gamma distribution with shape parameter $\alpha = 1$ and scale parameter $\beta = 1$. This is equivalent to an exponential distribution with mean 1. The number of observations used to perform the test is 100.

The null hypothesis H_0 to be tested is:

H_0 : The data generated can be regarded as a gamma (1, 1) distribution.

From the spreadsheet designed in Quattro Pro, the χ^2 chi-square statistic is:

$$\chi^2 = 2.396$$

Using a significance level of $\alpha' = 0.01$, the rejection region is defined by the following critical value:

$$\chi^2_{\alpha', k-1} = \chi^2_{0.99, 6} = 16.8119$$

Since the χ^2 chi-square statistic is not greater than the critical value $\chi^2_{0.99, 6}$, there is not statistical reasons to reject the null hypothesis that the 100 data values generated come from a gamma (1, 1) distribution.

1.3 Third test

Gamma distribution with shape parameter $\alpha = 2$ and scale parameter $\beta = 1$. The number of observations used to perform the test is 1000.

The null hypothesis H_0 to be tested is:

Ho: The data generated can be regarded as a gamma (2, 1) distribution.

From the spreadsheet designed in Quattro Pro, the χ^2 chi-square statistic is:

$$\chi^2 = 13.315$$

Using a significance level of $\alpha' = 0.01$, the rejection region is defined by the following critical value:

$$\chi^2_{\alpha', k-1} = \chi^2_{0.99, 14} = 29.1413$$

Since the χ^2 chi-square statistic is not greater than the critical value $\chi^2_{0.99, 14}$, there is not statistical reasons to reject the null hypothesis that the 1000 data values generated come from a gamma (2, 1) distribution.

1.4 Fourth test

Gamma distribution with shape parameter $\alpha = 2$ and scale parameter $\beta = 1$. The number of observations used to perform the test is 100.

The null hypothesis Ho to be tested is:

Ho: The data generated can be regarded as a gamma (2, 1) distribution.

From the spreadsheet designed in Quattro Pro, the χ^2 chi-square statistic is:

$$\chi^2 = 5.4973$$

Using a significance level of $\alpha' = 0.01$, the rejection region is defined by the following critical value:

$$\chi^2_{\alpha', k-1} = \chi^2_{0.99, 8} = 20.0902$$

Since the χ^2 chi-square statistic is not greater than the critical value $\chi^2_{0.99, 8}$, there is not statistical reasons to reject the null hypothesis that the 100 data values generated come from a gamma (2, 1) distribution.

2. Normal distribution

2.1 First test

Normal distribution with mean $\mu = 1$ and standard deviation $\sigma = 10$. The number of observations used to perform the test is 1000.

The null hypothesis H_0 to be tested is:

H_0 : The data generated can be regarded as a normal $(40, 10^2)$ distribution.

From the spreadsheet designed in Quattro Pro, the χ^2 chi-square statistic is:

$$\chi^2 = 17.80$$

Using a significance level of $\alpha' = 0.01$, the rejection region is defined by the following critical value:

$$\chi^2_{\alpha', k-1} = \chi^2_{0.99, 15} = 30.5779$$

Since the χ^2 chi-square statistic is not greater than the critical value $\chi^2_{0.99, 15}$, there is not statistical reasons to reject the null hypothesis that the 1000 data values generated come from a normal $(40, 10^2)$ distribution.

2.2 Second test

Normal distribution with mean $\mu = 1$ and standard deviation $\sigma = 10$. The number of observations used to perform the test is 100.

The null hypothesis H_0 to be tested is:

H_0 : The data generated can be regarded as a normal $(40, 10^2)$ distribution.

From the spreadsheet designed in Quattro Pro, the χ^2 chi-square statistic is:

$$\chi^2 = 2.624$$

Using a significance level of $\alpha' = 0.01$, the rejection region is defined by the following critical value:

$$\chi^2_{\alpha', k-1} = \chi^2_{0.99, 8} = 20.0902$$

Since the χ^2 chi-square statistic is not greater than the critical value $\chi^2_{0.99, 8}$, there is not statistical reasons to reject the null hypothesis that the 100 data values generated come from a normal $(40, 10^2)$ distribution.

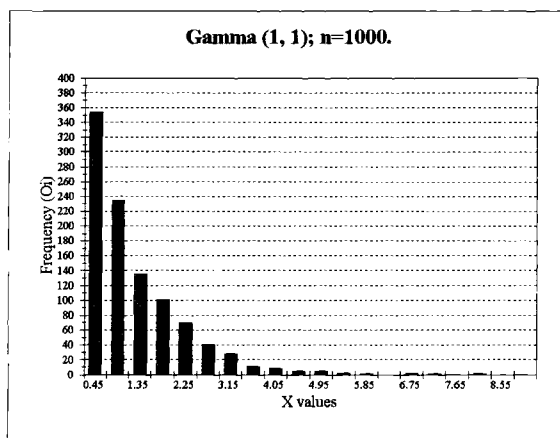
Appendix C - Goodness of fit tests

Gamma (1, 1); n=1000; k=20.

k	Interval	Collapsed		Collapsed		Expected		$(O_i - E_i)^2 / E_i$
		Interval	Frequency	Frequency	Cumulative Probability	Probability	Frequency	
1	0.45		354	354	0.362372	0.362372	362.372	0.19342108
2	0.9		235	235	0.59343	0.231058	231.058	0.06725309
3	1.35		135	135	0.74076	0.14733	147.33	1.03189371
4	1.8		101	101	0.834701	0.093941	93.941	0.53043379
5	2.25		70	70	0.894601	0.0599	59.9	1.70300501
6	2.7		41	41	0.932795	0.038194	38.194	0.20614851
7	3.15		28	28	0.957148	0.024353	24.353	0.54615895
8	3.6		11	11	0.972676	0.015528	15.528	1.32037506
9	4.05		8	8	0.982578	0.009902	9.902	0.36534074
10	4.5		5	5	0.988891	0.006313	6.313	0.27308237
11	4.95		5	5	0.992917	0.004026	4.026	0.23563736
12	5.4	*	2	5	0.995483	0.002566	5.912	0.14068742
13	5.85	*	1		0.99712	0.001637		
14	6.3	*	0		0.998164	0.001044		
15	6.75	*	2		0.998829	0.000665		
16	7.2	-	1	2	0.999253	0.000424	1.171	0.58688386
17	7.65	-	0		0.999524	0.000271		
18	8.1	-	1		0.999696	0.000172		
19	8.55	-	0		0.999806	0.00011		
20	9	-	0		0.999877	7.1E-05		
			=====	=====	=====		=====	=====
			1000	1000	0.999877		1000	7.20032094

Chi-square statistic = Sum [$(O_i - E_i)^2 / E_i$]

Chi-square statistic 7.200321



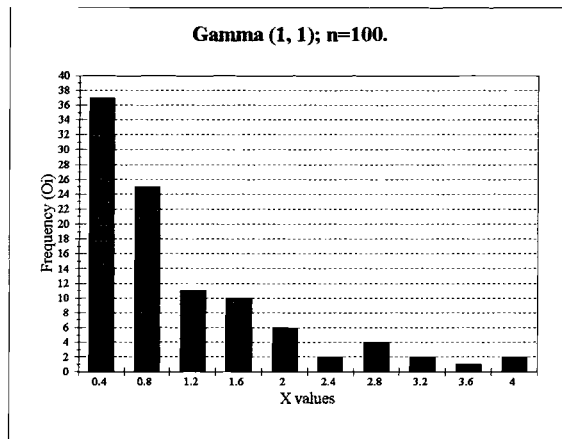
Appendix C - Goodness of fit tests

Gamma (1, 1); n=100; k=10.

k	Interval	Collapsed		Collapsed		Cumulative		Expected	
		Interval	Frequency	Frequency	O _i	Probability	Probability	E _i	(O _i -E _i) ² / E _i
1	0.4		37	37	37	0.32968	0.32968	32.968	0.49311526
2	0.8		25	25	25	0.550671	0.220991	22.0991	0.38079473
3	1.2		11	11	11	0.698806	0.148135	14.8135	0.98172493
4	1.6		10	10	10	0.798103	0.099297	9.9297	0.00049771
5	2		6	6	6	0.864665	0.066562	6.6562	0.06469133
6	2.4	*	2	2	6	0.909282	0.044617	7.4525	0.28309376
7	2.8	*	4	4	5	0.93919	0.029908		
8	3.2	-	2	2	5	0.959238	0.020048	6.081	0.19216593
9	3.6	-	1	1		0.972676	0.013438		
10	4	-	2	2		0.981684	0.009008		
			100	100		0.981684		100	2.39608365

Chi-square statistic = Sum [(O_i - E_i)² / E_i]

Chi-square statistic 2.396084



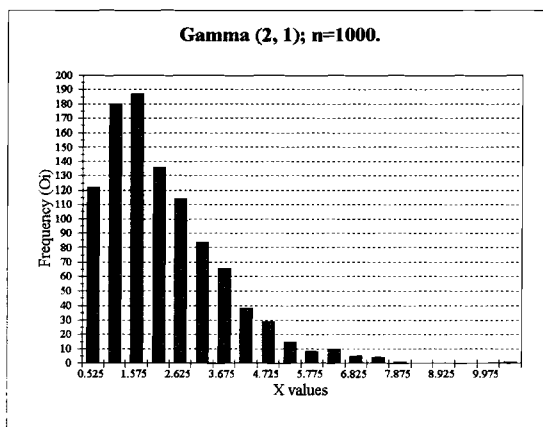
Appendix C - Goodness of fit tests

Gamma (2, 1) ; n=1000 ; k=20.

k	Interval	Colapsed Interval	Frequency	Colapsed Frequency Oi	Cumulative Probability	Probability	Expected Frequency Ei	(Oi-Ei)^2 / Ei
1	0.525		122	122	0.097878	0.097878	97.878	5.94485874
2	1.05		180	180	0.282628	0.18475	184.75	0.12212449
3	1.575		187	187	0.466956	0.184328	184.328	0.03873304
4	2.1		136	136	0.620385	0.153429	153.429	1.97987369
5	2.625		114	114	0.737406	0.117021	117.021	0.07798977
6	3.15		84	84	0.822164	0.084758	84.758	0.00677888
7	3.675		66	66	0.881492	0.059328	59.328	0.7503301
8	4.2		38	38	0.922023	0.040531	40.531	0.1580509
9	4.725		29	29	0.949215	0.027192	27.192	0.12021418
10	5.25		15	15	0.967203	0.017988	17.988	0.49633889
11	5.775		8	8	0.978969	0.011766	11.766	1.20540167
12	6.3		10	10	0.986595	0.007626	7.626	0.73903436
13	6.825		5	5	0.9915	0.004905	4.905	0.00183996
14	7.35	*	4		0.994634	0.003134		
15	7.875	*	1	5	0.996626	0.001992	5.126	0.00309715
16	8.4	-	0		0.997886	0.00126		
17	8.925	-	0		0.99868	0.000794		
18	9.45	-	0		0.999178	0.000498		
19	9.975	-	0		0.999489	0.000311		
20	10.5	-	1	1	0.999683	0.000194	3.374	1.67038411
			1000	1000		0.999683	1000	13.3150499

Chi-square statistic = $\sum [(O_i - E_i)^2 / E_i]$

Chi-square statistic = 13.31505



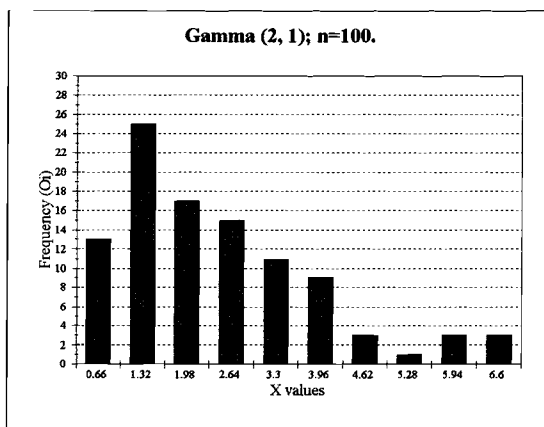
Appendix C - Goodness of fit tests

Gamma (2, 1); n=100 ; k=10.

k	Interval	Colapsed Interval	Frequency	Colapsed Frequency	O _i	Cumulative Probability	Probability	Expected Frequency	E _i	(O _i -E _i) ² / E _i
1	0.66		13		13	0.142027	0.142027	14.2027	14.2027	0.10184594
2	1.32		25		25	0.380246	0.238219	23.8219	23.8219	0.05826234
3	1.98		17		17	0.588554	0.208308	20.8308	20.8308	0.70448704
4	2.64		15		15	0.740245	0.151691	15.1691	15.1691	0.00188507
5	3.3		11		11	0.841402	0.101157	10.1157	10.1157	0.07730424
6	3.96		9		9	0.905447	0.064045	6.4045	6.4045	1.05185733
7	4.62	*	3			0.944627	0.03918			
8	5.28	*	1		4	0.968019	0.023392	6.2572	6.2572	0.81425427
9	5.94		3		3	0.981734	0.013715	1.3715	1.3715	1.93365822
10	6.6		3		3	0.989661	0.007927	1.8266	1.8266	0.75378712
			100	100	100	0.989661	0.007927	100	100	5.49734157

Chi-square statistic = Sum [(O_i - E_i)² / E_i]

Chi-square statistic = 5.497342



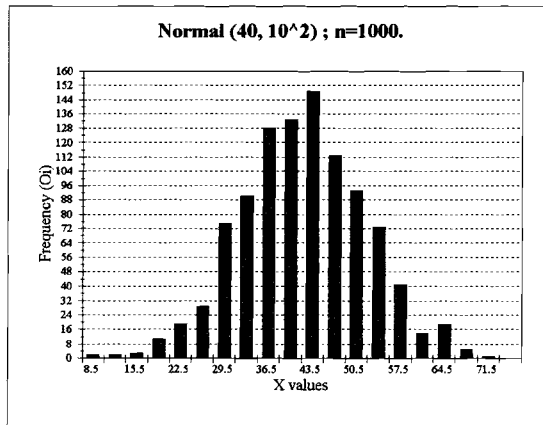
Appendix C - Goodness of fit tests

Normal (40, 10²) ; n=1000 ; k=20.

k	Interval	Collapsed Interval	Frequency	Collapsed Frequency O _i	Cumulative Probability	Probability	Expected Frequency E _i	(O _i -E _i) ² / E _i
1	8.5	*	2		0.000816	0.000816		
2	12	*	2		0.002555	0.001739		
3	15.5	*	3	7	0.007143	0.004588	7.143	0.0028628
4	19		11	11	0.017864	0.010721	10.721	0.00726061
5	22.5		19	19	0.040059	0.022195	22.195	0.45992453
6	26		29	29	0.080757	0.040698	40.698	3.36240611
7	29.5		75	75	0.146859	0.066102	66.102	1.1977611
8	33		90	90	0.241964	0.095105	95.105	0.27402371
9	36.5		128	128	0.363169	0.121205	121.205	0.38094159
10	40		133	133	0.5	0.136831	136.831	0.1072605
11	43.5		149	149	0.636831	0.136831	136.831	1.08224424
12	47		113	113	0.758036	0.121205	121.205	0.55543934
13	50.5		93	93	0.853141	0.095105	95.105	0.04659087
14	54		73	73	0.919243	0.066102	66.102	0.71983305
15	57.5		41	41	0.959941	0.040698	40.698	0.00224099
16	61		14	14	0.982136	0.022195	22.195	3.02581775
17	64.5		19	19	0.992857	0.010721	10.721	6.39323207
18	68	-	5		0.997445	0.004588		
19	71.5	-	1		0.999184	0.001739		
20	75	-	0	6	0.999767	0.000583	7.143	0.1828992
			1000	1000		0.999767	1000	17.8007385

Chi-square statistic = Sum [(O_i - E_i)² / E_i]

Chi-square statistic = 17.80074



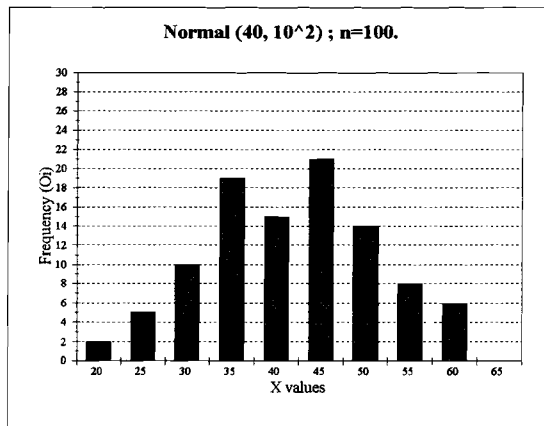
Appendix C - Goodness of fit tests

Normal (40, 10²) ; n=100 ; k=10.

k	Interval	Colapsed		Colapsed			Expected	
		Interval	Frequency	Frequency	Cumulative	Probability	Probability	Frequency
1	20		2	2	0.02275	0.02275	2.275	0.03324176
2	25		5	5	0.066807	0.044057	4.4057	0.08016717
3	30		10	10	0.158655	0.091848	9.1848	0.07235335
4	35		19	19	0.308538	0.149883	14.9883	1.07375332
5	40		15	15	0.5	0.191462	19.1462	0.89787918
6	45		21	21	0.691462	0.191462	19.1462	0.1794912
7	50		14	14	0.841345	0.149883	14.9883	0.06516662
8	55		8	8	0.933193	0.091848	9.1848	0.15283414
9	60	*	6	6	0.97725	0.044057	6.6807	0.06935688
10	65	*	0	6	0.99379	0.01654	6.6807	0.06935688
			100	100		0.99379	100	2.62424362

Chi-square statistic = Sum [(O_i - E_i)² / E_i]

Chi-square statistic = 2.624244



APPENDIX D

FORTY GAMMA DISTRIBUTION SPREADSHEETS

0.5 Alpha=0.5	0.5 Beta=0.5	MR	(MR-MR)*2	MR	(MR-MR)*2
0.07106	0.3058	3.3E-05	0.0118	0.03981	0.075675
0.37656	0.04939	0.03478	0.33445	0.000382	0.07638
0.12434	0.25252	0.003891	0.38478	0.33445	0.000382
0.02839	0.09595	0.047939	0.06282	0.31296	3E-05
0.10607	0.07968	0.055238	0.0159	0.04692	0.071813
0.06252	0.05555	0.067262	0.01426	0.03164	0.088132
0.46596	0.41704	0.04633	0.4561	0.2135	0.069881
0.03781	0.43185	0.013631	0.50023	0.25462	0.063334
0.00095	0.03726	0.077084	0.01124	0.48899	0.030307
0.67519	0.67454	0.129341	0.0013	0.00994	0.093001
0.93999	0.0752	0.057456	0.00142	0.00012	0.099006
0.30009	0.2999	0.000225	0.44383	0.44241	0.016259
0.12562	0.17427	0.019177	0.18145	0.22738	0.007186
0.41056	0.28474	0.00091	0.26645	0.01961	0.087196
0.45514	0.04458	0.03073	0.39737	0.16131	0.02359
0.07866	0.37628	0.007368	0.06872	0.32865	0.000189
0.27915	0.20029	0.013135	0.0796	0.10386	0.082428
0.37238	0.09323	0.049138	0.00336	0.07624	0.068959
0.03226	0.34012	0.000636	0.46684	0.04348	0.073669
0.03073	0.00153	0.08201	0.9369	0.89006	0.330809
0.18287	0.15214	0.026491	1.29106	0.35416	0.001541
0.06217	0.1307	0.03393	2.86992	1.02514	0.504441
0.00132	0.00132	0.08636	0.18736	0.06856	0.060883
0.00816	0.04533	0.072663	0.20053	0.09267	0.048369
0.06164	0.05348	0.06634	0.14152	0.14851	0.027866
0.2511	0.18946	0.015735	0.42176	0.28024	0.001201
0.09211	0.19899	0.013435	0.35583	0.08613	0.061817
0.0188	0.03331	0.079293	0.09805	0.05718	0.025319
0.00733	0.001147	0.08207	0.1781	0.02375	0.084768
0.00018	0.00715	0.09471	0.78221	0.80611	0.084803
0.40017	0.39999	0.00724	0.86009	0.73614	0.177443
0.01809	0.38408	0.004786	0.0	0.04607	0.07227
0.00596	0.01243	0.061488	0.30505	0.30505	0.71-05
0.00678	0.00309	0.087225	0.00736	0.00736	0.000238
0.05287	0.04612	0.072243	0.04234	0.03677	0.077356
2.97792	2.92505	8.812863	0.00313	0.00497	0.075038
1.19821	1.77871	2.145668	0.02591	0.02454	0.084309
0.05519	1.4302	0.685783	0.16878	0.14087	0.030286
0.042	0.01519	0.091029	1.09694	0.83016	0.178545
2.01538	1.97338	2.750556	0.2066	0.89034	0.331131
0.26307	1.75231	2.086147	0.24003	0.03343	0.079225
0.15992	0.10315	0.044838	0.99052	0.75049	0.189738
0.04044	0.11948	0.038189	0.10778	0.08878	0.329315
0.01114	0.0293	0.081567	0.08516	0.0166	0.088983
0.67781	0.66667	0.12742	1.20712	1.12106	0.651346
0.26085	0.41696	0.010416	1.09249	0.11463	0.040106
0.2164	0.04445	0.073143	0.00919	0.0833	0.590438
0.8486	0.4322	0.013759	0.93297	0.04378	0.073506
0.74857	0.09987	0.046195	0.06178	0.02881	0.081847
0.00958	0.73888	0.178844	0.0842	0.00242	0.087544
0.23855	0.22896	0.007386	0.2999	0.5457	0.053269
0.13573	0.10282	0.044978	0.1856	0.4443	0.016744
0.38473	0.749	0.004343	0.05542	0.13018	0.034121
0.07809	0.30654	8.8E-05	0.13316	0.0774	0.062645
0.09503	0.01694	0.08678	0.0088	0.03436	0.078703
0.05429	0.04074	0.075164	0.04377	0.00503	0.067532
0.04913	0.00516	0.095939	0.7808	0.73703	0.178194
0.2686	0.21947	0.009107	0.96889	0.18609	0.016592
0.12041	0.14819	0.027782	0.00508	0.96181	0.184893
1.39815	1.2774	0.927061	0.21315	0.22627	0.078655
0.05211	0.34604	0.16325	0.00318	0.00101	0.071021
0.82466	0.77275	0.206627	0.19965	0.19941	0.013338
0.01064	0.81422	0.24932	0.02785	0.32828	0.000179
0.20088	0.24025	0.005573	0.3638	0.18413	0.027232
0.02909	0.25089	0.004097	0.41356	0.01895	0.018985
0.02521	0.0221	0.085732	0.00318	0.00108	0.068496
0.01461	0.00749	0.094501	0.03889	0.03571	0.077947
0.07533	0.06072	0.084607	0.03284	0.00805	0.095388
0.00168	0.07365	0.058202	0.06742	0.03458	0.078579
0.64507	0.84338	0.107896	0.06289	0.01181	0.062897
0.4475	0.19787	0.013766	0.00231	0.00231	0.067447
0.23986	0.20764	0.011505	0.11226	0.00995	0.062995
0.44228	0.20242	0.012852	0.1306	0.11834	0.038638
0.71065	0.26837	0.002165	0.92335	0.79275	0.228341

0.5 Alpha=0.5	1 Beta=1	MR	(MR-MR)*2	MR	(MR-MR)*2	
0.64404	0.1745	0.47239	0.02732	0.14895	0.70426	0.004471
0.00957	0.07588	0.315317	1.42024	1.2705	0.400885	
0.01554	0.09023	0.104511	1.20115	9E-05	0.406178	
0.43338	0.41784	0.048211	6E-05	1.42009	0.612586	
0.00054	0.43284	0.041849	1.28552	1.28540	0.419967	
0.244839	2.44785	3.277698	0.69444	0.58906	0.002336	
3E-05	2.44836	3.279534	0.01481	0.60903	0.00332	
0.47367	4.73674	16.8045	0.06445	0.06304	0.329902	
0.00717	4.7296	16.74601	0.04734	0.01711	0.384773	
0.06001	0.05284	0.341723	0.11325	0.06591	0.328614	
0.07842	0.72411	0.007317	0.43344	0.32019	0.100629	
0.071069	0.07323	0.31883	0.08637	0.34017	0.084298	
0.18146	0.26391	0.139033	0.38974	0.38918	0.061619	
0.18331	0.00185	0.409338	0.23456	0.15518	0.232547	
0.17428	0.00893	0.394899	0.08226	0.1523	0.235333	
0.005365	0.12073	0.286959	0.27273	0.19004	0.200141	
0.18382	0.13017	0.257284	0.03944	0.23288	0.183662	
0.56905	0.38523	0.063995	0.83182	0.56238	0.002028	
0.92787	1.35882	0.520431	0.09515	0.57367	0.004063	
0.43758	1.49029	0.727402	0.98229	0.92414	0.082213	
0.15689	0.28069	0.12728	0.03068	0.94023	0.095369	
0.00753	0.14036	0.238194	0.15183	0.15187	0.212005	
0.03414	0.02961	0.373078	0.00691	0.14502	0.242449	
0.83206	0.79792	0.025763	0.18844	0.18153	0.207828	
0.02119	0.81087	0.000008	0.28035	0.09191	0.287571	
0.01697	0.00422	0.400831	0.08428	0.18908	0.203981	
0.06833	0.04138	0.395377	0.18125	0.68999	0.024558	
0.16599	0.10666	0.279578	0.48437	0.29688	0.115681	
0.26459	0.0976	0.291396	0.8683	0.38303	0.064253	
0.1745	0.09009	0.29996	0.01737	0.85093	0.04559	
0.09647	0.11803	0.269757	0.18073	0.14336	0.244087	
0.17145	0.1567	0.278959	1.38405	0.10714	0.184242	
0.134268	1.17119	0.28482	0.04386	0.16889	0.219512	
0.11378	1.2289	0.349859	0.07838	0.03241	0.368026	
0.32877	0.21499	0.17844	0.41854	0.34015	0.088384	
0.00225	0.32652	0.096653	0.30591	0.11083	0.277498	
0.21922	2.15697	2.308099	1.38405	0.10714	0.184242	
0.148528	0.67094	0.001124	12.437	1.25968	0.387219	
0.56596	0.92232	0.081173	0.78115	0.65678	0.000375	
0.00002	0.56576	0.005134	0.01322	0.78783	0.017035	
0.9378	0.9374	0.089993	0.39115	0.37793	0.08733	
0.65861	0.27899	0.128466	0.86441	0.47326	0.028946	
0.78821	0.1596	0.247812	0.7598	0.10461	0.283877	
0.88225	0.08404	0.002682	0.82263	0.06283	0.330143	
0.0543	0.82785	0.003605	0.58721	0.23542	0.161597	
0.26556	0.20128	0.102328	0.00014	0.86707	0.002534	
0.05789	0.19767	0.183372	1.30518	0.30504	0.445726	
0.34245	3.36656	7.488254	0.21101	1.00417	0.208629	
0.26231	3.16214	6.374256	0.11545	0.09556	0.293603	
0.02442	0.23789	0.159817	0.95304	0.83759	0.040072	
0.20253	0.17811	0.210958	0.00489	0.94815	0.096559	
0.36304	3.66051	9.139127	1.43005	1.43318	0.833218	
0.16392	0.06912	0.374061	0.15789	1.28086	0.413769	
0.83261	0.66689	0.000578	0.60684	0.44901	0.035495	
0.88634	0.05373	0.304864	1.57256	0.96816	0.108076	
0.00405	0.88229	0.009686	0.04488	1.52768	0.782579	
0.003484	0.03079	0.367899	0.00289	0.04223	0.354241	
0.002729	0.02119	0.379728	0.02315	0.39775	0.027437	
0.03795	0.01059	0.092204	0.28849	0.04814	0.388287	
0.00194	0.10087	0.287878	0.00959	0.58314	0.002945	
0.03419	0.03225	0.36622	0.00722	0.00237	0.403277	
0.16765	1.64144	1.008074	0.37769	0.37047	0.071258	
0.00061	1.66993	1.065299	0.4209	0.04321	0.353075	
0.02729	0.02119	0.379728	0.02315	0.39775	0.027437	
0.00128	0.01059	0.092204	0.28849	0.04814	0.388287	
0.44104	0.40309	0.064906	0.25673	0.22844	0.187257	
0.00053	0.44051	0.03877	0.0997	0.15703	0.230786	
0.57632	0.57579	0.003797	0.10728	0.00786	0.398712	
0.03338	0.54294	0.008925	1.1384	1.00658	0.136289	
0.44231	0.4081	0.092204	0.28849	0.04814	0.388287	
0.40361	0.0387	0.358485	0.14253	0.12993	0.281613	
0.15823	0.24538	0.153680	0.00113	0.1414	0.248627	
0.00135	0.16688	0.23091	0.08853	0.0874	0.302512	

0.5 Alpha=0.5	5 Beta=5	MR	(MR-MR)*2	MR	(MR-MR)*2
0.78209	0.78209	0.78209	0.3154	4.609	2.303749
0.7334	0.70775	15.97025	2.5781	2.8227	0.886395
2.25023	1.8169	8.418387	0.046	1.8169	0.315581
0.3066	1.3557	0.119723	0.6446	0.5886	6.213002
0.0075	3.9885	0.257364	0.1207	0.5238	6.590975
0.5255	0.518	6.213002	0.0282	0.0925	8.992138
0.45031	3.9776	0.785724	5.8922		

0.26728	0.08497	0.048369	0	0.14695	0.14644	0.241053	0	5.3528	0.3233	4.982318	0	4.2571	2.7899	12.57985
0.00746	0.25932	0.003086	0	0.14608	0.03087	0.367892	0	0.4334	4.9194	3.342354	0	6.4328	2.1757	17.14795
0.00589	0.05094	0.066675	0	0.04665	0.07043	0.321468	0	0.3315	0.1018	8.935951	0	0	6.4328	0.013477
0.00103	0.05787	0.066064	0	0.00074	0.04491	0.351058	0	8.0598	5.3281	5.003769	0	5.0368	5.0368	1.636164
0.00061	0.00042	0.098898	0	0.00852	0.00578	0.389558	0	5.9071	0.2475	8.066569	0	3.8022	1.2345	25.82782
1.22503	1.22442	0.827227	1	0.01814	0.00962	0.394122	0	4.4398	1.4873	2.637017	0	3.4909	0.3113	36.06492
0.00046	1.22457	0.827499	1	0.04122	0.02508	0.374949	0	3.3947	1.0451	4.186482	0	8.9614	5.4705	0.716068
0.01763	0.01717	0.068443	0	0.52534	0.48412	0.232498	0	3.2128	0.1819	8.463965	0	4.7674	4.194	4.505688
0.0643	0.04667	0.071947	0	0.06254	0.4644	0.029795	0	0.5642	2.6486	0.195585	0	1.0375	3.7299	8.891574
0.47852	0.41222	0.009471	0	0.01093	0.04961	0.34551	0	0.0706	0.4938	8.74747	0	0.0169	0.0218	28.03817
0.00573	0.47079	0.024302	0	0.02392	0.01299	0.389902	0	0	0.0708	8.12396	0	0.0446	0.0287	39.53904
0.39782	0.39209	0.005958	0	0.07898	0.05506	0.174431	0	0.0024	0.0024	5.40682	0	7.4549	7.4103	1.959494
0.32754	0.07028	0.0569839	0	0.00013	1.07855	0.194688	0	10.3817	10.3893	53.26242	1	0.8217	8.8332	0.266784
0.24568	0.08186	0.054308	0	0.23851	0.23838	0.180826	0	9.014	1.3777	2.938046	0	0.848	0.2283	37.09307
0.02827	0.21741	0.009504	0	0.03823	0.80172	0.029997	0	8.0367	0.9773	4.488528	0	4.888	4.04	5.183398
0.10633	0.07806	0.056093	0	1.46562	0.42739	0.044109	0	1.3875	6.6692	12.80218	0	2.9627	2.3253	15.93134
1.11819	1.00986	0.482969	0	0.20535	1.26027	0.387953	0	0.09	1.2775	3.289469	0	0.0195	2.5432	14.23936
0.08659	1.0178	0.493787	0	0.02964	0.17871	0.212248	0	0.8234	0.7334	5.959171	0	0.1291	0.1096	38.52819
2.40684	0.14205	0.029677	0	0.401	0.37238	0.070252	0	3.8397	3.0183	0.005608	0	0.3276	0.1985	37.43247
0.28272	0.05206	0.069074	0	0.97032	0.58932	0.004636	0	0.6323	2.2074	0.781584	0	0.3851	0.0595	39.18518
0.0157	0.27702	0.001435	0	0.8412	1.2912	0.25836	0	0.398	1.2463	3.403617	0	1.0502	7.0411	31.50137
0.1856	0.1899	0.021025	0	0.62992	0.21128	0.181588	0	0.335	0.051	9.242751	0	8.3582	7.286	0.901156
0.0782	0.1074	0.043056	0	0.03841	0.58151	0.002107	0	2.7819	2.4489	0.415109	0	0.6038	7.7526	0.061787
0.38733	0.30913	3.3E-05	0	1.84153	1.80312	0.932594	0	0.8554	1.2285	0.930626	0	3.6391	3.0355	10.78632
0.42803	0.0407	0.075186	0	1.1505	0.89103	0.288932	0	8.1751	3.2197	0.018515	0	1.8806	7.958	20.77726
0.22583	0.2022	0.012701	0	0.04852	1.50386	0.750942	0	0.5449	3.3302	0.057128	0	2.5643	0.8737	31.84354
0.05322	0.17261	0.020246	0	0.40498	0.35848	0.077814	0	1.3156	0.7707	5.368471	0	0.2135	2.3483	15.80784
0.04938	0.00786	0.084274	0	0.14349	0.28149	0.141317	0	1.5277	0.2121	8.289156	0	0.5718	0.3905	35.50262
0.01878	0.02856	0.081879	0	3.89638	3.55587	8.517402	1	0.0781	1.4496	2.894818	0	0.0712	0.5008	33.82711
0.00981	0.00367	0.084882	0	0.01376	3.8856	9.291455	0	0.6185	0.5384	5.187324	0	3.1785	3.1053	10.31314
0.31889	0.30898	3.5E-05	0	0.08565	0.07189	0.319814	0	3.0121	2.3856	0.483845	0	2.0218	1.1547	26.84832
0.02277	0.29812	0.000353	0	0.52762	0.44187	0.038236	0	4.4526	1.4406	2.724775	0	15.5271	13.5023	61.87586
0.04624	0.02347	0.084931	0	0.04531	0.48221	0.024087	0	0.4863	3.9843	0.872322	0	1.1407	14.3864	65.11993
0.18812	0.12188	0.037257	0	0.58023	0.50492	0.017554	0	4.0103	3.522	0.185588	0	1.1417	0.001	39.88818
0.08929	0.07883	0.095729	0	0.06747	0.46278	0.030603	0	0.3583	3.852	0.314508	0	5.8442	4.7025	2.905667
0.08795	0.00134	0.09832	0	1.06046	0.98299	0.128436	0	0.6158	0.1573	8.807707	0	0.0561	5.7881	0.279428
0.11786	0.02971	0.081333	0	1.18886	0.0384	0.388814	0	0.6219	0.1063	8.909564	0	3.9399	3.8838	5.91904
0.08629	0.05137	0.089448	0	0.0005	1.11838	0.231312	0	0.887	0.2851	7.986781	0	4.0953	0.1954	37.96171
0.62098	0.55489	0.057499	0	0.12362	1.12312	0.284495	0	0.6971	0.1899	8.41748	0	0.0299	4.0854	0.068387
0.00316	0.61782	0.084781	0	0.38875	0.24313	0.156458	0	5.1893	4.4822	1.962831	0	5.9317	5.5018	0.864078
0.14033	0.13717	0.031598	0	0.11702	0.24973	0.150297	0	0.3554	0.8339	3.03704	0	0.1304	5.4013	0.837971
0.00154	0.13879	0.031015	0	0.74135	0.62433	0.000171	0	3.7837	3.4283	0.113844	0	9.2713	9.1408	9.878062
0.01528	0.01374	0.080697	0	2.01392	1.27257	0.403427	0	0.3789	3.4048	0.088352	0	0.3394	8.9319	8.83923
0.75898	0.7437	0.183868	0	0.88367	1.12025	0.233133	0	0.849	0.4701	8.870109	0	1.8648	1.5254	22.95863
0.00316	0.61782	0.084781	0	1.36372	0.47005	0.02801	0	1.5418	0.6928	5.782272	0	8.822	8.7572	0.194033
0.04261	0.1822	0.023317	0	0.0048	1.35892	0.529575	0	0.2445	1.2973	3.218039	0	5.3383	3.2837	9.199136
0.15502	0.11241	0.041002	0	2.42208	2.41729	3.187989	1	0.017	0.2275	8.200717	0	0.0184	5.3199	0.993828
0.77809	0.82307	0.084988	0	0.86514	1.55895	0.845552	0	0.9991	0.8821	4.448258	0	0.0558	0.0374	38.42971
0.20794	0.57019	0.065153	0	0.05202	0.81312	0.030674	0	1.179	0.1796	8.475606	0	0.0331	0.0227	38.61453
0.00316	0.61782	0.084781	0	0.03038	0.02164	0.379174	0	0.4023	0.7787	5.358881	0	7.9899	7.5988	1.537829
0.02702	0.05038	0.089871	0	0.18832	0.13794	0.240471	0	0.0423	0.38	7.458395	0	4.7188	2.8711	11.87221
0.0312	0.00418	0.085547	0	0.35743	0.18911	0.200974	0	0.1983	0.156	8.816337	0	29.0987	24.3779	328.2067
0.38828	0.35708	0.001779	0	0.02357	0.33388	0.082143	0	0.0001	0.1982	8.369387	0	0.0159	29.0808	518.2038
0.28333	0.12495	0.038081	0	0.28544	0.28187	0.141031	0	1.8787	1.8786	2.001063	0	0.0648	0.0487	39.28792
0.28864	0.00531	0.065846	0	0.04517	0.24027	0.157721	0	0.2205	1.4562	2.67319	0	4.6089	4.5443	3.141429
0.05181	0.21703	0.009579	0	0.01984	0.02523	0.374786	0	4.9244	4.7038	2.800638	0	1.82	2.9889	11.0743
Sum MR > UCLmr				415	121			506	91			453	105	
Sum X	1984.884		491.3042	Sum X	4124.014		820.9386	Sum X	19753.38		4771.391	Sum X	39647.91	10041.17
X Average	0.247819			X Average	0.504495			X Average	2.452478			X Average	4.969906	
MR Average	0.3149			MR Average	0.837411			MR Average	3.091189			MR Average	6.316708	
Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128	
Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268	
Upper control limit for X (UCLx)	1.085119			Upper control limit for X (UCLx)	2.199737			Upper control limit for X (UCLx)	10.67373			Upper control limit for X (UCLx)	21.76966	
Lower control limit for X (LCLx)	-0.58988			Lower control limit for X (LCLx)	-1.19075			Lower control limit for X (LCLx)	-5.76877			Lower control limit for X (LCLx)	-11.8306	
Upper control limit MR (UCLmr)	1.029093			Upper control limit MR (UCLmr)	2.06306			Upper control limit MR (UCLmr)	10.10201			Upper control limit MR (UCLmr)	20.643	
MR Standard deviation	0.372561			MR Standard deviation	0.785767			MR Standard deviation	3.771874			MR Standard deviation	7.850387	
Process standard deviation	0.353553			Process standard deviation	0.7017			Process standard deviation	3.353544			Process standard deviation	7.071088	
Gamma d2	0.890672			Gamma d2	0.901435			Gamma d2	0.87432			Gamma d2	0.833117	
Gamma d3	1.05378			Gamma d3	1.111243			Gamma d3	1.086791			Gamma d3	1.081292	
Gamma D4	4.549322			Gamma D4	4.686245			Gamma D4	4.680411			Gamma D4	4.633406	
0.5 Alpha=0.5				0.5 Alpha=0.5				0.5 Alpha=0.5				0.5 Alpha=0.5		
0.5 Beta=0.5				0.5 Beta=1				0.5 Beta=5				0.5 Beta=10		
Proportion MR > UCLmr	0.053605			Proportion MR > UCLmr	0.058708			Proportion MR > UCLmr	0.055808			Proportion MR > UCLmr	0.057106	
Percentage	5.360538			Percentage	5.870597			Percentage	5.580558			Percentage	5.710571	
ARL for MR	18.85485			ARL for MR	18.74874			ARL for MR	17.91935			ARL for MR	17.51138	

1 Alpha=1
0.5 Beta=0.5

MR	(MR-MR)²	MR	(MR-MR)²
1.2252	0.07146	2.52468	4.096877
0.52487	0.70033	0.03969	1.00685
0.7399	0.21503	0.01780	0.00031
1.08785	0.34795	0.02304	0.00054
1.03371	0.05414	0.19332	0.2838
0.15638	0.87733	0.14192	0.099496
0.133	0.02338	0.22774	0.17197
0.13764	0.06454	0.04582	0.18922
0.77724	0.6398	0.01919	0.28184
0.13154	0.6457	0.02102	0.17395
0.00253	1.2901	0.13063	0.1858
0.09482	0.09209	0.16685	0.1897
0.361	0.28638	0.04593	0.09168
0.0907	0.29593	0.04086	1.44085
0.42174	0.32667	0.030254	1.25687
0.1819	0.23455	0.070768	0.86614
0.14584	0.04135	0.21018	0.14701
0.65994	0.51	0.81	0.22715
0.63399	0.19185	0.09533	0.01609
0.49649	0.0345	0.217255	0.31234
0.94378	0.44529	0.00306	0.07499
0.3926	0.55118	0.002556	0.26867
0.01359	0.37901	0.014786	0.42962
0.2151	0.26151	0.051767	0.2347
0.1871	0.088	0.17224	0.037518
1.2465	1.0594	0.31225	2.21577
1.20098	0.04522	0.207103	0.08611
0.52774	0.67324	0.029803	0.16301
0.42635	0.10139	0.03373	1.94221
0.5522	0.12585	0.140424	0.14354
0.09484	0.45736	0.00187	0.11815
0.28699	0.19215	0.095145	0.01134
0.00673	0.28026	0.048552	0.72898
0.11138	0.10485	0.56781	0.9037
0.30503	0.19365	0.08422	0.07031
0.17537	0.35722	0.733788	0.40233
0.19216	0.01679	0.234078	0.25231
0.04241	0.14875	0.11231	1.9857
0.46335	0.42094	0.008347	0.38331
0.57553	0.11218	0.150875	0.251
0.93983	0.3643	0.018579	0.17587
0.34845	0.59138	0.006284	1.97083
0.36048	0.21023	0.238708	0.81625
0.33208	0.02842	0.222959	0.25729
0.38522	0.05316	0.200208	0.28534
0.09931	0.29591	0.0418	0.06418
0.3525	0.26319	0.096386	0.29476
0.78853	0.43903	0.00417	0.37451
0.20712	0.58141	0.006529	0.59101
0.09928	0.11784	0.14851	0.49789
0.31695	0.22767	0.074494	0.7884
0.09421	0.22274	0.047209	0.5174
0.38309	0.29888	0.004693	0.38011
0.48077	0.09768	0.152499	0.63887
0.06281	0.42796	0.005277	1.74065
0.29372	0.23091	0.07236	0.34841
0.90279	0.69097	0.011764	0.32158
0.10696	0.79983	0.087157	0.55277
0.21572	0.18678	0.153453	0.47603
0.14042	0.0753	0.180885	0.02459
0.75089	0.61047	0.01207	0.9887
0.39779	0.3531	0.021758	0.05305
0.00187	0.39592	0.010995	1.2374
0.02956	0.32769	0.0299	1.003
0.74872	0.42018	0.006344	0.30944
0.28641	0.05211	0.00042	0.67715
0.12466	0.6417	0.190478	2.06095
0.02903	0.07459	0.181489	0.42172
0.02426	0.21477	0.081702	0.38469
0.05429	0.004498	0.40288	0.00498
0.49644	0.03664	0.011422	0.05837
0.3636	0.36036	0.019669	0.55258
0.46389	0.32781	0.029858	0.45288

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MR	(MR-MR)²	MR	(MR-MR)²
1.137167	0.88361	0.78731	0.044206
0.48332	0.88855	0.011884	0.35293
0.25879	0.22453	0.597578	0.18885
0.13897	1.13024	0.01305	0.26311
0.49134	0.89773	0.009966	0.12673
0.03369	0.15495	0.070969	0.77299
0.89571	0.59932	0.192056	2.88554
0.12067	0.22496	0.596913	0.08287
0.09543	0.52524	0.223088	0.28157
0.50646	0.06863	0.825793	0.24786
0.41145	0.09615	0.814347	0.22778
0.62585	0.2144	0.813342	1.47922
0.10218	0.38633	0.373604	0.81017
0.73483	0.27739	0.518705	0.4192
0.67209	1.20847	0.044482	0.21491
0.07441	0.59768	0.159905	0.33666
0.33206	0.25785	0.54499	0.05889
0.13122	0.20084	0.634765	0.66254
0.093737	0.80615	0.006838	0.46875
0.68229	0.29508	0.551279	1.71687
0.80551	1.12322	0.01579	0.72597
0.12579	0.18792	0.46534	1.26573
0.18923	0.06344	0.872653	0.33344
0.08501	0.28985	0.863349	1.45477
0.40888	0.87623	0.219728	0.01074
0.18464	0.43755	0.136133	1.49068
0.68848	0.18768	0.505385	0.15328
0.99901	0.31021	0.472452	0.95289
1.13871	0.3397	0.427482	0.25785
0.28615	0.10736	0.005776	0.94078
0.94706	0.88191	0.099636	0.31603
0.3065	0.21794	1.255248	1.04068
2.77974	0.28928	0.501891	0.7335
0.23888	0.4468	0.01769	0.11027
0.79437	1.56448	0.321398	1.71224
0.10581	0.26144	0.541875	1.53001
1.55307	0.49726	0.293002	0.14578
0.2111	1.34187	0.118617	1.8904
0.44737	0.32627	0.518715	0.92992
0.28936	0.73551	0.039561	0.02718
0.02715	0.66671	0.2786056	1.35118
0.44285	0.4155	0.338798	0.77987
0.50995	0.0673	0.863387	2.17198
0.50154	0.00841	0.978421	0.18331
0.10189	0.51655	0.231372	0.84658
0.27529	0.7428	0.064903	2.44894
1.88248	1.70717	0.503544	1.5917
0.20769	0.06343	0.817902	0.23683
0.18789	0.8051	0.011982	1.48424
0.83237	0.34207	0.400488	0.89412
0.18833	0.83464	0.31494	0.47424
1.95412	1.78579	0.590175	0.04949
0.21098	1.74314	0.555887	0.18588
0.89651	0.68753	0.59612	1.47815
0.16602	0.28751	0.504173	1.72943
0.9707	0.21532	0.811902	0.58507
2.42315	1.46245	0.216121	0.22742
0.84486	1.58829	0.34896	0.21984
0.15189	1.00683	0.88E-05	0.15713
0.40645	0.46524	0.305599	1.20565
2.11722	0.71077	0.082248	0.00574
1.137184	0.74538	0.68396	0.23814
0.0889	1.28294	0.881441	1.75653
0.28754	0.17864	0.870833	1.28432
0.46542	0.19718	0.639461	1.35884
0.70668	0.24126	0.571992	0.54028
1.40382	0.28174	0.090253	3.02229
1.14865	0.25517	0.551145	0.83787
0.09339	1.05528	0.003329	2.23263
0.58926	0.65167	0.459998	1.62987
0.57982	0.01524	0.984958	0.34883
4.37295	3.78313	0.781203	2.72804
0.62802	3.74493	0.7548033	1.28631

MR	(MR-MR)²	MR	(MR-MR)²
0.15777	3.3619	17.7885	184.9033
5.6364	5.4787	0.284801	5.9999
1.2583	4.3781	0.321413	1.7992
0.7458	0.5127	1.684557	3.3207
0.7887	0.0189	24.26678	13.5884
1.4854	0.7687	17.4515	1073
5.1774	3.682	1.592521	3.9747
2.7701	2.4073	6.440087	2.2683
0.9802	7.2101	5.13053	4.0929
9.7294	0.2588	22.03582	12.2909
0.4879	9.2415	18.49863	0.2726
8.866	6.3781	0.53882	10.4843
3.7308	0.5048	19.71567	0.5296
0.6861	6.8747	2.991749	1.845
0.6627	0.0334	24.1244	3.0078
9.3037	8.841	13.66017	0.6888
0.6156	6.6881	14.01055	3.9439
2.6990	2.0042	8.184363	2.4336
8.6786	4.1268	0.689505	6.1344
2.5184	4.3102	0.403012	0.1034
1.7907	0.7257	1.80277	2.3266
0.0267	3.236	3.920793	28.7187
3.8208	1.2059	13.98111	1.5076
3.5749	0.2459	22.08185	6.3522
2.8498	0.9253	16.15825	1.9567
0.9714	2.2783	2.119896	0.089
11.0167	10.6453	32.49305	8.3612
9.3573	1.8594	10.78688	16.4371
3.8007	5.7566	0.658842	3.8585
0.9068	1.4661	12.10287	10.7327
1.7377	1.1387	2.81568	4.8241
0.5531	1.8846	14.14085	17.0841
10.2238	8.8407	22.33193	0.0825
4.1807	6.0831	1.250075	13.1538
0.0827	4.078	0.751748	6.4773
1.2074	1.1247	14.94994	3.887
8.5792	5.3718	0.18213	1.1357
0.6966	5.8826	0.879033	1.4772
8.8357	0.1391	23.08698	0.1988
18.0893	15.2536	106.2866	18.8195
9.3707	8.7186	33.14541	7.5383
2.5032	6.6673	3.695881	0.7824
2.8432	0.14	23.08834	5.0219
2.1554	0.4878	19.89992	4.7925
2.2842	0.1086	23.1061	2.0982
3.8181	1.3318	13.95473	3.4447
3.8047	0.1886	22.62365	1.2862
5.3171	0.2876	21.69188	9.6022
1.7011	1.818	9.790845	0.7474
5.3209	3.6198	1.756242	2.8905
6.1022	2.8803	5.98833	1.0643
2.0771	5.8331	0.978277	10.1284
4.0467	1.8004	11.18657	3.2088
0.067	0.4097	20.58924	3.1659
3.1778	3.1106	3.365143	0.2304
8.8804	1.8828	0.543031	1.5739
6.548	2.5124	8.930755	23.7723
0.0863	6.4617	2.30028	2.0059
8.8005	4.7142	0.053284	0.0254
0.7384	0.0641	0.778042	0.7011
5.1174	4.3253	0.324495	3.08
1.1354	3.8783	0.938443	0.2082
1.1881	0.0527	23.94382	0.4756
2.3878	1.1975	14.19224	0.8618
11.8813	9.3137	18.08525	1.1758
8.8507	3.1008	3.292186	0.8227
11.86	3.4993	2.358475	3.378
1.8962	10.0708	28.27349	1.7529
1.3879	0.5213	19.59941	13.0846
0.8808	4.293	0.425147	0.7075
2.8627	2.9882	3.79157	11.7324
7.0085	4.4384	0.253318	3.7261
18.8487	11.5502	43.82824	10.7741
1.7315	16.9172	143.3288	1.6983

MR	(MR-MR)²	MR	(MR-MR)²
10.5765	0.0914	100.3815	1.77
0.9787	9.9978	0.012892	16.751
21.7563	21.1776	12.4816	1.655
8.8029	14.9534	23.45409	13.994
31.3354	24.5325	207.8953	46.972
0.327	31.0084	436.724	4.413
9.7001	9.3701	0.543698	33.738

0 15307	0 07049	0 185	0	0 53945	1 25489	0 068218	0	11 7813	10 9496	36 05483	0	15 7716	9 6943	0 173187
0 31838	1 65311	0 112423	0	0 29328	0 24617	0 954589	0	9 4845	2 2968	7 013136	0	6 1942	9 5774	0 28415
0 07225	0 24613	0 064758	0	0 79815	0 50487	0 242745	0	2 8367	6 6478	2 896417	0	3 1325	0 0617	48 68408
1 16362	1 09137	0 349002	0	0 00093	0 79722	0 040137	0	0 1275	2 7092	4 998948	0	2 7743	0 3582	95 10652
1 43514	0 27152	0 062448	0	1 160457	1 60364	0 367331	0	0 1984	0 0709	23 75717	0	0 478	2 2963	91 06106
2 17714	0 742	0 058271	0	0 42744	1 17713	0 032245	0	1 7445	1 5461	11 55274	0	28 3578	27 8798	315 7495
0 17035	2 00679	2 268591	1	0 75066	0 33322	0 441316	0	0 1567	1 5878	11 27101	0	8 2077	20 1501	100 7944
0 40487	0 23452	0 070802	0	0 49469	0 26597	0 535226	0	8 9385	8 7818	14 72078	0	3 9984	4 2093	34 82366
0 17197	0 2329	0 071668	0	0 15408	0 34061	0 431585	0	6 3904	2 5481	5 745286	0	4 0203	0 0219	101 779
0 00963	0 16334	0 113748	0	1 18945	1 03537	0 001429	0	16 1293	9 7389	22 98116	0	23 8567	19 6364	90 74358
3 1931	3 18447	7 203127	0	0 46504	0 72441	0 074812	0	5 8588	10 2705	28 3606	0	2 0488	21 6079	132 1912
0 7848	2 4282	3 71562	1	0 33544	0 1296	0 753357	0	2 4937	3 3651	2 496187	0	1 2431	0 8067	86 57851
0 0069	0 758	0 066252	0	0 41341	0 07797	0 845649	0	9 4494	6 9557	4 042783	0	54 482	53 2389	1860 963
1 23522	1 22832	0 529586	0	1 0007	0 58728	0 168323	0	2 6742	6 7752	3 349513	0	7 5993	66 8827	1352 198
0 00705	1 22817	0 52935	0	2 98183	1 98093	0 967013	0	0 0579	2 8163	5 422996	0	11 3126	3 7133	40 92362
0 19602	1 96091	0 095874	0	1 20059	1 78104	0 813838	0	5 9979	5 94	0 98996	0	5 2288	8 0838	16 21397
0 18418	0 01384	0 236941	0	0 66159	0 5385	0 21037	0	2 5392	3 4587	2 209185	0	6 9378	1 709	70 58449
0 49765	0 31347	0 03502	0	2 81626	95 457	0 916865	0	3 7645	1 2253	13 83641	0	2 2875	4 6503	29 81332
0 40483	0 45682	0 001917	0	0 3091	2 30716	1 715048	0	1 4284	2 3361	8 80653	0	1 7632	0 5243	81 89441
0 11851	0 07768	0 178866	0	1 30672	0 99762	3 4E-09	0	0 0343	11 2747	40 06469	0	0 6837	6 7627	11 20746
0 04292	0 07559	0 180638	0	0 08917	1 22155	0 050171	0	4 5574	4 0231	0 84996	0	8 5865	7 9028	4 873751
0 27974	0 23882	0 089583	0	0 80159	0 51642	0 231497	0	16 187	10 6296	32 31431	0	7 7584	0 8281	86 18218
2 41937	2 13963	2 886401	1	0 04838	0 55221	0 191748	0	8 1713	9 0187	16 57033	0	30 5315	22 7731	160 3425
0 05768	3 26169	3 463635	1	0 19405	0 14567	0 725719	0	0 1631	5 9832	1 088198	0	20 2282	10 3033	0 037188
0 92459	0 86691	0 134179	0	0 75089	0 56884	0 194238	0	12 7485	12 5854	56 07	0	2 5713	17 8569	56 9488
0 91879	0 0078	0 242858	0	0 22359	0 5273	0 221148	0	0 2355	12 913	57 27413	0	13 0632	10 4919	0 145408
0 98748	0 0707	0 184819	0	0 16845	0 06514	0 868198	0	2 5131	2 2778	7 115197	0	14 4679	1 4047	75 79021
0 06894	0 91855	0 174677	0	1 24905	1 0608	0 008595	0	7 7565	6 2434	0 08023	0	5 4947	8 9732	1 283354
1 17137	1 10243	0 362182	0	0 79841	0 45064	0 299123	0	2 5203	5 2382	0 084778	0	5 8641	0 3694	94 8842
0 22078	0 95061	0 202504	0	0 1946	0 60381	0 15504	0	1 379	1 1413	14 48838	0	20 1518	14 2877	17 44836
0 28395	0 08319	0 191333	0	1 24427	1 04867	0 002716	0	2 1505	0 7715	17 41837	0	10 2768	9 875	0 05544
0 8159	0 53185	0 000992	0	0 48524	0 75903	0 056897	0	2 1011	0 0494	23 98722	0	4 0692	6 2076	15 2323
0 04011	0 73759	0 075726	0	1 3619	0 87868	0 014817	0	0 7086	1 3945	12 82628	0	1 9254	2 2638	61 57003
1 21630	1 17820	0 456536	0	0 22302	1 13888	0 010971	0	1 0677	0 3811	21 01244	0	21 1852	19 3585	85 52629
1 30579	0 0094	0 18909	0	0 42297	0 19999	0 636164	0	1 7827	0 715	17 89318	0	13 8534	3 1105	7 853781
0 4257	0 89008	0 144008	0	1 26164	0 83867	0 025247	0	2 2495	0 4688	20 05457	0	0 4468	13 4086	10 86456
0 3388	0 0989	0 171152	0	2 08247	0 82083	0 031234	0	5 1499	2 8004	4 180523	0	9 431	8 9842	1 268456
0 58368	0 22488	0 078025	0	0 15274	1 32973	0 868938	0	1 8388	3 3111	2 699736	0	4 0499	5 3811	22 30962
0 15086	0 41279	0 007712	0	0 80722	0 45448	0 294938	0	0 3978	1 4412	12 27684	0	2 927	1 1229	80 77619
0 36824	0 21735	0 080234	0	0 25134	0 35588	0 411795	0	36 8582	36 4806	993 231	1	13 8225	10 8955	0 616292
0 82282	0 25458	0 080528	0	0 09682	0 15152	0 715786	0	4 9208	31 9374	728 5879	1	10 0011	3 8214	39 58224
0 10934	0 51348	0 000186	0	1 0188	0 91898	0 008175	0	4 7456	0 175	22 75321	0	32 0517	22 0506	142 567
0 04634	0 061	0 193253	0	2 50254	1 48374	0 236369	0	0 9437	3 8021	1 308295	0	14 4898	17 5821	56 82544
0 58657	0 51738	0 000281	0	0 855	1 64754	0 422472	0	12 3185	11 3748	41 34191	0	1 5764	12 8932	7 143657
0 79135	0 22965	0 076601	0	1 2928	0 4378	0 313333	0	10 1961	2 1224	7 987255	0	3 8386	2 2622	61 59514
0 26904	0 52231	0 000471	0	1 64271	0 34991	0 419453	0	4 3252	8 8709	0 85723	0	14 9137	11 0751	0 930535
0 4175	0 14846	0 124007	0	0 1309	1 51181	0 264451	0	10 6762	8 351	1 978744	0	6 8374	8 2763	3 384133
0 50768	0 09018	0 188449	0	0 2774	0 1485	0 724306	0	5 1656	5 5106	0 319866	0	1 5884	5 049	25 81835
0 30375	0 20393	0 088016	0	1 5064	1 229	0 053564	0	1 93	3 2356	2 92216	0	21 431	19 8426	94 7146
0 45744	0 15369	0 120351	0	0 4778	2 5714	2 478967	0	1 922	0 006	24 37429	0	2 1112	19 3188	84 81199
0 10113	0 36631	0 020621	0	1 40163	2 67617	2 817726	0	1 8147	0 1073	23 40366	0	4 2015	2 0903	64 32292
0 80587	0 70474	0 041671	0	1 83187	0 53024	0 218389	0	18 353	16 5383	134 4038	1	2 8887	1 5128	73 91971
2 53912	1 73325	1 519412	1	1 82728	0 00459	0 985993	0	9 0938	9 2582	18 81204	0	4 4778	1 7891	89 24499
0 59173	1 94189	2 077589	1	0 04278	1 88452	0 786995	0	0 3169	8 7769	14 85321	0	0 2283	4 2495	34 35082
0 59241	0 00472	0 245903	0	0 25588	0 21312	0 615349	0	21 1484	20 8315	252 3788	1	22 4542	22 2259	146 784
2 58614	2 00373	2 259383	1	0 0963	0 15958	0 702213	0							
Sum MR > UCLmr				331	85			307	79			327	86	
Sum X	4003 54		962 3167	Sum X	8114 779	2092 896		Sum X	39492 28	10079 35		Sum X	79849 85	18790 97
X Average	0 486586			X Average	1 020747			X Average	4 957163			X Average	9 964083	
MR Average	0 500606			MR Average	0 967562			MR Average	4 945033			MR Average	10 11046	
Normal constant d2 (n=2)	1 128			Normal constant d2 (n=2)	1 128			Normal constant d2 (n=2)	1 128			Normal constant d2 (n=2)	1 128	
Normal constant D4 (n=2)	3 268			Normal constant D4 (n=2)	3 268			Normal constant D4 (n=2)	3 268			Normal constant D4 (n=2)	3 268	
Upper contro limit for X (UCLx)	1 827984			Upper contro limit for X (UCLx)	3 873837			Upper contro limit for X (UCLx)	18 10885			Upper contro limit for X (UCLx)	36 8536	
Lower control limit for X (LCLx)	-0 83481			Lower control limit for X (LCLx)	-1 63234			Lower control limit for X (LCLx)	-8 19452			Lower control limit for X (LCLx)	-16 8254	
Upper control limit MR (UCLmr)	1 83598			Upper control limit MR (UCLmr)	3 280031			Upper control limit MR (UCLmr)	16 18037			Upper control limit MR (UCLmr)	33 04097	
MR Standard deviation	0 497896			MR Standard deviation	1 000811			MR Standard deviation	5 0334			MR Standard deviation	10 0519	
Process standard deviation	0 5			Process standard deviation	1			Process standard deviation	5			Process standard deviation	10	
Gamma d2	1 001211			Gamma d2	0 987562			Gamma d2	0 989007			Gamma d2	1 011046	
Gamma d3	0 999795			Gamma d3	1 000811			Gamma d3	1 006658			Gamma d3	1 00519	
Gamma D4	3 983771			Gamma D4	4 009771			Gamma D4	4 05361			Gamma D4	3 982624	
1 Alpha=1				1 Alpha=1				1 Alpha=1				1 Alpha=1		
5 Beta=0 5				5 Beta=0 5				5 Beta=0 5				5 Beta=0 5		
Proportion MR > UCLmr	0 039604			Proportion MR > UCLmr	0 038004			Proportion MR > UCLmr	0 041504			Proportion MR > UCLmr	0 038704	
Percentage	3 860396			Percentage	3 860396			Percentage	4 150415			Percentage	3 870367	
ARL for MR	25 25			ARL for MR	25 90415			ARL for MR	24 09398			ARL for MR	27 24523	

15 Alpha=1.5

0.5 Beta=1.0	MR	(MR-MR)²	MR	(MR-MR)²		
1.27555			0.13668	0.241346		
1.14682	0.12873	0.251222	2.37111	2.0554	2.031907	0
1.31959	0.17277	0.209014	0.44969	1.92203	1.66847	0
0.58522	0.73337	0.016496	1.8221	1.1702	0.284926	0
0.85396	0.28774	0.131196	0.43669	1.85541	3.038536	0
0.07728	0.77968	0.02153	0.2702	0.16649	0.214795	0
0.66472	0.58744	0.001807	0.11468	0.15552	0.225084	0
0.22639	0.43833	0.036718	0.18879	0.07411	0.308954	0
0.2611	0.03531	0.353597	0.69634	0.50735	0.014982	0
0.45093	0.18923	0.194234	0.38212	0.31422	0.098686	0
0.9134	0.46247	0.02805	0.91969	0.20957	0.178719	0
0.25192	0.66148	0.000994	1.23602	0.64433	0.000207	0
0.19727	0.05465	0.33097	0.19526	1.04078	0.18875	0
0.71153	0.51426	0.013384	0.47419	0.27893	0.123215	0
1.24352	0.53199	0.009596	0.74156	0.28737	0.131464	0
1.99225	0.74873	0.014109	0.42358	0.31798	0.087325	0
0.59073	1.40152	0.59532	2.03557	0.81199	0.964402	0
0.16057	0.43018	0.039916	0.78249	1.25308	0.388291	0
1.98	1.51944	0.970185	0.17525	0.80724	0.000516	0
0.42628	1.25371	0.389076	0.26951	0.09428	0.286984	0
1.38486	0.95857	0.107991	0.98494	0.72543	0.008118	0
0.11288	1.27198	0.412202	0.41287	0.58227	0.002273	0
1.66565	1.55277	0.851997	0.54781	0.13494	0.245035	0
0.05072	1.81483	0.970185	0.57827	0.03066	0.359149	0
0.33073	0.28001	0.124588	0.98962	0.41783	0.044951	0
1.477	1.14827	0.266588	1.02189	0.02549	0.385372	0
0.15782	1.31938	0.475314	1.70486	0.68329	0.022645	0
0.24691	0.08929	0.292313	2.30004	0.95906	0.001217	0
1.81527	1.36836	0.545249	0.81364	1.4862	0.373164	0
0.29236	1.32291	0.482383	0.32749	0.44108	0.038722	0
0.08984	0.20272	0.182528	1.27538	0.95208	0.071338	0
0.32463	0.23496	0.159964	0.26293	0.87713	0.026845	0
1.00888	0.68425	0.002448	0.51478	0.21853	0.170916	0
1.36898	0.37608	0.683439	0.38796	0.1268	0.25318	0
2.47541	1.08845	0.02222	0.47251	0.08455	0.297481	0
0.26171	2.1837	2.414138	0.6602	0.19698	0.187714	1
0.27961	0.0121	0.381739	0.4619	0.2073	0.178633	0
0.49305	0.21344	0.173481	0.68903	0.22713	0.162264	0
0.12219	0.37086	0.087128	1.44263	0.7539	0.015364	0
1.26995	1.4176	0.261949	0.8104	0.83253	0.041298	0
0.72737	0.53858	0.008718	0.25439	0.35091	0.070433	0
0.87457	0.24872	0.145336	0.10649	0.1323	0.203756	0
0.87457	0.39592	0.002477	0.57282	0.18593	0.197154	0
1.45888	0.58411	0.054701	0.21595	0.35687	0.074682	0
0.36547	0.6321	0.21481	0.12233	0.50838	0.01527	0
0.84184	0.27837	0.125019	0.5059	0.21843	0.170999	0
0.4149	0.22984	0.182417	0.09195	0.41395	0.046656	0
0.0993	0.3156	0.098816	0.85186	0.55991	0.004806	0
1.15037	1.05107	0.177342	0.026	0.42588	0.041853	0
0.10812	0.04125	0.189188	0.49352	0.28732	0.131358	0
0.1811	0.66747	0.001408	0.82649	0.38024	0.082355	0
0.32782	1.0057	0.280243	0.4302	0.03842	0.349898	0
0.95947	0.63165	0.29E-06	0.19033	0.23987	0.152163	0
0.42066	0.53881	0.006307	0.03498	0.15535	0.225245	0
0.84857	0.42791	0.04082	0.44825	0.41027	0.048259	0
0.22725	0.11813	0.28196	0.46882	0.02749	0.368086	0
0.23022	0.04912	0.337364	0.04513	0.78419	0.02379	0
0.96817	0.73795	0.016684	0.40338	0.36208	0.071754	0
0.99974	0.36843	0.068393	0.87155	0.46817	0.028173	0
0.23682	0.36292	0.071305	0.32349	0.53908	0.008261	0
1.56982	1.322	0.480318	0.42918	0.09888	0.284366	0
0.70151	0.85831	0.052148	0.94382	0.13444	0.244533	0
0.74096	0.03945	0.34889	0.19536	0.36826	0.068482	0
0.9616	0.22064	0.187355	2.72102	0.52588	0.393716	0
0.80084	0.16078	0.220139	0.74754	0.19738	1.805073	0
2.42128	1.62042	0.981031	0.86799	0.07955	0.302394	0
0.11587	2.30496	2.80819	1.86878	0.6887	0.081039	1
0.13278	0.01611	0.3768	0.39319	1.25387	0.389028	0
0.66578	0.53289	0.006403	1.90428	1.08991	0.271483	0
0.71999	0.05423	0.331454	0.29441	0.08513	0.319022	0
0.37128	0.34871	0.078096	0.15428	0.10513	0.279436	0
0.87378	0.02524	0.00098	0.04284	0.0796	0.04928	0
0.175	0.39878	0.05344	0.30823	0.25401	0.141331	0
0.52992	0.35092	0.077858	0.25967	0.30787	0.309448	0

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15 Alpha=1.5

1 Beta=1.0	MR	(MR-MR)²	MR	(MR-MR)²		
0.3433			1.8388	0.14818		
0.1862	0.1571	1.202878	0.5822	2.0683	0.660062	0
0.1115	0.0747	1.390414	1.7435	1.613	0.008567	0
0.4532	1.3417	0.007716	1.5305	0.213	1.083385	0
2.2032	0.7523	0.2958	5.0818	0.718	7.420995	0
0.909	1.2965	0.001818	0.3662	3.1423	3.566213	0
0.1206	0.1116	1.304753	1.2474	2.1886	0.931113	0
0.656	0.3646	0.79078	1.0568	0.9094	0.118651	0
0.18271	0.9711	0.078952	1.1913	0.1345	1.252962	0
2.9671	1.34	0.00742	0.9491	0.2422	1.023452	0
0.1384	0.8287	0.180759	2.5688	0.8197	0.13384	0
0.6873	1.4511	0.038904	1.20601	2.3602	3.9E-05	0
0.6844	0.0029	1.584896	0.4357	0.873	0.145053	0
0.1332	0.5512	0.493728	1.2639	0.8282	0.181185	0
1.2041	1.0709	0.033474	1.5228	0.2589	0.989941	0
0.1332	0.1291	1.28508	1.9487	0.4259	0.855514	0
0.7435	0.4121	0.708556	1.5777	1.361	0.011479	0
0.5331	1.1878	0.285094	1.418	0.8283	0.1811	0
0.4757	2.0574	0.84568	1.0008	0.4157	0.702509	0
0.12701	0.2058	1.088454	1.1206	1.203	1.294954	0
0.408	0.8621	0.153474	0.8338	0.2868	0.820521	0
0.7754	1.3874	0.012892	1.1204	0.5478	0.4988	0
0.6218	1.1536	0.000502	3.0478	1.6884	0.170191	0
0.3114	0.3104	0.810613	1.5217	1.5261	0.074118	0
0.21474	1.338	0.388897	1.5812	0.9605	0.088059	0
0.8556	1.1461	0.016112	0.1714	0.5438	1.589787	0
0.1813	0.18	1.153119	0.5616	1.3882	0.013843	0
0.5448	0.3635	0.792737	0.4006	0.753	0.250859	0
0.7422	0.1974	1.116103	0.5879	0.1873	1.137546	0
0.5343	1.2078	0.002112	0.8825	0.2948	0.920178	0
0.859	1.226	0.01645	0.811	0.0718	1.589787	0
0.8561	1.0038	0.082529	1.3073	2.9983	0.038104	0
0.4555	0.2008	1.108352	3.1526	0.6547	0.35899	0
0.3054	2.8498	0.254735	0.8028	2.2721	1.038817	0
0.5996	2.7088	2.118856	3.542	2.8615	1.981456	0
0.1738	0.4238	0.690957	0.8899	2.5521	1.854332	0
0.7297	1.5559	0.091229	0.5758	0.4144	0.70489	0
0.0532	3.3235	0.283418	0.3829	0.1926	1.126296	0
0.19047	3.1485	0.589888	1.0729	2.7902	2.380347	0
0.18415	0.2832	0.981403	2.3621	0.811	0.186123	0
1.236	0.4055	0.91711	0.6311	1.525	0.073518	0
0.1848	1.0711	0.0334	0.5337	0.3034	0.90337	0
0.4709	0.308	0.894385	0.7853	0.2316	1.045011	0
2.1788	1.7077	0.205973	0.1807	0.0838	1.389504	0
0.8501	1.5285	0.075428	0.5181	0.1828	1.190844	0
0.9239	0.2738	0.980514	3.5784	3.0583	0.259621	0
0.2707	0.7532	1.36079	2.5325	1.0459	0.043247	0
0.4875	1.1968	0.003256	3.9782	1.4457	0.038603	0
0.1348	0.1327	1.425985	0.9951	2.98027	0.000278	0
0.12826	0.0722	1.396316	2.3322	1.3371	0.006929	0
3.3283	2.0857	0.859887	1.7327	1.805	0.123301	0
0.9239	1.4342	0.032523	0.9112	0.218	1.077149	0
0.4762	1.2863	0.001052	0.7888	0.2326	1.042968	0
0.0128	3.4634	0.882078	1.1088	0.8302	0.179486	0
0.16136	1.8008	0.120369	1.9298	0.821	0.187396	0
0.4748	2.7812	2.27208	0.4954	0.432595	0	
0.1509	1.8639	0.812319	0.6889	0.1745	1.050114	0
0.2937	0.2172	1.07446	0.3843	0.2856	0.837523	0
0.9959	0.7022	0.304327	1.9604	1.5781	0.10384	0
0.4158	0.4199	0.895486	1.0632	0.8972	0.127205	0
0.10578	0.3582	0.802203	0.7931	0.2981	0.971719	0
0.256	0.198	1.114836	1.2874	0.4723	0.810833	0
0.0048	0.2508	1.008125	3.3321	0.6447	1.414007	0
1.0249	1.0201	0.054643	2.7022	1.9381	0.468187	0
0.29877	0.5728	0.46384	1.7595	1.5107	0.085998	0
0.6985	1.8892	0.416468	1.1393	0.8242	0.398499	0
0.5399	5.9205	14.9865	3.5018	2.185	0.688426	1
0.5119	4.2817	0.200577	7.448	8.7338	0.144444	1
0.2454	0.8335	0.178011	2.5346	0.0686	1.382491	0
0.6129	0.7325	0.271814	1.5587	0.2241	1.512305	0
0.9717	0.6412	0.37335	2.8688	1.1681	0.007184	0
0.409	0.2327	1.03339	0.8736	0.308	0.888435	0
0.28633	1.4563	0.040983	0.1661	0.9172	0.113339	0
0.16064	1.0689	0.038009				

0 0016	0 91177	0 079422
0 75854	0 75694	0 016126
0 57006	0 18848	0 194896
1 4802	0 91014	0 078506
0 3857	1 09415	0 215807
0 24831	0 13739	0 242615
0 85496	0 60665	0 000543
0 21792	0 63704	5E-05
0 24021	0 02229	0 369251
0 53323	0 29302	0 113522
0 66347	0 13024	0 214971
0 54477	0 1187	0 261377
1 49204	0 94727	0 100692
0 33967	1 15237	0 272923
1 53458	1 19491	0 31918
0 12688	1 4077	0 604895
0 3837	0 29682	0 139226
0 17369	0 21001	0 17635
1 09075	0 91706	0 082432
0 8579	0 23285	0 157689
1 53458	0 01243	0 381331
2 85433	1 80886	1 388828
0 17073	2 4836	3 436018
1 04069	0 86996	0 057605
0 6907	0 34999	0 078378
0 7697	3 079	5 997845
0 37848	3 39121	7 824558
0 15436	0 22413	0 18469
0 10117	0 05319	0 332652
0 86985	0 78668	0 019246
0 27494	0 58491	0 001228
0 45232	0 17738	0 20482
0 89287	0 44055	0 035872
1 33565	0 44278	0 035033
0 43916	0 89649	0 071044
0 42429	0 01487	0 378324
0 58126	0 15697	0 22371
0 80901	0 22775	0 161765
3 85159	3 04258	5 820783
1 71492	2 13667	2 270205
1 20854	0 50638	0 101827
0 47934	0 7292	0 009851
0 05394	0 4254	0 041841
0 59657	0 54463	0 00728
0 57512	0 02345	0 367842
0 48892	0 0862	0 295684
0 26946	0 21946	0 188502
2 17822	1 80576	1 837913
0 19206	1 98716	1 842019
1 18614	0 99408	0 132591
0 64037	0 54577	0 007066
0 925	0 28463	0 118246
0 77458	0 15042	0 229949
1 15322	0 37864	0 063157
0 50354	0 64968	0 000389
0 17703	0 32651	0 062076

0	1 0614	0 8187	0 189362
0	1 5135	0 4521	0 642816
0	0 4411	1 0724	0 032927
0	0 2911	0 15	1 218902
0	1 5841	1 283	0 001532
0	2 8301	1 246	8 2E-06
0	0 0119	2 8182	2 447166
0	2 1485	2 1366	0 779233
0	0 1828	1 9657	0 506719
0	0 5541	0 3713	0 778909
0	1 8375	1 0834	0 029056
0	2 295	0 6615	0 350888
0	1 999	0 3	0 909845
0	5 1584	3 1594	3 83109
0	0 4385	4 7199	12 01345
0	1 1477	0 7052	0 296852
0	0 8522	0 2955	0 91845
0	2 1599	1 3077	0 002899
0	1 243	0 9169	0 113541
0	0 1446	1 0984	0 024167
0	0 335	0 1804	1 130943
0	0 166	0 169	1 176917
1	0 6913	0 7253	0 276374
0	1 457	0 5657	0 473561
0	3 0771	1 6201	0 134133
1	2 8835	0 1938	1 124147
1	1 1726	1 1706	0 208887
0	1 8527	0 4801	0 598701
0	0 9697	0 853	0 38103
0	2 2391	1 2394	0 000209
0	1 5114	0 7277	0 278842
0	3 2384	1 727	0 223863
0	0 703	2 5354	1 84235
0	1 3488	0 6458	0 369734
0	1 1794	0 1694	1 176049
0	1 8635	0 4841	0 592527
0	0 8759	0 7876	0 217398
0	1 4134	0 5375	0 913189
1	1 4224	0 009	1 549671
0	0 9118	0 5106	0 552432
0	1 8581	0 7463	0 257815
0	0 838	0 8201	0 186146
0	1 457	0 819	0 403045
0	3 1495	1 8925	0 192407
0	0 5002	2 8493	1 847258
0	0 122	0 3782	0 786777
0	1 1251	1 0031	0 062888
0	0 7587	0 3684	0 784036
0	0 4596	0 2969	0 815769
0	2 2507	1 7809	0 288414
0	1 8965	0 3542	0 809384
0	0 2841	1 8124	0 128552
0	0 3047	0 0206	1 520925
0	0 3348	0 0301	1 497584
0	1 6301	1 2953	0 001717
0	1 0067	0 6204	0 401269

Sum X	11894 7
X Average	1 462002
MR Average	1 253858
Normal constant d2 (n=2)	1 128
Normal constant D4 (n=2)	3 268
Upper control limit for X (UCLx)	4 826731
Lower control limit for X (LCLx)	-1 84273
Upper control limit MR (UCLmr)	4 087608
MR Standard deviation	1 148045
Process standard deviation	1 224745
Gamma d2	1 023771
Gamma d3	0 937375
Gamma D4	3 746831
1.5 Alpha=1.5	
1 Beta=10	
Proportion MR > UCLmr	0 030103
Percentage	3 010301
ARL for MR	33 21927

0	5 8529	1 3814	24 83153
0	2 0574	3 7855	6 599888
0	2 3579	0 3005	36 7724
0	15 1303	12 7724	41 06087
0	3 3529	11 5774	27 17407
0	9 7395	6 1776	0 634941
0	19 6814	9 9509	12 86209
0	5 0246	14 6568	66 76183
0	3 9222	1 1044	27 66891
0	19 0629	15 1427	77 05636
0	7 234	11 8289	29 8594
0	13 3741	8 1401	0 050368
0	12 0196	1 3545	25 10035
0	13 1518	1 1322	27 37722
1	3 9464	9 2054	8 070572
0	8 9793	5 0329	1 73224
0	9 7067	0 8174	30 77059
0	2 0922	7 7045	1 795534
0	3 1824	1 0902	27 8185
0	11 1585	7 9741	2 590733
0	19 7827	8 6282	5 115175
0	2 8746	16 9081	111 187
0	11 3647	8 5101	4 803493
0	8 7935	2 5812	14 23798
0	0 8146	8 1789	3 291958
0	11 0271	10 4125	16 3861
0	4 7896	8 2375	0 018135
0	2 9478	1 842	20 45323
0	15 5309	12 5833	38 67317
0	0 3406	15 1903	77 89431
0	1 841	1 3004	25 64536
0	13 3353	11 8943	28 4065
0	9 7819	3 5534	7 902422
0	7 8035	1 9784	19 23809
0	3 8585	3 945	5 8541
0	5 8192	1 9607	19 39367
0	3 5588	2 2904	18 84384
0	5 1847	1 6299	22 45458
0	9 6041	4 4194	3 78351
0	3 2515	6 3526	0 000142
0	13 1083	9 8568	12 19599
0	12 2112	0 8971	29 89273
0	1 4936	10 7176	18 94626
0	3 5336	2 04	18 70151
0	5 1041	1 5705	22 98267
0	10 3252	5 2211	1 30742
0	8 1404	2 1848	17 4701
0	1 7829	8 3576	4 9E-05
0	32 4394	30 8565	590 1001
0	1 3555	31 0639	811 0475
0	10 7671	9 4118	9 284668
0	10 2371	0 53	34 04188
0	5 274	4 9631	1 963991
0	8 485	3 211	9 944718
0	9 3693	0 8843	30 03286
0	15 0543	5 685	0 461754

Sum X	60714 63
X Average	7 552796
MR Average	6 384525
Normal constant d2 (n=2)	1 128
Normal constant D4 (n=2)	3 298
Upper control limit for X (UCLx)	24 47972
Lower control limit for X (LCLx)	-8 37413
Upper control limit MR (UCLmr)	20 79927
MR Standard deviation	5 814637
Process standard deviation	6 123724
Gamma d2	1 039323
Gamma d3	0 949526
Gamma D4	3 740803
1.5 Alpha=1.5	
5 Beta=10	
Proportion MR > UCLmr	0 028803
Percentage	2 880288
ARL for MR	34 71875

0	2 149	0 748	142 8948
0	5 674	3 525	84 29154
0	5 886	0 212	156 1011
0	63 996	58 11	2061 519
0	27 83	36 366	559 7936
0	12 004	15 626	8 526152
0	5 2713	6 731	35 70113
0	23 137	17 864	26 60453
0	31 635	8 486	17 70782
0	8 413	23 222	110 5854
0	5 033	3 38	86 97907
0	0 589	4 444	68 26134
0	1 428	0 839	140 8267
0	2 504	1 076	135 2579
0	9 685	7 181	30 52609
0	18 082	8 397	18 56785
0	4 134	13 948	15 42459
0	4 808	0 674	144 77
0	27 68	22 872	103 3467
0	10 534	17 146	19 71322
0	3 847	6 687	36 22887
0	2 818	1 029	136 3533
0	4 456	1 838	122 5016
0	13 194	8 738	15 74536
0	19 216	6 022	44 87642
0	26 184	8 968	32 82513
0	11 545	14 839	3 786325
0	5 443	8 102	43 61338
0	6 888	1 245	131 3555
0	9 517	2 829	97 55997
0	1	8 517	17 54808
0	11 869	10 869	3 374725
0	39 98	28 111	237 3127
0	28 154	11 826	0 774475
0	49 722	21 568	78 53429
0	19 656	30 086	301 3681
0	15 224	4 432	88 45978
0	8 963	8 261	19 7584
0	15 716	6 753	15 82654
0	48 849	33 133	417 2606
0	11 271	37 578	818 8143
0	4 054	7 217	30 12959
0	5 04	0 886	137 3594
0	21 566	16 526	14 59208
0	41 809	20 043	53 83095
0	2 317	39 292	706 8131
0	15 494	13 177	0 221801
0	37 879	22 385	93 88222
1	3 531	34 348	468 3743
1	5 103	1 572	123 0636
0	13 769	8 866	16 32194
0	19 281	5 512	51 75425
0	7 007	12 274	0 186691
0	18 795	9 788	8 514971
0	39 344	22 549	96 88381
0	9 024	30 32	310 2515

Sum X	11894 7
X Average	1 462002
MR Average	1 253858
Normal constant d2 (n=2)	1 128
Normal constant D4 (n=2)	3 268
Upper control limit for X (UCLx)	4 826731
Lower control limit for X (LCLx)	-1 84273
Upper control limit MR (UCLmr)	4 087608
MR Standard deviation	1 148045
Process standard deviation	1 224745
Gamma d2	1 023771
Gamma d3	0 937375
Gamma D4	3 746831
1.5 Alpha=1.5	
1 Beta=10	
Proportion MR > UCLmr	0 030103
Percentage	3 010301
ARL for MR	33 21927

0	2 149	0 748	142 8948
0	5 674	3 525	84 29154
0	5 886	0 212	156 1011
0	63 996	58 11	2061 519
0	27 83	36 366	559 7936
0	12 004	15 626	8 526152
0	5 2713	6 731	35 70113
0	23 137	17 864	26 60453
0	31 635	8 486	17 70782
0	8 413	23 222	110 5854
0	5 033	3 38	86 97907
0	0 589	4 444	68 26134
0	1 428	0 839	140 8267
0	2 504	1 076	135 2579

2 Alpha=2				2 Alpha=2				2 Alpha=2				2 Alpha=2			
1 Beta=1				1 Beta=1				1 Beta=1				1 Beta=1			
MR	(MR1-MR)²	MR	(MR1-MR)²	MR	(MR1-MR)²	MR	(MR1-MR)²	MR	(MR1-MR)²	MR	(MR1-MR)²	MR	(MR1-MR)²	MR	(MR1-MR)²
1.4592	0	0.5365	0.7071	0.0006	0	0.4237	0.8358	0.8245	0.4520	0	0.16216	3.8634	2.7325	26.7326	0
0.507	0.63892	0.00927	1.42928	0.89283	0.24785	0	0.41794	0.0523	2.06879	0	0.2339	0.6019	8.002216	0	0
1.96461	1.45781	0.521878	1.28121	1.4807	0.34472	0	0.50677	3.8117	1.469556	1.3093	1.0754	0.178223	0	0	0
1.51783	0.44678	0.083185	2.20389	0.92268	0.035149	0	0.57825	5.1948	13.86995	0.503	0.8063	0.477847	1	0	0
0.7052	0.74231	5.1E-26	0.51133	1.89216	0.918924	0	0.2185	3.5775	43.28129	2.3796	1.8786	1.103467	0	0	0
0.33629	0.43923	0.087757	0.52713	0.016	0.517247	0	0.39743	1.7893	0.0851100	4.9514	2.2118	0.510319	0	0	0
0.29226	0.04403	0.047714	1.94762	1.42049	0.469624	0	0.63683	2.394	0.803596	1.7329	2.8585	1.852144	0	0	0
0.67991	0.38785	0.12079	0.77296	1.14668	1.93126	0	0.32771	3.0912	2.539672	4.4168	2.8659	1.41214	0	0	0
0.93086	0.98005	0.417508	0.39667	0.37629	0.128915	0	0.25168	0.7603	0.34356	0.7732	3.6456	4.614054	0	0	0
0.03181	0.55925	0.030958	0.0922	0.30467	1.85355	0	0.14661	1.7319	0.054913	0.268	0.3209	1.364541	0	0	0
0.70344	0.67183	0.040416	1.75333	0.43333	0.00844	0	0.31199	0.6812	0.666492	0.5889	1.0431	1.724665	0	0	0
0.6277	0.07574	0.004826	2.11478	1.37945	0.41508	0	0.20956	0.6295	0.735337	1.5082	1.2402	0.066237	0	0	0
2.13236	1.50466	0.592071	0.40891	1.0787	0.946089	0	0.19607	1.0449	1.82703	0.68	0.8282	0.44805	0	0	0
1.36036	0.772	0.001354	0.18493	0.22198	0.263393	0	0.95453	1.3964	0.102024	0.5582	0.1218	1.89273	0	0	0
0.45386	0.9065	0.029492	0.26985	0.08192	0.426773	0	0.39331	2.7988	1.693212	1.1226	0.5644	0.870797	0	0	0
0.45176	0.0021	0.537434	0.24741	0.01944	0.512311	0	0.13178	3.2153	0.959613	3.9492	2.8268	1.766334	0	0	0
0.55373	0.10197	0.400979	0.32642	0.07901	0.430584	0	0.29456	2.8108	1.724598	0.4304	3.5188	4.085391	0	0	0
0.65759	0.10386	0.398589	0.80426	0.27784	0.209177	0	0.4724	1.7754	0.071792	2.4314	2.001	0.253447	0	0	0
0.82748	0.18989	0.319574	0.32289	0.28137	0.20596	0	1.14466	3.5784	4.334037	2.8441	0.4127	1.176932	0	0	0
0.69111	0.66363	0.107867	0.30961	0.01328	0.521187	0	0.19522	0.8072	0.467002	0.839	2.0051	0.257992	0	0	0
0.52711	1.334	0.358563	1.10381	0.7942	0.003481	0	1.105	0.9026	0.35446	0.1777	0.2387	1.584741	0	0	0
1.0628	0.52549	0.043878	0.38139	0.74242	5.2E-05	0	0.15386	0.4866	1.02202	1.4385	0.3608	1.292235	0	0	0
1.24332	0.18072	0.330026	1.37568	1.01429	0.077892	0	0.4969	0.3024	2.355718	3.2787	1.8402	0.117399	0	0	0
0.10024	1.14306	0.166367	1.60878	1.0078	0.252103	0	0.3543	1.026	0.223374	1.075	2.2037	0.490627	0	0	0
1.27144	1.1712	0.190079	3.8339	3.23212	2.52785	0	0.28751	0.6879	0.888344	0.2431	0.9319	0.44311	0	0	0
0.89941	0.31703	0.131862	0.20244	3.73318	8.867772	0	0.57773	2.2978	0.640376	2.4937	2.2506	0.967082	0	0	0
0.24844	0.85097	0.007094	0.79551	0.94777	0.01872	0	0.12341	0.4558	1.083192	1.383	1.1107	0.148685	0	0	0
0.36229	0.11385	0.386274	1.70679	0.91128	0.031005	0	0.14777	0.4436	1.110842	1.5847	0.2017	1.879286	0	0	0
0.31208	0.05021	0.06901	0.37869	1.3271	0.350347	0	0.03218	1.1039	0.164972	2.4564	0.8717	0.391707	0	0	0
1.19	0.87782	0.020369	0.9073	0.52761	0.043093	0	0.17304	1.3566	0.019871	1.4357	1.0207	0.2274	0	0	0
1.48402	0.29402	0.029439	0.65437	0.42493	0.153875	0	0.4011	1.3293	0.028313	1.8291	0.3934	1.219181	0	0	0
0.89941	0.68431	0.016172	0.89117	1.288	0.370149	0	0.2174	0.7738	0.876187	0.4538	3.3763	0.29238	0	0	0
1.16667	0.37866	0.128335	0.54107	1.0501	0.34234	0	0.39884	1.4237	0.004556	1.4083	0.9645	0.29492	0	0	0
1.79678	0.83011	0.011044	0.65817	1.1171	0.145849	0	0.3575	0.0234	2.173163	3.5192	2.1078	0.372509	0	0	0
0.49117	1.30951	0.325369	1.40307	0.2951	0.230495	0	0.1811	1.764	0.070988	0.8213	2.6949	1.433811	0	0	0
1.22667	0.7385	0.00389	0.98228	0.31079	0.180123	0	0.25827	0.7717	0.526988	0.714	0.12701	1.932837	0	0	0
0.66346	0.42321	0.097337	0.73989	0.36922	1.046389	0	0.4852	2.0975	0.369922	1.851	1.137	0.130007	0	0	0
0.85942	0.05596	0.461365	1.09104	0.53135	0.14734	0	0.11336	0.6484	0.721081	3.4411	1.5901	0.008963	0	0	0
0.42042	0.439	0.087734	0.26311	0.82793	0.008599	0	0.21336	1.0	0.247571	1.6794	1.7817	0.068787	0	0	0
2.2198	1.79948	1.132695	0.18371	0.0794	0.430072	0	0.1177	1.0159	0.232001	1.7907	0.1113	1.921731	0	0	0
0.96019	1.25971	0.715112	0.21645	0.24888	0.493448	0	0.24888	1.1309	0.134443	0.6461	1.1446	0.124984	0	0	0
0.93978	0.0204	0.019337	1.01018	0.70371	0.003474	0	0.18795	0.5691	0.802407	0.4248	0.2215	1.628342	0	0	0
1.98921	1.04942	0.098735	0.57462	0.56446	0.029152	0	0.08995	0.40	0.28774	2.506	2.0814	0.340653	0	0	0
1.94906	0.04015	0.483093	0.20138	1.37326	0.407122	0	0.38056	3.1061	2.587385	1.4143	1.0917	0.164726	0	0	0
0.83006	1.119	0.147303	0.4737	0.27234	0.214238	0	0.30028	0.8028	0.482899	0.5185	0.8858	0.362121	0	0	0
0.13089	0.69917	0.001298	0.8698	0.39801	0.114988	0	0.19388	1.064	0.187879	0.9955	0.437	1.124798	0	0	0
0.36347	0.23206	0.252928	1.5379	0.6881	0.002218	0	0.92333	1.0155	0.323267	1.517	0.5619	0.876219	0	0	0
0.54269	0.17922	0.309112	0.38625	2.12185	0.236635	0	0.089	0.0293	0.195802	1.0986	0.4174	1.186757	0	0	0
0.99887	0.45618	0.077851	1.08401	0.74778	0.000158	0	0.1961	0.4921	0.100026	2.0308	0.931	0.320996	0	0	0
0.67385	0.32482	0.186329	0.59584	0.48817	0.061023	0	0.21868	0.7807	0.013898	2.2136	0.183	1.728801	0	0	0
0.55981	0.11434	0.385486	0.17414	0.4217	0.098281	0	0.39035	3.0335	2.359906	2.1562	0.0674	0.270478	0	0	0
0.29544	0.26417	0.221988	0.58456	0.41042	0.105481	0	0.2954	2.9345	0.264782	4.7932	1.56	1.128768	0	0	0
1.48479	1.18935	0.205253	1.84034	1.25578	0.271005	0	1.4483	0.8165	0.46395	3.0847	1.6218	0.01536	0	0	0
1.27113	0.89234	0.001837	0.83284	1.0075	0.074148	0	0.1014	0.3479	1.3173	0.6718	1.4228	0.856245	0	0	0
2.07418	0.10297	0.399713	2.31768	1.48482	0.561932	0	0.3128	0.2266	1.615352	2.9112	2.2394	0.550319	0	0	0
0.74309	1.33107	0.355063	0.42139	1.89447	1.34391	0	0.1322	1.7992	0.080848	1.0955	1.8157	1.01021	0	0	0
0.11962	0.62327	0.112528	1.42725	1.04066	0.072289	0	0.1778	1.3512	0.021423	1.1496	0.0541	0.285951	0	0	0
1.04836	0.92654	0.036611	0.82237	0.03355	0.173331	0	0.2184	1.0584	0.192866	0.338	0.818	0.470548	0	0	0
0.287	0.77036	0.09195	0.90831	0.08461	0.423266	0	0.24038	4.0308	1.137988	0.4853	0.1273	1.877628	0	0	0
0.61687	0.34987	0.148478	0.52452	0.38379	0.123488	0	0.2424	2.1612	0.414041	1.3187	0.8534	0.414949	0	0	0
0.37582	0.24105	0.244183	0.5653	0.40478	0.482217	0	0.59221	0.3497	1.175594	3.6286	2.3079	0.959843	0	0	0
0.1812	1.23818	0.082982	1.98087	1.14557	0.462905	0	0.34299	2.8378	1.78923	1.0673	2.5933	1.127281	0	0	0
1.16682	0.44538	0.083095	0.84681	1.32428	0.358474	0	0.2748	3.1851	2.74422	3.441	2.137	0.787812	0	0	0
0.88473	0.48189	0.064185	0.78561	0.139	0.359053	0	0.29324	2.1186	0.385894	1.5149	1.8261	1.183642	0	0	0
0.95824	0.27351	0.213156	0.63396	0.51585	0.340529	0	0.47738	2.3802	0.779044	1.3833	0.1316	1.869861	0	0	0
1.6932	0.73496	5.7E-06	1.06299	0.42903	0.893739	0	0.1741	3.0825	2.337842	0.314	1.0693	0.183411	0	0	0
1.18949	0.90371	0.093587	0.38389	0.6991	0.004369	0	0.63989	4.6498	9.946895	2.478	2.164	0.444135	0	0	0
0.2123	0.92826	0.23923	1.08071	0.1308	0.000489	0	0.2641	1.528	0.63284	1.9478	0.5628	0.8384	0	0	0
0.70442	0.48193	0.085269	0.63189	0.47508	0.067862	0	0.18458	0.9017	0.825754	2.2975	1.467	0.000928	0	0	0
0.39066	0.31376	0.177611	0.21448	0.41741	0.10099	0	0.46559	2.6201	1.260085	1.9677	0.9308	0.321223	0	0	0
0.57023															

176092	0.21436	0.271273	0	3.1981	0.159	1.791757	0	16.3204	9.3427	3.968025	0	6.78	9.159	36.42707	
131633	0.44459	0.084453	0	1.2343	1.9638	0.217375	0	8.7858	9.332	4.781032	0	12.888	6.106	82.66419	
185563	0.5393	0.038376	0	1.8901	0.8558	0.706568	0	10.6708	3.8824	11.992268	0	8.542	4.346	117.6886	
079604	1.05959	0.10523	0	0.0268	1.8633	0.133762	0	22.1041	11.4333	16.710055	0	4.196	4.344	117.733	
143775	0.84171	0.00674	0	1.8897	1.8629	0.13347	0	18.4468	3.8573	13.60242	0	18.252	14.054	1300704	
0.7011	0.73605	7.2E-07	0	3.0216	1.1319	0.133711	0	16.8171	1.6297	32.68975	0	18.786	0.514	215.9186	
124505	0.54335	0.036806	0	0.2774	2.7492	1.96659	0	4.7897	12.0474	22.10838	0	10.773	7.953	51.86137	
131938	0.07433	0.436747	0	0.8646	0.5922	0.818966	0	12.4005	7.6308	0.081827	0	52.707	41.834	715.0017	
0.87127	0.64811	0.007564	0	3.8849	2.8203	1.749628	0	8.0046	11.5959	18.06636	0	14.675	38.032	521.5521	
0.57905	0.09622	0.406294	0	1.3329	2.352	0.730059	0	20.5349	19.7303	153.3846	0	42.513	27.838	159.8585	
1.83525	1.0602	1.05628	0	1.2939	0.039	2.127412	0	11.9881	8.5468	1.443252	0	17.887	24.626	88.95349	
0.89308	0.66517	0.004765	0	0.2606	1.0333	0.215642	0	23.8869	11.8988	18.95169	0	12.317	5.57	92.8307	
0.56465	0.40443	0.109408	0	3.4561	3.2055	2.917042	0	5.9832	17.7237	107.7082	0	15.283	2.946	150.0254	
0.8341	0.26945	0.216922	0	1.2857	2.1804	0.466264	0	8.2003	2.2371	26.09519	0	33.58	18.417	10.38461	
1.04715	0.21306	0.272639	0	2.1711	0.8854	0.374746	0	14.0266	6.6263	2.307803	0	13.789	18.691	22.06726	
1.44734	0.40019	0.112231	0	2.4922	0.3211	1.384607	0	15.9795	1.9529	29.07955	0	13.5	0.289	222.1735	
0.31845	1.12889	0.154993	0	2.0153	0.4769	1.091757	0	12.3757	3.8038	13.99991	0	8.833	4.667	110.8279	
0.23343	0.08502	0.422732	0	2.5281	0.5108	0.973705	0	0.6254	11.7703	19.57834	0	48.129	37.296	486.477	
0.96544	0.33201	0.162561	0	3.4093	0.8832	0.377444	0	7.836	7.2306	0.01519	0	22.763	23.366	68.77367	
1.78874	1.2233	0.238243	0	5.2056	1.7963	0.089243	0	20.5912	12.7552	29.26544	0	22.455	0.306	221.6074	
0.58918	1.19956	0.215631	0	2.807	2.3986	0.811964	0	5.7443	14.8469	56.27182	0	44.986	22.531	53.82446	
1.22973	0.64255	0.008958	0	0.4505	2.3565	0.737769	0	23.8837	18.1194	116.0781	0	30.907	14.079	1.244305	
0.61296	0.81717	0.013931	0	2.8689	2.4184	0.847837	0	13.1698	10.6939	11.21215	0	8.392	22.515	53.96995	
1.18347	0.57091	0.026991	0	6.841	3.7721	5.173508	0	13.365	0.1852	0.12601	0	27.885	19.593	19.34694	
0.90959	0.99288	0.020255	0	1.833	4.808	10.95898	0	7.9035	5.4615	3.549251	0	17.277	10.708	20.12854	
1.01835	0.42578	0.095752	0	3.3611	1.5281	0.000932	0	3.8041	4.0994	10.53681	0	5.898	11.379	14.56792	
0.87289	0.14346	0.350155	0	4.2909	0.9298	0.322357	0	7.8234	4.0193	11.08324	0	7.248	1.35	191.6697	
0.10712	0.78571	0.000935	0	0.3387	3.9612	6.020324	0	24.5469	18.7235	87.9479	0	21.352	14.104	1.189196	
0.66686	0.58186	0.030046	0	2.2087	1.87	0.138708	0	9.0947	18.4522	65.71947	0	4.298	17.054	3.457789	
0.7787	0.11072	0.389974	0	3.7286	1.5189	0.000374	0	7.837	1.2577	37.08085	0	23.648	19.55	18.97052	
0.44147	0.33823	0.157584	0	0.4663	3.2603	3.107234	0	23.127	15.29	63.11594	0	28.53	4.682	110.5123	
1.53396	1.39249	0.432032	0	1.5604	1.0841	0.182784	0	1.3369	21.7901	208.848	0	20.612	7.818	52.84722	
0.22124	1.61272	0.770044	0	1.9799	0.1195	1.899693	0	8.734	7.3971	0.002688	0	56.571	35.959	431.1651	
0.72987	0.50673	0.061288	0	1.8047	0.1248	1.884484	0	12.595	3.861	12.14136	0	13.808	42.965	771.2015	
0.67822	0.05175	0.487102	0	4.0544	2.2497	0.685707	0	10.5363	2.0587	27.94988	0	19.376	5.97	85.02111	
0.55818	1.2004	0.37842	0	5.2976	1.2432	0.064702	0	1.4578	9.0785	3.003478	0	11.107	8.469	45.33214	
0.78741	0.22923	0.256004	0	0.1468	8.161	13.34759	1	3.8532	2.1954	26.52297	0	35.099	23.992	77.39628	
0.14046	0.64893	0.007791	0	0.7806	0.834	0.745745	0	8.7934	5.1402	4.863108	0	23.824	11.275	15.36236	
1.46323	1.32276	0.345217	0	2.8384	2.0578	0.313863	0	21.588	12.7846	29.69328	0	17.028	8.786	70.53454	
0.80192	0.56131	0.030237	0	0.8645	2.1738	0.457429	0	22.9703	1.3823	35.59811	0	12.778	4.252	119.738	
1.23725	0.33533	0.159895	0	0.4844	0.1801	1.735714	0	4.9308	18.6323	127.3931	0	59.333	46.557	883.6074	
0.86376	0.37349	0.130833	0	3.5383	3.0539	2.422178	0	19.2767	14.9387	57.85751	0	17.975	41.358	604.5296	
1.59055	0.72879	7.1E-05	0	0.3131	3.2252	2.984722	0	7.828	11.4487	16.8367	0	33.632	15.657	0.213921	
0.11405	1.478	0.048787	0	2.8953	2.8482	1.319362	0	4.6531	3.7749	12.7488	0	11.939	21.893	42.23071	
0.99247	0.87792	0.020369	0	2.1775	0.7816	0.81232	0	9.9669	5.5138	3.354826	0	20.494	8.555	44.08275	
1.48641	0.49394	0.058208	0	3.9934	1.6159	0.101337	0	15.5115	5.9448	1.982368	0	26.858	8.364	77.8745	
0.63539	0.85102	0.013415	0	2.2887	1.7047	0.042905	0	13.337	2.1745	26.73868	0	34.553	7.695	56.24226	
0.46758	0.16783	0.321907	0	4.8982	2.6095	1.236399	0	15.0864	1.7494	31.31573	0	18.285	16.268	1.152436	
0.08155	0.38801	0.121933	0	0.864	4.0142	6.333451	0	15.4657	0.3793	48.52718	0	19.377	1.092	198.8801	
0.03388	0.04757	0.472833	0	1.008	0.124	1.886881	0	5.7328	9.7329	5.699938	0	35.313	15.936	0.549646	
1.36929	1.33531	0.360134	0	4.8298	3.6218	4.512374	0	7.2878	1.535	33.78128	0	25.513	9.8	29.10046	
0.31498	0.05433	0.101845	0	0.8728	3.757	5.105046	0	7.4703	0.2025	51.02167	0	11.044	14.469	5.28327	
1.18704	0.85208	0.013661	0	2.3595	1.4867	0.000118	0	1.3445	6.1258	1.487535	0	20.788	9.744	29.70778	
2.57829	1.41126	0.487045	0	4.8014	2.2919	0.630968	0	2.0177	0.6732	44.51886	0	13.649	7.139	64.89083	
3.03371	0.45542	0.078272	0	1.0574	3.594	4.399339	0	8.1477	8.13	14.73738	0	31.208	17.589	5.590935	
1.24636	1.78735	1.107022	0	1.8603	0.8029	0.800428	0	6.5869	1.5608	33.46212	0	74.48	43.292	787.2242	
Sum X	7972.997		1998.12	Sum X	15756.53		3868.599	Sum X	79444.32		19578.22	Sum X	161813.4		39046.71
X Average	0.997112			X Average	1.962513			X Average	9.902254			X Average	20.08601		
MR Average	0.735109			MR Average	1.497655			MR Average	7.346446			MR Average	15.19448		
Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128		
Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268		
Upper control limit for X (UCLx)	2.952427			Upper control limit for X (UCLx)	5.945399			Upper control limit for X (UCLx)	49.23801			Upper control limit for X (UCLx)	60.49687		
Lower control limit for X (LCLx)	-0.9582			Lower control limit for X (LCLx)	-2.02037			Lower control limit for X (LCLx)	-9.63351			Lower control limit for X (LCLx)	-20.3249		
Upper control limit MR (UCLmr)	2.462529			Upper control limit MR (UCLmr)	4.894043			Upper control limit MR (UCLmr)	24.00492			Upper control limit MR (UCLmr)	49.65557		
MR Standard deviation	0.640816			MR Standard deviation	1.325774			MR Standard deviation	6.437374			MR Standard deviation	13.18933		
Process standard deviation	0.707107			Process standard deviation	1.412174			Process standard deviation	7.071068			Process standard deviation	14.14214		
Gamma d2	1.039728			Gamma d2	1.058938			Gamma d2	1.038803			Gamma d2	1.074412		
Gamma d3	0.909568			Gamma d3	0.937464			Gamma d3	0.910382			Gamma d3	0.932626		
Gamma D4	3.814053			Gamma D4	3.855586			Gamma D4	3.629129			Gamma D4	3.804102		
2 Alpha=2				2 Alpha=2				2 Alpha=2				2 Alpha=2			
5 Beta=0.5				1 Beta=1				5 Beta=5				10 Beta=10			
Proportion MR > UCLmr	0.022002			Proportion MR > UCLmr	0.027103			Proportion MR > UCLmr	0.029203			Proportion MR > UCLmr	0.024802		
Percentage	2.20022			Percentage	2.710271			Percentage	2.520252			Percentage	2.480248		
ARL for MR	45.45			ARL for MR	36.89668			ARL for MR	39.67857			ARL for MR	40.31855		

3 Alpha=3					
0.5 Beta=0.5	MR	(MR-MR)*2	MR	(MR-MR)*2	
113356		109656	26038	44289	
96098	0.16358	0.111993	18479	0.009153	
79284	0.17734	0.590554	114622	0.021896	
86384	0.0712	0.785065	3.0667	1.9178	0.944006
249578	1.63194	0.470678	0.553	2.5137	2.458059
119956	1.29622	1.02238	1.80332	1.25032	0.092684
198204	0.78248	0.0267	2.01896	0.21364	0.536176
805691	1.12513	0.032131	0.71546	1.3015	1.264568
196639	1.10948	0.026785	199996	1.28452	0.114677
15281	0.48289	0.257468	1.6397	0.36028	0.342927
45312	1.07498	0.016687	2.74853	1.10883	0.028553
0.9751	0.52198	0.119691	1.67423	1.0743	0.018492
190565	0.93055	0.002035	3.29093	1.6227	0.458085
113906	0.76759	0.031787	80944	2.48749	2.378561
219589	1.05783	0.012533	1.80053	1.09109	0.021086
82639	1.36959	0.179454	8.06579	1.37174	0.181357
135417	0.52778	0.174809	1.86026	1.33147	0.14968
150078	2.04659	0.489007	1.24244	0.81782	0.107623
40815	1.11261	0.027799	1.17627	0.06617	0.772389
255708	2.06893	1.82141	1.4547	0.27843	0.44549
82639	1.599	0.425666	2.10122	0.64852	0.089618
120081	0.24273	0.48442	0.81374	1.48748	0.29333
2.60251	1.40476	0.102057	2.51736	1.90362	0.187266
2.78974	1.01187	0.580202	1.17083	1.33763	1.0533
0.64741	2.14233	1.431492	1.57144	0.39161	0.307215
2.24591	1.5985	0.425913	1.84583	0.27439	0.450999
1.41048	0.83545	0.012195	0.49848	1.34735	0.161178
133855	0.0738	0.36206	2.3989	1.7415	0.633011
124498	0.98169	0.729641	0.43386	1.80812	0.740913
0.96757	2.27339	0.446879	0.49717	0.06331	0.77893
1.66166	0.69409	0.063398	0.77081	0.27384	0.451907
0.48705	1.17481	0.052317	0.60743	0.18298	0.612933
1.8358	1.15153	0.042292	1.70442	1.09659	0.022713
0.48815	0.20931	0.418053	1.7523	0.0281	0.857523
2.22081	0.88154	0.00414	0.17833	0.6459	0.089898
0.53827	1.68254	0.542666	0.75352	0.32581	0.384487
0.87871	0.14044	0.648734	0.63018	0.12334	0.678572
1.98768	1.00987	0.00398	1.23671	0.60653	0.119158
1.81483	0.07285	0.762182	0.48109	0.73562	0.028988
1.79942	1.8459	0.579563	2.48638	0.02529	1.185128
1.1228	0.67662	0.02595	0.55879	1.92758	0.963754
0.27541	0.84739	0.0097	1.75097	1.19218	0.080664
0.84291	0.3675	0.334524	3.1816	1.43063	0.234982
1.78019	1.11728	0.029378	1.07892	2.10568	1.345136
1.46323	2.0698	0.421057	2.37249	1.29657	0.122983
2.91375	1.45052	0.424959	2.16	0.21249	0.537861
2.61202	0.30173	0.419229	0.77687	1.38313	0.191187
103386	1.57816	0.395778	2.22836	1.45151	0.254662
1.27981	0.24875	0.490182	1.54356	0.84882	0.08152
0.94163	0.33798	0.389543	1.88897	0.82541	0.102701
2.58192	1.59749	0.424959	2.80612	0.43715	0.258806
1.1917	1.34742	0.161234	1.78263	0.82349	0.014979
1.4392	0.2478	0.487735	0.86857	0.91406	0.010113
0.76981	0.66939	0.076447	1.8212	0.75263	0.037346
3.19775	2.42794	1.198502	1.38299	0.23821	0.50797
0.78168	2.0607	1.23125	0.25867	1.12432	0.031841
1.87318	1.0815	0.018393	2.31379	2.05512	1.230413
3.62232	1.74914	0.845228	0.90547	1.40832	0.213851
1.08196	2.34086	0.543323	0.85312	0.05235	0.798396
0.49089	0.59077	0.128103	0.79062	0.0628	0.78038
4.02068	0.07021	0.718619	1.498	0.70738	0.056882
2.40364	1.98298	0.175535	0.86702	0.63098	0.099162
2.17771	0.22593	0.518328	0.79268	0.07434	0.759882
1.97377	0.20394	0.550479	0.8306	0.03792	0.824392
1.14768	0.82609	0.1035	1.6161	0.84864	0.091228
1.53538	0.3877	0.311565	0.81515	0.89289	0.082134
1.95598	0.4206	0.275919	1.98389	1.16874	0.049667
0.87223	1.08375	0.019008	1.53943	0.44446	0.251422
1.70926	0.83703	0.011848	1.42553	0.1139	0.692191
1.3883	0.319952	0.161	0.63692	0.161	0.033952
2.82771	0.87341	0.005252	1.827	1.16308	0.047179
0.57613	1.68658	0.546636	2.34587	0.51887	0.182338
0.21655	0.35958	0.343748	3.00239	0.87452	0.073638
1.8863	1.66975	0.523888	2.43521	0.58518	0.130105

3 Alpha=3					
1 Beta=1	MR	(MR-MR)*2	MR	(MR-MR)*2	
0.19923	0.2077	2.782846	5.1019	3.2043	1.784462
0.2	0.2	0.2	0.2	0.2	0.2
0.5929	1.3259	0.290843	1.9337	2.1782	0.091393
0.1752	2.2528	1.412819	1.9177	1.4778	0.488223
0.59257	4.6505	0.768448	3.2131	1.2214	0.428353
0.5994	0.0683	2.67369	2.4762	0.7369	1.29729
0.3137	2.6803	0.647081	0.682	1.8142	0.003006
0.43006	0.9869	0.790297	2.8747	2.1227	0.113443
0.6837	3.8369	1.011168	0.8675	0.0772	0.017243
0.36644	3.0007	1.26205	5.1936	4.3261	0.600546
0.4238	2.4006	2.674162	0.5229	0.3293	2.39193
0.1602	2.3636	0.237864	1.9578	3.5653	2.854118
0.33549	2.2947	0.173405	2.4418	0.4842	1.936791
0.20739	1.281	0.35389	9.8541	5.123	0.132195
0.6432	1.5693	0.093995	7.4573	3.5032	2.848149
0.2708	0.9372	0.681132	2.5104	0.9469	0.431124
0.20028	0.7032	1.375194	2.6553	0.1449	2.996314
0.47416	2.7388	0.74462	3.2378	0.5825	1.672849
0.36668	1.0748	0.64174	2.959	2.0431	0.027196
0.2008	1.6608	0.046282	1.3675	9.134	0.151461
0.6444	1.8384	0.0564	2.3111	0.9436	0.889158
0.23292	1.3152	0.314389	1.7732	0.5719	1.790208
0.25992	0.93	0.894701	0.9811	3.9179	1.169819
0.27731	0.7068	0.79172	3.2037	2.4874	0.373949
0.18347	0.4384	0.068366	2.4082	0.7975	1.162918
0.24145	0.5798	1.87984	0.9998	1.4064	0.220418
0.2861	0.2485	2.8549	1.0909	0.0811	3.185463
0.42768	0.1644	2.861127	2.3827	1.2718	0.304921
0.73366	3.2602	1.818324	3.1487	2.4874	1.181553
0.58589	0.1221	0.378787	8.6128	5.466	1.287828
0.28666	2.9723	1.202123	1.3542	2.7586	28.9736
0.19085	0.8881	0.875222	2.1179	0.7837	1.236959
0.32387	1.2402	0.404097	1.0101	1.0778	0.598997
0.62117	2.6117	0.0281	1.7785	0.6081	1.220526
0.8447	3.2023	0.318689	2.9892	2.9892	0.373284
0.7378	2.9069	1.062989	2.7971	1.7384	0.019068
0.5744	5.0062	0.798862	2.6204	0.1787	2.887235
0.30079	2.7361	0.739967	2.8836	0.8632	3.285832
0.52291	2.2212	0.19241	1.151	1.5328	0.117946
0.43821	0.837	0.108285	8.16	5.006	9.8164
0.13713	3.0208	0.727127	1.4828	4.6972	7.95881
0.4732	3.0019	1.267906	1.5572	0.9844	3.178994
0.51801	0.8069	1.142732	2.5294	0.9722	0.818649
0.44223	1.7378	0.019068	1.2809	1.2485	0.393614
0.1808	1.8333	0.001814	2.8017	1.3208	0.308121
0.19029	0.2939	0.202861	2.1572	0.4445	2.048887
0.60579	2.155	0.5194358	1.2407	0.9185	0.920423
0.8022	2.2557	1.44258	4.939	3.9883	3.321191
0.19355	1.8687	0.84605	5.1703	0.2313	2.704965
0.43628	2.0847	0.048903	2.9257	1.4088	0.21817
0.07114	1.8912	3.295363	4.8514	2.8057	0.532828
0.71392	6.4878	2.108587	6.131	1.4796	0.157043
0.33997	3.7395	0.473055	1.8019	4.3281	0.810256
0.49232	1.5235	0.124178	4.3345	2.5326	0.431273
0.28385	2.0847	0.048903	2.9257	1.4088	0.21817
0.1908	0.6477	1.508442	2.7548	1.079	2.908793
0.18364	0.5544	1.746327	3.3892	0.8344	1.541289
0.17115	0.1531	3.030338	8.588	2.4688	0.351546
0.34157	1.8402	0.053789	1.2084	3.4496	0.693486
0.17014	1.743	0.02811	1.592	2.3836	0.257773
0.35318	1.6345	0.00208	6.736	1.144	1.608112
0.36887	0.3688	2.368787	0.8856	8.5804	0.579876
0.38034	0.0653	2.782224	1.979	1.0834	0.612285
0.65714	2.786	0.799866	6.8874	3.7084	3.358108
0.1808	4.7634	0.337374	2.3225	3.369	2.297881
0.58486	3.7786	0.327212	1.7198	0.9089	1.620495
0.38759	1.7087	0.027951	3.6706	1.951	0.005642
0.28823	1.7936	0.006771	2.9002	0.7704	1.2221
0.4867	2.5847	0.502417	2.1282	0.772	2.118950
0.1241	2.5429	0.14907	3.4009	1.2727	0.363834
0.58486	1.1882	0.58486	1.9829	1.62089	1.62089
0.32509	2.295	0.179586	1.5444	1.2619	0.376793
0.41958	0.945	0.88655	3.3094	1.785	0.012296
0.27151	1.4808	0.158093	3.8081	0.4987	1.898643

3 Alpha=3					
5 Beta=5	MR	(MR-MR)*2	MR	(MR-MR)*2	
0.133879		12.0306	11.7059	5.220333	
0.282834	2.3395	12.07149	10.2652	1.7654	58.60967
0.114273	14.8551	29.53826	16.7025	5.9093	12.36682
0.49504	3.8868	12.01743	19.38023	3.2778	38.3568
0.87105	0.6201	17.45751	21.0185	1.6202	60.85396
2.21972	13.4887	16.52914	24.8735	3.855	30.98141
0.81446	14.0526	21.45084	16.2492	8.6243	0.34882
0.160519	1.8072	2.291878	14.9287	1.3225	65.58724
0.181699	15.0266	31.42168	16.0799	1.1532	68.35809
0.712836	13.7948	19.12929	24.4535	8.3736	1.097246
0.117275	5.5561	14.93819	10.		

0.71458	1.67817	0.533323	0
0.56205	0.15253	0.629404	0
1.62221	1.66016	0.01306	0
1.29478	0.32743	0.362481	0
2.90884	1.61406	0.446464	0
1.67492	1.03392	0.007751	0
1.34955	0.52537	0.176629	0
2.56729	2.1774	0.073908	0
2.38204	0.18525	0.578558	0
1.33123	1.05081	0.01101	0
1.78229	0.45106	0.244847	0
1.13752	0.64477	0.090667	0
1.28171	0.14419	0.642707	0
1.55817	0.27545	0.448123	0
0.25367	1.3045	0.129608	0
0.61058	0.35692	0.346874	0
1.24466	0.63407	0.097226	0
1.28567	0.04101	0.81879	0
1.62065	0.33498	0.373199	0
2.94951	1.32886	0.146674	0
1.64934	1.30017	0.125521	0
1.01459	0.63475	0.096802	0
0.91895	0.09564	0.722908	0
0.69208	0.29887	0.516976	0
1.5341	0.84202	0.010787	0
0.63815	0.89495	0.062966	0
2.00544	1.16629	0.048581	0
2.51472	0.50928	0.19062	0
1.82021	0.69451	0.063187	0
1.32668	0.49355	0.204621	0
2.27802	0.95134	3E-05	0
1.02396	1.25406	0.094975	0
0.51988	0.50408	0.195187	0
1.47757	0.95769	0.000139	0
1.29798	0.17981	0.58717	0
0.84257	0.85538	0.084384	0
1.23191	0.58934	0.127121	0
0.65648	0.57543	0.137233	0
1.79636	1.13988	0.037836	0
1.13284	0.66372	0.079614	0
0.69707	0.43557	0.260418	0
0.57729	1.19178	0.682441	0
2.45953	1.88224	0.876777	0
0.79264	1.66689	0.519855	0
1.3965	0.60368	0.118978	0
0.44483	0.95187	3.8E-05	0
0.43707	0.00756	0.860445	0
3.19362	2.75655	3.278526	0
0.78692	2.4067	2.133995	0
1.48713	0.70021	0.060354	0
0.88953	0.6098	0.115179	0
0.94447	0.06364	0.777995	0
0.50322	0.44125	0.254852	0
1.29948	0.79626	0.022386	0
1.26842	0.03106	0.836898	0
1.37696	0.10854	0.701138	0

3.9144	2.4928	0.380708	0
2.3808	1.5336	0.11718	0
2.7889	0.3881	2.13509	0
3.8881	0.9172	0.19308	0
3.9023	0.2162	2.754559	0
2.7319	1.1704	0.497711	0
3.6534	0.9215	0.910854	0
1.0221	2.6313	0.570685	0
3.3363	2.3142	0.192119	0
3.3264	0.3069	2.461719	0
1.8301	1.1993	0.457769	0
8.9613	17.1312	27.61832	0
5.1912	3.7701	3.589045	0
2.7365	2.4541	0.335025	0
1.4718	1.2647	0.373549	0
2.1088	1.6371	0.057019	0
3.8685	0.7598	1.248006	0
7.9485	4.08	4.858116	0
3.0764	4.8721	8.977295	0
4.0681	1.0117	0.746818	0
7.4049	3.1168	2.078232	0
0.7026	6.7023	23.29427	0
1.2723	0.5697	1.708123	0
6.3811	5.1088	10.45173	0
5.0999	1.2812	0.353652	0
4.1517	0.9492	0.869502	0
2.298	1.8537	0.000492	0
4.7064	2.4084	0.283571	0
1.2243	3.4821	2.579822	0
1.9303	0.708	1.368634	0
4.8383	3.008	1.281681	0
5.049	1.1107	3.115883	0
0.9541	4.0949	4.924021	0
2.5135	1.5594	0.100184	0
0.545	1.9685	0.008577	0
0.9892	4.512	2.029732	0
1.450	0.4828	1.958814	0
4.7589	3.2999	2.027814	0
3.5093	1.2486	0.392235	0
2.1256	2.2637	0.174568	0
2.5581	1.3425	0.284501	0
3.8284	1.2703	0.368735	0
1.6055	2.0229	0.021613	0
2.6853	0.8798	0.992188	0
8.1674	2.4821	0.367495	0
5.0457	0.1217	3.077117	0
3.2189	1.8268	0.002409	0
3.085	0.1339	3.034517	0
3.8758	0.5908	1.651447	0
3.7417	0.0659	3.270851	0
2.5459	1.1968	0.462518	0
7.308	4.7631	8.338302	0
5.3673	1.9417	0.004331	0
4.9064	0.4609	2.002187	0
5.4371	0.5307	1.809527	0
1.8978	3.5395	2.78781	0

6.6104	9.4676	0.002163	0
17.1597	10.5493	1.272846	0
24.5984	7.4387	3.929899	0
14.7002	9.8982	0.227629	0
10.42405	4.2757	26.47509	0
23.5732	13.1487	13.89504	0
7.5233	16.0499	43.94105	0
30.7557	23.2324	190.7521	0
7.9181	22.8396	180.0563	0
20.2379	12.3218	8.414089	0
17.0617	3.1752	38.99871	0
11.2694	5.7923	13.16815	1
17.3318	6.0624	11.28083	0
8.2014	9.1304	0.084504	0
8.8224	0.621	77.44767	0
22.8244	14.002	20.98469	0
81.0014	38.177	828.8021	1
22.2537	38.7477	860.0498	1
42.3601	20.1064	114.1757	0
89.1748	28.8147	302.5375	0
18.9089	50.2859	1658.298	1
10.484	8.4448	0.952957	1
36.2258	25.7818	267.0186	0
16.3902	19.8358	108.4619	0
9.0705	7.3197	4.415861	0
14.154	5.0835	18.81473	0
38.1643	24.0103	212.8449	0
7.4259	30.7394	454.4275	0
14.4282	7.0003	5.860249	0
9.0297	5.3985	16.19737	0
16.8269	5.7972	13.13262	0
14.0226	3.8044	31.54728	0
5.8393	5.1832	17.95975	0
30.0947	24.2954	220.0568	0
11.24	18.8547	88.9929	0
23.4336	12.1938	7.688783	0
3.3823	20.0513	113.0013	0
9.4407	8.0584	11.30772	0
33.9585	24.5178	227.9105	0
14.7544	19.2041	95.70718	0
2.1325	12.8219	10.24515	0
9.5583	7.4258	3.981202	0
10.2454	0.6871	78.28267	0
8.8925	1.3529	65.09577	0
4.545	4.3475	25.74137	0
22.2217	17.8767	88.15501	0
5.9323	18.2894	47.17361	0
32.0271	26.0948	278.0124	0
8.4406	23.5885	200.6587	0
8.6977	0.2571	83.97881	0
10.378	1.8803	59.91991	0
19.3601	8.9821	0.192717	0
12.0562	7.2981	4.507108	0
7.2341	4.8279	21.08744	0
9.3573	2.1232	53.25927	0
23.7365	14.3792	24.5828	0

26.748	27.672	78.98499	0
16.757	9.991	77.32826	0
23.3	6.543	149.858	0
13.241	10.059	76.13697	0
59.886	46.645	776.1991	0
33.332	26.554	60.9828	0
13.403	19.929	1.309537	0
50.127	36.724	321.8203	0
36.556	13.571	27.18215	0
15.025	21.531	7.942438	0
14.787	0.228	344.3483	0
10.743	4.054	216.9821	0
9.635	1.108	312.464	0
27.12	17.485	1.68909	0
24.084	3.036	248.02	0
27.346	3.262	240.9527	0
15.99	11.356	55.18484	1
36.186	20.196	1.991909	1
40.708	4.522	203.4232	0
59.95	19.242	0.209189	0
46.027	13.823	23.63564	0
23.659	22.368	12.8404	0
37.938	14.279	20.30085	0
40.918	2.98	249.787	0
53.771	5.147	185.9855	0
44.171	6.4	107.841	0
25.843	18.328	0.208529	0
55.247	29.404	112.7706	0
42.543	12.704	36.97431	0
32.205	10.338	71.3459	0
31.574	0.831	329.565	0
39.133	7.559	126.0152	0
19.132	20.001	1.479507	0
12.307	6.825	143.0332	0
15.179	2.722	253.2124	0
10.039	5.14	186.1785	0
22.874	12.835	35.39834	0
35.772	12.898	34.65265	0
20.86	14.812	15.78195	0
82.085	81.125	1792.705	0
15.379	66.706	2296.456	0
29.951	14.572	17.74842	0
21.894	8.057	115.0825	0
28.675	8.781	144.0878	0
28.153	2.522	264.4738	0
33.624	7.471	127.9987	0
44.862	11.038	80.01959	0
31.595	13.067	32.69152	0
45.148	13.551	27.39109	0
27.714	17.432	1.829662	0
18.513	9.201	81.84635	0
19.85	1.337	304.4205	0
30.818	10.768	84.26868	0
17.836	12.782	36.03181	0
24.952	7.116	136.1574	0
14.548	10.406	70.20178	0

Sum X	12068.98	2948.656
X Average	1.501763	
MR Average	0.94588	
Normal constant d2 (n=2)	1.128	
Normal constant D4 (n=2)	3.268	
Upper control limit for X (UCLx)	4.017402	
Lower control limit for X (LCLx)	-1.01386	
Upper control limit MR (UCLmr)	3.081136	
MR Standard deviation	0.806728	
Process standard deviation	0.866025	
Gamma d2	1.092204	
Gamma d3	0.931529	
Gamma D4	3.558658	
3 Alpha=3		
0.5 Beta=0.5		
Proportion MR > UCLmr	0.021802	
Percentage	2.180218	
ARL for MR	45.86697	

Sum X	24155.91	6124.122
X Average	3.028003	
MR Average	1.875887	
Normal constant d2 (n=2)	1.128	
Normal constant D4 (n=2)	3.268	
Upper control limit for X (UCLx)	8.017093	
Lower control limit for X (LCLx)	-1.96106	
Upper control limit MR (UCLmr)	6.130397	
MR Standard deviation	1.583547	
Process standard deviation	1.732051	
Gamma d2	1.083044	
Gamma d3	0.920035	
Gamma D4	3.548471	
3 Alpha=3		
1 Beta=1		
Proportion MR > UCLmr	0.022002	
Percentage	2.20022	
ARL for MR	45.45	

Sum X	121403.4	29768.68
X Average	15.11721	
MR Average	9.421095	
Normal constant d2 (n=2)	1.128	
Normal constant D4 (n=2)	3.268	
Upper control limit for X (UCLx)	40.17311	
Lower control limit for X (LCLx)	-9.9389	
Upper control limit MR (UCLmr)	30.78814	
MR Standard deviation	7.820973	
Process standard deviation	8.860254	
Gamma d2	1.087854	
Gamma d3	0.903208	
Gamma D4	3.940466	
3 Alpha=3		
0.5 Beta=0.5		
Proportion MR > UCLmr	0.019402	
Percentage	1.940124	
ARL for MR	51.5414	

4 Alpha=4
0.5 Beta=0

MR	(MRi-MRj)^2	MR	(MRi-MRj)^2
108946	1.34045	68319	0.175901
113115	0.04169	100343	1.55346
147313	0.34198	0.81361	0.205996
802663	0.6463	0.190353	0.96071
116462	0.13779	0.55473	3.04077
294196	1.77734	0.48267	0.83027
523558	2.29362	1.466581	2.70156
233648	2.89912	0.299763	2.88078
358373	1.25727	0.030511	2.33833
112683	2.3769	1.675225	1.44737
119222	0.05761	1.050594	1.866204
146999	0.31077	0.995714	2.33357
249841	1.02842	0.002935	0.76878
0.70547	1.79294	0.50459	0.89671
177908	0.07361	8.1E-05	1.06790
100674	0.77234	0.096258	2.0267
148416	0.47742	0.366237	1.32002
119302	0.29114	0.626401	2.72489
2.80826	1.61524	0.283171	1.60333
253636	0.2718	0.657227	0.1834
333891	0.80255	0.076425	2.49703
268104	0.65787	0.180391	1.70884
107385	1.60719	0.2792	1.44575
2.08329	1.00844	0.065352	2.45072
2.18497	0.10188	0.962194	1.97449
2.77367	0.5887	0.243392	8.0167
2.39259	0.36108	0.492123	1.23461
1.2379	1.15469	0.005198	0.8733
2.09223	0.67133	0.04633	3.57618
1.72733	0.3819	0.490574	4.96337
1.46819	0.25914	0.67078	2.07248
2.93792	1.46973	0.194873	3.23038
2.08138	0.85654	0.051101	3.21117
1.70211	0.37827	0.499966	1.08442
4.77567	0.17356	0.43134	1.866169
2.54224	2.33343	1.564588	3.01971
2.21068	0.33156	0.046053	3.03136
2.2303	0.01962	1.299918	1.04378
2.59721	0.36891	0.512205	2.53077
3.54445	0.94724	0.018221	2.49459
3.97745	0.433	0.421714	1.78034
1.36549	2.61196	2.338957	3.0738
1.317	0.04849	1.069373	2.90567
0.66613	0.65087	0.186387	0.78488
1.46879	0.83296	0.062468	0.83805
2.96458	1.49579	1.07173	3.52927
2.25696	0.73762	0.119008	1.47305
1.5285	0.72846	0.125412	0.71918
0.81919	0.96931	0.030028	0.52272
0.77358	0.15439	0.881565	2.81603
2.84908	0.0735	0.985586	2.58664
3.67856	0.62948	0.064067	2.33107
1.48657	2.19199	1.230757	2.4291
3.30469	0.18188	0.811288	1.30265
2.30223	0.99754	0.007234	4.40551
2.24514	0.05709	0.051681	1.73743
1.35183	0.89331	0.035629	1.612
1.24376	1.08007	0.949699	1.83923
1.94909	0.70533	0.142329	1.42026
2.88619	1.0371	0.00207	1.73665
1.28216	1.72403	0.411439	1.80326
1.9826	0.72044	0.13156	1.06558
1.53041	0.45219	0.397411	2.18458
1.54924	0.01883	1.131598	1.56645
1.93452	0.38528	0.496248	3.28174
1.27457	0.85965	0.178529	3.07741
0.94892	0.32655	0.572968	0.87458
0.67764	0.27128	0.658232	1.96619
1.02262	0.34896	0.531891	1.81528
1.66785	0.94123	0.194003	0.73358
0.42869	0.71859	0.1128	1.46934
2.00635	1.05709	0.000951	2.71227
1.74905	0.2573	0.891112	1.52909
2.78042	1.04137	0.0017	1.44646

4 Alpha=4
1 Beta=1

MR	(MRi-MRj)^2	MR	(MRi-MRj)^2
5.9191	2.8703	4.9493	7.57269
0.85601	2.641	0.1978	2.3969
0.39627	5.1974	0.006885	2.7078
1.734	1.371	0.467196	2.4747
2.755	1.9791	0.047156	1.8631
0.37963	1.0311	1.427912	5.5821
0.7214	2.0549	0.019981	1.8335
1.1519	3.5737	1.897361	9.3948
0.30275	3.8532	2.126959	2.3486
0.4231	1.4046	0.625132	5.0867
0.4183	0.0138	4.7631	4.8722
0.3123	1.106	1.186651	2.1746
0.3494	0.0371	8.070413	2.7239
0.0424	7.8252	31.65605	2.0804
2.9484	2.4242	0.05196	7.3297
0.1457	1.973	0.997907	8.276
0.2014	2.0943	0.010394	2.28
0.40247	1.9733	0.049708	9.6012
2.1546	1.8701	0.106376	5.4373
0.4864	2.7094	0.26332	2.1985
0.28031	2.0609	0.01832	2.5079
2.2338	0.5693	2.646975	11.1522
0.24126	0.1788	4.070118	11.4548
2.3651	0.0473	6.17999	9.9874
0.67094	5.7141	12.37525	2.9267
0.307	5.0094	1.973197	1.6125
0.812	2.942	0.556138	1.7486
0.75884	0.2438	3.812853	4.0559
0.58406	0.0722	4.5118	3.1339
0.7656	1.8154	0.145048	4.0187
0.62762	1.3798	0.666595	2.026
0.2546	2.7518	0.308411	6.9374
0.37272	0.2028	3.974853	2.1477
0.7241	0.37274	0.708443	4.2387
0.1357	0.7823	0.203354	4.998
0.76537	2.618	0.17871	1.378
0.54638	0.0289	3.634181	0.5563
1.2845	4.1793	3.932476	6.4828
0.4158	2.8735	0.458664	3.7712
0.32135	0.8445	1.068895	2.0302
0.38424	0.4289	3.123536	3.1689
0.63801	2.7377	0.293185	4.2687
0.41444	2.2387	0.301458	2.8445
0.46882	0.5238	2.780958	9.4206
0.09449	0.9833	2.211309	7.2992
1.39689	0.412	1.823558	2.8343
0.4943	3.5461	8.120088	1.0473
0.44831	0.4599	3.014921	4.0385
0.55282	1.9401	1.325153	2.502
0.7923	1.7589	0.291219	3.5442
0.41001	1.3078	0.899348	1.8077
0.56635	1.9634	0.005003	2.5012
0.39088	2.5727	0.141713	4.8553
0.81772	0.5984	3.352951	5.1857
0.80877	2.0895	0.111396	2.9882
0.113659	1.5492	8.179887	5.4539
1.06703	9.6296	0.952865	5.5999
0.49816	3.2843	1.83847	4.843
1.7328	3.1588	0.928497	6.8429
1.4416	0.2912	3.822226	6.4316
0.26074	1.1658	0.108313	3.2852
0.862	4.2546	4.266783	5.7014
0.50478	1.8142	1.45864	5.5838
0.52498	0.2018	3.978628	2.4698
0.2201	3.0293	0.839386	6.2303
0.5637	3.4169	1.408922	7.7115
0.37578	1.8792	0.100522	4.9011
0.7827	1.9651	0.053432	3.1191
0.36811	1.8684	0.107487	1.1054
0.42889	0.4678	2.987459	5.2472
0.93781	5.642	1.344449	1.6387
0.48949	0.8932	3.344644	3.5473
0.3072	1.8229	1.39392	3.5759

4 Alpha=4
5 Beta=1

MR	(MRi-MRj)^2	MR	(MRi-MRj)^2
8.31246	13.2859	1.7718	86.5393
10.13439	18.3897	5.51281	29.7812
0.20192	4.7771	12.94096	20.6818
10.11011	9.8309	1.546429	19.9392
0.87744	2.3067	7.67302	58.6367
18.4734	10.699	0.140966	16.1637
18.1879	2.6555	70.8788	19.9367
1.42.8362	26.0203	223.3783	1.4637
22.7488	20.0884	81.25121	18.5256
0.86333	14.1865	9.864825	25.1769
29.20205	20.4572	88.03591	16.9174
17.8397	11.8608	0.011309	16.4946
14.7516	3.0081	83.78188	18.6537
6.6745	0.0771	9.984136	14.9024
10.0826	3.4081	58.773	5.5235
10.0395	0.0431	121.8908	7.2329
20.5736	15.0341	15.67878	28.9462
19.968	10.506	35.62223	9.9477
1.23.8243	3.5143	57.15584	17.701
18.413	0.0693	36.08188	24.5246
20.9994	2.5864	72.04707	31.8795
10.3019	10.6875	0.142095	15.8229
1.35.816	25.3141	202.7675	14.1548
9.3391	28.2789	231.1143	24.9122
17.9138	8.5747	8.248774	39.9358
0.83058	9.874	4.41092	19.9607
11.1277	3.0878	83.78506	13.108
29.09	17.9613	47.42864	16.2936
4.4855	24.6035	183.0351	9.3678
14.9487	10.4632	0.373632	7.0622
18.3578	8.6909	20.12226	14.2217
27.963	18.8052	72.7736	23.5553
12.2016	15.7614	21.96746	31.911
17.1267	4.9251	37.81456	7.1705
0.3367	9.78	1.848824	22.3429
20.6859	13.3492	5.174668	17.1818
35.1836	14.2077	1.178717	15.1251
9.1941	25.9995	222.757	10.6143
18.8565	7.8824	11.64212	15.9884
10.5338	8.3227	22.57917	11.8931
25.8295	15.2957	11.81891	21.6898
20.8556	4.8735	37.81877	9.0851
18.0121	2.8435	67.74862	36.8902
7.1999	10.8122	0.068778	20.8644
13.1438	5.9438	26.32259	20.5818
17.0054	3.8616	52.02527	37.225
18.0759	1.0705	10.07891	24.0638
31.5157	13.4396	5.594585	7.3441
5.7347	25.784	216.3709	19.3309
18.6429	12.9112	3.373633	18.8582
9.1052	8.5377	2.361615	32.4868
22.644	13.9388	6.072997	20.3307
19.7796	2.8644	67.405	23.0188
8.5011	11.4758	0.041634	18.3871
8.3955	10.1286	1.19.4399	21.0992
15.4507	7.0652	15.83447	28.1871
19.9442	4.6328	43.30897	15.2048
38.5544	10.6162	56.87446	29.8872
11.1458	27.8026	296.8043	14.1052
20.7709	9.4626	2.100629	21.5419
14.4916	8.2783	22.99351	10.7817
13.472	1.0198	101.0001	22.0743
35.9719	13.4197	5.001755	11.2576
26.123	0.8489	1.501985	10.7434
15.427	1.628	3.461386	23.4835
37.3383	11.2153	0.019837	28.8249
20.266	0.2458	1.19.1014	16.3366
21.1284	20.3747	334.859	14.4285
41.7987	20.8693	92.08105	9.8749
22.6213	19.1744	65.60919	37.8056
22.0502	0.5711	11.0.3205	5.5391
22.266	0.2458	1.19.1014	16.3366
28.2638	0.9676	25.77445	15.338
11.3412	16.9224	34.1046	11.8621

4 Alpha=4
10 Beta=10

MR	(MRi-MRj)^2	MR	(MRi-MRj)^2
38.562	29.241	3.927	8.2156
74.721	36.359	204.9885	32.353
49.915	25.006	8.789657	26.431
38.589	14.056	83.76391	58.676
62.044	26.195	17.	

1 09858	1 88549	0 84464	0	3 2761	0 5312	2 772401	0	16 7155	7 295	14 28428	0	36 421	3 452	345 5595		
1 49762	0 39924	0 466974	0	3 1285	0 1476	4 196978	0	30 7051	13 9896	8 498072	0	21 957	14 464	57 41443		
1 83378	0 38596	0 4853	0	1 4922	1 6363	0 313547	0	15 1981	15 507	19 64746	0	25 704	3 747	334 8788		
3 03508	1 1513	0 00472	0	6 0798	4 5876	5 718542	0	8 8178	6 3802	22 03603	0	44 879	18 975	9 401771		
1 75961	1 27547	0 037201	0	4 696	1 3838	0 660079	0	26 4512	17 6433	43 14973	0	36 613	8 066	195 3071		
1 5944	0 16521	0 841595	0	4 3288	0 3672	3 345434	0	33 7005	7 2393	14 70841	0	53 11	16 487	30 73849		
1 21829	0 37811	0 499121	0	3 8791	0 4497	3 050447	0	18 6952	14 0053	8 589584	0	21 115	31 995	99 07752		
1 15226	0 06603	0 1033405	0	2 0694	1 8097	0 149423	0	10 614	9 0612	3 873065	0	19 005	2 11	397 254		
2 23471	1 08245	2 1E-08	0	2 0337	0 0357	4 667988	0	45 5969	32 2204	447 151	0	57 185	38 18	280 4699		
1 79203	0 44268	0 409491	0	5 1436	3 7099	2 291128	0	1 1762	4 4207	920 8946	0	66 876	9 691	152 3282		
2 75685	9 96482	0 013871	0	5 0857	0 6579	2 366529	0	10 0273	5 8511	27 28344	0	21 38	9 079	168 0194		
0 80837	1 94848	0 749757	0	0 9591	4 1266	3 72624	0	13 046	3 0187	64 89519	0	34 662	11 549	110 0869		
1 08262	0 27425	0 853422	0	4 2454	3 2863	1 188203	0	16 9719	9 9259	51 10184	0	46 395	11 733	106 2596		
2 46288	1 38028	0 089604	0	3 8627	0 2827	3 861684	0	11 651	5 3209	33 10339	0	64 622	18 227	14 54836		
2 51487	0 05189	1 062147	0	2 3286	1 6101	0 343575	0	26 8786	15 2278	17 248682	0	30 459	34 163	146 9373		
1 99665	0 51622	0 320781	0	2 0817	0 2709	3 709693	0	2 6785	6 6549	19 53246	0	103 012	81 632	355 106		
1 14976	0 84889	0 054618	0	4 3722	2 2900	0 008883	0	15 435	4 7867	39 51071	0	24 124	76 888	3231 555		
1 84378	0 694	0 151006	0	7 0573	2 6851	0 238972	0	21 8837	6 4487	21 39781	0	93 344	69 22	2225 836		
1 38625	0 45781	0 380731	0	2 2399	4 7274	6 406708	0	18 3961	10 4781	0 355639	0	65 973	27 371	28 40644		
0 99044	0 39581	0 47 1674	0	0 072	4 51245	0	0	14 1157	4 2804	46 15918	0	44 852	21 121	0 848825		
0 66207	0 32837	0 568856	0	1 9361	0 3218	3 513573	0	16 4282	4 3128	45 72403	0	9 053	30 023	63 70664		
2 50843	1 84636	0 583337	0	1 73	0 2061	3 960708	0	14 2888	4 1394	48 09499	0	14 39	5 337	279 0313		
1 20405	1 30438	0 049189	0	6 5186	4 7866	6 709899	0	28 5099	14 2211	9 901376	0	18 361	3 971	326 5332		
0 74158	0 46247	0 384955	0	1 2897	5 2269	9 184823	0	10 2418	16 2681	51 74853	0	73 307	56 946	1218 343		
1 67927	0 93769	0 020997	0	6 0664	5 1167	8 529012	0	31 4489	21 2071	102 8706	0	42 113	33 194	124 3843		
2 31728	0 83801	0 187856	0	6 0654	0 341	3 441963	0	17 6751	13 7738	7 286465	0	69 118	27 005	24 83901		
1 09186	1 22542	0 020399	0	1 0819	4 9835	7 788747	0	12 5696	5 1055	35 62842	0	57 831	11 287	115 8535		
2 00821	0 91635	0 027837	0	1 2922	0 2103	3 944006	0	14 7748	2 2052	78 86368	0	31 742	26 089	16 38444		
0 97124	1 03687	0 002082	0	1 7081	4 0159	3 169656	0	22 8217	6 0469	9 169086	0	15 106	16 636	29 21652		
0 74158	0 17186	0 829438	0	7 6412	5 9331	13 86403	0	17 503	12 0963	10 7254	0	44 537	29 431	54 80669		
0 69055	0 10883	0 948219	0	4 5603	2 81	0 370688	0	2 7 551	15 4547	19 18655	0	79 945	35 408	178 6705		
3 06813	2 37558	1 67 8181	0	6 3403	1 78	0 173266	0	20 1547	7 3963	13 52862	0	20 713	69 232	1383 153		
1 79335	1 27278	0 03617	0	3 0188	3 3235	1 270886	0	30 5482	10 3815	0 468427	0	33 663	12 95	82 65048		
2 44329	0 64994	0 18719	0	8 5428	5 526	11 08722	0	14 8512	15 695	21 34944	0	27 065	6 608	238 1846		
2 95335	0 51006	0 327796	0	4 5103	4 0325	3 37 1804	0	9 5059	5 3509	32 75908	0	12 18	14 876	51 35486		
1 65227	1 27108	0 035527	0	4 2375	0 2728	3 699671	0	20 3689	10 8666	0 043204	0	36 104	23 924	3 54482		
1 85034	0 16807	0 836356	0	2 2727	1 9648	0 063557	0	10 4831	9 8838	4 178559	0	56 701	22 597	0 308879		
0 56348	1 28686	0 041724	0	2 5465	0 2738	3 695825	0	33 3854	22 9023	139 8979	0	14 104	44 597	506 7827		
1 35134	0 78786	0 086669	0	3 8363	1 2896	0 821657	0	22 8604	10 505	0 324279	0	16 189	2 085	398 2511		
1 80622	0 45488	0 394028	0	3 5781	0 2582	3 796049	0	26 0935	3 2131	61 8009	0	63 841	47 652	856 91115		
2 58392	0 7777	0 092961	0	0 9972	2 5808	0 147953	0	17 1817	8 9318	4 59087	0	41 203	22 638	0 306 134		
0 95846	1 62546	0 294702	0	2 6834	1 6862	0 260154	0	13 9575	3 2042	61 94091	0	57 402	16 199	34 13166		
0 64936	0 3091	0 596295	0	4 552	1 8688	0 107356	0	21 1978	2 2777	77 38289	0	21 949	35 453	179 8756		
3 02393	2 37457	1 869199	0	1 2117	3 3403	1 308844	0	27 9659	16 2861	27 16125	0	19 993	1 956	403 4185		
1 97857	1 04736	0 001242	0	4 1822	2 8705	0 589459	0	18 3537	8 4122	7 067901	0	50 875	30 882	78 15992		
1 0942	3 5969	8 321728	1	1 5155	2 9887	0 221321	0	13 6083	5 9474	26 28669	0	25 827	25 048	9 040661		
1 24324	2 86618	3 181175	0	0 5349	4 0194	3 323686	0	26 4122	12 8059	2 997902	0	31 175	5 348	278 864		
2 4238	1 18056	0 009597	0	2 201	3 3339	1 294241	0	17 0756	9 3366	3 020139	0	13 014	18 161	15 05619		
1 50753	0 91627	0 027664	0	2 6806	0 4796	2 946897	0	17 5518	4 0782	112 323	0	49 548	36 534	210 0404		
1 15817	0 35136	0 534705	0	4 4864	2 2058	9 1E-05	0	37 7903	20 2385	63 97972	0	24 474	25 074	9 197689		
1 81836	0 86219	0 17674	0	3 5321	1 3543	0 708884	0	19 7048	18 0855	48 15475	0	67 481	43 007	439 5635		
2 13654	0 31818	0 58433	0	8 8331	3 301	1 220468	0	17 0197	2 6851	70 38127	0	45 469	22 012	0 000054		
1 86201	0 27453	0 852989	0	7 32	0 4869	2 921887	0	9 7843	7 2554	14 58518	0	90 28	44 811	516 4624		
2 87165	1 00964	0 005322	0	7 0927	0 2273	3 878775	0	11 5141	1 7498	86 94919	0	54 168	36 112	197 8665		
0 67726	2 19439	1 236088	0	7 8196	0 7269	2 158998	0	15 9671	19 9492	39788 14	0	321468 8	40684 96			
Sum MR > UCLmr	15905		4063 998	Sum X	128	50	3208 97	7944 791	Sum X	146	31	159671	Sum X	124	27	321468 8
Sum X	1 9669			X Average	4 003177				X Average	19 9492			X Average	40 21518		
X Average	1 082595			MR Average	2 196253				MR Average	11 07445			MR Average	22 04123		
Normal constant d2 (n=2)	1 128			Normal constant d2 (n=2)	1 128				Normal constant d2 (n=2)	1 128			Normal constant d2 (n=2)	1 128		
Normal constant D4 (n=2)	3 268			Normal constant D4 (n=2)	3 268				Normal constant D4 (n=2)	3 268			Normal constant D4 (n=2)	3 268		
Upper control limit for X (UCLx)	4 876142			Upper control limit for X (UCLx)	9 844274				Upper control limit for X (UCLx)	49 39925			Upper control limit for X (UCLx)	98 83547		
Lower control limit for X (LCLx)	-0 88234			Lower control limit for X (LCLx)	-1 83782				Lower control limit for X (LCLx)	-9 50742			Lower control limit for X (LCLx)	-18 4051		
Upper control limit MR (UCLmr)	3 537921			Upper control limit MR (UCLmr)	7 177354				Upper control limit MR (UCLmr)	36 19132			Upper control limit MR (UCLmr)	72 03074		
MR Standard deviation	0 883636			MR Standard deviation	1 796333				MR Standard deviation	8 964618			MR Standard deviation	18 15019		
Process standard deviation	1			Process standard deviation	2				Process standard deviation	10			Process standard deviation	20		
Gamma d2	1 082595			Gamma d2	1 086128				Gamma d2	1 107445			Gamma d2	1 102062		
Gamma d3	0 883636			Gamma d3	0 898167				Gamma d3	0 896462			Gamma d3	0 907509		
Gamma D4	3 448661			Gamma D4	3 453725				Gamma D4	3 428458			Gamma D4	3 470396		
4 Alpha=4				4 Alpha=4					4 Alpha=4				4 Alpha=4			
5 Beta=0 5				5 Beta=1					5 Beta=5				10 Beta=10			
Proportion MR > UCLmr	0 017802			Proportion MR > UCLmr	0 017702				Proportion MR > UCLmr	0 015102			Proportion MR > UCLmr	0 018702		
Percentage	1 760178			Percentage	1 770177				Percentage	1 51011			Percentage	1 870187		
ARL for MR	56 8125			ARL for MR	56 49153				ARL for MR	68 21854			ARL for MR	53 47099		

5 Alpha=5
0.5 Beta=0.5

MR	(MR1-MR)²	MR	(MR1-MR)²
107974		3.54696	2.0556
1.56595	0.48621	5.060114	1.4186
3.26401	0.69806	0.214779	2.60927
1.14765	2.11836	0.7747	1.41834
3.33162	2.18367	0.90127	2.24629
3.38343	0.05181	1.399034	2.25183
1.66348	1.71995	0.235546	1.27574
1.55128	0.1122	1.259821	0.66318
4.66223	1.91095	0.457426	2.80084
4.91497	1.45274	0.047577	2.32513
2.12517	3.6334	5.754157	2.13463
2.00249	0.72092	0.263885	1.57571
4.21315	2.21066	0.952659	1.73857
2.30045	1.81927	0.459796	3.08138
2.80184	0.50139	0.537623	1.38824
1.73687	1.06317	0.292394	6.66489
2.09337	0.3207	0.835245	6.83814
2.5696	0.51023	0.524737	1.40689
3.37311	0.80351	0.185854	1.77205
2.90405	0.46995	0.589094	2.38566
3.81113	1.52293	0.083124	2.89921
1.96881	0.56768	0.185226	0.01506
2.50294	0.53413	0.490683	1.31052
1.52343	0.97961	0.09508	3.22294
1.19606	0.32355	0.628767	1.07073
0.73216	1.33468	0.49041	1.32842
2.6296	3.10296	1.490704	2.80962
0.81147	1.81833	0.34072	3.35291
2.19976	1.34829	0.012921	1.56148
1.76522	0.39454	0.70573	1.59813
2.2527	0.48748	0.686214	1.10888
1.85925	0.59345	0.411096	1.85438
1.33464	0.32461	0.828114	3.84904
3.45224	2.1176	0.779656	0.37281
1.88209	1.57015	0.112582	0.94076
2.38122	0.49913	0.549842	3.84916
4.30565	1.92443	0.475841	1.09845
2.24181	2.06384	0.68761	2.96343
2.89748	0.65567	0.33518	2.2063
1.87849	1.01899	0.046495	1.33428
0.99817	0.68032	0.125527	3.70307
0.03124	3.03307	3.234431	1.9855
2.64701	1.38423	0.022384	1.42879
1.00248	1.64453	0.168028	2.63034
3.00951	2.03003	0.590458	3.02965
1.71786	1.28785	0.028112	2.30793
1.09607	0.62179	0.376558	1.81462
3.25642	1.22935	2.9E-06	2.13612
1.12327	1.20215	0.001054	1.01064
3.31009	2.18682	0.90689	1.9222
2.47822	0.83187	0.162206	0.77492
3.60301	1.12479	0.012062	4.06389
2.02372	1.57929	0.118799	3.10839
4.4306	2.40688	1.374199	1.85345
0.47303	3.95797	7.41447	3.18142
4.06493	3.9919	0.556781	3.26808
4.81555	0.55682	0.467853	3.3447
3.77706	0.83849	0.156917	2.89318
1.7330	2.04346	0.654226	3.08006
1.61067	0.12293	1.235849	2.9304
2.03211	0.42144	0.681258	1.57448
1.48172	0.54039	0.481962	1.844
2.12446	0.62974	0.385877	4.93944
2.89361	0.72115	0.213876	2.71643
1.87355	0.10126	0.04756	2.17464
0.70458	1.19721	0.001399	1.24451
4.23446	1.1689	0.02019	1.99316
2.28265	0.35172	0.859332	1.38552
2.84001	0.51376	0.519636	1.02385
1.22033	1.81968	0.148273	3.18004
2.09993	0.8796	0.126037	2.08508
9.96812	3.86619	0.839704	3.30394
3.00291	2.95517	0.94881	1.70218
4.03388	1.80089	0.0044	4.16786

5 Alpha=5
1 Beta=1

MR	(MR1-MR)²	MR	(MR1-MR)²
1.9053	1.107	1.894332	4.8731
1.5059	0.832	2.726948	7.6612
0.5033	0.2033	1.920588	4.6775
1.965	0.3861	1.965	0.29995
9.5113	6.1312	13.30683	5.7102
4.7212	4.7901	5.321106	3.1602
0.8319	0.1107	5.629456	3.4271
0.1617	1.3298	1.330672	3.8513
0.3017	3.1	0.41818	3.548
0.0751	0.30434	0.313659	8.1861
4.5401	1.535	0.899363	6.243
3.0363	1.5038	0.959513	2.7294
2.418	0.6183	3.478402	3.9991
7.2346	4.8186	5.444088	4.4425
5.485	1.7698	0.509435	6.798
8.3506	0.8856	2.552787	4.6819
8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
4.4622	4.9277	5.91486	4.1417
8.3506	0.8856	2.552787	4.6819
8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
4.4622	4.9277	5.91486	4.1417
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8.4171	2.0665	0.173762	6.2089
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0.4099	6.7505	16.20859	7.2148
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8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
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8.4171	2.0665	0.173762	6.2089
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0.4099	6.7505	16.20859	7.2148
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8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
4.4622	4.9277	5.91486	4.1417
8.3506	0.8856	2.552787	4.6819
8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
4.4622	4.9277	5.91486	4.1417
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4.4622	4.9277	5.91486	4.1417
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8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
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2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
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8.3506	0.8856	2.552787	4.6819
8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
4.4622	4.9277	5.91486	4.1417
8.3506	0.8856	2.552787	4.6819
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2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
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8.3506	0.8856	2.552787	4.6819
8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
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8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
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4.4622	4.9277	5.91486	4.1417
8.3506	0.8856	2.552787	4.6819
8.4171	2.0665	0.173762	6.2089
2.8594	5.757	10.72139	6.9431
0.4099	6.7505	16.20859	7.2148
4.4622	4.9277	5.91486	4.1417
8.3506	0.8856	2.552787	4.6819
8.4171			

1 58767	2 25556	1 042324	0	3 9194	3 7792	1 879234	0	13 4849	2 0713	105 5847	0	30 246	34 317	100 0423	
6 32837	4 7407	12 29261	1	8 5446	4 6252	4 587533	0	20 253	6 7681	31 12125	0	54 826	24 56	0 070286	
0 70575	5 82292	19 25457	1	3 9542	4 5904	4 439671	0	32 172	11 919	18 28264	0	59 22	4 394	396 8417	
1 28319	0 57744	0 431882	0	10 1961	6 2439	14 41678	0	20 9338	11 2384	1 228423	0	40 853	18 367	35 37734	
1 29391	0 01072	1 49725	0	6 0758	4 1223	2 686166	0	38 0435	17 1059	22 68767	0	62 386	21 533	7 738887	
3 92502	2 63111	1 950191	0	6 4503	0 3745	4 447237	0	8 7951	29 2484	285 696	0	80 037	17 851	44 403737	
2 00296	1 92206	0 472577	0	1 7938	4 8565	4 722592	0	35 6191	26 824	209 591	0	40 173	39 864	241 775	
2 4507	0 44774	0 619176	0	8 3348	6 541	16 46454	0	16 1141	19 505	51 24065	0	69 802	29 628	28 23981	
2 89182	0 44112	0 629638	0	3 0281	5 3067	7 97132	0	21 924	5 8099	42 73031	0	61 27	8 532	249 0995	
5 89255	2 80073	2 452706	0	5 0815	2 0534	0 164855	0	17 6107	4 3133	64 5362	0	47 771	13 499	116 9834	
2 86582	2 82673	2 534822	0	6 5176	1 4361	1 096727	0	12 8699	4 7206	58 155	0	34 46	13 311	121 0855	
0 92787	1 93785	0 494536	0	8 3015	1 7839	0 489227	0	25 8902	12 8003	0 205714	0	29 591	4 769	382 0416	
4 47004	3 54207	5 324337	0	4 7321	3 5694	1 17951	0	21 1203	4 5699	80 47928	0	49 714	20 023	16 42026	
2 97975	1 49029	0 905368	0	4 2538	0 4783	4 020215	0	27 1342	6 0139	40 1049	0	37 709	12 005	151 5333	
5 23164	2 25189	1 034843	0	3 9871	0 2667	4 913526	0	10 9298	16 2076	14 90822	0	32 587	5 112	368 7508	
2 68479	2 54685	1 721954	0	4 4203	0 4332	4 203104	0	40 4286	29 502	294 3029	0	65 364	32 787	71 77672	
2 6106	0 07419	1 246592	0	2 0929	2 3274	0 02432	0	25 9821	14 4465	4 408981	0	17 275	5 891	339 4396	
3 24484	0 63404	0 380693	0	12 4445	10 3516	81 9094	1	14 2669	11 7152	0 398846	0	62 262	9 013	234 1477	
2 83602	0 40862	0 682272	0	6 8579	5 5866	9 630177	0	16 49	4 2231	85 99357	0	22 855	39 407	227 7719	
2 18898	0 64714	0 34513	0	4 8303	2 0276	0 207708	0	8 4126	10 0774	5 149916	0	64 992	42 137	317 8278	
1 83126	0 35782	0 769125	0	3 0771	1 7532	0 533115	0	11 0242	2 6116	84 773	0	43 534	21 458	8 16795	
4 00033	2 16907	0 873201	0	13 1329	10 0558	57 34204	0	33 84	22 8158	109 6012	0	85 901	42 367	325 8788	
3 60401	0 39632	0 702743	0	1 806	11 3269	78 20842	1	18 9113	14 8287	8 666504	0	36 85	49 051	611 8754	
1 62835	1 97766	0 952112	0	2 0614	0 2554	4 96375	0	15 8663	3 055	86 33648	0	54 433	17 563	45 31828	
3 46141	1 83506	0 306531	0	4 5561	2 4947	0 000129	0	26 8714	10 6151	2 345929	0	66 709	12 276	144 9348	
2 00324	1 45817	0 048978	0	3 4022	1 1539	1 76743	0	21 5148	5 1566	51 09815	0	64 848	2 243	487 1681	
1 94828	0 05496	1 391592	0	7 0557	3 8535	1 388257	0	38 7034	17 1886	23 44358	0	44 838	19 828	21 9659	
2 41004	0 46176	0 997309	0	1 9579	6 0978	8 835363	0	30 1095	8 5939	14 08363	0	30 388	14 45	97 31597	
1 87374	0 0 5383	0 487647	0	4 4673	2 5084	0 000678	0	21 8454	8 4641	15 07491	0	87 045	66 657	1792 855	
1 409	0 46474	0 592711	0	1 897	2 5703	0 007561	0	56 3517	34 7063	499 9498	0	26 781	70 264	2111 321	
2 36462	0 85622	0 07784	0	4 8838	2 9666	0 253485	0	31 7285	24 8252	150 7605	0	48 452	21 671	6 990131	
5 9116	3 54698	5 34702	0	1 8504	3 0334	0 302558	0	33 3052	1 5787	115 9507	0	34 14	24 312	8 3E-062	
1 05545	4 85615	13 1155	1	6 8296	6 9792	20 21269	0	17 1896	16 1156	14 20428	0	12 873	11 467	165 0682	
2 88096	1 82541	0 349036	0	3 5065	5 3241	0 069875	0	11 417	5 7726	43 21935	0	88 473	75	8 2650 717	
2 0714	0 80946	0 180759	0	6 2189	2 7142	0 063293	0	20 1894	8 7724	12 77593	0	37 673	50 0	701 4613	
2 87818	0 80878	0 183045	0	4 6808	1 3569	1 264382	0	18 6211	3 5683	77 06106	0	30 864	7 009	299 4937	
4 10763	1 22945	2 7E-05	0	3 0568	1 801	0 469598	0	82 747	36 1206	585 1963	0	30 304	0 36	573 8365	
1 92318	1 18445	0 902182	0	1 7304	1 3294	1 331595	0	33 704	19 0377	44 76891	0	36 688	6 382	321 5884	
2 26336	0 34018	0 800018	0	1 6601	0 0703	5 822798	0	23 8455	10 0585	5 236054	0	56 366	21 68	6 942622	
4 83043	2 58707	1 77543	0	6 7943	5 1342	7 02702	0	13 9967	9 6488	7 278894	0	33 825	24 541	0 051128	
1 25407	3 57836	5 483758	0	6 2525	0 5418	3 789606	0	18 7171	4 7204	58 1611	0	62 961	29 136	23 24314	
1 80311	0 54904	0 470017	0	7 8566	1 8041	0 773076	0	19 0236	0 3065	144 9674	0	85 149	2 188	489 5991	
2 01204	0 20693	1 052035	0	5 2234	2 6332	0 022456	0	43 8729	24 8493	156 3138	0	102 156	37 007	161 0858	
2 77569	0 76385	0 22181	0	2 8746	2 2488	0 065012	0	18 1313	27 7416	237 0016	0	25 738	76 416	2714 735	
2 25416	0 52153	0 908494	0	3 3522	0 3778	4 434172	0	28 5668	12 4255	0 006203	0	78 351	52 613	800 7833	
1 33296	0 9212	0 998231	0	1 4025	1 8497	0 28478	0	31 1521	2 5953	95 09063	0	49 263	29 088	22 78262	
2 37064	1 03768	0 038784	0	3 6647	2 2822	0 048906	0	17 2871	13 885	2 366236	0	85 194	15 931	70 28994	
2 34732	0 02332	1 467242	0	4 5439	0 8792	2 573286	0	35 868	18 4209	36 99539	0	32 615	32 579	68 29559	
1 13815	1 20917	0 006648	0	3 8428	0 7011	3 176406	0	27 7642	7 9238	19 56242	0	29 003	3 812	428 8095	
0 98563	0 15252	1 170935	0	8 1281	4 2833	3 239829	0	7 6298	20 1344	80 64781	0	70 977	41 974	311 8443	
3 71177	2 72614	2 224639	0	5 875	2 4511	0 00104	0	29 7713	22 1415	95 93727	0	40 713	30 264	35 39196	
Gamma d2	2 5404	1 18337	0 005078	4 4511	1 2239	1 588208	0	33 3711	3 5998	76 50901	0	27 775	12 938	129 4335	
Gamma d3	1 64925	0 89915	0 112538	0	3 8726	0 5765	3 628444	0	14 9362	18 4349	37 06566	0	45 725	17 95	40 51177
Gamma d4	2 06215	0 4129	0 87522	0	4 0164	0 1438	5 473482	0	17 0003	2 0641	105 7327	0	27 068	18 659	31 98904
0 5 Alpha=5	0 5 Beta=0 5	0 018602	0 1520162	61 72222	1 51116	1 43557	0 040382	1 51116	1 43557	0 040382	0	69 83	42 764	340 3698	
Sum MR > UCLmr	20148	4931.105	Sum X	40133 31	9931 408	Sum X	122	41	200090 1	49861 53	Sum X	143	40	399443 5	
Sum X	2 508	MR Average	1 234618	X Average	5 006472	MR Average	2 29616	1 234674	X Average	2 29616	MR Average	24 31489	X Average	49 84228	
Normal constant d2 (n=2)	1 128	Normal constant D4 (n=2)	3 268	Upper control limit for X (UCLx)	11 81112	Lower control limit for X (LCLx)	-1 59818	Upper control limit MR (UCLmr)	8 115579	MR Standard deviation	2 001456	Process standard deviation	1 118034	Gamma d2	1 104275
Upper control limit for X (UCLx)	0 791557	Lower control limit for X (LCLx)	-0 77556	Upper control limit MR (UCLmr)	8 115579	MR Standard deviation	2 001456	Process standard deviation	1 118034	Gamma d2	1 104275	Gamma d3	0 891881	Gamma D4	3 422985
Lower control limit for X (LCLx)	-0 77556	Upper control limit MR (UCLmr)	8 115579	MR Standard deviation	2 001456	Process standard deviation	1 118034	Gamma d2	1 104275	Gamma d3	0 891881	Gamma D4	3 422985	5 Alpha=5	0 5 Beta=0 5
Upper control limit MR (UCLmr)	8 115579	MR Standard deviation	2 001456	Process standard deviation	1 118034	Gamma d2	1 104275	Gamma d3	0 891881	Gamma D4	3 422985	5 Alpha=5	0 5 Beta=0 5	0 018602	1 520162
MR Standard deviation	2 001456	Process standard deviation	1 118034	Gamma d2	1 104275	Gamma d3	0 891881	Gamma D4	3 422985	5 Alpha=5	0 5 Beta=0 5	0 018602	1 520162	61 72222	
Process standard deviation	1 118034	Gamma d2	1 104275	Gamma d3	0 891881	Gamma D4	3 422985	5 Alpha=5	0 5 Beta=0 5	0 018602	1 520162	61 72222	1 520162	61 72222	
Gamma d2	1 104275	Gamma d3	0 891881	Gamma D4	3 422985	5 Alpha=5	0 5 Beta=0 5	0 018602	1 520162	61 72222	1 520162	61 72222	1 520162	61 72222	
Gamma d3	0 891881	Gamma D4	3 422985	5 Alpha=5	0 5 Beta=0 5	0 018602	1 520162	61 72222	1 520162	61 72222	1 520162	61 72222	1 520162	61 72222	
Gamma D4	3 422985	5 Alpha=5	0 5 Beta=0 5	0 018602	1 520162	61 72222	1 520162	61 72222	1 520162	61 72222	1 520162	61 72222	1 520162	61 72222	
5 Alpha=5	0 5 Beta=0 5	0 018602	1 520162	61 72222	1 520162	61 72222	1								

10 Alpha=10
0.5 Beta=0.5

MR	(MR-MR)*2	MR	(MR-MR)*2
3.4831	0	6.4753	0
5.8622	2.2241	6.4753	0
6.4087	0.7215	6.4753	0
7.6504	1.2417	6.4753	0
8.4734	5.813	6.4753	0
4.6458	2.8084	6.4753	0
3.7845	0.8613	6.4753	0
4.7956	1.0111	6.4753	0
8.9884	4.1928	6.4753	0
3.9825	5.0509	6.4753	0
4.2194	0.2869	6.4753	0
5.0726	0.8532	6.4753	0
4.8661	0.2065	6.4753	0
5.6818	0.8157	6.4753	0
10.263	4.4445	6.4753	0
10.0258	0.1005	6.4753	0
8.4517	3.5741	6.4753	0
2.9275	3.5242	6.4753	0
4.0081	1.0806	6.4753	0
4.1126	0.1045	6.4753	0
4.6884	0.7558	6.4753	0
3.478	2.4792	6.4753	0
3.6901	3.6575	6.4753	0
4.0165	0.3264	6.4753	0
7.5475	3.531	6.4753	0
7.8327	0.2852	6.4753	0
3.1433	4.6894	6.4753	0
6.9937	3.5804	6.4753	0
4.3953	2.5084	6.4753	0
4.3332	0.0621	6.4753	0
3.4475	0.8857	6.4753	0
4.8889	0.7558	6.4753	0
4.3171	1.8298	6.4753	0
5.739	1.4219	6.4753	0
3.7707	1.9683	6.4753	0
4.1532	0.3825	6.4753	0
4.9869	0.8337	6.4753	0
4.0562	0.9307	6.4753	0
6.103	0.5541	6.4753	0
5.4626	0.8523	6.4753	0
4.5328	0.9296	6.4753	0
4.8151	0.0823	6.4753	0
1.0537	0.5614	6.4753	0
5.8604	1.8267	6.4753	0
8.4349	2.7545	6.4753	0
4.798	3.6369	6.4753	0
7.4941	2.6861	6.4753	0
4.1843	3.3098	6.4753	0
6.5995	2.4552	6.4753	0
3.4332	3.1663	6.4753	0
6.1768	2.7436	6.4753	0
7.1914	1.0146	6.4753	0
2.4044	4.787	6.4753	0
5.1159	3.1115	6.4753	0
7.3511	1.8352	6.4753	0
5.1222	3.8389	6.4753	0
3.682	0.1998	6.4753	0
3.6544	0.0176	6.4753	0
6.2587	2.5953	6.4753	0
5.2849	0.9948	6.4753	0
5.197	0.0679	6.4753	0
7.6037	2.4067	6.4753	0
2.8585	0.1752	6.4753	0
4.3194	0.073432	6.4753	0
2.6693	1.6501	6.4753	0
5.2386	2.5693	6.4753	0
5.1559	0.0827	6.4753	0
6.3998	1.2439	6.4753	0
4.6938	1.7892	6.4753	0
4.8348	0.141	6.4753	0
6.7811	1.9465	6.4753	0
5.1102	1.6709	6.4753	0
4.9005	0.2097	6.4753	0
6.1949	3.2944	6.4753	0

166

1 Beta=1	(MR-MR)*2	1 Beta=1	(MR-MR)*2
0	15.3092	0	15.3092
0	8.6521	0	8.6521
0	8.527	0	8.527
0	16.4922	0	16.4922
1	10.4939	1	10.4939
0	10.2233	0	10.2233
0	3.8104	0	3.8104
0	18.0181	0	18.0181
0	8.7146	0	8.7146
0	8.1937	0	8.1937
0	13.4176	0	13.4176
0	8.6124	0	8.6124
0	14.9947	0	14.9947
0	6.43	0	6.43
0	12.9856	0	12.9856
0	10.9177	0	10.9177
0	11.7932	0	11.7932
0	6.607	0	6.607
0	9.362	0	9.362
0	8.8768	0	8.8768
0	7.723	0	7.723
0	13.8242	0	13.8242
0	12.1658	0	12.1658
0	4.0165	0	4.0165
0	9.9537	0	9.9537
0	4.5687	0	4.5687
0	11.4538	0	11.4538
0	8.3419	0	8.3419
0	8.2432	0	8.2432
0	8.9166	0	8.9166
0	4.4786	0	4.4786
0	6.9951	0	6.9951
0	10.2038	0	10.2038
0	20.4229	0	20.4229
0	15.3697	0	15.3697
0	6.3717	0	6.3717
0	14.2603	0	14.2603
0	4.0562	0	4.0562
0	6.103	0	6.103
0	5.4626	0	5.4626
0	4.5328	0	4.5328
0	4.8151	0	4.8151
0	1.0537	0	1.0537
0	5.8604	0	5.8604
0	8.4349	0	8.4349
0	4.798	0	4.798
0	7.4941	0	7.4941
0	4.1843	0	4.1843
0	6.5995	0	6.5995
0	3.4332	0	3.4332
0	6.1768	0	6.1768
0	7.1914	0	7.1914
0	2.4044	0	2.4044
0	5.1159	0	5.1159
0	7.3511	0	7.3511
0	5.1222	0	5.1222
0	3.682	0	3.682
0	3.6544	0	3.6544
0	6.2587	0	6.2587
0	5.2849	0	5.2849
0	5.197	0	5.197
0	7.6037	0	7.6037
0	2.8585	0	2.8585
0	4.3194	0	4.3194
0	2.6693	0	2.6693
0	5.2386	0	5.2386
0	5.1559	0	5.1559
0	6.3998	0	6.3998
0	4.6938	0	4.6938
0	4.8348	0	4.8348
0	6.7811	0	6.7811
0	5.1102	0	5.1102
0	4.9005	0	4.9005
0	6.1949	0	6.1949

MR	(MR-MR)*2	MR	(MR-MR)*2
13.6984	2.3218	13.6984	2.3218
7.9952	0.7092	7.9952	0.7092
10.857	3.0578	10.857	3.0578
10.2296	0.6274	10.2296	0.6274
14.879	1.4879	14.879	1.4879
11.4685	2.7268	11.4685	2.7268
11.7721	0.3036	11.7721	0.3036
8.0543	3.7178	8.0543	3.7178
8.5631	0.5337	8.5631	0.5337
11.7271	4.2085	11.7271	4.2085
11.6934	0.0337	11.6934	0.0337
7.3	4.3934	7.3	4.3934
8.4187	2.1187	8.4187	2.1187
8.2417	1.177	8.2417	1.177
6.4489	1.7848	6.4489	1.7848
11.0813	4.6344	11.0813	4.6344
7.1037	3.9776	7.1037	3.9776
6.2671	0.35001	6.2671	0.35001
7.9001	2.6233	7.9001	2.6233
7.8938	0.4287	7.8938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671	6.233
7.6938	0.4287	7.6938	0.4287
10.2511	0.2673	10.2511	0.2673
8.2777	1.8234	8.2777	1.8234
12.6009	4.2732	12.6009	4.2732
7.2671	6.233	7.2671</	

3.259	0.2833	2.098393	0	9.6511	3.2039	0.093467	0	47.702	0.149	300.8478	0	111.505	30.404	20.97324	
3.676	0.417	1.72618	0	9.2422	0.4089	9.614688	0	50.638	3.136	206.1511	0	99.214	12.291	514.9566	
3.4962	0.1798	2.408962	0	7.3127	1.9295	2.466933	0	61.078	10.24	52.91999	0	96.403	2.811	1035.08	
1.5129	1.9833	0.06321	0	8.4681	1.1554	5.542522	0	45.517	15.501	3.971905	0	65.8	29.503	30.57358	
7.0403	5.5274	14.40595	0	6.4769	1.9912	2.305709	0	64.466	18.889	1.946126	0	177.802	110.702	5.733.268	
5.128	1.9123	0.03255	0	7.034	0.9571	8.717587	0	76.4	11.934	30.9132	0	91.959	85.643	2.566.369	
4.4491	0.6789	1.08773	0	7.6448	0.6108	8.403366	0	36.387	40.013	507.107	0	94.643	2.684	1043.268	
5.4935	1.0444	0.472633	0	12.6102	5.1654	2.741486	0	27.909	8.478	81.2876	0	134.229	39.586	21.18158	
7.1851	1.6946	0.00139	0	7.0009	5.8093	5.288362	0	49.492	21.583	16.72022	0	117.974	16.255	350.7825	
3.8026	3.3855	2.734449	0	6.9108	0.0901	11.69336	0	37.267	12.225	27.76198	0	112.948	5.026	897.4811	
4.6459	0.8433	0.78958	0	7.4307	0.5199	8.938641	0	55.124	17.857	0.131795	0	125.011	12.063	525.3564	
6.1427	1.4968	0.055264	0	10.8845	3.4538	0.00312	0	38.173	16.951	0.29481	0	129.624	4.613	922.3767	
5.5737	0.569	1.352297	0	8.3832	2.5013	1.016782	0	72.356	34.183	278.5239	0	83.146	46.478	132.12	
6.8372	1.0635	0.446736	0	10.2557	1.8725	2.68028	0	58.569	13.787	13.74158	0	39.719	43.427	71.29008	
4.6567	1.9805	0.06181	0	14.9374	4.6817	1.373687	0	48.348	10.223	52.86992	0	70.257	30.538	19.76385	
6.95	2.2933	0.315189	0	4.3835	10.5539	49.62137	0	34.092	14.254	10.49737	0	99.848	29.591	29.08073	
5.9605	0.9895	0.551133	0	16.0784	11.6949	66.98822	1	45.833	11.541	35.43778	0	106.252	8.404	706.478	
5.1238	0.8367	0.801353	0	7.0234	9.055	30.75084	0	55.277	9.644	61.62193	0	95.762	12.49	505.9645	
4.3936	0.7302	1.003369	0	7.0283	0.0049	12.28332	0	34.313	20.964	12.04115	0	117.18	21.418	184.027	
6.6256	2.232	0.250117	0	8.94	1.9117	2.553464	0	47.471	13.158	18.80058	0	82.321	24.859	102.5086	
4.8778	1.7478	0.000253	0	9.6149	0.6749	8.035842	0	54.358	8.887	112.5077	0	113.768	21.447	183.247	
3.4141	1.4837	0.071922	0	9.0306	0.5843	8.557708	0	33.932	20.426	8.596836	0	132.773	19.005	255.3174	
6.8063	3.1922	2.132525	0	12.8177	3.7871	0.076975	0	61.515	27.583	101.7886	0	121.134	11.639	544.9729	
3.5438	3.0625	1.770541	0	12.251	0.5387	8.838468	0	49.499	12.018	30.00699	0	128.677	7.543	762.9895	
5.3057	1.7619	0.000901	0	9.2929	2.9881	0.272021	0	67.288	17.787	0.08587	0	82.37	46.307	128.2181	
3.3877	1.918	0.034639	0	9.4785	20.769	11.06285	0	46.517	20.769	10.72586	0	79.231	3.139	1014.082	
2.818	0.4697	1.593106	0	9.5456	0.0691	11.83743	0	44.033	2.484	225.299	0	48.026	31.205	14.27823	
6.5211	3.6031	3.501453	0	8.417	3.1288	0.145204	0	27.888	16.145	1.819704	0	236.414	188.388	23532.89	
3.0681	3.453	2.962243	0	10.9997	4.5827	1.151423	0	65.181	37.293	392.0018	0	95.951	140.863	11210.44	
6.5546	3.4865	3.07868	0	7.7571	3.2421	0.071586	0	51.187	13.994	12.24975	0	92.338	3.213	1008.375	
6.1113	0.4433	1.660446	0	14.5698	8.8122	10.9068	0	34.87	16.317	1.305244	0	65.268	27.07	62.62593	
6.6735	0.5822	1.368159	0	9.6202	4.9496	2.073439	0	49.242	14.372	7.46659	0	122.227	56.959	482.9158	
5.1995	1.474	0.066504	0	7.1201	2.5001	0.109203	0	48.788	0.454	290.3604	0	81.375	40.852	34.43748	
6.4329	1.2334	0.248485	0	9.824	2.7039	0.649243	0	37.935	10.853	44.1024	0	83.809	2.234	1072.54	
4.0949	1.749	0.002293	0	9.6833	0.7407	7.687118	0	36.117	1.818	245.7358	0	70.995	13.014	482.8657	
4.3588	1.8348	0.010592	0	14.894	5.5013	3.986646	0	44.054	9.009	155.8743	0	104.27	33.875	1.712577	
5.5136	1.1548	0.333025	0	5.3894	9.1952	32.32541	0	43.198	2.07	237.8987	0	62.743	41.527	42.81537	
3.8297	1.6839	0.002302	0	11.1724	5.783	5.168093	0	52.856	9.86	61.37099	0	133.775	17.032	1299.483	
5.8439	2.0142	0.078703	0	3.9349	7.2375	13.89682	0	35.088	17.788	0.075096	0	88.21	45.565	111.9649	
4.0949	1.749	0.002293	0	14.2016	10.2867	45.85764	0	44.555	9.487	84.43215	0	109.265	21.055	184.0074	
2.358	1.7389	2.5E-05	0	10.6324	3.8692	0.025454	0	48.776	4.221	176.1716	0	81.194	28.071	47.7848	
4.2639	1.9059	0.030282	0	7.3107	3.2217	0.082919	0	42.772	8.004	132.0193	0	109.498	28.304	44.61779	
4.9046	0.6407	1.180681	0	8.5364	1.2257	5.216455	0	49.83	7.058	108.9093	0	74.886	34.812	0.029465	
3.7388	1.1856	0.32045	0	10.3886	1.8534	2.743184	0	74.884	25.054	57.15415	0	89.043	14.357	425.4589	
6.3068	2.588	0.899991	0	8.9717	3.4161	0.008383	0	69.026	5.858	135.3957	0	85.833	23.21	138.6189	
5.1172	1.1896	0.294071	0	3.9768	2.9949	0.264974	0	49.364	19.842	4.614059	0	85.456	19.623	235.9497	
1.9019	3.2153	2.200526	0	8.7765	4.7997	1.664213	0	68.014	18.63	1.290578	0	103.41	17.964	290.0291	
5.2358	3.3339	2.568458	0	5.478	3.2985	0.044587	0	36.894	31.12	185.6689	0	64.627	38.783	14.43502	
3.538	1.8978	0.001162	0	10.7198	5.2418	3.000323	0	50.628	13.734	14.13733	0	85.651	21.024	194.872	
3.0371	0.5009	1.516319	0	15.2875	4.5677	1.119457	0	36.552	14.078	11.68248	0	90.195	4.544	926.5726	
6.4853	3.4282	2.877481	0	8.6207	8.6668	9.967558	0	52.89	16.338	1.336253	0	62.881	27.514	55.79574	
3.826	2.8393	0.823405	0	14.4055	5.7848	5.17828	0	37.48	18.41	34.29906	0	113.72	51.039	257.7741	
2.8897	0.9383	0.632952	0	8.8207	5.5848	4.306222	0	34.635	2.845	214.5921	0	114.409	0.689	1176.123	
6.3917	3.502	3.133314	0	8.9654	0.1447	11.32293	0	86.211	31.578	198.3037	0	123.757	9.348	657.1868	
3.9775	2.4142	0.465556	0	12.1043	3.1389	0.13748	0	64.472	1.739	248.2189	0	91.759	31.998	8.914134	
10.6477	6.6702	24.38697	1	1	11.3766	0.7277	7.73928	0	40.617	23.655	37.95837	0	82.118	9.641	642.2502
Sum MR > UCLmr				103	26			99	30			113	30		
Sum X	39805.68		10046.13	Sum X	79561.21		19661.73	Sum X	399638.6		101670.2	Sum X	806414.7	199957.1	
X Average	4.985181			X Average	9.942294			X Average	100.13088			X Average	100.6372		
MR Average	1.731883			MR Average	3.509656			MR Average	17.49396			MR Average	34.98365		
Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128			Normal constant d2 (n=2)	1.128		
Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268			Normal constant D4 (n=2)	3.268		
Upper control limit for X (UCLx)	9.519253			Upper control limit for X (UCLx)	18.27649			Upper control limit for X (UCLx)	96.65738			Upper control limit for X (UCLx)	193.6788		
Lower control limit for X (LCLx)	0.379109			Lower control limit for X (LCLx)	0.608102			Lower control limit for X (LCLx)	3.604379			Lower control limit for X (LCLx)	7.595542		
Upper control limit MR (UCLmr)	5.859794			Upper control limit MR (UCLmr)	11.469956			Upper control limit MR (UCLmr)	57.17027			Upper control limit MR (UCLmr)	114.3286		
MR Standard deviation	1.357156			MR Standard deviation	2.747928			MR Standard deviation	13.72334			MR Standard deviation	27.5976		
Process standard deviation	1.581138			Process standard deviation	3.162278			Process standard deviation	15.81138			Process standard deviation	31.62278		
Gamma d2	1.095339			Gamma d2	1.106851			Gamma d2	1.106416			Gamma d2	1.10628		
Gamma d3	0.858341			Gamma d3	0.868971			Gamma d3	0.86794			Gamma d3	0.87271		
Gamma D4	3.350891			Gamma D4	3.348887			Gamma D4	3.353385			Gamma D4	3.366605		
10 Alpha=10				10 Alpha=10				10 Alpha=10				10 Alpha=10			
0.5 Beta=0.5				1 Beta=1				5 Beta=0				10 Beta=10			
Proportion MR > UCLmr	0.012901			Proportion MR > UCLmr	0.012901			Proportion MR > UCLmr	0.014301			Proportion MR > UCLmr	0.015602		
Percentage	1.290129			Percentage	1.290129			Percentage	1.430143			Percentage	1.560156		
ARL for MR	77.51163			ARL for MR	77.51163			ARL for MR	89.92308			ARL for MR	64.09615		

50 Alpha=50				50 Alpha=50				50 Alpha=50				50 Alpha=50			
0.5 Beta=0.5	MR	(MR-MR) ²	MR	(MR-MR) ²	1 Beta=1	MR	(MR-MR) ²	MR	(MR-MR) ²	1 Beta=1	MR	(MR-MR) ²	MR	(MR-MR) ²	
23 0138			23 2331	2 2212	3 148789			52 8053			52 8053				
23 0601	0 0463	15 59762	19 0161	4 217	0 048891	0	0	56 0993	3 294	21 33751	51 7471	1 7575	37 91996	0	
27 2329	4 8638	0 753627	22 6801	3 664	0 110014	0	0	52 9364	3 1629	22 58643	52 4583	0 7112	51 90078	0	
29 3374	1 4135	15 36496	24 3987	1 7726	5 148737	0	0	53 7075	0 7171	51 04128	57 0872	4 6289	10 8012	0	
33 9659	4 6291	0 601217	30 7967	6 389	0 727967	0	0	58 4356	0 4836	10 155	49 6025	7 4448	1 85516	0	
25 0522	8 9143	24 29719	20 2402	0 5555	11 83486	0	0	54 2687	4 1689	14 05139	48 7821	0 8204	50 33928	0	
26 1158	1 0636	36 1927	26 8944	3 3458	4 223488	0	0	53 4664	0 8023	50 59645	42 8609	5 9212	3 978908	0	
27 3085	1 1927	7 586713	19 4029	7 4915	12 22074	0	0	55 5256	2 0952	34 29529	38 5331	4 3278	12 8781	0	
19 3172	7 9913	15 36496	21 946	2 5431	2 109997	0	0	58 5568	3 0312	23 85559	48 7305	10 1075	5 207441	0	
26 2268	6 8096	8 489913	24 7966	2 8506	1 311215	0	0	53 3667	5 1901	7 427359	48 8322	0 1017	81 05419	0	
26 2825	0 0557	15 52346	28 3409	3 5443	0 203746	0	0	40 9188	12 4478	20 54339	54 6841	5 8519	4 258107	0	
34 4624	8 1799	17 56767	27 2293	1 111	8 321395	0	0	45 1186	4 1998	13 80582	57 384	2 6999	27 20163	0	
18 5971	15 8653	140 8878	33 0112	5 7813	3 188429	0	0	58 5254	13 6336	32 69989	57 5218	0 1378	60 49134	0	
25 9471	7 25	10 59058	29 8079	7 8377	10 28881	0	0	52 5147	6 8377	1 632964	50 1662	7 3596	0 313396	0	
21 2117	5 3646	8 179304	26 114	0 3061	13 61302	0	0	55 2993	3 1636	22 3901	41 2478	8 9196	1 006374	0	
31 2038	0 0079	15 80241	23 1941	2 1919	1 157309	0	0	60 4081	11 0958	10 20408	47 0064	5 7588	4 651022	0	
28 727	2 4768	2 307005	28 1811	1 987	4 034807	0	0	54 8853	11 5228	13 0312	51 8222	4 8158	9 807633	0	
26 6547	2 0723	3 699402	33 8266	1 3545	6 975847	0	0	49 207	5 6783	5 004698	41 1781	10 6461	7 456623	0	
26 9616	0 0931	15 23015	26 2604	2 4338	2 439478	0	0	47 8126	1 3944	42 52368	51 324	10 1479	2 983975	0	
24 4137	2 1479	3 414301	25 1022	1 1582	8 051309	0	0	47 8092	0 0034	62 60003	47 4297	3 8943	16 18399	0	
27 936	3 6223	23 22491	27 3722	2 227	2 877981	0	0	47 9287	0 1195	60 77834	40 1159	7 1338	0 361944	0	
21 2278	8 0782	7 357749	23 5007	3 8715	0 015421	0	0	47 0219	0 9068	49 12073	49 9963	8 8404	3 961153	0	
33 8269	2 5991	1 950444	20 7937	2 707	1 660703	0	0	58 4958	11 4739	12 66279	49 9039	0 0924	61 19961	0	
27 7138	3 3869	0 370817	34 4327	1 4750	0 929357	0	0	47 5074	10 9884	9 443217	48 0727	1 8312	37 01771	0	
24 3348	2 878	17 32654	24 8044	8 6283	31 72638	0	0	49 1222	1 8148	38 89779	50 1009	2 0782	34 85934	0	
27 463	3 1282	0 735266	23 0968	1 3771	5 098486	0	0	27 0945	33 86309	42 6563	7 4448	4 22167		0	
21 0221	6 8409	6 997174	30 2036	7 1369	9 867245	0	0	40 9355	10 2812	5 598924	43 4272	0 7298	10 0414	0	
25 8962	4 6741	0 46025	20 6129	9 5907	31 30422	0	0	53 6845	12 712	23 0072	54 9139	11 4867	12 75408	0	
26 1491	0 4529	12 55131	24 3869	3 774	0 049143	0	0	56 0479	3 0374	23 79500	40 6301	14 2838	40 55629	0	
29 5001	3 351	0 415616	21 3925	2 9944	1 002626	0	0	41 1817	15 4832	57 42278	40 5248	0 1053	60 99796	0	
29 0812	0 4189	12 78338	24 327	9 2945	1 126109	0	0	40 2328	19 0609	104 2218	58 7884	5 87403	0 68423	0	
29 1017	0 0205	15 80206	26 0715	1 7445	5 067824	0	0	45 0918	15 1681	52 4406	50 0581	8 6423	2 227312	0	
24 906	4 1967	0 400007	27 1126	10 411	8 72956	0	0	50 7162	5 8246	5 247848	54 5274	4 4693	11 87573	0	
22 9951	1 9469	1 97511	22 4368	4 678	0 462831	0	0	37 1089	13 5973	32 28378	49 1187	5 4087	6 28386	0	
23 3944	0 4353	13 304833	28 8734	4 4007	0 184039	0	0	38 8641	1 5452	40 57968	59 7192	6 8725	2 544864	0	
22 9224	0 472	12 41634	20 9241	5 1232	3 678872	0	0	42 2958	3 5417	18 8911	49 3682	8 423	2 227312	0	
27 1432	4 2208	0 006878	26 4704	5 5463	2 404414	0	0	60 5173	11 1135	108 0785	50 3084	0 9382	48 68157	0	
32 2708	5 1276	1 281236	22 7817	3 8887	0 094238	0	0	38 5367	20 8906	170 699	54 5849	4 2785	13 22717	0	
33 8718	8 599	21 19053	19 5211	3 2696	0 540347	0	0	52 6311	13 0944	26 82185	50 0291	4 5558	11 28703	0	
19 124	4 5478	0 304833	29 9869	6 4658	8 101478	0	0	48 9139	5 7172	4 832183	55 1284	5 0993	7 930522	0	
28 4042	9 2802	27 92612	24 8976	1 0994	8 44848	0	0	48 8764	13 9675	36 5872	48 2499	6 8785	1 076199	0	
20 8572	7 147	14 07238	21 9535	2 944	1 109337	0	0	40 9184	19 9957	144 9997	48 1147	0 1352	60 5318	0	
26 338	5 6806	2 83962	20 7495	1 204	7 793493	0	0	51 5808	10 6612	7 539318	48 0388	0 0759	61 45805	0	
27 5462	1 2082	7 770061	27 0286	8 2771	5 204864	0	0	82 0214	10 4318	6 332177	55 2781	7 2373	0 459844	0	
31 0232	3 477	0 269032	22 7031	4 3236	0 107464	0	0	85 3128	3 3004	21 99839	42 9084	12 3677	19 82281	0	
22 8604	8 0628	18 54144	19 1721	3 531	0 21593	0	0	44 1051	21 2077	176 5848	50 1142	7 2058	0 503558	0	
31 1213	8 2609	18 19208	22 5606	3 3885	0 368871	0	0	40 0065	4 0086	14 5681	56 2498	6 1356	3 187753	0	
26 408	4 8133	0 668498	26 6488	4 0882	0 008559	0	0	51 5116	11 5051	12 88582	45 5569	10 8929	7 714405	0	
27 0966	0 6886	10 9368	27 9413	1 2929	7 307197	0	0	37 2679	14 2437	40 04715	55 6782	10 1213	4 885915	0	
22 2136	4 883	0 787332	20 1383	7 803	14 49566	0	0	53 8294	16 5615	74 75473	51 5855	4 0927	14 81317	0	
27 7292	5 5186	3 10148	29 394	9 2557	27 66778	0	0	56 6811	28 5117	26 64124	48 4898	3 0599	23 22778	0	
25 7202	2 459	3 381394	25 9664	3 4378	0 311456	0	0	43 8518	12 8293	24 14623	50 3703	1 8807	36 41782	0	
20 0268	5 2434	1 566796	22 3353	3 6211	0 140312	0	0	51 6947	7 8429	0 005259	48 6404	1 7299	38 26084	0	
32 9457	12 9189	79 6238	32 1902	9 8549	34 33043	0	0	52 9118	1 2171	44 86747	45 0817	3 5587	18 98099	0	
29 8439	3 1018	0 799027	20 4905	11 6997	59 35188	0	0	47 9026	5 0092	8 446104	48 6384	3 5587	18 98842	0	
22 0394	7 8045	14 30709	27 4005	6 1891	8 493245	0	0	55 9961	8 0935	0 031713	51 9058	3 2674	21 80407	0	
23 6955	1 5661	5 902873	28 3543	0 9538	9 253051	0	0	48 7976	6 1985	2 947808	59 9259	8 0201	0 102958	0	
23 5067	0 0988	15 1857	24 4005	3 9538	0 001754	0	0	49 5902	0 2074	59 41354	26 8763	33 0496	831 7271	0	
28 7396	5 2329	1 530706	25 5544	1 1539	8 07573	0	0	56 6413	7 0511	0 747046	45 1797	38 3034	923 4294	0	
26 5278	0 2118	14 31777	32 5664	6 012	9 098169	0	0	41 8005	15 3608	55 43371	47 7935	17 3862	9 89571	0	
30 9647	2 4389	2 429804	26 3391	7 2773	4 980115	0	0	47 6963	8 4158	2 248855	57 1062	9 3127	1 952396	0	
24 7436	6 5187	6 36116	26 8017	0 4628	12 48267	0	0	44 7055	2 8906	14 26186	40 1945	16 6867	7 93538	0	
26 9435	2 4975	2 244552	35 2994	8 4877	20 26818	0	0	53 1201	8 8146	0 248182	52 4513	12 0318	11 36939	0	
22 5851	4 3584	0 131564	24 7531	10 5463	42 91059	0	0	52 3114	0 0407	50 50544	49 9496	2 5014	29 31159	0	
21 9199	0 6652	11 0992	30 5168	3 7633	3 125885	0	0	38 3235	13 9879	36 87504	50 0086	0 0587	61 72802	0	
26 2472	4 3273	0 0827	26 1013	4 1455	0 176246	0	0	53 8069	15 483	57 28829	55 8247	5 8181	4 407137	0	
25 3012	0 946	0 30565	26 8039	0 021	15 7981	0	0	53 0535	0 752	51 30023	35 9118	19 9129	143 9396	0	
21 2613	4 0399	0 01905	20 9081	5 1722	1 384193	0	0	60 7247	2 3389	31 2103	40 0387	4 5449	11 36939	0	
24 5174	3 2561	0 456983	20 4895	0 4186	12 79552	0	0	47 3522	3 3695	20 66637	56 4732	16 0185	8 62753	0	
29 5649	5 0475	1 106319	29 8841	9 3946	29 14831	0	0	39 9843	7 3709	0 2965	52 2109	4 2623	13 34527	0	
22 7202	6 8447	8 116899	23 5136	6 3705	5 639757	0	0	46 6556	6 8713	1 54783	42 1543	10 0586	6 548659	0	
25 0088	2 2886	2 91132	21 8625	1 6311	5 591252	0	0	47 3407	0 8851	52 2775	54 0623	11 908	15 94071	0	
23 7172	1 2913	17 31264	27 1774	2 8454	4 82248	0	0	45 2661	36 584	8 23782	50 784	7 7431	10 8748	0	
20 4096															

25 8875	3 6774	0 101304	0	45 9415	5 6333	5 208603	0	287 049	7 145	1043 874	0	400 819	142 61	3888 583
22 9539	2 9336	1 12802	0	54 2768	8 3353	0 173201	0	236 268	50 781	128 3002	0	569 881	169 062	7887 305
21 0238	1 9301	4 266632	0	56 7891	2 5123	29 18369	0	213 171	23 097	287 6526	0	431 916	137 965	3330 958
19 7915	1 2323	7 636285	0	44 1125	12 6766	22 66885	0	214 094	0 923	1484 841	0	523 052	81 136	118 4724
16 2803	1 5112	6 172655	0	59 5968	15 4843	57 28797	0	244 011	29 917	90 95501	0	429 152	93 9	186 2816
18 9351	0 6548	11 1615	0	52 7226	6 8742	1 084135	0	306 386	62 375	525 3707	0	530 667	101 515	452 1366
28 3862	9 4611	29 87078	0	56 9424	4 2198	13 65759	0	291 257	15 128	591 7073	0	522 807	8 06	521 612
22 1974	8 1986	4 653725	0	51 3713	5 5711	6 495826	0	281 718	9 538	894 9052	0	463 075	59 532	429 2976
32 9361	10 7387	45 46828	0	49 9244	1 4469	41 84173	0	227 279	54 439	224 5492	0	507 701	44 626	1269 176
21 3682	11 5679	57 33847	0	58 1562	8 2318	0 100097	0	234 732	7 453	1024 066	0	515 218	7 517	5230 307
18 0137	3 3545	0 411115	0	45 1442	13 012	25 97515	0	271 92	37 188	5 134906	0	497 537	17 681	3815 067
22 2931	4 2794	0 080496	0	55 9913	10 4471	6 409413	0	260 898	8 978	928 7888	0	462 391	15 146	4238 726
27 4463	5 1522	1 337532	0	54 1757	1 4156	42 24764	0	308 98	28 082	129 3231	0	482 848	0 457	6367 162
22 0199	5 4254	2 044091	0	43 5666	10 6091	7 255922	0	251 808	57 172	313 9263	0	659 356	176 506	9265 314
26 3911	4 3712	0 141013	0	45 5961	2 0295	34 64403	0	214 611	37 197	5 0942	0	502 543	156 813	3861 954
23 3819	3 0092	0 973148	0	39 5812	6 0149	3 611969	0	283 328	68 717	856 3212	0	550 168	47 625	1064 488
25 5077	2 1258	3 496462	0	49 5085	9 9273	4 047668	0	242 107	41 221	3 122711	0	538 721	11 447	4734 059
23 6796	1 8281	4 658415	0	62 2885	12 78	23 96416	0	228 373	13 734	661 5201	0	495 889	42 832	1400 219
25 1585	1 4789	6 334196	0	55 5072	6 7813	1 286224	0	162 308	66 065	708 1435	0	480 287	15 802	4179 957
23 2011	1 9574	4 154597	0	53 5702	1 937	35 74148	0	251 712	89 404	2494 999	0	493 716	13 429	4465 246
25 8193	2 6182	1 897459	0	55 044	1 4738	41 49445	0	237 793	13 919	652 0379	0	633 088	139 372	3495 234
20 7501	5 0692	1 152439	0	48 7122	6 3318	2 507847	0	193 902	43 891	19 68667	0	490 846	142 242	3882 822
26 0146	5 2645	1 603997	0	43 8483	5 0639	6 131196	0	221 063	27 151	151 3646	0	614 281	123 435	1864 815
29 8192	3 6046	0 152946	0	37 4552	6 1931	2 966308	0	204 154	16 899	508 7295	0	441 597	172 884	8543 788
21 1769	8 4423	19 7724	0	47 8223	10 4671	6 51108	0	223 564	19 41	401 7633	0	435 168	8 429	5449 761
18 6655	2 5114	2 203096	0	54 8326	7 0103	0 819239	0	274 2	50 636	125 0364	0	645 562	210 384	18937 307
28 2787	9 8142	31 56774	0	40 2408	14 6918	45 91935	0	223 311	50 889	130 7585	0	332 897	312 665	54015 04
22 1594	6 0857	4 368172	0	49 3083	9 0675	1 327292	0	210 178	13 133	692 7968	0	522 139	189 242	11878 93
24 5376	2 3436	2 729378	0	51 8529	2 5446	26 84569	0	254 067	43 889	19 66893	0	487 668	34 471	2095 854
21 7251	2 8125	1 399922	0	41 0853	10 7876	6 248428	0	265 91	11 843	782 3692	0	510 048	22 38	3349 11
19 929	1 7961	4 838165	0	45 7819	4 7166	10 23244	0	286 695	22 785	277 8567	0	443 238	66 81	180 8738
24 3417	4 4127	0 173903	0	51 2925	5 5106	5 783151	0	229 788	58 907	378 4179	0	557 388	114 15	1148 108
25 7328	1 3911	6 763852	0	48 9871	2 3054	31 4723	0	184 757	45 031	31 10256	0	428 196	129 192	2395 173
29 4727	3 7399	0 065425	0	44 9569	4 0302	15 98492	0	231 686	48 929	58 87512	0	501 896	73 7	42 92211
26 9881	2 4846	2 283371	0	58 3347	13 3778	29 83761	0	116 658	15 018	597 1187	0	412 443	10 547	4658 717
23 2675	3 7206	0 075871	0	55 0977	3 237	21 8876	0	248 618	31 96	56 31052	0	470 831	41 512	1500 749
22 9608	0 3067	13 60659	0	38 183	16 9147	80 98707	0	240 084	6 534	956 0485	0	597 786	126 857	2172 073
24 3396	1 3748	6 869927	0	48 4465	8 2635	0 121161	0	213 717	26 367	171 2704	0	383 519	214 269	17960 69
25 6848	1 3462	7 003871	0	37 9842	8 4823	0 321355	0	252 317	31 6	81 68584	0	458 574	75 055	27 00358
25 5403	0 1445	14 83161	0	47 0203	9 0561	1 301155	0	287 645	42 328	8 259683	0	429 468	29 106	26 15 862
16 0021	7 5382	12 54943	0	44 7558	2 2645	31 83288	0	232 333	55 212	251 4751	0	469 725	40 251	1598 56
25 3836	7 3815	11 46378	0	56 1939	11 4381	12 40829	0	271 259	38 926	0 278619	0	483 3	13 575	4445 758
27 4515	2 0879	3 716347	0	54 8679	1 528	40 82466	0	269 809	1 65	1429 145	0	562 872	79 372	0 273515
26 1261	1 3254	7 13041	0	40 2996	14 3683	41 83968	0	214 483	95 126	245 6105	0	431 848	130 824	2557 578
19 3415	3 2154	0 608841	0	44 5375	4 2279	13 99779	0	343 48	128 997	8017 943	1	466 271	34 423	2100 251
25 9595	3 382	0 376607	0	47 7249	3 1974	22 2597	0	267 503	85 977	2184 386	0	508 978	40 707	1583 767
26 8901	1 0206	8 851118	0	57 1525	9 4276	2 286694	0	264 528	7 025	1051 642	0	535 629	28 651	2662 611
21 0246	5 9655	3 840883	0	43 83	13 3225	29 23853	0	201 487	63 031	555 8733	0	466 663	66 966	176 5044
25 9906	4 966	0 841515	0	48 8367	5 0067	6 460842	0	224 847	23 35	259 3399	0	519 524	50 861	863 8013
25 438	0 5526	11 85482	0	54 4537	5 617	5 282726	0	192 922	31 925	56 88635	0	422 884	96 64	268 583
26 2063	0 7683	10 416	0	54 2568	0 1989	99 57592	0	227 342	34 42	25 34149	0	445 091	22 207	3369 184
21 1683	5 038	1 086425	0	44 1296	10 1273	4 892421	0	311 182	83 84	1870 114	0	672 942	227 851	21783 61
27 4567	6 2884	5 256552	0	43 8945	0 435	55 95666	0	312 087	0 905	1488 026	0	545 795	127 147	2199 180
25 0258	2 4309	2 448545	0	51 2583	7 5638	0 123635	0	254 432	57 855	331 2752	0	722 253	176 458	9255 691
30 0607	5 0349	1 079972	0	50 7683	0 49	55 13684	0	287 505	33 073	40 71759	0	639 979	182 274	10408 59
21 0119	9 0458	25 53399	0	82 614	11 8457	15 14612	0	229 55	57 965	342 2858	0	635 812	95 833	242 7832
Sum MR > UCLmr			64	36			91	21			501134 9	84	20	
Sum X	199499 8	50184 43	Sum X	399485 3	100121 8	Sum X	2001378	2001378	501134 9	Sum X	4005407	1002796		
X Average	24 96842		X Average	49 95871		X Average	250 2511			X Average	500 6203			
MR Average	3 995683		MR Average	7 915416		MR Average	39 45403			MR Average	80 2515			
Normal constant d2 (n=2)	1 128		Normal constant d2 (n=2)	1 128		Normal constant d2 (n=2)	1 128			Normal constant d2 (n=2)	1 128			
Normal constant D4 (n=2)	3 268		Normal constant D4 (n=2)	3 268		Normal constant D4 (n=2)	3 268			Normal constant D4 (n=2)	3 268			
Upper control limit for X (UCLx)	35 59524		Upper control limit for X (UCLx)	71 01036		Upper control limit for X (UCLx)	355 182			Upper control limit for X (UCLx)	714 2551			
Lower control limit for X (LCLx)	14 34161		Lower control limit for X (LCLx)	28 90707		Lower control limit for X (LCLx)	145 3202			Lower control limit for X (LCLx)	287 3855			
Upper control limit MR (UCLmr)	13 05789		Upper control limit MR (UCLmr)	25 86759		Upper control limit MR (UCLmr)	128 9356			Upper control limit MR (UCLmr)	262 2619			
MR Standard deviation	3 020592		MR Standard deviation	6 047264		MR Standard deviation	29 95058			MR Standard deviation	60 69407			
Process standard deviation	3 535534		Process standard deviation	7 071068		Process standard deviation	35 35324			Process standard deviation	70 71066			
Gamma d2	1 13015		Gamma d2	1 119409		Gamma d2	1 115929			Gamma d2	1 134928			
Gamma d3	0 854352		Gamma d3	0 855212		Gamma d3	0 847258			Gamma d3	0 858344			
Gamma D4	3 267891		Gamma D4	3 291956		Gamma D4	3 27772			Gamma D4	3 268895			
50 Alpha=50			50 Alpha=50			50 Alpha=50				50 Alpha=50				
0 5 Beta=0 5			1 Beta=1			5 Beta=5				10 Beta=10				
Proportion MR > UCLmr	0 010001		Proportion MR > UCLmr	0 011201		Proportion MR > UCLmr	0 010401			Proportion MR > UCLmr	0 008901			
Percentage	1 0001		Percentage	1 120112		Percentage	1 040104			Percentage	0 890089			
ARL for MR	99 99		ARL for MR	89 27679		ARL for MR	96 14423			ARL for MR	112 3483			

100 Alpha=100
0.5 Beta=0.5

Table with 5 columns: ID, MR, (MR)-MR)^2, MR, (MR)-MR)^2. Contains 100 rows of numerical data.

100 Alpha=100
1 Beta=1

Table with 5 columns: ID, MR, (MR)-MR)^2, MR, (MR)-MR)^2. Contains 100 rows of numerical data.

100 Alpha=100
5 Beta=5

Table with 5 columns: ID, MR, (MR)-MR)^2, MR, (MR)-MR)^2. Contains 100 rows of numerical data.

100 Alpha=100
10 Beta=10

Table with 5 columns: ID, MR, (MR)-MR)^2, MR, (MR)-MR)^2. Contains 100 rows of numerical data.

170

50 8	0 6376	25 0832	0	97 099	5 514	32 77471	0	456 2	3 573	2698 292	0	846 04	415 79	91703 77
39 8012	10 9988	28 65339	0	85 061	9 038	4 844051	0	467 923	11 723	1918 01	0	1020 7	174 66	3608 434
51 1327	11 3315	32 32589	0	103 579	15 516	18 31052	0	480 057	12 134	1882 179	0	966 67	54 03	3473 181
46 5228	4 6099	1 073324	0	94 946	4 633	43 63819	0	949 365	14 308	1996 272	0	947 98	18 69	8887 53
63 4832	16 9604	128 0176	0	105 138	8 192	25 47141	0	447 83	46 535	80 6959	0	1119 86	171 88	3471 131
49 0508	14 4324	77 20235	0	84 24	20 898	93 29782	0	593 979	146 149	8213 962	0	1104 06	15 8	9440 784
61 2407	12 1899	42 82376	0	95 836	11 506	0 127606	0	381 083	202 896	21720 25	1	894 47	209 59	9336 843
52 3248	8 9159	10 69281	0	120 193	24 357	172 084	0	415 123	74 04	890 8701	0	826 41	88 06	2818 342
54 1882	1 8634	14 30741	0	99 735	20 458	84 99143	0	553 509	138 386	6867 091	0	789 49	36 92	5782 644
37 4604	3 2722	5 634515	0	98 648	1 087	103 0615	0	420 2	133 309	6051 426	0	1051 03	261 54	22074 92
50 2378	7 2228	2 486571	0	101 178	2 53	75 8453	0	535 256	115 055	3544 844	0	963 51	87 52	647 3817
39 7905	10 4471	23 05139	0	106 319	5 141	37 18463	0	578 793	43 538	143 5226	0	972 57	9 06	10795 98
42 5873	2 7968	8 117447	0	101 625	4 694	42 83598	0	410 57	168 223	12702 4	0	928 13	44 44	4695 497
48 5994	8 0121	0 134093	0	84 563	17 062	33 90825	0	493 422	82 852	747 1426	0	840 38	87 75	635 7306
40 8235	7 7759	4 536843	0	109 673	25 11	182 4088	0	484 925	8 497	2210 983	0	1077 72	237 34	15469 46
43 4051	2 5818	9 390917	0	108 14	3 533	59 38121	0	565 427	80 502	624 1958	0	1016 32	81 4	2658 815
40 8649	2 5402	9 645456	0	104 724	1 416	96 48977	0	501 774	83 853	86 17676	0	965 82	60 5	2752 439
45 7194	4 8545	0 826335	0	106 319	5 141	37 18463	0	489 618	12 156	1860 271	0	1006 71	50 89	3853 144
48 7091	2 9897	7 05547	0	100 423	8 977	5 116284	0	447 107	42 511	189 1844	0	850 4	156 31	1878 902
51 6244	2 9153	7 45625	0	95 785	4 638	43 57215	0	545 368	98 281	1828 666	0	944 53	94 13	354 7081
48 6022	3 0222	6 883872	0	95 7	0 085	124 4099	0	491 489	53 899	2 621452	0	1146 8	202 27	7875 816
57 8276	8 2254	12 81272	0	93 365	2 335	79 2798	0	436 122	35 367	0 222828	0	921 75	225 05	12563 34
96 0327	1 7949	14 8303	0	112 542	19 177	63 03111	0	514 237	78 115	510 6294	0	1078 88	155 13	1777 997
52 021	4 0117	2 670854	0	96 944	15 568	19 00157	0	465 108	49 131	40 75492	0	870 86	206 02	8658 476
52 6619	0 6409	25 05016	0	101 268	4 324	47 81813	0	509 586	44 48	121 8394	0	1195 16	32 4	44683 03
48 3452	4 3167	1 766808	0	101 501	0 233	121 1303	0	566 107	56 521	1 006829	0	1022 13	173 03	3807 961
41 3711	8 9741	1 76406	0	95 488	6 013	27 31024	0	552 812	13 295	1782 789	0	1022 7	0 57	12632 34
55 0652	13 8941	64 77331	0	99 3	9 812	95 15915	0	499 077	53 735	3 179409	0	876 89	146 01	1092 058
42 3529	12 7123	49 93382	0	94 512	4 788	41 81438	0	457 323	41 756	189 3951	0	929 9	33 21	3570 504
50 8694	8 5165	0 240267	0	98 22	3 706	56 17476	0	515 273	57 952	5 823819	0	989 21	59 31	2878 719
46 9979	3 8715	3 148543	0	92 583	5 637	31 38151	0	469 177	46 096	80 75758	0	832 79	56 42	3187 19
48 7157	1 7178	15 43008	0	94 701	2 118	83 19119	0	492 681	23 504	1024 902	0	1205 98	273 2	26675 67
43 7879	4 3278	5 915687	0	95 09	0 389	117 7206	0	491 281	1 4	2928 788	0	931	274 96	26252 52
46 8038	3 0159	6 816971	0	88 769	6 321	24 10594	0	518 732	25 451	904 0299	0	1046 31	115 31	5305 141
48 936	2 1322	12 34618	0	107 275	18 506	52 81044	0	469 713	47 019	72 23453	0	1177 45	131 24	334 0233
54 7885	8 8525	0 042678	0	114 382	7 107	17 07277	0	465 712	4 001	2654 011	0	1013 1	184 45	2659 839
95 5382	10 7487	26 04864	0	114 225	0 043	125 3486	0	462 067	3 645	2690 817	0	1187 59	154 48	1724 434
48 5581	16 9821	128 5091	0	116 824	18 821	54 4951	0	574 447	112 38	3233 277	0	897 32	270 27	24745 27
42 6657	5 8904	0 059774	0	103 444	7 64	12 95223	0	513 311	61 136	31 96091	0	1048 84	151 52	1486 589
51 8809	9 2152	12 73981	0	107 053	3 609	58 21569	0	468 082	45 229	105 8654	0	1016 61	32 23	8517 83
43 4357	8 4452	7 836005	0	101 477	5 576	32 08867	0	538 615	71 533	256 4773	0	1239 58	222 85	12096 99
44 4385	1 0028	21 5585	0	98 72	2 787	71 94298	0	472 917	86 898	124 9904	0	1214 58	24 98	7741 131
50 0458	5 6103	0 001268	0	90 287	8 433	7 87319	0	447 41	29 507	900 6855	0	1069 28	145 10	1045 637
47 5075	2 5413	9 836625	0	95 109	4 822	41 17887	0	498 058	50 648	23 17177	0	888 47	180 81	4603 121
51 421	3 9135	3 001256	0	126 567	31 458	408 8112	0	587 528	89 468	1152 596	0	946 75	58 28	2590 307
49 8073	1 8137	14 68588	0	98 552	30 015	352 5412	0	529 275	86 251	7 468798	0	1037 89	91 14	478 2737
50 8998	1 2925	18 95221	0	113 346	16 794	30 85891	0	548 927	17 652	1433 841	0	794 47	243 42	17018 85
45 2947	5 8051	0 001696	0	119 68	6 334	24 05825	0	530 515	18 412	1529 286	0	1151 38	356 91	59509 8
52 373	7 0783	2 051731	0	88 354	31 316	403 0891	0	579 008	48 493	49 35189	0	1007 4	143 98	962 0111
45 5679	6 8051	1 343714	0	104 842	16 476	27 44795	0	431 373	147 635	8485 525	0	901 96	106 32	44 1387
53 747	8 1791	6 417034	0	86 909	17 933	44 8107	0	462 59	31 217	590 543	0	1136 13	235 05	14905 07
47 2843	8 4627	0 66714	0	100 872	13 963	7 420609	0	471 261	8 671	2184 65	0	983 18	152 95	1598 904
52 5739	5 2896	0 126959	0	114 327	13 455	4 911008	0	548 399	77 138	487 4205	0	943 53	39 65	5374 898
44 243	8 3439	7 279132	0	80 498	33 829	510 3117	0	525 894	22 505	1089 864	0	957 7	14 17	9760 194
47 509	3 279	5 802279	0	118 183	38 865	753 2873	0	398 848	127 046	5118 742	0	904 04	53 66	3518 928
50 3036	2 7946	8 129888	0	104 589	14 594	11 25656	0	496 288	97 44	1757 447	0	1030 4	126 36	179 461
Sum MR > UCLmr			85	22			70	17				81	18	
Sum X	399933 1	99910 3	Sum X	800969 6	200262 9	Sum X	3998864	1000351	Sum X	8005017	Sum X	8005017	2008214	
X Average	49 98434		X Average	100 1233		X Average	459 9215		X Average	112 9637	X Average	1001 323		
MR Average	5 645913		MR Average	11 23892		MR Average	55 51809		MR Average	11 23892	MR Average	11 23892		
Normal constant d2 (n=2)	1 128		Normal constant d2 (n=2)	1 128		Normal constant d2 (n=2)	1 128		Normal constant d2 (n=2)	1 128	Normal constant d2 (n=2)	1 128		
Normal constant D4 (n=2)	3 268		Normal constant D4 (n=2)	3 268		Normal constant D4 (n=2)	3 268		Normal constant D4 (n=2)	3 268	Normal constant D4 (n=2)	3 268		
Upper control limit for X (UCLx)	65 00006		Upper control limit for X (UCLx)	130 014		Upper control limit for X (UCLx)	647 578		Upper control limit for X (UCLx)	1301 758	Upper control limit for X (UCLx)	1301 758		
Lower control limit for X (LCLx)	34 96861		Lower control limit for X (LCLx)	70 23251		Lower control limit for X (LCLx)	352 267		Lower control limit for X (LCLx)	700 8875	Lower control limit for X (LCLx)	700 8875		
Upper control limit MR (UCLmr)	18 45085		Upper control limit MR (UCLmr)	36 72819		Upper control limit MR (UCLmr)	181 5331		Upper control limit MR (UCLmr)	369 1854	Upper control limit MR (UCLmr)	369 1854		
MR Standard deviation	4 289753		MR Standard deviation	8 451407		MR Standard deviation	42 17469		MR Standard deviation	85 69263	MR Standard deviation	85 69263		
Process standard deviation	5		Process standard deviation	10		Process standard deviation	50		Process standard deviation	100	Process standard deviation	100		
Gamma d2	1 129183		Gamma d2	1 123892		Gamma d2	1 110362		Gamma d2	1 129637	Gamma d2	1 129637		
Gamma d3	0 857951		Gamma d3	0 849141		Gamma d3	0 843494		Gamma d3	0 856926	Gamma d3	0 856926		
Gamma D4	3 279394		Gamma D4	3 266607		Gamma D4	3 278897		Gamma D4	3 275757	Gamma D4	3 275757		
100 Alpha=100			100 Alpha=100			100 Alpha=100			100 Alpha=100		100 Alpha=100			
0 5 Beta=0 5			1 Beta=1			5 Beta=5			10 Beta=10		10 Beta=10			
Proportion MR > UCLmr	0 010701		Proportion MR > UCLmr	0 008701		Proportion MR > UCLmr	0 009901		Proportion MR > UCLmr	0 009601	Proportion MR > UCLmr	0 009601		
Percentage	1 070107		Percentage	0 870087		Percentage	0 990039		Percentage	0 960096	Percentage	0 960096		
ARL for MR	93 4486		ARL for MR	114 931		ARL for MR	101		ARL for MR	104 1562	ARL for MR	104 1562		

APPENDIX E

PROBABILITIES OF DETECTION AND AVERAGE RUN LENGTH (ARL)

SPREADSHEETS

Objective #1 ARL curves

Alpha Beta Shift k	0.5		Average run length							Alpha Beta Shift k	0.5		Average run length						
	Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	Probabil. of detection P	ARL (1/P)	Sigma		k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	Probabil. of detection P	ARL (1/P)		
0	0.353553	0	1.085119	-0.58988	1.085119	-0.58988	0.037216	26.87016	0	0.707107	0	2.199737	-1.19075	2.199737	-1.19075	0.03595	27.81641		
0.2	0.353553	0.070711	1.085119	-0.58988	1.014408	-0.66059	0.043972	22.74174	0.2	0.707107	0.141421	2.199737	-1.19075	2.058316	-1.33217	0.042464	23.54936		
0.4	0.353553	0.141421	1.085119	-0.58988	0.943698	-0.7313	0.05203	19.21968	0.4	0.707107	0.282843	2.199737	-1.19075	1.916894	-1.47359	0.050229	19.90882		
0.6	0.353553	0.212132	1.085119	-0.58988	0.872987	-0.80201	0.061668	16.21587	0.6	0.707107	0.424264	2.199737	-1.19075	1.775473	-1.61501	0.059512	16.80333		
0.8	0.353553	0.282843	1.085119	-0.58988	0.802276	-0.87272	0.07323	13.65561	0.8	0.707107	0.565685	2.199737	-1.19075	1.634052	-1.75644	0.070639	14.15649		
1	0.353553	0.353553	1.085119	-0.58988	0.731566	-0.94343	0.087149	11.4746	1	0.707107	0.707107	2.199737	-1.19075	1.49263	-1.89786	0.084026	11.90108		
1.2	0.353553	0.424264	1.085119	-0.58988	0.660855	-1.01414	0.10398	9.617234	1.2	0.707107	0.848528	2.199737	-1.19075	1.351209	-2.03928	0.100196	9.980438		
1.4	0.353553	0.494975	1.085119	-0.58988	0.590144	-1.08485	0.124436	8.03626	1.4	0.707107	0.989949	2.199737	-1.19075	1.209788	-2.1807	0.119827	8.345365		
1.6	0.353553	0.565685	1.085119	-0.58988	0.519434	-1.15557	0.149462	6.890664	1.6	0.707107	1.131371	2.199737	-1.19075	1.068366	-2.32212	0.143808	6.953716		
1.8	0.353553	0.636396	1.085119	-0.58988	0.448723	-1.22628	0.180331	5.545358	1.8	0.707107	1.272792	2.199737	-1.19075	0.926945	-2.46354	0.173332	5.769275		
2	0.353553	0.707107	1.085119	-0.58988	0.378012	-1.29699	0.218827	4.56982	2	0.707107	1.414214	2.199737	-1.19075	0.785523	-2.60496	0.210055	4.760658		
2.2	0.353553	0.777817	1.085119	-0.58988	0.307302	-1.3677	0.267562	3.737452	2.2	0.707107	1.555635	2.199737	-1.19075	0.644102	-2.74638	0.256379	3.900475		
2.4	0.353553	0.848528	1.085119	-0.58988	0.236591	-1.43841	0.330647	3.024373	2.4	0.707107	1.697056	2.199737	-1.19075	0.502881	-2.88781	0.316017	3.164387		
2.6	0.353553	0.919239	1.085119	-0.58988	0.16588	-1.50912	0.41532	2.407782	2.6	0.707107	1.838478	2.199737	-1.19075	0.361259	-3.02923	0.395319	2.529603		
2.8	0.353553	0.989949	1.085119	-0.58988	0.09517	-1.57983	0.537241	1.861362	2.8	0.707107	1.979899	2.199737	-1.19075	0.219838	-3.17065	0.507279	1.971302		
3	0.353553	1.06066	1.085119	-0.58988	0.024459	-1.65054	0.754443	1.325481	3	0.707107	2.12132	2.199737	-1.19075	0.078417	-3.31207	0.692089	1.444901		

Alpha Beta Shift k	0.5		Average							Alpha Beta Shift k	0.5		Average								
	5		UCLx		LCLx		Altered		Probabil. of detection		10		UCLx		LCLx		Altered		Probabil. of detection		Average run length
	Sigma	k*Sigma	UCLx	LCLx	UCLx	LCLx	UCLx	LCLx	P	ARL (1/P)		Sigma	k*Sigma	UCLx	LCLx	UCLx	LCLx	UCLx	LCLx	P	ARL (1/P)
0	3.535534	0	10.67373	-5.76877	10.67373	-5.76877	10.67373	-5.76877	0.038803	25.7712	0	7.071068	0	21.76866	-11.8308	21.76866	-11.8308	0.036928	27.07972		
0.2	3.535534	0.707107	10.67373	-5.76877	9.966623	-6.47588	0.045862	21.80454			0.2	7.071068	1.414214	21.76866	-11.8308	20.35445	-13.245	0.043628	22.92106		
0.4	3.535534	1.414214	10.67373	-5.76877	9.259516	-7.18298	0.054289	18.41994			0.4	7.071068	2.828427	21.76866	-11.8308	18.94023	-14.6592	0.05162	19.37234		
0.6	3.535534	2.12132	10.67373	-5.76877	8.55241	-7.89009	0.064373	15.53446			0.6	7.071068	4.242641	21.76866	-11.8308	17.52602	-16.0734	0.061176	16.34628		
0.8	3.535534	2.828427	10.67373	-5.76877	7.845303	-8.5972	0.076482	13.07497			0.8	7.071068	5.656854	21.76866	-11.8308	16.11181	-17.4877	0.072639	13.76671		
1	3.535534	3.535534	10.67373	-5.76877	7.138196	-9.3043	0.091074	10.98008			1	7.071068	7.071068	21.76866	-11.8308	14.69759	-18.9019	0.086437	11.56912		
1.2	3.535534	4.242641	10.67373	-5.76877	6.431089	-10.0114	0.10874	9.196248			1.2	7.071068	8.485281	21.76866	-11.8308	13.28338	-20.3161	0.103116	9.697816		
1.4	3.535534	4.949747	10.67373	-5.76877	5.723983	-10.7185	0.130244	7.677897			1.4	7.071068	9.899495	21.76866	-11.8308	11.86917	-21.7303	0.123384	8.104779		
1.6	3.535534	5.656854	10.67373	-5.76877	5.016876	-11.4256	0.1566	6.385696			1.6	7.071068	11.31371	21.76866	-11.8308	10.45495	-23.1445	0.14817	6.749005		
1.8	3.535534	6.363961	10.67373	-5.76877	4.309769	-12.1327	0.189191	5.285664			1.8	7.071068	12.72792	21.76866	-11.8308	9.040738	-24.5587	0.178731	5.595		
2	3.535534	7.071068	10.67373	-5.76877	3.602662	-12.8398	0.229967	4.34845			2	7.071068	14.14214	21.76866	-11.8308	7.626524	-25.9729	0.216818	4.612163		
2.2	3.535534	7.778175	10.67373	-5.76877	2.895555	-13.5469	0.281835	3.548175			2.2	7.071068	15.55635	21.76866	-11.8308	6.212311	-27.3871	0.264997	3.773628		
2.4	3.535534	8.485281	10.67373	-5.76877	2.188449	-14.2541	0.34947	2.861476			2.4	7.071068	16.97056	21.76866	-11.8308	4.798097	-28.8014	0.327283	3.05546		
2.6	3.535534	9.192388	10.67373	-5.76877	1.481342	-14.9612	0.44144	2.265314			2.6	7.071068	18.38478	21.76866	-11.8308	3.383884	-30.2156	0.410699	2.434873		
2.8	3.535534	9.899495	10.67373	-5.76877	0.774235	-15.6683	0.577868	1.730499			2.8	7.071068	19.79899	21.76866	-11.8308	1.96967	-31.6298	0.530239	1.885942		
3	3.535534	10.6066	10.67373	-5.76877	0.067128	-16.3754	0.869838	1.149639			3	7.071068	21.2132	21.76866	-11.8308	0.555457	-33.044	0.738905	1.353354		

Alpha Beta Shift k	1 0.5						Probabil. of detection P	Average run length ARL (1/P)
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	
0		0.5	0	1.827984	-0.83481	1.827984	-0.83481	0.025836
0.2		0.5	0.1	1.827984	-0.83481	1.727984	-0.93481	0.031557
0.4		0.5	0.2	1.827984	-0.83481	1.627984	-1.03481	0.038544
0.6		0.5	0.3	1.827984	-0.83481	1.527984	-1.13481	0.047077
0.8		0.5	0.4	1.827984	-0.83481	1.427984	-1.23481	0.0575
1		0.5	0.5	1.827984	-0.83481	1.327984	-1.33481	0.070231
1.2		0.5	0.6	1.827984	-0.83481	1.227984	-1.43481	0.08578
1.4		0.5	0.7	1.827984	-0.83481	1.127984	-1.53481	0.104772
1.6		0.5	0.8	1.827984	-0.83481	1.027984	-1.63481	0.127969
1.8		0.5	0.9	1.827984	-0.83481	0.927984	-1.73481	0.156302
2		0.5	1	1.827984	-0.83481	0.827984	-1.83481	0.190907
2.2		0.5	1.1	1.827984	-0.83481	0.727984	-1.93481	0.233175
2.4		0.5	1.2	1.827984	-0.83481	0.627984	-2.03481	0.2848
2.6		0.5	1.3	1.827984	-0.83481	0.527984	-2.13481	0.347856
2.8		0.5	1.4	1.827984	-0.83481	0.427984	-2.23481	0.424872
3		0.5	1.5	1.827984	-0.83481	0.327984	-2.33481	0.518939

Alpha Beta Shift k	1 1						Probabil. of detection P	Average run length ARL (1/P)
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	
0		1	0	3.673837	-1.63234	3.673837	-1.63234	0.025379
0.2		1	0.2	3.673837	-1.63234	3.473837	-1.83234	0.030998
0.4		1	0.4	3.673837	-1.63234	3.273837	-2.03234	0.037881
0.6		1	0.6	3.673837	-1.63234	3.073837	-2.23234	0.046243
0.8		1	0.8	3.673837	-1.63234	2.873837	-2.43234	0.056482
1		1	1	3.673837	-1.63234	2.673837	-2.63234	0.068987
1.2		1	1.2	3.673837	-1.63234	2.473837	-2.83234	0.084261
1.4		1	1.4	3.673837	-1.63234	2.273837	-3.03234	0.102917
1.6		1	1.6	3.673837	-1.63234	2.073837	-3.23234	0.125703
1.8		1	1.8	3.673837	-1.63234	1.873837	-3.43234	0.153533
2		1	2	3.673837	-1.63234	1.673837	-3.63234	0.187526
2.2		1	2.2	3.673837	-1.63234	1.473837	-3.83234	0.229045
2.4		1	2.4	3.673837	-1.63234	1.273837	-4.03234	0.279756
2.6		1	2.6	3.673837	-1.63234	1.073837	-4.23234	0.341695
2.8		1	2.8	3.673837	-1.63234	0.873837	-4.43234	0.417347
3		1	3	3.673837	-1.63234	0.673837	-4.63234	0.509749

Alpha Beta Shift	1 5						Probabil. of detection	Average run length ARL (1/P)
k	Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	P	
0	5	0	18.10885	-8.19452	18.10885	-8.19452	0.026735	37.40415
0.2	5	1	18.10885	-8.19452	17.10885	-9.19452	0.032654	30.62412
0.4	5	2	18.10885	-8.19452	16.10885	-10.1945	0.039884	25.07271
0.6	5	3	18.10885	-8.19452	15.10885	-11.1945	0.048715	20.52756
0.8	5	4	18.10885	-8.19452	14.10885	-12.1945	0.0595	16.80672
1	5	5	18.10885	-8.19452	13.10885	-13.1945	0.072674	13.76008
1.2	5	6	18.10885	-8.19452	12.10885	-14.1945	0.088764	11.26583
1.4	5	7	18.10885	-8.19452	11.10885	-15.1945	0.108417	9.223646
1.6	5	8	18.10885	-8.19452	10.10885	-16.1945	0.132421	7.551672
1.8	5	9	18.10885	-8.19452	9.10885	-17.1945	0.161739	6.182801
2	5	10	18.10885	-8.19452	8.10885	-18.1945	0.197549	5.062035
2.2	5	11	18.10885	-8.19452	7.10885	-19.1945	0.241286	4.144459
2.4	5	12	18.10885	-8.19452	6.10885	-20.1945	0.294708	3.393189
2.6	5	13	18.10885	-8.19452	5.10885	-21.1945	0.359957	2.77811
2.8	5	14	18.10885	-8.19452	4.10885	-22.1945	0.439653	2.274521
3	5	15	18.10885	-8.19452	3.10885	-23.1945	0.536993	1.862222

Alpha Beta Shift	1 10						Probabil. of detection	Average run length ARL (1/P)
k	Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	P	
0	10	0	36.8536	-16.9254	36.8536	-16.9254	0.025088	39.85969
0.2	10	2	36.8536	-16.9254	34.8536	-18.9254	0.030643	32.63388
0.4	10	4	36.8536	-16.9254	32.8536	-20.9254	0.037427	26.71868
0.6	10	6	36.8536	-16.9254	30.8536	-22.9254	0.045714	21.87514
0.8	10	8	36.8536	-16.9254	28.8536	-24.9254	0.055835	17.90991
1	10	10	36.8536	-16.9254	26.8536	-26.9254	0.068197	14.6634
1.2	10	12	36.8536	-16.9254	24.8536	-28.9254	0.083296	12.00538
1.4	10	14	36.8536	-16.9254	22.8536	-30.9254	0.101737	9.829266
1.6	10	16	36.8536	-16.9254	20.8536	-32.9254	0.124262	8.047513
1.8	10	18	36.8536	-16.9254	18.8536	-34.9254	0.151774	6.588744
2	10	20	36.8536	-16.9254	16.8536	-36.9254	0.185378	5.394383
2.2	10	22	36.8536	-16.9254	14.8536	-38.9254	0.226421	4.416551
2.4	10	24	36.8536	-16.9254	12.8536	-40.9254	0.278551	3.61597
2.6	10	26	36.8536	-16.9254	10.8536	-42.9254	0.33778	2.960507
2.8	10	28	36.8536	-16.9254	8.8536	-44.9254	0.412568	2.423855
3	10	30	36.8536	-16.9254	6.8536	-46.9254	0.503909	1.984485

Alpha Beta Shift	1.5 0.5						Probabil. of detection P	Average run length ARL (1/P)		Alpha Beta Shift	1.5 1						Probabil. of detection P	Average run length ARL (1/P)	
k		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx			k		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx		
0		0.612372	0	2.420511	-0.93029	2.420511	-0.93029	0.021472	46.57228	0		1.224745	0	4.826731	-1.84273	4.826731	-1.84273	0.021754	45.96856
0.2		0.612372	0.122474	2.420511	-0.93029	2.298037	-1.05276	0.026842	37.25505	0.2		1.224745	0.244949	4.826731	-1.84273	4.581782	-2.08768	0.027193	36.77417
0.4		0.612372	0.244949	2.420511	-0.93029	2.175562	-1.17524	0.033523	29.83027	0.4		1.224745	0.489898	4.826731	-1.84273	4.336833	-2.33263	0.03396	29.44641
0.6		0.612372	0.367423	2.420511	-0.93029	2.053088	-1.29771	0.041821	23.91143	0.6		1.224745	0.734847	4.826731	-1.84273	4.091884	-2.57758	0.042363	23.6055
0.8		0.612372	0.489898	2.420511	-0.93029	1.930613	-1.42019	0.05211	19.19017	0.8		1.224745	0.979796	4.826731	-1.84273	3.846935	-2.82253	0.052781	18.94621
1		0.612372	0.612372	2.420511	-0.93029	1.808139	-1.54266	0.064843	15.42187	1		1.224745	1.224745	4.826731	-1.84273	3.601986	-3.06747	0.065673	15.22696
1.2		0.612372	0.734847	2.420511	-0.93029	1.685664	-1.66514	0.080569	12.41172	1.2		1.224745	1.469694	4.826731	-1.84273	3.357037	-3.31242	0.081592	12.2561
1.4		0.612372	0.857321	2.420511	-0.93029	1.56319	-1.78761	0.09994	10.006	1.4		1.224745	1.714643	4.826731	-1.84273	3.112088	-3.55737	0.101199	9.881521
1.6		0.612372	0.979796	2.420511	-0.93029	1.440715	-1.91009	0.123735	8.081788	1.6		1.224745	1.959592	4.826731	-1.84273	2.867139	-3.80232	0.125279	7.982184
1.8		0.612372	1.10227	2.420511	-0.93029	1.318241	-2.03256	0.152866	6.541677	1.8		1.224745	2.204541	4.826731	-1.84273	2.62219	-4.04727	0.154752	6.461952
2		0.612372	1.224745	2.420511	-0.93029	1.195766	-2.15503	0.188389	5.308166	2		1.224745	2.44949	4.826731	-1.84273	2.377241	-4.29222	0.190684	5.244278
2.2		0.612372	1.347219	2.420511	-0.93029	1.073292	-2.27751	0.231498	4.319692	2.2		1.224745	2.694439	4.826731	-1.84273	2.132292	-4.53717	0.234275	4.268488
2.4		0.612372	1.469694	2.420511	-0.93029	0.950817	-2.39998	0.283506	3.527262	2.4		1.224745	2.939386	4.826731	-1.84273	1.887343	-4.78212	0.286844	3.486216
2.6		0.612372	1.592168	2.420511	-0.93029	0.828343	-2.52246	0.345786	2.891962	2.6		1.224745	3.184337	4.826731	-1.84273	1.642394	-5.02707	0.349765	2.859063
2.8		0.612372	1.714643	2.420511	-0.93029	0.705868	-2.64493	0.41965	2.382938	2.8		1.224745	3.429286	4.826731	-1.84273	1.397445	-5.27202	0.424342	2.35659
3		0.612372	1.837117	2.420511	-0.93029	0.583394	-2.76741	0.506119	1.97582	3		1.224745	3.674235	4.826731	-1.84273	1.152496	-5.51696	0.511565	1.954786

Alpha Beta Shift k	1.5								Average run length ARL (1/P)	Alpha Beta Shift k	1.5								Average run length ARL (1/P)
	5										10								
	Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	Probabil. of detection P			Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	Probabil. of detection P			
0	6.123724	0	24.47972	-9.37413	24.47972	-9.37413	0.020421	48.9692	0	12.24745	0	48.82124	-18.7641	48.82124	-18.7641	0.020688	48.3559		
0.2	6.123724	1.224745	24.47972	-9.37413	23.25498	-10.5989	0.025534	39.16347	0.2	12.24745	2.44949	48.82124	-18.7641	46.37175	-21.2136	0.025857	38.67425		
0.4	6.123724	2.44949	24.47972	-9.37413	22.03023	-11.8236	0.031896	31.35189	0.4	12.24745	4.898979	48.82124	-18.7641	43.92226	-23.6631	0.032298	30.96187		
0.6	6.123724	3.674235	24.47972	-9.37413	20.80549	-13.0484	0.039802	25.12437	0.6	12.24745	7.348469	48.82124	-18.7641	41.47277	-26.1126	0.040301	24.81328		
0.8	6.123724	4.898979	24.47972	-9.37413	19.58074	-14.2731	0.049608	20.15804	0.8	12.24745	9.797959	48.82124	-18.7641	39.02328	-28.5621	0.050226	19.91001		
1	6.123724	6.123724	24.47972	-9.37413	18.356	-15.4979	0.061749	16.19459	1	12.24745	12.24745	48.82124	-18.7641	36.57379	-31.0115	0.062514	15.99642		
1.2	6.123724	7.348469	24.47972	-9.37413	17.13125	-16.7226	0.076751	13.02915	1.2	12.24745	14.69694	48.82124	-18.7641	34.1243	-33.461	0.077695	12.87084		
1.4	6.123724	8.573214	24.47972	-9.37413	15.90651	-17.9473	0.095241	10.49968	1.4	12.24745	17.14643	48.82124	-18.7641	31.67481	-35.9105	0.096403	10.37312		
1.6	6.123724	9.797959	24.47972	-9.37413	14.68176	-19.1721	0.11797	8.476731	1.6	12.24745	19.59592	48.82124	-18.7641	29.22532	-38.36	0.119396	8.37549		
1.8	6.123724	11.0227	24.47972	-9.37413	13.45702	-20.3968	0.145818	6.857864	1.8	12.24745	22.04541	48.82124	-18.7641	26.77583	-40.8095	0.147563	6.776767		
2	6.123724	12.24745	24.47972	-9.37413	12.23227	-21.6216	0.179809	5.561457	2	12.24745	24.4949	48.82124	-18.7641	24.32634	-43.259	0.181933	5.496529		
2.2	6.123724	13.47219	24.47972	-9.37413	11.00753	-22.8463	0.221106	4.522718	2.2	12.24745	26.94439	48.82124	-18.7641	21.87685	-45.7085	0.22368	4.470672		
2.4	6.123724	14.69694	24.47972	-9.37413	9.782782	-24.0711	0.271001	3.690023	2.4	12.24745	29.39388	48.82124	-18.7641	19.42736	-48.158	0.2741	3.648304		
2.6	6.123724	15.92168	24.47972	-9.37413	8.558037	-25.2958	0.330858	3.022445	2.6	12.24745	31.84337	48.82124	-18.7641	16.97787	-50.6075	0.33456	2.989		
2.8	6.123724	17.14643	24.47972	-9.37413	7.333292	-26.5206	0.402021	2.487432	2.8	12.24745	34.29286	48.82124	-18.7641	14.52838	-53.057	0.406398	2.460642		
3	6.123724	18.37117	24.47972	-9.37413	6.108547	-27.7453	0.485603	2.059295	3	12.24745	36.74235	48.82124	-18.7641	12.07889	-55.5064	0.490704	2.037888		

Alpha Beta Shift	2 0.5						Probabil. of detection P	Average run length ARL (1/P)	Alpha Beta Shift	2 1						Probabil. of detection P	Average run length ARL (1/P)
k		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx		k		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	
0		0.707107	0	2.952427	-0.9582	2.952427	-0.9582	0.018824	0	1.414214	0	5.945399	-2.02037	5.945399	-2.02037	0.018182	54.99945
0.2		0.707107	0.141421	2.952427	-0.9582	2.811006	-1.09962	0.023954	0.2	1.414214	0.282843	5.945399	-2.02037	5.662556	-2.30321	0.023143	43.20961
0.4		0.707107	0.282843	2.952427	-0.9582	2.669584	-1.24104	0.030427	0.4	1.414214	0.565685	5.945399	-2.02037	5.379714	-2.58606	0.029405	34.00782
0.6		0.707107	0.424264	2.952427	-0.9582	2.528163	-1.38246	0.038572	0.6	1.414214	0.848528	5.945399	-2.02037	5.096871	-2.86689	0.037288	26.81828
0.8		0.707107	0.565685	2.952427	-0.9582	2.386742	-1.52389	0.048791	0.8	1.414214	1.131371	5.945399	-2.02037	4.814028	-3.15174	0.047182	21.19452
1		0.707107	0.707107	2.952427	-0.9582	2.24532	-1.66531	0.061569	1	1.414214	1.414214	5.945399	-2.02037	4.531185	-3.43458	0.059559	16.79007
1.2		0.707107	0.848528	2.952427	-0.9582	2.103899	-1.80673	0.077487	1.2	1.414214	1.697056	5.945399	-2.02037	4.248343	-3.71743	0.074988	13.33547
1.4		0.707107	0.989949	2.952427	-0.9582	1.962478	-1.94815	0.097233	1.4	1.414214	1.979899	5.945399	-2.02037	3.9655	-4.00027	0.094139	10.62259
1.6		0.707107	1.131371	2.952427	-0.9582	1.821056	-2.08957	0.121609	1.6	1.414214	2.262742	5.945399	-2.02037	3.682657	-4.28311	0.117797	8.489181
1.8		0.707107	1.272792	2.952427	-0.9582	1.679635	-2.23099	0.151531	1.8	1.414214	2.545584	5.945399	-2.02037	3.399815	-4.56595	0.146863	6.809067
2		0.707107	1.414214	2.952427	-0.9582	1.538213	-2.37241	0.18802	2	1.414214	2.828427	5.945399	-2.02037	3.116972	-4.8488	0.182345	5.48411
2.2		0.707107	1.555635	2.952427	-0.9582	1.396792	-2.51383	0.232173	2.2	1.414214	3.111127	5.945399	-2.02037	2.834129	-5.13164	0.225331	4.437916
2.4		0.707107	1.697056	2.952427	-0.9582	1.255371	-2.65526	0.2851	2.4	1.414214	3.394113	5.945399	-2.02037	2.551286	-5.41448	0.276934	3.610969
2.6		0.707107	1.838478	2.952427	-0.9582	1.113949	-2.79668	0.347821	2.6	1.414214	3.676955	5.945399	-2.02037	2.268444	-5.69733	0.338196	2.956865
2.8		0.707107	1.979899	2.952427	-0.9582	0.972528	-2.9381	0.421082	2.8	1.414214	3.959798	5.945399	-2.02037	1.985601	-5.98017	0.409917	2.439518
3		0.707107	2.12132	2.952427	-0.9582	0.831107	-3.07952	0.505071	3	1.414214	4.242641	5.945399	-2.02037	1.702758	-6.26301	0.492389	2.030915

Alpha Beta Shift k	2 5	Average run length ARL (1/P)						
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	Probabil. of detection P
0	7.071068	0	29.43801	-9.63351	29.43801	-9.63351	0.019104	52.34506
0.2	7.071068	1.414214	29.43801	-9.63351	28.0238	-11.0477	0.024307	41.14041
0.4	7.071068	2.828427	29.43801	-9.63351	26.60958	-12.4619	0.030872	32.39181
0.6	7.071068	4.242641	29.43801	-9.63351	25.19537	-13.8762	0.039132	25.55453
0.8	7.071068	5.656854	29.43801	-9.63351	23.78116	-15.2904	0.049492	20.20529
1	7.071068	7.071068	29.43801	-9.63351	22.36694	-16.7046	0.062444	16.01435
1.2	7.071068	8.485281	29.43801	-9.63351	20.95273	-18.1188	0.078575	12.72869
1.4	7.071068	9.899495	29.43801	-9.63351	19.53852	-19.533	0.098579	10.14415
1.6	7.071068	11.31371	29.43801	-9.63351	18.1243	-20.9472	0.123266	8.112537
1.8	7.071068	12.72792	29.43801	-9.63351	16.71009	-22.3614	0.153558	6.512197
2	7.071068	14.14214	29.43801	-9.63351	15.29587	-23.7756	0.190482	5.24984
2.2	7.071068	15.55635	29.43801	-9.63351	13.88166	-25.1899	0.235139	4.252804
2.4	7.071068	16.97056	29.43801	-9.63351	12.46745	-26.6041	0.288636	3.464571
2.6	7.071068	18.38478	29.43801	-9.63351	11.05323	-28.0183	0.351982	2.841054
2.8	7.071068	19.79899	29.43801	-9.63351	9.63902	-29.4325	0.4259	2.347969
3	7.071068	21.2132	29.43801	-9.63351	8.224807	-30.8467	0.51053	1.958749

Alpha Beta Shift k	2 10	Average run length ARL (1/P)						
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	Probabil. of detection P
0	14.14214	0	60.49687	-20.3249	60.49687	-20.3249	0.016627	60.14314
0.2	14.14214	2.828427	60.49687	-20.3249	57.66844	-23.1533	0.021178	47.21881
0.4	14.14214	5.656854	60.49687	-20.3249	54.84002	-25.9818	0.026926	37.13882
0.6	14.14214	8.485281	60.49687	-20.3249	52.01159	-28.8102	0.03417	29.26544
0.8	14.14214	11.31371	60.49687	-20.3249	49.18316	-31.6386	0.043271	23.11017
1	14.14214	14.14214	60.49687	-20.3249	46.35473	-34.467	0.054673	18.29056
1.2	14.14214	16.97056	60.49687	-20.3249	43.52631	-37.2955	0.068904	14.51295
1.4	14.14214	19.79899	60.49687	-20.3249	40.69788	-40.1239	0.086597	11.54774
1.6	14.14214	22.62742	60.49687	-20.3249	37.86945	-42.9523	0.108495	9.217015
1.8	14.14214	25.45584	60.49687	-20.3249	35.04103	-45.7807	0.135455	7.382526
2	14.14214	28.28427	60.49687	-20.3249	32.2126	-48.6092	0.168448	5.93655
2.2	14.14214	31.1127	60.49687	-20.3249	29.38417	-51.4376	0.208537	4.795312
2.4	14.14214	33.94113	60.49687	-20.3249	26.55574	-54.266	0.256835	3.89355
2.6	14.14214	36.76955	60.49687	-20.3249	23.72732	-57.0945	0.314425	3.180409
2.8	14.14214	39.59798	60.49687	-20.3249	20.89889	-59.9229	0.382222	2.616281
3	14.14214	42.42641	60.49687	-20.3249	18.07046	-62.7513	0.460744	2.170403

Alpha Beta Shift k	3 0.5					Altered	Altered	Probabil. of detection P	Average run length ARL (1/P)
		Sigma	k*Sigma	UCLx	LCLx	UCLx	LCLx		
0		0.866025	0	4.017402	-1.01388	4.017402	-1.01388	0.013385	74.7105
0.2		0.866025	0.173205	4.017402	-1.01388	3.844197	-1.18709	0.01752	57.07763
0.4		0.866025	0.34641	4.017402	-1.01388	3.670992	-1.36029	0.022862	43.74071
0.6		0.866025	0.519615	4.017402	-1.01388	3.497787	-1.5335	0.029735	33.6304
0.8		0.866025	0.69282	4.017402	-1.01388	3.324582	-1.7067	0.038536	25.94976
1		0.866025	0.866025	4.017402	-1.01388	3.151377	-1.87991	0.049746	20.10212
1.2		0.866025	1.03923	4.017402	-1.01388	2.978172	-2.05311	0.063945	15.63844
1.4		0.866025	1.212436	4.017402	-1.01388	2.804966	-2.22632	0.081814	12.22285
1.6		0.866025	1.385641	4.017402	-1.01388	2.631761	-2.39952	0.10414	9.602458
1.8		0.866025	1.558846	4.017402	-1.01388	2.458556	-2.57273	0.131809	7.586735
2		0.866025	1.732051	4.017402	-1.01388	2.285351	-2.74593	0.16578	6.032091
2.2		0.866025	1.905256	4.017402	-1.01388	2.112146	-2.91914	0.207045	4.829868
2.4		0.866025	2.078461	4.017402	-1.01388	1.938941	-3.09234	0.256548	3.897906
2.6		0.866025	2.251666	4.017402	-1.01388	1.765736	-3.26555	0.315065	3.173948
2.8		0.866025	2.424871	4.017402	-1.01388	1.592531	-3.43875	0.38303	2.610762
3		0.866025	2.598076	4.017402	-1.01388	1.419326	-3.61196	0.460291	2.172539

Alpha Beta Shift k	3 1					Altered	Altered	Probabil. of detection P	Average run length ARL (1/P)
		Sigma	k*Sigma	UCLx	LCLx	UCLx	LCLx		
0		1.732051	0	8.017063	-1.96106	8.017063	-1.96106	0.013572	73.68111
0.2		1.732051	0.34641	8.017063	-1.96106	7.670653	-2.30747	0.017762	56.29997
0.4		1.732051	0.69282	8.017063	-1.96106	7.324243	-2.65388	0.023174	43.15181
0.6		1.732051	1.03923	8.017063	-1.96106	6.977833	-3.00029	0.030135	33.18401
0.8		1.732051	1.385641	8.017063	-1.96106	6.631422	-3.3467	0.039047	25.61016
1		1.732051	1.732051	8.017063	-1.96106	6.285012	-3.69311	0.050395	19.84324
1.2		1.732051	2.078461	8.017063	-1.96106	5.938602	-4.03952	0.064765	15.44044
1.4		1.732051	2.424871	8.017063	-1.96106	5.592192	-4.38593	0.082842	12.07117
1.6		1.732051	2.771281	8.017063	-1.96106	5.245782	-4.73234	0.10542	9.485866
1.8		1.732051	3.117691	8.017063	-1.96106	4.899372	-5.07875	0.133387	7.496982
2		1.732051	3.464102	8.017063	-1.96106	4.552961	-5.42516	0.167708	5.962745
2.2		1.732051	3.810512	8.017063	-1.96106	4.206551	-5.77157	0.209373	4.776165
2.4		1.732051	4.156922	8.017063	-1.96106	3.860141	-6.11798	0.259321	3.856225
2.6		1.732051	4.503332	8.017063	-1.96106	3.513731	-6.46439	0.318315	3.141542
2.8		1.732051	4.849742	8.017063	-1.96106	3.167321	-6.8108	0.386766	2.585543
3		1.732051	5.196152	8.017063	-1.96106	2.820911	-7.15721	0.464484	2.152927

Alpha Beta Shift k	3							Average run length ARL (1/P)	Alpha Beta Shift k	10							Average run length ARL (1/P)
	Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	Probabil. of detection P			Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	Probabil. of detection P	
0	8.660254	0	40.17331	-9.9389	40.17331	-9.9389	0.013387	74.69934	0	17.32051	0	80.08158	-19.8368	80.08158	-19.8368	0.013667	73.16895
0.2	8.660254	1.732051	40.17331	-9.9389	38.44126	-11.671	0.017522	57.07111	0.2	17.32051	3.464102	80.08158	-19.8368	76.61748	-23.3009	0.017884	55.9159
0.4	8.660254	3.464102	40.17331	-9.9389	36.70921	-13.403	0.022865	43.73497	0.4	17.32051	6.928203	80.08158	-19.8368	73.15338	-26.765	0.023332	42.85959
0.6	8.660254	5.196152	40.17331	-9.9389	34.97716	-15.1351	0.029738	33.62701	0.6	17.32051	10.3923	80.08158	-19.8368	69.68928	-30.2291	0.030338	32.96196
0.8	8.660254	6.928203	40.17331	-9.9389	33.24511	-16.8671	0.03854	25.94707	0.8	17.32051	13.85641	80.08158	-19.8368	66.22517	-33.6932	0.039306	25.44141
1	8.660254	8.660254	40.17331	-9.9389	31.51306	-18.5992	0.049751	20.1001	1	17.32051	17.32051	80.08158	-19.8368	62.76107	-37.1573	0.050724	19.71453
1.2	8.660254	10.3923	40.17331	-9.9389	29.78101	-20.3312	0.063952	15.63673	1.2	17.32051	20.78461	80.08158	-19.8368	59.29697	-40.6214	0.06518	15.34213
1.4	8.660254	12.12436	40.17331	-9.9389	28.04895	-22.0633	0.081822	12.22165	1.4	17.32051	24.24871	80.08158	-19.8368	55.83287	-44.0855	0.083363	11.99573
1.6	8.660254	13.85641	40.17331	-9.9389	26.3169	-23.7953	0.104151	9.601444	1.6	17.32051	27.71281	80.08158	-19.8368	52.36877	-47.5496	0.106067	9.428003
1.8	8.660254	15.58846	40.17331	-9.9389	24.58485	-25.5274	0.131822	7.585987	1.8	17.32051	31.17691	80.08158	-19.8368	48.90467	-51.0137	0.134186	7.452342
2	8.660254	17.32051	40.17331	-9.9389	22.8528	-27.2594	0.165796	6.031509	2	17.32051	34.64102	80.08158	-19.8368	45.44056	-54.4778	0.168683	5.92828
2.2	8.660254	19.05256	40.17331	-9.9389	21.12075	-28.9915	0.207064	4.829425	2.2	17.32051	38.10512	80.08158	-19.8368	41.97646	-57.9419	0.210549	4.749488
2.4	8.660254	20.78461	40.17331	-9.9389	19.3887	-30.7235	0.25657	3.897572	2.4	17.32051	41.56922	80.08158	-19.8368	38.51236	-61.406	0.260721	3.835518
2.6	8.660254	22.51666	40.17331	-9.9389	17.65665	-32.4556	0.315091	3.173686	2.6	17.32051	45.03332	80.08158	-19.8368	35.04826	-64.8701	0.319955	3.12544
2.8	8.660254	24.24871	40.17331	-9.9389	15.9246	-34.1876	0.38306	2.610557	2.8	17.32051	48.49742	80.08158	-19.8368	31.58416	-68.3342	0.38865	2.573009
3	8.660254	25.98076	40.17331	-9.9389	14.19255	-35.9197	0.460325	2.172378	3	17.32051	51.96152	80.08158	-19.8368	28.12006	-71.7983	0.466597	2.143177

Alpha Beta Shift k	4 0.5				Altered UCLx	Altered LCLx	Probabil. of detection P	Average run length ARL (1/P)
0	1	0	4.876142	-0.88234	4.876142	-0.88234	0.012382	80.7624
0.2	1	0.2	4.876142	-0.88234	4.676142	-1.08234	0.016522	60.52536
0.4	1	0.4	4.876142	-0.88234	4.476142	-1.28234	0.021954	45.54979
0.6	1	0.6	4.876142	-0.88234	4.276142	-1.48234	0.029039	34.43645
0.8	1	0.8	4.876142	-0.88234	4.076142	-1.68234	0.038223	26.16226
1	1	1	4.876142	-0.88234	3.876142	-1.88234	0.050046	19.98162
1.2	1	1.2	4.876142	-0.88234	3.676142	-2.08234	0.065151	15.34896
1.4	1	1.4	4.876142	-0.88234	3.476142	-2.28234	0.084287	11.86423
1.6	1	1.6	4.876142	-0.88234	3.276142	-2.48234	0.108302	9.23344
1.8	1	1.8	4.876142	-0.88234	3.076142	-2.68234	0.138123	7.239924
2	1	2	4.876142	-0.88234	2.876142	-2.88234	0.174715	5.723607
2.2	1	2.2	4.876142	-0.88234	2.676142	-3.08234	0.219007	4.566064
2.4	1	2.4	4.876142	-0.88234	2.476142	-3.28234	0.271788	3.679338
2.6	1	2.6	4.876142	-0.88234	2.276142	-3.48234	0.333552	2.998033
2.8	1	2.8	4.876142	-0.88234	2.076142	-3.68234	0.404299	2.473417
3	1	3	4.876142	-0.88234	1.876142	-3.88234	0.483295	2.06913

Alpha Beta Shift k	4 1				Altered UCLx	Altered LCLx	Probabil. of detection P	Average run length ARL (1/P)
0	2	0	9.844274	-1.83792	9.844274	-1.83792	0.011581	86.34833
0.2	2	0.4	9.844274	-1.83792	9.444274	-2.23792	0.015467	64.65378
0.4	2	0.8	9.844274	-1.83792	9.044274	-2.63792	0.020573	48.6074
0.6	2	1.2	9.844274	-1.83792	8.644274	-3.03792	0.027241	36.70937
0.8	2	1.6	9.844274	-1.83792	8.244274	-3.43792	0.035898	27.85671
1	2	2	9.844274	-1.83792	7.844274	-3.83792	0.047061	21.24902
1.2	2	2.4	9.844274	-1.83792	7.444274	-4.23792	0.061349	16.30018
1.4	2	2.8	9.844274	-1.83792	7.044274	-4.63792	0.079486	12.58083
1.6	2	3.2	9.844274	-1.83792	6.644274	-5.03792	0.1023	9.775171
1.8	2	3.6	9.844274	-1.83792	6.244274	-5.43792	0.130701	7.651051
2	2	4	9.844274	-1.83792	5.844274	-5.83792	0.16565	6.036825
2.2	2	4.4	9.844274	-1.83792	5.444274	-6.23792	0.208095	4.805497
2.4	2	4.8	9.844274	-1.83792	5.044274	-6.63792	0.258866	3.863002
2.6	2	5.2	9.844274	-1.83792	4.644274	-7.03792	0.318542	3.139303
2.8	2	5.6	9.844274	-1.83792	4.244274	-7.43792	0.387257	2.582264
3	2	6	9.844274	-1.83792	3.844274	-7.83792	0.46447	2.152992

Alpha	4							Probabil.	Average
Beta	5							of	run
Shift			UCLx	LCLx	Altered	Altered	detection	length	
k	Sigma	k*Sigma	UCLx	LCLx	UCLx	LCLx	P	ARL	
								(1/P)	
0	10	0	49.39925	-9.50742	49.39925	-9.50742	0.011285	88.6132	
0.2	10	2	49.39925	-9.50742	47.39925	-11.5074	0.015077	66.32619	
0.4	10	4	49.39925	-9.50742	45.39925	-13.5074	0.020061	49.84796	
0.6	10	6	49.39925	-9.50742	43.39925	-15.5074	0.026574	37.63077	
0.8	10	8	49.39925	-9.50742	41.39925	-17.5074	0.035035	28.54289	
1	10	10	49.39925	-9.50742	39.39925	-19.5074	0.045952	21.76184	
1.2	10	12	49.39925	-9.50742	37.39925	-21.5074	0.059934	16.68502	
1.4	10	14	49.39925	-9.50742	35.39925	-23.5074	0.077697	12.87051	
1.6	10	16	49.39925	-9.50742	33.39925	-25.5074	0.100058	9.994203	
1.8	10	18	49.39925	-9.50742	31.39925	-27.5074	0.127924	7.817141	
2	10	20	49.39925	-9.50742	29.39925	-29.5074	0.162252	6.163252	
2.2	10	22	49.39925	-9.50742	27.39925	-31.5074	0.203993	4.902129	
2.4	10	24	49.39925	-9.50742	25.39925	-33.5074	0.253996	3.93707	
2.6	10	26	49.39925	-9.50742	23.39925	-35.5074	0.312866	3.196257	
2.8	10	28	49.39925	-9.50742	21.39925	-37.5074	0.380788	2.626133	
3	10	30	49.39925	-9.50742	19.39925	-39.5074	0.457289	2.186801	

Alpha Beta Shift k	4 10					Altered UCLx	Altered LCLx	Probabil. of detection P	Average run length ARL (1/P)
		Sigma	k*Sigma	UCLx	LCLx				
0		20	0	98.83547	-18.4051	98.83547	-18.4051	0.011254	88.8573
0.2		20	4	98.83547	-18.4051	94.83547	-22.4051	0.015037	66.50263
0.4		20	8	98.83547	-18.4051	90.83547	-26.4051	0.020008	49.98001
0.6		20	12	98.83547	-18.4051	86.83547	-30.4051	0.026506	37.72731
0.8		20	16	98.83547	-18.4051	82.83547	-34.4051	0.034946	28.61558
1		20	20	98.83547	-18.4051	78.83547	-38.4051	0.045838	21.81596
1.2		20	24	98.83547	-18.4051	74.83547	-42.4051	0.059788	16.72576
1.4		20	28	98.83547	-18.4051	70.83547	-46.4051	0.077513	12.90106
1.6		20	32	98.83547	-18.4051	66.83547	-50.4051	0.099828	10.01723
1.8		20	36	98.83547	-18.4051	62.83547	-54.4051	0.127638	7.834657
2		20	40	98.83547	-18.4051	58.83547	-58.4051	0.161902	6.176576
2.2		20	44	98.83547	-18.4051	54.83547	-62.4051	0.203571	4.912291
2.4		20	48	98.83547	-18.4051	50.83547	-66.4051	0.253494	3.944867
2.6		20	52	98.83547	-18.4051	46.83547	-70.4051	0.312281	3.202244
2.8		20	56	98.83547	-18.4051	42.83547	-74.4051	0.38012	2.630748
3		20	60	98.83547	-18.4051	38.83547	-78.4051	0.456546	2.19036

Alpha Beta Shift k	5 0.5						Probabil. of detection P	Average run length ARL (1/P)	Alpha Beta Shift k	5 1						Probabil. of detection P	Average run length ARL (1/P)	
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx					Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	
0	1.118034	0	5.791557	-0.77556	5.791557	-0.77556	0.010149	98.53188	0	2.236068	0	11.61112	-1.59818	11.61112	-1.59818	0.009956	100.4419	
0.2	1.118034	0.223607	5.791557	-0.77556	5.56795	-0.99917	0.013778	72.57947	0.2	2.236068	0.447214	11.61112	-1.59818	11.16391	-2.04539	0.013519	73.96997	
0.4	1.118034	0.447214	5.791557	-0.77556	5.344343	-1.22277	0.018611	53.73166	0.4	2.236068	0.894427	11.61112	-1.59818	10.71669	-2.49261	0.018268	54.74652	
0.6	1.118034	0.67082	5.791557	-0.77556	5.120737	-1.44638	0.025002	39.9968	0.6	2.236068	1.341641	11.61112	-1.59818	10.26948	-2.93982	0.024548	40.73652	
0.8	1.118034	0.894427	5.791557	-0.77556	4.89713	-1.66999	0.033394	29.9455	0.8	2.236068	1.788854	11.61112	-1.59818	9.822266	-3.38703	0.0328	30.4878	
1	1.118034	1.118034	5.791557	-0.77556	4.673523	-1.89359	0.044325	22.56063	1	2.236068	2.236068	11.61112	-1.59818	9.375052	-3.83425	0.043554	22.96	
1.2	1.118034	1.341641	5.791557	-0.77556	4.449918	-2.1172	0.058439	17.11186	1.2	2.236068	2.683282	11.61112	-1.59818	8.927838	-4.28146	0.057447	17.40735	
1.4	1.118034	1.565248	5.791557	-0.77556	4.226309	-2.34081	0.076487	13.07412	1.4	2.236068	3.130495	11.61112	-1.59818	8.480625	-4.72868	0.075226	13.29328	
1.6	1.118034	1.788854	5.791557	-0.77556	4.002703	-2.56441	0.099323	10.06816	1.6	2.236068	3.577709	11.61112	-1.59818	8.033411	-5.17589	0.097736	10.23164	
1.8	1.118034	2.012461	5.791557	-0.77556	3.779096	-2.78802	0.127876	7.820076	1.8	2.236068	4.024922	11.61112	-1.59818	7.586198	-5.6231	0.125902	7.942686	
2	1.118034	2.236068	5.791557	-0.77556	3.555489	-3.01163	0.163106	6.130982	2	2.236068	4.472136	11.61112	-1.59818	7.138984	-6.07032	0.160686	6.223318	
2.2	1.118034	2.459675	5.791557	-0.77556	3.331882	-3.23523	0.205931	4.855995	2.2	2.236068	4.91935	11.61112	-1.59818	6.69177	-6.51753	0.203011	4.925841	
2.4	1.118034	2.683282	5.791557	-0.77556	3.108275	-3.45884	0.257115	3.88931	2.4	2.236068	5.366563	11.61112	-1.59818	6.244557	-6.96474	0.253654	3.942378	
2.6	1.118034	2.906888	5.791557	-0.77556	2.884669	-3.68245	0.317116	3.15342	2.6	2.236068	5.813777	11.61112	-1.59818	5.797343	-7.41196	0.313098	3.193888	
2.8	1.118034	3.130495	5.791557	-0.77556	2.661062	-3.90606	0.385897	2.591365	2.8	2.236068	6.26099	11.61112	-1.59818	5.35013	-7.85917	0.381342	2.622318	
3	1.118034	3.354102	5.791557	-0.77556	2.437455	-4.12966	0.46271	2.161181	3	2.236068	6.708204	11.61112	-1.59818	4.902916	-8.30638	0.45769	2.184885	

Alpha Beta Shift k	5 5						Probabil. of detection P	Average run length ARL (1/P)
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	
0		11.18034	0	57.83224	-7.84192	57.83224	-7.84192	0.010267
0.2		11.18034	2.236068	57.83224	-7.84192	55.59617	-10.078	0.013935
0.4		11.18034	4.472136	57.83224	-7.84192	53.3601	-12.3141	0.018818
0.6		11.18034	6.708204	57.83224	-7.84192	51.12404	-14.5501	0.025276
0.8		11.18034	8.944272	57.83224	-7.84192	48.88797	-16.7862	0.033752
1		11.18034	11.18034	57.83224	-7.84192	46.6519	-19.0223	0.044789
1.2		11.18034	13.41641	57.83224	-7.84192	44.41583	-21.2583	0.059036
1.4		11.18034	15.65248	57.83224	-7.84192	42.17976	-23.4944	0.077247
1.6		11.18034	17.88854	57.83224	-7.84192	39.9437	-25.7305	0.100279
1.8		11.18034	20.12461	57.83224	-7.84192	37.70763	-27.9685	0.129063
2		11.18034	22.36068	57.83224	-7.84192	35.47156	-30.2028	0.16456
2.2		11.18034	24.59675	57.83224	-7.84192	33.23549	-32.4387	0.207685
2.4		11.18034	26.83282	57.83224	-7.84192	30.99942	-34.6747	0.259192
2.6		11.18034	29.06888	57.83224	-7.84192	28.76336	-36.9108	0.319524
2.8		11.18034	31.30495	57.83224	-7.84192	26.52729	-39.1469	0.388623
3		11.18034	33.54102	57.83224	-7.84192	24.29122	-41.3829	0.465709

Alpha Beta Shift k	5 10						Probabil. of detection P	Average run length ARL (1/P)
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	
0		22.36068	0	114.5095	-14.825	114.5095	-14.825	0.011115
0.2		22.36068	4.472136	114.5095	-14.825	110.0374	-19.2971	0.015067
0.4		22.36068	8.944272	114.5095	-14.825	105.5652	-23.7693	0.020319
0.6		22.36068	13.41641	114.5095	-14.825	101.0931	-28.2414	0.027251
0.8		22.36068	17.88854	114.5095	-14.825	96.62096	-32.7135	0.036333
1		22.36068	22.36068	114.5095	-14.825	92.14882	-37.1857	0.048133
1.2		22.36068	26.83282	114.5095	-14.825	87.67668	-41.6578	0.063326
1.4		22.36068	31.30495	114.5095	-14.825	83.20455	-46.13	0.082697
1.6		22.36068	35.77709	114.5095	-14.825	78.73241	-50.6021	0.107123
1.8		22.36068	40.24922	114.5095	-14.825	74.26028	-55.0742	0.13755
2		22.36068	44.72136	114.5095	-14.825	69.78814	-59.5464	0.174933
2.2		22.36068	49.1935	114.5095	-14.825	65.316	-64.0185	0.22016
2.4		22.36068	53.66563	114.5095	-14.825	60.84387	-68.4906	0.27392
2.6		22.36068	58.13777	114.5095	-14.825	56.37173	-72.9628	0.336547
2.8		22.36068	62.6099	114.5095	-14.825	51.8996	-77.4349	0.407817
3		22.36068	67.08204	114.5095	-14.825	47.42746	-81.907	0.48673

Alpha Beta Shift	10 0.5						UCLx Probabil. of detection	LCLx Probabil. of detection	Average run length ARL	
k		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	P1	P2	(1/(P1+P2))
0		1.581139	0	9.591253	0.379109	9.591253	0.379109	0.007989	*	125.1721
0.2		1.581139	0.316228	9.591253	0.379109	9.275025	0.062881	0.011383	*	87.8503
0.4		1.581139	0.632456	9.591253	0.379109	8.958797	-0.25335	0.016082		62.18132
0.6		1.581139	0.948683	9.591253	0.379109	8.64257	-0.56957	0.022516		44.41286
0.8		1.581139	1.264911	9.591253	0.379109	8.326342	-0.8858	0.031227		32.02357
1		1.581139	1.581139	9.591253	0.379109	8.010114	-1.20203	0.04287		23.32634
1.2		1.581139	1.897367	9.591253	0.379109	7.693886	-1.51826	0.058225		17.17475
1.4		1.581139	2.213594	9.591253	0.379109	7.377659	-1.83449	0.078181		12.79083
1.6		1.581139	2.529822	9.591253	0.379109	7.061431	-2.15071	0.103709		9.642365
1.8		1.581139	2.84605	9.591253	0.379109	6.745203	-2.46694	0.135805		7.363499
2		1.581139	3.162278	9.591253	0.379109	6.428975	-2.78317	0.175402		5.701189
2.2		1.581139	3.478505	9.591253	0.379109	6.112748	-3.0994	0.223247		4.479344
2.4		1.581139	3.794733	9.591253	0.379109	5.79652	-3.41562	0.279751		3.574607
2.6		1.581139	4.110961	9.591253	0.379109	5.480292	-3.73185	0.344804		2.900198
2.8		1.581139	4.427189	9.591253	0.379109	5.164064	-4.04808	0.417608		2.39459
3		1.581139	4.743416	9.591253	0.379109	4.847837	-4.36431	0.496535		2.013957

Alpha Beta Shift	10 1						UCLx Probabil. of detection	LCLx Probabil. of detection	Average run length ARL	
k		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	P1	P2	(1/(P1+P2))
0		3.162278	0	19.27649	0.608102	19.27649	0.608102	0.007574	*	132.0306
0.2		3.162278	0.632456	19.27649	0.608102	18.64403	-0.02435	0.010805		92.54975
0.4		3.162278	1.264911	19.27649	0.608102	18.01158	-0.85681	0.015285		65.42362
0.6		3.162278	1.897367	19.27649	0.608102	17.37912	-1.28926	0.021431		46.66138
0.8		3.162278	2.529822	19.27649	0.608102	16.74667	-1.92172	0.029764		33.59763
1		3.162278	3.162278	19.27649	0.608102	16.11421	-2.55418	0.040925		24.43494
1.2		3.162278	3.794733	19.27649	0.608102	15.48176	-3.18663	0.055674		17.96171
1.4		3.162278	4.427189	19.27649	0.608102	14.8493	-3.81909	0.074887		13.35345
1.6		3.162278	5.059644	19.27649	0.608102	14.21685	-4.45154	0.099524		10.04783
1.8		3.162278	5.6921	19.27649	0.608102	13.58439	-5.084	0.130582		7.658023
2		3.162278	6.324555	19.27649	0.608102	12.95193	-5.71645	0.169011		5.916775
2.2		3.162278	6.957011	19.27649	0.608102	12.31948	-6.34891	0.215595		4.638326
2.4		3.162278	7.589466	19.27649	0.608102	11.68702	-6.98136	0.270804		3.692708
2.6		3.162278	8.221922	19.27649	0.608102	11.05457	-7.61382	0.334618		2.988482
2.8		3.162278	8.854377	19.27649	0.608102	10.42211	-8.24628	0.406352		2.460921
3		3.162278	9.486833	19.27649	0.608102	9.789657	-8.87873	0.484507		2.063954

Alpha Beta Shift	10 5					UCLx Probabil. of detection	LCLx Probabil. of detection	Average run length ARL
k		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	P1 (1/(P1+P2))
0		15.81139	0	96.65738	3.604379	96.65738	3.604379	0.007341 *
0.2		15.81139	3.162278	96.65738	3.604379	93.4951	0.442101	0.01048 *
0.4		15.81139	6.324555	96.65738	3.604379	90.33282	-2.72018	0.014837
0.6		15.81139	9.486833	96.65738	3.604379	87.17055	-5.88245	0.020818
0.8		15.81139	12.64911	96.65738	3.604379	84.00827	-9.04473	0.028937
1		15.81139	15.81139	96.65738	3.604379	80.84599	-12.207	0.039824
1.2		15.81139	18.97367	96.65738	3.604379	77.68371	-15.3693	0.054228
1.4		15.81139	22.13594	96.65738	3.604379	74.52144	-18.5316	0.073016
1.6		15.81139	25.29822	96.65738	3.604379	71.35916	-21.6938	0.097142
1.8		15.81139	28.4605	96.65738	3.604379	68.19688	-24.8561	0.127602
2		15.81139	31.62278	96.65738	3.604379	65.0346	-28.0184	0.165356
2.2		15.81139	34.78505	96.65738	3.604379	61.87233	-31.1807	0.211206
2.4		15.81139	37.94733	96.65738	3.604379	58.71005	-34.343	0.265658
2.6		15.81139	41.10961	96.65738	3.604379	55.54777	-37.5052	0.328741
2.8		15.81139	44.27189	96.65738	3.604379	52.38549	-40.6675	0.398833
3		15.81139	47.43416	96.65738	3.604379	49.22322	-43.8298	0.477511

Alpha Beta Shift	10 10					UCLx Probabil. of detection	LCLx Probabil. of detection	Average run length ARL
k		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	P1 (1/(P1+P2))
0		31.62278	0	193.6788	7.595542	193.6788	7.595542	0.00719 *
0.2		31.62278	6.324555	193.6788	7.595542	187.3542	1.270987	0.01027 *
0.4		31.62278	12.64911	193.6788	7.595542	181.0297	-5.05357	0.014546
0.6		31.62278	18.97367	193.6788	7.595542	174.7051	-11.3781	0.020421
0.8		31.62278	25.29822	193.6788	7.595542	168.3806	-17.7027	0.028401
1		31.62278	31.62278	193.6788	7.595542	162.056	-24.0272	0.03911
1.2		31.62278	37.94733	193.6788	7.595542	155.7315	-30.3518	0.053289
1.4		31.62278	44.27189	193.6788	7.595542	149.4069	-36.6763	0.071799
1.6		31.62278	50.59644	193.6788	7.595542	143.0824	-43.0009	0.095591
1.8		31.62278	56.921	193.6788	7.595542	136.7578	-49.3255	0.12566
2		31.62278	63.24555	193.6788	7.595542	130.4332	-55.65	0.162969
2.2		31.62278	69.57011	193.6788	7.595542	124.1087	-61.9746	0.208337
2.4		31.62278	75.89466	193.6788	7.595542	117.7841	-68.2991	0.262287
2.6		31.62278	82.21922	193.6788	7.595542	111.4596	-74.6237	0.324883
2.8		31.62278	88.54377	193.6788	7.595542	105.135	-80.9482	0.395545
3		31.62278	94.86833	193.6788	7.595542	98.81047	-87.2728	0.472898

Alpha Beta Shift k	50 0.5					UCLx Probabil. of detection	LCLx Probabil. of detection	Average run length ARL (1/(P1+P2))
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	
0		3.535534	0	35.59524	14.34161	35.59524	14.34161	0.003464
0.2		3.535534	0.707107	35.59524	14.34161	34.88813	13.6345	0.00553
0.4		3.535534	1.414214	35.59524	14.34161	34.18103	12.9274	0.008675
0.6		3.535534	2.12132	35.59524	14.34161	33.47392	12.22029	0.013367
0.8		3.535534	2.828427	35.59524	14.34161	32.76681	11.51318	0.020215
1		3.535534	3.535534	35.59524	14.34161	32.05971	10.80608	0.029992
1.2		3.535534	4.242641	35.59524	14.34161	31.3526	10.09897	0.04363
1.4		3.535534	4.949747	35.59524	14.34161	30.64549	9.391863	0.062192
1.6		3.535534	5.656854	35.59524	14.34161	29.93839	8.684756	0.086824
1.8		3.535534	6.363961	35.59524	14.34161	29.23128	7.977649	0.118649
2		3.535534	7.071068	35.59524	14.34161	28.52417	7.270542	0.158634
2.2		3.535534	7.778175	35.59524	14.34161	27.81707	6.563435	0.207417
2.4		3.535534	8.485281	35.59524	14.34161	27.10996	5.856329	0.265128
2.6		3.535534	9.192388	35.59524	14.34161	26.40285	5.149222	0.331221
2.8		3.535534	9.899495	35.59524	14.34161	25.69575	4.442115	0.404373
3		3.535534	10.6066	35.59524	14.34161	24.98864	3.735008	0.482472

Alpha Beta Shift k	50 1					UCLx Probabil. of detection	LCLx Probabil. of detection	Average run length ARL (1/(P1+P2))
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	
0		7.071068	0	71.01036	28.90707	71.01036	28.90707	0.00368
0.2		7.071068	1.414214	71.01036	28.90707	69.59615	27.49286	0.005862
0.4		7.071068	2.828427	71.01036	28.90707	68.18193	26.07864	0.009175
0.6		7.071068	4.242641	71.01036	28.90707	66.76772	24.66443	0.014104
0.8		7.071068	5.656854	71.01036	28.90707	65.35351	23.25022	0.02128
1		7.071068	7.071068	71.01036	28.90707	63.93929	21.836	0.031494
1.2		7.071068	8.485281	71.01036	28.90707	62.52508	20.42179	0.045697
1.4		7.071068	9.899495	71.01036	28.90707	61.11087	19.00758	0.064968
1.6		7.071068	11.31371	71.01036	28.90707	59.69665	17.59336	0.090456
1.8		7.071068	12.72792	71.01036	28.90707	58.28244	16.17915	0.123271
2		7.071068	14.14214	71.01036	28.90707	56.86822	14.76493	0.16435
2.2		7.071068	15.55635	71.01036	28.90707	55.45401	13.35072	0.214275
2.4		7.071068	16.97056	71.01036	28.90707	54.0398	11.93651	0.273099
2.6		7.071068	18.38478	71.01036	28.90707	52.62558	10.52229	0.340183
2.8		7.071068	19.79899	71.01036	28.90707	51.21137	9.10808	0.414098
3		7.071068	21.2132	71.01036	28.90707	49.79716	7.693867	0.492639

Alpha Beta Shift k	50 5					UCLx Probabil. of detection P1	LCLx Probabil. of detection P2	Average run length ARL (1/(P1+P2))
0		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	255.8395
0.2		35.35534	0	355.182	145.3202	355.182	145.3202	0.003648
0.4		35.35534	7.071068	355.182	145.3202	348.1109	138.2491	0.005813
0.6		35.35534	14.14214	355.182	145.3202	341.0399	131.1781	0.009102
0.8		35.35534	21.2132	355.182	145.3202	333.9688	124.107	0.013995
1		35.35534	28.28427	355.182	145.3202	326.8977	117.0359	0.021123 *
1.2		35.35534	35.35534	355.182	145.3202	319.8267	109.9649	0.031273 *
1.4		35.35534	42.42641	355.182	145.3202	312.7556	102.8938	0.045393 *
1.6		35.35534	49.49747	355.182	145.3202	305.6845	95.82273	0.064561 *
1.8		35.35534	56.56854	355.182	145.3202	298.6135	88.75166	0.089923 *
2		35.35534	63.63961	355.182	145.3202	291.5424	81.68059	0.122595 *
2.2		35.35534	70.71068	355.182	145.3202	284.4713	74.60952	0.163514 *
2.4		35.35534	77.78175	355.182	145.3202	277.4003	67.53845	0.213275 *
2.6		35.35534	84.85281	355.182	145.3202	270.3292	60.46739	0.271939 *
2.8		35.35534	91.92388	355.182	145.3202	263.2581	53.39632	0.33888 *
3		35.35534	98.99495	355.182	145.3202	256.1871	46.32525	0.412687 *
		35.35534	106.066	355.182	145.3202	249.116	39.25418	0.491167 *

Alpha Beta Shift k	50 10					UCLx Probabil. of detection P1	LCLx Probabil. of detection P2	Average run length ARL (1/(P1+P2))
0		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	287.3855
0.2		70.71068	0	714.2551	287.3855	714.2551	287.3855	0.0032
0.4		70.71068	14.14214	714.2551	287.3855	700.113	273.2434	0.005123
0.6		70.71068	28.28427	714.2551	287.3855	685.9708	259.1012	0.00806
0.8		70.71068	42.42641	714.2551	287.3855	671.8287	244.9591	0.012456
1		70.71068	56.56854	714.2551	287.3855	657.6866	230.817	0.018897 *
1.2		70.71068	70.71068	714.2551	287.3855	643.5444	216.6748	0.028127 *
1.4		70.71068	84.85281	714.2551	287.3855	629.4023	202.5327	0.041052 *
1.6		70.71068	98.99495	714.2551	287.3855	615.2602	188.3906	0.058718 *
1.8		70.71068	113.1371	714.2551	287.3855	601.118	174.2484	0.082261 *
2		70.71068	127.2792	714.2551	287.3855	586.9759	160.1063	0.112818 *
2.2		70.71068	141.4214	714.2551	287.3855	572.8337	145.9641	0.151391 *
2.4		70.71068	155.5635	714.2551	287.3855	558.6916	131.822	0.198687 *
2.6		70.71068	169.7056	714.2551	287.3855	544.5495	117.6799	0.254932 *
2.8		70.71068	183.8478	714.2551	287.3855	530.4073	103.5377	0.319702 *
3		70.71068	197.9899	714.2551	287.3855	516.2652	89.3956	0.391807 *
		70.71068	212.132	714.2551	287.3855	502.1231	75.25347	0.469261 *

Alpha Beta Shift k	100 0.5					UCLx Probabil. of detection P1	LCLx Probabil. of detection P2	Average run length ARL (1/(P1+P2))	Alpha Beta Shift k	100 1						UCLx Probabil. of detection P1	LCLx Probabil. of detection P2	Average run length ARL (1/(P1+P2))		
		Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx				Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx				
0		5	0	65.00006	34.96861	65.00006	34.96861	0.002755	0.000418	315.1393	0	10	0	130.014	70.23251	130.014	70.23251	0.002745	0.000478	310.2314
0.2		5	1	65.00006	34.96861	64.00006	33.96861	0.00457	0.000161	211.3941	0.2	10	2	130.014	70.23251	128.014	68.23251	0.004554	0.000186	210.9749
0.4		5	2	65.00006	34.96861	63.00006	32.96861	0.007422	5.66E-05	133.7149	0.4	10	4	130.014	70.23251	126.014	66.23251	0.007398	6.64E-05	133.9692
0.6		5	3	65.00006	34.96861	62.00006	31.96861	0.011797	1.83E-05	84.63602	0.6	10	6	130.014	70.23251	124.014	64.23251	0.011776	2.17E-05	84.87739
0.8		5	4	65.00006	34.96861	61.00006	30.96861	0.018339	5.4E-06	54.51255	0.8	10	8	130.014	70.23251	122.014	62.23251	0.018284	6.5E-06	54.67319
1		5	5	65.00006	34.96861	60.00006	29.96861	0.027867	*	35.88474	1	10	10	130.014	70.23251	120.014	60.23251	0.027788	*	35.96676
1.2		5	6	65.00006	34.96861	59.00006	28.96861	0.041377	*	24.16802	1.2	10	12	130.014	70.23251	118.014	58.23251	0.041267	*	24.23244
1.4		5	7	65.00006	34.96861	58.00006	27.96861	0.060005	*	16.66528	1.4	10	14	130.014	70.23251	116.014	56.23251	0.059855	*	16.70704
1.6		5	8	65.00006	34.96861	57.00006	26.96861	0.084956	*	11.7708	1.6	10	16	130.014	70.23251	114.014	54.23251	0.084758	*	11.7983
1.8		5	9	65.00006	34.96861	56.00006	25.96861	0.117394	*	8.518323	1.8	10	18	130.014	70.23251	112.014	52.23251	0.117141	*	8.536721
2		5	10	65.00006	34.96861	55.00006	24.96861	0.15828	*	6.317918	2	10	20	130.014	70.23251	110.014	50.23251	0.157965	*	6.330516
2.2		5	11	65.00006	34.96861	54.00006	23.96861	0.208185	*	4.80342	2.2	10	22	130.014	70.23251	108.014	48.23251	0.207807	*	4.812157
2.4		5	12	65.00006	34.96861	53.00006	22.96861	0.267108	*	3.743804	2.4	10	24	130.014	70.23251	106.014	46.23251	0.266669	*	3.749967
2.6		5	13	65.00006	34.96861	52.00006	21.96861	0.334317	*	2.991173	2.6	10	26	130.014	70.23251	104.014	44.23251	0.333825	*	2.995582
2.8		5	14	65.00006	34.96861	51.00006	20.96861	0.408279	*	2.449305	2.8	10	28	130.014	70.23251	102.014	42.23251	0.407748	*	2.452495
3		5	15	65.00006	34.96861	50.00006	19.96861	0.486699	*	2.054658	3	10	30	130.014	70.23251	100.014	40.23251	0.486145	*	2.056999

Alpha Beta Shift	100 5							UCLx Probabil. of detection	LCLx Probabil. of detection	Average run length ARL	Alpha Beta Shift	100 10						UCLx Probabil. of detection	LCLx Probabil. of detection	Average run length ARL
k	Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	P1	P2	(1/(P1+P2))	k	Sigma	k*Sigma	UCLx	LCLx	Altered UCLx	Altered LCLx	P1	P2	(1/(P1+P2))	
0	50	0	647.576	352.267	647.576	352.267	0.00312	0.000529	274.0852	0	100	0	1301.758	700.8875	1301.758	700.8875	0.002632	0.000448	324.6542	
0.2	50	10	647.576	352.267	637.576	342.267	0.00515	0.000207	186.6682	0.2	100	20	1301.758	700.8875	1281.758	680.8875	0.004374	0.000173	219.9204	
0.4	50	20	647.576	352.267	627.576	332.267	0.008322	7.47E-05	119.0944	0.4	100	40	1301.758	700.8875	1261.758	660.8875	0.007119	6.15E-05	139.2661	
0.6	50	30	647.576	352.267	617.576	322.267	0.013156	2.47E-05	75.8685	0.6	100	60	1301.758	700.8875	1241.758	640.8875	0.011337	2E-05	88.05142	
0.8	50	40	647.576	352.267	607.576	312.267	0.02034	7.4E-06	49.14633	0.8	100	80	1301.758	700.8875	1221.758	620.8875	0.017658	5.9E-06	56.61264	
1	50	50	647.576	352.267	597.576	302.267	0.030736 *		32.53514	1	100	100	1301.758	700.8875	1201.758	600.8875	0.026887 *		37.1927	
1.2	50	60	647.576	352.267	587.576	292.267	0.045378 *		22.03711	1.2	100	120	1301.758	700.8875	1181.758	580.8875	0.040003 *		24.99813	
1.4	50	70	647.576	352.267	577.576	282.267	0.065428 *		15.28398	1.4	100	140	1301.758	700.8875	1161.758	560.8875	0.058132 *		17.20223	
1.6	50	80	647.576	352.267	567.576	272.267	0.092092 *		10.85871	1.6	100	160	1301.758	700.8875	1141.758	540.8875	0.08248 *		12.12415	
1.8	50	90	647.576	352.267	557.576	262.267	0.126502 *		7.905013	1.8	100	180	1301.758	700.8875	1121.758	520.8875	0.114218 *		8.755187	
2	50	100	647.576	352.267	547.576	252.267	0.169541 *		5.898278	2	100	200	1301.758	700.8875	1101.758	500.8875	0.154332 *		6.479538	
2.2	50	110	647.576	352.267	537.576	242.267	0.221658 *		4.511455	2.2	100	220	1301.758	700.8875	1081.758	480.8875	0.203435 *		4.915575	
2.4	50	120	647.576	352.267	527.576	232.267	0.282687 *		3.537481	2.4	100	240	1301.758	700.8875	1061.758	460.8875	0.261584 *		3.822864	
2.6	50	130	647.576	352.267	517.576	222.267	0.351705 *		2.843292	2.6	100	260	1301.758	700.8875	1041.758	440.8875	0.328115 *		3.047712	
2.8	50	140	647.576	352.267	507.576	212.267	0.426985 *		2.342003	2.8	100	280	1301.758	700.8875	1021.758	420.8875	0.401566 *		2.490251	
3	50	150	647.576	352.267	497.576	202.267	0.506067 *		1.976023	3	100	300	1301.758	700.8875	1001.758	400.8875	0.479703 *		2.084623	

APPENDIX F

MINITAB OUTPUTS - MULTIPLE REGRESSION MODELS FOR

d_2 , d_3 , AND D_4

1. Bias correction factor d_2

1.1 Regression Analysis

The regression equation is

$$d_2 = 1.03 + 0.00433 \text{ Alpha} - 0.0021 \text{ Beta} - 0.000035 \text{ Alpha}^2 + 0.00025 \text{ Beta}^2 - 0.000003 \text{ Alph*Bet}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	1.02930	0.02107	48.86	0.000
Alpha	0.004333	0.001404	3.09	0.004
Beta	-0.00211	0.01125	-0.19	0.852
Alpha ²	-0.00003451	0.00001400	-2.47	0.019
Beta ²	0.000254	0.001044	0.24	0.810
Alph*Bet	-0.00000306	0.00008224	-0.04	0.971

$$s = 0.06122 \quad R\text{-sq} = 33.9\% \quad R\text{-sq(adj)} = 24.2\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	5	0.065297	0.013059	3.48	0.012
Error	34	0.127431	0.003748		
Total	39	0.192729			

1.2 Regression Analysis

- * NOTE * Alpha is highly correlated with other predictor variables
- * NOTE * Alpha² is highly correlated with other predictor variables
- * NOTE * Alpha³ is highly correlated with other predictor variables

The regression equation is

$$d_2 = 0.979 + 0.0234 \text{ Alpha} - 0.00211 \text{ Beta} - 0.000600 \text{ Alpha}^2 + 0.000254 \text{ Beta}^2 - 0.000003 \text{ Alph*Bet} + 0.000004 \text{ Alpha}^3$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.97908	0.01905	51.39	0.000
Alpha	0.023366	0.003973	5.88	0.000
Beta	-0.002112	0.008631	-0.24	0.808
Alpha^2	-0.0005997	0.0001141	-5.26	0.000
Beta^2	0.0002535	0.0008011	0.32	0.754
Alph*Bet	-0.00000306	0.00006309	-0.05	0.962
Alpha^3	0.00000381	0.00000076	4.98	0.000

s = 0.04697 R-sq = 62.2% R-sq(adj) = 55.4%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	6	0.119937	0.019989	9.06	0.000
Error	33	0.072792	0.002206		
Total	39	0.192729			

1.3 Regression Analysis

The regression equation is

$$d2 = 1.03 + 0.00432 \text{ Alpha} - 0.000035 \text{ Alpha}^2$$

Predictor	Coef	Stdev	t-ratio	p
Constant	1.02859	0.01205	85.36	0.000
Alpha	0.004321	0.001308	3.30	0.002
Alpha^2	-0.00003451	0.00001344	-2.57	0.014

s = 0.05877 R-sq = 33.7% R-sq(adj) = 30.1%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	0.064942	0.032471	9.40	0.000
Error	37	0.127787	0.003454		
Total	39	0.192729			

1.4 Regression Analysis

- * NOTE * Alpha is highly correlated with other predictor variables
- * NOTE * Alpha^2 is highly correlated with other predictor variables
- * NOTE * Alpha^3 is highly correlated with other predictor variables

The regression equation is

$$d2 = 0.978 + 0.0234 \text{ Alpha} - 0.000600 \text{ Alpha}^2 + 0.000004 \text{ Alpha}^3$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.97836	0.01339	73.08	0.000
Alpha	0.023354	0.003805	6.14	0.000
Alpha^2	-0.0005997	0.0001095	-5.48	0.000
Alpha^3	0.00000381	0.00000073	5.19	0.000

s = 0.04508 R-sq = 62.0% R-sq(adj) = 58.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	0.119582	0.039861	19.62	0.000
Error	36	0.073147	0.002032		
Total	39	0.192729			

1.5 Regression Analysis

The regression equation is

$$d2 = 0.760 + 0.351 1-e^{-x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.759508	0.009801	77.49	0.000
1-e^-x	0.35134	0.01113	31.58	0.000

s = 0.01365 R-sq = 96.3% R-sq(adj) = 96.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.18565	0.18565	996.99	0.000
Error	38	0.00708	0.00019		

Total 39 0.19273

1.6 Regression Analysis

The regression equation is

$$d2 = 0.878 + 0.250 1-e^{-.5x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.87794	0.01042	84.22	0.000
1-e ^{-.5x}	0.25036	0.01339	18.70	0.000

s = 0.02230 R-sq = 90.2% R-sq(adj) = 89.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.17384	0.17384	349.67	0.000
Error	38	0.01889	0.00050		
Total	39	0.19273			

1.7 Regression Analysis

The regression equation is

$$d2 = 0.505 + 0.591 1-e^{-2x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.50466	0.03128	16.13	0.000
1-e ^{-2x}	0.59064	0.03296	17.92	0.000

s = 0.02316 R-sq = 89.4% R-sq(adj) = 89.1%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.17234	0.17234	321.18	0.000
Error	38	0.02039	0.00054		
Total	39	0.19273			

1.8 Regression Analysis

The regression equation is

$$d2 = 1.00 + 0.149 1-e^{-.1x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	1.00080	0.01234	81.13	0.000
1-e ^{-0.1x}	0.14879	0.02339	6.36	0.000

$$s = 0.04957 \quad R\text{-sq} = 51.6\% \quad R\text{-sq(adj)} = 50.3\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.099374	0.099374	40.45	0.000
Error	38	0.093355	0.002457		
Total	39	0.192729			

1.9 Regression Analysis

The regression equation is

$$d2 = 0.772 + 0.308 1-e^{-x} + 0.0331 1-e^{-.5x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.77222	0.01452	53.17	0.000
1-e ^{-x}	0.30830	0.03809	8.09	0.000
1-e ^{-0.5x}	0.03312	0.02805	1.18	0.245

$$s = 0.01358 \quad R\text{-sq} = 96.5\% \quad R\text{-sq(adj)} = 96.3\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	0.185909	0.092955	504.36	0.000
Error	37	0.006819	0.000184		
Total	39	0.192729			

1.10 Regression Analysis

* NOTE * $1-e^{-x}$ is highly correlated with other predictor variables

The regression equation is

$$d2 = 0.624 - 0.150 1-e^{-x} + 0.197 1-e^{-.5x} + 0.448 1-e^{-2x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.62359	0.03547	17.58	0.000
$1-e^{-x}$	-0.1499	0.1077	-1.39	0.172
$1-e^{-.5x}$	0.19707	0.04339	4.54	0.000
$1-e^{-2x}$	0.4479	0.1008	4.44	0.000

$$s = 0.01106 \quad R\text{-sq} = 97.7\% \quad R\text{-sq(adj)} = 97.5\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	0.188325	0.062775	513.25	0.000
Error	36	0.004403	0.000122		
Total	39	0.192729			

2. Bias correction factor d_3

2.1 Regression Analysis

The regression equation is

$$d3 = 0.959 - 0.00457 \text{ Alpha} - 0.0016 \text{ Beta} + 0.000036 \text{ Alpha}^2 + 0.000192 \text{ Beta}^2 - 0.000002 \text{ Alph*Bet}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.95909	0.01966	48.78	0.000
Alpha	-0.004572	0.001311	-3.49	0.001
Beta	-0.00164	0.01050	-0.16	0.877
Alpha^2	0.00003595	0.00001306	2.75	0.009
Beta^2	0.0001919	0.0009747	0.20	0.845
Alph*Bet	-0.00000222	0.00007676	-0.03	0.977

$$s = 0.05714 \quad R\text{-sq} = 41.1\% \quad R\text{-sq(adj)} = 32.5\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	5	0.077564	0.015513	4.75	0.002
Error	34	0.111016	0.003265		
Total	39	0.188580			

2.2 Regression Analysis

- * NOTE * Alpha is highly correlated with other predictor variables
- * NOTE * Alpha² is highly correlated with other predictor variables
- * NOTE * Alpha³ is highly correlated with other predictor variables

The regression equation is

$$d3 = 1.01 - 0.0240 \text{ Alpha} - 0.00164 \text{ Beta} + 0.000614 \text{ Alpha}^2 + 0.000192 \text{ Beta}^2 - 0.000002 \text{ Alph*Bet} - 0.000004 \text{ Alpha}^3$$

Predictor	Coef	Stdev	t-ratio	p
Constant	1.01046	0.01639	61.66	0.000
Alpha	-0.024039	0.003417	-7.03	0.000
Beta	-0.001636	0.007424	-0.22	0.827
Alpha ²	0.00061404	0.00009811	6.26	0.000
Beta ²	0.0001919	0.0006890	0.28	0.782
Alph*Bet	-0.00000222	0.00005426	-0.04	0.968
Alpha ³	-0.00000389	0.00000066	-5.92	0.000

$$s = 0.04040 \quad R\text{-sq} = 71.4\% \quad R\text{-sq(adj)} = 66.3\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	6	0.134729	0.022455	13.76	0.000
Error	33	0.053851	0.001632		
Total	39	0.188580			

2.3 Regression Analysis

The regression equation is

$$d3 = 0.958 - 0.00458 \text{ Alpha} + 0.000036 \text{ Alpha}^2$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.95840	0.01124	85.25	0.000
Alpha	-0.004581	0.001220	-3.75	0.001
Alpha^2	0.00003595	0.00001253	2.87	0.007

s = 0.05482 R-sq = 41.0% R-sq(adj) = 37.8%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	0.077375	0.038687	12.87	0.000
Error	37	0.111206	0.003006		
Total	39	0.188580			

2.4 Regression Analysis

* NOTE * Alpha is highly correlated with other predictor variables

* NOTE * Alpha^2 is highly correlated with other predictor variables

* NOTE * Alpha^3 is highly correlated with other predictor variables

The regression equation is

$$d3 = 1.01 - 0.0240 \text{ Alpha} + 0.000614 \text{ Alpha}^2 - 0.000004 \text{ Alpha}^3$$

Predictor	Coef	Stdev	t-ratio	p
Constant	1.00977	0.01151	87.75	0.000
Alpha	-0.024048	0.003270	-7.35	0.000
Alpha^2	0.00061404	0.00009410	6.53	0.000
Alpha^3	-0.00000389	0.00000063	-6.17	0.000

s = 0.03874 R-sq = 71.3% R-sq(adj) = 69.0%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	0.134540	0.044847	29.88	0.000
Error	36	0.054040	0.001501		
Total	39	0.188580			

2.5 Regression Analysis

The regression equation is

$$d3 = 0.875 + 0.342 e^{-x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.874665	0.003557	245.91	0.000
e^{-x}	0.34207	0.01484	23.05	0.000

$$s = 0.01820 \quad R\text{-sq} = 93.3\% \quad R\text{-sq(adj)} = 93.1\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.17599	0.17599	531.09	0.000
Error	38	0.01259	0.00033		
Total	39	0.18858			

2.6 Regression Analysis

The regression equation is

$$d3 = 0.856 + 0.251 e^{-0.5x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.855758	0.004323	197.95	0.000
$e^{-0.5x}$	0.25090	0.01152	21.78	0.000

$$s = 0.01919 \quad R\text{-sq} = 92.6\% \quad R\text{-sq(adj)} = 92.4\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.17459	0.17459	474.17	0.000
Error	38	0.01399	0.00037		
Total	39	0.18858			

2.7 Regression Analysis

The regression equation is

$$d3 = 0.891 + 0.561 e^{-2x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.890590	0.005238	170.04	0.000
e^{-2x}	0.56135	0.04187	13.41	0.000

$$s = 0.02943 \quad R\text{-sq} = 82.5\% \quad R\text{-sq(adj)} = 82.1\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.15567	0.15567	179.74	0.000
Error	38	0.03291	0.00087		
Total	39	0.18858			

2.8 Regression Analysis

The regression equation is

$$d3 = 0.828 + 0.160 e^{-0.1x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.82809	0.01418	58.41	0.000
$e^{-0.1x}$	0.15981	0.02082	7.68	0.000

$$s = 0.04411 \quad R\text{-sq} = 60.8\% \quad R\text{-sq(adj)} = 59.8\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.11464	0.11464	58.92	0.000
Error	38	0.07394	0.00195		
Total	39	0.18858			

2.9 Regression Analysis

The regression equation is

$$d3 = 0.865 + 0.190 e^{-x} + 0.117 e^{-0.5x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.864807	0.004173	207.26	0.000
e^{-x}	0.19025	0.04471	4.26	0.000
$e^{-0.5x}$	0.11685	0.03293	3.55	0.001

$s = 0.01593$ $R\text{-sq} = 95.0\%$ $R\text{-sq}(\text{adj}) = 94.7\%$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	0.179186	0.089593	352.86	0.000
Error	37	0.009394	0.000254		
Total	39	0.188580			

2.10 Regression Analysis

The regression equation is

$$d3 = 0.857 + 0.255 e^{-0.5x} - 0.0044 e^{-0.1x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	0.857133	0.006655	128.80	0.000
$e^{-0.5x}$	0.25545	0.02026	12.61	0.000
$e^{-0.1x}$	-0.00437	0.01593	-0.27	0.785

$s = 0.01943$ $R\text{-sq} = 92.6\%$ $R\text{-sq}(\text{adj}) = 92.2\%$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	0.174617	0.087308	231.35	0.000
Error	37	0.013963	0.000377		
Total	39	0.188580			

3. Control chart constant D_4

3.1 Regression Analysis

The regression equation is

$$D4 = 3.82 - 0.0246 \text{ Alpha} + 0.0021 \text{ Beta} + 0.000196 \text{ Alpha}^2 - 0.00022 \text{ Beta}^2 + 0.000002 \text{ Alph*Bet}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	3.8201	0.1213	31.50	0.000
Alpha	-0.024634	0.008083	-3.05	0.004
Beta	0.00211	0.06477	0.03	0.974
Alpha^2	0.00019614	0.00008059	2.43	0.020
Beta^2	-0.000223	0.006012	-0.04	0.971
Alph*Bet	0.0000016	0.0004734	0.00	0.997

s = 0.3525 R-sq = 33.5% R-sq(adj) = 23.7%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	5	2.1300	0.4260	3.43	0.013
Error	34	4.2236	0.1242		
Total	39	6.3536			

3.2 Regression Analysis

- * NOTE * Alpha is highly correlated with other predictor variables
- * NOTE * Alpha^2 is highly correlated with other predictor variables
- * NOTE * Alpha^3 is highly correlated with other predictor variables

The regression equation is

$$D4 = 4.11 - 0.136 \text{ Alpha} + 0.0021 \text{ Beta} + 0.00351 \text{ Alpha}^2 - 0.00022 \text{ Beta}^2 + 0.000002 \text{ Alph*Bet} - 0.000022 \text{ Alpha}^3$$

Predictor	Coef	Stdev	t-ratio	p
Constant	4.1143	0.1082	38.02	0.000
Alpha	-0.13613	0.02257	-6.03	0.000
Beta	0.00211	0.04902	0.04	0.966
Alpha^2	0.0035071	0.0006479	5.41	0.000
Beta^2	-0.000223	0.004550	-0.05	0.961
Alph*Bet	0.0000016	0.0003583	0.00	0.996
Alpha^3	-0.00002230	0.00000434	-5.13	0.000

s = 0.2668 R-sq = 63.0% R-sq(adj) = 56.3%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	6	4.00517	0.66753	9.38	0.000
Error	33	2.34844	0.07116		
Total	39	6.35361			

3.3 Regression Analysis

The regression equation is

$$D4 = 3.82 - 0.0246 \text{ Alpha} + 0.000196 \text{ Alpha}^2$$

Predictor	Coef	Stdev	t-ratio	p
Constant	3.82174	0.06928	55.16	0.000
Alpha	-0.024627	0.007519	-3.28	0.002
Alpha^2	0.00019614	0.00007725	2.54	0.015

$$s = 0.3379 \quad R\text{-sq} = 33.5\% \quad R\text{-sq(adj)} = 29.9\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	2.1298	1.0649	9.33	0.001
Error	37	4.2238	0.1142		
Total	39	6.3536			

3.4 Regression Analysis

- * NOTE * Alpha is highly correlated with other predictor variables
- * NOTE * Alpha^2 is highly correlated with other predictor variables
- * NOTE * Alpha^3 is highly correlated with other predictor variables

The regression equation is

$$D4 = 4.12 - 0.136 \text{ Alpha} + 0.00351 \text{ Alpha}^2 - 0.000022 \text{ Alpha}^3$$

Predictor	Coef	Stdev	t-ratio	p
Constant	4.11598	0.07586	54.26	0.000
Alpha	-0.13613	0.02156	-6.31	0.000
Alpha^2	0.0035071	0.0006203	5.65	0.000

Alpha^3 -0.00002230 0.00000416 -5.36 0.000

s = 0.2554 R-sq = 63.0% R-sq(adj) = 60.0%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	4.0050	1.3350	20.46	0.000
Error	36	2.3486	0.0652		
Total	39	6.3536			

3.5 Regression Analysis

The regression equation is

$$D4 = 3.35 + 2.03 e^{-x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	3.34869	0.01312	255.19	0.000
e ^{-x}	2.02745	0.05476	37.02	0.000

s = 0.06716 R-sq = 97.3% R-sq(adj) = 97.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	6.1822	6.1822	1370.67	0.000
Error	38	0.1714	0.0045		
Total	39	6.3536			

3.6 Regression Analysis

The regression equation is

$$D4 = 3.25 + 1.44 e^{-0.5x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	3.24989	0.02885	112.64	0.000
e ^{-0.5x}	1.43744	0.07690	18.69	0.000

s = 0.1281 R-sq = 90.2% R-sq(adj) = 89.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	5.7304	5.7304	349.41	0.000
Error	38	0.6232	0.0164		
Total	39	6.3536			

3.7 Regression Analysis

The regression equation is

$$D4 = 3.44 + 3.44 e^{-2x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	3.43660	0.02059	166.93	0.000
e^{-2x}	3.4398	0.1646	20.90	0.000

$$s = 0.1157 \quad R\text{-sq} = 92.0\% \quad R\text{-sq(adj)} = 91.8\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	5.8451	5.8451	436.82	0.000
Error	38	0.5085	0.0134		
Total	39	6.3536			

3.8 Regression Analysis

The regression equation is

$$D4 = 3.13 + 0.856 e^{-0.1x}$$

Predictor	Coef	Stdev	t-ratio	p
Constant	3.12678	0.09128	34.25	0.000
$e^{-0.1x}$	0.8558	0.1341	6.38	0.000

$$s = 0.2840 \quad R\text{-sq} = 51.7\% \quad R\text{-sq(adj)} = 50.5\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	3.2878	3.2878	40.75	0.000
Error	38	3.0658	0.0807		
Total	39	6.3536			

APPENDIX G

**MINITAB OUTPUTS - MULTIPLE REGRESSION MODELS SELECTED FOR
d₂, d₃, AND D₄**

1. Bias correction factor d_2

The regression equation is

$$d_2 = 0.643 + 0.0977 1 - e^{-.5x} + 0.357 1 - e^{-2x} + 0.0248 1 - e^{-.1x}$$

Where x represents the shape parameter α .

Predictor	Coef	Stdev	t-ratio	p
Constant	0.64282	0.02142	30.01	0.000
$1 - e^{-.5x}$	0.09775	0.02083	4.69	0.000
$1 - e^{-2x}$	0.35736	0.03317	10.77	0.000
$1 - e^{-.1x}$	0.02483	0.01035	2.40	0.022

$$s = 0.01054 \quad R\text{-sq} = 97.9\% \quad R\text{-sq(adj)} = 97.8\%$$

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	0.188728	0.062909	566.07	0.000
Error	36	0.004001	0.000111		
Total	39	0.192729			

SOURCE	DF	SEQ SS
$1 - e^{-.5x}$	1	0.173837
$1 - e^{-2x}$	1	0.014251
$1 - e^{-.1x}$	1	0.000639

2. Bias correction factor d_3

The regression equation is

$$d_3 = 0.859 - 0.296 e^{-x} + 0.291 e^{-0.5x} + 0.476 e^{-2x}$$

Where x represents the shape parameter α .

Predictor	Coef	Stdev	t-ratio	p
Constant	0.859457	0.003827	224.56	0.000
e^{-x}	-0.2964	0.1325	-2.24	0.032
$e^{-0.5x}$	0.29099	0.05340	5.45	0.000
e^{-2x}	0.4758	0.1240	3.84	0.000

s = 0.01361 R-sq = 96.5% R-sq(adj) = 96.2%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	0.181911	0.060637	327.34	0.000
Error	36	0.006669	0.000185		
Total	39	0.188580			

SOURCE	DF	SEQ SS
e ^{-x}	1	0.175988
e ^{-0.5x}	1	0.003198
e ^{-2x}	1	0.002726

3. Control chart constant D₄

The regression equation is

$$D_4 = 3.29 + 1.87 e^{-x} + 0.137 e^{-0.1x}$$

Where x represents the shape parameter α .

Predictor	Coef	Stdev	t-ratio	p
Constant	3.28976	0.01945	169.18	0.000
e ^{-x}	1.87067	0.06329	29.56	0.000
e ^{-0.1x}	0.13663	0.03663	3.73	0.001

s = 0.05802 R-sq = 98.0% R-sq(adj) = 97.9%

Analysis of Variance

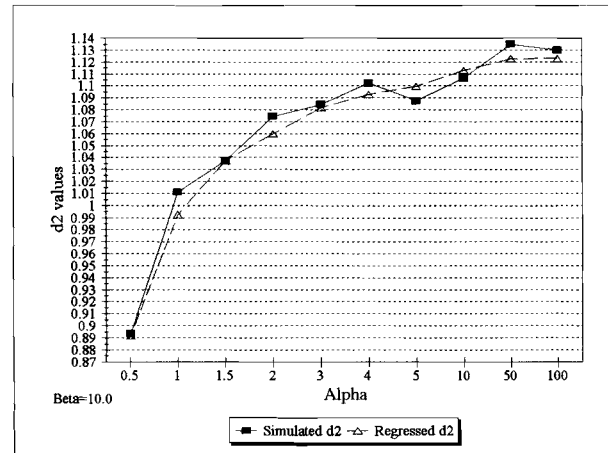
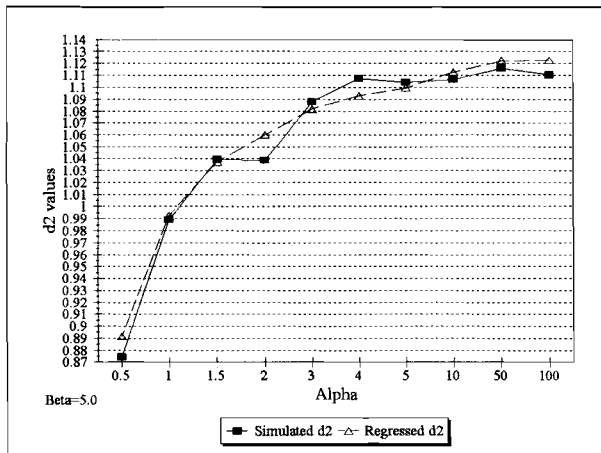
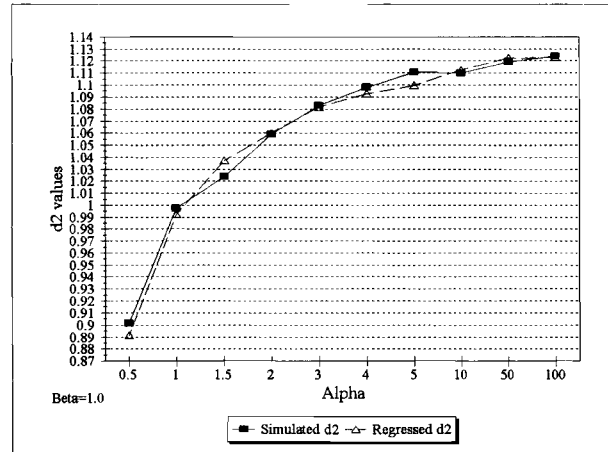
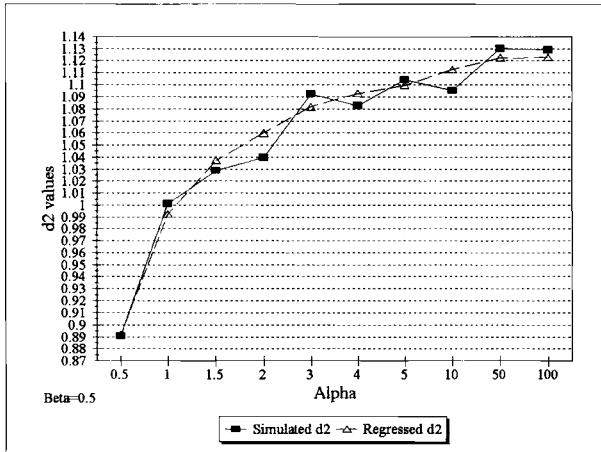
SOURCE	DF	SS	MS	F	p
Regression	2	6.2290	3.1145	925.12	0.000
Error	37	0.1246	0.0034		
Total	39	6.3536			

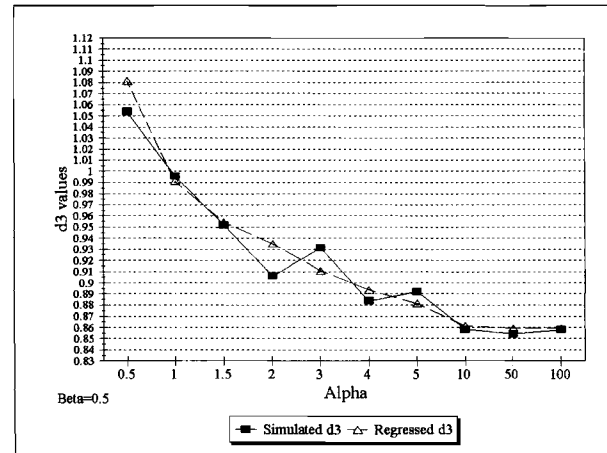
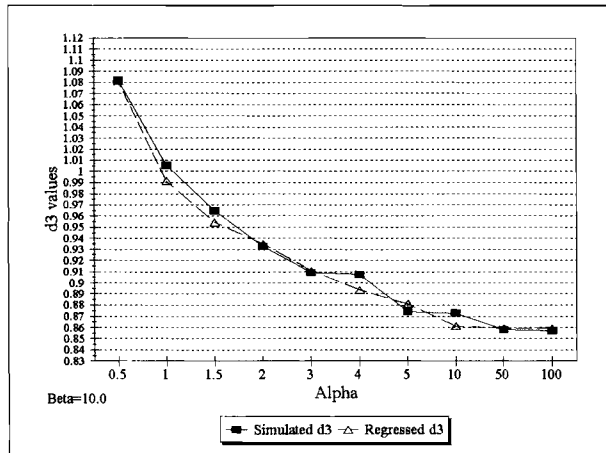
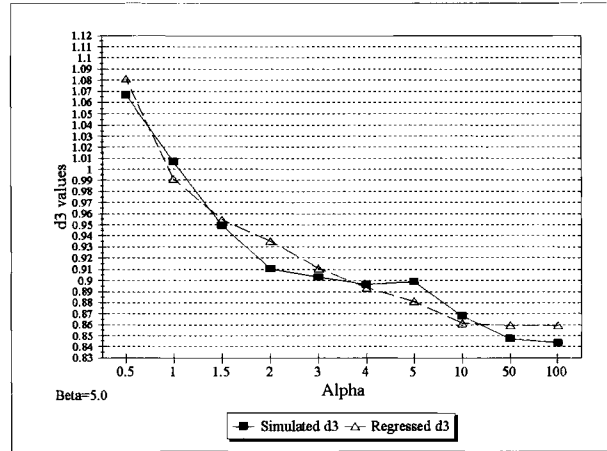
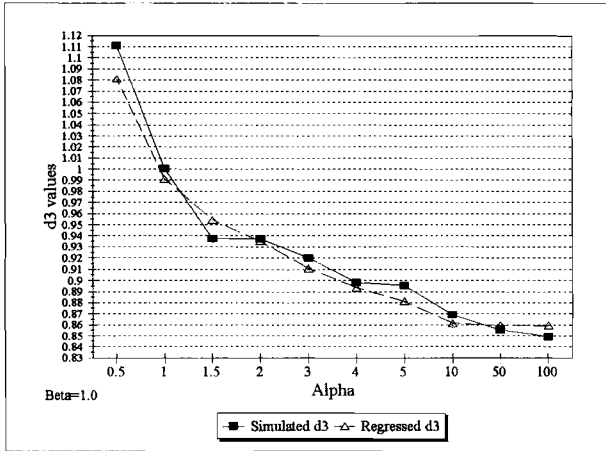
SOURCE	DF	SEQ SS
e ^{-x}	1	6.1822
e ^{-0.1x}	1	0.0468

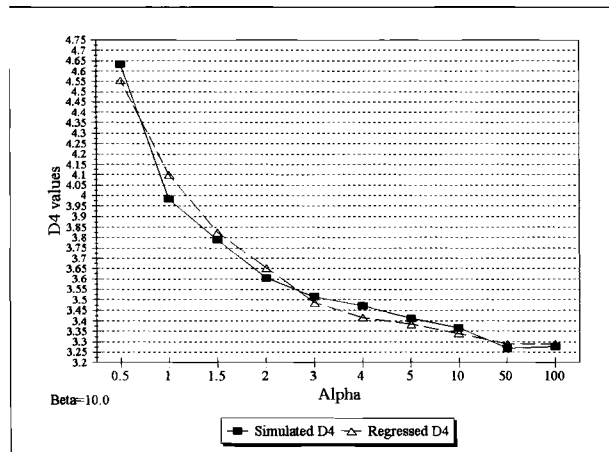
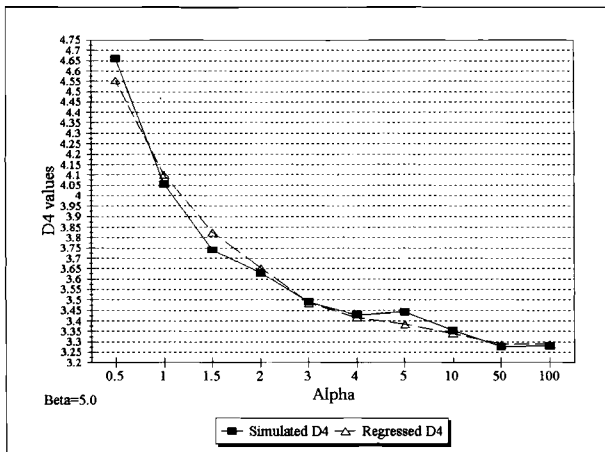
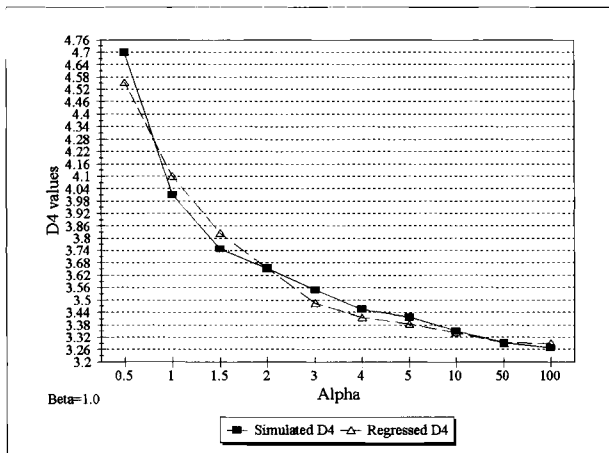
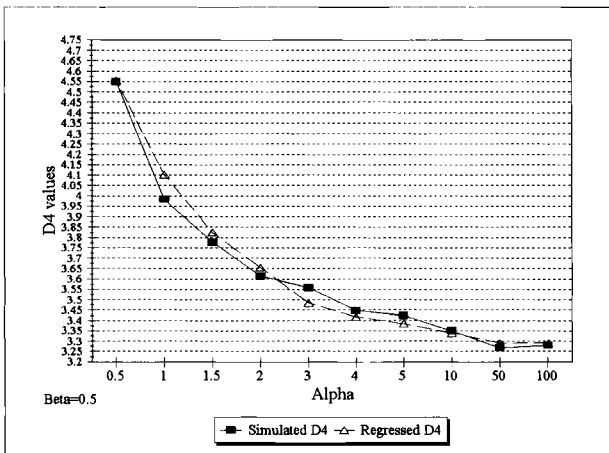
APPENDIX H

**GRAPHS - SIMULATED VALUES OF d_2 , d_3 , AND D_4 VERSUS REGRESSED
VALUES OF d_2 , d_3 , AND D_4**

Alpha	Beta	d2	Fits	d3	Fits	D4	Fits
0.5	0.5	0.89067	0.89154	1.05376	1.08131	4.54932	4.55435
1	0.5	1.00121	0.99263	0.9958	0.99129	3.98377	4.10157
1.5	0.5	1.0287	1.03741	0.95218	0.95446	3.77682	3.82477
2	0.5	1.03973	1.05992	0.90597	0.9351	3.61405	3.6548
3	0.5	1.09221	1.08166	0.93153	0.91081	3.55866	3.48412
4	0.5	1.08259	1.09276	0.88364	0.89357	3.44866	3.41561
5	0.5	1.10427	1.09965	0.89188	0.88137	3.42299	3.38524
10	0.5	1.09534	1.11296	0.85834	0.8614	3.35089	3.34011
50	0.5	1.13015	1.12259	0.85435	0.85946	3.26789	3.29069
100	0.5	1.12918	1.12275	0.85795	0.85946	3.27939	3.28977
0.5	1	0.90144	0.89154	1.11124	1.08131	4.69825	4.55435
1	1	0.99756	0.99263	1.00081	0.99129	4.00977	4.10157
1.5	1	1.02377	1.03741	0.93738	0.95446	3.74683	3.82477
2	1	1.05894	1.05992	0.93746	0.9351	3.65586	3.6548
3	1	1.08304	1.08166	0.92004	0.91081	3.54847	3.48412
4	1	1.09813	1.09276	0.89817	0.89357	3.45373	3.41561
5	1	1.11059	1.09965	0.89508	0.88137	3.41785	3.38524
10	1	1.10985	1.11296	0.86897	0.8614	3.34889	3.34011
50	1	1.11941	1.12259	0.85521	0.85946	3.29196	3.29069
100	1	1.12389	1.12275	0.84914	0.85946	3.26661	3.28977
0.5	5	0.87432	0.89154	1.06679	1.08131	4.66041	4.55435
1	5	0.98901	0.99263	1.00668	0.99129	4.05361	4.10157
1.5	5	1.03932	1.03741	0.94953	0.95446	3.7408	3.82477
2	5	1.0388	1.05992	0.91038	0.9351	3.62913	3.6548
3	5	1.08785	1.08166	0.90309	0.91081	3.49047	3.48412
4	5	1.10745	1.09276	0.89646	0.89357	3.42846	3.41561
5	5	1.10433	1.09965	0.89902	0.88137	3.44228	3.38524
10	5	1.10642	1.11296	0.86794	0.8614	3.35338	3.34011
50	5	1.11593	1.12259	0.84726	0.85946	3.27772	3.29069
100	5	1.11036	1.12275	0.84349	0.85946	3.27897	3.28977
0.5	10	0.89332	0.89154	1.08193	1.08131	4.63341	4.55435
1	10	1.01105	0.99263	1.00519	0.99129	3.98262	4.10157
1.5	10	1.03744	1.03741	0.96472	0.95446	3.7897	3.82477
2	10	1.07441	1.05992	0.93263	0.9351	3.6041	3.6548
3	10	1.08453	1.08166	0.90925	0.91081	3.51513	3.48412
4	10	1.10206	1.09276	0.90751	0.89357	3.4704	3.41561
5	10	1.08739	1.09965	0.87412	0.88137	3.41159	3.38524
10	10	1.10628	1.11296	0.87271	0.8614	3.36661	3.34011
50	10	1.13493	1.12259	0.85834	0.85946	3.26889	3.29069
100	10	1.12964	1.12275	0.85693	0.85946	3.27576	3.28977







APPENDIX I

SPREADSHEETS OBJECTIVE THREE (#3)

OBJECTIVE #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34						
Lognormal (0, 1%)	k = 10 obs																																							
	Shft 2 process standard deviation																																							
	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180			
New observations k =	1	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175	180			
Estimator shape parameter	0.82517	4.32587	1.04861	0.07184	0.047683	2.0878	1.03656	0.051217	0.041581	1.043222	2.25837	2.536188	0.846451	1.563674	1.08022	1.732207	1.56124	1.823481	1.285204	1.013472	2.102118	2.842727	1.328808	0.885402	1.154108	4.236022	0.972341	2.278258	0.962875	1.528788	1.043247									
Estimator scale parameter	1.50241	0.227236	2.14424	0.753004	1.806428	0.818488	1.211874	1.925484	0.847708	1.082706	2.089541	0.888227	2.870147	1.261573	0.845583	0.825707	0.312880	0.708125	1.487805	0.442180	0.348218	1.267202	2.380787	1.452271	0.228411	1.892563	3.358442	1.858497	2.170718	2.015521										
Control chart constant d2	0.887475	1.085348	0.888773	1.001387	0.885814	1.062504	0.887582	0.888894	0.817173	0.888011	1.073748	0.810783	1.041133	0.832643	0.845548	0.825780	1.044344	1.022178	0.884359	0.883131	1.074265	1.02587	0.87848	1.01021	1.084874	0.912121	1.021705	0.887492	1.03814	0.888014										
Control chart constant d3	0.100337	0.88048	0.82008	0.355259	0.701151	0.729359	0.11886	0.72645	0.60082	0.58889	0.72886	0.47628	0.68864	0.428118	0.548708	0.13885	0.00116	0.26701	0.26213	0.64178	0.52088	1.18832	0.89518	0.37188	0.75318	0.88258	0.80981	0.48855	0.48773	0.50643	1.18804	0.88084	0.78025	0.85888	0.45328					
Control chart constant D4	4.145202	3.40314	4.087489	4.052815	3.139184	4.074323	4.090578	4.433887	4.071825	3.848484	4.543883	4.212524	3.788286	4.038823	3.725586	3.801974	3.774828	3.827328	4.002184	3.50158	4.08843	4.188241	4.001381	4.068282	4.42227	3.830242	4.128023	3.812574	4.071808											
Upper control limit X	0.887081	2.21812	1.17954	2.585579	2.584028	0.887578	0.885575	2.10014	2.586133	0.854422	4.818882	1.38778	2.545387	2.573887	2.928887	4.188288	4.45898	2.588925	2.888745	2.84884	4.088473	2.138818	2.288122	2.278822	4.428822	3.888822	4.088822	4.088822	4.088822	4.088822	4.088822	4.088822	4.088822	4.088822	4.088822	4.088822	4.088822	4.088822	4.088822	
Lower control limit X	-0.08007	-0.25382	-1.85555	-0.84848	-0.22838	-0.827217	-0.07281	-0.180243	-18.87404	-0.880831	-3.712887	-0.405813	-0.320232	-1.82884	-1.00875	-3.296144	-3.951515	-0.827884	-2.270185	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	
Upper control limit MR	7.448880	1.535743	0.844550	2.377842	7.88148	3.18715	0.07782	10.18184	22.88838	7.880183	8.881887	4.02883	10.84243	7.345753	5.178828	3.847384	4.088881	3.741488	4.748584	0.818885	2.521015	2.335145	7.084378	0.843548	7.481878	1.648471	28.17188	0.844578	11.88004	12.82884	11.88004	12.82884	11.88004	12.82884	11.88004	12.82884	11.88004	12.82884		
False alarm in X (Y=I=0)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
False alarm in X (Y=I=0)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
False alarm in X (Y=I=0)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
Total false alarm in X	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
False alarm in MR (Y=I=0)	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
False alarm in MR (Y=I=0)	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
False alarm in MR (Y=I=0)	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total false alarm in MR	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Sub-total X control charts with at least one false alarm k=30																																								
Sub-total MR control charts with at least one false alarm k=30																																								
Normal upper control limit X	0.182245	2.182242	0.584535	2.388274	0.851828	0.818854	4.574344	0.584007	17.84335	0.808801	4.678442	4.786402	0.278878	7.118244	4.835595	3.730888	4.128135	3.488443	4.125878	7.280631	2.777015	2.788315	0.548183	6.528757	0.848152	2.247014	19.10144	0.871858	9.900158	12.2964	12.2964	12.2964	12.2964	12.2964	12.2964	12.2964	12.2964			
Normal lower control limit X	-3.37523	-2.31884	-2.08238	-0.75218	-3.27843	-1.056	-2.0549	-4.73582	-8.82808	-3.38803	-1.18878	-1.7885	-4.40288	-3.17888	-1.82788	-1.62888	-1.80373	-2.38851	-4.77454	-0.81884	-0.78888	-3.10148	-2.28238	-3.78881	-0.23877	-12.24288	-3.82823	-5.82823	-12.24288	-3.82823	-5.82823	-12.24288	-3.82823	-5.82823	-12.24288	-3.82823	-5.82823			
Normal upper control limit MR	0.886488	1.474758	0.538278	1.917143	0.22838	0.827217	4.07281	0.180243	18.87404	0.880831	3.712887	0.405813	0.320232	-1.82884	-1.00875	-3.296144	-3.951515	-0.827884	-2.270185	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781	-0.420188	-0.116832	-1.82781		
False alarm in X (Y=I=0)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
False alarm in MR (Y=I=0)	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Sub-total MR control charts with at least one false alarm k=10																																								
False alarm in X (Y=I=0)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
False alarm in MR (Y=I=0)	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Sub-total X control charts with at least one false alarm k=10																																								

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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

OBJECTIVE # 3

Normal (40, 10%)

k = 30 sigma SRR = process standard deviation

Table with 34 columns and multiple rows of numerical data. Rows are grouped under 'New observations k ='. Each group starts with a row number (e.g., 10, 20, 30) and contains 34 columns of values ranging from 0 to 1000.

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Estimator shape parameter table. Columns include parameter names (e.g., Estimator shape parameter, Estimator scale parameter) and 34 columns of numerical values. Rows include various control charts like 'Sub-total X control charts with at least one false alarm k = 30' and 'Sub-total normal MR control charts with at least one false alarm k = 30'.

OBJECTIVE # 3

Exponential (Mean=1) k = 10 obs Shift: 1 process standard deviation

New observations k = 17

1	1.4883	3.7803	3.2374	1.2839	1.5866	1.2582	8.5504	1.8967	1.0853	1.2512	1.8912	3.0476	4.0715	1.0171	1.1385	1.8007	1.8031	4.1087	1.3481	4.2878	3.4578	1.2420	1.0136	1.6848	1.0438	1.1282	1.4801	1.8639	1.3052	1.0042	1.8711	
1	1.3285	1.8218	3.2646	1.8211	1.4276	1.3028	1.3083	1.5174	1.1488	1.8025	1.3478	1.1754	1.2878	1.3836	1.1886	1.1865	1.3293	1.8624	1.2091	1.8647	1.1233	1.2818	1.2459	1.1580	1.4893	1.7351	1.1876	1.2648	1.2183	1.9048	1.2183	
1	1.3485	1.3118	2.0911	1.3903	1.8724	2.2014	1.8413	1.2894	1.0812	3.8720	1.0154	1.0208	1.4692	1.8035	1.8514	1.4823	1.0440	1.0743	1.4041	1.3216	1.1039	1.2881	1.2281	1.1214	1.0271	1.2261	1.2765	1.2536	1.5291	1.5731	1.7008	1.0471
1	1.4552	1.8573	1.0130	2.2027	3.0824	5.9705	1.9348	1.8247	2.0972	1.2295	1.5879	1.5879	1.8013	2.4885	1.7280	1.8843	1.4823	2.5378	1.7528	1.3053	1.3818	1.7188	1.4438	1.3042	1.2085	1.4382	1.2879	1.4872	2.1135	1.9780	1.8184	
1	1.8015	1.3548	1.0923	1.8087	2.3380	2.4465	1.4042	1.8117	1.8254	1.5448	2.1280	1.5817	1.4888	1.8403	1.1438	1.8510	1.0891	1.0891	1.2735	1.2351	2.8958	1.2718	1.9451	1.0587	1.8118	1.4845	1.2281	1.2281	1.3231	1.0348	1.183	
1	1.5305	2.1183	1.3583	1.3091	1.9137	1.2890	1.0878	1.7018	1.4453	1.5288	1.1878	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	1.2844	
2	1.085	1.2584	1.8456	1.1880	1.1557	2.2314	1.8239	1.2382	2.8437	1.3121	1.1823	1.4537	1.8478	1.3207	1.8808	1.3172	1.0745	1.1824	1.8852	2.5804	2.2845	2.2742	1.3748	1.4251	1.8802	1.4825	1.2594	1.4825	1.2594	1.4825	1.4825	
3	1.2848	1.4788	1.3242	1.5288	1.8937	1.3123	2.1804	1.3308	1.1723	2.5827	2.2823	1.8979	1.3648	1.5471	2.8421	1.1513	1.1812	1.6811	1.3182	1.3147	2.1814	1.3743	1.3888	1.3488	1.5822	2.4881	1.4513	1.7588	2.6571	1.3111	1.7183	
3	1.8178	1.2184	2.8078	1.1298	2.8158	1.1404	1.8781	2.0258	1.1782	1.8031	2.5553	2.1288	1.2101	2.8024	1.8984	1.6884	2.8612	1.4058	1.2807	1.7217	1.4574	1.2861	1.2984	1.2831	1.8453	2.8518	1.4584	1.1588	1.4584	1.2838	2.8108	
4	1.2731	1.3698	1.8035	1.7455	1.2813	1.5817	1.5814	1.2274	1.1302	2.5342	1.0528	1.8028	1.0723	2.1188	1.0728	2.1188	1.0728	2.1188	1.0728	2.1188	1.0728	2.1188	1.0728	2.1188	1.0728	2.1188	1.0728	2.1188	1.0728	2.1188	1.0728	
5	1.7708	1.8294	1.0903	3.0187	1.5813	1.1848	1.2889	1.8108	1.1575	1.7891	1.8004	1.8582	1.0588	1.8318	1.4542	1.3238	2.5822	2.1088	1.1383	1.1452	1.7248	1.1028	1.8818	1.2882	1.2487	1.1243	1.3707	1.5584	1.7298	1.1038	1.0548	
5	1.1332	1.5785	1.3451	1.2283	1.1238	1.5848	2.8878	1.1865	1.1774	1.1811	1.0785	1.5588	1.0807	1.0288	1.8013	2.8785	1.5458	1.3843	1.2701	1.2211	1.2748	1.0731	1.1071	1.2715	1.2545	2.0287	1.2737	2.8277	3.2882	1.7817	1.3595	
5	1.0211	1.1701	2.3784	2.7298	1.1838	1.2058	1.1838	1.5182	1.2883	1.0878	1.6583	1.2878	1.6583	1.2878	1.6583	1.2878	1.6583	1.2878	1.6583	1.2878	1.6583	1.2878	1.6583	1.2878	1.6583	1.2878	1.6583	1.2878	1.6583	1.2878	1.6583	
5	1.2182	2.2578	1.4871	1.4158	1.8938	3.3128	1.0838	1.0714	1.1485	1.3587	3.7875	1.1781	1.0884	1.3348	1.0553	1.0281	1.8708	1.8348	1.5204	1.4887	1.0737	2.4878	1.8478	1.8478	1.8478	1.8478	1.8478	1.8478	1.8478	1.8478	1.8478	
5	1.4858	1.4803	2.7282	1.2457	1.1833	1.4858	1.5858	1.8988	2.8101	1.4853	1.1828	1.4827	1.1885	1.2888	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	1.1885	
5	1.9225	1.5781	1.0375	1.0485	1.3928	1.5288	2.8208	1.4018	2.8874	1.2811	1.0843	1.4882	1.4825	1.1148	1.4521	1.8113	1.5147	2.8411	1.0054	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	
5	1.2812	1.7147	1.2818	1.8127	1.1123	1.4138	1.0858	1.0418	1.8788	1.0882	2.8835	1.4041	1.4528	2.7878	1.3988	1.5102	1.2041	1.1824	1.5384	1.8834	1.2483	2.2712	2.2832	1.8714	1.7243	1.0888	1.3283	1.2418	1.4817	1.8812	1.8812	
5	1.0888	1.1541	2.3895	1.2841	1.2882	1.7284	2.8833	1.0818	1.4275	1.8787	1.8888	1.5888	1.5888	1.7241	2.8533	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	1.2818	
5	1.518	1.5478	2.1105	1.5823	1.1182	1.0158	2.8835	1.2818	1.1888	1.3283	1.5888	1.2843	1.5873	1.0888	1.8238	1.1173	2.3518	1.9513	1.0843	1.5888	2.7487	2.8735	2.8258	1.3818	1.3818	1.8201	1.8614	1.5828	2.4705	2.1842	1.4053	
5	1.3857	1.5207	1.0474	1.1263	1.2038	1.4358	1.0813	1.1874	1.8344	1.4547	2.8455	2.2423	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	
5	4.2107	1.7218	1.8535	1.0785	1.7855	1.7855	1.1438	1.1438	1.8718	1.0228	1.3454	1.5488	1.8718	1.0228	1.3454	1.5488	1.8718	1.0228	1.3454	1.5488	1.8718	1.0228	1.3454	1.5488	1.8718	1.0228	1.3454	1.5488	1.8718	1.0228	1.3454	
5	1.8478	3.3433	1.9281	1.3382	1.8922	2.8801	1.4818	2.1524	1.2158	1.8879	1.1828	1.1232	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	1.1828	
5	2.0174	1.2738	1.8507	2.8828	1.0881	1.0181	1.8207	1.7788	1.3478	1.0415	1.2207	1.8784	1.2418	1.7808	1.0428	1.8388	1.8888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	
5	1.8812	1.2144	1.5432	1.048	2.3878	1.5278	1.8788	1.4328	1.3853	1.1548	1.1838	1.2418	1.8418	1.8418	1.1724	1.1882	2.8814	1.4481	1.8051	1.2888	1.3853	1.3783	1.8888	1.0808	1.4481	1.8888	1.0808	1.4481	1.8888	1.0808	1.4481	
5	1.7435	1.58124	1.5803	1.7885	1.1544	1.8158	1.1722	1.2184	1.0888	1.4881	1.71418	2.8915	1.8258	1.4888	1.3381	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	
5	2.1714	1.2738	1.8507	2.8828	1.0881	1.0181	1.8207	1.7788	1.3478	1.0415	1.2207	1.8784	1.2418	1.7808	1.0428	1.8388	1.8888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	
5	1.8812	1.2144	1.5432	1.048	2.3878	1.5278	1.8788	1.4328	1.3853	1.1548	1.1838	1.2418	1.8418	1.8418	1.1724	1.1882	2.8814	1.4481	1.8051	1.2888	1.3853	1.3783	1.8888	1.0808	1.4481	1.8888	1.0808	1.4481	1.8888	1.0808	1.4481	
5	1.7435	1.58124	1.5803	1.7885	1.1544	1.8158	1.1722	1.2184	1.0888	1.4881	1.71418	2.8915	1.8258	1.4888	1.3381	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	
5	2.1714	1.2738	1.8507	2.8828	1.0881	1.0181	1.8207	1.7788	1.3478	1.0415	1.2207	1.8784	1.2418	1.7808	1.0428	1.8388	1.8888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	
5	1.8812	1.2144	1.5432	1.048	2.3878	1.5278	1.8788	1.4328	1.3853	1.1548	1.1838	1.2418	1.8418	1.8418	1.1724	1.1882	2.8814	1.4481	1.8051	1.2888	1.3853	1.3783	1.8888	1.0808	1.4481	1.8888	1.0808	1.4481	1.8888	1.0808	1.4481	
5	1.7435	1.58124	1.5803	1.7885	1.1544	1.8158	1.1722	1.2184	1.0888	1.4881	1.71418	2.8915	1.8258	1.4888	1.3381	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	
5	2.1714	1.2738	1.8507	2.8828	1.0881	1.0181	1.8207	1.7788	1.3478	1.0415	1.2207	1.8784	1.2418	1.7808	1.0428	1.8388	1.8888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	
5	1.8812	1.2144	1.5432	1.048	2.3878	1.5278	1.8788	1.4328	1.3853	1.1548	1.1838	1.2418	1.8418	1.8418	1.1724	1.1882	2.8814	1.4481	1.8051	1.2888	1.3853	1.3783	1.8888	1.0808	1.4481	1.8888	1.0808	1.4481	1.8888	1.0808	1.4481	
5	1.7435	1.58124	1.5803	1.7885	1.1544	1.8158	1.1722	1.2184	1.0888	1.4881	1.71418	2.8915	1.8258	1.4888	1.3381	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	1.3878	
5	2.1714	1.2738	1.8507	2.8828	1.0881	1.0181	1.8207	1.7788	1.3478	1.0415	1.2207	1.8784	1.2418	1.7808	1.0428	1.8388	1.8888	1.4888	1.0888	1.4888	1.0888	1.4888	1.0888	1.4888	1.088							

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

OBJECTIVE # 3

Exponential (Mean=1) k = 30 obs

Table with 34 columns and multiple rows of numerical data. Rows include 'New observations k =', 'Estimator shape parameter', 'Estimator scale parameter', 'Control chart constants G2', 'Control chart constant d3', 'Control chart constant D4', 'Upper control limit X', 'Lower control limit X', 'Upper control limit MR', 'False alarm in X', 'Total false alarm in X', 'False alarm in MR', 'Sub-total X control charts with at least one false alarm k = 30', 'Normal upper control limit X', 'Normal lower control limit X', 'Normal upper control limit MR', 'Normal lower control limit MR', 'Sub-total MR control charts with at least one false alarm k = 30', 'False alarm in X (Y=1-N=0)', 'Sub-total X control charts with at least one false alarm k = 1', 'False alarm in MR (Y=1-N=0)', 'Sub-total MR control charts with at least one false alarm k = 1', 'False alarm in X (Y=1-N=0)', 'Sub-total X control charts with at least one false alarm k = 10', 'False alarm in MR (Y=1-N=0)', 'Sub-total MR control charts with at least one false alarm k = 10'.

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Table with 34 columns and multiple rows of numerical data. Rows include 'Estimator shape parameter', 'Estimator scale parameter', 'Control chart constants G2', 'Control chart constant d3', 'Control chart constant D4', 'Upper control limit X', 'Lower control limit X', 'Upper control limit MR', 'False alarm in X', 'Total false alarm in X', 'False alarm in MR', 'Sub-total X control charts with at least one false alarm k = 30', 'Normal upper control limit X', 'Normal lower control limit X', 'Normal upper control limit MR', 'Normal lower control limit MR', 'Sub-total MR control charts with at least one false alarm k = 30', 'False alarm in X (Y=1-N=0)', 'Sub-total X control charts with at least one false alarm k = 1', 'False alarm in MR (Y=1-N=0)', 'Sub-total MR control charts with at least one false alarm k = 1', 'False alarm in X (Y=1-N=0)', 'Sub-total X control charts with at least one false alarm k = 10', 'False alarm in MR (Y=1-N=0)', 'Sub-total MR control charts with at least one false alarm k = 10'.

OBJECTIVE # 3

Exponential (Mean=1) k = 30 obs SNR = 1 process standard deviation

Table with 32 columns (1-32) and 32 rows (1-32). Contains numerical data for exponential distribution parameters and observations.

Table with 32 columns (1-32) and 32 rows (1-32). Contains numerical data for estimator shape and scale parameters, and various control charts (Normal upper/lower control limit, MR, X, etc.).

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OBJECTIVE #3

Exponential (Mean=1)

k = 30 obs

SNR = 2 process standard deviation

Table with 34 columns (1-34) and 34 rows (1-34). Each cell contains numerical data representing the results of the experiment for various SNR and process standard deviation values.

Table with 34 columns (1-34) and 34 rows (1-34). Each cell contains numerical data representing the results of the experiment for various SNR and process standard deviation values.

Table with 34 columns (1-34) and 34 rows (1-34). Each cell contains numerical data representing the results of the experiment for various SNR and process standard deviation values.

Table with 34 columns (1-34) and 34 rows (1-34). Each cell contains numerical data representing the results of the experiment for various SNR and process standard deviation values.

Table with 34 columns (1-34) and 34 rows (1-34). Each cell contains numerical data representing the results of the experiment for various SNR and process standard deviation values.

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OBJECTIVE # 3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	
Gamma (1.5 1.0)																																			
k = 10 obs																																			
1	70790	0.6493	1.9862	0.5872	0.7908	1.05894	1.36781	1.64793	0.44148	1.20924	0.84356	2.19482	2.4138	2.32365	0.56132	3.74088	0.13818	0.20521	1.63917	1.07735	2.62637	1.13603	1.03681	0.2703	0.86692	0.25472	1.45367	2.71934	1.23676	0.54902	0.25646				
10	81208	2.20662	0.41628	1.0322	1.82933	0.41878	1.25598	1.30304	1.30035	0.12711	0.89727	0.35341	0.48138	2.19119	2.31495	0.17124	0.56864	0.68822	0.67609	2.43380	0.41467	0.63611	1.05094	0.0866	0.86121	4.9708	1.20118	0.61682	2.32663	1.07308	1.26818				
10	0.14143	0.63458	0.22837	0.286	0.72657	2.20814	1.00488	1.80014	1.00984	2.80953	1.53745	1.32115	2.55777	0.56215	1.20217	0.05373	1.50104	2.08219	2.48043	0.85026	0.50535	0.16343	4.82488	1.03288	2.47088	2.66878	1.95185	2.81141	0.88821	2.49604					
10	0.095864	0.91287	1.23723	2.5042	2.82622	1.14412	0.82163	1.88894	4.77073	0.26458	1.85416	0.74902	1.29584	0.83213	2.06967	0.22056	1.97531	1.27151	1.01230	2.96883	1.29279	1.79548	0.75773	2.1776	1.81818	0.70712	2.20318	1.45574	2.48022	0.8208	0.48805				
10	1.50087	1.04918	0.26818	0.56838	2.78424	2.56338	0.73377	3.92062	0.9706	1.44478	0.31854	2.47745	0.62148	1.69382	0.58842	0.40151	1.01236	1.30225	1.08242	0.19001	0.75767	0.47484	0.85133	3.80078	2.07977	0.56738	0.24259	3.85585	1.38881	3.38037	0.7285				
10	1.94105	0.08173	0.75517	0.47894	0.31452	0.51575	0.62558	3.98805	0.07128	0.74988	0.95858	2.12426	0.5515	3.33244	0.01824	1.13074	0.88434	0.51158	1.28808	0.81907	0.20805	0.79829	4.7158	2.14184	1.30847	0.88133	1.07364	0.07364	0.01077	0.92904	4.45842				
10	1.80987	1.6322	2.80508	0.54205	0.28772	2.87734	1.54115	2.08411	0.24861	0.36558	1.02871	0.83268	2.01228	2.01228	1.02871	0.2864	0.84641	0.54532	2.85	0.88227	1.10084	1.2866	0.80756	2.6471	3.48634	2.6471	5.0818	2.38822	1.56435	1.87738	2.32183	1.24618			
10	0.89898	0.70108	0.13831	0.78714	1.70189	0.22358	0.84052	1.88334	1.74285	0.95718	1.29858	0.27363	0.91038	0.43314	0.34675	1.2738	0.111	0.5254	2.26387	0.70587	1.27835	1.17592	1.13408	1.9333	0.81895	0.85947	0.01277	0.85318	3.98843	1.17719	0.82521				
10	0.14882	2.2187	0.20525	0.46888	1.17452	1.86773	0.28217	0.68332	1.82436	2.44241	3.82892	0.83888	1.43407	2.13744	1.8838	0.97333	2.4137	1.84088	1.83424	1.67064	0.77022	2.01282	2.3505	0.87894	1.12445	0.83168	0.77681	3.09513	0.40287	1.43807	3.07789				
10	0.27312	0.54537	0.38181	2.38388	0.55848	1.27501	0.7038	0.88808	0.086	0.88207	0.80457	0.00120	0.3123	0.08843	3.2508	1.172	3.20844	0.3467	1.88684	2.84828	1.03234	2.35844	0.81737	1.48064	0.81737	1.48064	0.81737	1.48064	0.81737	1.48064	0.81737	1.48064			
10	1.12411	0.18852	0.80586	1.75574	0.54181	0.34574	0.86288	0.68752	0.44888	0.89843	1.73889	2.84847	3.01832	1.23105	0.4887	0.88032	0.31232	0.8146	0.7285	0.49776	1.45888	0.88981	0.4133	0.88918	1.84052	0.84887	2.49175	0.843	4.88958	0.45056	1.00816				
10	0.03584	1.48221	0.8638	0.36785	1.02354	0.38985	0.25847	1.28977	0.48723	1.13233	0.02358	1.3288	1.33678	1.34887	1.31428	1.25433	1.18458	0.31012	0.37555	0.48228	0.44411	0.32778	2.18814	0.00964	1.84887	1.8888	0.2183	0.1652	0.62784						
10	0.95408	0.62614	0.07248	1.04842	1.8431	0.08187	0.2524	1.28848	2.75885	0.70848	0.41845	0.87054	0.28003	0.87883	0.28003	0.5144	0.47033	0.8088	0.90818	1.45887	1.48685	1.78218	0.40848	0.13438	2.53758	2.20077	0.8547	0.06837	2.58954	2.47547					
10	0.62748	0.88588	0.78322	0.88011	2.27431	1.10388	1.67295	2.52224	3.83055	0.41545	1.88345	0.41872	2.23788	1.83887	2.10242	0.83232	0.21444	2.86853	3.28154	0.23358	0.80855	0.23335	1.30823	0.37258	0.00804	0.48373	0.83831	0.23733	0.51044	2.36288	0.13178				
10	0.31008	0.98012	0.82275	1.1838	1.34382	0.86442	3.5888	2.1814	1.45957	4.82555	3.3818	0.02884	1.71424	0.70884	1.18248	0.3388	0.58884	0.18488	0.3141	0.87458	0.50432	2.14843	1.58131	1.1088	0.75848	0.54075	1.80131	1.58888	1.5883	2.26584	0.19181				
10	0.23288	1.24044	0.48382	0.77811	0.77114	1.80184	1.87788	1.01078	0.47083	1.28248	1.38652	0.88821	0.38882	1.87853	0.77188	3.18375	0.83784	1.111	3.48888	0.87858	0.84181	0.83381	2.15818	2.88564	2.88887	0.81437	1.37888	1.75324	1.45818	1.68488	0.88324				
10	0.41817	2.89951	0.08854	1.41351	2.08828	2.38848	0.95281	2.74353	0.08833	0.75174	1.81758	1.37579	1.0827	0.57501	0.87887	0.11337	2.8657	1.38383	0.38288	2.4872	0.87172	0.87288	0.80187	1.71284	1.04441	0.88585	0.84207	3.44803	0.07781	0.88182					
10	0.81803	2.2088	0.57871	0.88588	0.28775	0.55851	3.04438	3.02187	1.95887	2.28982	3.01882	1.48881	2.8888	0.02878	0.78503	0.78707	1.02228	1.0743	1.48814	1.98331	2.70384	1.79585	0.85881	0.13437	1.22704	0.81502	4.83403	1.1818	2.88883	1.1818	2.88883				
10	0.58188	0.8731	1.48838	0.02845	1.78878	0.42548	1.05114	1.02888	0.54288	0.73888	0.38088	0.9544	0.98788	2.13344	1.82842	0.48188	1.81688	2.48834	1.52858	2.0854	0.73888	0.52557	0.8045	1.08188	1.01213	0.83818	0.83444	2.2313	1.28884	1.2588	2.4873				
10	0.8218	2.21078	1.78838	0.74837	0.28837	0.40124	3.88551	0.52855	2.37788	0.84837	0.52785	0.35883	0.48852	1.97845	0.84888	0.72212	1.13722	0.45755	2.01842	0.858	0.86218	1.88832	0.98038	0.07478	2.1854	0.04878	0.88524	0.28888	0.28385	0.72951					
10	1.58401	2.44027	2.38048	1.2482	0.7847	3.33535	1.23783	1.88788	0.31822	0.17254	3.20888	1.88315	2.48231	3.10283	1.00875	0.48888	0.82852	0.88218	0.82852	0.88218	1.75425	0.83148	0.88784	1.75425	0.83148	0.88784	1.75425	0.83148	0.88784	1.75425	0.83148	0.88784			
10	0.99518	1.91013	2.88888	0.808	1.08874	0.42522	0.88858	2.14443	0.18288	2.41188	1.12188	0.88434	1.43587	0.81481	1.78282	1.11889	1.84835	0.87254	2.88883	1.52188	0.88833	1.10472	0.88883	1.10472	0.88883	1.10472	0.88883	1.10472	0.88883	1.10472	0.88883	1.10472			
10	1.87848	3.48585	1.388	1.01858	0.32733	0.41287	0.88387	0.88385	0.27138	2.2821	0.08173	0.37542	0.88388	2.8888	1.88472	0.88038	0.88032	3.18183	0.8481	0.53885	0.84812	4.35878	1.45184	1.72231	4.27373	3.28888	1.884	0.83818	1.88715	1.78388					
10	1.81838	0.21508	0.14384	1.58584	1.8818	1.03831	0.88871	0.84888	2.10788	2.40882	0.28848	1.58385	1.31084	1.30184	1.88888	1.88101	1.03884	1.88287	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848	0.28848			
10	0.32485	0.84878	3.28881	0.88433	1.28885	0.18283	0.32752	0.88884	1.1508	0.53778	0.82885	2.01822	1.41474	0.02258	0.38284	1.30748	0.88687	0.57422	0.81771	0.11188	1.46753	0.88548	0.88028	0.88883	0.18338	0.88888	1.02288	0.34281	1.57387	0.47887	0.70772				
10	3.58188	0.8731	1.48838	0.02845	1.78878	0.42548	1.05114	1.02888	0.54288	0.73888	0.38088	0.9544	0.98788	2.13344	1.82842	0.48188	1.81688	2.48834	1.52858	2.0854	0.73888	0.52557	0.8045	1.08188	1.01213	0.83818	0.83444	2.2313	1.28884	1.2588	2.4873				
10	5.0884	0.80528	3.88881	0.28881	0.45182	0.23772	0.88145	0.35788	1.83758	0.78885	0.31108	0.78888	0.48888	4.7372	1.88844	0.1444	0.05183	0.58848	0.27618	1.88425	1.7377	0.2408	0.7408	0.87882	0.88883	0.12288	1.32888	1.52888	0.84885	1.14752	0.88887	1.08883			
10	1.2074	0.307431	2.3828	2.4874	2.32183	1.22181	0.88377	0.38185	0.38185	2.87282	2.87282	1.08354	1.475	1.1385	0.81788	0.42738	3.55214	1.00154	1.8844	2.08881	0.88425	0.38788	1.88818	0.88818	0.88818	0.88818	0.88818	0.88818	0.88818	0.88818	0.88818	0.88818			
10	5.8181	1.32418	2.18837	0.31788	0.31788	0.88785	0.48884	2.02541	0.88848	1.88382	4.40588	0.53183	0.72885	0.88888	0.88812	1.4782	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285	2.20285			
10	3.01123	2.88882	2.38558	2.18218	2.7438	3.68488	2.12582	0.88732	1.97884	0.88875	1.48888	0.88885	1.78888	0.88888	1.88844	0.28848	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883	0.88883			
Estimator shape parameter	2.114283	1.828188	2.884925	3.887345	1.018478	2.188249	1.38888	1.828048	3.885397	1.87832	1.831483	0.8482879	2.388585	2.728128	1.875432	1.887043	1.788858</																		

OBJECTIVE #3

Gamma (1.5 | 0)

k = 10 obs SNR 1 process standard deviation

Table with 34 columns (1-34) and 30 rows of numerical data. The data is organized into three main sections: 'New observations n =', 'Estimator shape parameter', and 'Estimator scale parameter'. Each section contains a series of values for each of the 34 objectives.

Table with 34 columns (1-34) and 30 rows of binary data (0s and 1s). The data is organized into three main sections: 'Sub-total X control charts with at least one false alarm k=30', 'Sub-total MR control charts with at least one false alarm k=30', and 'Sub-total X control charts with at least one false alarm k=10'. Each section contains a series of binary values for each of the 34 objectives.

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OBJECTIVE # 3

Gamma (1.5, 10) k = 30 obs

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34		
1	1.78790	0.84831	1.98822	0.59722	0.79088	1.95884	1.36781	1.04783	0.44468	1.20824	0.84356	2.91482	2.41230	2.32295	0.56132	3.74088	0.13618	0.20521	1.83817	1.87735	2.82637	1.13803	1.03861	0.2703	0.86632	0.25472	1.43587	2.71934	1.21876	0.54802	0.25948				
10	0.27312	0.54537	0.39181	3.38398	0.55648	1.27501	0.70208	3.08006	0.699	0.58248	0.87327	0.80457	0.81103	0.3183	0.82485	1.43717	3.29044	0.3407	0.08874	1.2604	0.80758	0.24671	1.46834	1.28042	0.17867	0.35678	0.24258	3.65485	1.36881	2.56037	0.2385				
New observations n =	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
Estimator shape parameter	1.32028	1.37928	1.30816	1.20451	1.43384	2.50821	1.50810	0.82815	1.85595	1.49870	2.20862	1.65892	1.40485	1.84522	1.81848	1.84883	1.83682	1.80287	1.87447	2.57338	1.88385	1.40864	1.00811	1.35724	2.95457	1.78286	2.27310	2.58838	1.88026	1.42048	1.84173	0.00000	0.00000		
Estimator scale parameter	1.34445	0.86215	1.14534	1.58442	1.23623	0.49838	0.85848	1.81845	1.01290	1.86878	1.00577	1.77861	1.81202	0.94864	0.86889	0.84002	0.82802	1.00527	0.77914	0.97284	1.30588	1.22473	1.13882	0.86954	0.82897	0.80821	0.57641	0.00341	0.00341	0.00341	0.00000	0.00000			
Control chart constant d2	1.02494	0.29421	0.24026	1.01551	0.33221	1.07378	0.52719	0.8871	0.65894	1.03591	0.51584	1.04813	0.33228	0.94501	1.05213	0.68183	0.68183	0.94501	1.05213	0.68183	0.68183	0.94501	1.05213	0.68183	0.68183	0.94501	1.05213	0.68183	0.68183	0.94501	1.05213	0.68183	0.68183		
Control chart constant D4	3.00910	3.87882	3.91533	3.87248	3.95491	3.54838	3.82126	3.23275	3.85180	3.38288	3.99900	3.78159	3.88768	3.70812	3.89881	3.70527	3.78282	3.88892	3.79528	3.88073	3.96102	3.79528	3.88073	3.96102	3.79528	3.88073	3.96102	3.79528	3.88073	3.96102	3.79528	3.88073	3.96102		
Upper control limit X	6.49526	6.89982	6.88078	7.24378	6.87543	6.98236	6.89108	6.28815	6.72014	6.58178	6.48403	6.50287	6.01082	6.38829	6.39870	6.39722	6.22225	6.41748	6.48737	6.44858	6.37725	6.39781	6.32018	6.43415	6.22321	6.45183	6.31838	6.34547	6.39615	6.39371	6.40816	6.40816			
Lower control limit X	-2.84637	-1.88095	-1.84637	-1.40395	-1.32216	-1.48253	-2.31355	-2.9106	-1.21362	-1.13168	-1.28435	-1.91522	-2.71228	-2.32788	-2.26281	-2.33385	-1.3121	-2.74583	-1.79808	-1.93275	-3.7446	-2.52451	-2.67851	-3.19729	-1.88415	-1.15334	-0.88587	-1.09732	-1.15882	-1.97048	0.00000	0.00000			
Upper control limit MR	6.20512	4.44058	4.52732	7.15478	7.78818	3.46944	4.78108	3.88877	4.27212	4.78734	5.28835	4.20833	5.83181	4.30586	3.28187	3.20888	3.20888	4.50245	3.48572	2.99045	6.74144	4.78251	4.32987	3.24196	1.24826	2.80828	5.87078	1.24826	2.80828	5.87078	1.24826	2.80828			
False alarm in X (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
False alarm in MR (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Total false alarm in X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
False alarm in MR2 (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
False alarm in MR2 (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
False alarm in MR2 (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Sub-total normal X control charts with at least one false alarm k=30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Sub-total normal MR control charts with at least one false alarm k=30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
False alarm in X (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
False alarm in MR2 (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sub-total X control charts with at least one false alarm k=10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sub-total MR control charts with at least one false alarm k=10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sub-total X control charts with at least one false alarm k=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sub-total MR control charts with at least one false alarm k=1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
False alarm in X (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
False alarm in MR2 (Y=1=0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sub-total X control charts with at least one false alarm k=10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Sub-total MR control charts with at least one false alarm k=10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

OBJECTIVE # 3

Gamma (1.5, 10) k = 30 obs Shift 1 process standard deviation

Table with 34 columns and 34 rows of numerical data. The first row is labeled '1' and the last row is labeled '30'. The data represents a time series with a shift in standard deviation at observation 10.

Table with 34 columns and 34 rows of binary data (0s and 1s). The first row is labeled 'Estimator shape parameter' and the last row is labeled 'Sub-total MR control charts with at least one false alarm k = 10'. The data represents the results of various control chart tests.

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OBJECTIVE # 3

Gamma (1.5, 1.0) k = 50 obs

Table with 33 columns and 30 rows of numerical data. The first row is labeled '1' and the last row is labeled '30'. The data represents the results of 30 observations for Objective #3.

Table with 33 columns and 30 rows of numerical data. The first row is labeled '1' and the last row is labeled '30'. The data represents the results of 30 observations for Objective #3.

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OBJECTIVE # 1

Chi-square (df=4) k = 10 obs

Table with 32 columns (numbered 1-32) and 32 rows (numbered 1-32). Each cell contains numerical data representing chi-square values for various observations.

Table with 32 columns (numbered 1-32) and 32 rows (numbered 1-32). Each cell contains numerical data representing estimator shape parameters and control chart constants.

OBJECTIVE # 3

Chi-square (df=4)

k = 10 obs

Shift process standard deviation

Table with 32 columns (1-32) and 30 rows (1-30). Row 1: 0.7784 5.1566 6.2922 5.178 3.8007 4.843 7.4781 12.1422 4.4383 7.903 10.5594 10.2000 3.3354 0.9881 3.3748 4.3382 5.7500 3.8724 3.348 3.1552 4.4438 3.0688 5.8814 7.5841 13.4887 4.588 4.2371 4.8927 4.1731 3.1821 6.4058. Rows 2-30 contain similar numerical data.

Table with 32 columns (1-32) and 30 rows (1-30). Row 1: Estimator scale parameter 4.207037 3.265556 2.247263 3.806636 1.658603 2.845008 5.168882 1.872453 2.025138 2.951564 1.437863 2.878002 1.514096 3.915188 4.113446 2.882404 3.828012 2.272556 2.036228 1.868995 3.210607 1.491888 5.388431 2.838843 1.463427 3.677577 3.182674 4.11328 7.10142 1.413881. Rows 2-30 contain similar numerical data.

OBJECTIVE # 3

Chi-square (df=4) k = 10 obs SStd 2 process standard deviation

Table with 33 columns (1-33) and 33 rows (1-33) of numerical data. The data represents process standard deviations for various observations.

Table with 33 columns (1-33) and 33 rows (1-33) of numerical data. This section contains estimator scale parameters and control chart constants.

Table with 33 columns (1-33) and 33 rows (1-33) of numerical data. This section contains upper and lower control limits for various control charts.

Table with 33 columns (1-33) and 33 rows (1-33) of numerical data. This section contains sub-total X control charts with at least one false alarm k=30.

Table with 33 columns (1-33) and 33 rows (1-33) of numerical data. This section contains sub-total MR control charts with at least one false alarm k=30.

Table with 33 columns (1-33) and 33 rows (1-33) of numerical data. This section contains sub-total X control charts with at least one false alarm k=10.

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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34

OBJECTIVE #3

Chi-square (df=4) k = 30 obs SNR 2 process standard deviation

Table with 34 columns and 34 rows of numerical data. The first row contains values from 8.7400 to 15.7821. The table is divided into sections by 'New observations k ='. The last row contains values from 13.3097 to 6.945.

Table with 34 columns and 34 rows of numerical data. The first row contains values from 3.498813 to 1.71017. The table lists various parameters and their values, including Estimator scale parameter, Normal upper control limit, and various alarm conditions.

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Table with columns for Objective # 3, k=50 obs, and 34 numbered variables. Rows include parameter estimates (Estimator shape parameter, Estimator scale parameter, Control chart constant d3, Control chart constant d4, Upper control limit X, Lower control limit X, Upper control limit MR, False alarm in X7, Total false alarm in X, False alarm in MR7, Sub-total MRR control charts with at least one false alarm k=30, Normal upper control limit X, Normal lower control limit X, Normal upper control limit MR, False alarm in X7, Total false alarm in X, False alarm in MR7, Sub-total MRR control charts with at least one false alarm k=10, False alarm in X7, False alarm in MR7, Sub-total MRR control charts with at least one false alarm k=10, Sub-total MRR control charts with at least one false alarm k=10, Sub-total MRR control charts with at least one false alarm k=10, Sub-total MRR control charts with at least one false alarm k=10, Sub-total MRR control charts with at least one false alarm k=10.

OBJECTIVE # 3

Chi-square (d=4) k = 50 obs Shift 1 process standard deviation

Table with 32 columns (1-32) and 30 rows (1-30). Each cell contains numerical data points, likely representing process standard deviations for various observations.

Table with 32 columns (1-32) and 30 rows (1-30). Each cell contains numerical data points, likely representing estimator shape parameters for various observations.

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OBJECTIVE #3			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34		
Chi-square (d=4)	k = 50 obs		SNR = 2 process standard deviation																																			
1	8.7408	8.1871	7.6153	8.0890	8.2712	8.6700	7.5834	14.4818	10.0782	11.7121	8.1134	8.5108	12.0751	8.9223	8.7895	17.5812	11.8334	11.8884	8.1083	8.5714	8.1501	8.8004	10.4275	10.8803	8.2327	8.3982	12.7207	10.0772	7.8039	8.7388	15.7821							
10	8.1371	11.8783	10.0834	8.9081	14.2828	11.0007	8.4762	11.9818	14.5043	12.9533	8.9857	8.9849	8.9800	7.5289	11.0582	8.2331	7.0151	14.7182	10.1280	10.1823	11.0832	8.8026	8.6831	8.9016	8.4880	7.9871	12.7072	7.7155	13.7488	13.8586	8.1812							
20	14.8825	8.5679	11.0878	8.3854	15.8696	9.0631	8.1888	8.4451	21.2893	7.8504	7.5813	8.2322	7.7250	12.8087	5.8643	13.8470	10.1935	7.8884	8.4156	8.0683	8.5672	8.171	8.5523	8.8385	13.1882	7.8461	7.7883	8.824	12.2488									
30	8.7129	11.4444	8.6242	7.8022	8.8253	8.7912	8.3835	8.4786	8.8524	7.0557	7.5833	7.8312	7.1854	8.2608	8.8093	7.8759	8.1157	14.4884	8.0882	12.1051	8.8262	8.8061	8.805	8.7218	8.1708	7.5566	11.0081	12.0834	13.8386	10.8801								
40	7.3961	10.2985	14.8653	10.3781	5.8863	8.3082	7.506	8.8426	12.0449	17.7575	8.4149	11.8115	10.1171	8.5180	8.0280	8.0634	14.0875	7.0778	8.4541	10.2023	11.3383	7.2223	7.7023	7.8311	8.8031	9.885	8.5854	8.1025	8.1888	12.2287	11.423							
50	8.1264	8.7830	10.1274	8.4141	10.3172	13.3481	7.8808	11.2524	8.8833	8.3838	11.0237	8.9228	8.7818	8.1395	10.8601	12.0815	15.2508	10.2833	8.1188	8.1558	7.8483	21.0884	12.7863	8.3281	11.2384	7.3579	7.0528	7.4785	7.5129	8.824								
60	11.1742	8.2527	8.858	8.2028	7.8307	13.9555	8.9531	13.4898	8.482	10.4257	7.2115	8.1878	7.882	8.1884	8.3258	15.2822	10.1131	17.181	12.5445	4.8854	7.3548	8.8648	10.2824	8.4024	12.0888	4.8862	7.2203	7.095	14.717	7.2081	8.9118							
70	8.8113	10.287	10.8598	8.8484	7.7972	7.8757	10.5523	18.5965	7.1188	10.8037	12.533	8.8628	7.3275	10.803	8.7187	11.8708	12.8328	8.8351	7.7154	8.3588	8.8085	14.3063	8.5157	8.2028	13.783	7.4807	10.3858	8.8085	7.774	12.4882								
80	8.5732	10.8868	8.4525	7.1542	8.3611	11.8953	10.8213	12.8545	8.1898	10.0717	8.4274	7.5424	15.1021	8.1898	8.8785	14.0857	12.3480	7.281	13.3151	10.3252	8.0887	8.4872	11.1444	8.5785	11.7525	11.4827	8.2731	7.181	8.8077	8.6353								
90	7.7572	8.1187	14.1707	8.0811	8.8474	12.1048	15.3038	10.8378	10.331	11.6118	13.8035	8.3118	11.8556	8.3744	8.8887	12.8589	11.1	9.2957	8.1481	11.3285	8.2553	13.2427	7.0534	8.8548	8.8841	7.882	15.0738	7.0083	8.8024	8.6438								
100	7.4278	11.8422	11.5641	8.0875	8.3721	8.8053	8.3188	12.3485	8.2388	8.9585	14.8807	8.0023	11.8542	12.2895	7.85	8.8214	8.4258	8.3184	8.8882	7.8103	7.88	7.0372	8.8668	7.3887	8.8851	20.8871	7.0887	10.583	15.1148	7.8835								
110	8.833	8.7718	10.3718	8.3721	10.5007	8.8495	5.7882	10.1202	8.8357	7.3504	8.3732	8.8748	7.2554	11.4788	14.7888	8.7108	11.1423	7.4223	8.1821	7.5848	11.1472	21.8428	8.3282	7.8314	14.3188	13.3273	8.58	16.345	8.7119	8.8844	8.818							
120	10.2158	8.3734	7.2584	10.5305	7.9052	8.9884	10.7807	8.8255	5.7188	15.4707	11.2241	8.8284	11.4838	11.1432	13.2653	8.8083	8.8028	8.8287	13.788	10.152	9.531	27.272	15.8788	7.471	7.8852	10.8852	8.4835	8.0784	8.444	11.527	18.5805							
130	8.3208	8.4343	10.3108	18.4222	8.777	8.8575	8.7714	10.2845	8.9808	10.7838	1.8802	8.8855	11.8883	8.8432	7.0885	8.9818	13.1003	10.9741	8.8185	8.8408	18.1102	7.0508	10.1887	8.371	8.7225	8.9442	27.2728	8.8828	11.9571	8.8732								
140	7.5213	7.1831	8.8774	7.4458	8.4128	10.2433	18.8138	12.8873	7.0554	11.8572	8.2558	7.3438	8.884	8.8888	10.7887	8.484	13.8844	10.4443	10.4618	7.381	8.4584	8.8818	7.5885	8.881	8.3715	11.0338	8.4478	7.7187	17.2441	18.42	13.7312							
150	8.3885	8.7881	8.8772	12.2388	8.1827	15.2588	8.1131	8.8848	7.3812	8.3858	8.0154	8.2428	7.8853	12.2883	11.2488	8.8888	8.2888	7.28	8.8407	18.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888							
160	10.0174	8.8853	8.8828	12.3151	8.8171	8.8108	8.288	11.7318	11.8107	8.8828	8.888	8.8828	8.181	10.8858	10.8854	8.1231	7.4017	8.8884	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854	12.8854						
170	7.5435	7.0478	11.8158	8.3885	10.8188	8.7488	8.2888	7.8834	7.8812	8.4833	7.8284	17.5347	8.8183	8.8228	13.1581	8.3388	7.8858	13.8858	11.3254	11.3842	8.8771	21.7884	12.7872	7.758	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888						
180	8.8885	10.7288	8.8888	14.1512	8.5338	13.5801	12.5484	8.4352	10.7833	8.878	8.4037	12.004	7.2885	13.1883	8.3332	8.1	17.2402	13.8818	10.581	8.8888	11.8415	10.7714	11.0514	11.5213	7.5778	7.382	7.8888	8.8888	8.8888	8.8888	8.8888	8.8888						
190	13.18108	8.8844	13.7372	13.2717	8.8882	8.8848	8.7813	8.7342	8.8842	10.8213	8.8731	8.8828	10.5848	7.432	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888					
200	7.8008	8.2503	8.4814	10.5577	8.388	8.3584	8.5847	11.3878	8.2471	7.2148	8.4817	10.4318	8.838	10.1102	10.5118	8.4713	7.8218	8.883	8.7374	7.3334	8.8888	10.2585	13.9078	8.4778	12.1718	7.8118	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888					
210	7.8885	13.7018	8.8887	10.4578	14.5418	13.1507	13.5113	13.4575	8.884	14.8148	15.8752	10.8288	8.8883	8.8828	7.1385	8.7873	8.7775	10.8181	8.8281	8.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888				
220	14.1122	11.3284	7.8248	8.4788	8.7801	8.5751	8.884	14.222	8.783	12.1214	8.5545	8.3458	8.8883	8.8257	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888	8.8888			
230	12.3885	7.2388	7.3888	8.8885	8.8888	7.88	8.8831	8.515	8.5785	8.8141	12.8485	8.0181	15.2485	10.8233	8.8285	10.2171	17.4438	8.8872	10.0271	20.1885	8.4451	10.4323	8.1883	15.7417	12.8851	7.4715	8.2538	8.5741	17.0714	8.5741	17.0714	8.5741	17.0714	8.5741				
240	13.1887	8.8857	11.8275	7.1754	8.2827	15.2012	8.1191	8.8887	8.7447	14.8888	10.3888	8.1111	8.7223	7.8784	11.3188	8.2278	14.5787	8.8887	8.8811	8.1851	8.8888	13.8183	8.0538	8.8888	8.1851	8.8888	8.1851	8.8888	8.1851	8.8888	8.1851	8.8888	8.1851	8.8888	8.1851			
Estimator shape parameter	2.820531	1.555534	1.74118	1.588556	2.044303	2.815888	1.99594	2.138202	2.053185	2.570543	1.783742	1.728086	2.087658	2.082817	1.88011	2.003788	1.728184	2.288324	1.78208	1.85605	2.022708	1.88348	1.884747	1.578854	1.848935	2.318105	2.328374	1.888083	2.814837	2.072344	1.782537							
Estimator scale parameter	1.458278	2.377153	2.218181	2.788874	1.887822	2.118882	2.382008	2.158922	2.08842	1.53341	2.804038	2.418863	2.035178	1.873286	2.188848	2.287338	2.488181	1.8182	2.241071	2.141882	2.187078	2.332884	1.82857	2.031028	1.90182	1.50832	1.433718	2.001878	1.888888	1.835541	1.858467							
Control chart constant c2	1.02138	1.040691	1.04888	1.04148	1.08158	1.075822	1.082854	1.0841	1.08188	1.074837	1.07248	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188	1.07188				
Control chart constant d3	0.918003	0.951782	0.943898	0.951133	0.833782	0.919104	0.826373	0.831118	0.832487	0.818885	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888	0.814888				
Control chart constant d4	3.531011	3.01582	3.732508	3.843235	3.531725	3.888888	3.821273	3.84108	3.837825	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115	3.781115				
Upper control limit X	10.76888	13.1883	13.8788	18.20214	12.7888	16.4888	16.5488	14.878	12.8882	10.2883	14.0888	15.2882	13.8888	12.2248	12.7833	14.8842	12.8837	13.2788	13.2788	14.8842	18.847	13.2125	11.3784	7.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888	10.8888				
Lower control limit X	-3.10883	-8.10115	-8.28445	-7.81031	-4.87238	-5.38884	-7.1501	-5.2187	-4.874	-2.38818	-5.0738	-4.848																										

APPENDIX J

TABLE "T" STATISTIC

TABLE 6.19
 $\hat{\alpha}$ as a function of T , gamma distribution

T	$\hat{\alpha}$	T	$\hat{\alpha}$	T	$\hat{\alpha}$	T	$\hat{\alpha}$
0.01	0.010	1.40	0.827	5.00	2.655	13.00	6.662
0.02	0.019	1.50	0.879	5.20	2.755	13.50	6.912
0.03	0.027	1.60	0.931	5.40	2.856	14.00	7.163
0.04	0.036	1.70	0.983	5.60	2.956	14.50	7.413
0.05	0.044	1.80	1.035	5.80	3.057	15.00	7.663
0.06	0.052	1.90	1.086	6.00	3.157	15.50	7.913
0.07	0.060	2.00	1.138	6.20	3.257	16.00	8.163
0.08	0.068	2.10	1.189	6.40	3.357	16.50	8.413
0.09	0.076	2.20	1.240	6.60	3.458	17.00	8.663
0.10	0.083	2.30	1.291	6.80	3.558	17.50	8.913
0.11	0.090	2.40	1.342	7.00	3.658	18.00	9.163
0.12	0.098	2.50	1.393	7.20	3.759	18.50	9.414
0.13	0.105	2.60	1.444	7.40	3.859	19.00	9.664
0.14	0.112	2.70	1.495	7.60	3.959	19.50	9.914
0.15	0.119	2.80	1.546	7.80	4.059	20.00	10.164
0.16	0.126	2.90	1.596	8.00	4.159	20.50	10.414
0.17	0.133	3.00	1.647	8.20	4.260	21.00	10.664
0.18	0.140	3.10	1.698	8.40	4.360	21.50	10.914
0.19	0.147	3.20	1.748	8.60	4.460	22.00	11.164
0.20	0.153	3.30	1.799	8.80	4.560	22.50	11.414
0.30	0.218	3.40	1.849	9.00	4.660	23.00	11.664
0.40	0.279	3.50	1.900	9.20	4.760	23.50	11.914
0.50	0.338	3.60	1.950	9.40	4.860	24.00	12.164
0.60	0.396	3.70	2.001	9.60	4.961	24.50	12.414
0.70	0.452	3.80	2.051	9.80	5.061	25.00	12.664
0.80	0.507	3.90	2.101	10.00	5.161	30.00	15.165
0.90	0.562	4.00	2.152	10.50	5.411	35.00	17.665
1.00	0.616	4.20	2.253	11.00	5.661	40.00	20.165
1.10	0.669	4.40	2.353	11.50	5.912	45.00	22.665
1.20	0.722	4.60	2.454	12.00	6.162	50.00	25.166
1.30	0.775	4.80	2.554	12.50	6.412		

VITA

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