THE EFFECT OF HETEROSCEDASTICITY ON AN OPERATING CHARACTERISTIC CURVE IN A DESIGNED EXPERIMENT

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I. The Problem and Its Setting

Introduction

Designed experiments provide an organized means for scientifically determining the relationships of inputs to outputs in a given process. Statistical techniques can be used to determine which inputs are most critical to the final product and which inputs do not significantly impact the final product. This knowledge can be invaluable when an individual is setting tolerances, determining which input processes to try to improve, and trying to control the output, or the final product.

Designing experiments requires several steps: 1) brainstorm to determine which input factors are likely to be significant, 2) decide at what range of levels the significant input factors should be set, and 3) arrange all, or a set of, the factors and levels in such a way as to ensure consideration of all the possible combinations. Each of the combinations of factors and settings the experimenter decides to run is called a treatment combination. Once the three steps are completed, the process is run for a set number of times, or replications, for each of the different combinations of factors and settings, and the output characteristic of interest is measured and recorded. The experimenter determines the number of replications to run by deciding how much error is acceptable. Two types of error exist: 1) declaring that an input factor *does* have an effect on the output when it really does *not* (Type I error), and 2) declaring that an input factor does *not* have an effect on the output when it really *does* (Type II error). The experimenter must reconcile these risks with the amount of data she or he is willing to collect. Once all the data is collected, an analysis of variance (ANOVA) is performed to determine how the input factors affect the output. With this information, an individual can adjust the input factors to get the desired results for the output.

Definition of Terms

Factor. Any one of several inputs to a process that can be manipulated during experimentation (Schmidt, 1994).

Interaction. A combination of factors wherein one factor's effect on the response is dependent on the levels of other factor(s).

Heterogeneity of variance. Unequal variance. Heterogeneity of variance can also be called heteroscedasticity.

Treatment Combination. A combination of factors and levels at which the experiment is performed.

Replication. A repetition of the experiment.

Assumptions with ANOVA

To perform the ANOVA one must make two primary assumptions: 1) the data from the process is independent and normally distributed, and 2) the variance of the replications within a given treatment combination is equal to variance of the replication within every other treatment combination. The second assumption is sometimes referred to as homogeneity of variance, or homoscedasticity. In addition, the process must be in a state of statistical control (SOSC). According to Shewhart (1980), "A phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future." The data must be in a SOSC so the experimenter can be certain that the variation in the data was due only to the changing treatment combinations and not to a special cause variation in the process itself.

In practice, however, these assumptions may not hold true. If the data are not normally distributed, averaging the data in each treatment combination can make the averages approach normality, due to the central limit theorem. If the assumption of homogeneity of variance is not true the analyst typically transforms the data in an effort to make the variances become more equal. After the variances are transformed, a regular ANOVA can be performed. If the process is not in a SOSC, methods should be used to control the process before a designed experiment is ever performed.

Extensive research has been done on how to test for homogeneity of variance. Research has also been done to determine robust methods for testing for homogeneity of variance in case the normality assumption does not hold true (Conover, Johnson, & Johnson, 1981). However, little research has been performed on the effect of performing an ANOVA in a two way classification (two factors with two levels) when the homogeneity of variance assumption is not true. Box (1954a) addresses the effect of inequality of variance on an ANOVA but restricts it to a one-way classification (one factor at several levels). Dudewicz and Bishop (1981) developed a new procedure for performing an ANOVA with unequal variances in an *r*-way layout, where *r* is some integer, but did not address the effects of performing a traditional ANOVA. Box (1954b) discusses the effects of unequal variance on the ANOVA in the two-way classification, but only with one observation per treatment combination, which assumes no interactions exist. The literature holds a distinct lack of information about the effect of inequality of variances on an ANOVA for a classification greater than one-way. This research proposes to begin to fill that gap by examining the case of the two-way classification.

Statement of the Problem

The impact of unequal variances between treatment combinations on a two-way ANOVA are unknown. This research will examine a case in which the variance of the data coming from the process is heterogeneous. This could happen for a number of reasons. When performing a designed experiment, a set of treatment combinations must be defined. A combination required by the experiment may never have been run before. Changing the setting of a factor(s) could increase or decrease the variance, either directly or through some interaction. This increase or decrease causes the assumption of equal variances to be untrue. When the false assumption occurs, the effect on the ANOVA is unknown. An example will illustrate the problem.

Example of a Designed Experiment Containing Heterogeneity of Variance

Let A and B be two factors that could be significant to some process. Assume that the two factors contribute a linear effect to the process at hand. Also assume that the two factors each have two settings. Let a "+" indicate the high level setting of each factor, and let a "-" indicate a lower level setting of each factor. The two factors can only have one interaction, A×B. Then, a set of treatment combinations can be defined as in Table 1.

Table 1: Treatment Combinations for Example

Treatment Combinations (TC)	A	B	AxB
1	-	-	+
2	-	+	-
3	+	-	-
4	+	+	+

The next step is to determine the number of replications necessary for statistically valid results, which for this design will be nine. Running nine replications gives a Type I (α) probability of 0.05 and a Type II (β) probability of .25 that a factor or interaction identified as significant truly does belong in the calculation for variance (Schmidt, 1994). The next step is to run the process and gather the data for each of the nine replications. This data is in Table 2, below.

Table 2: Data for Example

				Replications										
TC	A	B	AxB	1	2	3	4	5	6	7	8	9	mean	variance
1	-	-	+	9.99	10.33	8.33	9.62	11.44	8.84	14.58	10.92	9.80	10.43	3.34
2	-	+	-	8.85	9.26	9.23	9.80	12.15	12.72	11.78	11.93	11.97	10.86	2.34
3	+	-	-	16.09	13.91	6.71	6.53	12.49	11.16	6.71	11.76	1.86	9.69	20.25
4	+	+	+	8.91	3.36	13.23	12.29	7.83	3.77	10.78	3.24	0.81	7.14	20.17

The mean and variance have been found for this data, and it can be seen that the variances for the four treatment combinations are not equal. It looks as though setting factor A at the "+" level increases the variance greatly. The question now is what effect these variances will have on the ANOVA. There are 2 possible effects: 1) the values of the Type I and Type II errors for the variance could be something other than the values at which the analyst has set them, or 2) absolutely nothing. If, after some research, it is determined that the former effect occurs, the severity of any change must be determined. Without knowing the probability of a Type I (α) and Type II (β) error occurring, the ANOVA will not produce meaningful results.

Operating Characteristic Curve

A graphical representation of both the α and β error can be seen in an operating characteristic (OC) curve. When analyzing the results of an experiment, the first step is to hypothesize that the mean of a factor at its lower setting is the same as the mean at its higher setting. An OC curve of this process would be a graph with the probability of accepting the hypothesis on the vertical axis and the actual values of the mean on the horizontal axis. For the above example, an OC curve could look like Figure 1.

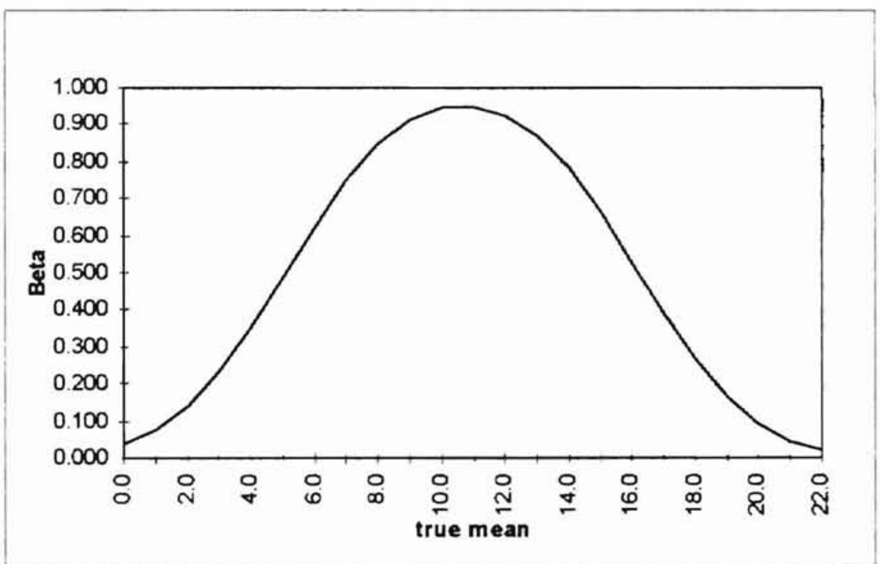

Figure 1: Operating Characteristic Curve for Example

General Approach

In order to determine if the probabilities of error are something other than they were set to be when the ANOVA was performed, an individual could design an experiment and generate the data for the replications. Since the individual would be generating the data, the true means and variances for all the levels of all the factors would be known. By knowing the data, the individual would know what results an ANOVA should give. Then the individual could perform an ANOVA and compare what they know the result should be to the result they obtained. By doing this several times, the individual could determine if the error probabilities they were finding were equal to the error probabilities they set when they performed the ANOVA. In this manner, an entire OC curve could be made for the results of the ANOVA on the unequal variances. If this OC curve were compared to an OC curve where the variances were equal, a difference could clearly be seen if one existed.

Importance of the Study

Currently, the effect of the heterogeneity of variance of a factor or group of factors on the OC curve for the mean of any factor or interaction is neglected. If the effect is significant, then correcting for it could change the outcome of a designed experiment. For example, an experimenter might set alpha equal to 0.05. The effect of heterogeneity of variance could cause the alpha value for the ANOVA to be significantly higher or lower. If this were the case, the results of the ANOVA -- the significant factors and interactions -- would have more or less chance for error than the experimenter intended. The different alpha level could change the experimenter's decision about which factors were important to the process being studied.

Objective Statement

This research proposes to determine the effect of the heterogeneity of variance of a particular factor or group of factors on the operating characteristic (OC) curve for the mean of any factor or interaction.

Subobjectives

The first subobjective. The first subobjective is to set the parameters of the study. Subobjective one has three parts: 1) determining which combinations of factors and levels or groups of factors and levels should be used in the study, 2) deciding at what levels of mean and standard deviation shift to set those combinations of factors and levels, and 3) defining precisely what data will be collected and how it will be calculated.

The second subobjective. The second subobjective is to write a simulation program to perform the experiments at the correct settings of combinations, analyze the data to determine significance, and record the results. The simulation program will be written in FORTRAN. This will require several steps: 1) designing the program, 2) writing the simulation code, 3) finding a random number generator, 4) validating and verifying the program code, 5) determining an equation for sample size, and 6) calculating the number of simulation replications.

The third subobjective. The third subobjective is to analyze the data. Analyzing the data will include: 1) calculating and graphing the OC curves, and 2) determining if the shifting variance affects the OC curves. The analysis will determine the effect of the heterogeneity of variance of a particular factor or group of factors on the OC curve for the mean of any factor or interaction.

Assumptions

The effect of the heterogeneity of variance of a particular factor or group of factors on the OC curve for the mean of any factor or interaction for a two factor, two level designed experiment will be approximately the same as in an f factor, l level designed experiment.

II. Review of the Literature

Assumptions

In a two factor, two level ANOVA, it is assumed that the response variable is normally and independently distributed, the terms in the model are linear, and that the variance of the replications within each treatment combination is equal (Anderson and McLean 1974). To be normally distributed means that the response data, if enough were collected, would take on all the characteristics of a normal distribution. That is, the response data would fit the equation for the normal probability density with two parameters, μ = population mean and σ^2 = population variance. To be independent means that the value of one response has no bearing on the value of the next response. To have equal variances means that the variance of a given set of responses for one treatment combination is the same as the variance of a set of responses for another treatment combinations. In the example in Chapter 1, the response data were normally and independently distributed but did not have equal variances. This research addresses the assumption of homogeneous variance so the author will now concentrate only on this assumption.

Testing for Homogeneity of Variance

In an effort to conform to the assumption of homogeneity of variance necessary to perform the traditional ANOVA, several methods have been developed to test the equality of variances. With several tests in existence, the question arises as to which of the existing tests is the most robust, or less likely to be wrong if any of the assumptions for performing the tests are not true. Several comparison studies have been done, the most extensive of which was by Conover, Johnson, and Johnson (1981) who compared 56 tests for equality of variance. The purpose of their comparison was to find a list of tests that had a stable Type I error rate when the normality assumption was not true, sample sizes were small or unequal, and the distributions were skewed or heavy tailed. The study was performed using simulation. A test was defined to be robust if the Type I error rate was less than 10 percent for a 5 percent test. The tests that showed a stable Type I error rate were then compared on the basis of power. (Power = 1 - β.) For each test of equal variance, the authors computed each test statistic 1000 times in each of 91 situations representing various deviations from the assumptions. From the simulation, the authors found that three tests appeared to be most robust in terms of Type I error rate and power.

New Methods of Performing ANOVA

Although Conover, Johnson, and Johnson determined which tests were most robust for determining equal variances, they did not explore options in case the tests determined that the variances were unequal. The usual approach for dealing with this inequality is to transform the variances in some way. According to Bishop and Dudewicz (1981) these transformations can be useful, but are only approximate in terms of equal variances. To combat this problem, Bishop and Dudewicz developed a new method for performing an exact ANOVA with unequal variances. They retained the assumptions of normality and independence but did not consider variance at all in the computation. Because they did not consider variance, their method applies to data with both equal and unequal variances. Their theory is based on two-stage sampling instead of the more traditional one-stage procedures. They replaced the traditional F statistic with their own, called $\widetilde{F}$. The test statistic $\widetilde{F}$ replaces the standard error $\left(\frac{s}{\sqrt{n}}\right)$ with some value $z > 0$ chosen to obtain the desired power. This z value eliminated the need for any variance term. In addition, they replaced $\overline{X}$ with $\widetilde{X}$, so the mean did not have to be known. Bishop and Dudewicz mathematically proved the validity of the above substitutions and went on to compare the results of this test to those of one-stage test statistics that have been proposed to test the equality of means when the variances were unequal.

Bishop and Dudewicz compared their $\widetilde{F}$ with the usual F statistic, Welch's W, and Brown and Forsythe's F^*. The equations for Welch's W and James' test statistic can be found in Brown and Forsythe (1974). They are both modifications of the usual F statistic. Brown and Forsythe's F^* changed the denominator of the F statistic so that it had the same expected value as the numerator when the population means were equal (Brown and Forsythe, 1974). Brown and Forsythe considered only a one-way layout and compared the four tests at equal and unequal sample sizes and equal and unequal variances using a Monte Carlo simulation. They recommended using F^* or W instead of James' statistic or the usual F statistic regardless of the equality of the variances. Bishop and Dudewicz did not compare their $\widetilde{F}$ to James' test statistic based on the results of Brown and Forsythe (1974).

Bishop and Dudewicz also compared the tests with equal and unequal sample sizes and equal and unequal variances. They found that their $\widetilde{F}$ statistic was superior to Welch's W and Brown and Forsythe's F^*. They also found that $\widetilde{F}$ performed better than F when the variances were unequal. With equal variance, $\widetilde{F}$ performed adequately as long as the sample size was small (Bishop and Dudewicz, 1981).

Effects of Unequal Variances

Numerous studies have been performed on the effects of an incorrect homogeneity of variance assumption in an analysis of variance. Some of the more notable ones are Rogan and Keselman, 1977; Boneau, 1960; Lindquist, 1953; and Box, 1954a. Glass, Peckham, and Sanders, 1972, have a survey paper that discusses most of this research. The one overwhelming similarity among these papers is that they all consider the problem with a one-way analysis of variance. The conclusion among all these papers is that when sample sizes are equal, the ANOVA F-test is insensitive to heterogeneity of variance (Glass and Stanley, 1970). However, when sample sizes are unequal, The F-test can become much more sensitive.

Of all the research on the effect of heterogeneity of variance, Box (1954b) is the only researcher who considered a two-way analysis of variance. Scheffe (1959) discusses the problem, but bases all of his discussion on Box's 1954b paper. Box (1954b) considers a two-way layout with one observation per cell. In this case, it is generally assumed that no interactions exist. For the purposes of this discussion, call the two variables A and B. Box varies the number of levels for both A and B, then sets unequal variances among the levels of A. The levels of B have equal variances. Table 3 below shows how he set his factors, levels, and variances.

Table 3: Box's 1954b Levels and Settings

B

		L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	
	L_1									σ^2_{1A}
A	L_2									σ^2_{2A}
	L_3									σ^2_{3A}

$$\sigma^2_{1B} = \sigma^2_{2B} = \sigma^2_{3B} = \sigma^2_{4B} = \sigma^2_{5B} = \sigma^2_{6B} = \sigma^2_{7B} = \sigma^2_{8B}$$

where $\sigma^2_{1A} : \sigma^2_{2A} : \sigma^2_{3A}$ was in the ratio of 1:2:3. He found the probability of Type I error for equality of row means and column means. His hypothesis for factor A was H_o: No row effect, or equivalently H_o: $\mu_{A1} = \mu_{A2} = \mu_{A3}$. He failed to reject his null hypothesis when sample sizes were equal.

Working with Unequal Variances

Schmidt and Launsby(1994) also accepted the non-homogeneity of variance in designed experiments. In an effort to minimize the variance of a process, they studied five measures of dispersion (variance). The measures of dispersion were supposed to identify which factor(s) was causing high variance in the process. Once this information was known, the factors could be set at levels which minimized their variance. Schmidt and Launsby performed a Monte Carlo simulation to determine the dispersion effect detection capabilities of the five techniques. Their results showed that the standard deviation of responses for different factor combinations must change by at least a factor of 3 to be detected by the five techniques. Even with a factor of 3, only two of the five techniques tested proved to detect the non-homogeneity of variance consistently.

Summary

It is a recognized fact that the assumption of homogeneity of variance necessary to perform a traditional ANOVA is often false. New methods to perform an ANOVA have been developed. These methods primarily involve changing the F statistic in some way Attempts have been made to determine the effect of unequal variance on the results of the ANOVA under certain conditions. Box's 1954b work considers a two-way classification, but he uses one observation per cell which assumes no interaction exists. Box also only varies the variance for one-factor, letting the levels of the other factor have equal variances. When an ANOVA is performed on a two factor, two level designed experiment where both factors have unequal variances between levels and/or where an interaction exists, the effect of the unequal variances is unknown.

III. Methodology

This chapter begins by explaining how to analyze a designed experiment. Although the specific format shown is not used in this research, understanding how to analyze a designed experiment is the first step in understanding the rest of the methodology for this research. The explanation below is aimed at analyzing a single experiment by hand, but the same methodology is used in the simulation program. The rest of the chapter addresses each of the subobjectives in turn.

Steps in Analyzing a Designed Experiment

The explanation below assumes a factorial arrangement of treatments, or that each level of a given factor is combined with all levels of every other factor, for the designed experiment and an equal number of replications for each treatment combination. Schmidt and Launsby (1994) outlined the process of analyzing a designed experiment in five steps.

Step 1. Define the hypothesis. When performing an ANOVA, the null hypothesis is that the mean of the data obtained when a factor is set at its upper level is equal to the mean of the data obtained when the same factor is set at its lower level. The alternative is that the two means are unequal. The hypotheses are mathematically stated as follows:

$$H_o:\ \mu_{(+)} = \mu_{(-)}$$

$$H_a:\ \mu_{(+)} \neq \mu_{(-)}$$

Step 2. Select a level for α, the probability of a Type I error.

Step 3. Compute the mean square error (MSE) and the mean square between (MSB). This is done in the following manner. Perform the experiment and calculate the mean and variance as shown in Table 4 below. To find the mean square error, use Equation 1.

Table 4: Symbolic Representation of Experimental Design

Treatment combinations (TC_i)	Replications	Mean ($\bar{y}_i$)	Variance (s^2_i)
TC_1	$y_{i1}, y_{i2}, \ldots, y_{1n}$	$\sum_{j=1}^{n} \frac{y_{1j}}{n}$	$\frac{\sum_{j=1}^{n}(y_{1j} - \bar{y}_1)^2}{n-1}$
TC_2	$y_{21}, y_{22}, \ldots, y_{2n}$	$\sum_{j=1}^{n} \frac{y_{2j}}{n}$	$\frac{\sum_{j=1}^{n}(y_{2j} - \bar{y}_2)^2}{n-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
TC_i	$y_{i1}, y_{i2}, \ldots, y_{mn}$	$\sum_{j=1}^{n} \frac{y_{mj}}{n}$	$\frac{\sum_{j=1}^{n}(y_{mj} - \bar{y}_m)^2}{n-1}$

$$\bar{\bar{y}} = \frac{\sum_{j=1}^{i} \bar{y}_j}{m} \qquad \overline{S^2} = \frac{\sum_{j=1}^{i} s_j^2}{m}$$

Equation 1: Mean Square Error

$$MSE = \frac{SSE}{df_{(E)}} = \frac{\sum_{k=1}^{TC}(n_{TC} - 1)S_{TC}^2}{\sum_{k=1}^{TC}(n_{TC} - 1)} \quad \text{where}$$

MSE = mean square error

SSE = sum of squares error

$df_{(E)}$ = degrees of freedom for error

TC = number of treatment combinations

n_{TC} = number of data values in treatment combination TC

S_r^2 = variance of the data in treatment combination TC

If the replication sizes are equal, as in this example, then the equation for MSE can be simplified to be equal to the average variance, or $\overline{S^2}$ from above. To find the MSB, use the appropriate equation below. Compute the test statistic $F_o = \dfrac{MSB}{MSE}$.

Equation 2: Mean Square Between for a Factor

$$MSB_f = \frac{SSB_f}{df_f} = \frac{\sum_{k=1}^{l_f} n_{fk}(\overline{y}_{fk} - \overline{\overline{y}})^2}{l_f - 1} \quad \text{where}$$

MSB_f = the mean square between for factor f

l_f = the number of levels for factor f

n_{fk} = the number of data values for factor f at level k

$\overline{y}_{fk}$ = the average response for factor f at level k

$\overline{\overline{y}}$ = the grand average of all the replications

Equation 3: Mean Square Between for an AxB Interaction

$$MSB_{(AxB)} = \frac{SSB_{(AxB)}}{df_{(AxB)}} = \frac{SS_{(TOT)} - SSB_{(A)} - SSB_{(B)} - SSE}{df_{(TOT)} - df_{(A)} - df_{(B)} - df_E} \quad \text{where}$$

$MSB_{(AxB)}$ = mean square between for the interaction AxB

$SSB_{(AxB)}$ = sum of squares between for AxB

$SS_{(TOT)}$ = sum of squares total, which is $S^2(N-1)$ where S^2 is the variance of all N response values

$df_{(TOT)}$ = degrees of freedom total which is N-1

$df_{(AxB)}$ = degrees of freedom for AxB, which is found from the expression in the denominator of the $MSB_{(AxB)}$ equation

Step 4. Determine the critical F value, F_c, to compare against the above test statistic. The format for the F_c value is $F_{1-\alpha,dfb,dfe}$ where α is the probability of a Type I error, *dfb* is the degrees of freedom for the MSB and *dfe* is the degrees of freedom for the MSE. The value for α is determined in step 2. The $dfb = l - 1$ where l is the number of levels, and $dfe = \sum_{i=1}^{m}(n-1)$ where m is the total number of treatment combinations and n is the number of replications. The F_c value must be found in an F table.

Step 5. Compare the value of F_o from step 3 to F_c from step 4. If $F_o \leq F_c$, fail to reject H_o. If $F_o \geq F_c$, reject H_o with $(1-\alpha)100\%$ confidence. If H_o is rejected for any factor or interaction, that factor or interaction is significant to the process of interest.

Subobjective One: Setting the Parameters

Subobjective one has three parts: 1) determining which combinations of factors and levels, or groups of factors and levels, should be used in the study, 2) deciding at what levels of mean and standard deviation shift to set those combinations of factors and levels, and 3) defining precisely what data will be collected and how it will be calculated.

Combinations of Factors to Test

Determining on which combinations of factors or groups of factors to perform the study requires two steps: 1) find the entire set of combinations, and 2) remove the redundant combinations.

Finding the entire set of possible combinations can be done by holding one combination of levels and factors constant and changing every other combination of levels and factors, then repeating this for every combination. The combinations are A-, A+, B-, B+, A-B-, A-B+, A+B-, and A+B+, where a + means the high level and a - means the low level. An AB combination means that both A and B are set at some experimental level. The interaction AB cannot be independently set at any level because it is dependent on A and B. The interaction can be controlled only by controlling A and B. An example of how to find every possible combination is in Table 5, below.

Table 5: A Subset of All Possible Combinations

Mean	Standard Deviation
A-	A-
A-	A+
A-	B-
A-	B+
A-	A-B-
A-	A-B+
A-	A+B-
A-	A+B+

Mean	Standard Deviation
A+	A-
A+	A+
A+	B-
A+	B+
A+	A-B-
A+	A-B+
A+	A+B-
A+	A+B+

The process illustrated above would be continued for every combination. This method gives 64 combinations. If a level of a factor is not included in a particular combination, it is assumed that level of that factor will be set at the base level, which is a normal distribution with a mean of zero and a standard deviation of one, $\sim N(0,1)$.

To recognize the redundant combinations, it is important to note that each factor is independent of all other factors and each level is independent of the other level. Because of this independence, setting A+ against B- is not different from setting A+ against B+. This means that only 26 non-redundant combinations exist, as shown in Table 6.

Table 6: Non-redundant Combinations

Mean	SD	Mean	SD
A-	A-	A-B-	A+B+
A-	A+	A+B-	A-B-
A-	A-B-	A+B-	A+B-
A-	A+B-	A+B-	A-B+
A-	A-B+	A+B-	A+B+
A-	A+B+	A-B+	A-B-
A-B-	A-	A-B+	A+B-
A+B-	A-	A-B+	A-B+
A-B+	A-	A-B+	A+B+
A+B+	A-	A+B+	A-B-
A-B-	A-B-	A+B+	A+B-
A-B-	A+B-	A+B+	A-B+
A-B-	A-B+	A+B+	A+B+

Levels of Mean and Standard Deviation Shift

In Box's study, the largest variance ratio between levels of a factor is 1:3, which gives a standard deviation ratio between levels of a factor of 1:1.47. This study goes well beyond that. The decision of how many levels of mean and standard deviation shift to simulate was influenced by how other literature handles the problem and the amount of calculation required (additional levels of mean and standard deviation shift increase the number of calculations exponentially). A mixture of these factors leads to the levels in Table 7.

Table 7: Matrix for Levels of Mean and Standard Deviation Shift

		Standard Deviation Shift			
		1σ	2σ	3σ	4σ
	0σ				
Mean	1σ				
Shift	2σ				
	3σ				
	4σ				

In Table 7, the mean shift of xσ means that the mean shifts x base level process standard deviations. The base level process standard deviation is one in this case. The simulation model fills in the blank matrix.

Definition of Data Collected

The data collected is the proportion of detection of significance for each factor and the interaction. A cell is the proportion of detection of significance, which is equal to the number of times a factor is detected to be significant divided by the number of runs that were made. One run is defined as performing the experiment one time.

Subobjective Two: Writing the Simulation Program

Subobjective two requires several steps: 1) designing the program, 2) writing the simulation code, 3) finding a random number generator, 4) validating and verifying the program code, 5) determining an equation for sample size, and 6) calculating the number of simulation replications.

The Design of the Program

This program simulates running a designed experiment just as would be done in real time. It gathers data, fills in the response matrix, analyzes the data with an ANOVA, and performs an F-test for significance. The differences are that the data is generated, not collected from a real-world setting, and that the experiment is performed thousands of times with different factors and levels at different mean and standard deviation shifts.

The combinations of factors and levels that are changing are put into two arrays, one for the mean and one for the standard deviation. At the beginning of the program, the arrays are read and the appropriate levels of mean and standard deviation shift are assigned to the appropriate levels of factors. If a level of a factor is not specifically assigned, its mean is set to zero and its standard deviation to one. Once both the levels of both the factors are assigned a mean and standard deviation value, two uniform (0,1) random numbers, U_1 and U_2, are generated for each level of each factor (for a total of 8 random numbers). These uniform random numbers, U_1 and U_2, along with the mean, μ, and standard deviation, σ, values, are used to generate a normal random variate, X, for each level of each factor with the equation $X = \mu + \sigma\sqrt{-2\ln U_1}\cos(2\pi U_2)$ [Law and Kelton, 1991].

The settings in the designed experiment are in Table 8. The experiment has

Table 8: Settings in Designed Experiment

A	B	AxB
-	-	+
-	+	-
+	-	-
+	+	+

nine replications, as recommended by Schmidt and Launsby [1994]. The simulation program is run two ways: 1) with no interaction, and 2) with an interaction. With no interaction, the values for the cells in the response matrix in row one were obtained by summing the normal random numbers for the low levels of factors A and B. In row two, the low level of A and the high level of B were added, and so on for all four rows. With the interaction, the values for the cells in the response matrix in row one were obtained by summing the normal random numbers for the low levels of factors A and B and adding the product of the low levels of factors A and B, and so on for all four rows. For example, the first row response is calculated using (A-) and (B-). If $(A-) = 5$ and $(B-) = 10$ for a particular replication of a response, the calculation for the no-interaction term would be $(A-) + (B-) = 5 + 10 = 15$. With an interaction, the first row response is calculated by $(A-) + (B-) + (A-)*(B-) = 5 + 10 + 5*10 = 65$. For the second cell in row one, eight more uniform random numbers are calculated, and the process begins again.

Generating eight random numbers for each cell in the response matrix is somewhat wasteful because for any cell only four of the numbers are needed (since it takes 2 uniform random numbers to get one normal random number for one level of a factor, and only two levels are ever used at once.) However, as discussed later in the section about the random number generator, no shortage of random numbers exists. The improvement in efficiency that would be obtained by only generating the numbers necessary for use is not really needed.

After the program has filled in the response matrix, it analyzes the data as shown in the first section of this chapter. The program performs an ANOVA and uses an F-test to determine which factors are significant, if any. The program keeps a count of the number of times any factor is significant. Then, the program starts the process again with a different setting of mean and standard deviation shift until the settings from the entire mean and standard deviation shift matrix have been executed.

After the matrix is completely filled in, the program repeats the process 500 times. The 500 repetitions are called simulation replications. The reasoning behind choosing 500 is discussed in a later section. After the program has completed the 500 replications, it divides the number of times a factor was significant by 500. This gives a proportion of significance. The program then prints the proportion matrices for factors A and B and interaction AB. Afterward, it returns to the beginning of the program and reads the next combination of factors and levels to shift, and begins the process again. After the program has gone through all 26 combinations of factors and levels to shift, it ends. A copy of the program is in Appendix A.

The Random Number Generator

The program uses Marse's and Roberts's random number generator. The generation routine "accepts an integer seed in the range [1, 2^{31}-2] and produces a new seed using a Prime Modulus Multiplicative Congruential Generator" [Marse and Roberts, 1983]. The new seed is returned as the function parameter. The function converts the seed to a uniform (0,1) single precision random number and returns the random number as the value of the function. The formal definition of the generator is $Z_n = (aZ_{n-1})\text{mod } m$ where the prime modulus used is $2^{31}-1$, Z_0 is the starting seed, a is the multiplier 630360016, and the uniform (0,1) value returned is Z_n/m.

All random number generators have a period. A period is the number of random numbers a generator can generate before it repeats itself. When the generator starts repeating the same numbers, the numbers are no longer random and could produce correlated data. The longest period of this random number generator is equal to $m = 2^{31} - 1$. It generates 2,147,483,647 random numbers before beginning to repeat itself. This simulation uses 93,600,000 random numbers. There are over 2 billion random numbers left before the generator starts repeating itself. Therefore, the period length will produce numbers that are random and will not produce correlated data.

The number of random numbers used in this simulation is calculated by the following: 8 random numbers are generated for each cell of the designed experiment matrix, the designed experiment matrix has 4 rows and 9 columns, a designed experiment is performed for each of the 5 levels of mean shift and 5 levels of standard deviation shift, 500 simulation replications are performed for each setting of mean and standard deviation shift, and there are 26 settings of mean and standard deviation shift. The product of these numbers is 8 * 4 * 9 * 5 * 5 * 500 * 26 = 93,600,000.

Validation and Verification

Pritsker [1995] defines validation as "the process of establishing that a desired accuracy or correspondence exists between the simulation model and the real system." Verification is "the process of establishing that the computer program executes as intended" [Pritsker, 1995]. Two areas need to be considered separately in this application: the main program and the random number generator.

Main Program

Since this simulation is not attempting to model a particular real-world system, validation is not a significant issue. The typical methods of validating (e.g., comparing simulation output to real-system output with the same set of input parameters) are not applicable here.

Verifying that the program works correctly is a significant issue. Verification was performed by a piecemeal process in which the program printed out the numbers it had calculated at any given point and the same numbers were calculated by hand. When the numbers matched, the program was working correctly.

The verification process started by modifying the program to print the values of mean and standard deviation for the levels of both the factors to make sure that the arrays of combinations had been entered properly and that the assignment process was working. Then, the program was set to print the normal random numbers it generated and the data it calculated for the designed experiment response matrix. The same data was then calculated by hand from the random numbers. When these matched, the program was modified to print the mean and variance of each row. When the hand calculations for the mean and variance matched, the program was modified to print the MSE, MSB, and F-value for each factor. The verification of these numbers by hand calculations meant that the main body of the program was working correctly for the first set of combinations at the first setting of mean and standard deviation shift.

To determine if the program was still working properly for later combinations of factors and later settings of mean and standard deviation shift, the above process was repeated for the tenth combination of factors and levels and other levels of mean and standard deviation shift. When this was first done, the program's numbers were incorrect due to the placement of an initialization loop. After the initialization was correctly placed, the program produced the correct data.

To ensure that the counting mechanism for the number of significant factors was working properly, the program was modified to print the F-values for each factor, and then set to run for 10 replications. The number of significant As, Bs, and ABs was counted by hand to make sure it matched the computer's count. The last step was to make sure the program was dividing the count of the significant factors by the number of simulation replications. To do this, the data from checking the counting mechanism was divided by ten and compared to the final results of the program. This matched, so the program was working correctly.

Random Number Generator

Two areas need to be considered for the random number generator: 1) technical accuracy, or the ability to actually generate random numbers (validation), and 2) correct implementation (verification).

Fishman and Moore [1982] tested several multiplicative congruential random number generators with modulus 2^{31}-1. In all tests, they failed to reject the hypotheses for the multiplier used in this random number generator, which means that this random number generator was not deficient in any of the areas tested. For more details on the tests performed and the methods of performing them, see Fishman and Moore [1982]. This multiplier's most common use is in the SIMSCRIPT II simulation programming language.

To verify that the generation routine had been implemented in this program correctly, a list was obtained of the first 10 numbers the routine should generate, given a certain seed. When the first ten numbers generated by this program matched the list of numbers the routine should generate, the routine was verified.

An Equation for Sample Size

The number of simulation replications necessary for an accurate result is dependent on the maximum error of the estimate one is willing to accept. The maximum error of the estimate, E, is the difference between the sample mean and the population mean. Assuming that the data comes from a normal population, the equation for E can be obtained from the mathematical expression of the value of a random variable having an approximately standard normal distribution. The expression is $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ where $\bar{x}$ is a sample mean, μ is the population mean, σ is the standard deviation of the population, and n is the sample size. So, with probability 1-α where α is the probability of a Type I error, it can be said that $-z_{\alpha/2} \leq \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \leq z_{\alpha/2}$, or that $\frac{|\bar{x}-\mu|}{\sigma/\sqrt{n}} \leq z_{\alpha/2}$ where $z_{\alpha/2}$ is a value such that the normal curve area to its right equals $\alpha/2$. Then, if E is set to the maximum value of $|\bar{x}-\mu|$ the equation can be rearranged to get $E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ with probability 1-α. This equation can be used to determine the maximum error of an estimate at any value of n, assuming the data come from a normal population.

The data that go into the results matrix for this research are not normally distributed. The number in each cell of the matrix is a sum of ones and zeroes divided by the total number of runs. The values of one and zero are arbitrary -- they come from trials where two results are possible: not significant and significant. A zero is assigned to a 'not significant' result and a one is assigned to a 'significant' result. These trials are called Bernoulli trials. Data from Bernoulli trials take on a binomial distribution. A binomial distribution has the parameters n and p, where n is the number of trials and p is the probability of success. The mean of a binomial distribution is $\mu = n*p$. The variance is $\sigma^2 = n*p*(1-p)$. The quantity (1-p) is often assigned to the variable q, which gives $\sigma^2 = n*p*q$.

The actual value in each cell of the matrix is a sample proportion, $\hat{p}$, equal to X/n, where X is the total number of significant results and n is the total number of runs. The expected value of a proportion is equal to p. This can be seen by the following: $E(\hat{p}) = E(X/n) = (1/n)(np) = p$. The expected variance of a proportion is equal to (pq)/n. The equation for E from above cannot be used until it is adjusted to reflect a binomial distribution of proportions.

The normal approximation to the binomial distribution states the following: If x is a value of a random variable having the binomial distribution with parameters n and p, and if $z = \frac{x - np}{\sqrt{npq}}$, then the probability density corresponding to z is the standard normal density. Using the same substitution and rearranging as above, the equation for E of a binomial distribution is $E = z_{\alpha/2}\sqrt{npq}$. Substituting for the variance of a proportion, Equation 4 is derived.

Equation 4: Sample Size

$$E = \frac{z_{\alpha/2}\sqrt{pq}}{\sqrt{n}}$$

Using this equation, the values of E are found for several values of n and an appropriate number of simulation runs is chosen.

Calculation of the Sample Size

$Cell_{11}$, or the cell in the first row and first column of the results matrix, is based on all factors having a N(0,1) distribution. Since the alpha value for this experiment is equal to 0.05, $cell_{11}$ should always have a value of around 0.05. Since the cell is the proportion of successes, the value of p will also be 0.05, which leaves q = 0.95. Since $\alpha = 0.05$ the value of $z_{\alpha/2}$ is equal to $z_{0.025} = 1.96$. Substituting these values into the equation for E gives $E = \frac{1.96\sqrt{0.05(0.95)}}{\sqrt{n}} = \frac{0.42717}{\sqrt{n}}$. Table 9 shows the values of E for some values of n.

Table 9: Maximum Error for Some Values of n

n	E
100	0.042717
200	0.030205
500	0.019104
1000	0.013508

Five hundred simulation replications are executed in this research. The errors for n equal to 100 and 200 are too high for this application. The decrease in error for n equal to 1000 is not large enough to justify the time it would take to double the number of replications. An error of 0.019 with only 500 replications seems to be a good balance.

Subobjective Three: Analysis

There are two parts to the analysis: 1) calculating and graphing the OC curves, and 2) determining if the shifting variance affects the OC curves.

Operating Characteristic Curves

The data collected by the simulation program can be used to make OC curves. The simulation's final output is the proportion of significance for each factor and the interaction. A graph of OC curves for this data for a particular factor has a vertical axis of the probability of accepting the null hypothesis given that it is false. This is also called the β value, or the probability of a Type II error, as discussed in Chapter 1. The null hypothesis states that the means of a factor at its upper and lower levels are equal. The horizontal axis shows the actual values of the mean. An OC curve is drawn for every value of the standard deviation shift. To determine if the changing standard deviation affects the OC curve, a test must be performed to determine if the curves are significantly different. An example of what an OC curve might look like for factor A is in Figure 2, below. These curves are for A- shifting both means and standard deviations.

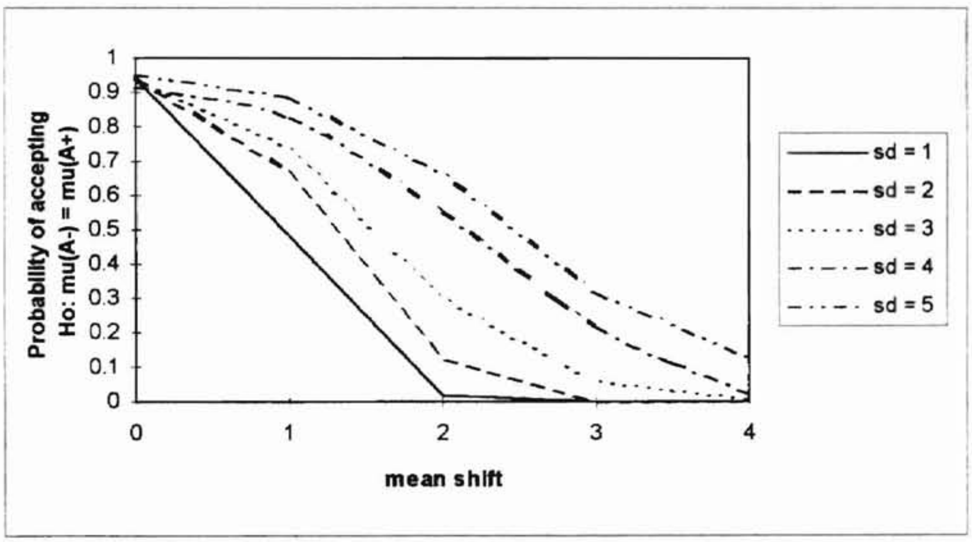

Figure 2: Example of OC Curve for Factor A

The OC curve is read as follows:

When the sd shift of A- equals one: if the mean shift of A- equals zero, the probability of accepting the null hypothesis is about 0.95; if the mean shift of A- equals one, the probability of accepting the null hypothesis is approximately 0.50; if the mean shift of A- is equal to two, three, or four, the probability of accepting the null hypothesis is approximately zero.

When the sd shift of A- equals two: if the mean shift of A- equals zero, the probability of accepting the null hypothesis is about 0.95; if the mean shift of A- equals one, the probability of accepting the null hypothesis is about .0.67; if the mean shift of A- equals two, the probability of accepting the null hypothesis is about 0.11; if the mean shift of A- is equal to three or four, the probability of accepting the null hypothesis is approximately equal to zero.

The curves for the other values of the standard deviation shift can be read in a similar manner.

A Test for Significant Difference of Curves

A median test, with one adjustment, is used to test for a significant difference between curves. The median test compares the matrices row by row with an r×c contingency table. A median test is designed to examine whether several samples came from populations having the same median. Rather than separating this data by the median, it is separated according to the proportion of detection of significance. All of the assumptions, the test statistic, and the decision rule for the median test will still hold. The table for each row has the format shown in Table 10.

Table 10: Sample Table for Chi-Square Test

	SD = 1	SD = 2	SD = 3	SD = 4	SD = 5	Totals
Amount Detected	O_{11}	O_{12}	O_{13}	O_{14}	O_{15}	a
Amount not Detected	O_{21}	O_{22}	O_{23}	O_{24}	O_{25}	b
Totals	n	n	n	n	n	N

The values of O_{11} through O_{15} are calculated by multiplying the proportion of detection of significance in the results matrix by the sample size $n = 500$. The values of

O_{21} through O_{25} are calculated by subtracting the value in the corresponding cell in the top row from the sample size. The value of n will be the same in every column. The value of a is the sum of O_{11} through O_{15}. The value of b is the sum of O_{21} through O_{25}. The value N is the sum of a and b. The hypotheses are as follows:

> H_o: All five populations have the same number of significant detections
>
> H_a: At least two of the populations have different numbers of significant detections

The test statistic is in Equation 5. If T is greater than the $(1-\alpha)$ quantile of a chi-square

Equation 5: Test Statistic for Median Test

$$T = \frac{N^2}{ab}\sum_{i=1}^{5}\frac{O_{1i}^{\ 2}}{n} - \frac{Na}{b}$$

random variable with 4 degrees of freedom, H_o is rejected. Otherwise, the analyst fails to reject H_o (Conover, 1980). In this application, α is set to 0.05, which gives a chi-square value of 9.488. Table 11, below, shows an example. The data is from the no interaction simulation for Factor A with the mean and standard deviation (sd) shifts shown.

Table 11: Hypothesis Test Example

Means: A-
SDs: A-

	Factor A -- Raw Data				
	sd = 1	sd = 2	sd = 3	sd = 4	sd = 5
mean = 0	0.0520	0.0480	0.0480	0.0660	0.0400
mean = 1	0.2680	0.1740	0.1320	0.0960	0.0680
mean = 2	0.6000	0.4720	0.3160	0.2020	0.1600
mean = 3	0.7780	0.6640	0.5520	0.3640	0.3260
mean = 4	0.8720	0.7900	0.7120	0.6460	0.4960

Hypothesis Test	
Calc. Test Value	Result
3.782	Accept
97.108	Reject
300.096	Reject
299.391	Reject
197.185	Reject

This method can not compare entire curves, but it can compare the points that make up the curves, which is a very close approximation. If the shifting variances significantly affect the operating characteristic curves, they should also significantly affect the points that comprise the operating characteristic curves.

IV. Analysis

The analysis contains three major sections: 1) the analysis and interpretation of the results of the simulation without interactions, 2) the analysis and interpretation of the results of the simulation with interactions, and 3) a discussion of the differences between the no-interaction and interaction simulation results.

No-Interaction Results

As explained in Chapter 3, the no-interaction results are obtained from the simulation when no interaction term is included in the generation of data. The result of not including an interaction term in the simulation program is that no forced interaction is present in the data generation process. However, the interaction term can still be significant, which is illustrated by the fact that data obtained from the simulation for the interaction term are significant. Therefore, the interaction term AB is included in the analysis and tested just like the factors A and B. The raw data from the simulation is in Appendix B.

OC Curves

As discussed in Chapter 3, the vertical axis of an OC curve is the probability of accepting the null hypothesis given that it is false. This is also called the β value, or the Type II error. The null hypothesis states that the mean of the data obtained when a factor is set at its upper level is equal to the mean of the data obtained when the same factor is set at its lower level. The horizontal axis is the actual value of the mean. An OC curve exists for each factor and interaction at each value of the standard deviation shift. The horizontal axis in this case is the value of the mean shift.

Since 26 different combinations of factor settings were simulated, 26 sets of fifteen OC curves exist. The fifteen OC curves come from 5 levels of standard deviation shift for A, B, and AB. To simplify the data interpretation and provide similar scales among the OC curves for A, B, and AB, all fifteen curves have been plotted on one graph for each combination of factor settings. An example of one of these graphs is in Figure 3.

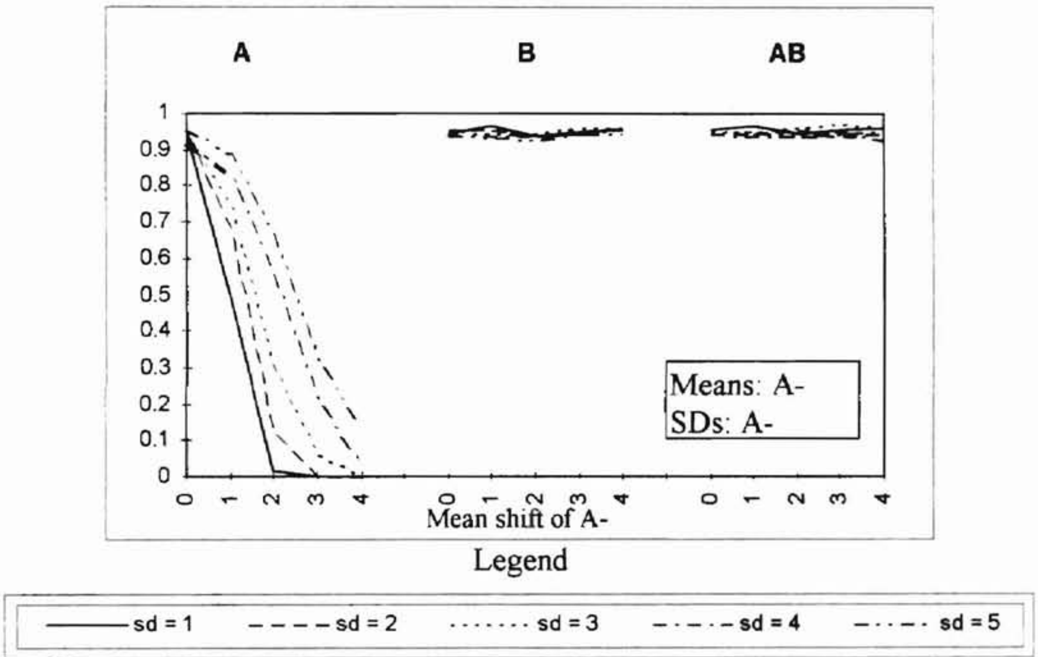

Figure 3: OC Curves for No-Interaction Data where A is Shifting Means

The A, B, and AB at the top of the graph indicate which set of curves goes with which factor. The numbers on the horizontal axis repeat from 0 to 4 because the values of the curve are known at each level of mean shift (0 through 4) for each factor. The values are repeated to make the graph easier to read. The box in the lower right-hand corner notes that the low level of factor A is shifting means and standard deviations.

Although each set of OC curves for a particular combination is slightly different from all the other sets of OC curves, two primary patterns appear. The set of curves in Figure 3 above is representative of all the sets of OC curves where only factor A shifted means regardless of which factors and levels shifted standard deviations. The other pattern is in Figure 4, below. This pattern is representative of all the sets of OC curves where factors A and B, at any combinations of levels, shifted means regardless of which factors and levels shifted standard deviations. The complete set of 26 graphs is in Appendix C.

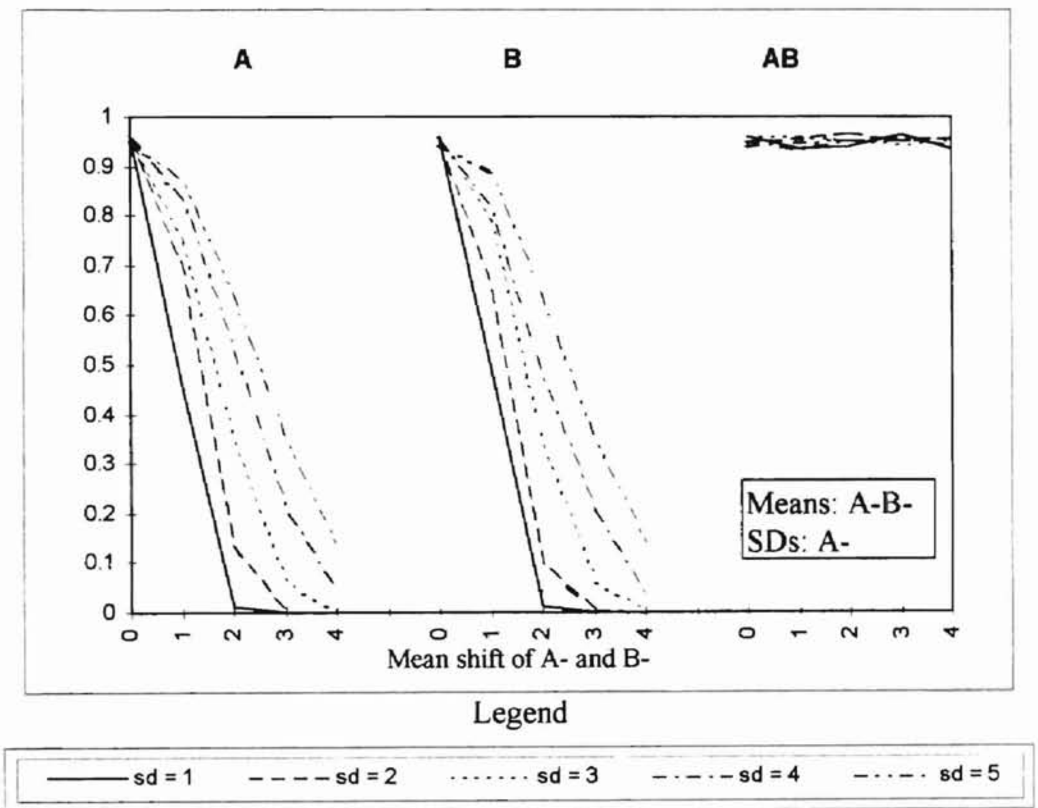

Figure 4: OC Curves for No-Interaction Data where A and B are Shifting Means

Some observations can be made about the OC curves. These observations are obtained by noting the repeated patterns and drawing conclusions from them. While repeated patterns can make a strong statement, they are not necessarily statistically convincing. In the next section, each of these observations will be addressed and tested statistically. The observations are as follows:

1. Shifting the standard deviation of any combination of factors does not meaningfully affect the OC curve for a particular factor when the mean of that factor does not shift.
2. When the mean of a particular factor is shifted, shifting the standard deviation value does meaningfully affect its OC curve.
3. Neither shifting means nor shifting standard deviations in any combination affects the OC curve for the interaction. The vertical axis value (β) is not meaningfully different for any value of the mean or standard deviation. It remains constant at approximately 0.95 ($\alpha = 0.05$).

Statistical Tests

The method of testing for significant differences of curves is presented in Chapter 3. The test has to be performed for each factor and the interaction at each value of the mean. The null hypothesis states that the proportion of significance at all levels of standard deviation are equal at a particular mean (i.e., the values within a particular row of the results matrix are equal). If the null hypothesis is not rejected, it can be concluded that shifting the standard deviation, for a particular mean, does not have a statistically significant effect on the proportion of significance. This conclusion implies that the heterogeneity of variance of the factor for which the conclusion was drawn has no significant effect on the OC curve for the mean of any factor or interaction, as long as the mean of the factor that was tested does not shift.

The results of the hypothesis tests for the two sets of OC curves shown above are in Table 12. The factors and levels shifting means and standard deviations are at the top of each table. An "Accept" in a cell means that the null hypothesis was not rejected for that factor at that level of mean shift. A "Reject" means that the null hypothesis was rejected for that factor at that level of mean shift.

Table 12: Results of Hypothesis Tests for OC Curves for No-Interaction Data

Means: A-
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

Means: A-B-
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

Just as the sets of OC curves had two primary patterns, so do the hypothesis tests. The results of the hypothesis tests for the six combinations of factor settings where A shifted means and B did not are identical to the results above where A- is shifting means, with one exception. That exception, for a mean shift of A- and a standard deviation shift of A+B+, is that the AB term rejected the null hypothesis at mean shift = 0. The calculated test value was 10.3172 against a table value of 9.488. Because this rejection only happened one time and the calculated test value was relatively close to the table value, it is safe to assume that the rejection was due to random chance rather than the shifting standard deviation value of A+B+.

The results of the hypothesis test for the twenty combinations of factor settings where both A and B shifted means are all very similar to the results in Table 12 where A-and B- are shifting means. Among all twenty tables, only eleven out of the 300 results are different. Since no pattern appears to exist and the calculated test values are all very close to the table value, it is again assumed that the aberrations are due to random chance rather than a significant effect of shifting means or standard deviations. The test results for all 26 sets of data are in Appendix D. The calculated test values are in Appendix B with the raw data. The results of the hypothesis tests can be used to statistically validate the observations about the OC curves in the previous section.

Validation of Observations

The first observation from above is as follows: Shifting the standard deviation of any combination of factors does not meaningfully affect the OC curve for a particular factor when the mean of that factor does not shift. The hypothesis tests conclusively prove this observation to be true. In every combination of settings where only factor A shifts means, the null hypothesis is never rejected for factor B. This means that no matter how the standard deviation shifts for factor B, the points that make up the OC curve are not significantly different at any level of factor A's mean shift.

The second observation is: When the mean of a particular factor is shifted, the shifting standard deviation value meaningfully affects its OC curve. This observation is seen to be true from the hypothesis test results. In every instance when a factor shifts means, the null hypothesis for that factor is rejected for mean shifts equal to 1, 2, 3, or 4 (see Table 12). This pattern indicates that the changing standard deviation is significantly affecting the value of the proportion of significance at these levels of mean shift. Because the values for the OC curves were calculated by subtracting the proportion of significance from one, anything that significantly affects the proportions of significance significantly affects the OC curve. Once this is accepted, it can be observed that the probability of accepting H_0: $\mu_{(+)} = \mu_{(-)}$ for a particular factor whose mean is shifting increases as the standard deviation shift increases. This statement restates observation two, but makes the conclusion directional.

The third observation: Neither shifting means nor shifting standard deviations in any combination affects the OC curve for the interaction. The vertical axis value (β) is not significantly different for any value of the mean or standard deviation. It remains constant at approximately 0.95 ($\alpha = 0.05$). Neither shifting means nor shifting standard deviations in any combination affects the OC curve for the interaction because the null hypothesis is never rejected for the interaction, except for some random occurrences. The numerical value can be seen from the raw data in Appendix B.

With-Interaction Results

The "with-interaction" results are the data obtained from the simulation where an interaction term is included in the generation of the data. The interaction term is the product of the appropriate levels of factors A and B. This forced interaction increases the probability of the interaction term being significant in the analysis. It also increases the probability that the shifting standard deviation will affect the OC curve for the interaction. The raw data from the "with-interaction" simulation is in Appendix E.

OC Curves

The axes of the OC curves are the same as they are in the no-interaction case. As in the no-interaction case, two primary patterns appear in the OC curves for the with-interaction data. The first pattern is similar to the first pattern for the no-interaction results. The set of curves in Figure 5, below, is representative of all the sets of OC curves where only factor A is shifting means regardless of which factors and levels are shifting standard deviations. The shape of the OC curves for A varies more in this set of data than with the no-interaction data set, but A is still the only factor to have an OC curve with values that deviate significantly from 0.95.

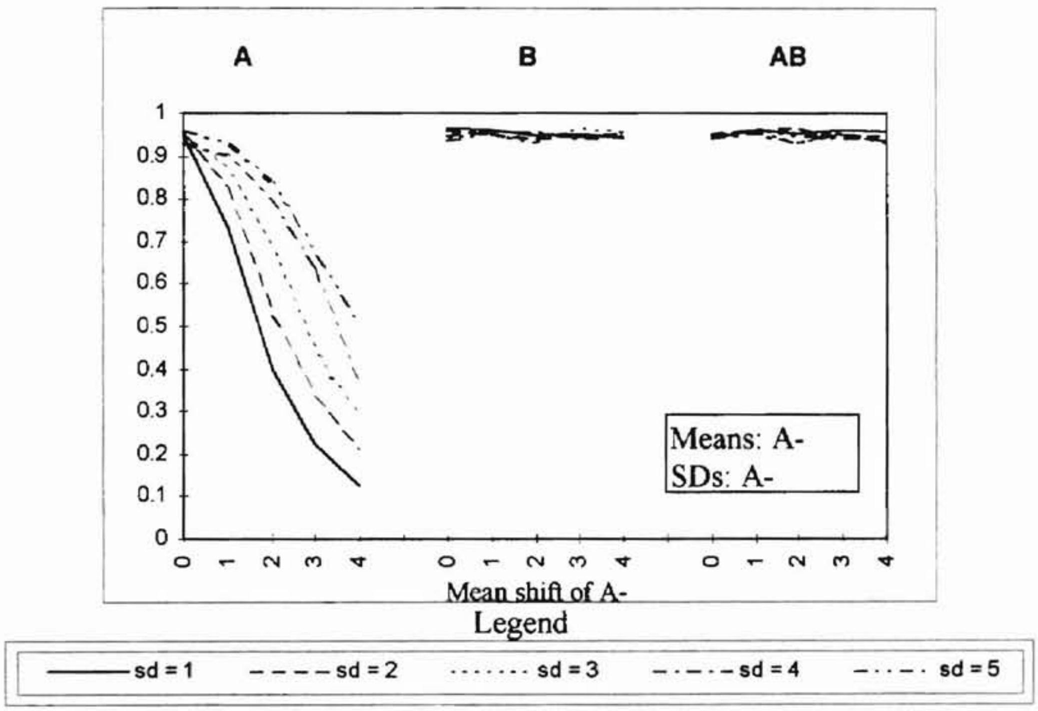

Figure 5: OC Curves for With-Interaction Data where A is Shifting Means

The second pattern of OC curves for the with-interaction data is in Figure 6. When compared to the second pattern for the no-interaction data, it is obvious that the OC curve for the interaction term AB has changed. It now has values for its OC curves that are significantly different from 0.95. Forcing an interaction into the data generation by including the product of factors A and B has affected the OC curve significantly. The complete set of OC curves for the with-interaction data is in Appendix F.

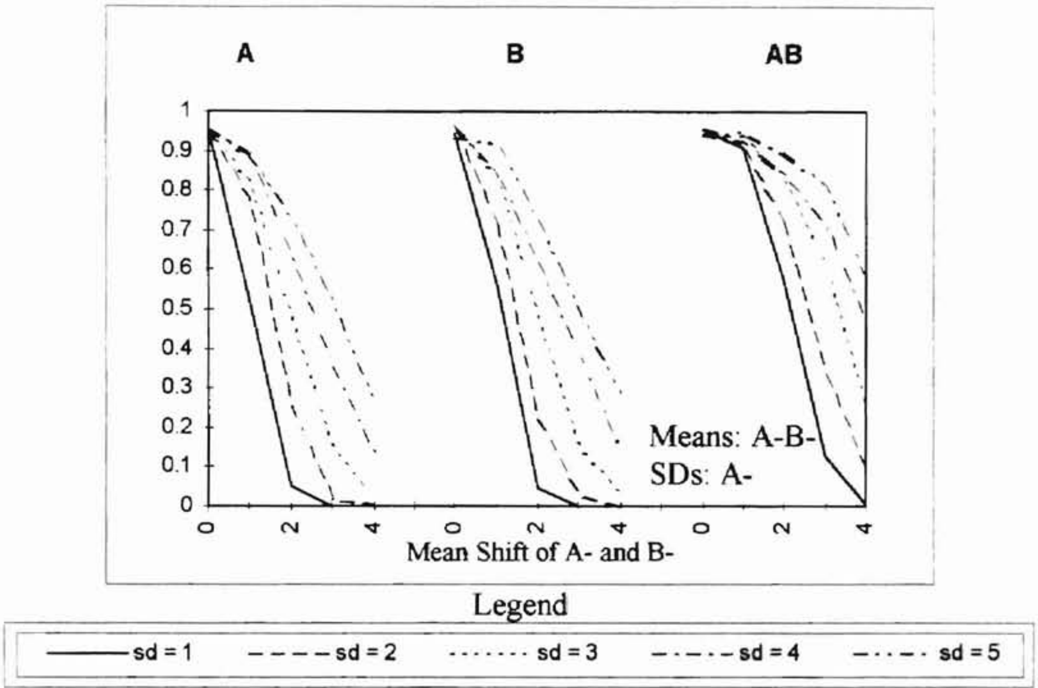

Figure 6: OC Curves for With-Interaction Data where A and B are Shifting Means

Again, observations can be made from the OC curves, but the same caveat still applies that observations are not as strong as statistical conclusions. As before, the statistical tests will be presented in the next section. The observations are as follows, with the first two being identical to the first two from the no-interaction section:

1. Shifting the standard deviation of any combination of factors does not meaningfully affect the OC curve for a particular factor when the mean of that factor does not shift.
2. When the mean of a particular factor is shifted, the shifting standard deviation value does meaningfully affect its OC curve.

3. To effect a change on the OC curve for an interaction, all the terms of that interaction must shift means.

4. The magnitude of the slope of the OC curve for the interaction is less likely to increase meaningfully between the mean shifts of 0 and 1 than the magnitudes of the slopes of the OC curves for the factors that make up the interaction.

Statistical Tests

The hypothesis tests for the with-interaction data were performed exactly the same way as for the no-interaction data. The results of the hypothesis tests for the OC curves above are in Table 13.

Table 13: Results of Hypothesis Tests for OC Curves for With-Interaction Data

Means: A-
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

Means: A-B-
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

Two patterns exist in the hypothesis test results that match the patterns in the OC curves. The pattern for only A shifting means is the same as in the no-interaction results analysis. A few of the results do not match, but for the reasons explained in the previous section, these can be attributed to random chance.

The pattern for both A and B shifting means is different from the no-interaction results analysis. The forced interaction in the data generation causes the interaction term to reject the null hypothesis for the last three mean shifts. An interesting result from this test is in $cell_{23}$, the cell for a mean shift of 1 under the interaction term. In the tests results shown above, the result was to accept the null hypothesis. However, of the twenty sets of combinations where both A and B shifted means, eight of the tests resulted in a conclusion to reject the null hypothesis. The number of occurrences suggest that this result is meaningful, but no obvious pattern exists that suggests what the meaning may be. The results of all the hypothesis tests are in Appendix G. The calculated test values for the hypothesis tests are with the raw data in Appendix E.

Validation of Observations

The first two observations are identical to the first two observation from the No-Interaction Results section and can be shown to be true in this case with the same reasoning.

The third observation is: To effect a change on the OC curve for an interaction, all the terms of that interaction must shift means. This statement is intuitive from looking at the graphs but can also be shown by the hypothesis tests. The only time the null hypothesis is rejected for the interaction is when a level of both factors is shifting means.

The fourth observation: The magnitude of the slope of the OC curve for the interaction is less likely to increase meaningfully between the mean shifts of 0 and 1 than the magnitudes of the slopes of the OC curves for the factors that make up the interaction. This observation is seen in the hypothesis test results by the frequency (twelve out of twenty times) that the null hypothesis for the interaction term where the mean shift is equal to 1 is accepted. It suggests that the mean shift must be more extreme to cause a significant effect for the interaction than for the main factors. This indicates that the interaction responds more slowly to the data than the main factors do.

Differences

The primary differences between the no-interaction and with-interaction results are the OC curves for the interaction. In the former case, the OC curves do not vary significantly whereas in the latter case they do. To determine precisely where the data between the two cases differed significantly, the results of the hypothesis tests can be compared. The comparisons for the hypothesis test results shown earlier are in Table 14, below. The tables serve as an easy way to compare how the results for the two sets of data differed.

Table 14: Difference in Hypothesis Tests

Means: A-
SDs: A-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	0
mean = 3	0	0	0
mean = 4	0	0	0

Means: A-B-
SDs: A-

A	B	AB
0	0	0
0	0	0
0	0	Reject
0	0	Reject
0	0	Reject

The zero means no difference exists between the two tests. A "Reject" or "Accept" is the result of the test for the with-interaction data. In these cases, the no-interaction data obviously had the opposite result. The complete set of tables that show the differences are in Appendix H.

V. Summary and Conclusions

Summary

The objective of this research was to determine the effect of the heterogeneity of variance of a particular factor or group of factors on the operating characteristic (OC) curve for the mean of any factor or interaction in a designed experiment. To accomplish this, a simulation program was written to simulate a designed experiment. The simulation forced particular factors and groups of factors to shift variances and means, then performed an analysis of variance to determine which, if any, interactions or factors were significant in the experiment. The program simulated two cases: 1) no interaction effect in the system, and 2) with an interaction effect in the system.

The program executed the experiment five hundred times for each case and kept a count of the interactions and factors that were detected to be significant. The final output of the simulation was a proportion of detection of significance for each factor and interaction for 26 combinations of factors and levels shifting means and standard deviations. This output was obtained for each of the two cases above.

The data from the simulation were used to plot operating characteristic curves by subtracting the output value from one and plotting it. Then, hypothesis tests were performed to determine if the points that made up the operating characteristic curves were significantly different from each other. These hypothesis tests were performed by using a modified median test, where the median was substituted with the proportion of detection of significance, in effect making a 2×5 contingency table. In the course of making the OC curves, several observations were made based on the repeated patterns made by the OC curves. These observations were then validated statistically using the results of the hypotheses tests and the patterns they formed. These statistically validated conclusions became the basis for the conclusions of the research.

Conclusions

The primary conclusions answer the question that was the basis of this research: What is the effect of heteroscedasticity on the operating characteristic curve in a designed experiment? The two conclusions that apply to the factors of a designed experiment follow:

1. The data did not provide enough evidence to support the statement that, in a designed experiment, the heterogeneity of variance of a particular factor or group of factors has a significant effect on the OC curve of the mean of any factor or interaction that is not shifting means. That is, unless a given factor or interaction were shifting means, any factor or combination of factors, including the given factor, could shift standard deviations without significantly affecting the OC curve of the mean for the given factor.
2. In the situation when the mean of a given factor does shift, shifting the standard deviation value does significantly affect the OC curve. The more the standard deviation shifts, the more probable is the acceptance of the null hypothesis.

The two conclusions that consider the effect of heteroscedasticity on the OC curve for the interaction term in a designed experiment follow:

3. To cause the OC curve of a two-way interaction to be significantly different at various levels of standard deviation shift, both of the factors that make up the interaction must shift means.
4. The OC curve for an interaction responds to a mean shift more slowly than the factors that make up the interaction.

Directions for Future Research

This study examined a 2^2 full factorial experiment. Two level experiments with more than two factors could be addressed, as could two and three level fractional factorials. It would be interesting to see how the OC curve of a three-way interaction, when the simulation program was coded to force a three-way interaction into the experiment's response, would respond if exactly two of the factors in the interaction shifted means.

Other designs for experiments are excellent areas for future research: central composite designs, Latin squares, and split-plot designs are just a few. This research centers around a completely randomized design. Any design with blocking variables could consider different combinations of unequal variances with the blocking variable and the treatment variable. Mixed models of any design type would add another level to the study.

Research could also be done to determine the effect of heteroscedasticity on an analysis of variance for any design-type with missing data. Different estimation procedures for the missing data affect the results of the ANOVA; adding unequal variances could affect the ANOVA even more.

The existing literature shows that heteroscedasticity with unequal sample sizes in a one-way analysis affects the F-test significantly. Further research could be performed to find the effect of unequal sample sizes in a two-way or higher classification with any of the various experimental designs discussed above.

Finally, this research concentrated on the effect of violating only one of the three major assumptions in an ANOVA. Along with unequal variance, the assumptions of normality and independence are also required to perform an ANOVA. In the literature search for this research, no study was found that concentrated on any of the assumptions in a classification higher than one-way. The effects of violating the normality and independence assumptions could be investigated in a manner similar to this research, or the problem could be paired with any of the variations listed above for another direction of study.

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Appendix A: Simulation Program

```
      PROGRAM THESIS
      CHARACTER*2 MEAN(26,2),SD(26,2)
*
* The MEAN array has the factors and levels that will shift means.  The
* SD array has the factors and levels that will shift sds.  The four R
* variables hold uniform random numbers from the function UNIRAN.
*
      REAL MEVAR(4,2),RSPNSE(4,9),SHIFT(6,6,3),R1,R2,R3,R4,UNIRAN
*
* The MEVAR array will hold the mean and variance of the rows in the
* DOE response matrix.  RSPNSE will hold the raw data of the response
* matrix.  SHIFT holds the levels of mean and sd shift and will also
* hold the type II error for A, B, and AB.
*
      REAL AP,AM,BP,BM,APM,APSD,AMM,AMSD,BPM,BPSD,BMM,BMSD
*
* AP, AM, BP, and BM stand for A+, A-, B+, and B-, respectively.
* An M afterward stands for Mean and an SD afterward stand for the
* standard deviaiton.
*
      REAL FA,FB,FAB,FC,PI,R5,R6,R7,R8
*
* FA is the F test value for factor A and so on for B and AB.      FC is
* the critical F value with alpha=.05, v1=1, and v2=32.  The value is 4.152.
*
      REAL Q,GRNDAV,SBARSQ,MSE,MSBA,MSBB,MSBAB
*
* I, R, and C, and are counters for do loops. R stands for row and C stands
* for column.  Q is the value of a single response.  GRNDAV is first the
* sum of the averages of the DOE response matrix and then the Grand Average.
* SBARSQ is the sum of the squared variances.     MSE is the mean square error.
* MSBA is the mean square between for factor A; likewise for B and AB.
* Progseed holds the first seed and is used to send the seed to
* Marse and Roberts's random number function UNIRAN.
*
      INTEGER      X,Y,SIMREP,I,R,C,L,PROGSEED,A,B,D,E
*
* These integers are all DO loop counters.  SIMREP stands for simulation
* replications.
*
*
* These data statements initialize the mean and sd arrays.  Column 1
* is listed first, followed by column 2.
*
```

```
      DATA MEAN/'A-','A-','A-','A-','A-','A-','A-','A+',
     +          'A-','A+','A-','A-','A-','A-','A+','A+',
     +          'A+','A+','A-','A-','A-','A-','A+','A+',
     +          'A+','A+',' ',' ',' ',' ',' ',' ','B-','B-',
     +          'B+','B+','B-','B-','B-','B-','B-','B-','B-',
     +          'B-','B+','B+','B+','B+','B+','B+','B+','B+'/
      DATA SD/'A-','A+','A-','A+','A-','A+','A-','A-','A-','A-',
     +       'A-','A+','A-','A+','A-','A+','A-','A+',
     +       'A-','A+','A-','A+','A-','A+','A-','A+',
     +       ' ',' ','B-','B-','B+','B+',' ',' ',' ',' ',
     +       'B-','B-','B+','B+','B-','B-','B+','B+',
     +       'B-','B-','B+','B+','B-','B-','B+','B+'/
      PROGSEED = 1973272912
*
* This do loop initializes the SHIFT array with the correct numbers of
* mean and sd shift that each factor and level from the mean and sd arrays
* will go through.
*
      DO 10 X = 0 , 4
            SHIFT(X+2,1,1)=X
            SHIFT(1,X+2,1)=X + 1
   10 CONTINUE
      PRINT*,'THIS VERSION HAS CROSS-PRODUCTS FOR THE INTERACTION.'
*
* The I loop is to make the program execute for every value in the MEAN
* and SD arrays. Many of the variables are initialized to zero after
* this loop to reset them after each set of factors and levels shifts.
*
      DO 220 I = 1,26
      SBARSQ = 0
* These loops initialize the rest of the SHIFT array to zero.
      DO 13 Z = 1 , 3
          DO 12 X = 2 , 6
            DO 11 Y = 2 , 6
              SHIFT(X,Y,Z) = 0
   11       CONTINUE
   12     CONTINUE
   13 CONTINUE
* These loops initialize the RESPNSE array to zero.
      DO 15 X = 1 , 4
          DO 14 Y = 1 , 9
            RSPNSE(X,Y) = 0
   14     CONTINUE
   15 CONTINUE
      PI = 3.141593
```

```
*
* The SIMREP loop is for the simulation replications.        The X loop makes the
* program go through every row (mean shift) of the array SHIFT.  The Y loop
* makes the program go through every column (sd shift) of the array SHIFT.
*
      DO 100 SIMREP = 1 , 500
       DO 90 X = 1 , 5
        DO 80 Y = 1 , 5
* These loops initialize the MEVAR array to zero.
   DO 18 R = 1 , 4
      DO 16 C = 1 , 2
       MEVAR(R,C) = 0
  16    CONTINUE
  18 CONTINUE
   SBARSQ = 0
*
* Assigns the appropriate mean shift to the appropriate factor
* and level.  The default is mean = 0.
*
         IF (MEAN(I,1).EQ.'A+') THEN
             APM = SHIFT(X+1,1,1)
          ELSE
             APM = 0.0
         ENDIF
         IF (MEAN(I,1).EQ 'A-') THEN
             AMM = SHIFT(X+1,1,1)
          ELSE
             AMM = 0.0
         ENDIF
         IF (MEAN(I,2).EQ.'B+') THEN
             BPM = SHIFT(X+1,1,1)
          ELSE
             BPM = 0.0
         ENDIF
         IF (MEAN(I,2).EQ.'B-') THEN
             BMM = SHIFT(X+1,1,1)
          ELSE
             BMM = 0.0
         ENDIF
*
* Assigns the appropriate sd shift to the appropriate factor
* and level.  The default is sd = 1.
*
         IF (SD(I,1).EQ.'A+') THEN
             APSD = SHIFT(1,Y+1,1)
```

```
           ELSE
              APSD = 1.0
          ENDIF
          IF (SD(I,1).EQ.'A-') THEN
              AMSD = SHIFT(1,Y+1,1)
           ELSE
              AMSD = 1.0
          ENDIF
          IF (SD(I,2).EQ.'B+') THEN
              BPSD = SHIFT(1,Y+1,1)
           ELSE
              BPSD = 1.0
          ENDIF
          IF (SD(I,2).EQ.'B-') THEN
              BMSD = SHIFT(1,Y+1,1)
           ELSE
              BMSD = 1.0
          ENDIF
*
* These loops generate random numbers and add the appropriate numbers
* together to get the response.  The "truth" function is the sum of the
* appropriate levels of factors for the no-interaction case and the sum plus the product for
* the with-interaction case.  R is for the rows of the DOE response matrix and C is for the
* columns.  UNIRAN is the function for the uniform random numbers.
*
          DO 30 R = 1 , 4
           DO 25 C = 1 , 9
              R1 = UNIRAN(PROGSEED)
              R2 = UNIRAN(PROGSEED)
              R3 = UNIRAN(PROGSEED)
              R4 = UNIRAN(PROGSEED)
              R5 = UNIRAN(PROGSEED)
              R6 = UNIRAN(PROGSEED)
              R7 = UNIRAN(PROGSEED)
              R8 = UNIRAN(PROGSEED)
              AP = APM + APSD*SQRT(-2*LOG(R1))*COS(2*PI*R2)
              AM = AMM + AMSD*SQRT(-2*LOG(R3))*COS(2*PI*R4)
              BP = BPM + BPSD*SQRT(-2*LOG(R5))*COS(2*PI*R6)
              BM = BMM + BMSD*SQRT(-2*LOG(R7))*COS(2*PI*R8)
*
* The settings of the levels change with each row.
*
              IF (R.EQ.1) Q = AM + BM + AM*BM
* For the no-interaction case, Q = AM + BM
              IF (R.EQ.2) Q = AM + BP + AM*BP
```

```
* For the no-interaction case, Q = AM + BP
            IF (R.EQ.3) Q = AP + BM + AP*BM
* For the no-interaction case, Q = AP + BM
            IF (R.EQ.4) Q = AP + BP + AP*BP
* For the no-interaction case, Q = AP + BP
            RSPNSE(R,C) = Q
*
* Column 1 of MEVAR is being used to add the raw data.
* Here, it just holds sums.
*
            MEVAR(R,1)=MEVAR(R,1)+Q
 25       CONTINUE
 30      CONTINUE
         DO 40 L = 1 , 4
*
* Now, column 1 of MEVAR holds the means of each row.
*
          MEVAR(L,1)=MEVAR(L,1)/9.0
40       CONTINUE
         DO 60 R=1,4
          DO 50 C=1,9
*
* Column 2 of MEVAR is collecting the sum for the numerator of the
* equation for variance.
*
            MEVAR(R,2) = MEVAR(R,2)+((RSPNSE(R,C)-MEVAR(R,1))**2)
 50       CONTINUE
 60      CONTINUE
         DO 70 R = 1 , 4
*
* Column 2 of MEVAR will now hold the variances of each row.
* SBARSQ is the sum of the variances.
*
          MEVAR(R,2) = MEVAR(R,2) / 8.0
          SBARSQ = SBARSQ + MEVAR(R,2)
 70      CONTINUE
*
* MSE is the average of the variances for each row.
* MSB is N/4 times delta squared.  In all cases, N = 36.
* So, the delta squareds are multiplied by 9.
*
         MSE = SBARSQ / 4.0
         MSBA = 9.0 * ((((MEVAR(3,1) + MEVAR(4,1)) / 2.0) -
   +            ((MEVAR(1,1) + MEVAR(2,1)) / 2.0))**2)
         MSBB = 9.0 * ((((MEVAR(2,1) + MEVAR(4,1)) /2.0 ) -
```

```
     +          ((MEVAR(1,1) + MEVAR(3,1)) / 2.0))**2)
          MSBAB = 9.0 * ((((MEVAR(1,1) + MEVAR(4,1)) / 2.0) -
     +          ((MEVAR(2,1) + MEVAR(3,1)) / 2.0))**2)
*
* The observed F effect for any factor or interaction is the MSB of that
* factor or interaction divided by the MSE.
*
          FA = MSBA / MSE
          FB = MSBB / MSE
          FAB = MSBAB / MSE
          FC = 4.152
*
* The IF statements are equivalent to the last step of a hypothesis
* test. If the observed F is greater than the critical F, then the
* conclusion is that the factor or interaction is significant. If this is
* the case, 1 is added to the appropriate cell in the SHIFT matrix.
* At this point, the matrix values are the number of times a factor
* or interaction was signigicant.
*
          IF (FA.GT.FC)
     +      SHIFT(X+1,Y+1,1) = SHIFT(X+1,Y+1,1) + 1.0
          IF (FB.GT.FC)
     +      SHIFT(X+1,Y+1,2) = SHIFT(X+1,Y+1,2) + 1.0
          IF (FAB.GT.FC)
     +      SHIFT(X+1,Y+1,3) = SHIFT(X+1,Y+1,3) + 1.0
*
* The end of the X, Y, and SIMREP loops.
*
 80     CONTINUE
 90    CONTINUE
 100 CONTINUE
*
* This loop divides the sums in the SHIFT array by the number of simulation
* replications, which gives the proportion of time each factor or
* interaction was significant. These are the values that will be compared
* to the theoretical calculations. OC curves will aslo be made from these
* values.
*
      DO 130 Z = 1 , 3
         DO 120 R = 1 , 5
          DO 110 C = 1 , 5
           SHIFT(R+1,C+1,Z) = SHIFT(R+1,C+1,Z) / (REAL(SIMREP)-1)
 110      CONTINUE
 120     CONTINUE
 130   CONTINUE
```

```
*
* This section prints the data. It is more complicated than necessary because it
* printed so the Excel could read it as matrices.
*
      PRINT 140,MEAN(I,1),MEAN(I,2)
  140 FORMAT (1X,'MEANS. ',A,A)
      PRINT 145,SD(I,1),SD(I,2)
  145 FORMAT (1X,' SDs: ',A,A)
      DO 210 Z = 1 , 3
        IF (Z.EQ.1) PRINT*,'A'
        IF (Z.EQ.2) PRINT*,'B'
        IF (Z.EQ.3) PRINT*,'AB'
        DO 200 R = 2 , 6
         A=2
         B=3
         C=4
         D=5
         E=6
    PRINT 190,SHIFT(R,A,Z),SHIFT(R,B,Z),SHIFT(R,C,Z),SHIFT(R,D,Z),
     +    SHIFT(R,E,Z)
  190 FORMAT (1X,F7.5,' ',F7.5,' ',F7.5,' ',F7.5,' 'F7.5)
  200   CONTINUE
  210 CONTINUE
*
* This continues the I loop. Now, the program will go on to the next
* set of factors and levels that need to be shifted.
*
  220 CONTINUE
      END
*
* This is Marse and Roberts random number generator [Marse and Roberts, 1983].
*
      REAL FUNCTION UNIRAN(SEED)
      INTEGER B2E15,B2E16,HI15,HI31,LOW15,LOWPRD,MODLUS,
     + MULT1,MULT2,OVFLOW,SEED
      DATA MULT1,MULT2/24112,26143/
      DATA B2E15,B2E16,MODLUS/32768,65536,2147483647/
      HI15 = SEED/B2E16
      LOWPRD = (SEED-HI15*B2E16)*MULT1
      LOW15 = LOWPRD/B2E16
      HI31 = HI15*MULT1 + LOW15
      OVFLOW = HI31/B2E15
      SEED = (((LOWPRD - LOW15*B2E16) - MODLUS) +
     +    (HI31 - OVFLOW*B2E15)*B2E16) + OVFLOW
      IF (SEED.LT.0) SEED = SEED + MODLUS
```

```
HI15 = SEED/B2E16
LOWPRD = (SEED - HI15*B2E16)*MULT2
LOW15 = LOWPRD/B2E16
HI31 = HI15 * MULT2 + LOW15
OVFLOW = HI31/B2E15
SEED = (((LOWPRD - LOW15*B2E16) - MODLUS) +
+    (HI31 - OVFLOW*B2E15)*B2E16) + OVFLOW
IF (SEED.LT.0) SEED = SEED + MODLUS
UNIRAN = (2*(SEED/256) + 1)/16777216.0
RETURN
END
```

Appendix B: No-Interaction Data

MEANS: A- Reject hypothesis if
SDs: A- test value > 9.488

						Hypothesis Test	
						Calc. Test Value	Result
A							
mean = 0	0.0560	0.0580	0.0560	0.0840	0.0480	6.654	Accept
mean = 1	0.5160	0.3300	0.2600	0.1680	0.1140	246.006	Reject
mean = 2	0.9840	0.8780	0.6980	0.4480	0.3340	687.622	Reject
mean = 3	1.0000	1.0000	0.9400	0.7840	0.6800	388.664	Reject
mean = 4	1.0000	1.0000	0.9960	0.9740	0.8720	197.429	Reject
B							
mean = 0	0.0560	0.0480	0.0500	0.0600	0.0560	0.940	Accept
mean = 1	0.0360	0.0460	0.0540	0.0600	0.0660	5.591	Accept
mean = 2	0.0600	0.0600	0.0540	0.0620	0.0720	1.481	Accept
mean = 3	0.0520	0.0540	0.0420	0.0580	0.0540	1.461	Accept
mean = 4	0.0480	0.0400	0.0400	0.0560	0.0440	2.059	Accept
AB							
mean = 0	0.0460	0.0560	0.0500	0.0540	0.0560	0.757	Accept
mean = 1	0.0340	0.0580	0.0500	0.0620	0.0500	4.778	Accept
mean = 2	0.0580	0.0440	0.0420	0.0620	0.0620	3.817	Accept
mean = 3	0.0460	0.0640	0.0300	0.0520	0.0540	6.721	Accept
mean = 4	0.0380	0.0580	0.0400	0.0760	0.0520	9.486	Accept

MEANS: A- Reject hypothesis if
SDs: A+ test value > 9.488

						Hypothesis Test	
						Calc. Test Value	Result
A							
mean = 0	0.0520	0.0500	0.0540	0.0460	0.0620	1.408	Accept
mean = 1	0.5280	0.3560	0.2300	0.1800	0.1300	250.786	Reject
mean = 2	0.9840	0.8780	0.6440	0.4920	0.3660	605.516	Reject
mean = 3	1.0000	0.9980	0.9540	0.8080	0.6520	438.176	Reject
mean = 4	1.0000	1.0000	0.9960	0.9620	0.8640	198.342	Reject
B							
mean = 0	0.0700	0.0420	0.0720	0.0560	0.0840	8.620	Accept
mean = 1	0.0540	0.0400	0.0660	0.0740	0.0520	6.423	Accept
mean = 2	0.0640	0.0600	0.0640	0.0520	0.0660	1.086	Accept
mean = 3	0.0440	0.0600	0.0640	0.0520	0.0440	3.327	Accept
mean = 4	0.0580	0.0500	0.0560	0.0660	0.0420	3.141	Accept
AB							
mean = 0	0.0640	0.0460	0.0580	0.0720	0.0840	6.772	Accept
mean = 1	0.0520	0.0380	0.0540	0.0620	0.0540	3.083	Accept
mean = 2	0.0460	0.0520	0.0760	0.0740	0.0660	6.021	Accept
mean = 3	0.0560	0.0460	0.0620	0.0500	0.0400	3.036	Accept
mean = 4	0.0520	0.0580	0.0700	0.0560	0.0660	1.931	Accept

MEANS: A- Reject hypothesis if
SDs: A-B- test value > 9.488

						Hypothesis Test	
						Calc. Test Value	Result
A							
mean = 0	0.0560	0.0460	0.0680	0.0400	0.0540	4.527	Accept
mean = 1	0.5320	0.2560	0.1400	0.1120	0.1000	371.512	Reject
mean = 2	0.9960	0.7580	0.4380	0.3220	0.1800	891.329	Reject
mean = 3	1.0000	0.9720	0.7660	0.5720	0.4020	691.634	Reject
mean = 4	1.0000	1.0000	0.9520	0.7960	0.6360	465.299	Reject
B							
mean = 0	0.0480	0.0340	0.0440	0.0500	0.0560	3.019	Accept
mean = 1	0.0300	0.0320	0.0340	0.0420	0.0500	3.803	Accept
mean = 2	0.0500	0.0480	0.0520	0.0560	0.0540	0.406	Accept
mean = 3	0.0400	0.0380	0.0340	0.0600	0.0360	5.558	Accept
mean = 4	0.0500	0.0540	0.0420	0.0700	0.0480	4.447	Accept
AB							
mean = 0	0.0640	0.0600	0.0600	0.0540	0.0540	0.684	Accept
mean = 1	0.0500	0.0540	0.0360	0.0600	0.0480	3.343	Accept
mean = 2	0.0380	0.0400	0.0460	0.0620	0.0660	6.887	Accept
mean = 3	0.0460	0.0640	0.0660	0.0480	0.0380	5.913	Accept
mean = 4	0.0380	0.0620	0.0520	0.0360	0.0560	5.567	Accept

MEANS: A- Reject hypothesis if
SDs: A+B- test value > 9.488

						Hypothesis Test	
						Calc. Test Value	Result
A							
mean = 0	0.0520	0.0680	0.0620	0.0520	0.0500	2.285	Accept
mean = 1	0.5340	0.2500	0.1860	0.1120	0.1160	332.647	Reject
mean = 2	0.9860	0.7280	0.4720	0.3100	0.2100	806.832	Reject
mean = 3	1.0000	0.9720	0.8260	0.5300	0.3700	804.341	Reject
mean = 4	1.0000	0.9980	0.9560	0.8200	0.6060	518.744	Reject
B							
mean = 0	0.0520	0.0520	0.0600	0.0660	0.0680	2.027	Accept
mean = 1	0.0540	0.0460	0.0720	0.0440	0.0640	5.372	Accept
mean = 2	0.0400	0.0600	0.0340	0.0540	0.0620	6.484	Accept
mean = 3	0.0540	0.0500	0.0320	0.0560	0.0480	3.939	Accept
mean = 4	0.0440	0.0500	0.0360	0.0420	0.0420	1.230	Accept
AB							
mean = 0	0.0560	0.0360	0.0420	0.0480	0.0640	5.267	Accept
mean = 1	0.0400	0.0520	0.0320	0.0560	0.0600	5.952	Accept
mean = 2	0.0600	0.0520	0.0360	0.0480	0.0680	5.887	Accept
mean = 3	0.0580	0.0540	0.0420	0.0540	0.0560	1.568	Accept
mean = 4	0.0580	0.0460	0.0420	0.0560	0.0600	2.529	Accept

	MEANS:	A-	Reject hypothesis if		
	SDs:	A-B+	test value > 9.488		
A					
mean = 0	0.0480	0.0420	0.0480	0.0540	0.0600
mean = 1	0.5820	0.2840	0.1620	0.1500	0.1040
mean = 2	0.9920	0.7280	0.4440	0.2880	0.2240
mean = 3	1.0000	0.9640	0.8000	0.5560	0.4060
mean = 4	1.0000	1.0000	0.9480	0.7920	0.6640
B					
mean = 0	0.0500	0.0640	0.0460	0.0380	0.0660
mean = 1	0.0400	0.0620	0.0460	0.0480	0.0640
mean = 2	0.0300	0.0600	0.0560	0.0540	0.0400
mean = 3	0.0440	0.0620	0.0460	0.0440	0.0520
mean = 4	0.0420	0.0620	0.0300	0.0500	0.0580
AB					
mean = 0	0.0680	0.0580	0.0700	0.0740	0.0440
mean = 1	0.0460	0.0480	0.0560	0.0680	0.0740
mean = 2	0.0440	0.0440	0.0460	0.0560	0.0560
mean = 3	0.0400	0.0600	0.0620	0.0380	0.0460
mean = 4	0.0560	0.0460	0.0580	0.0840	0.0560

Hypothesis Test	
Calc. Test Value	Result
1.956	Accept
393.989	Reject
828.323	Reject
702.173	Reject
418.229	Reject
5.727	Accept
4.463	Accept
6.915	Accept
2.495	Accept
7.156	Accept
4.934	Accept
5.485	Accept
1.676	Accept
5.353	Accept
7.163	Accept

	MEANS:	A-	Reject hypothesis if		
	SDs:	A+B+	test value > 9.488		
A					
mean = 0	0.0440	0.0480	0.0460	0.0580	0.0660
mean = 1	0.5500	0.2900	0.1540	0.0840	0.1120
mean = 2	0.9700	0.7300	0.5100	0.2860	0.2220
mean = 3	1.0000	0.9620	0.7820	0.5840	0.4080
mean = 4	1.0000	1.0000	0.9560	0.8080	0.6160
B					
mean = 0	0.0300	0.0520	0.0480	0.0660	0.0480
mean = 1	0.0660	0.0520	0.0540	0.0400	0.0760
mean = 2	0.0720	0.0540	0.0700	0.0420	0.0620
mean = 3	0.0500	0.0780	0.0420	0.0540	0.0580
mean = 4	0.0440	0.0560	0.0540	0.0480	0.0440
AB					
mean = 0	0.0320	0.0560	0.0460	0.0720	0.0680
mean = 1	0.0460	0.0520	0.0420	0.0480	0.0600
mean = 2	0.0580	0.0400	0.0520	0.0520	0.0660
mean = 3	0.0480	0.0500	0.0540	0.0580	0.0520
mean = 4	0.0380	0.0440	0.0440	0.0540	0.0480

Hypothesis Test	
Calc. Test Value	Result
3.496	Accept
404.442	Reject
780.888	Reject
669.559	Reject
503.461	Reject
7.118	Accept
7.030	Accept
5.390	Accept
6.795	Accept
1.334	Accept
10.317	Reject
1.986	Accept
3.580	Accept
0.596	Accept
1.599	Accept

MEANS: A-B- SDs: A-

Reject hypothesis if test value > 9.488

						Hypothesis Test	
						Calc. Test Value	Result
A							
mean = 0	0.0400	0.0440	0.0500	0.0420	0.0500	0.982	Accept
mean = 1	0.5480	0.3100	0.2500	0.1680	0.1320	267.075	Reject
mean = 2	0.9860	0.8660	0.6540	0.4660	0.3620	618.777	Reject
mean = 3	1.0000	1.0000	0.9420	0.7920	0.6540	425.604	Reject
mean = 4	1.0000	1.0000	0.9960	0.9560	0.8700	182.264	Reject
B							
mean = 0	0.0420	0.0520	0.0400	0.0440	0.0580	2.544	Accept
mean = 1	0.5120	0.3460	0.2100	0.1880	0.1140	248.638	Reject
mean = 2	0.9880	0.9040	0.6640	0.5260	0.3640	626.612	Reject
mean = 3	1.0000	0.9960	0.9400	0.7960	0.6560	412.682	Reject
mean = 4	1.0000	1.0000	1.0000	0.9680	0.8620	217.513	Reject
AB							
mean = 0	0.0400	0.0540	0.0400	0.0500	0.0620	3.814	Accept
mean = 1	0.0640	0.0460	0.0400	0.0640	0.0540	4.526	Accept
mean = 2	0.0600	0.0320	0.0500	0.0500	0.0500	4.464	Accept
mean = 3	0.0360	0.0480	0.0580	0.0440	0.0480	2.833	Accept
mean = 4	0.0640	0.0460	0.0480	0.0440	0.0580	3.002	Accept

MEANS: A+B- SDs: A-

Reject hypothesis if test value > 9.488

						Hypothesis Test	
						Calc. Test Value	Result
A							
mean = 0	0.0480	0.0360	0.0540	0.0360	0.0720	9.543	Reject
mean = 1	0.5260	0.3420	0.2520	0.1540	0.1280	258.017	Reject
mean = 2	0.9860	0.8380	0.6660	0.4740	0.3380	615.938	Reject
mean = 3	1.0000	1.0000	0.9500	0.8100	0.6700	408.499	Reject
mean = 4	1.0000	1.0000	0.9960	0.9620	0.8740	180.025	Reject
B							
mean = 0	0.0540	0.0460	0.0600	0.0580	0.0460	1.728	Accept
mean = 1	0.5680	0.3720	0.1900	0.1600	0.1360	329.903	Reject
mean = 2	0.9800	0.8600	0.6720	0.4560	0.3620	613.152	Reject
mean = 3	1.0000	0.9920	0.9500	0.8100	0.6300	457.794	Reject
mean = 4	1.0000	1.0000	0.9980	0.9540	0.8680	188.220	Reject
AB							
mean = 0	0.0520	0.0540	0.0500	0.0520	0.0560	0.208	Accept
mean = 1	0.0440	0.0500	0.0700	0.0740	0.0520	6.369	Accept
mean = 2	0.0380	0.0400	0.0460	0.0480	0.0560	2.335	Accept
mean = 3	0.0440	0.0460	0.0460	0.0620	0.0600	3.057	Accept
mean = 4	0.0520	0.0540	0.0440	0.0440	0.0460	0.963	Accept

	MEANS:	A-B+	Reject hypothesis if		
	SDs:	A-	test value > 9.488		
A					
mean = 0	0.0380	0.0580	0.0560	0.0880	0.0560
mean = 1	0.5740	0.3740	0.2380	0.1760	0.1300
mean = 2	0.9800	0.8740	0.6760	0.4980	0.3160
mean = 3	1.0000	0.9960	0.9380	0.8180	0.6560
mean = 4	1.0000	1.0000	1.0000	0.9640	0.8540
B					
mean = 0	0.0620	0.0500	0.0620	0.0460	0.0700
mean = 1	0.5400	0.3220	0.2420	0.1460	0.1500
mean = 2	0.9820	0.8840	0.6380	0.5060	0.3680
mean = 3	1.0000	0.9920	0.9440	0.8200	0.6320
mean = 4	1.0000	1.0000	1.0000	0.9640	0.8740
AB					
mean = 0	0.0520	0.0660	0.0540	0.0420	0.0760
mean = 1	0.0580	0.0520	0.0460	0.0460	0.0740
mean = 2	0.0680	0.0620	0.0460	0.0580	0.0600
mean = 3	0.0580	0.0480	0.0600	0.0400	0.0600
mean = 4	0.0440	0.0300	0.0460	0.0440	0.0720

Hypothesis Test	
Calc. Test Value	Result
11.678	Reject
307.272	Reject
660.578	Reject
408.249	Reject
227.900	Reject
3.514	Accept
262.063	Reject
597.969	Reject
447.388	Reject
190.161	Reject
6.369	Accept
5.185	Accept
2.356	Accept
3.145	Accept
10.371	Reject

	MEANS:	A+B+	Reject hypothesis if		
	SDs:	A-	test value > 9.488		
A					
mean = 0	0.0480	0.0340	0.0580	0.0560	0.0660
mean = 1	0.5140	0.3820	0.2260	0.1500	0.1340
mean = 2	0.9800	0.8820	0.6560	0.4080	0.3380
mean = 3	1.0000	0.9940	0.9400	0.7760	0.6560
mean = 4	1.0000	1.0000	0.9940	0.9640	0.8400
B					
mean = 0	0.0620	0.0520	0.0680	0.0640	0.0520
mean = 1	0.5380	0.3640	0.2320	0.1840	0.1400
mean = 2	0.9920	0.8560	0.6740	0.4580	0.3820
mean = 3	1.0000	0.9940	0.9340	0.7840	0.6680
mean = 4	1.0000	1.0000	0.9960	0.9820	0.8760
AB					
mean = 0	0.0340	0.0520	0.0660	0.0620	0.0640
mean = 1	0.0360	0.0720	0.0640	0.0580	0.0560
mean = 2	0.0520	0.0540	0.0480	0.0520	0.0600
mean = 3	0.0420	0.0320	0.0660	0.0500	0.0560
mean = 4	0.0440	0.0620	0.0440	0.0520	0.0540

Hypothesis Test	
Calc. Test Value	Result
5.913	Accept
262.916	Reject
702.903	Reject
414.358	Reject
242.098	Reject
1.884	Accept
251.894	Reject
604.134	Reject
388.459	Reject
202.008	Reject
6.658	Accept
6.646	Accept
0.762	Accept
7.234	Accept
2.355	Accept

	MEANS:	A-B-	Reject hypothesis if				Hypothesis Test	
	SDs:	A-B-	test value > 9.488				Calc. Test Value	Result
			A					
mean = 0	0.0440	0.0620	0.0520	0.0500	0.0600		2.161	Accept
mean = 1	0.5340	0.2560	0.1580	0.1300	0.0860		361.058	Reject
mean = 2	0.9900	0.7340	0.5040	0.2960	0.1800		866.902	Reject
mean = 3	1.0000	0.9740	0.7980	0.5640	0.4080		708.492	Reject
mean = 4	1.0000	1.0000	0.9480	0.8160	0.6500		436.773	Reject
			B					
mean = 0	0.0420	0.0560	0.0380	0.0600	0.0440		3.939	Accept
mean = 1	0.5540	0.2260	0.1560	0.1440	0.1020		371.280	Reject
mean = 2	0.9720	0.7520	0.4400	0.3100	0.2100		811.027	Reject
mean = 3	1.0000	0.9560	0.7800	0.5600	0.3840		707.428	Reject
mean = 4	1.0000	1.0000	0.9440	0.8240	0.6520		428.772	Reject
			AB					
mean = 0	0.0600	0.0520	0.0540	0.0580	0.0560		0.378	Accept
mean = 1	0.0620	0.0420	0.0320	0.0700	0.0600		9.736	Reject
mean = 2	0.0600	0.0420	0.0600	0.0540	0.0500		2.271	Accept
mean = 3	0.0600	0.0500	0.0460	0.0400	0.0400		3.077	Accept
mean = 4	0.0480	0.0480	0.0640	0.0440	0.0520		2.437	Accept

	MEANS:	A-B-	Reject hypothesis if				Hypothesis Test	
	SDs:	A+B-	test value > 9.488				Calc. Test Value	Result
			A					
mean = 0	0.0480	0.0460	0.0300	0.0480	0.0720		9.703	Reject
mean = 1	0.5200	0.2560	0.1580	0.0980	0.0640		392.509	Reject
mean = 2	0.9920	0.7620	0.4180	0.3000	0.2460		831.207	Reject
mean = 3	1.0000	0.9560	0.7900	0.6140	0.4080		651.449	Reject
mean = 4	1.0000	1.0000	0.9540	0.8260	0.6600		428.692	Reject
			B					
mean = 0	0.0500	0.0500	0.0500	0.0460	0.0380		1.219	Accept
mean = 1	0.5280	0.2640	0.1300	0.1340	0.0840		370.625	Reject
mean = 2	0.9880	0.6980	0.4980	0.3060	0.2440		745.678	Reject
mean = 3	1.0000	0.9660	0.7960	0.5780	0.4660		607.579	Reject
mean = 4	1.0000	0.9980	0.9600	0.7640	0.6000		534.768	Reject
			AB					
mean = 0	0.0600	0.0280	0.0360	0.0340	0.0620		11.887	Reject
mean = 1	0.0500	0.0560	0.0400	0.0780	0.0460		8.378	Accept
mean = 2	0.0540	0.0520	0.0640	0.0440	0.0740		4.967	Accept
mean = 3	0.0500	0.0480	0.0420	0.0520	0.0580		1.432	Accept
mean = 4	0.0380	0.0340	0.0300	0.0560	0.0740		15.224	Reject

MEANS: A-B- Reject hypothesis if
SDs: A-B+ test value > 9.488

						Hypothesis Test: Calc. Test Value	Hypothesis Test: Result
A							
mean = 0	0.0480	0.0380	0.0540	0.0560	0.0480	2.120	Accept
mean = 1	0.5500	0.2440	0.1660	0.1100	0.0940	390.665	Reject
mean = 2	0.9900	0.7840	0.4560	0.3160	0.2040	872.048	Reject
mean = 3	1.0000	0.9820	0.8220	0.5340	0.3900	784.654	Reject
mean = 4	1.0000	1.0000	0.9640	0.8100	0.6360	481.725	Reject
B							
mean = 0	0.0540	0.0520	0.0580	0.0520	0.0600	0.506	Accept
mean = 1	0.5360	0.2160	0.1640	0.1180	0.1240	342.645	Reject
mean = 2	0.9900	0.7500	0.4460	0.3080	0.2280	819.476	Reject
mean = 3	1.0000	0.9820	0.7860	0.5560	0.3760	765.676	Reject
mean = 4	1.0000	1.0000	0.9460	0.7620	0.6040	512.813	Reject
AB							
mean = 0	0.0380	0.0500	0.0380	0.0340	0.0400	1.875	Accept
mean = 1	0.0500	0.0440	0.0540	0.0440	0.0580	1.600	Accept
mean = 2	0.0300	0.0540	0.0680	0.0520	0.0600	8.046	Accept
mean = 3	0.0480	0.0640	0.0400	0.0540	0.0600	3.621	Accept
mean = 4	0.0520	0.0500	0.0340	0.0500	0.0600	3.814	Accept

MEANS: A-B- Reject hypothesis if
SDs: A+B+ test value > 9.488

						Hypothesis Test: Calc. Test Value	Hypothesis Test: Result
A							
mean = 0	0.0660	0.0360	0.0560	0.0400	0.0520	6.232	Accept
mean = 1	0.5620	0.2560	0.1660	0.1240	0.0860	403.194	Reject
mean = 2	0.9840	0.7080	0.4240	0.2680	0.2160	832.885	Reject
mean = 3	1.0000	0.9880	0.7940	0.5340	0.4280	721.464	Reject
mean = 4	1.0000	1.0000	0.9580	0.8240	0.6500	450.445	Reject
B							
mean = 0	0.0520	0.0700	0.0380	0.0420	0.0580	6.654	Accept
mean = 1	0.5340	0.2500	0.1640	0.1100	0.0740	387.806	Reject
mean = 2	0.9900	0.7460	0.4040	0.2920	0.2180	859.334	Reject
mean = 3	1.0000	0.9820	0.8140	0.5700	0.4240	705.358	Reject
mean = 4	1.0000	0.9980	0.9660	0.8180	0.6020	540.617	Reject
AB							
mean = 0	0.0360	0.0460	0.0460	0.0440	0.0500	1.263	Accept
mean = 1	0.0520	0.0660	0.0600	0.0620	0.0480	2.019	Accept
mean = 2	0.0440	0.0500	0.0580	0.0540	0.0320	4.535	Accept
mean = 3	0.0400	0.0440	0.0560	0.0480	0.0560	2.206	Accept
mean = 4	0.0420	0.0380	0.0740	0.0440	0.0480	8.773	Accept

MEANS: A+B- Reject hypothesis if
SDs: A-B- test value > 9.488

						Calc. Test Value	Result
A							
mean = 0	0.0300	0.0540	0.0480	0.0440	0.0540	4.466	Accept
mean = 1	0.5820	0.2280	0.1460	0.1200	0.0840	461.229	Reject
mean = 2	0.9780	0.7200	0.4520	0.2900	0.2200	796.398	Reject
mean = 3	1.0000	0.9700	0.8120	0.5460	0.3900	749.388	Reject
mean = 4	1.0000	1.0000	0.9440	0.7740	0.5880	533.857	Reject
B							
mean = 0	0.0320	0.0480	0.0500	0.0560	0.0380	4.356	Accept
mean = 1	0.5760	0.2280	0.1540	0.0960	0.0980	453.640	Reject
mean = 2	0.9900	0.7360	0.4400	0.3340	0.1800	850.151	Reject
mean = 3	1.0000	0.9860	0.7460	0.5560	0.3540	784.324	Reject
mean = 4	1.0000	1.0000	0.9640	0.7800	0.6440	472.560	Reject
AB							
mean = 0	0.0600	0.0560	0.0380	0.0640	0.0580	3.881	Accept
mean = 1	0.0700	0.0460	0.0420	0.0360	0.0540	7.331	Accept
mean = 2	0.0500	0.0300	0.0680	0.0440	0.0460	8.241	Accept
mean = 3	0.0380	0.0800	0.0640	0.0600	0.0460	9.830	Reject
mean = 4	0.0500	0.0600	0.0340	0.0600	0.0520	4.660	Accept

Hypothesis Test

MEANS: A+B- Reject hypothesis if
SDs: A+B- test value > 9.488

						Calc. Test Value	Result
A							
mean = 0	0.0640	0.0580	0.0420	0.0600	0.0640	3.051	Accept
mean = 1	0.5200	0.2600	0.1380	0.1180	0.0760	374.224	Reject
mean = 2	0.9840	0.7200	0.4440	0.2560	0.1900	880.567	Reject
mean = 3	1.0000	0.9700	0.8260	0.5800	0.3780	758.840	Reject
mean = 4	1.0000	0.9980	0.9560	0.8040	0.6500	444.347	Reject
B							
mean = 0	0.0500	0.0500	0.0480	0.0580	0.0620	1.451	Accept
mean = 1	0.5340	0.2300	0.1480	0.1100	0.0860	388.198	Reject
mean = 2	0.9920	0.7340	0.4460	0.3040	0.2140	830.252	Reject
mean = 3	1.0000	0.9760	0.7940	0.5780	0.4540	637.161	Reject
mean = 4	1.0000	0.9980	0.9460	0.8380	0.6300	466.970	Reject
AB							
mean = 0	0.0540	0.0540	0.0460	0.0500	0.0560	0.649	Accept
mean = 1	0.0620	0.0600	0.0580	0.0500	0.0480	1.478	Accept
mean = 2	0.0460	0.0600	0.0440	0.0540	0.0700	4.371	Accept
mean = 3	0.0520	0.0540	0.0400	0.0620	0.0520	2.515	Accept
mean = 4	0.0620	0.0620	0.0640	0.0480	0.0500	2.121	Accept

Hypothesis Test

MEANS: A+B- SDs: A-B+

Reject hypothesis if test value > 9.488

						Hypothesis Test: Calc. Test Value	Hypothesis Test: Result
A							
mean = 0	0.0640	0.0760	0.0520	0.0640	0.0760	3.252	Accept
mean = 1	0.5300	0.2740	0.1400	0.1240	0.0960	361.662	Reject
mean = 2	0.9920	0.7440	0.4260	0.2860	0.2120	864.935	Reject
mean = 3	1.0000	0.9740	0.7540	0.5700	0.4120	676.508	Reject
mean = 4	1.0000	1.0000	0.9520	0.8020	0.6120	504.241	Reject
B							
mean = 0	0.0460	0.0580	0.0540	0.0680	0.0580	2.359	Accept
mean = 1	0.5360	0.2580	0.1580	0.0960	0.0760	404.960	Reject
mean = 2	0.9820	0.7160	0.4560	0.2780	0.2460	780.716	Reject
mean = 3	1.0000	0.9640	0.8120	0.5960	0.4080	684.795	Reject
mean = 4	1.0000	0.9980	0.9500	0.8180	0.6740	398.086	Reject
AB							
mean = 0	0.0400	0.0480	0.0460	0.0760	0.0760	11.245	Reject
mean = 1	0.0500	0.0620	0.0420	0.0500	0.0500	2.124	Accept
mean = 2	0.0560	0.0420	0.0500	0.0580	0.0500	1.614	Accept
mean = 3	0.0480	0.0720	0.0420	0.0500	0.0220	14.400	Reject
mean = 4	0.0600	0.0460	0.0480	0.0540	0.0560	1.328	Accept

MEANS: A+B- SDs: A+B+

Reject hypothesis if test value > 9.488

						Hypothesis Test: Calc. Test Value	Hypothesis Test: Result
A							
mean = 0	0.0400	0.0440	0.0320	0.0560	0.0500	3.997	Accept
mean = 1	0.5860	0.2820	0.1640	0.0940	0.0780	478.113	Reject
mean = 2	0.9800	0.7200	0.4420	0.2840	0.2480	776.298	Reject
mean = 3	1.0000	0.9760	0.8100	0.5600	0.4140	713.560	Reject
mean = 4	1.0000	1.0000	0.9520	0.7540	0.6300	484.269	Reject
B							
mean = 0	0.0560	0.0500	0.0460	0.0480	0.0600	1.379	Accept
mean = 1	0.5280	0.2520	0.1680	0.1260	0.1080	328.580	Reject
mean = 2	0.9880	0.7320	0.4500	0.2740	0.2340	824.104	Reject
mean = 3	1.0000	0.9760	0.7900	0.5900	0.4160	680.684	Reject
mean = 4	1.0000	1.0000	0.9760	0.8460	0.6340	510.820	Reject
AB							
mean = 0	0.0500	0.0480	0.0560	0.0760	0.0440	6.147	Accept
mean = 1	0.0620	0.0660	0.0380	0.0480	0.0700	6.690	Accept
mean = 2	0.0560	0.0660	0.0540	0.0460	0.0440	3.065	Accept
mean = 3	0.0580	0.0560	0.0660	0.0560	0.0460	1.909	Accept
mean = 4	0.0520	0.0500	0.0580	0.0520	0.0480	0.568	Accept

	MEANS:	A-B+	Reject hypothesis if			Hypothesis Test	
	SDs:	A-B-	test value > 9.488			Calc. Test Value	Result
A							
mean = 0	0.0400	0.0540	0.0580	0.0360	0.0620	5.474	Accept
mean = 1	0.5340	0.2420	0.1380	0.0980	0.1200	372.505	Reject
mean = 2	0.9840	0.7680	0.4780	0.3020	0.2080	846.828	Reject
mean = 3	1.0000	0.9780	0.8180	0.5680	0.3920	750.110	Reject
mean = 4	1.0000	1.0000	0.9540	0.8000	0.6280	480.577	Reject
B							
mean = 0	0.0320	0.0540	0.0380	0.0640	0.0620	8.674	Accept
mean = 1	0.5400	0.2480	0.1540	0.1280	0.0780	383.605	Reject
mean = 2	0.9820	0.7240	0.4920	0.3360	0.2060	775.903	Reject
mean = 3	1.0000	0.9740	0.7960	0.5760	0.3780	745.284	Reject
mean = 4	1.0000	1.0000	0.9540	0.8060	0.6280	480.492	Reject
AB							
mean = 0	0.0480	0.0860	0.0420	0.0680	0.0640	10.477	Reject
mean = 1	0.0660	0.0500	0.0600	0.0540	0.0540	1.463	Accept
mean = 2	0.0420	0.0440	0.0520	0.0580	0.0380	2.923	Accept
mean = 3	0.0660	0.0380	0.0420	0.0620	0.0560	6.047	Accept
mean = 4	0.0460	0.0740	0.0340	0.0540	0.0500	8.697	Accept

	MEANS:	A-B+	Reject hypothesis if			Hypothesis Test	
	SDs:	A+B-	test value > 9.488			Calc. Test Value	Result
A							
mean = 0	0.0460	0.0360	0.0400	0.0340	0.0500	2.288	Accept
mean = 1	0.5480	0.2460	0.1460	0.1140	0.1000	390.885	Reject
mean = 2	0.9800	0.7360	0.4700	0.2660	0.1880	873.090	Reject
mean = 3	1.0000	0.9680	0.7760	0.5660	0.4040	690.959	Reject
mean = 4	1.0000	1.0000	0.9460	0.7940	0.6140	492.789	Reject
B							
mean = 0	0.0400	0.0500	0.0380	0.0660	0.0380	6.545	Accept
mean = 1	0.5620	0.2640	0.1720	0.1100	0.0980	398.696	Reject
mean = 2	0.9880	0.7360	0.4560	0.2620	0.2060	872.006	Reject
mean = 3	1.0000	0.9760	0.7640	0.5860	0.4200	664.092	Reject
mean = 4	1.0000	0.9980	0.9660	0.8240	0.6220	506.361	Reject
AB							
mean = 0	0.0460	0.0500	0.0420	0.0520	0.0640	2.870	Accept
mean = 1	0.0560	0.0400	0.0580	0.0540	0.0700	4.373	Accept
mean = 2	0.0620	0.0520	0.0460	0.0640	0.0600	2.135	Accept
mean = 3	0.0460	0.0480	0.0560	0.0480	0.0420	1.138	Accept
mean = 4	0.0600	0.0440	0.0640	0.0500	0.0480	2.827	Accept

	MEANS: A-B+		Reject hypothesis if				Hypothesis Test	
	SDs: A-B+		test value > 9.488				Calc. Test Value	Result
A								
mean = 0	0.0540	0.0540	0.0580	0.0360	0.0520		3.036	Accept
mean = 1	0.5440	0.2500	0.1540	0.1180	0.1100		365.744	Reject
mean = 2	0.9960	0.7380	0.4760	0.3060	0.2020		846.781	Reject
mean = 3	1.0000	0.9640	0.8120	0.5580	0.4520		646.046	Reject
mean = 4	1.0000	0.9980	0.9560	0.7820	0.5900		543.213	Reject
B								
mean = 0	0.0520	0.0480	0.0560	0.0640	0.0720		3.375	Accept
mean = 1	0.5060	0.2740	0.1440	0.1220	0.0860		336.420	Reject
mean = 2	0.9740	0.7480	0.4820	0.2840	0.2200		811.431	Reject
mean = 3	1.0000	0.9740	0.8140	0.5800	0.4340		672.333	Reject
mean = 4	1.0000	0.9980	0.9540	0.7720	0.6780		409.642	Reject
AB								
mean = 0	0.0400	0.0460	0.0500	0.0520	0.0620		2.779	Accept
mean = 1	0.0520	0.0340	0.0280	0.0440	0.0440		4.581	Accept
mean = 2	0.0360	0.0560	0.0520	0.0500	0.0500		2.465	Accept
mean = 3	0.0540	0.0480	0.0580	0.0600	0.0600		0.984	Accept
mean = 4	0.0240	0.0620	0.0560	0.0460	0.0640		11.149	Reject

	MEANS: A-B+		Reject hypothesis if				Hypothesis Test	
	SDs: A+B+		test value > 9.488				Calc. Test Value	Result
A								
mean = 0	0.0520	0.0420	0.0520	0.0520	0.0460		0.913	Accept
mean = 1	0.5160	0.2500	0.1540	0.1080	0.0900		352.100	Reject
mean = 2	0.9860	0.7300	0.4380	0.2580	0.2320		841.152	Reject
mean = 3	1.0000	0.9720	0.7840	0.5620	0.4200		681.407	Reject
mean = 4	1.0000	0.9980	0.9380	0.8000	0.6300		453.194	Reject
B								
mean = 0	0.0440	0.0560	0.0480	0.0660	0.0400		4.446	Accept
mean = 1	0.5720	0.2840	0.1520	0.1140	0.0900		430.868	Reject
mean = 2	0.9880	0.7680	0.4460	0.2780	0.1980		898.626	Reject
mean = 3	1.0000	0.9660	0.7940	0.5480	0.4000		715.034	Reject
mean = 4	1.0000	0.9980	0.9360	0.8080	0.6280		453.888	Reject
AB								
mean = 0	0.0640	0.0740	0.0540	0.0560	0.0420		5.198	Accept
mean = 1	0.0380	0.0420	0.0540	0.0400	0.0580		3.652	Accept
mean = 2	0.0480	0.0600	0.0480	0.0380	0.0520		2.702	Accept
mean = 3	0.0600	0.0580	0.0460	0.0440	0.0420		2.947	Accept
mean = 4	0.0560	0.0440	0.0560	0.0580	0.0540		1.214	Accept

	MEANS:	A+B+	Reject hypothesis if			Hypothesis Test	
	SDs:	A-B-	test value > 9.488			Calc. Test Value	Result
A							
mean = 0	0.0580	0.0500	0.0400	0.0580	0.0440	2.779	Accept
mean = 1	0.5580	0.2340	0.1480	0.0900	0.0840	447.507	Reject
mean = 2	0.9860	0.7240	0.4380	0.2660	0.2380	821.025	Reject
mean = 3	1.0000	0.9820	0.7840	0.5480	0.4200	706.365	Reject
mean = 4	1.0000	0.9960	0.9360	0.8180	0.5940	508.853	Reject
B							
mean = 0	0.0500	0.0680	0.0600	0.0440	0.0640	3.679	Accept
mean = 1	0.5680	0.2520	0.1160	0.1240	0.0860	451.959	Reject
mean = 2	0.9780	0.7240	0.4800	0.2880	0.1780	849.592	Reject
mean = 3	1.0000	0.9740	0.8080	0.5700	0.3900	738.266	Reject
mean = 4	1.0000	0.9960	0.9560	0.8100	0.6300	472.877	Reject
AB							
mean = 0	0.0560	0.0660	0.0500	0.0620	0.0480	2.210	Accept
mean = 1	0.0540	0.0460	0.0480	0.0360	0.0540	2.418	Accept
mean = 2	0.0540	0.0440	0.0620	0.0360	0.0440	4.464	Accept
mean = 3	0.0520	0.0620	0.0500	0.0520	0.0740	3.734	Accept
mean = 4	0.0440	0.0560	0.0400	0.0440	0.0640	4.277	Accept

	MEANS:	A+B+	Reject hypothesis if			Hypothesis Test	
	SDs:	A+B-	test value > 9.488			Calc. Test Value	Result
A							
mean = 0	0.0540	0.0440	0.0460	0.0540	0.0520	0.926	Accept
mean = 1	0.5420	0.2700	0.1260	0.1320	0.0960	383.823	Reject
mean = 2	0.9880	0.7560	0.4820	0.3060	0.1980	851.737	Reject
mean = 3	1.0000	0.9600	0.7960	0.5520	0.3720	745.800	Reject
mean = 4	1.0000	1.0000	0.9260	0.8100	0.6620	393.466	Reject
B							
mean = 0	0.0420	0.0440	0.0560	0.0440	0.0640	3.874	Accept
mean = 1	0.5360	0.2460	0.1320	0.1100	0.0740	413.393	Reject
mean = 2	0.9840	0.7860	0.4600	0.2780	0.2120	887.271	Reject
mean = 3	1.0000	0.9620	0.7680	0.5360	0.4200	675.261	Reject
mean = 4	1.0000	1.0000	0.9340	0.8080	0.6660	396.063	Reject
AB							
mean = 0	0.0500	0.0500	0.0560	0.0620	0.0460	1.568	Accept
mean = 1	0.0480	0.0300	0.0500	0.0440	0.0440	2.961	Accept
mean = 2	0.0500	0.0580	0.0560	0.0540	0.0480	0.683	Accept
mean = 3	0.0520	0.0600	0.0520	0.0500	0.0460	1.055	Accept
mean = 4	0.0680	0.0440	0.0760	0.0680	0.0460	7.358	Accept

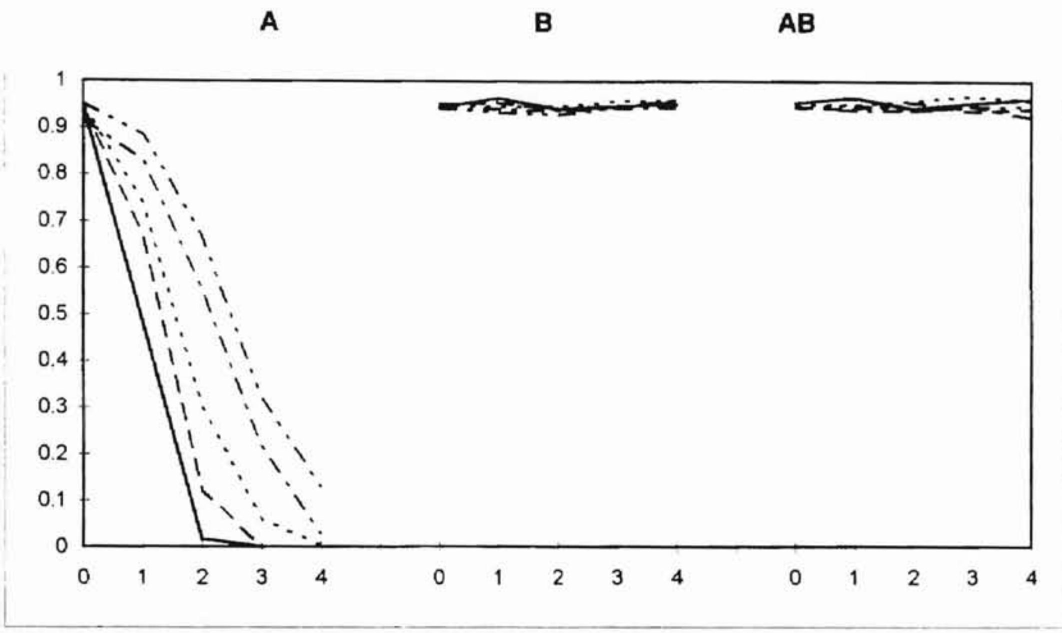
	MEANS:	A+B+	Reject hypothesis if			Hypothesis Test	
	SDs:	A-B+	test value > 9.488			Calc. Test Value	Result
A							
mean = 0	0.0520	0.0460	0.0560	0.0540	0.0620	1.331	Accept
mean = 1	0.5580	0.2620	0.1320	0.1020	0.0820	445.554	Reject
mean = 2	0.9840	0.7240	0.4460	0.2780	0.1720	885.638	Reject
mean = 3	1.0000	0.9920	0.7460	0.5680	0.4300	679.130	Reject
mean = 4	1.0000	1.0000	0.9720	0.8020	0.6340	497.460	Reject
B							
mean = 0	0.0480	0.0400	0.0580	0.0620	0.0640	3.997	Accept
mean = 1	0.5340	0.2540	0.1640	0.1140	0.0800	377.265	Reject
mean = 2	0.9860	0.7660	0.4560	0.2900	0.1880	891.762	Reject
mean = 3	1.0000	0.9780	0.7560	0.5680	0.4380	649.796	Reject
mean = 4	1.0000	1.0000	0.9660	0.8320	0.6380	483.656	Reject
AB							
mean = 0	0.0500	0.0560	0.0420	0.0620	0.0420	3.209	Accept
mean = 1	0.0600	0.0400	0.0460	0.0520	0.0540	2.457	Accept
mean = 2	0.0540	0.0500	0.0540	0.0500	0.0440	0.702	Accept
mean = 3	0.0500	0.0360	0.0420	0.0520	0.0460	1.909	Accept
mean = 4	0.0480	0.0580	0.0420	0.0720	0.0460	5.765	Accept

	MEANS:	A+B+	Reject hypothesis if			Hypothesis Test	
	SDs:	A+B+	test value > 9.488			Calc. Test Value	Result
A							
mean = 0	0.0560	0.0360	0.0440	0.0560	0.0760	9.021	Accept
mean = 1	0.5360	0.2600	0.1440	0.1180	0.0840	385.225	Reject
mean = 2	0.9720	0.7200	0.4620	0.2580	0.2120	824.149	Reject
mean = 3	1.0000	0.9700	0.8060	0.5760	0.3680	761.446	Reject
mean = 4	1.0000	1.0000	0.9440	0.8260	0.6160	489.550	Reject
B							
mean = 0	0.0540	0.0520	0.0660	0.0620	0.0600	1.200	Accept
mean = 1	0.5540	0.2620	0.1780	0.0960	0.0980	396.904	Reject
mean = 2	0.9900	0.7300	0.5020	0.2680	0.2040	859.006	Reject
mean = 3	1.0000	0.9660	0.7620	0.5420	0.4300	661.289	Reject
mean = 4	1.0000	1.0000	0.9440	0.8120	0.6160	487.410	Reject
AB							
mean = 0	0.0420	0.0500	0.0640	0.0720	0.0640	5.339	Accept
mean = 1	0.0540	0.0520	0.0400	0.0420	0.0540	2.032	Accept
mean = 2	0.0540	0.0540	0.0540	0.0360	0.0720	6.342	Accept
mean = 3	0.0480	0.0580	0.0400	0.0220	0.0400	8.769	Accept
mean = 4	0.0600	0.0440	0.0500	0.0760	0.0640	5.609	Accept

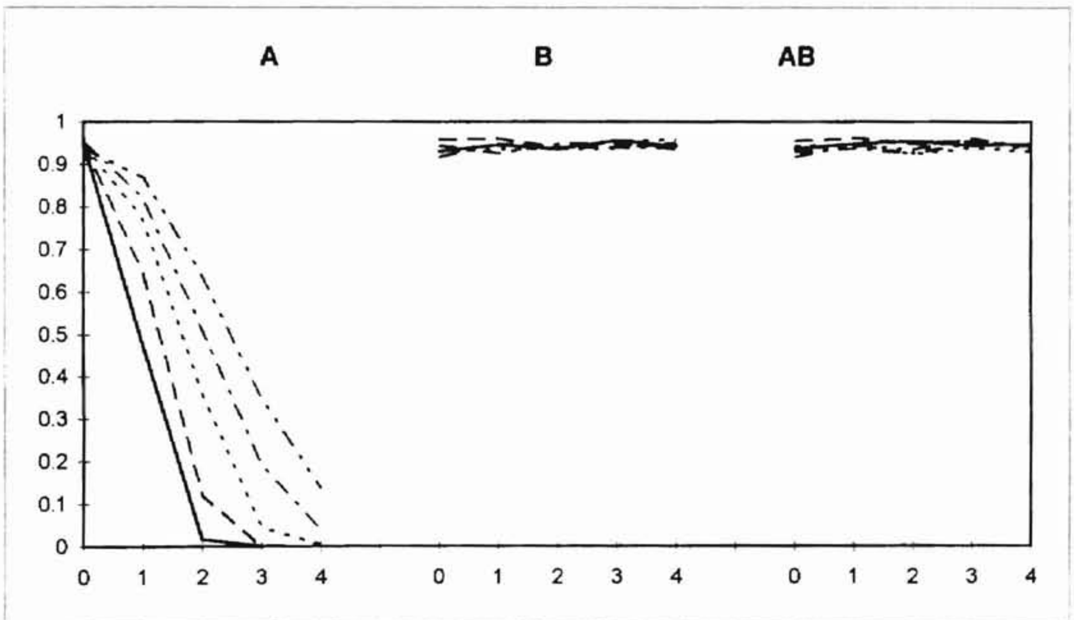
Appendix C: OC Curves for No-Interaction Data

Operating Characteristic curves
for 'no-interaction' data

Mean	A-
SD	A-



Mean	A-
SD	A+

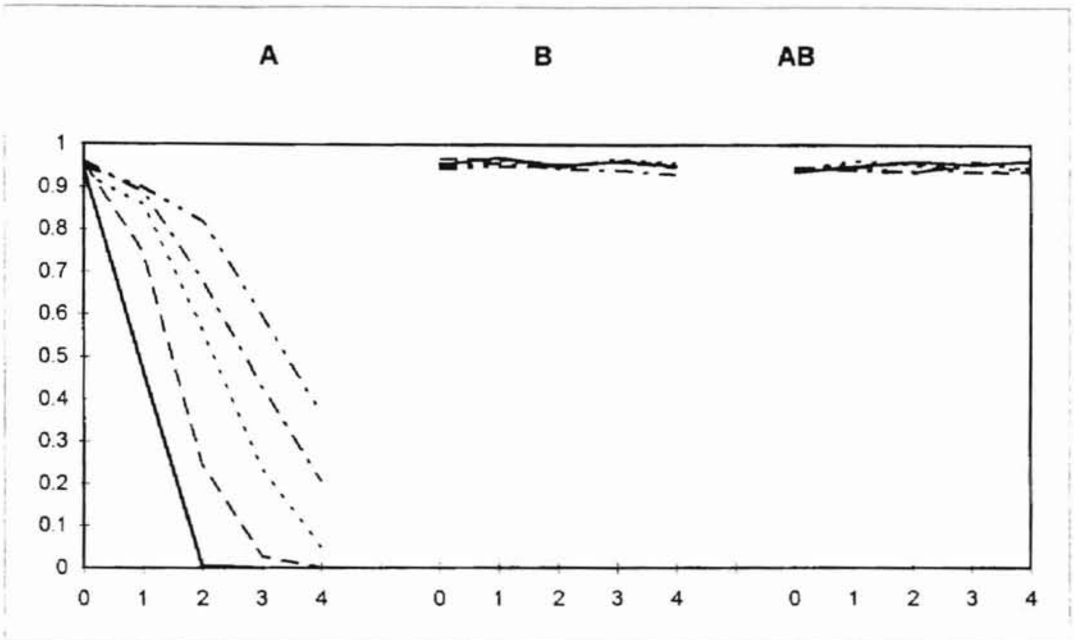


Legend

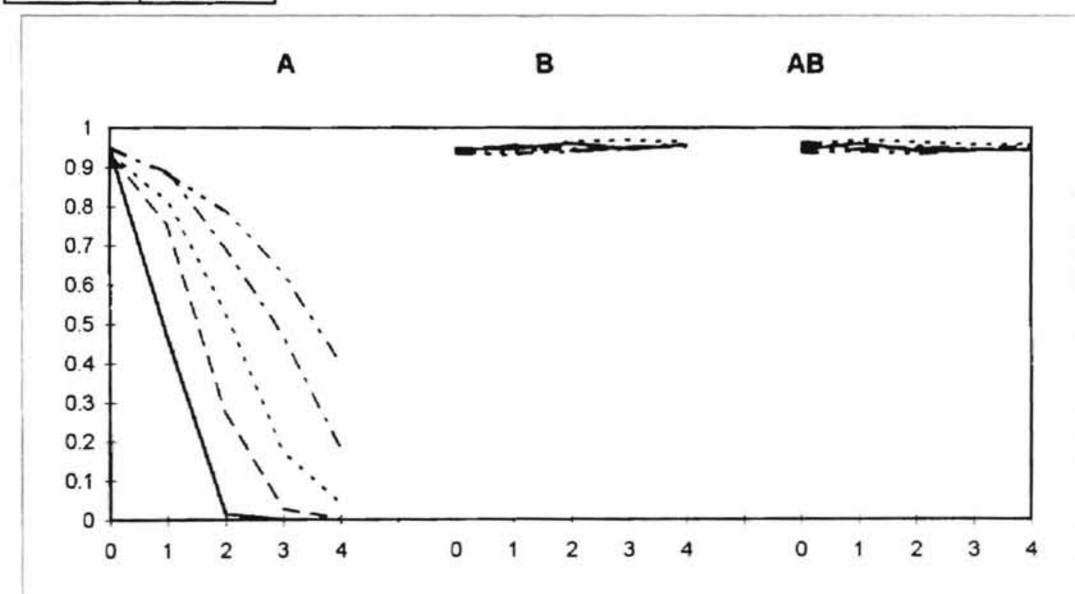
sd = 1	sd = 2	sd = 3	sd = 4	sd = 5

Operating Characteristic curves
for 'no-interaction' data

Mean	A-
SD	A-B-



Mean	A-
SD	A+B-

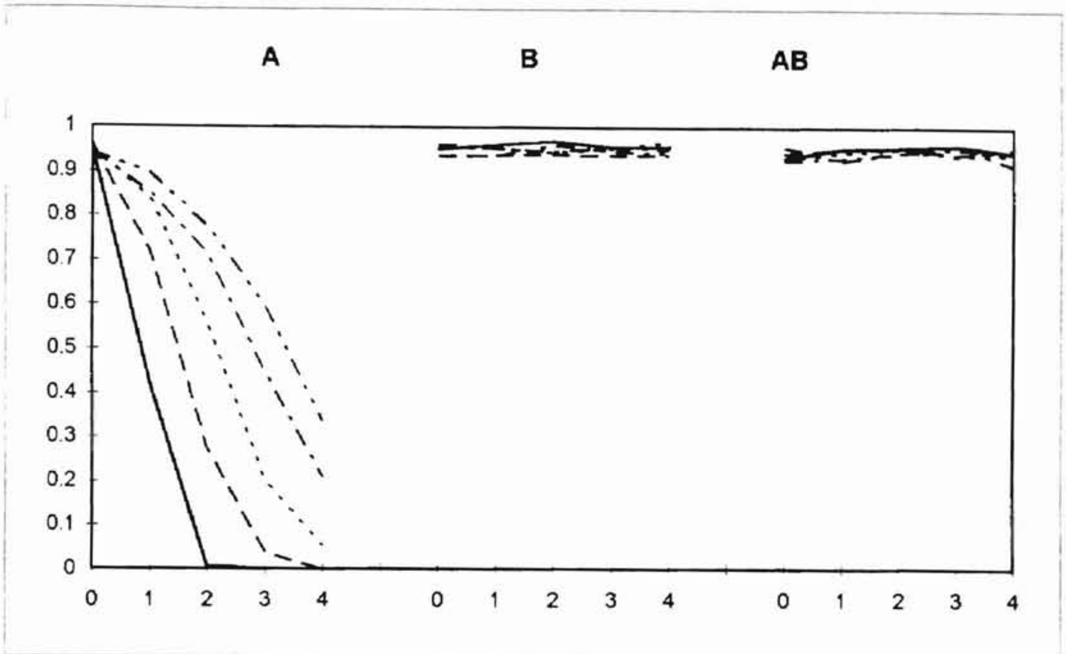


Legend

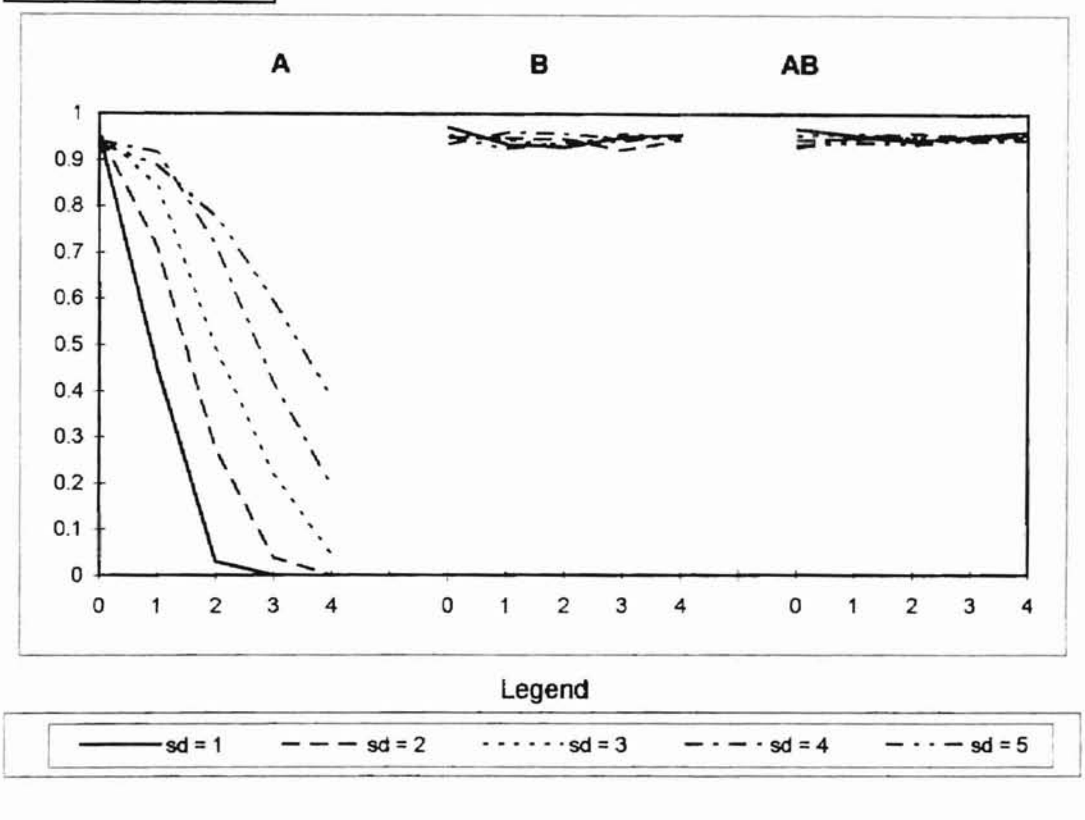
sd = 1 | sd = 2 | sd = 3 | sd = 4 | sd = 5

Operating Characteristic curves
for 'no-interaction' data

Mean	A-
SD	A-B+

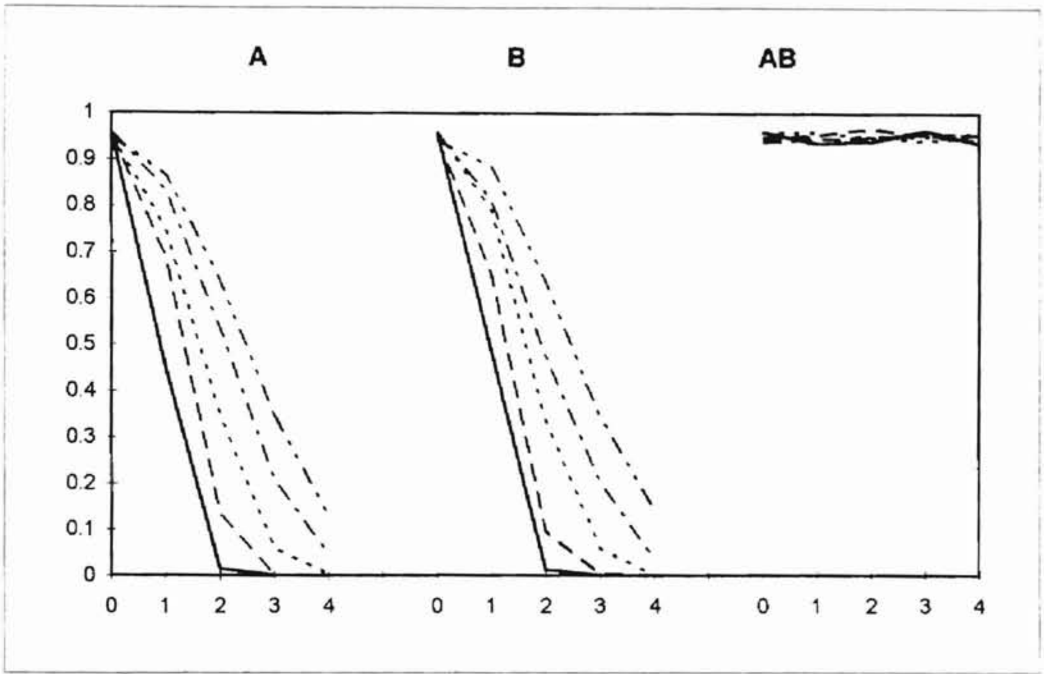


Mean	A-
SD	A+B+

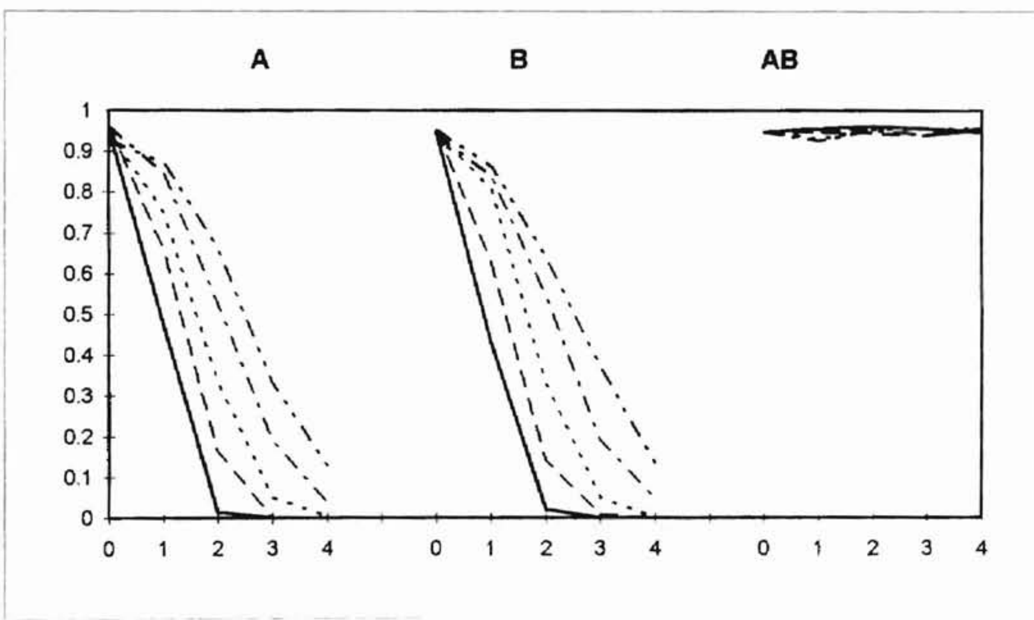


Operating Characteristic curves
for 'no-interaction' data

Mean	A-B-
SD	A-



Mean	A+B-
SD	A-

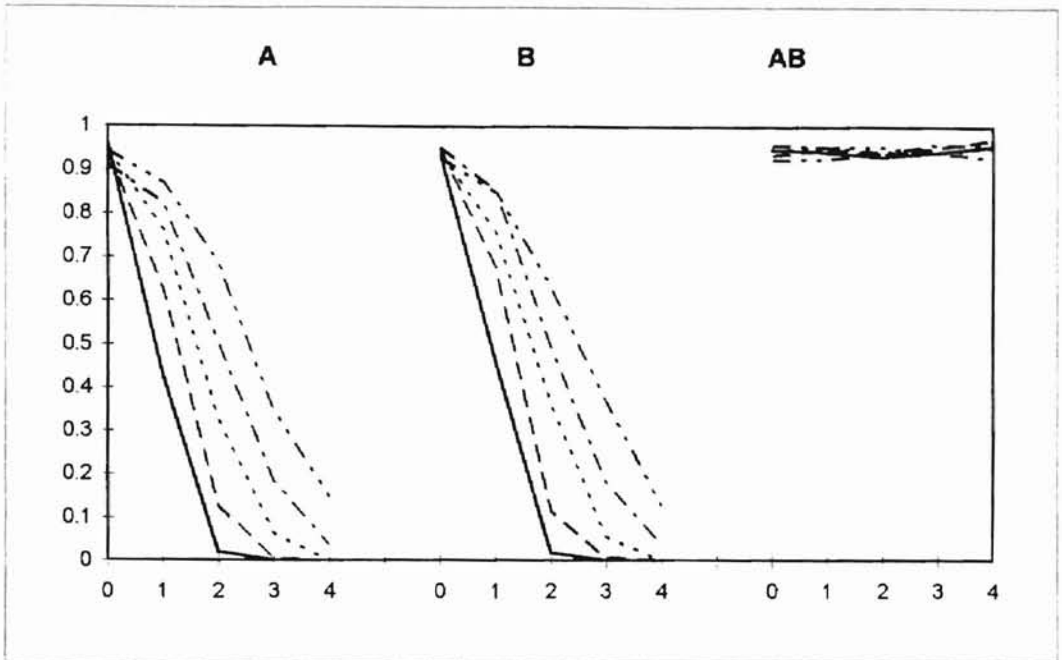


Legend

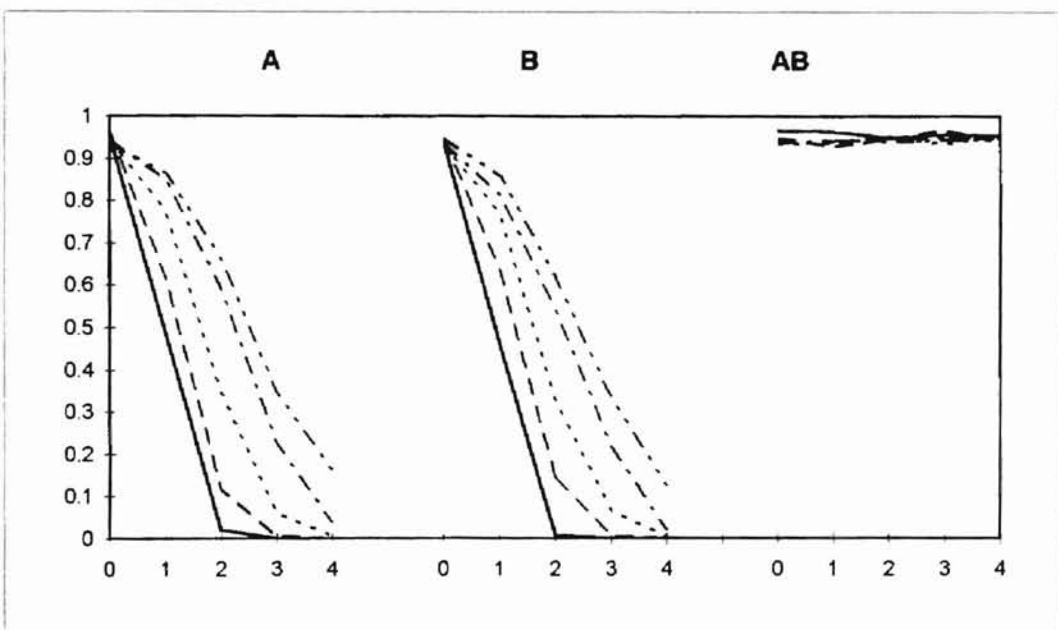
sd = 1 sd = 2 sd = 3 sd = 4 sd = 5

Operating Characteristic curves
for 'no-interaction' data

Mean	A-B+
SD	A-



Mean	A+B+
SD	A-

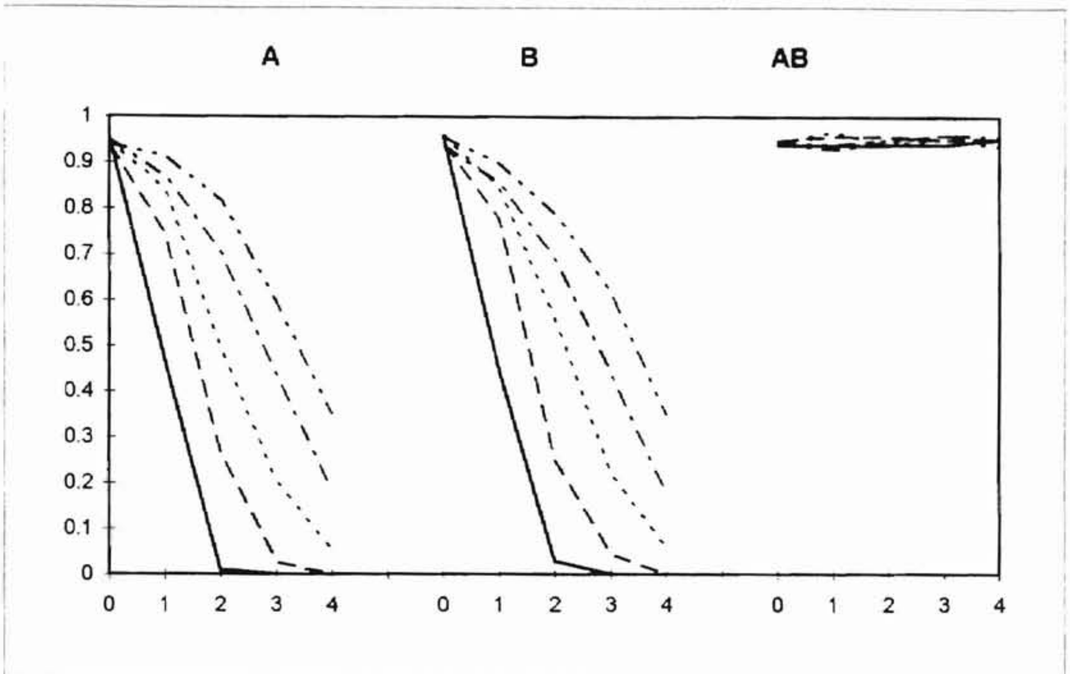


Legend

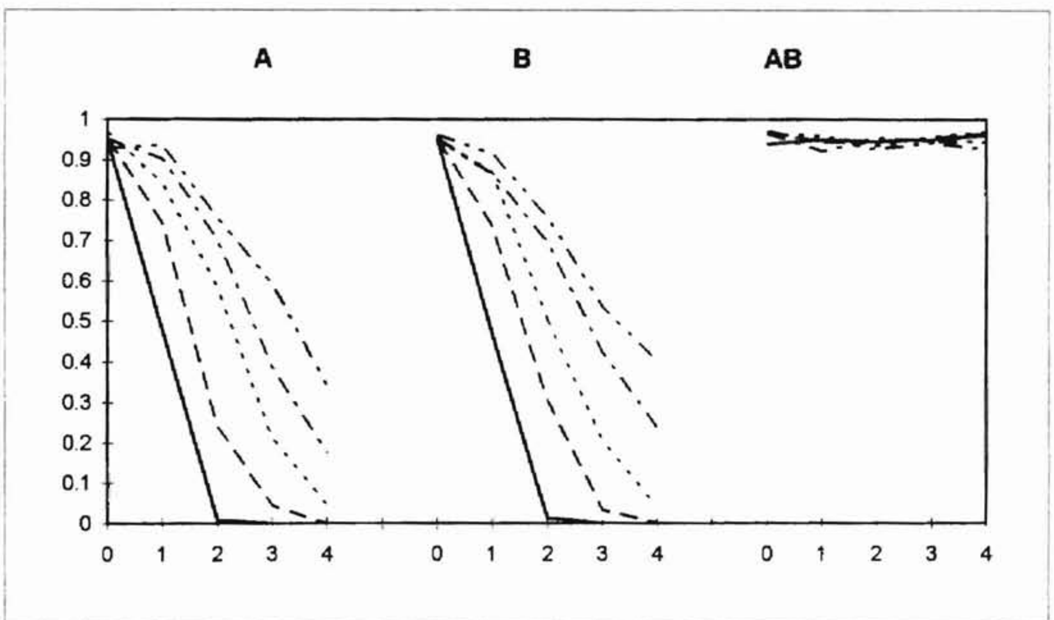
sd = 1 sd = 2 sd = 3 sd = 4 sd = 5

Operating Characteristic curves
for 'no-interaction' data

Mean	A-B-
SD	A-B-



Mean	A-B-
SD	A+B-

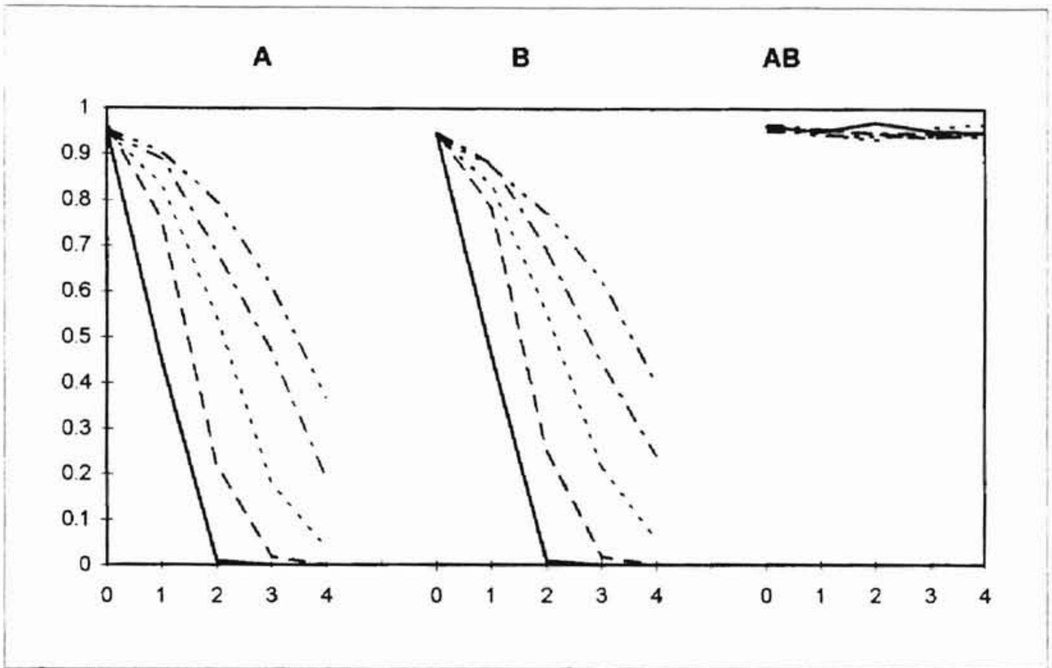


Legend

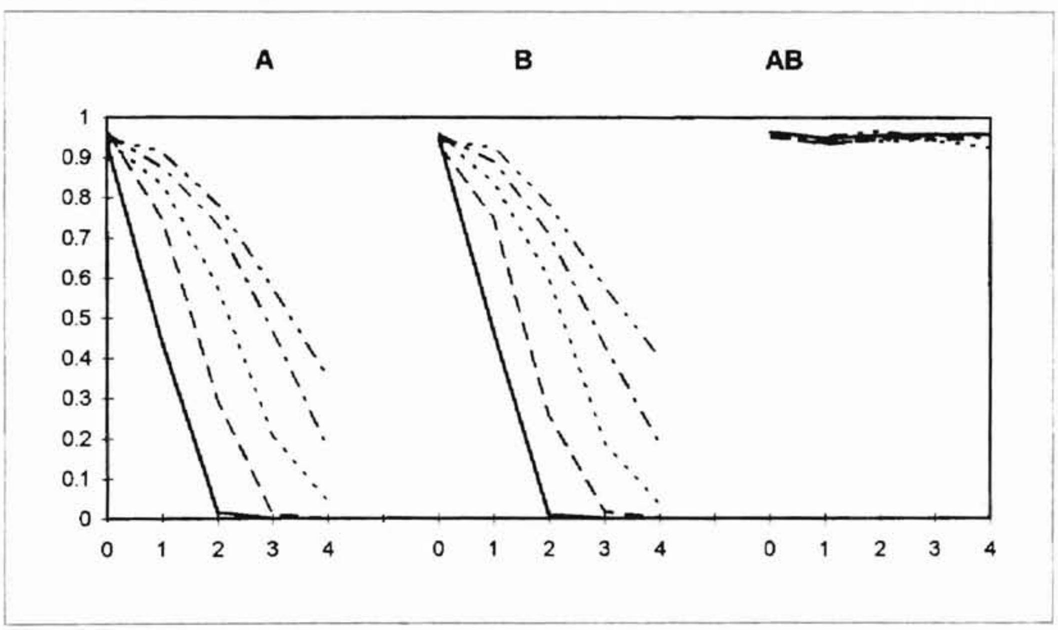
sd = 1 sd = 2 sd = 3 sd = 4 sd = 5

Operating Characteristic curves
for 'no-interaction' data

Mean	A-B-
SD	A-B+



Mean	A-B-
SD	A+B+

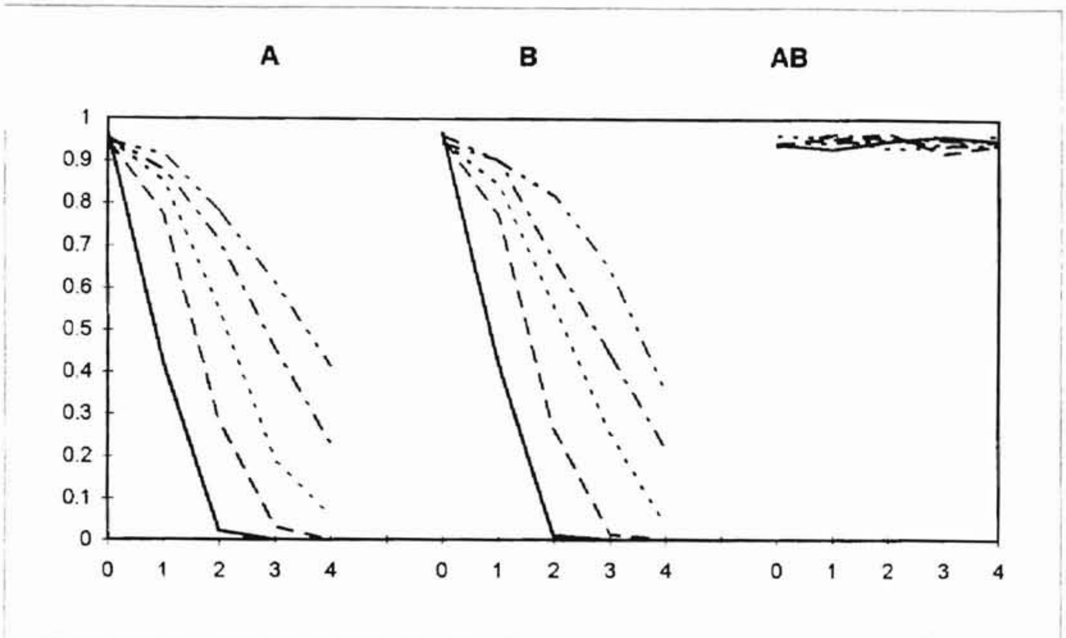


Legend

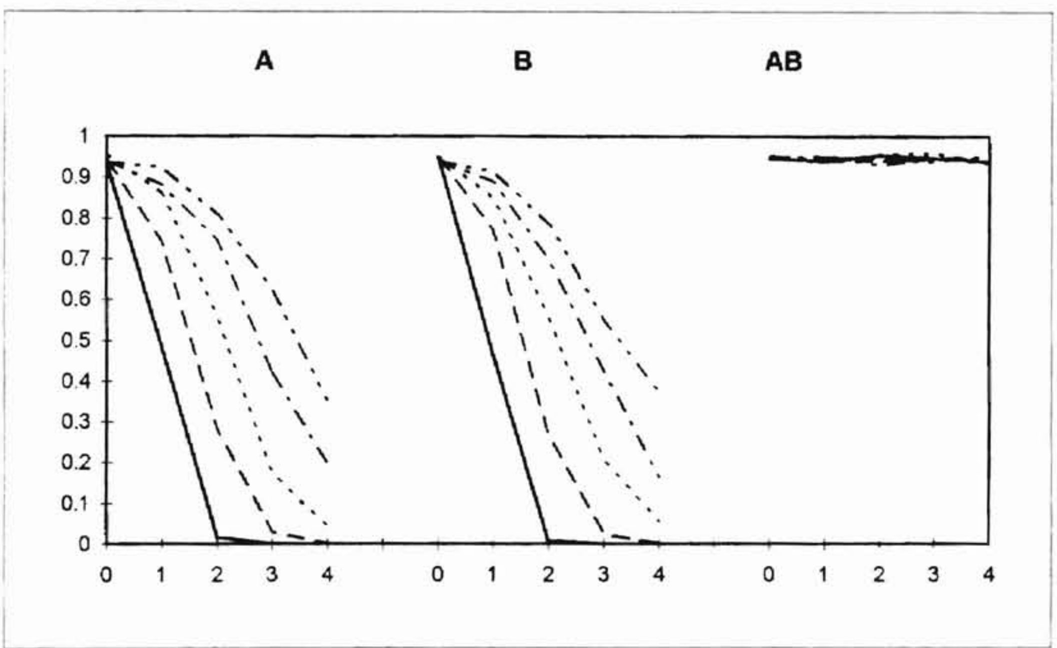
sd = 1 sd = 2 sd = 3 sd = 4 sd = 5

Operating Characteristic curves
for 'no-interaction' data

Mean	A+B-
SD	A-B-



Mean	A+B-
SD	A+B-

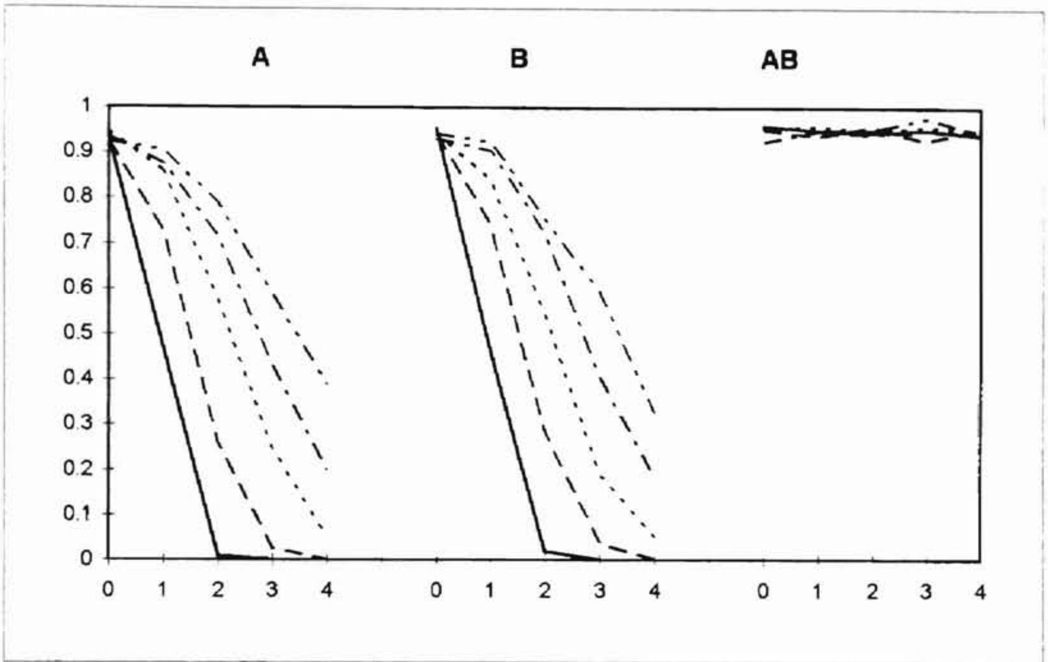


Legend

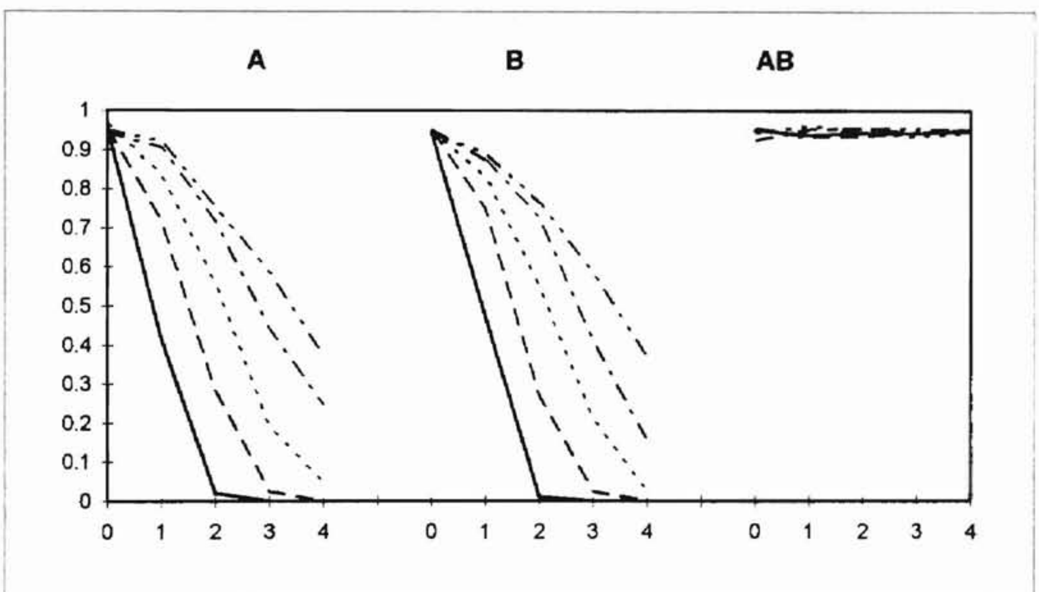
sd = 1   sd = 2   sd = 3   sd = 4   sd = 5

Operating Characteristic curves
for 'no-interaction' data

Mean	A+B-
SD	A-B+



Mean	A+B-
SD	A+B+

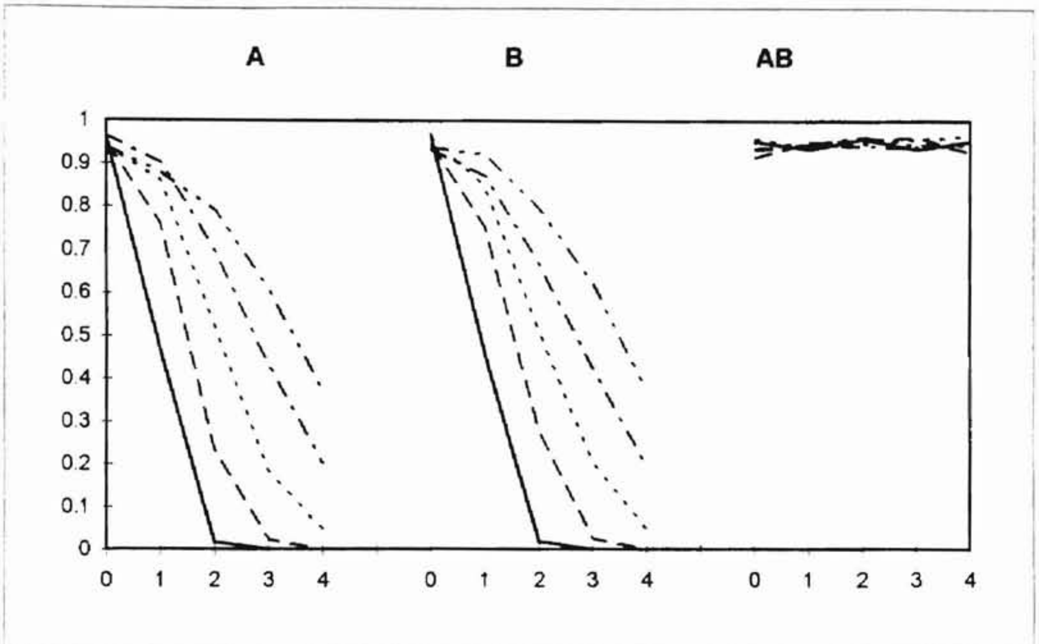


Legend

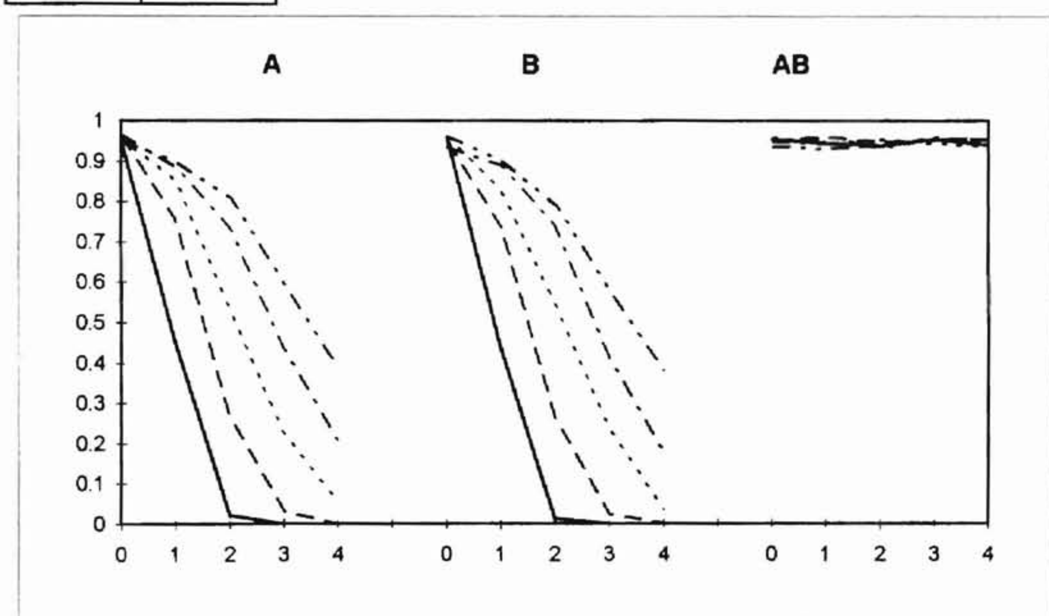
sd = 1 — sd = 2 — sd = 3 — sd = 4 — sd = 5

Operating Characteristic curves
for 'no-interaction' data

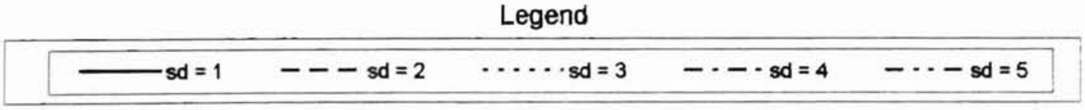
Mean	A-B+
SD	A-B-



Mean	A-B+
SD	A+B-

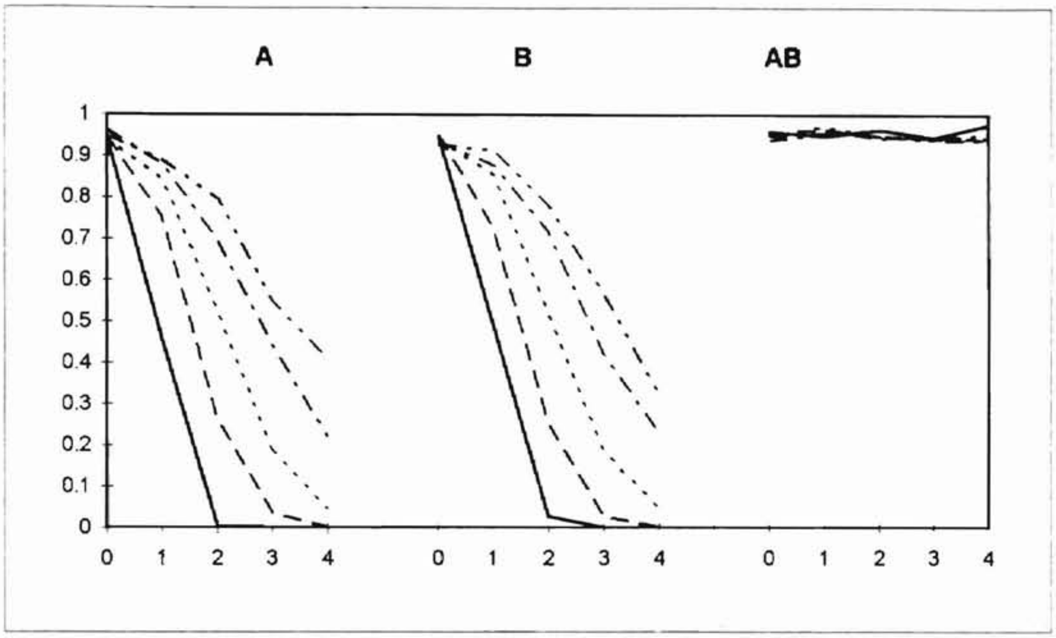


Legend

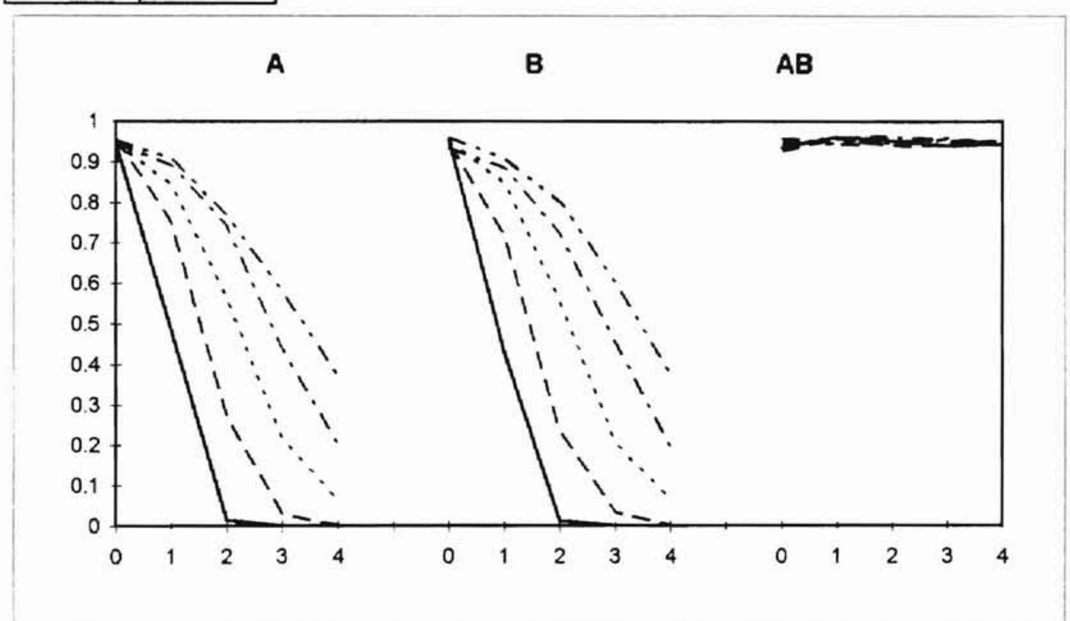


Operating Characteristic curves
for 'no-interaction' data

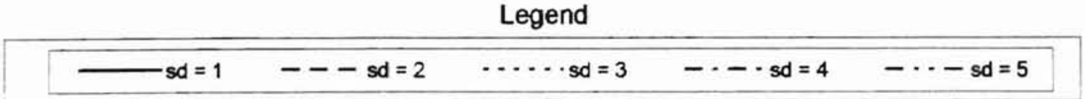
Mean	A-B+
SD	A-B+



Mean	A-B+
SD	A+B+

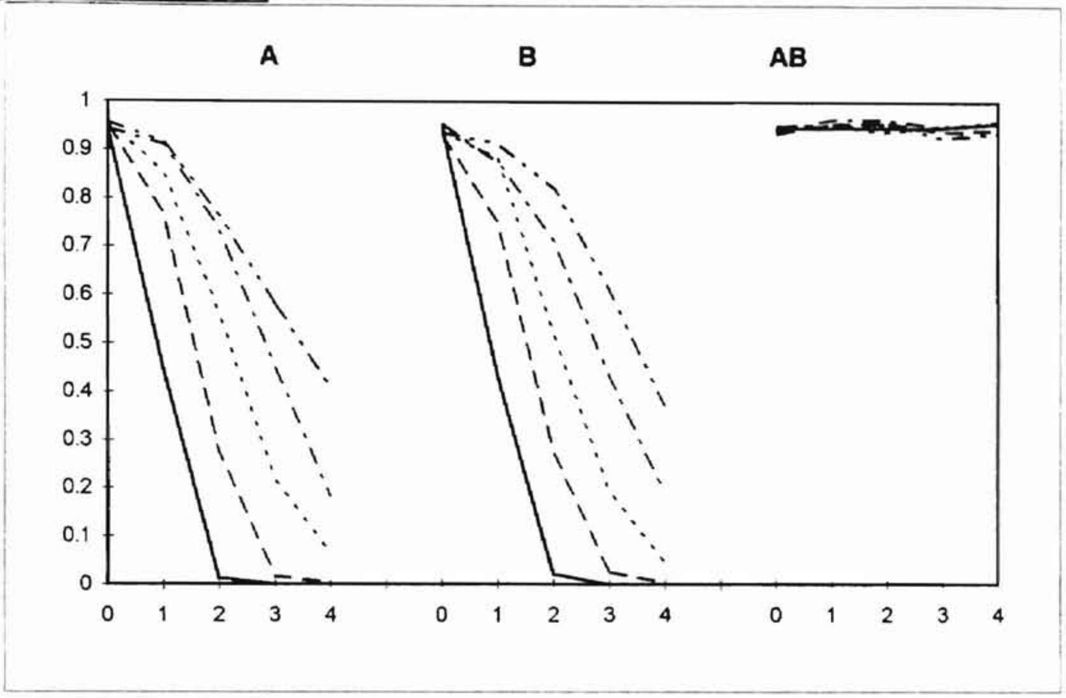


Legend

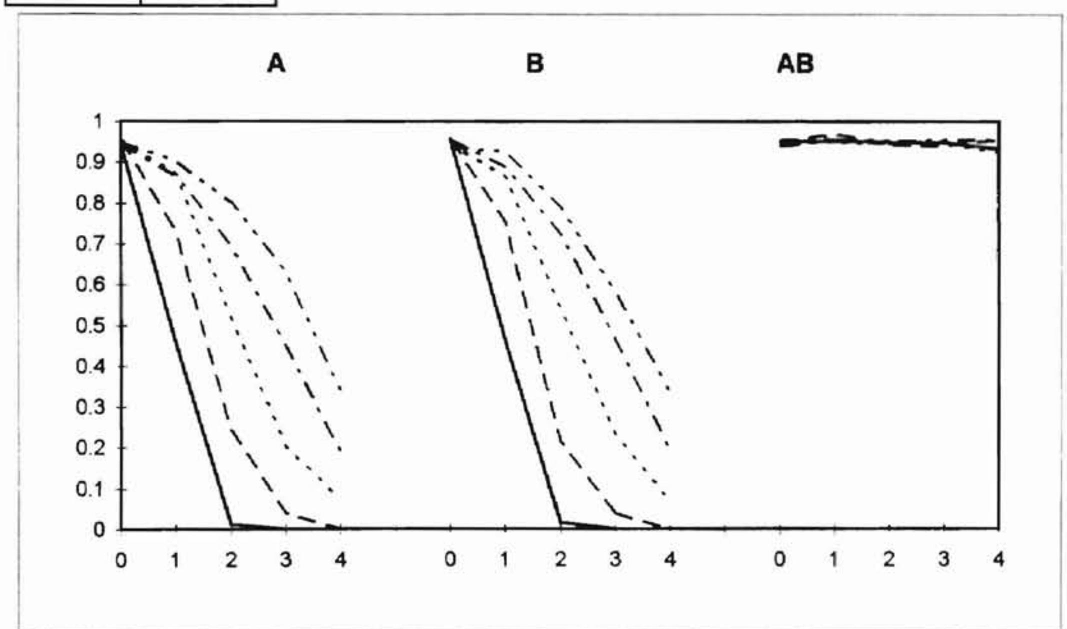


Operating Characteristic curves
for 'no-interaction' data

Mean	A+B+
SD	A-B-



Mean	A+B+
SD	A+B-

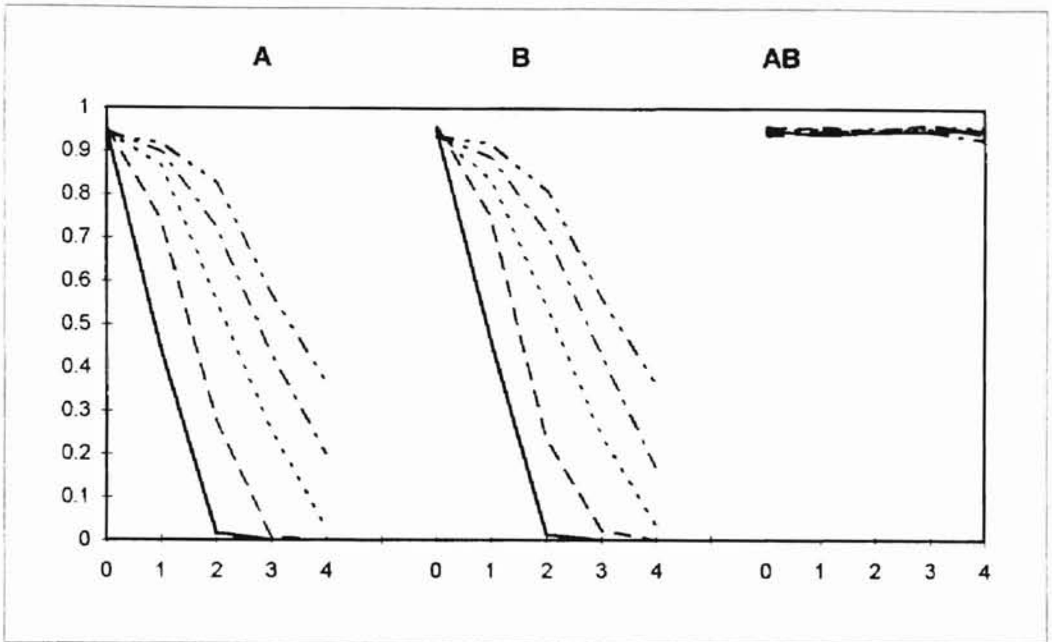


Legend

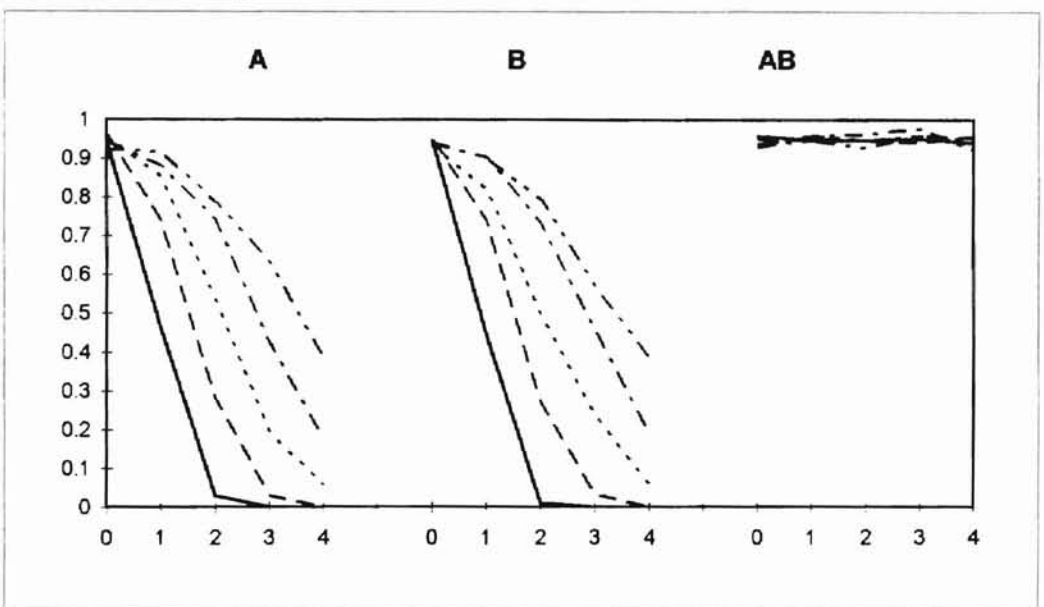
sd = 1 sd = 2 sd = 3 sd = 4 sd = 5

Operating Characteristic curves
for 'no-interaction' data

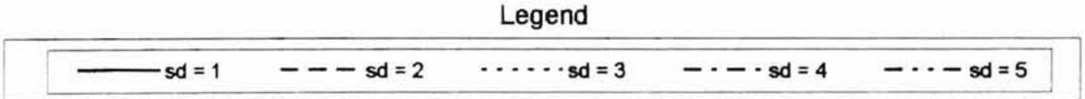
Mean	A+B+
SD	A-B+



Mean	A+B+
SD	A+B+



Legend



Appendix D: Hypothesis Test Results for No-Interaction Data

MEANS: A-
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-
SDs: A+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-
SDs: A+B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-
SDs: A+B+

	A	B	AB
mean = 0	Accept	Accept	Reject
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-B-
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A+B-
SDs: A-

	A	B	AB
mean = 0	Reject	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A-B+
SDs: A-

	A	B	AB
mean = 0	Reject	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Reject

MEANS: A+B+
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A-B-
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A-B-
SDs: A+B-

A	B	AB
Reject	Accept	Reject
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Reject

MEANS: A-B-
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A-B-
SDs: A+B+

A	B	AB
Accept	Accept	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept

MEANS: A+B-
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Accept

MEANS: A+B-
SDs: A+B-

A	B	AB
Accept	Accept	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept

MEANS: A+B-
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Reject
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Accept

MEANS: A+B-
SDs: A+B+

A	B	AB
Accept	Accept	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept

MEANS: A-B+
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Reject
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A-B+
SDs: A+B-

A	B	AB
Accept	Accept	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept
Reject	Reject	Accept

MEANS: A-B+
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Reject

MEANS: A-B+
SDs: A+B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A+B+
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A+B+
SDs: A+B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A+B+
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

MEANS: A+B+
SDs: A+B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Reject	Accept

Appendix E: With-Interaction Data

	MEANS:	A-		Reject hypothesis if		Hypothesis Test	
	SDs:	A-		test value > 9.488		Calc. Test Value	Result
A							
mean = 0	0.0520	0.0480	0.0480	0.0660	0.0400	3.783	Accept
mean = 1	0.2680	0.1740	0.1320	0.0960	0.0680	97.108	Reject
mean = 2	0.6000	0.4720	0.3160	0.2020	0.1600	300.097	Reject
mean = 3	0.7780	0.6640	0.5520	0.3640	0.3260	299.391	Reject
mean = 4	0.8720	0.7900	0.7120	0.6460	0.4960	197.185	Reject
B							
mean = 0	0.0320	0.0380	0.0540	0.0640	0.0540	7.417	Accept
mean = 1	0.0380	0.0480	0.0480	0.0460	0.0400	1.046	Accept
mean = 2	0.0520	0.0600	0.0680	0.0460	0.0500	2.961	Accept
mean = 3	0.0480	0.0520	0.0340	0.0600	0.0520	3.899	Accept
mean = 4	0.0580	0.0480	0.0440	0.0540	0.0540	1.259	Accept
AB							
mean = 0	0.0560	0.0500	0.0500	0.0580	0.0540	0.505	Accept
mean = 1	0.0380	0.0400	0.0440	0.0500	0.0340	1.883	Accept
mean = 2	0.0480	0.0340	0.0660	0.0660	0.0540	7.128	Accept
mean = 3	0.0400	0.0580	0.0520	0.0440	0.0500	2.120	Accept
mean = 4	0.0440	0.0540	0.0620	0.0700	0.0620	3.521	Accept

	MEANS:	A-		Reject hypothesis if		Hypothesis Test	
	SDs:	A+		test value > 9.488		Calc. Test Value	Result
A							
mean = 0	0.0540	0.0540	0.0560	0.0520	0.0640	0.832	Accept
mean = 1	0.2700	0.1860	0.1820	0.1420	0.1160	46.492	Reject
mean = 2	0.6580	0.5220	0.3500	0.2840	0.2360	255.891	Reject
mean = 3	0.7660	0.6860	0.5280	0.4380	0.3880	210.751	Reject
mean = 4	0.8640	0.7920	0.7220	0.5760	0.4900	220.879	Reject
B							
mean = 0	0.0480	0.0540	0.0620	0.0380	0.0580	3.570	Accept
mean = 1	0.0420	0.0440	0.0540	0.0560	0.0500	1.590	Accept
mean = 2	0.0720	0.0600	0.0560	0.0540	0.0500	2.575	Accept
mean = 3	0.0520	0.0580	0.0560	0.0400	0.0480	2.124	Accept
mean = 4	0.0580	0.0640	0.0640	0.0540	0.0400	3.708	Accept
AB							
mean = 0	0.0580	0.0620	0.0400	0.0500	0.0620	3.453	Accept
mean = 1	0.0480	0.0340	0.0440	0.0460	0.0440	1.413	Accept
mean = 2	0.0600	0.0420	0.0600	0.0460	0.0480	2.849	Accept
mean = 3	0.0460	0.0620	0.0540	0.0520	0.0440	2.076	Accept
mean = 4	0.0600	0.0520	0.0380	0.0280	0.0480	7.192	Accept

	MEANS:	A-		Reject hypothesis if	
	SDs:	A-B-		test value > 9.488	
A					
mean = 0	0.0560	0.0400	0.0480	0.0420	0.0640
mean = 1	0.2680	0.1060	0.0680	0.0440	0.0640
mean = 2	0.6060	0.2360	0.1200	0.0900	0.0860
mean = 3	0.7800	0.3780	0.1800	0.1180	0.0800
mean = 4	0.8700	0.4360	0.2480	0.1660	0.1120
B					
mean = 0	0.0460	0.0420	0.0540	0.0500	0.0660
mean = 1	0.0340	0.0300	0.0580	0.0320	0.0560
mean = 2	0.0580	0.0520	0.0640	0.0580	0.0620
mean = 3	0.0540	0.0440	0.0500	0.0460	0.0580
mean = 4	0.0560	0.0880	0.0520	0.0660	0.0680
AB					
mean = 0	0.0380	0.0380	0.0460	0.0500	0.0620
mean = 1	0.0600	0.0540	0.0680	0.0460	0.0620
mean = 2	0.0380	0.0420	0.0640	0.0700	0.0640
mean = 3	0.0500	0.0500	0.0520	0.0420	0.0560
mean = 4	0.0440	0.0600	0.0540	0.0680	0.0640

Hypothesis Test	
Calc. Test Value	Result
4.211	Accept
169.642	Reject
551.255	Reject
780.322	Reject
812.736	Reject
3.466	Accept
9.444	Accept
0.766	Accept
1.371	Accept
6.359	Accept
4.447	Accept
2.562	Accept
8.029	Accept
1.095	Accept
3.221	Accept

	MEANS:	A-		Reject hypothesis if	
	SDs:	A+B-		test value > 9.488	
A					
mean = 0	0.0440	0.0580	0.0540	0.0500	0.0480
mean = 1	0.2800	0.1260	0.1020	0.0500	0.0820
mean = 2	0.5900	0.2760	0.1320	0.1240	0.0900
mean = 3	0.7660	0.3680	0.2280	0.1140	0.0740
mean = 4	0.8580	0.4360	0.2160	0.1840	0.1220
B					
mean = 0	0.0380	0.0460	0.0460	0.0600	0.0360
mean = 1	0.0460	0.0620	0.0580	0.0560	0.0580
mean = 2	0.0400	0.0360	0.0380	0.0460	0.0720
mean = 3	0.0540	0.0600	0.0400	0.0600	0.0520
mean = 4	0.0420	0.0620	0.0520	0.0460	0.0620
AB					
mean = 0	0.0760	0.0460	0.0380	0.0380	0.0500
mean = 1	0.0460	0.0420	0.0360	0.0400	0.0520
mean = 2	0.0840	0.0460	0.0540	0.0560	0.0640
mean = 3	0.0680	0.0660	0.0580	0.0560	0.0640
mean = 4	0.0440	0.0540	0.0500	0.0540	0.0560

Hypothesis Test	
Calc. Test Value	Result
1.211	Accept
143.277	Reject
466.633	Reject
729.631	Reject
782.765	Reject
4.134	Accept
1.362	Accept
9.890	Reject
2.668	Accept
3.327	Accept
10.386	Reject
1.800	Accept
7.327	Accept
0.916	Accept
0.932	Accept

	MEANS:	A-		Reject hypothesis if		Hypothesis Test	
	SDs:	A-B+		test value > 9.488		Calc. Test Value	Result
			A				
mean = 0	0.0500	0.0560	0.0700	0.0540	0.0660	2.557	Accept
mean = 1	0.3100	0.0940	0.0580	0.0540	0.0820	220.423	Reject
mean = 2	0.5980	0.2440	0.1440	0.0760	0.0800	536.842	Reject
mean = 3	0.7520	0.3620	0.1920	0.0860	0.0720	766.252	Reject
mean = 4	0.8520	0.4620	0.2220	0.1620	0.1380	774.088	Reject
			B				
mean = 0	0.0460	0.0600	0.0420	0.0800	0.0520	8.550	Accept
mean = 1	0.0320	0.0640	0.0480	0.0440	0.0840	15.739	Reject
mean = 2	0.0400	0.0420	0.0680	0.0480	0.0540	5.299	Accept
mean = 3	0.0280	0.0620	0.0380	0.0540	0.0640	10.483	Reject
mean = 4	0.0600	0.0640	0.0480	0.0740	0.0540	3.475	Accept
			AB				
mean = 0	0.0420	0.0420	0.0540	0.0500	0.0440	1.302	Accept
mean = 1	0.0420	0.0460	0.0340	0.0600	0.0740	10.342	Reject
mean = 2	0.0440	0.0560	0.0500	0.0520	0.0440	1.163	Accept
mean = 3	0.0500	0.0600	0.0560	0.0540	0.0500	0.705	Accept
mean = 4	0.0660	0.0640	0.0620	0.0720	0.0640	0.483	Accept

	MEANS:	A-		Reject hypothesis if		Hypothesis Test	
	SDs:	A+B+		test value > 9.488		Calc. Test Value	Result
			A				
mean = 0	0.0300	0.0440	0.0320	0.0520	0.0480	4.820	Accept
mean = 1	0.2500	0.1600	0.0740	0.0720	0.0700	115.429	Reject
mean = 2	0.6360	0.2420	0.1480	0.1020	0.0980	552.086	Reject
mean = 3	0.8040	0.3780	0.2160	0.1200	0.0760	801.147	Reject
mean = 4	0.8540	0.4440	0.2500	0.1540	0.1140	791.430	Reject
			B				
mean = 0	0.0360	0.0540	0.0580	0.0400	0.0560	4.360	Accept
mean = 1	0.0540	0.0520	0.0420	0.0340	0.0660	6.313	Accept
mean = 2	0.0720	0.0520	0.0600	0.0540	0.0780	4.364	Accept
mean = 3	0.0360	0.0460	0.0420	0.0500	0.0720	8.089	Accept
mean = 4	0.0560	0.0500	0.0460	0.0660	0.0540	2.208	Accept
			AB				
mean = 0	0.0340	0.0520	0.0440	0.0620	0.0460	4.712	Accept
mean = 1	0.0420	0.0500	0.0500	0.0560	0.0600	1.913	Accept
mean = 2	0.0660	0.0360	0.0620	0.0400	0.0620	7.830	Accept
mean = 3	0.0440	0.0540	0.0600	0.0600	0.0620	2.043	Accept
mean = 4	0.0620	0.0740	0.0640	0.0500	0.0680	2.646	Accept

MEANS: A-B- Reject hypothesis if
SDs: A- test value > 9.488

						Calc. Test Value	Result
A							
mean = 0	0.0480	0.0520	0.0520	0.0440	0.0480	0.483	Accept
mean = 1	0.4760	0.2200	0.1740	0.1060	0.1180	266.327	Reject
mean = 2	0.9500	0.7480	0.5240	0.3680	0.2680	633.379	Reject
mean = 3	1.0000	0.9800	0.8480	0.6480	0.4860	596.087	Reject
mean = 4	1.0000	0.9960	0.9840	0.8660	0.7380	342.569	Reject
B							
mean = 0	0.0540	0.0480	0.0400	0.0420	0.0640	4.022	Accept
mean = 1	0.4380	0.2860	0.1620	0.1560	0.0800	226.503	Reject
mean = 2	0.9540	0.7740	0.5200	0.3840	0.2780	638.600	Reject
mean = 3	1.0000	0.9700	0.8440	0.6140	0.4980	579.686	Reject
mean = 4	1.0000	0.9980	0.9700	0.8620	0.7180	356.395	Reject
AB							
mean = 0	0.0440	0.0580	0.0540	0.0400	0.0600	3.178	Accept
mean = 1	0.0920	0.0860	0.0620	0.0760	0.0520	8.061	Accept
mean = 2	0.4220	0.2800	0.1660	0.1600	0.1060	181.581	Reject
mean = 3	0.8720	0.6560	0.3920	0.2840	0.1900	630.920	Reject
mean = 4	0.9940	0.9060	0.7520	0.5300	0.4240	582.892	Reject

Hypothesis Test

MEANS: A+B- Reject hypothesis if
SDs: A- test value > 9.488

						Calc. Test Value	Result
A							
mean = 0	0.0440	0.0360	0.0540	0.0420	0.0500	2.280	Accept
mean = 1	0.4320	0.2860	0.2060	0.1380	0.1280	173.868	Reject
mean = 2	0.9380	0.7340	0.5340	0.3600	0.2800	595.023	Reject
mean = 3	1.0000	0.9660	0.8320	0.6300	0.4800	580.508	Reject
mean = 4	1.0000	0.9980	0.9620	0.8300	0.6960	379.070	Reject
B							
mean = 0	0.0400	0.0500	0.0500	0.0520	0.0400	1.573	Accept
mean = 1	0.4400	0.2740	0.1540	0.1200	0.0980	236.780	Reject
mean = 2	0.9380	0.7400	0.5140	0.3420	0.2800	616.292	Reject
mean = 3	1.0000	0.9600	0.8520	0.6760	0.5020	534.951	Reject
mean = 4	1.0000	1.0000	0.9640	0.8620	0.7540	296.163	Reject
AB							
mean = 0	0.0380	0.0500	0.0560	0.0460	0.0440	2.026	Accept
mean = 1	0.0800	0.0780	0.0860	0.0940	0.0580	4.914	Accept
mean = 2	0.4300	0.2500	0.1840	0.1320	0.0940	205.002	Reject
mean = 3	0.8700	0.6260	0.4340	0.3060	0.2300	534.973	Reject
mean = 4	1.0000	0.9220	0.7280	0.5500	0.4360	576.078	Reject

Hypothesis Test

MEANS: A-B+ Reject hypothesis if
SDs: A- test value > 9.488

						Hypothesis Test	
						Calc. Test Value	Result
			A				
mean = 0	0.0360	0.0500	0.0600	0.0840	0.0380	15.093	Reject
mean = 1	0.4760	0.2760	0.1720	0.1300	0.0720	288.978	Reject
mean = 2	0.9340	0.7720	0.4820	0.3800	0.2300	673.983	Reject
mean = 3	1.0000	0.9720	0.8280	0.6680	0.4780	573.722	Reject
mean = 4	1.0000	1.0000	0.9720	0.8520	0.7120	371.625	Reject
			B				
mean = 0	0.0580	0.0560	0.0460	0.0560	0.0500	1.001	Accept
mean = 1	0.4400	0.2480	0.2040	0.1160	0.1280	194.956	Reject
mean = 2	0.9260	0.7300	0.4880	0.3820	0.2500	598.715	Reject
mean = 3	1.0000	0.9780	0.8240	0.6760	0.4800	574.904	Reject
mean = 4	1.0000	1.0000	0.9800	0.8680	0.6900	424.716	Reject
			AB				
mean = 0	0.0480	0.0580	0.0480	0.0400	0.0600	2.787	Accept
mean = 1	0.1120	0.0740	0.0660	0.0620	0.0720	11.263	Reject
mean = 2	0.4140	0.2220	0.1660	0.1320	0.1020	186.904	Reject
mean = 3	0.8700	0.6500	0.4140	0.3080	0.2040	581.657	Reject
mean = 4	0.9940	0.9320	0.7820	0.5340	0.4240	626.745	Reject

MEANS: A+B+ Reject hypothesis if
SDs: A- test value > 9.488

						Hypothesis Test	
						Calc. Test Value	Result
			A				
mean = 0	0.0500	0.0380	0.0420	0.0600	0.0560	3.643	Accept
mean = 1	0.4100	0.2920	0.1820	0.1420	0.1220	163.823	Reject
mean = 2	0.9540	0.7500	0.5200	0.3380	0.2860	646.191	Reject
mean = 3	1.0000	0.9720	0.8220	0.6320	0.5060	547.115	Reject
mean = 4	1.0000	0.9960	0.9680	0.8200	0.6840	403.302	Reject
			B				
mean = 0	0.0500	0.0520	0.0480	0.0560	0.0580	0.688	Accept
mean = 1	0.3960	0.2580	0.1640	0.1560	0.1120	150.793	Reject
mean = 2	0.9500	0.7340	0.5220	0.3820	0.3100	566.525	Reject
mean = 3	0.9980	0.9700	0.8480	0.6380	0.4900	576.624	Reject
mean = 4	1.0000	1.0000	0.9800	0.8880	0.7300	361.957	Reject
			AB				
mean = 0	0.0380	0.0440	0.0580	0.0600	0.0440	4.016	Accept
mean = 1	0.0980	0.0860	0.0760	0.0560	0.0580	9.398	Accept
mean = 2	0.4260	0.2780	0.1720	0.1600	0.1280	166.146	Reject
mean = 3	0.9000	0.6780	0.4540	0.3060	0.2400	596.219	Reject
mean = 4	0.9880	0.8880	0.7160	0.5060	0.3880	600.356	Reject

	MEANS:	A-B-		Reject hypothesis if	
	SDs:	A-B-		test value > 9.488	
A					
mean = 0	0.0540	0.0460	0.0480	0.0460	0.0460
mean = 1	0.4480	0.1380	0.0920	0.0520	0.0500
mean = 2	0.9420	0.5840	0.2820	0.1720	0.0960
mean = 3	1.0000	0.9100	0.5620	0.3320	0.1860
mean = 4	1.0000	0.9940	0.8680	0.5720	0.3580
B					
mean = 0	0.0380	0.0400	0.0460	0.0520	0.0400
mean = 1	0.4400	0.1540	0.0780	0.0700	0.0560
mean = 2	0.9300	0.5980	0.2700	0.1540	0.0900
mean = 3	1.0000	0.9080	0.5640	0.3360	0.1660
mean = 4	1.0000	0.9940	0.8500	0.6000	0.3320
AB					
mean = 0	0.0400	0.0460	0.0440	0.0700	0.0480
mean = 1	0.1020	0.0480	0.0420	0.0580	0.0600
mean = 2	0.4260	0.1800	0.0800	0.0820	0.0520
mean = 3	0.8920	0.4920	0.1960	0.1280	0.0720
mean = 4	0.9880	0.8180	0.4820	0.2900	0.1460

Hypothesis Test	
Calc. Test Value	Result
0.525	Accept
424.323	Reject
998.293	Reject
1041.498	Reject
875.829	Reject
1.606	Accept
387.970	Reject
1020.795	Reject
1066.492	Reject
890.138	Reject
5.889	Accept
19.052	Reject
347.269	Reject
1012.004	Reject
1006.022	Reject

	MEANS:	A-B-		Reject hypothesis if	
	SDs:	A+B-		test value > 9.488	
A					
mean = 0	0.0340	0.0600	0.0440	0.0580	0.0500
mean = 1	0.4320	0.2060	0.1200	0.0820	0.0740
mean = 2	0.9500	0.5960	0.2900	0.1960	0.1580
mean = 3	0.9980	0.8460	0.6140	0.3940	0.2660
mean = 4	1.0000	0.9860	0.7780	0.6140	0.4040
B					
mean = 0	0.0360	0.0420	0.0400	0.0560	0.0360
mean = 1	0.4360	0.1360	0.0840	0.0460	0.0500
mean = 2	0.9560	0.5300	0.3040	0.1720	0.1180
mean = 3	1.0000	0.9140	0.5660	0.3500	0.2240
mean = 4	1.0000	0.9860	0.8280	0.5880	0.3860
AB					
mean = 0	0.0480	0.0340	0.0480	0.0620	0.0500
mean = 1	0.0920	0.0760	0.0520	0.0640	0.0660
mean = 2	0.4420	0.2000	0.1200	0.1020	0.1020
mean = 3	0.9140	0.4620	0.2740	0.2080	0.1460
mean = 4	0.9940	0.7960	0.4820	0.3280	0.2320

Hypothesis Test	
Calc. Test Value	Result
4.840	Accept
296.487	Reject
905.881	Reject
788.851	Reject
697.353	Reject
3.380	Accept
419.327	Reject
957.999	Reject
973.915	Reject
769.788	Reject
4.290	Accept
6.882	Accept
269.259	Reject
802.161	Reject
837.447	Reject

	MEANS: A-B- SDs: A-B+			Reject hypothesis if test value > 9.488		Hypothesis Test Calc. Test Value	Result
			A				
mean = 0	0.0380	0.0280	0.0640	0.0540	0.0560	9.366	Accept
mean = 1	0.4860	0.1720	0.0860	0.0600	0.0600	458.502	Reject
mean = 2	0.9520	0.5780	0.2780	0.1560	0.1040	1025.623	Reject
mean = 3	1.0000	0.8920	0.5980	0.3400	0.2260	951.286	Reject
mean = 4	1.0000	0.9980	0.8180	0.5760	0.3960	773.055	Reject
			B				
mean = 0	0.0600	0.0540	0.0600	0.0500	0.0480	1.197	Accept
mean = 1	0.4120	0.1960	0.1180	0.0640	0.0680	293.947	Reject
mean = 2	0.9440	0.5920	0.3140	0.2180	0.1360	881.746	Reject
mean = 3	1.0000	0.8900	0.5680	0.3860	0.2200	915.601	Reject
mean = 4	1.0000	0.9780	0.7860	0.5280	0.3900	756.611	Reject
			AB				
mean = 0	0.0520	0.0360	0.0380	0.0540	0.0620	5.332	Accept
mean = 1	0.0900	0.0600	0.0520	0.0600	0.0520	8.400	Accept
mean = 2	0.4080	0.2040	0.1640	0.0820	0.0880	229.659	Reject
mean = 3	0.8660	0.4880	0.2660	0.1760	0.1060	800.920	Reject
mean = 4	0.9940	0.7920	0.4780	0.3200	0.2200	856.872	Reject

	MEANS: A-B- SDs: A+B+			Reject hypothesis if test value > 9.488		Hypothesis Test Calc. Test Value	Result
			A				
mean = 0	0.0480	0.0580	0.0520	0.0340	0.0460	3.476	Accept
mean = 1	0.4480	0.1840	0.1180	0.0760	0.0660	339.363	Reject
mean = 2	0.9520	0.5420	0.2680	0.1500	0.1300	987.832	Reject
mean = 3	1.0000	0.8820	0.5540	0.3220	0.2380	936.805	Reject
mean = 4	1.0000	0.9740	0.8140	0.5760	0.4140	703.387	Reject
			B				
mean = 0	0.0480	0.0500	0.0400	0.0540	0.0700	4.946	Accept
mean = 1	0.4280	0.1960	0.1280	0.1040	0.0820	260.997	Reject
mean = 2	0.9540	0.5280	0.2760	0.1520	0.1360	970.925	Reject
mean = 3	0.9960	0.8720	0.5640	0.3640	0.2180	905.636	Reject
mean = 4	1.0000	0.9620	0.8060	0.5780	0.3500	776.988	Reject
			AB				
mean = 0	0.0480	0.0540	0.0500	0.0420	0.0580	1.538	Accept
mean = 1	0.0680	0.0640	0.0560	0.0560	0.0480	2.211	Accept
mean = 2	0.4280	0.1940	0.1000	0.1000	0.0700	295.371	Reject
mean = 3	0.8800	0.4800	0.2340	0.1480	0.0940	895.398	Reject
mean = 4	0.9980	0.7800	0.4600	0.2760	0.1960	930.115	Reject

MEANS: A+B- Reject hypothesis if

SDs: A-B- test value > 9.488

						Hypothesis Test: Calc. Test Value	Hypothesis Test: Result
A							
mean = 0	0.0340	0.0560	0.0600	0.0460	0.0600	5.155	Accept
mean = 1	0.4940	0.1920	0.1180	0.1000	0.0720	380.315	Reject
mean = 2	0.9300	0.5780	0.3100	0.2220	0.1380	843.341	Reject
mean = 3	1.0000	0.8720	0.5980	0.3180	0.2180	958.107	Reject
mean = 4	1.0000	0.9900	0.8320	0.5340	0.4200	763.348	Reject
B							
mean = 0	0.0300	0.0420	0.0460	0.0580	0.0380	5.233	Accept
mean = 1	0.4380	0.1540	0.0960	0.0620	0.0540	378.873	Reject
mean = 2	0.9500	0.5620	0.2700	0.1760	0.1000	1001.603	Reject
mean = 3	1.0000	0.8820	0.5860	0.3140	0.2220	968.141	Reject
mean = 4	1.0000	0.9860	0.8040	0.5280	0.3720	808.250	Reject
AB							
mean = 0	0.0560	0.0580	0.0360	0.0560	0.0340	6.215	Accept
mean = 1	0.1140	0.0580	0.0520	0.0540	0.0580	22.053	Reject
mean = 2	0.4140	0.2060	0.1380	0.0940	0.0780	247.074	Reject
mean = 3	0.8780	0.4720	0.2600	0.1620	0.1060	838.656	Reject
mean = 4	1.0000	0.7900	0.5200	0.3040	0.2500	835.547	Reject

MEANS: A+B- Reject hypothesis if

SDs: A+B- test value > 9.488

						Hypothesis Test: Calc. Test Value	Hypothesis Test: Result
A							
mean = 0	0.0500	0.0560	0.0320	0.0540	0.0480	3.939	Accept
mean = 1	0.3760	0.1540	0.1060	0.0540	0.0440	293.130	Reject
mean = 2	0.9540	0.5640	0.2660	0.1420	0.0980	1057.166	Reject
mean = 3	1.0000	0.9360	0.5760	0.3040	0.1820	1115.902	Reject
mean = 4	1.0000	0.9880	0.8740	0.5880	0.3800	826.761	Reject
B							
mean = 0	0.0500	0.0440	0.0480	0.0620	0.0500	1.875	Accept
mean = 1	0.4360	0.1380	0.0800	0.0480	0.0600	407.648	Reject
mean = 2	0.9500	0.5480	0.2360	0.1480	0.0980	1057.613	Reject
mean = 3	1.0000	0.9060	0.5700	0.3260	0.2120	1002.616	Reject
mean = 4	1.0000	0.9940	0.8660	0.6180	0.3680	836.463	Reject
AB							
mean = 0	0.0440	0.0560	0.0380	0.0540	0.0440	2.544	Accept
mean = 1	0.0800	0.0520	0.0580	0.0500	0.0700	5.571	Accept
mean = 2	0.4220	0.1580	0.0900	0.0780	0.0680	324.696	Reject
mean = 3	0.9100	0.4760	0.2420	0.1240	0.0840	993.038	Reject
mean = 4	0.9980	0.7940	0.4740	0.2480	0.1960	971.285	Reject

	MEANS:	A+B-		Reject hypothesis if	
	SDs:	A-B+		test value > 9.488	
A					
mean = 0	0.0680	0.0640	0.0660	0.0500	0.0580
mean = 1	0.4060	0.1940	0.0960	0.1000	0.0820
mean = 2	0.9560	0.5680	0.2960	0.1980	0.1220
mean = 3	0.9960	0.8660	0.5360	0.3160	0.1880
mean = 4	1.0000	0.9900	0.8180	0.5380	0.3640
B					
mean = 0	0.0500	0.0460	0.0640	0.0620	0.0500
mean = 1	0.4460	0.1980	0.1180	0.0780	0.0680
mean = 2	0.9360	0.5480	0.3100	0.1860	0.1220
mean = 3	0.9980	0.8720	0.5800	0.3340	0.2240
mean = 4	1.0000	0.9760	0.7900	0.5680	0.4080
AB					
mean = 0	0.0300	0.0580	0.0440	0.0660	0.0500
mean = 1	0.0720	0.0620	0.0420	0.0540	0.0600
mean = 2	0.4400	0.1840	0.1140	0.0680	0.0540
mean = 3	0.8660	0.5060	0.2800	0.1620	0.0860
mean = 4	0.9960	0.8140	0.5020	0.2600	0.1980

Hypothesis Test	
Calc. Test Value	Result
1.852	Accept
256.400	Reject
944.268	Reject
985.783	Reject
831.463	Reject
2.519	Accept
329.223	Reject
899.393	Reject
928.137	Reject
704.341	Reject
8.010	Accept
4.466	Accept
351.337	Reject
840.475	Reject
968.982	Reject

	MEANS:	A+B-		Reject hypothesis if	
	SDs:	A+B+		test value > 9.488	
A					
mean = 0	0.0480	0.0380	0.0460	0.0440	0.0460
mean = 1	0.4780	0.1580	0.0980	0.0660	0.0740
mean = 2	0.9440	0.5480	0.2480	0.1900	0.1560
mean = 3	1.0000	0.8720	0.5920	0.3400	0.1840
mean = 4	1.0000	0.9860	0.8200	0.5340	0.3420
B					
mean = 0	0.0400	0.0440	0.0440	0.0240	0.0420
mean = 1	0.4520	0.1880	0.1160	0.0920	0.0880
mean = 2	0.9620	0.5700	0.2980	0.2000	0.1420
mean = 3	0.9980	0.8640	0.6360	0.4020	0.2680
mean = 4	1.0000	0.9840	0.8100	0.5900	0.4260
AB					
mean = 0	0.0620	0.0400	0.0360	0.0280	0.0420
mean = 1	0.1120	0.0780	0.0560	0.0540	0.0820
mean = 2	0.4120	0.2080	0.1240	0.0800	0.0700
mean = 3	0.8960	0.5080	0.3160	0.1800	0.1380
mean = 4	0.9980	0.7740	0.4880	0.3340	0.2300

Hypothesis Test	
Calc. Test Value	Result
0.698	Accept
416.336	Reject
911.189	Reject
986.956	Reject
863.640	Reject
3.818	Accept
309.198	Reject
927.557	Reject
803.742	Reject
691.215	Reject
7.966	Accept
15.725	Reject
271.867	Reject
789.960	Reject
819.128	Reject

	MEANS: A-B+ SDs: A-B-			Reject hypothesis if test value > 9.488		Hypothesis Test: Calc. Test Value	Result
			A				
mean = 0	0.0360	0.0520	0.0460	0.0540	0.0480	2.188	Accept
mean = 1	0.4520	0.1440	0.0820	0.0620	0.0440	433.437	Reject
mean = 2	0.9500	0.5900	0.3300	0.1860	0.1120	949.943	Reject
mean = 3	1.0000	0.8980	0.5600	0.3640	0.2120	956.464	Reject
mean = 4	1.0000	0.9860	0.8280	0.5520	0.3520	843.609	Reject
			B				
mean = 0	0.0280	0.0660	0.0540	0.0440	0.0640	10.095	Reject
mean = 1	0.4700	0.2220	0.1480	0.0760	0.0600	353.122	Reject
mean = 2	0.9240	0.5620	0.3260	0.2120	0.1420	820.022	Reject
mean = 3	1.0000	0.8580	0.6000	0.3720	0.2500	845.018	Reject
mean = 4	1.0000	0.9800	0.7900	0.5900	0.4060	701.719	Reject
			AB				
mean = 0	0.0460	0.0680	0.0540	0.0540	0.0400	4.463	Accept
mean = 1	0.1320	0.0560	0.0560	0.0460	0.0480	41.783	Reject
mean = 2	0.4060	0.1960	0.1140	0.0900	0.0700	262.181	Reject
mean = 3	0.8960	0.4660	0.2700	0.1780	0.1440	801.571	Reject
mean = 4	0.9940	0.7760	0.4980	0.3080	0.2580	800.962	Reject

	MEANS: A-B+ SDs: A+B-			Reject hypothesis if test value > 9.488		Hypothesis Test: Calc. Test Value	Result
			A				
mean = 0	0.0500	0.0300	0.0360	0.0480	0.0480	3.882	Accept
mean = 1	0.4540	0.2000	0.1260	0.0720	0.0580	352.374	Reject
mean = 2	0.9360	0.5660	0.3080	0.1520	0.1020	975.273	Reject
mean = 3	1.0000	0.8580	0.5640	0.3240	0.1920	969.212	Reject
mean = 4	1.0000	0.9900	0.7880	0.5640	0.3660	791.223	Reject
			B				
mean = 0	0.0420	0.0420	0.0560	0.0540	0.0460	1.926	Accept
mean = 1	0.4480	0.1980	0.1260	0.0740	0.0580	341.570	Reject
mean = 2	0.9580	0.5400	0.3220	0.1780	0.1260	937.881	Reject
mean = 3	0.9960	0.8640	0.5320	0.3660	0.2240	887.392	Reject
mean = 4	1.0000	0.9820	0.8280	0.5260	0.3740	819.177	Reject
			AB				
mean = 0	0.0480	0.0740	0.0580	0.0460	0.0660	5.121	Accept
mean = 1	0.0820	0.0460	0.0640	0.0680	0.0560	6.121	Accept
mean = 2	0.4460	0.2000	0.0820	0.1140	0.0680	327.924	Reject
mean = 3	0.8780	0.5000	0.2600	0.1560	0.1040	855.942	Reject
mean = 4	0.9920	0.7920	0.4680	0.3020	0.1860	917.722	Reject

	MEANS: A-B+ SDs: A-B+			Reject hypothesis if test value > 9.488		Hypothesis Test: Calc. Test Value	Result
A							
mean = 0	0.0600	0.0420	0.0560	0.0460	0.0640	3.422	Accept
mean = 1	0.4360	0.1720	0.0780	0.0720	0.0400	392.115	Reject
mean = 2	0.9540	0.5680	0.2780	0.1480	0.0780	1072.716	Reject
mean = 3	1.0000	0.9220	0.6060	0.3300	0.1880	1062.564	Reject
mean = 4	1.0000	0.9940	0.8620	0.5720	0.3280	921.197	Reject
B							
mean = 0	0.0540	0.0600	0.0420	0.0480	0.0540	1.913	Accept
mean = 1	0.3900	0.1560	0.0800	0.0760	0.0460	309.807	Reject
mean = 2	0.9400	0.5720	0.2760	0.1300	0.0940	1045.990	Reject
mean = 3	0.9980	0.8840	0.5620	0.3300	0.2080	970.066	Reject
mean = 4	1.0000	0.9920	0.8640	0.5500	0.3720	857.718	Reject
AB							
mean = 0	0.0280	0.0420	0.0340	0.0580	0.0560	8.384	Accept
mean = 1	0.0780	0.0420	0.0460	0.0440	0.0320	13.149	Reject
mean = 2	0.4100	0.1580	0.0840	0.0840	0.0440	332.027	Reject
mean = 3	0.8680	0.4560	0.2260	0.1160	0.0900	915.666	Reject
mean = 4	1.0000	0.7940	0.4600	0.2320	0.1600	1045.961	Reject

	MEANS: A-B+ SDs: A+B+			Reject hypothesis if test value > 9.488		Hypothesis Test: Calc. Test Value	Result
A							
mean = 0	0.0500	0.0540	0.0520	0.0380	0.0620	3.096	Accept
mean = 1	0.4260	0.2060	0.1100	0.0920	0.0760	284.237	Reject
mean = 2	0.9640	0.5520	0.2840	0.2220	0.1460	906.408	Reject
mean = 3	0.9980	0.8880	0.5820	0.3660	0.2260	917.189	Reject
mean = 4	1.0000	0.9860	0.8020	0.5680	0.3960	745.701	Reject
B							
mean = 0	0.0580	0.0740	0.0480	0.0500	0.0600	3.880	Accept
mean = 1	0.4720	0.1700	0.0720	0.0820	0.0540	432.346	Reject
mean = 2	0.9300	0.5940	0.2960	0.1640	0.1160	951.773	Reject
mean = 3	1.0000	0.8820	0.5720	0.3160	0.2720	898.628	Reject
mean = 4	1.0000	0.9900	0.8200	0.5580	0.3560	833.798	Reject
AB							
mean = 0	0.0380	0.0480	0.0380	0.0500	0.0680	6.548	Accept
mean = 1	0.0940	0.0780	0.0640	0.0520	0.0560	9.278	Accept
mean = 2	0.4260	0.1900	0.1340	0.0920	0.0780	269.075	Reject
mean = 3	0.8720	0.4700	0.2600	0.1540	0.1360	797.495	Reject
mean = 4	0.9960	0.8140	0.4940	0.3240	0.2200	876.592	Reject

	MEANS:	A+B+		Reject hypothesis if	
	SDs:	A-B-		test value > 9.488	
A					
mean = 0	0.0760	0.0400	0.0620	0.0620	0.0680
mean = 1	0.4540	0.1900	0.1200	0.0760	0.0740
mean = 2	0.9340	0.5260	0.2800	0.1740	0.1180
mean = 3	0.9980	0.8600	0.5640	0.3580	0.2300
mean = 4	1.0000	0.9820	0.7900	0.5300	0.3540
B					
mean = 0	0.0360	0.0580	0.0420	0.0400	0.0620
mean = 1	0.4420	0.1860	0.0980	0.0720	0.0780
mean = 2	0.9440	0.5460	0.3260	0.1800	0.1280
mean = 3	0.9940	0.8580	0.5700	0.3200	0.1940
mean = 4	1.0000	0.9800	0.8000	0.5660	0.3840
AB					
mean = 0	0.0420	0.0500	0.0560	0.0620	0.0780
mean = 1	0.0600	0.0640	0.0620	0.0300	0.0480
mean = 2	0.4520	0.1680	0.1320	0.0700	0.0660
mean = 3	0.8740	0.4920	0.2480	0.1740	0.1200
mean = 4	0.9940	0.7460	0.4720	0.3020	0.2200

Hypothesis Test	
Calc. Test Value	Result
6.186	Accept
337.348	Reject
924.036	Reject
882.202	Reject
817.549	Reject
5.947	Accept
336.863	Reject
904.534	Reject
959.415	Reject
753.710	Reject
6.809	Accept
8.046	Accept
347.461	Reject
813.681	Reject
831.260	Reject

	MEANS:	A+B+		Reject hypothesis if	
	SDs:	A+B-		test value > 9.488	
A					
mean = 0	0.0440	0.0280	0.0520	0.0420	0.0340
mean = 1	0.3940	0.1620	0.0700	0.0700	0.0580
mean = 2	0.9420	0.5840	0.2880	0.1580	0.1000
mean = 3	0.9960	0.8900	0.5980	0.3260	0.2220
mean = 4	1.0000	0.9880	0.7960	0.5500	0.3800
B					
mean = 0	0.0360	0.0460	0.0500	0.0480	0.0400
mean = 1	0.4220	0.1880	0.1100	0.0820	0.0580
mean = 2	0.9460	0.6000	0.3300	0.1820	0.1160
mean = 3	1.0000	0.8720	0.5720	0.3600	0.2560
mean = 4	1.0000	0.9860	0.8000	0.5640	0.4200
AB					
mean = 0	0.0640	0.0480	0.0500	0.0540	0.0460
mean = 1	0.0960	0.0400	0.0520	0.0460	0.0500
mean = 2	0.4620	0.2060	0.1380	0.0800	0.0740
mean = 3	0.8820	0.4740	0.2620	0.1800	0.1460
mean = 4	0.9920	0.7860	0.4780	0.3560	0.2520

Hypothesis Test	
Calc. Test Value	Result
4.479	Accept
316.028	Reject
1004.834	Reject
960.862	Reject
779.648	Reject
1.617	Accept
307.887	Reject
946.374	Reject
863.284	Reject
711.954	Reject
2.046	Accept
18.711	Reject
330.291	Reject
776.682	Reject
776.634	Reject

	MEANS:	A+B+		Reject hypothesis if	
	SDs:	A-B+		test value > 9.488	
A					
mean = 0	0.0480	0.0480	0.0320	0.0480	0.0640
mean = 1	0.4560	0.2360	0.1260	0.0860	0.0520
mean = 2	0.9640	0.5740	0.2780	0.1800	0.1180
mean = 3	1.0000	0.8920	0.5460	0.3980	0.2540
mean = 4	1.0000	0.9860	0.7720	0.5800	0.3940
B					
mean = 0	0.0360	0.0500	0.0660	0.0400	0.0720
mean = 1	0.4860	0.1720	0.0920	0.0680	0.0720
mean = 2	0.9540	0.6420	0.2860	0.1660	0.0980
mean = 3	1.0000	0.8880	0.5700	0.3280	0.2140
mean = 4	1.0000	0.9880	0.8240	0.5200	0.3940
AB					
mean = 0	0.0660	0.0540	0.0440	0.0440	0.0700
mean = 1	0.0920	0.0580	0.0440	0.0560	0.0500
mean = 2	0.4220	0.1980	0.1260	0.0860	0.0600
mean = 3	0.8660	0.4840	0.2380	0.2180	0.1320
mean = 4	0.9840	0.7880	0.5080	0.3220	0.2060

Hypothesis Test	
Calc. Test Value	Result
5.602	Accept
345.380	Reject
1001.025	Reject
862.180	Reject
724.432	Reject
9.966	Reject
429.317	Reject
1061.632	Reject
972.550	Reject
800.889	Reject
5.591	Accept
12.411	Reject
290.050	Reject
747.021	Reject
845.658	Reject

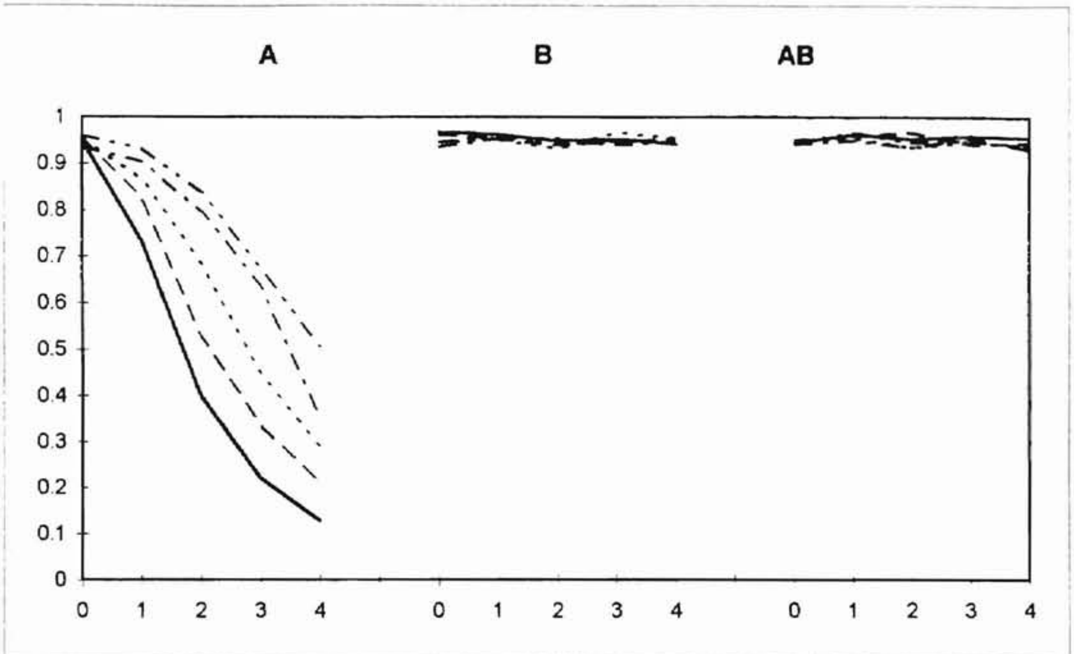
	MEANS:	A+B+		Reject hypothesis if	
	SDs:	A+B+		test value > 9.488	
A					
mean = 0	0.0500	0.0440	0.0480	0.0460	0.0500
mean = 1	0.4260	0.1520	0.0720	0.0700	0.0420
mean = 2	0.9320	0.5520	0.2600	0.1440	0.1000
mean = 3	1.0000	0.8940	0.6080	0.3000	0.1880
mean = 4	1.0000	1.0000	0.8620	0.5840	0.3800
B					
mean = 0	0.0500	0.0580	0.0500	0.0640	0.0500
mean = 1	0.4420	0.1460	0.1100	0.0560	0.0400
mean = 2	0.9560	0.5440	0.2800	0.1460	0.1020
mean = 3	1.0000	0.9140	0.5600	0.2860	0.2140
mean = 4	1.0000	0.9960	0.8500	0.5540	0.3820
AB					
mean = 0	0.0540	0.0480	0.0380	0.0520	0.0540
mean = 1	0.0900	0.0580	0.0560	0.0620	0.0560
mean = 2	0.4480	0.1460	0.0900	0.0480	0.0720
mean = 3	0.8700	0.4620	0.1880	0.1120	0.0880
mean = 4	0.9980	0.8400	0.4660	0.2980	0.1840

Hypothesis Test	
Calc. Test Value	Result
0.300	Accept
388.232	Reject
1004.607	Reject
1052.896	Reject
837.220	Reject
1.586	Accept
402.106	Reject
1031.644	Reject
1053.165	Reject
832.120	Reject
1.932	Accept
6.997	Accept
401.371	Reject
962.262	Reject
991.092	Reject

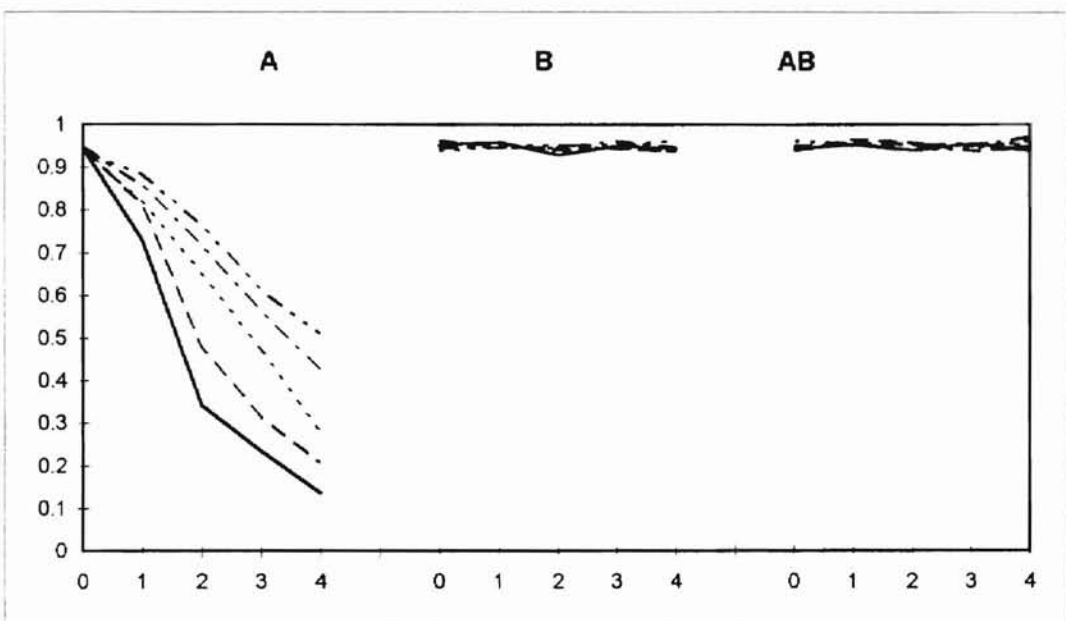
Appendix F: OC Curves for With-Interaction Data

Operating Characteristic curves
for 'with-interaction' data

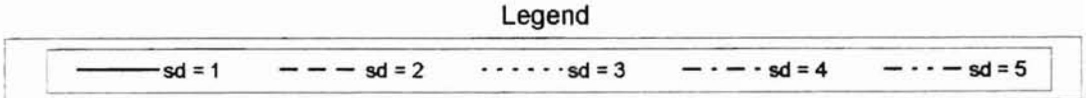
Mean	A-
SD	A-



Mean	A-
SD	A+

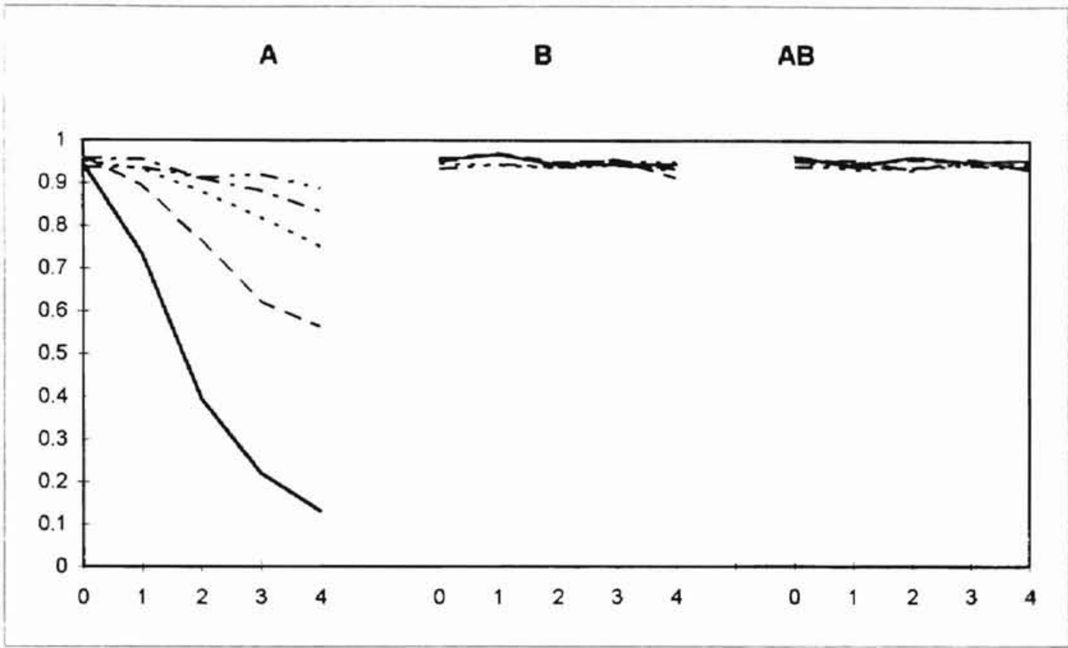


Legend

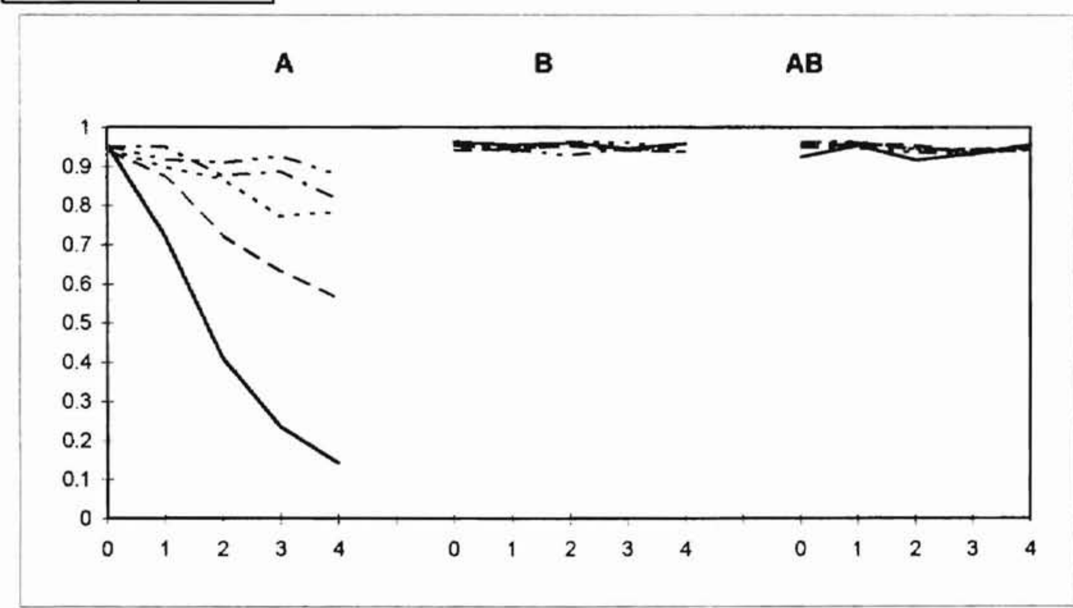


Operating Characteristic curves
for 'with-interaction' data

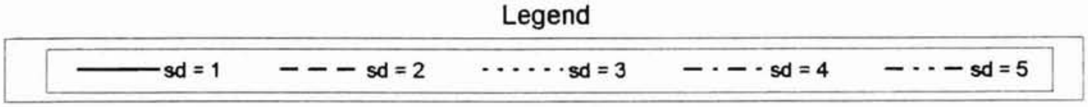
Mean	A-
SD	A-B-



Mean	A-
SD	A+B-

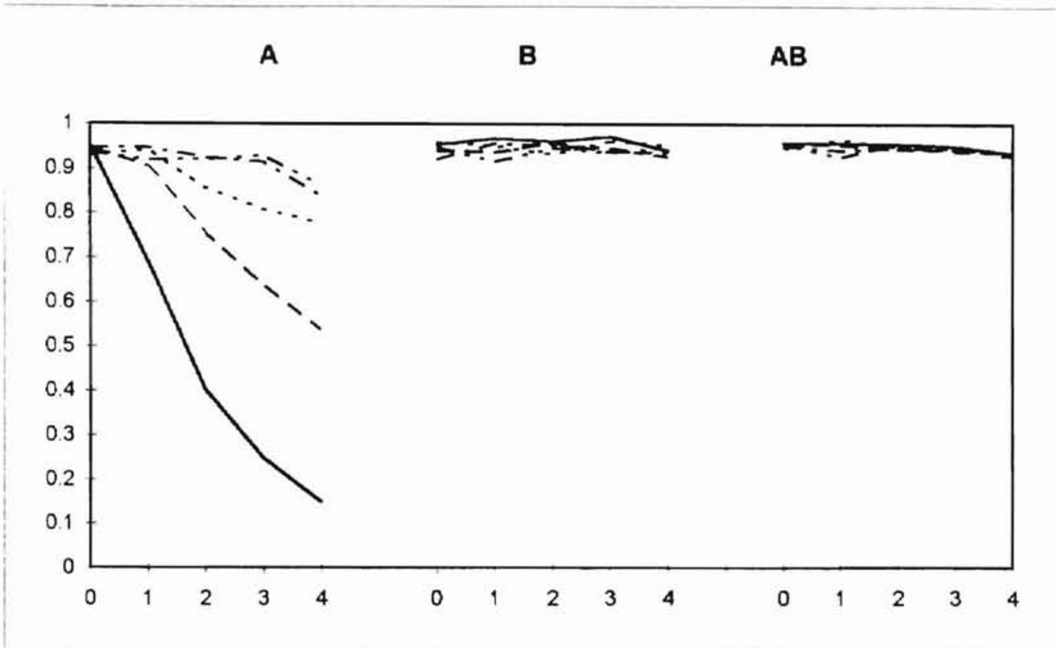


Legend

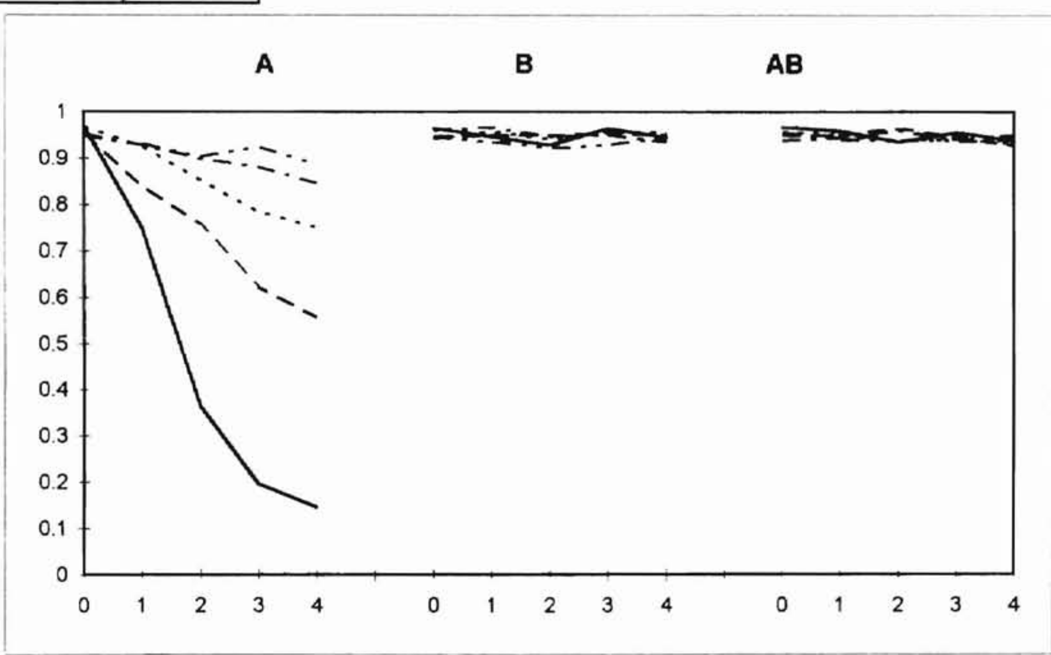


Operating Characteristic curves
for 'with-interaction' data

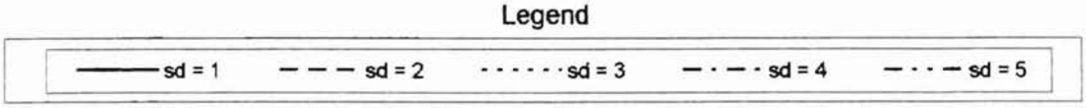
Mean	A-
SD	A-B+



Mean	A-
SD	A+B+

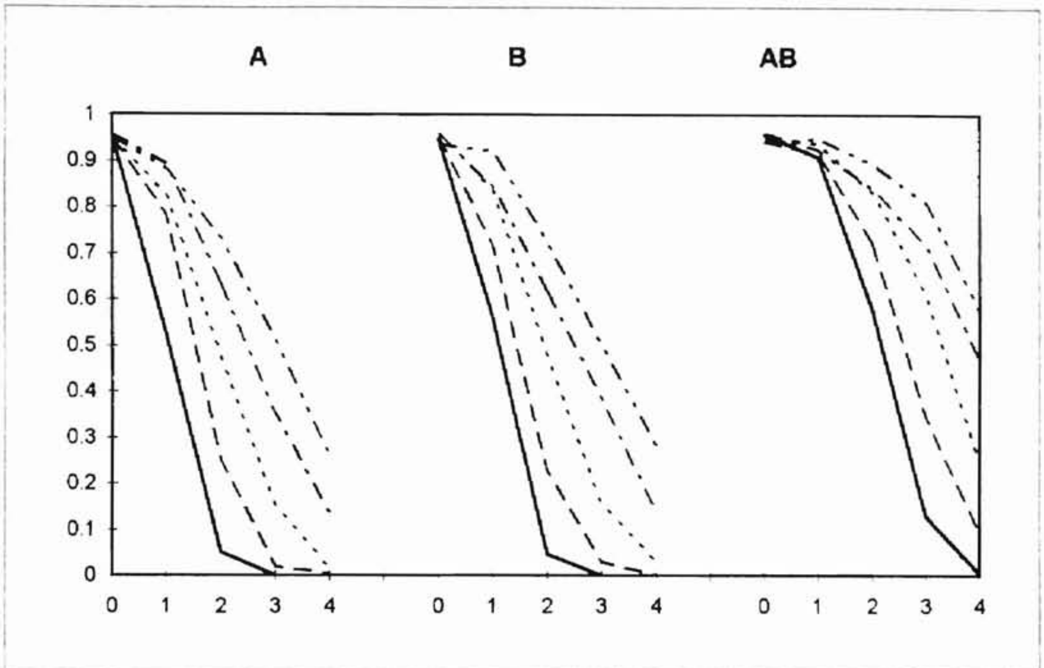


Legend

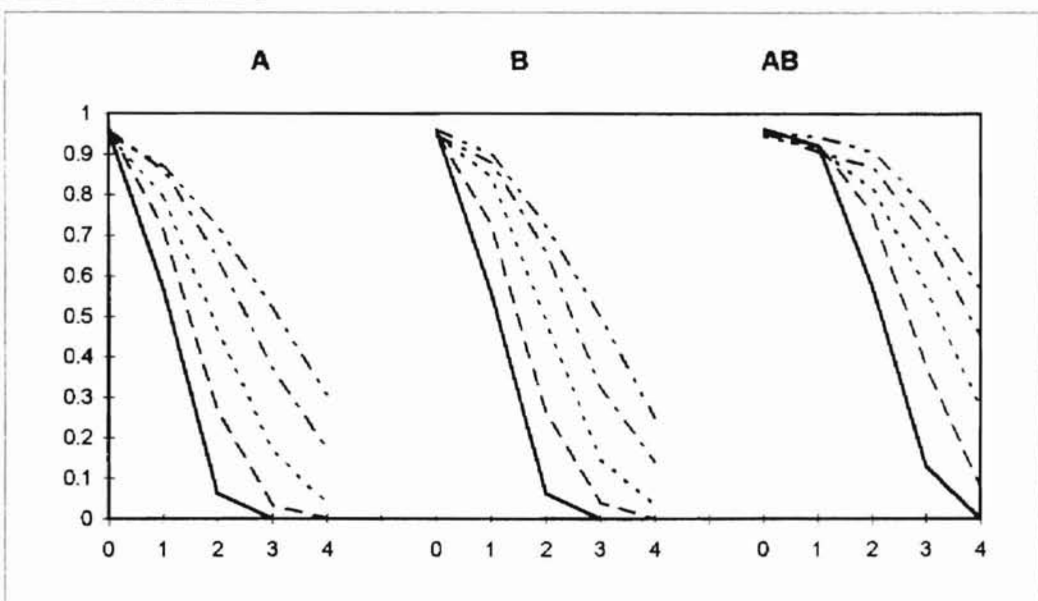


Operating Characteristic curves
for 'with-interaction' data

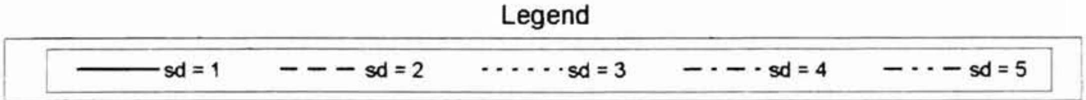
Mean	A-B-
SD	A-



Mean	A+B-
SD	A-

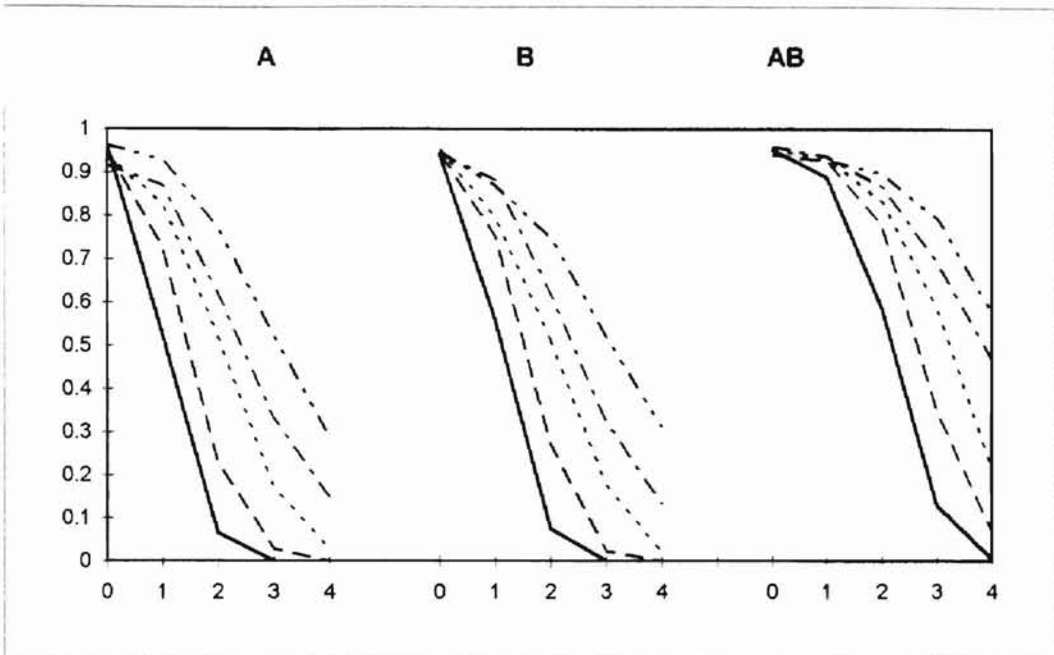


Legend

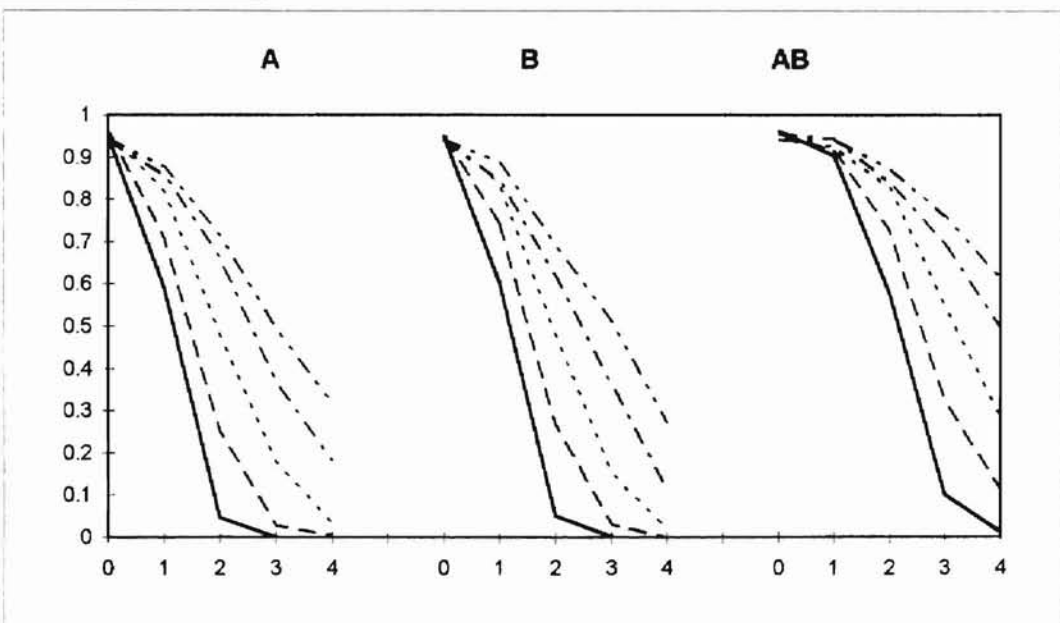


Operating Characteristic curves
for 'with-interaction' data

Mean	A-B+
SD	A-



Mean	A+B+
SD	A-

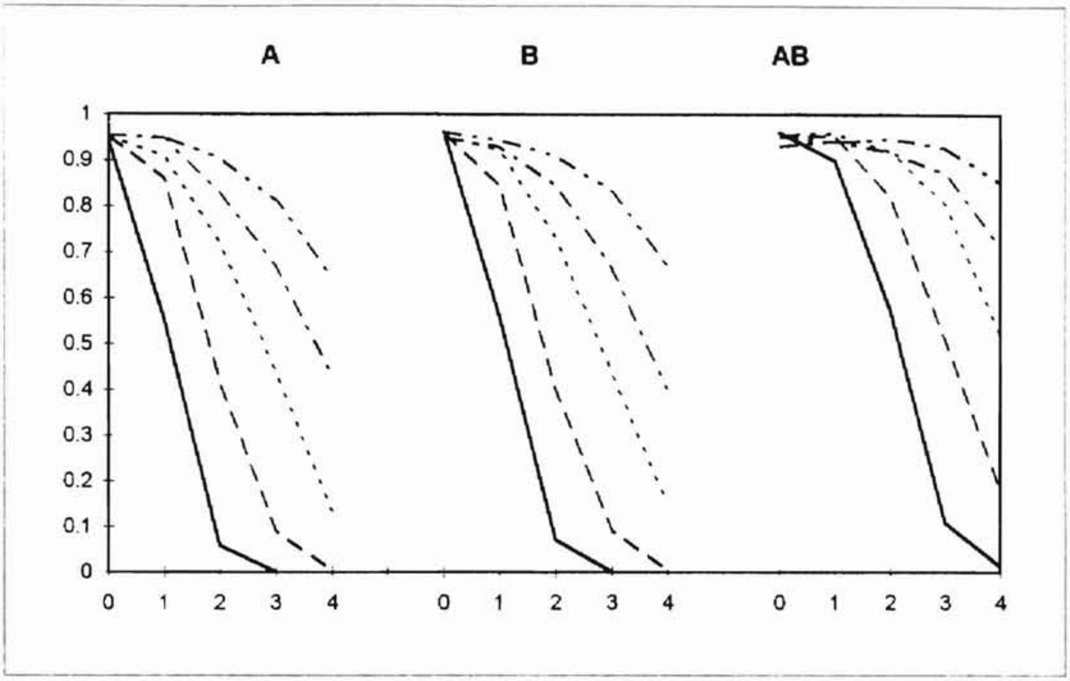


Legend

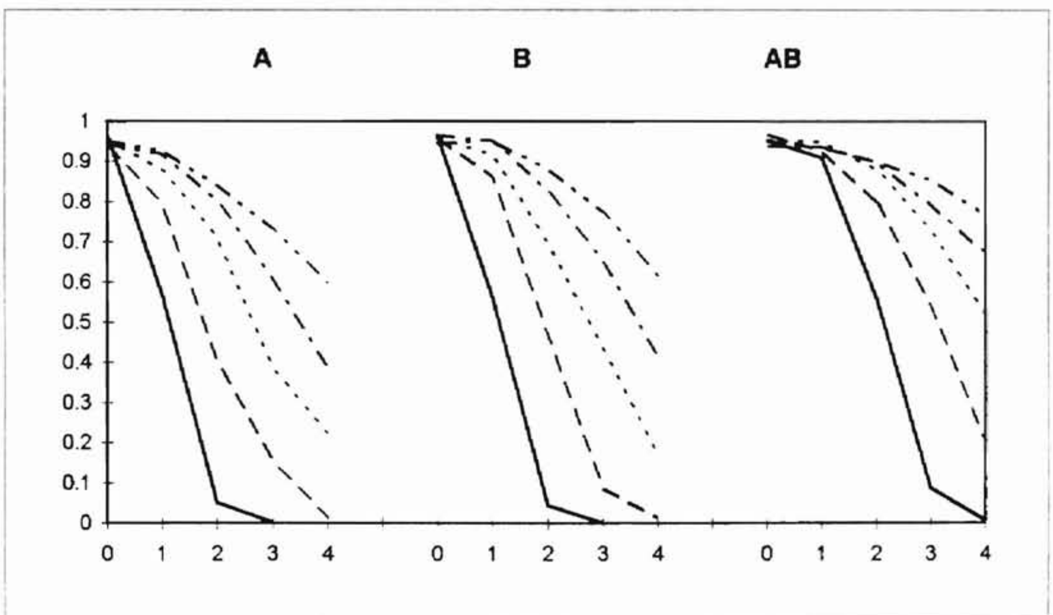
sd = 1 — sd = 2 — sd = 3 — sd = 4 — sd = 5

Operating Characteristic curves
for 'with-interaction' data

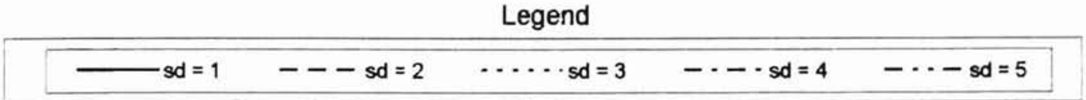
Mean	A-B-
SD	A-B-



Mean	A-B-
SD	A+B-

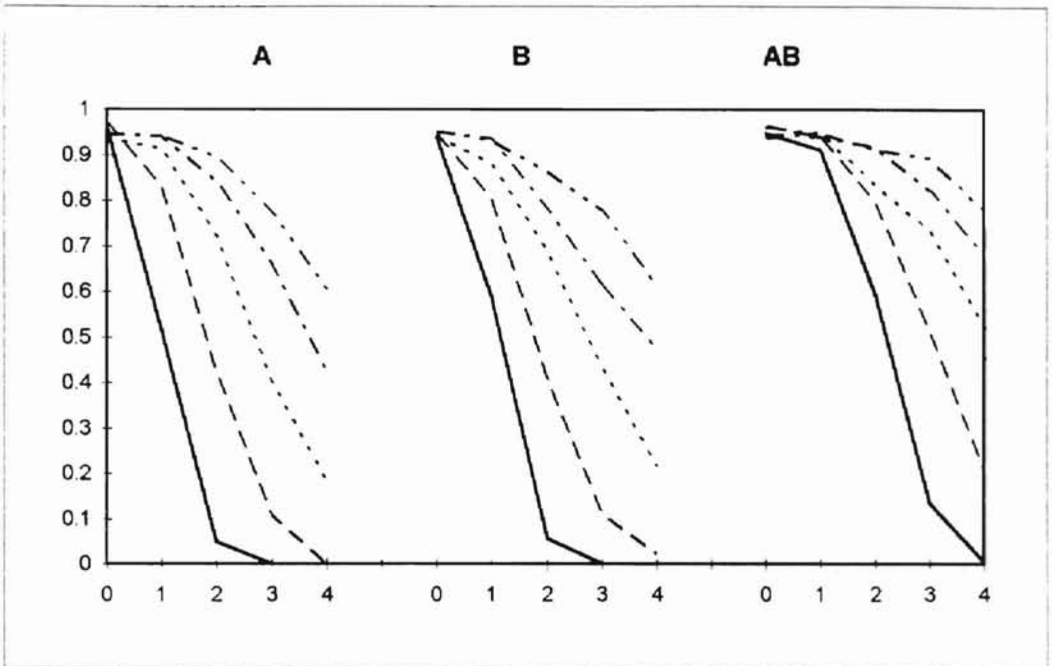


Legend

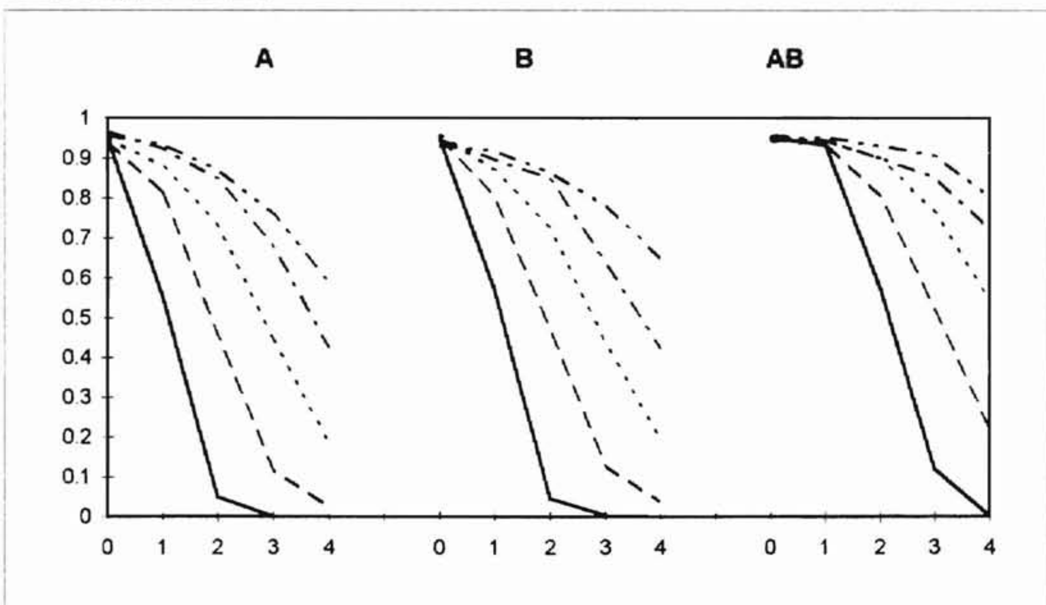


Operating Characteristic curves
for 'with-interaction' data

Mean	A-B-
SD	A-B+



Mean	A-B-
SD	A+B+

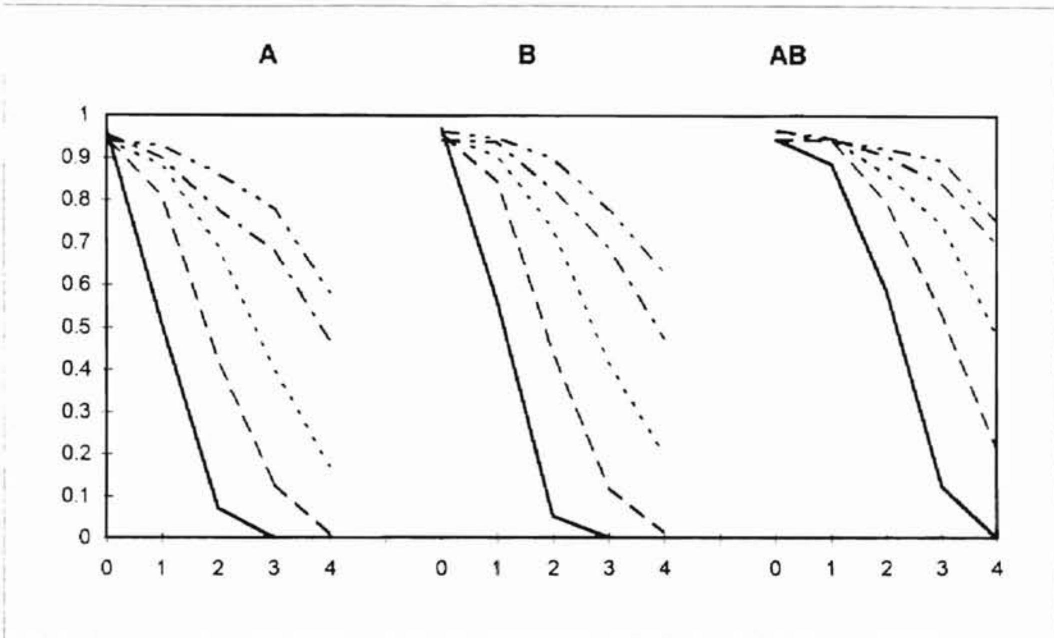


Legend

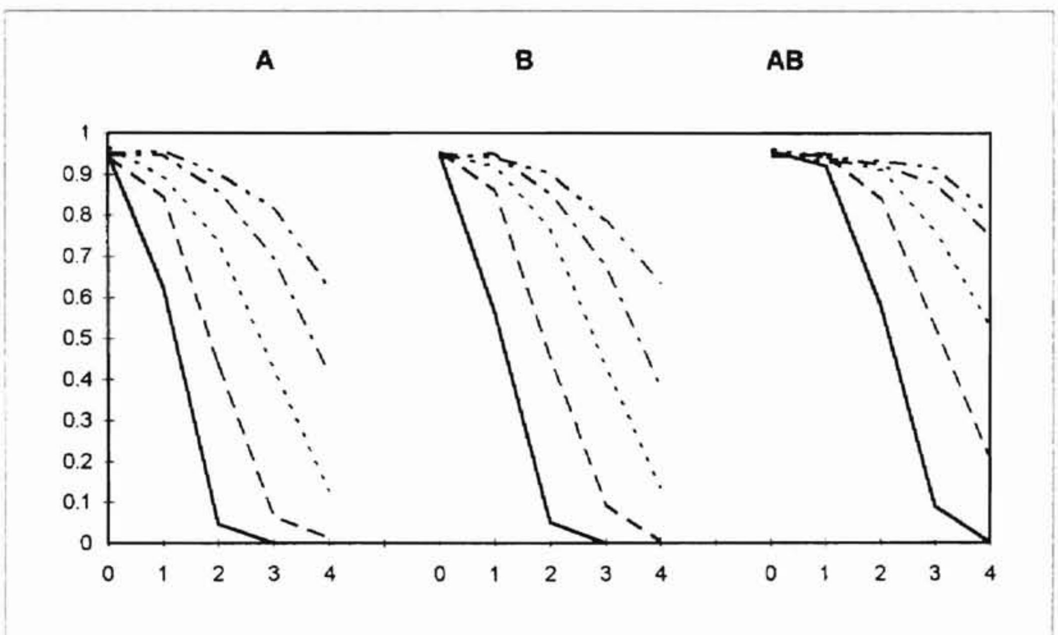
sd = 1 sd = 2 sd = 3 sd = 4 sd = 5

Operating Characteristic curves
for 'with-interaction' data

Mean	A+B-
SD	A-B-



Mean	A+B-
SD	A+B-

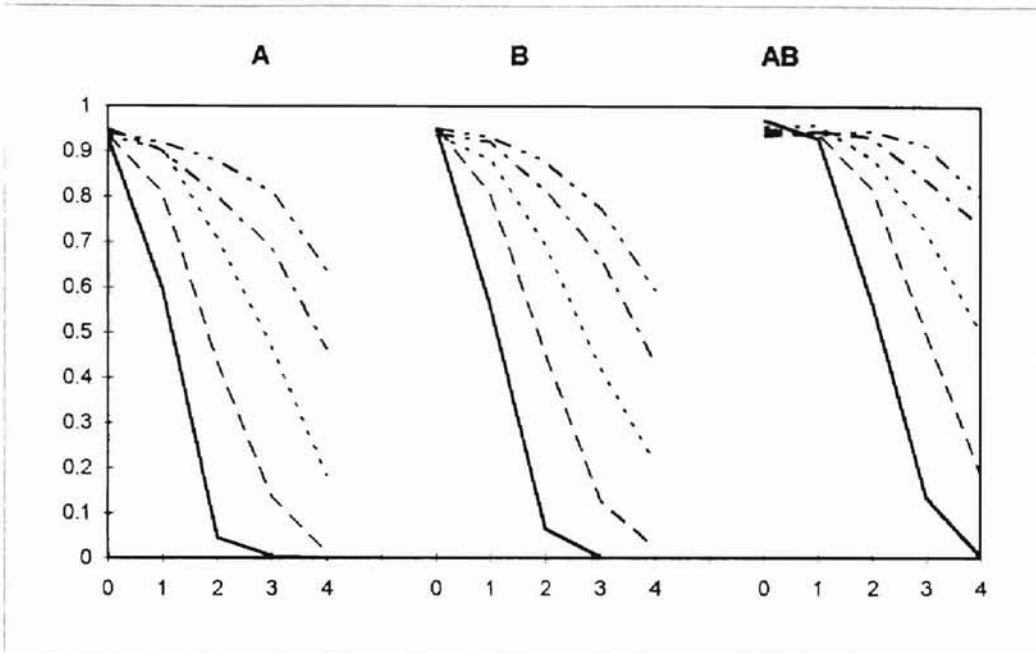


Legend

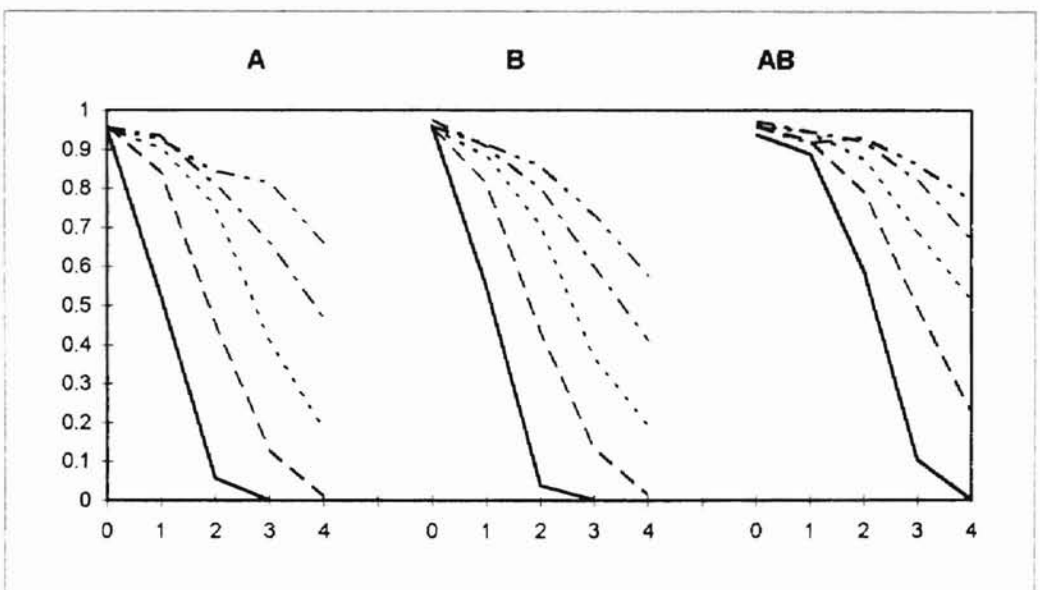
sd = 1 sd = 2 sd = 3 sd = 4 sd = 5

Operating Characteristic curves
for 'with-interaction' data

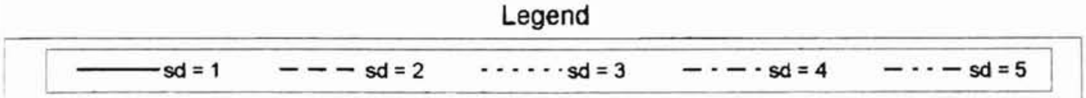
Mean	A+B-
SD	A-B+



Mean	A+B-
SD	A+B+

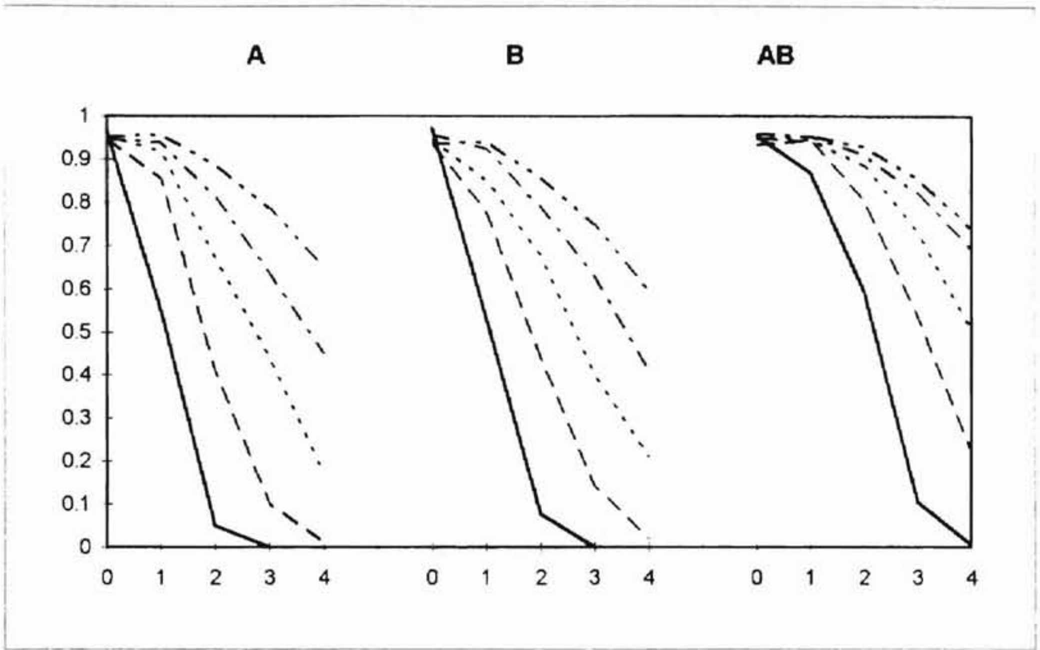


Legend

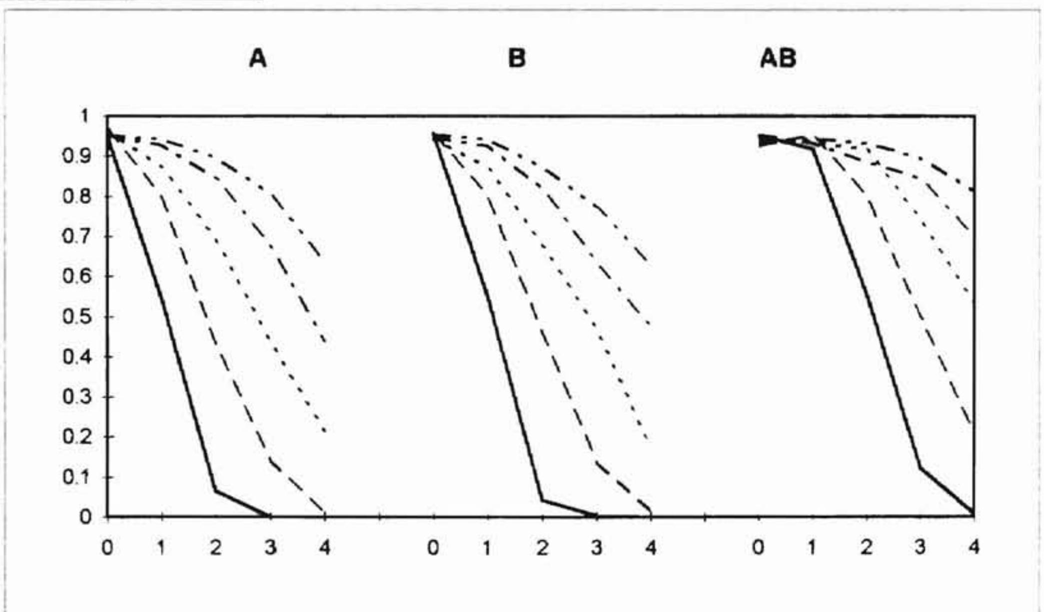


Operating Characteristic curves
for 'with-interaction' data

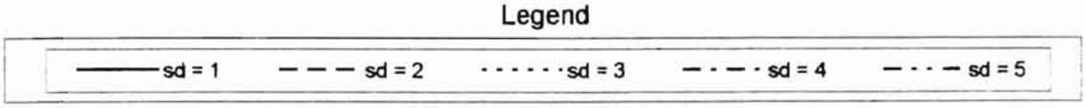
Mean	A-B+
SD	A-B-



Mean	A-B+
SD	A+B-

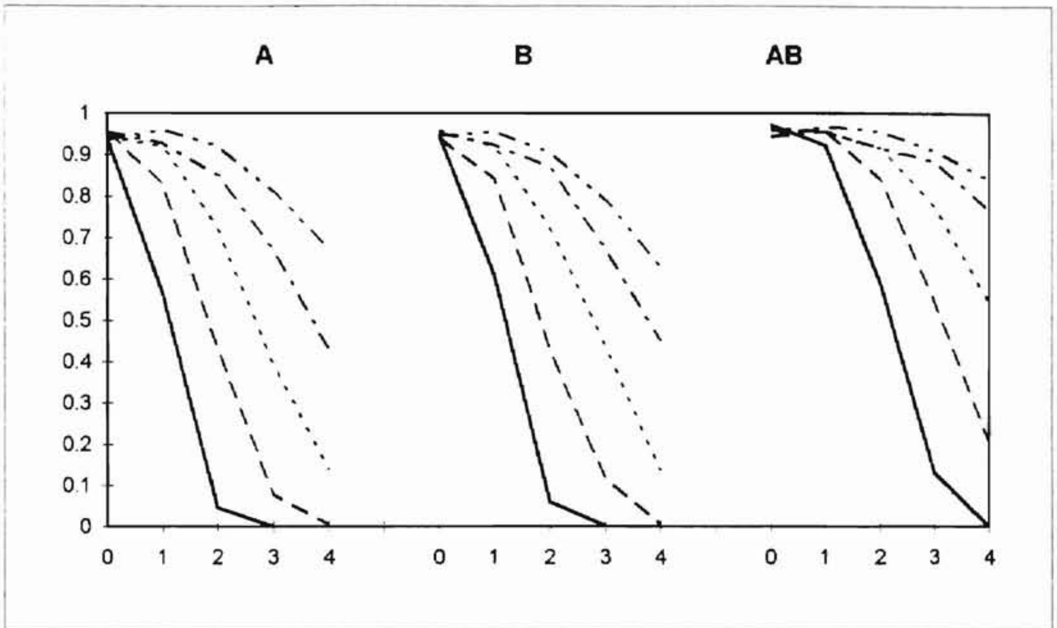


Legend

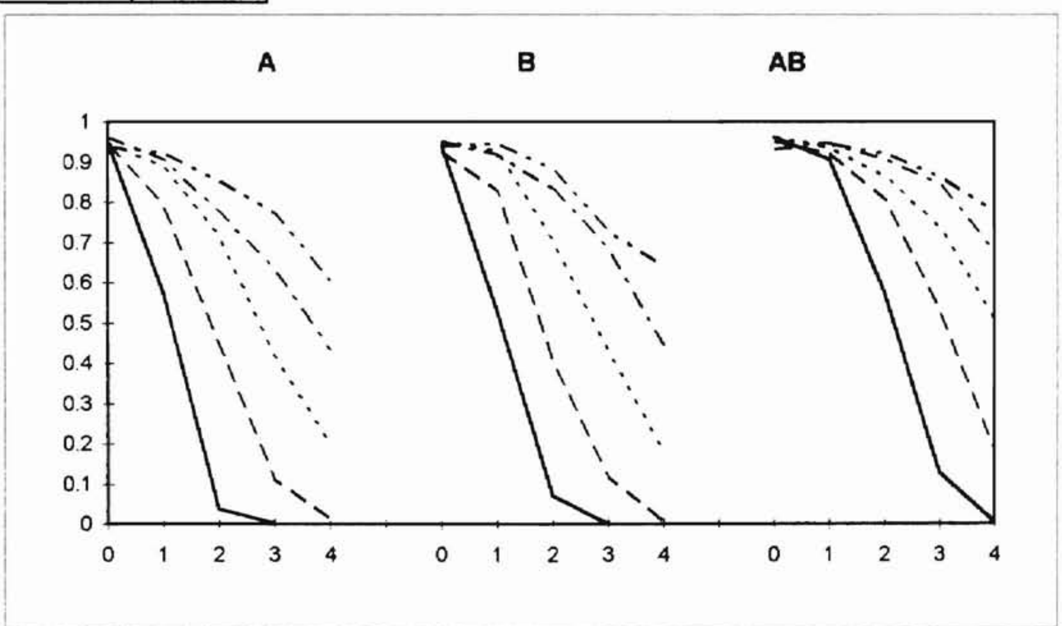


Operating Characteristic curves
for 'with-interaction' data

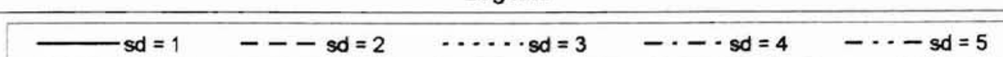
Mean	A-B+
SD	A-B+



Mean	A-B+
SD	A+B+

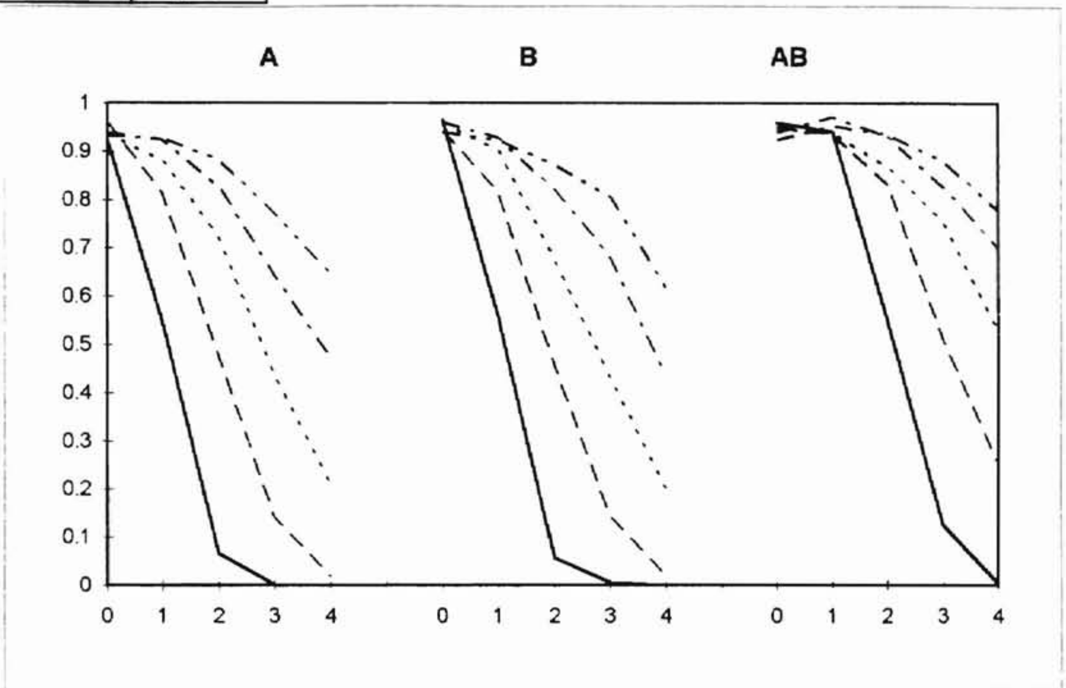


Legend

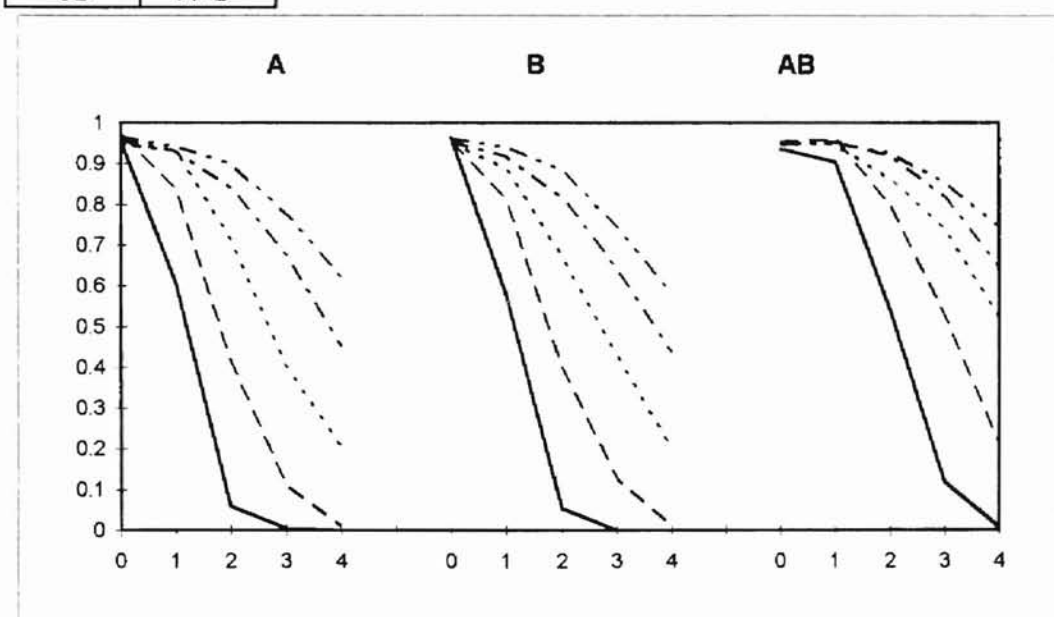


Operating Characteristic curves
for 'with-interaction' data

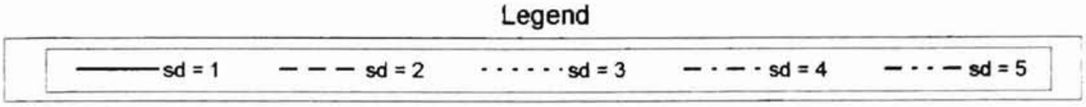
Mean	A+B+
SD	A-B-



Mean	A+B+
SD	A+B-

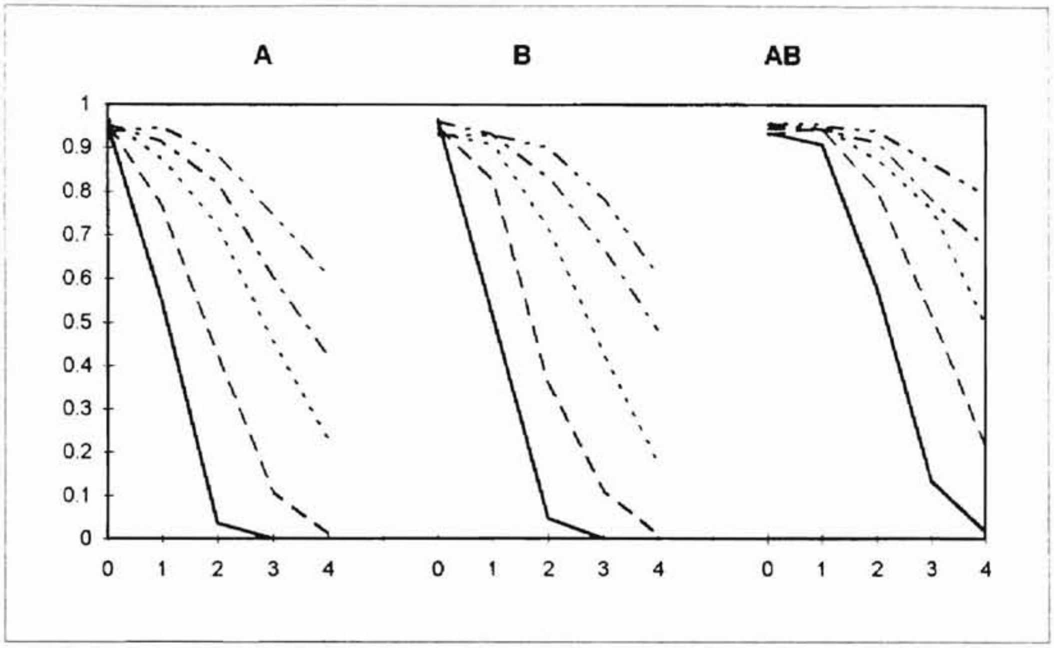


Legend

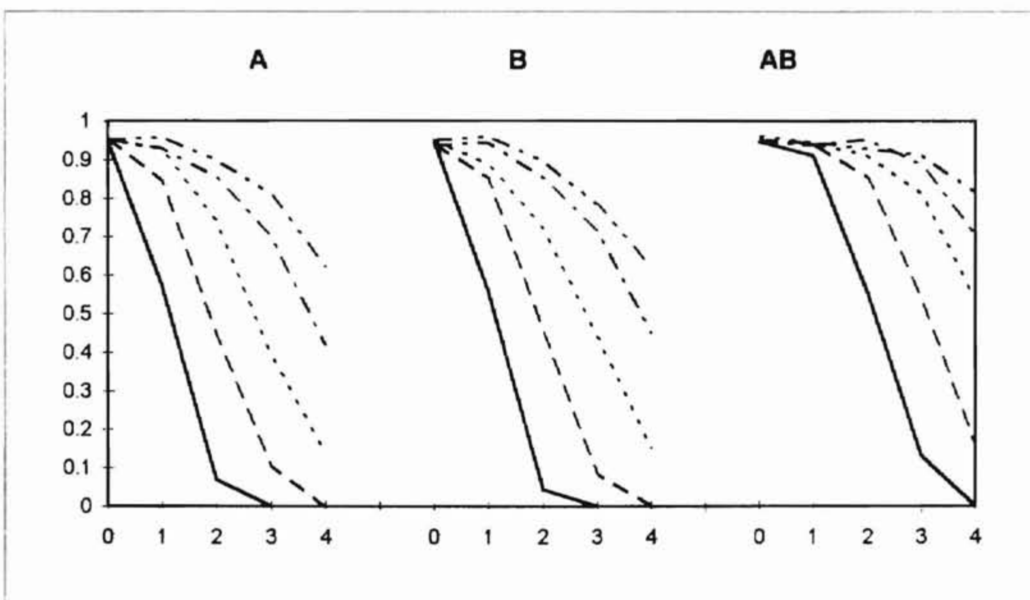


Operating Characteristic curves
for 'with-interaction' data

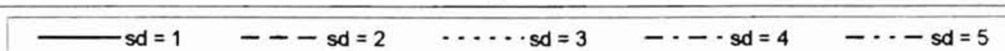
Mean	A+B+
SD	A-B+



Mean	A+B+
SD	A+B+



Legend



Appendix G: Hypothesis Test Results for With-Interaction Data

MEANS: A-
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Accept	Accept
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Accept	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Accept	Accept
mean = 3	Reject	Reject	Accept
mean = 4	Reject	Accept	Accept

MEANS: A-B-
SDs: A-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-B+
SDs: A-

	A	B	AB
mean = 0	Reject	Accept	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-
SDs: A+

A	B	AB
Accept	Accept	Accept
Reject	Accept	Accept
Reject	Accept	Accept
Reject	Accept	Accept
Reject	Accept	Accept

MEANS: A-
SDs: A+B-

A	B	AB
Accept	Accept	Reject
Reject	Accept	Accept
Reject	Reject	Accept
Reject	Accept	Accept
Reject	Accept	Accept

MEANS: A-
SDs: A+B+

A	B	AB
Accept	Accept	Accept
Reject	Accept	Accept
Reject	Accept	Accept
Reject	Accept	Accept
Reject	Accept	Accept

MEANS: A+B-
SDs: A-

A	B	AB
Accept	Accept	Accept
Reject	Reject	Accept
Reject	Reject	Reject
Reject	Reject	Reject
Reject	Reject	Reject

MEANS: A+B+
SDs: A-

A	B	AB
Accept	Accept	Accept
Reject	Reject	Accept
Reject	Reject	Reject
Reject	Reject	Reject
Reject	Reject	Reject

MEANS: A-B-
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-B-
SDs: A+B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-B-
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-B-
SDs: A+B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A+B-
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A+B-
SDs: A+B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A+B-
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A+B-
SDs: A+B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-B+
SDs: A-B-

	A	B	AB
mean = 0	Accept	Reject	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-B+
SDs: A+B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-B+
SDs: A-B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A-B+
SDs: A+B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A+B+
SDs: A-B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A+B+
SDs: A+B-

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A+B+
SDs: A-B+

	A	B	AB
mean = 0	Accept	Reject	Accept
mean = 1	Reject	Reject	Reject
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

MEANS: A+B+
SDs: A+B+

	A	B	AB
mean = 0	Accept	Accept	Accept
mean = 1	Reject	Reject	Accept
mean = 2	Reject	Reject	Reject
mean = 3	Reject	Reject	Reject
mean = 4	Reject	Reject	Reject

Appendix H: Differences in Hypothesis Tests

A zero means that the two data sets had the same result. A "Reject" or "Accept" is the test result for the with-interaction data. The no-interaction data would have the opposite result.

MEANS: A-
SDs: A-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	0
mean = 3	0	0	0
mean = 4	0	0	0

MEANS: A-
SDs: A+

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	0
mean = 3	0	0	0
mean = 4	0	0	0

MEANS: A-
SDs: A-B-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	0
mean = 3	0	0	0
mean = 4	0	0	0

MEANS: A-
SDs: A+B-

	A	B	AB
mean = 0	0	0	Reject
mean = 1	0	0	0
mean = 2	0	Reject	0
mean = 3	0	0	0
mean = 4	0	0	0

MEANS: A-
SDs: A-B+

	A	B	AB
mean = 0	0	0	0
mean = 1	0	Reject	Reject
mean = 2	0	0	0
mean = 3	0	Reject	0
mean = 4	0	0	0

MEANS: A-
SDs: A+B+

	A	B	AB
mean = 0	0	0	Accept
mean = 1	0	0	0
mean = 2	0	0	0
mean = 3	0	0	0
mean = 4	0	0	0

MEANS: A-B-
SDs: A-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A+B-
SDs: A-

	A	B	AB
mean = 0	Accept	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A-B+
SDs: A-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	Reject
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	0

MEANS: A+B+
SDs: A-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A-B-
SDs: A-B-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A-B-
SDs: A+B-

	A	B	AB
mean = 0	Accept	0	Accept
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	0

MEANS: A-B-
SDs: A-B+

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A-B-
SDs: A+B+

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A+B-
SDs: A-B-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	Reject
mean = 2	0	0	Reject
mean = 3	0	0	0
mean = 4	0	0	Reject

MEANS: A+B-
SDs: A+B-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A+B-
SDs: A-B+

	A	B	AB
mean = 0	0	0	Accept
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	0
mean = 4	0	0	Reject

MEANS: A+B-
SDs: A+B+

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	Reject
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A-B+
SDs: A-B-

	A	B	AB
mean = 0	0	Reject	Accept
mean = 1	0	0	Reject
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS: A-B+
SDs: A+B-

	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

	MEANS:	A-B+	
	SDs:	A-B+	
	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	Reject
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	0

MEANS:	A-B+	
SDs:	A+B+	
A	B	AB
0	0	0
0	0	0
0	0	Reject
0	0	Reject
0	0	Reject

	MEANS:	A+B+	
	SDs:	A-B-	
	A	B	AB
mean = 0	0	0	0
mean = 1	0	0	0
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS:	A+B+	
SDs:	A+B-	
A	B	AB
0	0	0
0	0	Reject
0	0	Reject
0	0	Reject
0	0	Reject

	MEANS:	A+B+	
	SDs:	A-B+	
	A	B	AB
mean = 0	0	Reject	0
mean = 1	0	0	Reject
mean = 2	0	0	Reject
mean = 3	0	0	Reject
mean = 4	0	0	Reject

MEANS:	A+B+	
SDs:	A+B+	
A	B	AB
0	0	0
0	0	0
0	0	Reject
0	0	Reject
0	0	Reject

2

VITA

Sarah Johnson

Candidate for the Degree of

Master of Science

Thesis: THE EFFECT OF HETEROSCEDASTICITY ON AN OPERATING CHARACTERISTIC CURVE IN A DESIGNED EXPERIMENT

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Experience: Employed by Oklahoma State University, Department of Industrial Engineering and Management as a graduate teaching assistant; employed as Quality Technician at National Standard, Inc., from 6-95 to 12-95; employed by Conoco, Inc., as Summer Enginering Intern for the summer of 1994.

Professional Memberships: American Society for Quality Control, Institute of Industrial Engineers, Alpha Pi Mu, Tau Beta Pi.