A DISTANCE-INDEPENDENT INDIVIDUAL TREE BASAL AREA GROWTH MODEL FOR NATURAL UNEVEN-AGED STANDS OF SHORTLEAF PINE (*Pinus echinata* MILL.) IN THE OUACHITA MOUNTAINS

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure page

CHAPTER I

INTRODUCTION

Shortleaf pine (*Pinus echinata* Mill) is one of the most important and widely represented pine species of the southern region of the United States. Found in 22 states from Pennsylvania to Texas on more than 440,000 square miles, shortleaf pine accounts for more than a quarter of the volume of southern pine volume (Murphy and Farrar 1985). Shortleaf pine has its greatest concentration in the Interior Highlands of Arkansas and east Oklahoma where it represents more than one half of the softwood growing stock (Sternitzke and Nelson 1970).

Although shortleaf pine is important in terms of area and growing stock, relatively little research has been initiated relative to other southern pines (Murphy and Farrar 1985). However, according to Baker and others (in press), substantial data relating to growth and yield of shortleaf pine has been accumulating since the late 1980's; several growth and yield models have recently been derived from experimental studies (Reynolds et al. 1984; Farrar et al. 1984a; Murphy et al. 1991).

In its natural range, shortleaf pine is usually established naturally after one of four regeneration cuttings: clearcutting, seed tree, shelterwood, and selection methods. The first three regeneration methods result in even-aged stands. The selection method results in uneven-aged stands and is comprised of two variations: single tree selection and group selection stands (Smith 1986). There has been a tendency for forest industry to replant loblolly pine (*Pinus taeda*) rather than shortleaf pine in order to achieve greater growth in short rotations. However, small landowners often prefer natural regeneration due to the high cost of artificial regeneration (site preparation and plantation establishment). This also emphasizes the development of uneven-aged stands.

In past years, forests were usually managed under even-aged systems. Recently, however, clearcutting on National Forests has been viewed as inappropriate by some members of the public because of the growing interest in managing the entire ecosystems of the National Forests. The increasing desire for the public of noncommodity products from the forest has influenced forest management practices. Therefore, the public has indicated a preference for uneven-aged management as the best silvicultural alternative for providing both commodity and noncommodity products (Baker et al. in press; Hamilton 1991). Because even-aged management was predominately used until recently, there are few adequate growth models which can predict future growths and yields for all-aged stands of shortleaf pine, especially in the Ouachita Mountains of Arkansas and Oklahoma. Consequently, there is a need to focus research on growth and yield models of shortleaf pine in this region to provide the best tools to forest managers and planners.

Growth and yield information, which is indispensable for forest production planning, was traditionally in the form of tables; these have now been replaced by mathematical formulae. For yield estimates to be predicted by these models, variables to be incorporated should be easy to measure and should be the readily collected during forest inventories.

The purpose of this study is to develop model equations to estimate current tree and stand basal area of shortleaf pine. Parameters in these equations will be estimated from data collected from permanent plots established in a continuous forest inventory system (CFI) in Arkansas. The objectives of this study are:

- to develop a distance-independent individual tree basal area growth model for natural uneven-aged stands of shortleaf pine in the Ouachita Mountains. The model will include variables representing site index, stand density, and individual tree characteristics such as diameter at breast height (dbh) and crown position. Age will not be included in the model as it would be difficult to assess it for each individual tree in an uneven-aged forest;
- to validate the model with data from an independent data set; and
- to make recommendations regarding use of the model.

CHAPTER II

LITERATURE REVIEW

Modeling growth and yield of shortleaf pine and other species in the southern United States has been largely concentrated on even-aged stands. Very few studies have been done on uneven-aged modeling relative to the great number of models for even-aged stands. However forest management practices are now shifting from even-aged to uneven-aged systems in many areas of the United States. Uneven-aged management systems need increased research on modeling of growth and yield in order to help forest managers and planners reach short and long term decisions.

Shortleaf pine (*Pinus enchinata* Mill.) is the southern yellow pine represented by the largest land area and accounts for a quarter of the total growing stock on approximately 114,000,000 hectares (281,600,000 acres). This is approximately one half of growing stock in the Interior Highlands of Arkansas and Oklahoma (Murphy 1985). In order to maintain a balanced harvesting in the new uneven-aged management scheme, knowledge of the growth dynamics of individual trees that make up these all-aged stands is required. Thus, to get reliable information concerning the growth in uneven-aged stands, understanding the relationships between tree growth and tree characteristics (such as initial diameter at breast height, crown position, and site quality) is crucial for model development.

Uneven-aged stand is defined as a stand composed of three or more age classes that are spatially intermingled. Each age class within an uneven-aged stand makes a significant contribution to the stand's stocking or aggregate volume (Murphy et al. 1994). Gering (1985) suggested that stocking involves a normative evaluation of the growing stock present while stand density is a numerical expression of the degree of crowding of the stand. *An uneven-aged silvicultural system* involves the manipulation of a stand for a continuous high-forest cover, recurring regeneration of desirable species, and the orderly growth and development of trees through a range of age or diameter classes to provide a sustained yield of forest resources and values. Therefore, *uneven-aged forest management* is a forest management system that involves frequent partial cuttings with the objective of producing uneven-aged stands.

Williston (1975) compiled a complete bibliography on growth and yield of the four major southern pines. This bibliography included research published through 1974. It included nine studies related to all-aged shortleaf pine stands, 17 studies on even-aged, and 33 studies on planted shortleaf. Baker and others. (in press) did an extensive review on work relating to the growth and yield and stand development of uneven-aged shortleaf-loblolly (*Pinus taeda*) pine stands in the southern United States. Their review revealed that most of the studies were conducted in Arkansas, Georgia, Mississippi and Texas. Two studies were on pure stands of shortleaf pine and eight other were conducted on stands where loblolly pine was mixed with shortleaf pine. Length of cutting cycles for these studies varied from one year to ten years while the durations of study varied from six to 41 years.

Forests are dynamic biological systems that change over time, and it is important to be able to predict these changes to obtain relevant information for wise decision making (Avery and Burkhart 1994). Future growth, mortality, and reproduction can be predicted by direct or indirect methods. Direct methods such as stand table projection techniques involve field observations in existing stands. Past growth trends are used to predict future trends in these observed stands. However, direct methods are not satisfactory in many situations. Avery and Burkhart (1994) stated that diameter growth, mortality, and ingrowth relationships developed through stand-table projection method are not reliable for long periods of time. Managers also may wish to evaluate a broad range of treatment alternatives. Given that inferences from past growth are limited to the conditions under which that growth occurred and that costs associated with direct observations are often prohibitive, foresters often rely on indirect methods of predicting stand dynamics. For example it may be more appropriate to infer a stand's mortality, growth, and ingrowth from the study of other stands possessing similar characteristics. The technique of predicting stand dynamics is refered to as growth and yield modeling.

Although empirical growth models differ widely, there are basic elements common in most of them. Estimates are made of the changes over time for diameter at breast height (dbh), basal area, height, volume, or all of these variables. Changes in the number of trees per unit area may also be considered. Regardless of the structural complexity and amount of output detail provided, all growth and yield models have a common purpose: to produce estimates of stand characteristics at specified points in time.

Dependent and independent variables used in developing models relate to the stand characteristic being estimated and on the overall objective of the model. For growth models, dependent variables may be volume, basal area, or any of their associated properties. Independent variables may be site index, age, number of trees per unit area, height, and dbh. The following implicit equation was developed by MacKinney and Chaiken (1939) to predict natural logarithm of yield for loblolly pine .

$$\log Y = b_0 + b_1A^{-1} + b_2S + b_3\log SDI + b_4C \tag{1}$$

where $\log Y$ = logarithm of yield (total cu ft per acre of loblolly pine)
A^{-1} = reciprocal of stand age (years)
S = site index (feet at index age 50 years)
SDI = stand density index
C = competition index (basal area per acre of loblolly pine divided by total stand basal area)

This model was among the first models that used a multiple regression techniques and is an example of a model whose yield (cubic feet per acre) is a dependent variable and age, site index, stand density, and competition are independent variables.

Data Classification

Murphy and Farrar (1982, 1988) suggested that stands should be inventoried towards the end of each cutting cycle. However, for well-stocked and well-structured stands, growth and yield models can be used to project stand development at the end of future cutting cycles, provided the stands were inventoried prior to harvest. Depending upon the timing of initial and subsequent measurements relative to harvest, there are three types of data used to develop forest growth and yield models. The different types are classified based on the method of collection. Additionally, the type of data may suggest the level of accuracy of the models derived from that particular data. Finally the cost for collecting the data also depends upon the type of information required. The three types are: real growth series, abstract growth series, and approximated real growth series. Extensive descriptions of these data types may be found in several forest management and forest measurements texts (Bruce and Schumacher 1950; Husch et al. 1982; Davis and Johnson 1987; Avery and Burkhart 1994).

Real growth series data are collected from the time of stand establishment until final harvest. It is rare to have such a dataset. They are very expensive to collect because they come from permanent plots which are often difficult to establish and maintain. However, data from real growth series provide an understanding of the dynamics of stand growth. They outweigh any other type of data in this regard. The major drawback of this type of data is that it takes a long time (an entire rotation) to collect.

In the literature, no studies have ever used real growth series for modeling either even-aged or uneven-aged stands. One reason might be that monitoring of the stand must begin at the time of stand establishment, a characteristic that cannot be determined in uneven-aged stands.

Abstract growth series data are obtained from several temporary plots covering a number of sites and ages. This type of data can be obtained in a relatively short time compared to the real growth series data. In order to cover all ages, the plots are often extended to a large area in order to include all present stand characteristics. Although time required to compile data may be reduced significantly, the complete history of the stand can never be completely determined. This data type is not suited to development of individual tree growth models because there is no remeasurement of individual trees.

Most of the data used to construct growth and yield tables prior to the 1960's were abstract growth series. Several of these yield and tables (including tables for shortleaf pine) were published in Miscellaneous Publication 50 of the USDA Forest Service (1929). Schumacher and Coile (1960) conducted a study on the growth of fully stocked "or normally stocked" stands of the southern pine species including shortleaf pine. The abstract growth series data of this study were collected from stands located in North Carolina. The solutions of different equations derived from multiple regression techniques were predictions of change in stocking level, average height, and number of trees per acre. Future basal area could also be inferred once predicted stocking level and height were known.

Approximated real growth series data are the third type of data and can be considered as an intermediate between real growth series and abstract growth series. They are obtained from permanent plots established in already existing stands. After the plots are established, they are remeasured through time and undergo the same management activities as the rest of the stand in which they occur. As with the real growth series, the approximated real growth series are expensive to obtain and the history of the stand before the establishment of the plots is unknown. However, this type of data is important in that it is possible to model growth of individual trees and that the history of the stand is known for the period during which the stand is monitored. The recent study by Hitch (1994) involved approximated real growth series data to create a distance-independent individual tree basal area model for shortleaf pine. Data were collected on permanent research plots in Eastern Oklahoma and Western Arkansas. These plots were established from 1985-87 as part of USDA Forest Service Study 48 conducted cooperatively by the Southern Forest Experiment Station in Monticello, Arkansas and the Department of Forestry at Oklahoma State University. The final model for predicting basal area growth was one of the forms initially investigated for this present study.

Wykoff (1990) used an approximated real growth series to model the growth of eleven western individual conifers. The model predicts the natural log of squared diameter increment as a function of tree size, site, and a competition factor. Another study where approximated real growth series data were used is that of Buckman (1962). He created a stand periodic net annual basal area increment for red pine (*Pinus resinosa* Ait) in Minnesota. Permanent sample plots were scattered over a wide range to cover the maximum variety of conditions found in the area. The resulting equations predicted basal area growth of all trees as large or larger than 3.6 inch dbh as a function of age, site index and stand density. The following explicit function was obtained:

$$\begin{aligned} Y = {} & 1.6689 + 0.041066BA - 0.00016303BA^2 - 0.076958A \\ & + 0.00022741A^2 + 0.06441SI \end{aligned} \tag{2}$$

where Y = periodic net annual basal area increment (ft^2 / acre / year)
BA = basal area, in square feet per acre
A = age, (years)
SI = site index (feet at index age (50 years)

Other studies involving approximated real growth series on even-aged stands of shortleaf pine at the stand level are those of Murphy and Beltz (1981) and Murphy et al. (1985).

Model Classification

Whole Stand Models

The early works on growth and yield were based on natural whole stands in even-aged, fully stocked forests. They formed the basis for the concept of normal yield tables. These normal yield tables were developed using values interpreted from relationships illustrated by graphical means (Bruce 1926). In 1929, USDA Miscellaneous Publication 50 was published and included volume, yield and stand tables for second growth stands of southern pine species. Most of the growth and yield relationships developed during the 1930's were also presented in the form of tables created from interpretation of graphs. However, these methods were not very efficient in prediction of future yields, especially for understocked stands (Bruce and Shumacher 1950).

Statistical methods proved to be more efficient and accurate than graph-based methods. Shumacher (1939) used multiple regression methods and differential equations to characterize a relatively simple method for preparation of yield tables. Since then, many researchers have used the Schumacher model type as the foundation for developing new models.

One important aspect of the early work in development of growth and yield models is that growth was not directly related to yield even though the biological relationships could be readily expressed (Avery and Burkhart 1994). The first researchers that attempted to relate growth to yield through mathematical integration were Buckman (1962) and Clutter (1963). By ensuring that the algebraic form of the yield model could be obtained by mathematical integration of the growth model, Clutter (1963) derived compatible growth and yield models for loblolly pine. From Clutter's models, yield and cumulative growth can be estimated as a function of initial age, initial basal area, site index, and future age (Sullivan and Clutter 1972). Beck and Dellabianca (1972) adapted Sullivan and Clutter's models and produced compatible growth and yield models for thinned stands of yellow poplar (*Liriodendron tulipifera* L.). The general form of the equation includes a projection of future basal area:

$$\ln BA_2 = (A_1/A_2)(\ln BA_1) + b_1(1 - A_1/A_2) + b_2(S)(1 - A_1/A_2) \qquad (3)$$

where BA_1 = current basal area, sq ft per acre
BA_2 = basal area, sq ft per acre, at future age 2
A_1 = current age (years)
A_2 = future age (years)
S = site index, ft (base age 50)
b_i = are parameters to be estimated.

Buckman (1962) and Clutter (1963) developed the first growth and yield models based on differential equations for whole stand even-aged forests. These differential equations related rates of change in forest yields to stand age, site index, and stand density. Until the late 1960's, all the models were for even-aged stands and most models included a measure of stand age. This characteristic is theoretically meaningless for all-aged stands where age of individual trees can vary from regeneration to rotation age.

The approach by Buckman and Clutter was not directly applicable to uneven-aged conditions as age and site index are hard to define for these conditions. However, Moser and Hall (1969) used similar techniques for uneven-aged stands where age was not an independent variable. They used data obtained from permanently established plots located in northern hardwood forests in Wisconsin. The general form of the model was:

$$dB / dt = b_1 B^{b_2} - b_3 B \tag{4}$$

where dB/dt = rate of change in basal area per acre with respect to time
B = basal area per acre
b_i = parameters to be estimated

Diameter Class Models

The basic concept of diameter distribution models is that it is possible to predict stand tables as of the number of trees per acre by dbh class and stock tables as of volume per acre by dbh class. Since estimation is on a per stand basis within each diameter class, these models are also classified as whole stand models. For even-aged stands, diameter class distribution has central tendency whereas it follows an inverse-J distribution for uneven-aged stands. The major mathematical function used to describe diameter distribution is the Weibull distribution function but other functions also exist, such as the exponential and beta distributions. The Weibull distribution function is flexible and can be used to describe diameter distribution for all forest types. For even-aged stands, diameter distribution is often positively skewed due to mortality in the smaller diameter classes, therefore the normal distribution is usually not an accurate model of the diameter distribution. Competition results in mortality of less vigorous trees and therefore reduces the number of smaller trees.

Meyer (1953) used the exponential distribution function to characterize the inverse-J distribution of balanced uneven-aged forest stands. His function predicts number of trees by dbh class as:

$$Y = ke^{-aX} \tag{5}$$

where Y = number of trees in each diameter class with midpoint X
X = midpoint of the dbh class
a, k = parameters

All diameter class functions require the following components in order to estimate future growth and yield:

- a model for the diameter distribution;
- mortality function; and
- individual tree or volume equation.

Smalley and Bailey (1974) presented yield tables and stand structure for shortleaf plantations in Tennessee, Alabama, and Georgia Highlands. Other studies involving diameter distribution methods on stand plantations include Bennett and Clutter (1968); Lenhart and Clutter (1971); Smith (1978); Dell et al. (1979); and, for natural stands, McGee and Della Bianca (1967); and Schreuder et al. (1979).

Individual Tree Models

The wide range of model varieties available today reflect different silvicultural practices, modeling philosophies and level of mathematical complexity (Clutter et al.

1983). In uneven-aged stands, given the large variability in tree sizes, it seems reasonable that modeling individual trees provides more accurate results on growth and yield estimates. Individual tree modeling is further justified by the fact that younger trees grow differently than older ones; but trees of different ages are pooled when modeling, especially when parameters of individual tree models are fitted to mathematical Models.

Individual tree modeling requires large amounts of time for data collection and analyses. These models also require extensive datasets because values for independent variables are needed for each tree. However, this drawback is currently overcome by the ever-increasing availability of computing technology. Compared to whole stand models and diameter class models, individual tree models are the most complex. These models predict the growth of individual trees based on the characteristics of that particular tree and the forest stand in which it is located. The stand estimates are obtained by summation of all individual tree estimates.

As opposed to stand density variable models, individual tree models partition growth and yield into different products or by species components. Many individual tree models are developed by calculating a competition index (CI) for each tree; it is often used as an independent variable to predict probability of mortality for that tree. If it lives, its growth is then determined as a function of its diameter, height, crown size and competition index. The method of calculating the competition index determines whether an individual tree model is distance-dependent or distance-independent form.

Distance-dependent models include the calculation of the crown competition index based on the distances from the subject tree to all neighboring trees. Distance-independent models do not include calculating the competition index based on distances from the subject tree to all of its neighbors. Obviously, distance-dependent models require more data than distance-independent models. Studies have repeatedly shown that the two types of model do not perform significantly differently.

Distance-Dependent Models PTAEDA (Daniels and Burkhart 1975) and FOREST (Ek and Monserud 1974) are the most comprehensive computer-driven models for distance-dependent, individual tree modeling. PTAEDA was designed for modeling loblolly pine plantations but can be modified to adapt to all-aged stand conditions. PTAEDA performs well for predicting the future stand conditions from any given initial condition; plantations are handled in two stages (Avery and Burkhart 1994). The first stage begins from initial planting "until between-tree" competition starts. This is the precompetitive stage. During this stage, a set of stand equations are used until the crown competition factor reaches a value of 100 percent, a point where the stand is assumed to have reached the initial between-tree competition.

In the second stage, the post competition stage, diameter, height, and crown growth of each tree are annually incremented. Growth in height and diameter is assumed to follow some theoretical growth potential. This potential is adjusted by applying a modifier function based on individual tree's competitive status and vigor (Avery and Burkhart 1994). The crown ratio is used to express the photosynthetic potential of each tree. It is usually used in conjunction with competition index to compute an adjustment factor for height growth.

FOREST is a model that simulates growth and reproduction for both even and uneven-aged mixed species stands. This distance-dependent model includes variables reflecting tree coordinates (spatial location) and tree characteristics such as height, diameter, age, clear bole length, and species. Future tree coordinates and tree characteristics can be generated by the program. The competition index is normally determined from assessment of tree size, density, and species (tolerance to shade). Survival, mortality, and ingrowth can also be modeled from the program. Solutions to the system of equations or output of the model are in the form of periodic stand tables with yield and mortality for different types of product.

Distance-Independent Models Distance-independent models are the most widely used at this time because of their performance and the relatively inexpensive data requirements. They are developed by using individual tree data as well as stand characteristics. Since tree spatial distributions are not known, it is necessary to express indirect measures of competition. These measures often are functions of the size of the subject tree in relation to other trees in stand. Measures commonly used include the ratio of quadratic mean diameter of the stand to the dbh of the subject tree and basal area per acre of trees as large or larger than the subject tree. Examples of distance-independent individual tree model include PROGNOSIS (Wykoff et al. 1982) and a basal area growth model for natural even-aged stands of shortleaf pine (Hitch 1994).

Even Vs Uneven-Aged Stands

Two methods have been used to model individual tree growth in even and uneven-aged stands. The first approach is to develop an individual tree model directly as a function of individual tree and stand characteristics. This method is termed as a "direct model of stem growth". The second method includes two steps. The first is to develop a *potential growth model* based on site quality and individual tree maturity. Then a *modifier model* is developed and is used to adjust the potential growth. This method is termed the "modified potential of tree growth". These two methods may also be considered empirical and semi-empirical respectively (Martin and Ek 1984). In general, these methods produce acceptable results if relationships in the model are based on generally accepted principles of tree growth (Wykoff 1990).

Using the direct model of stem growth method, several growth and yield models have been developed for both even and uneven-aged stand conditions. Stage (1973) used this concept to develop PROGNOSIS, a comprehensive model that simulates growth in natural and managed forest stands of the Northern Rocky Mountains. PROGNOSIS can initiate analysis with existing stands in almost any condition of size, stocking, species, and vigor. Another important characteristic of the model is that the user can collect increment data from any subject stand and incorporate this new growth information into the model (Davis and Johnson 1982). The original model had four main functions to predict diameter growth, height growth, crown ratio, and probable mortality respectively. The model has been modified by several researchers in order to meet their needs. Wykoff (1986) used the direct method to model the diameter increment for western conifers.

Solution of the model gave a natural log of the squared diameter of individual tree. An important aspect of the model is that it does not contain site index or age because the stands were irregular and had mixed species. Site effects were modeled as a function of slope, aspect, geographic location, elevation, and habitat. Also crown ratio was modeled as a function of crown ratio and basal area of all trees as large or larger than the subject tree.

The other method, "semi-empirical" or more commonly referred to as the "modified potential stem growth method" has been widely used (Hahn and Leary 1979; Leary and Holdaway 1979; Belcher et al. 1982; Shifley 1987; Fairweather 1988). As previously stated, in this method growth models are developed using two distinct stages. First, a potential growth model for individual trees is developed based on site quality and maturity. Then a modifier function is applied to the potential growth. The modifier takes into account of the environmental conditions such as competition and management practices.

When developing an individual tree growth model for Pennsylvania, Fairweather (1988) used this potential-modifier method. His model may be used for several species; parameters differ from species to species. Tree basal area is the dependent variable as it is in the STEMS model of Shifley (1987). However, unlike STEMS, site quality is represented by site class (1, 2, or 3) rather than site index. The other major difference was that data restrictions precluded using any type of crown measurement such as crown ratio or crown class as used in STEMS. The data used were from 431 permanent plots located in state forests. All the plots were 1/5th-acre and were remeasured at irregular intervals. Growing season varied from five to nine years and the oldest plots in the data set were established in the early 1960's. The major species modeled were sugar maple (*Acer saccharum* Marsh), black cherry (*Prinus serotina* Ehrh.), red oak (*Quercus Rubra*), and white oak (*Q. alba* L.). The potential growth equation was of the form :

$$\mathrm{POT} = ((\mathrm{a} \cdot \mathrm{TBA}^{\mathrm{b}}) - \mathrm{c} \cdot \mathrm{TBA}) \cdot (\mathrm{d} + \mathrm{eX}_1 + \mathrm{fX}_2) \tag{6}$$

where POT = potential annual basal area growth (ft^2)
TBA = tree basal area (ft^2)
Xi = site classes
a, b,...,f are coefficients to be estimated for each species

The modifier function was of the form:

$$\mathrm{MOD} = \mathrm{g}\left[1.0 - \exp\left(-\left((\mathrm{h}/\mathrm{BAL}) + (\mathrm{i} \cdot \mathrm{DBH}^2)\right) \cdot (\mathrm{BATERM})^{1/2}\right)\right] \tag{7}$$

where MOD = a fraction of the potential basal area growth
BAL = basal area per acre(ft^2) in trees larger in dbh than the subject tree
BATERM = (1.0-(BAA/250)), and BAA is total basal area per acre (ft^2)
g, h, i = are coefficients to be estimated for each species

Basal area growth for any particular tree is then predicted as the product of the potential growth and the modifier function. Although the work of Farr and Johnson (1988) is a height-growth model of Western hemlock (*Tsuga heterophylla*) and Sitka spruce (*Picea sitchensis*), it is worth mentioning because of the model structure. It is a PROGNOSIS type model applied to Southeast Alaskan forests. It is referred to as SEAPROG (Southeast Alaska PROGnosis). In SEAPROG, site index was used because the model was applied to even-aged stands, as opposed to habitat type used in PROGNOSIS. In this study, linear and nonlinear regression techniques were compared for height increments. It was concluded that height increments were best estimated by multiple-regression techniques where the dependent variable was not transformed. By the same token, nonlinear, modified-potential growth models had appeal, but more research on the stand dynamic of hemlock and spruce was recommended before they could be used with more precision.

Goodwin (1987) also used the potential-modifier model techniques to apply the STEMS growth model to Eucalypt (*Eucalyptus* spp.) forests. The model was distance-independent type and age was not used in his study due to the difficulty for taking increment cores in hardwoods. This study is of particular interest as it was the first attempt to model growth of uneven-aged stands in Tasmania. As age and site index are not inherently meaningful in the all-aged stands, other alternatives were considered. One of the attempts was the use of a diameter-height function. The theory behind this was to estimate an asymptote for height for which could be interpreted as mature height. When the asymptote is found, height at age 50 could be estimated using anamorphic site index curves presented by Goodwin (1987).

Shortleaf Pine Models

Relative to other major species of the south, shortleaf pine has received less attention in terms of research. However, information on shortleaf has been accumulating since the early 1980's. Murphy and Beltz (1981) developed a variable-density growth and yield model for shortleaf pine in the West Gulf Region which includes Oklahoma, Arkansas, Texas, and Louisiana. This growth and yield model is applicable to natural even-aged stands and data used were from permanent inventory plots installed by USDA Forest Service. Future basal area per acre was predicted as a function of density and age. Using the projected future basal area, future volume was predicted as a function of site index and stand density. With the same data Murphy (1982) predicted sawtimber volumes for stands in the West Gulf Region.

Lynch and others (1991) used the projected stand basal area to predict merchantable cubic-foot volume, sawtimber cubic-foot volume, and Doyle, Scribner, and International board-foot volume for shortleaf pine in the Ouachita region of Oklahoma and Arkansas. Data used to develop the models were ordinary inventory data from non-managed natural even-aged stands. Therefore, the models may not be suitable for managed stands.

Murphy (1992) reported the effects of different measures of stand structure on periodic growth of natural even-aged stands of shortleaf pine in the Ozarks in Arkansas. In this study, equations to predict future stand basal area were a function of initial age, initial basal area, and age at time of prediction; equations to predict volume were also developed.

Smalley and Bailey (1974) developed a model using a Weibull distribution function to predict yield of shortleaf by diameter class in old field plantations in Tennessee, Alabama, and Georgia Highlands. Stand tables and stock tables were generated from information of diameter class distribution, mortality, and density .

At the level of individual tree models, Shifley (1987) developed a generalized system of models for forecasting tree growth in central States (Missouri, Indiana, and

Ohio) by fitting approximated real growth data to a modification of the TWIGS model (Belcher et al. 1982) to estimate individual tree growth. Shifley used a potential-modifier function to develop the equations. The potential was a version of Chapman-Richards function with the addition of a competition factor. The modifier was a variant of STEMS, which was then combined with the potential function. The combined equation was fit to the entire dataset and the model was evaluated for several species. For shortleaf pine, it explained less than 31 percent of the variation in growth.

A more recent study was that of Hitch (1994). This distance-independent model predicts individual tree basal area growth for even-aged stands in eastern Oklahoma and western Arkansas. Data for this study were collected from permanent research plots. The resulting model was a potential-modifier type where the annual average basal area growth was a function of initial individual basal area, initial stand basal area, age, and some competition factor. The potential used was a version of Chapman-Richards growth function which was restricted to maximum tree size in the region. The function was of the following form:

$$\mathrm{AABAG} = \frac{\beta_1 \cdot \mathrm{B}^{\beta_2} - \beta_1 / \mathrm{M}^{(1-\beta_2)} \cdot \mathrm{B}}{\{1 + \exp[\beta_3 + \beta_4 \cdot \mathrm{BA} + \beta_5 \cdot \mathrm{AGE} + \beta_6 \cdot \mathrm{BAL}]} \tag{8}$$

where AABAG = Individual tree Annual average basal area growth
B = individual tree basal area
M = maximum individual tree basal area
BA = plot basal area
BAL = Proportion of basal area of all trees as large or larger than the subject tree
β_i = parameters to be estimated

When this model was fit to the entire dataset, 60.9 percent of the total variation in individual tree basal area growth was explained and mean square error value was 0.000044 square feet per tree.

A survey of research on shortleaf pine reveals that prior to Hitch (1994) no individual tree basal area growth model applicable to the conditions of the Interior Highlands of Arkansas and Oklahoma has yet been developed. It can be also concluded that the only major study modeling individual tree basal area growth was conducted by Hitch (1994). While this study was for even-aged stands, it can serve as the basis for future studies investigating the modeling of uneven-aged stands.

CHAPTER III

DATA DESCRIPTION

The data used for this study came from 1/5th-acre continuous forest inventory (CFI) plots established by the Deltic Farm & Timber Co., Inc. on forest lands located in the Ouachita Mountains of Arkansas. There were 452 permanent plots established in 1965-66; these were remeasured every five years until 1993. There were 132 and 236 additional plots installed in 1983 and 1988 respectively, to cover other stand conditions in order to get a representative data sample of all conditions found in study area. A total of 820 permanent plots were installed in the uneven-aged forest stands. These plots were established in order to determine:

- net forest growth between measurements, made at 5-year intervals;
- the total volume of the forest as a separate and completely independent check of Deltic's forest inventory system;
- annual and periodic growth rate;
- stand and stock tables; and
- stand conditions and timber quality.

A detailed description of the data collection procedure is given in the Deltic Farm & Timber Co., Inc. CFI procedure manual (1987). Individual tree records were maintained for all trees 5.0 inches diameter at breast height (dbh) and larger. Pertinent information collected on shortleaf pine included the following: (1) dbh to the nearest 0.1 inch; (2) total merchantable height; (3) sawlog height; (4) merchantable height for sawtimber trees (5) tree history; and (6) crown position. The derived independent variables used in the study design included stand basal area (BA) and site index.

Site index (base age 50) for shortleaf pine was also determined for each plot using Misc. Publ. 50 (USDA Forest Service 1929). The site index values used are averages of multiple values assigned through time. Stand basal area was estimated by aggregating all individual shortleaf pine basal area estimates per plot and then multiplying by a factor of five for values on a per acre basis.

For each merchantable basal area and site index variables, six and four distinct levels, respectively, were selected to cover all major conditions found in Ouachita Mountains region of Arkansas. The summary is given in Table I.

Table I illustrates a potential of 24 treatment combinations, similar to Murphy et al. (1985) with slight variations. Unlike Murphy and Farrar (1985), sawtimber basal area was not included in the combination because information about sawtimber basal area was not available in the dataset. However, the purpose of the present study was to develop a individual tree basal area growth model regardless of tree classification whereas Murphy and Farrar (1995). developed a model for the whole stand and were concerned with different tree categories.

TABLE I

SUMMARY OF TREATMENT COMBINATIONS COMPRISED OF SIX CLASSES OF BASAL AREA PER ACRE AND FOUR CLASSES OF SITE INDEX (BASE AGE 50) FOR DELTIC DATASET

Variable	Unit	Class Range	Class Midpoint
Merchantable	ft^2	<11	-
Basal area		11 - 29	20
		30 -49	40
		50 - 69	60
		70 - 89	80
		>89	-
Site index	feet	<45	40
(base age 50)		46 - 55	50
		56 - 65	60
		>65	70

Dataset For Model Development

Plots retained for model development had to satisfy the following criteria:

- natural stands with relative uniform spacing of trees;
- uneven-aged composition;

- plots consisting of at least 70 percent of shortleaf pine basal area;
- plots having less than 10 percent mortality of initial plot basal area; and
- plots that were not harvested or thinned during the entire growth period.

There were a total of 820 plots available. However, after eliminating all the plots that did not meet the selection criteria, only 319 plots remained, but many of them had basal area per acre less than 30 square feet or more than 90 square feet per acre. Plots are not equally represented in each combination (Table II). Very few plots were present in merchantable basal area classes less than 11 square feet per acre and classes greater than 89 square feet per acre. The same is true for all combinations of site index less than 45 feet and greater than 65 feet. Baker and others (in press) suggested that uneven-aged stands having less than 30 square feet of basal area per acre are understocked while those with more than 90 square feet are overstocked. The data were balanced by eliminating plots having less than 30 or more than 90 square feet basal area per acre of shortleaf at beginning of the growth period. The remaining plots were randomly assigned identification numbers. For each merchantable basal area class, the maximum number of plots allowed was restricted to 21. Because the data were mainly concentrated in a few site index and basal area classes, a restricted set of observations were randomly chosen to obtain a more uniform sample across the range of data. This procedure was also adopted by Murphy and Farrar (1985). This unbalanced condition of the data was expected since the data were collected from ordinary forest inventory operations, not from controlled permanent research plots.

TABLE II

NUMBER OF PLOTS BY SITE INDEX CLASS AND MERCHANTABLE BASAL AREA (PER ACRE) CLASS FOR PLOTS RETAINED FOR MODEL DEVELOPMENT PRIOR TO BALANCING THE DATA

Site Index		Merchantable Basal Area						Total
Range	Midpoint	<11	11-29	30-49	50-69	70-89	>89	
<45	40	3	6	1	2	0	0	12
46-55	50	8	36	66	26	8	0	144
56-65	60	8	39	52	28	8	3	138
>65	70	1	4	10	6	1	3	25
Total		20	85	129	62	17	6	319

When establishing research plots, all forest conditions are represented with equal frequency in each condition, consequently requiring no further data balancing. The final selection reduced the total plots to 118 (Table III). The process of plot identification also gives us information on the total number of individual trees (or observations) that will be used for model development. The total number of observations from the 118 plots is 2713.

TABLE III

NUMBER OF PLOTS BY SITE INDEX CLASS AND MERCHANTABLE BASAL AREA (PER ACRE) CLASS FOR PLOTS RETAINED FOR MODEL DEVELOPMENT AFTER BALANCING DATASET

Site Index		Merchantable Basal Area			Total
Range	Midpoint	30-49	50-69	70-89	
<45	40	1	2	0	3
46-55	50	21	20	8	49
56-65	60	21	20	8	49
>65	70	10	6	1	17
Total		53	48	17	118

Calibration and Validation Datasets

From this dataset of 2713 observations, 75 percent of the total observations (2063 observations) were randomly selected for calibrating the model and the remaining 25 percent (650 observations) were used for validation. Summary statistics for the seven variables required for individual tree basal area growth model development for the entire, calibration, and validation datasets are shown in Tables IV, V, and VI respectively.

TABLE IV

SUMMARY OF DESCRIPTIVE STATISTICS FOR SEVEN VARIABLES INCLUDED IN THE COMPLETE DATASET (2713 OBSERVATIONS) USED FOR INDIVIDUAL TREE BASAL AREA GROWTH MODEL DEVELOPMENT

VARIABLE AT MIDPOINT	MINIMUM	MAXIMUM	MEAN	STD DEV	CV (%)
Site index (feet at base age 50)	42.00	73.5	56.87	6.95	12.22
Stand basal area (ft^2 /acre)	37.47	101.82	74.60	15.03	20.15
Diameter at breast height for shortleaf pine (inches)	5.20	20.55	9.16	2.50	27.30
Quadratic mean diameter (inches)	7.67	12.60	9.33	1.09	11.68
Individual tree basal area (ft^2)	0.1475	2.3034	0.49205	0.2822	57.35
Average annual individual tree basal area growth (ft^2)	-0.004189	0.06849	0.01596	0.0111	69.32
Proportion of plot basal area of all trees as large or larger than the subject tree	0	0.990149	0.60015	0.2879	47.96

2713 observations

TABLE V

SUMMARY OF DESCRIPTIVE STATISTICS FOR SEVEN VARIABLES INCLUDED IN THE CALIBRATION DATASET (2063 OBSERVATIONS) USED FOR INDIVIDUAL TREE BASAL AREA GROWTH MODEL DEVELOPMENT

VARIABLE AT MIDPOINT	MINIMUM	MAXIMUM	MEAN	STD DEV	CV (%)
Site index (feet at base age 50)	42	73.5	56.82	6.90	12.14
Stand basal area (ft^2/acre)	37.47	101.82	70.58	14.89	21.09
Diameter at breast height for shortleaf pine (inches)	5.2	20.10	9.0977	2.46	27.05
Quadratic mean diameter (inches)	7.67	12.60	9.31	1.0958	11.77
Individual tree basal area (ft^2)	0.1475	2.2044	0.48446	0.2822	58.25
Average annual individual tree basal area growth (ft^2)	-0.0025436	0.06849	0.015768	0.0110	69.88
Proportion of plot basal area of all trees as large or larger than the subject tree	0	0.996203	0.60514	0.2876	47.53

2063 observations

TABLE VI

SUMMARY OF DESCRIPTIVE STATISTICS FOR SEVEN VARIABLES INCLUDED IN THE VALIDATION DATASET (650 OBSERVATIONS) USED FOR INDIVIDUAL TREE BASAL AREA GROWTH MODEL DEVELOPMENT

VARIABLE AT MIDPOINT	MINIMUM	MAXIMUM	MEAN	STD DEV	CV (%)
Site index (feet at base age 50)	42	73.5	57.04	7.12	12.47
Stand basal area (ft^2/acre)	37.47	101.82	68.55	15.41	22.48
Diameter at breast height for shortleaf pine (inches)	5.2	20.55	9.37	2.60	27.75
Quadratic mean diameter (inches)	7.67	12.60	9.41	1.1225	13.02
Individual tree basal area (ft^2)	0.153	2.3034	0.51616	0.30392	58.88
Average annual individual tree basal area growth (ft^2)	-0.004188	0.06480	0.01656	0.0119	71.80
Proportion of plot basal area of all trees as large or larger than the subject tree	0	0.98839	0.584301	0.29532	50.52

650 observations

The purpose of dividing the original dataset into two subsets is to allow the development of estimates of the regression parameters using the calibration dataset. Once the final model has been selected, the validation dataset can be used to determine the robustness of the model. It is important that two datasets contain variables that have common statistical properties while remaining independent and mutually exclusive of each other.

Test of hypothesis about the mean for each variable concluded that there were no significant differences (0.05 level) between corresponding variables. Therefore it was appropriate to build the model with one dataset and validate with the other dataset.

The data come from uneven-aged stands and dbh can be graphed as a frequency distribution to reveal the inverse-J characteristics. In other words, we would expect to observe a graph that has many trees occurring in the smaller dbh classes and fewer trees in the larger classes. The frequency distribution of number of trees per dbh classes for the entire, calibration, and validation datasets are shown in Figure 1. The datasets are combined in one graph for ease of comparisons.

The original dataset clearly illustrates the inverse-J characteristic of the uneven-aged population. This characteristic is also shown in the calibration dataset. While it appears that the validation dataset has less trees in the smaller dbh classes than expected (a flattening of the frequency for the five to nine inch dbh classes), this was the result of a random selection of observations. Statistical tests also show no significant differences.

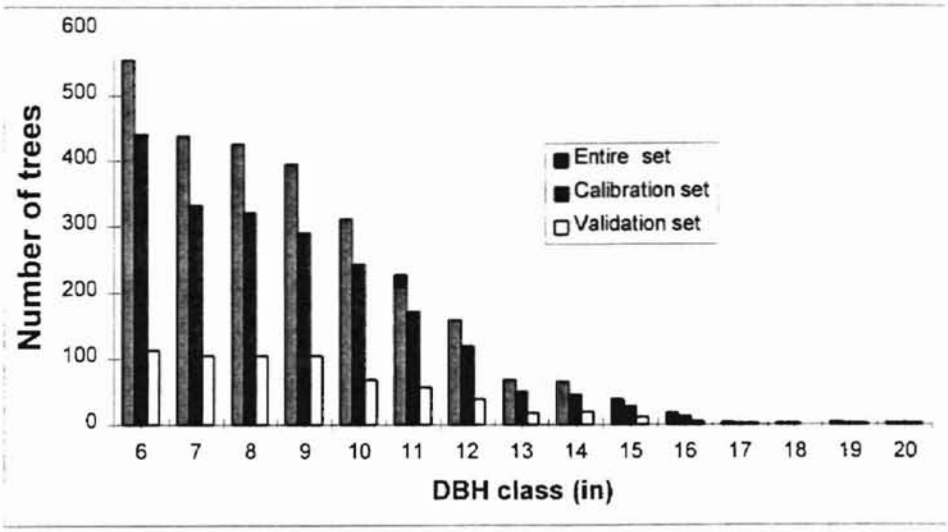


Figure 1. Frequency distribution of trees per diameter class for the entire, calibration, and validation datasets used for model development

CHAPTER IV

ANALYSES AND RESULTS

Several individual tree growth models used in previous studies were examined. Most of these models were used either for natural even-aged stands of shortleaf pine or other species. As no individual tree basal area growth model exists for uneven-aged conditions, models used for even-aged conditions were modified to fit an uneven-aged context. Variables which are not applicable to uneven-aged stands, such as age, were removed, while still preserving the model structure. For instance, a potential function could be maintained while variables of the modifier function could be changed, keeping only those which have a meaning for uneven-aged stands. Selection of models was based on criteria such as:

- dependent variable,
- independent variables,
- growth theories of organism or biological growth, and
- fit of the model to the data.

In the past, several researchers have studied different dependent variables to be used when fitting data to a given model. The most studied variables are dbh and basal area increments. These variables are directly related as basal area is obtained by multiplying the square root of dbh with a constant. Other forms of the these variables can be used such as natural logarithms or their inverses. The STEMS models have featured the prediction of either diameter growth (eg. the Lake States version) or basal area growth (the Central States version).

West (1979) is among the researchers who investigated the subject. He concluded that there were no differences in the precision of predicted future growth values when basal area and dbh are used as dependent variables. However, he found that basal area increment was highly correlated with various independent variables used in equations that predict growth. Many researchers prefer the use of basal area as a dependent variable. Shifley (1987) preferred basal area because, when calculating tree volume, tree basal area and tree height are commonly used. Fairweather (1987) used basal area increment for the same reasons, arguing that the correlation between basal area growth and tree basal area is stronger than that between diameter growth and dbh. He reported correlations for white oak ($r = 0.658$ vs. $r = 0.333$) and eastern hemlock ($r = 0.647$ vs. $r = 0.230$). Hilt and Teck (1987) also reported that the relationship between tree basal area growth and tree size is graphically more distinct than the relationship between dbh growth and tree size. Therefore, the choice of basal area increment as dependent variable is reasonable.

The independent variables used in these studies were those that are readily available from ordinary forest inventories. This study uses inventory data from natural stands. It would be very difficult to assess the geographic locations of all trees in such stands, therefore distances and directions between subject trees and neighboring trees were not recorded. The lack of these data leaves only one modeling approach, the distance-independent method. The independent variables were individual midpoint tree basal area, midpoint stand basal area, site index, midpoint dbh, midpoint stand quadratic mean diameter, proportion of basal area of all trees as large or larger than the subject tree, and midpoint stand basal area of all species. Age, a commonly used independent variable in modeling growth of even-aged was not considered in this study as it is inestimable in uneven aged stands.

Growth models are only valid when they predict growth to reasonable individual or population sizes. For instance, a model that would grow a tree indefinitely would be violating established biological concepts of growth. A living organism grows faster in the early ages, then levels off until it reaches a maximum size, which is arrived at an asymptotic rate. When the tree size approaches its maximum, growth approaches zero. A good model is expected to reflect this scenario.

Making the analogy with the logistic growth, an individual tree growth is proportional both to its size and to the difference between the maximum size and the actual size. In symbols:

$$GR = k\,(S\,(M - S)) \tag{9}$$

where GR = growth rate
S = actual tree size
M = maximum tree size
k = constant of proportionality

The quantities GR and S vary with time; k and M are parameters.

Most important, the model chosen must fit the data. Shifley (1987) used a fit index to assess the fit of models to data. The fit index is equivalent to r^2 and can be evaluated by the following formula:

$$\text{Fit Index} = 1 - \left\{ \frac{\text{Error Sum of Squares}}{\text{Corrected Total Sum of Squares}} \right\} \qquad (10)$$

This statistic measures the proportion of variation of the dependent variable accounted for by the model. Models chosen for their fit were then compared to evaluate their performances on a independent data set. Hitch (1994) described different statistics used in evaluating models used in modeling individual tree basal area of natural even aged stands of shortleaf pine. Criteria used in the evaluation were average deviations which are calculated as the sum of the differences of predicted average annual basal area growth and actual average annual basal area growth, divided by the number of observations. Mean square errors were calculated as the sum of the squared differences of predicted average annual basal area growth and actual average annual basal area growth, divided by the number of observations. Average error percent was calculated as the average predicted annual basal area growth minus the average actual basal area growth, divided by the product of actual average annual basal area growth and the number of observations, the result multiplied by 100. Plots of residuals against different independent variables were analyzed to check for any bias associated with the models.

Development of Models

There are two main methods used for modeling individual-tree growth. These are the direct approach and indirect approach. The direct approach predicts growth directly as a function of several independent variables and the model derived explains variations in tree growth by modeling deviations from the mean growth of trees. The first model tried was of the direct approach adapted to the PROGNOSIS model form (Wykoff 1986). The natural logarithm of basal area growth is predicted as a function of site index and additional tree characteristics. The resulting equation was on the form:

$$\ln(\mathrm{BAG}) = b_1 + b_2\mathrm{BALS} + b_3\mathrm{SBA} + b_4 \ln \mathrm{dbh} + b_5\mathrm{BAS} + b_6\mathrm{SI} \tag{11}$$

where BALS = proportion of total basal area of all shortleaf pine trees per acre as large or larger than the subject tree
SBA = stand basal area (ft^2 /acre)
BAS = individual tree basal area (ft^2)
SI = site Index, feet at base age 50
dbh = diameter at breast height (in)
b_i = parameters to be estimated

The logarithmic transformation of annual basal area growth of individual tree was fitted to the data using the Reg procedure in SAS (SAS Institute Inc. 1989). The dependent variable was fit to several independent variables by the techniques of multiple regression, where the selection option was used to retain the best explanatory variables based on mean square error, r^2, and Mallow's C(p).

This technique was applied in conjunction with stepwise regression which could cause all significant independent variables to be included in the model. Based on the results of these procedures, several models having different independent variable combinations were compared by studying the scatter plots of the residuals and by analyzing different independent variables used in the model. For example, we were concerned by having many variables which are highly correlated in the same model unless having both variables resulted in significant improvement of the model performance expressed by higher r^2, lower mean square error, and improvement of variance of the residuals.

Equation (11) provided an r^2 value of 0.4463. When transformed to predict average annual basal area growth directly, the model provided a similar fit index with a value of 0.4457 and mean square error of 0.0000687. The fit index was low compared to other studies. Hitch (1994) found a fit index of 0.593. However, given that Hitch's study was based on data from research plots, fit index obtained here was considered acceptable. The parameter values associated with independent variables are reasonable in sign since growth increases with an increase in tree size and site index, and growth decreases with an increase of competition (BALS and plot basal area) as shown in Table VII. However, the model contained two independent variables highly correlated with each other; the natural logarithm of dbh and individual tree basal area. Elimination of either variable reduced the model performance by considerably decreasing r^2.

<u>Use of modified potential growth models</u> dominates many recent studies. Theoretically, these models set up an upper limit size that a given tree cannot exceed based an growth theories (Hann and Leary 1979). The growth model is created in two stages. In the first step, a potential growth model is developed based on forest growth theories. In the second step, a modifier function is developed to adjust the potential growth to actual growth achieved. The modifier function reflects the environmental stress on the tree. These two steps produce a model having a following form:

Individual-tree growth = (potential)*(modifier)

TABLE VII

PARAMETER VALUES FOR EQUATION # 11 WHEN FITTED TO THE CALIBRATION DATASET

Parameters	Parameter Estimates	Standard Error
b_1	-8.794061	0.37427861
b_2	-0.314270	0.08279128
b_3	-0.764954	0.15410207
b_4	2.269143	0.18183499
b_5	-0.050609	0.00441713
b_6	0.013279	0.00190541

Among potential functions tried was the version of Smith et al. (1992). This model was developed to be used for both even and uneven-aged stands of shortleaf pine and was explicitly expressed as follow:

$$PAIBBAG = 0.0031 + 0.03165(IIBBA)^{0.46922} - 0.03809(IIBBA) \qquad (12)$$

where PAIBBAG = potential annual inside bark basal area growth (meters^2)
IIBBA = initial inside bark basal area (meters^2)

The parameters for this function were obtained by fitting growth data collected from open grown trees. The above function is a modification of the growth function of Chapman-Richards model where an intercept was added. The model of Smith and others (1992) uses basal area inside bark as the independent variable and may not be directly applicable to ordinary forest inventories. Values commonly collected in the field are outside bark diameter. For this reason, Smith et al. developed an equation to predict a double bark thickness for individual tree:

$$\mathrm{DB} = 0.75631 + 0.8879\ \mathrm{dbh} \tag{13}$$

where DB = double bark thickness (centimeter)
dbh = diameter at breast height (centimeter)

Equations 15 and 16 permit expression of potential basal area growth in ft^2 as a function of dbh (inches). In Hitch (1994), the algorithm designed to estimate potential growth outside bark and in English system was as follows:

1. Calculate current inside bark basal area
2. Calculate future inside basal area

$$\mathrm{FIBBA} = 0.00031 + (0.03165\ \mathrm{CIBBA}^{0.46922}) + 0.96191\ \mathrm{CIBBA} \tag{14}$$

3. Calculate potential annual basal area growth

$$\mathrm{PABAG} = 0.000582 + 0.17378\ \mathrm{FIIBA}^{0.5} + 12.96343\ \mathrm{FIBBA} - \mathrm{B} \tag{15}$$

where CIBBA = current inside bark basal area (meters^2)
dbh = diameter at breast height in inches
FIBBA = future inside bark basal area
PABAG = potential annual basal area growth
BA = current tree basal area (ft^2)

Hann and Leary (1979) suggested another potential growth based on the principle that growth of trees in the upper 95th percentile are not subject to competition, therefore can represent the potential growth. Following the Hann and Leary procedure, the data were divided into cells by 1-inch dbh class and site index class, where the classes are as previously defined. Within each cell, average and standard deviation of basal area growth were calculated. The potential growth was then estimated by the average basal area growth in each cell plus 1.65 multiplied by the standard deviation of the mean. The results correspond to the 95th percentile of growth in each cell. This potential growth was then fitted to the potential growth model using the SAS procedure NLIN. The potential growth was of the form:

$$\text{Potential} = b_1 + b_2 BA^{b_3} + b_4 \left((SI)(BA)^{b_5}\right) \qquad (16)$$

where BA = basal area of individual tree
SI = site index
b_i = parameters to be estimated

Equation 16 differed slightly from that of Hann and Leary because crown ratio was not included since these values were not available for the dataset used here. When residuals were plotted against tree size, there was a pronounced heteroscedasticity. Variance of the residuals increased as the tree size increased. This was apparently a result of fewer trees represented in larger dbh classes. In order to reduce heteroscedasticity, a weight (number of observations in each cell) option was added to the NLIN procedure. This resulted in improvement of the variance of the residuals, there was no trend when wheighted residuals were plotted against individual tree basal area. The resulting model explained 75 percent of the total variation in potential basal area growth and provided a mean square error value of 0.000568.

TABLE VIII

PARAMETER VALUES FOR EQUATION # 16 WHEN FITTED TO THE DATA OF STUDY

Parameter	Parameter Estimates	Asymptotic Standard Error
b_1	0.005819294	0.00826532710
b_2	-0.025945197	0.00891475773
b_3	1.727449	0.30921293766
b_4	0.00114511	0.00016549935
b_5	0.965828403	0.25306658777

We observe that b_1 is negative and b_3 is positive. Normally, we would expect the opposite in order to be in agreement with Chapman-Richards growth function. However, Richards (1959) suggested that b_1 could be negative and b_3 be positive in models fitted from empirical data. Therefore these parameters values in equation 19 are allowed.

Modifier Functions are used to adjust the potential growth to the actually achieved growth by individual trees. Due to competitive pressure at typical levels of forest stocking, individual trees do not grow to their potential. There are several factors that affect this reduced growth such as crowding or density which can be expressed in terms of basal area per acre or number of trees per unit area. Other factors can be genetic or environmental, in general, modifier models are functions of site, stand characteristics, and measures of competition. Selection of models was based on initial performance and simplicity in application. Modifier functions that were selected and analyzed are presented below.

The first analyzed function was proposed for even and uneven-aged stands of loblolly pine by Murphy and Shelton (1993). The model adjusts potential growth by incorporating several stand and individual tree characteristics. The modifier function is of the form (with SI, BA, BAL previously defined):

$$\text{Modifier} = 1/\{1 + \exp(b_1\text{BAL} + b_2\text{SI} + b_3\ \text{BA} + \ldots)\} \qquad (17)$$

For a tree growing without competition, its actual growth equals the potential growth. Consequently the modifier has a maximum value of one which is arrived at at an asymptotic rate, when the exponential term approaches zero. Alternatively, if the tree receives extensive competition. the modifier function approaches zero; this occurs when the exponential term grows to infinity. Parameters of the function may be estimated by the method of linear or nonlinear regression. The use of linear regression is accomplished by rearranging the equation as:

$$\text{BAG} = \text{POTBAG}/\{1 + \exp[b_1\text{BAL} + b_2\text{BA} + b_3\text{SI} + \ldots]\} \qquad (18)$$

and,

$$\ln[(\text{POTBAG}/\text{BAG}) - 1] = (b_1\text{BAL} = b_2\text{SI} + b_3\text{BA}+ \ldots) \qquad (19)$$

where POTABAG = potential basal area growth (ft^2/acre/yr)
BAG = individual tree basal area growth (ft^2/acre/yr)

Equation (19) is in a form where multiple linear regression can be applied and is flexible so that more variables that explain basal area growth can be included and those not significant may be removed through stepwise techniques.

Another type of modifier function tested on this dataset was that described by Shifley (1987) Shifley proposed a modifier function which is a variation of the STEMS and TWIGS modifiers. This type uses individual tree competitive status in the stand, expressed by the total basal area of all trees per acre as large or larger than the subject tree (BAL) to adjust the potential growth to the actually achieved growth. Other competition factors can also be incorporated such as stand density and crown position. As applied to our dataset, the modifier function was of the form:

$$\mathrm{Mod} = b_3\ \{1 - \exp[- b_4/(\mathrm{BAL} + 1) + b_5 B(1 - \mathrm{BA}/\mathrm{BA}_{\max})^{1/2}]\} \qquad (20)$$

where
Mod = proportion of potential growth actually achieved by individual tree
$\mathrm{BA}_{\max}$ = maximum stand basal area (ft^2/acre)
BA = stand basal area (ft^2/acre)
B = individual tree basal area (ft^2)
BAL = total basal area of all trees as large or larger than the subject tree
b_i = parameters to be estimated

The maximum basal area used was 200 ft^2/acre as suggested by Shifley. The maximum basal area used in model development was 90 ft^2 and the maximum basal area observed on field was 136 ft^2 per acre. For even-aged stands of shortleaf pine, Hitch observed a maximum of 144 ft^2 whereas Miscellaneous Publication No. 50 reported a maximum of 174 ft^2 per acre. For equation 23, as maximum basal area approaches 200 ft^2/acre, its value approaches zero. The structure of this equation is such that the maximum value is equal to b_3 which is arrived at an asymptotic rate as BAL and basal area approach zero and individual tree basal area increases. This modifier did not perform well in initial analyses and consequently was not used in model comparisons.

Comparison of Models Used

Several different potential and modifier function combinations along with a direct function based on PROGNOSIS were used to create several models to predict annual basal area growth for uneven-aged stands of shortleaf pine. Four models were selected for comparison based on initial performance and simplicity. Parameter estimates and fit indices of these models when fitted to the independent dataset are presented in Table IX. Model 1 was a variation of the PROGNOSIS model (Wykoff 1990) and used equation 11 (reiterated here).

$$\ln(bai) = b_1 + b_2\, bal + b_3\, Ba + b_4 \ln(dbh) + b_5\, SBA + b_6\, SI \tag{11}$$

where bai = annual individual tree basal area growth (ft^2/acre/yr)
Ba = individual tree basal area at midpoint
bal = proportion of total basal area of all trees as large or larger than the subject tree
dbh = diameter at breast height (inches)
SBA = stand basal area at midpoint.
SI = site index (base age 50)
b_i parameters to be estimated

This model has a fit index value of 0.44574 and a mean square error value of 0.0003456. The model residuals appear to be approximately distributed with a slight skewness to the right.

Model 2 used a potential growth function described by Shifley (1987). The first version tried for this model contained bal, dd, and crown position at the same time. Bal and dd are highly correlated while crown position is difficult to predict in the future. Different variable combinations were studied by eliminating crown position and dropping either bal or dd_mid. Crown position could be dropped without affecting seriosly the model performance; but dbh lost significance ($p = 0.94$). When crown position, dd, and qmd were dropped at the same time, the model performed and all parameters were significant at 0.05 level. Model 2 has a fit index value of 0.48142 and has the following implicit form:

$$\text{aabag} = \text{pot}/\{1 + \exp(b_3 + b_4\ \text{SBA} + b_5\ \text{bal} + b_6\ \text{si} + b_7\ \text{dbh})\} \tag{21}$$

where $\text{pot} = b_1\,(\text{Ba})^{b_2} - b_1 / M^{(1-b_1)} * \text{Ba}$

aabag = annual average basal area growth

SBA = stand basal area (ft^2/acre)

si = site index (feet at age index 50)

dbh = diameter at breast height at mid point

bal = proportion of total basal area of all trees as large and larger than the subject tree

Model 3 was based on a potential growth function described by Smith et al. (1992). Although crown position variable was significant in the first version of this model, removing crown position did not significantly hamper the model performance; the fit index dropped from 0.4734 to 0.46257 and the variance of residual was not substantially changed. Model 3 has the following implicit form:

$$\text{aabag} = \text{pot}/\{1 + \exp(b_5 + b_6\ \text{dd} + b_7\ \text{SBA} + b_8\ \text{qmd} + b_{10}\ \text{si})\} \tag{22}$$

where aabag = average annual basal area growth (ft^2)

qmd = quadratic mean diameter at midpoint (inch)

dd = ratio of mean diameter over quadratic mean diameter

SBA = stand basal area (ft^2/acre)
si = site index (fett at age 50 years)
pot is a potential basal area growth derived from equations 14 and 15

Model 4 was based on the potential described by Hahn and Leary (1979) given by equation (15) and a modifier proposed by Murphy and Shelton (1993). For this model, the drop of crown position variable resulted in an increase of the variance of residuals and reducing considerably the fit index. It was therefore left in the model although it is not a continuous variable. Only four possibilities were available for each tree, dominant, codominant, intermediate, and overtopped. Because of the nominal characteristic of the crown position variable, dummy variables were assigned to each crown status as:

Dominant = 1 then $c_1 = 1$ $c_2 = 1$ $c_3 = 1$
Codominant = 2 then $c_1 = 1$ $c_2 = 1$ $c_3 = 0$
Intermediate = 3 then $c_1 = 1$ $c_2 = 0$ $c_3 = 0$
Overtopped = 4 then $c_1 = 0$ $c_2 = 0$ $c_3 = 0$

The model provided a fit index value of 0.4714 and a mean square error value of 0.00002036. Model 4 has the following implicit form:

$$aabag = pot/\{1+\exp(b_6+b_7 dbh+b_8\, qmd+b_9\, SBA+ b_{10}\, bal+ b_{11}\, c_1= b_{12}\, c_2=b_{13}c_3)\} \tag{23}$$

where aabag, dbh. qmd, SBA, bal, c_1, c_2, and c_3, as previously defined.

TABLE IX

PARAMETER ESTIMATES AND FIT INDICES FOR MODELS FITTED TO THE CALIBRATION DATASET

Parameter	Model # 1	Model # 2	Model # 3	Model # 4
b_1	-8.79406	0.179337	0.00031	0.005819
b_2	-0.31427	0.861911	0.03165	-0.02545
b_3	-0.76495	0.293189	0.46922	1.727449
b_4	2.269143	0.018418	0.03809	0.001145
b_5	-0.05061	-0.024241	1.073813	0.965828
b_6	0.013279	0.646854	1.534472	-.0.569946
b_7	-	-0.076309	0.012237	0.019417
b_8	-	-	-0.133048	-0.063180
b_9	-	-	-0.017942	0.01315
b_{10}	-	-	-	0.782989
b_{11}	-	-	-	-0.497736
b_{12}	-	-	-	-0.299839
b_{13}				0.066432
Fit index	0.44574	0.48142	0.46257	0.47140

All four models retained were compared and evaluated with an independent dataset of 650 observations which was reserved for validation purpose. Using parameters estimated with the calibration dataset, predicted values of annual basal area growth were generated with the calibration dataset for each model. With predicted values, average deviations, mean square errors, and mean absolute deviations were calculated. These statistics were used to compare all the models for their performance by dbh class, site index class, and by stand density. For a complete evaluation, box plot analysis of the residuals by diameter, site index, and stand density classes were performed instead of the simple scatter plots. The residuals were calculated by subtracting observed annual growth from predicted growth given by each model. Positive values of residuals indicated an overestimation while negative values indicated an underestimation of the model. Figure 2 illustrates the box plots used to detect bias of different models.

Average Deviations were calculated for each model by 2-inch dbh classes and the results are shown in Table X.

TABLE X

AVERAGE DEVIATIONS OF MODELS BY DBH CLASS WHEN EVALUATED USING THE VALIDATION DATASET

Class	Obs	Model 1	Model 2	Model 3	Model 4
6	169	-0.002788	-0.00058279	-0.0017030	-0.00078076
8	204	-0.0033006	-0.00045857	-0.0010425	-.00081931
10	148	-0.0018588	0.0010894	0.00033825	0.00089453
12	75	-0.0017695	0.0018339	0.00060407	0.00085280
14	38	-0.00043814	0.00016803	-0.00097975	-0.0022905
16	16	-0.00088184	0.0031527	0.00062599	-0.00011118
ALL	650				

Average deviations were also calculated by site index class and stand density level for all the models. Results are shown in Table XI. Based on average deviations criterion. Model 2, 3, and 4 performed similarly and have small average deviations across all diameter classes, site index, and all stand densities. Model 1 seemed to perform poorly compared to all other models particularly for dbh classes where the annual average basal area is underestimated. (TABLE XI)

TABLE XI

AVERAGE DEVIATIONS OF MODELS BY SITE INDEX AND BASAL AREA PER ACRE WHEN EVALUATED USING THE VALIDATION DATASET

Site Index	Obs	Model 1	Model 2	Model 3	Model 4
40	31	0.0019248	0.0041308	0.0028918	0.0016536
50	217	-0.0041394	-0.0011544	-0.0021392	-0.0027725
60	271	-0.0020839	0.00084013	-0.00025756	0.00046909
70	131	-0.0023019	0.00069160	0.00030710	0.0019039

BA/acre	Obs	Model 1	Model 2	Model 3	Model 4
40	195	0.0026800	-0.0012014	-0.0013201	-0.0012438
60	342	-0.0024546	0.00072022	-0.00047775	0.000073690
80	113	0.0030397	0.0013877	-0.00006899	0.00028914

Mean Square Errors were calculated for each model by 2-inch dbh classes and the results are shown in Table XII. For this criterion, all the models performed similarly well although Model tended to have slightly heigher mean square errors value for several diameter and all stand basal area classes.

TABLE XII

MEAN SQUARE ERRORS OF MODELS BY DBH CLASS WHEN EVALUATED USING THE VALIDATION DATASET

DBH Class	Obs	Model 1	Model 2	Model 3	Model 4
6	169	0.000045473	0.000030703	0.000032425	0.000029003
8	204	0.000068554	0.000049293	0.000050054	0.000049063
10	148	0.000084168	0.000069676	0.000069084	0.000074709
12	75	0.00011195	0.000092811	0.000099469	0.00010126
14	38	0.00017480	0.00012110	0.00012469	0.00015129
16	13	0.00030573	0.00023792	0.00020875	0.00018927
ALL	650				

Mean square errors were also calculated for each model by site index and stand basal area levels. The results are shown in Table XIII. With respect to this criteria, Models 2 performs slightly better than all other models. Model 1 had larger values of mean square errors for larger classes compared to other models.

TABLE XIII

MEAN SQUARE ERRORS OF MODELS BY SITE INDEX AND BASAL AREA PER ACRE WHEN EVALUATED USING THE VALIDATION DATASET

Site Index	Obs	Model 1	Model 2	Model 3	Model 4
40	31	0.000061266	0.000072812	0.000056400	0.000049521
50	217	0.000088104	0.000064356	0.000065507	0.000068680
60	271	0.000060543	0.000045136	0.000047588	0.000046177
70	131	0.00012744	0.000096603	0.000096610	0.00010327

BA/acre	Obs	Model 1	Model 2	Model 3	Model 4
40	195	0.00012076	0.0000835	0.000088549	0.000088188
60	342	0.000060122	0.000059520	0.000059045	0.000058067
80	113	0.0030397	0.000038637	0.000036016	0.000046002

Mean Absolute Deviations were also calculated for each model by 2-inch dbh class and the results are reported in Table XIV. For this criterion, Models 2 and 3 performed equally and have the lowest mean absolute deviations. However, the mean absolute deviation alone does not give a clear indication of the best model among the four.

TABLE XIV

MEAN ABSOLUTE DEVIATIONS OF MODELS BY DBH CLASS WHEN EVALUATED USING THE VALIDATION DATASET

DBH Class	Obs	Model 1	Model 2	Model 3	Model 4
6	169	0.0047213	0.0040686	0.0040077	0.0039137
8	204	0.0059963	0.0052231	0.0051791	0.0051000
10	148	0.0072635	0.0066516	0.0066956	0.0067944
12	75	0.0086126	0.0078255	0.0080224	0.0082199
14	38	0.0098843	0.0087411	0.0085887	0.0094156
16	13	0.013781	0.012218	0.011818	0.010526
ALL	650				

Average mean absolute deviations were also calculated for each model by site index and stand density. All the models performed equally well except Model 1 which present slightly heigher values (Table XV).

Residuals for each model by DBH class were examined to detect any model biases. Figure 2 shows that Model 2 performs well throughout all dbh classes. Model 1 understimates basal area growth for all dbh classes whereas Models 3 and 4 understimate basal area growth of larger trees (dbh 14 inches and larger). The underprediction of larger trees is understandable since large trees were underpresented in the dataset used for model development. Figure 3 also shows residuals plots distribution by stand basal area classes. Models 2, 3, and 4 performed reasonably well across all stand basal area classes

TABLE XV

MEAN ABSOLUTE DEVIATIONS OF MODELS BY SITE INDEX AND BASAL AREA PER ACRE WHEN EVALUATED USING THE VALIDATION DATASET

Site Index	Obs	Model 1	Model 2	Model 3	Model 4
40	31	0.0060760	0.0065321	0.0059682	0.0056068
50	217	0.0068195	0.0058860	0.0058913	0.0058049
60	271	0.0059023	0.0052510	0.0052840	0.0053088
70	131	0.0082688	0.0073254	0.0072764	0.0075199

BA/acre	Obs	Model 1	Model 2	Model 3	Model 4
40	195	0.0080837	0.0068770	0.0070308	0.0069276
60	342	0.0067318	0.0058262	0.0058438	0.0057411
80	113	0.0056625	0.0046216	0.0042050	0.0046789

while Model 1 underpredicts individual tree basal area gowth in stands having basal area between 30 and 70 ft^2/acre. Residual plots were also generated for all models by site index classes. Figure 4 shows that Model 2 performs fairly well based on this criterion. All the models are biased with stands having low site indices where annual average basal area growth is overestimated. In general, Model 1 and 3 underpredict the basal area growth of individual trees in all site index classes except site index class 40.

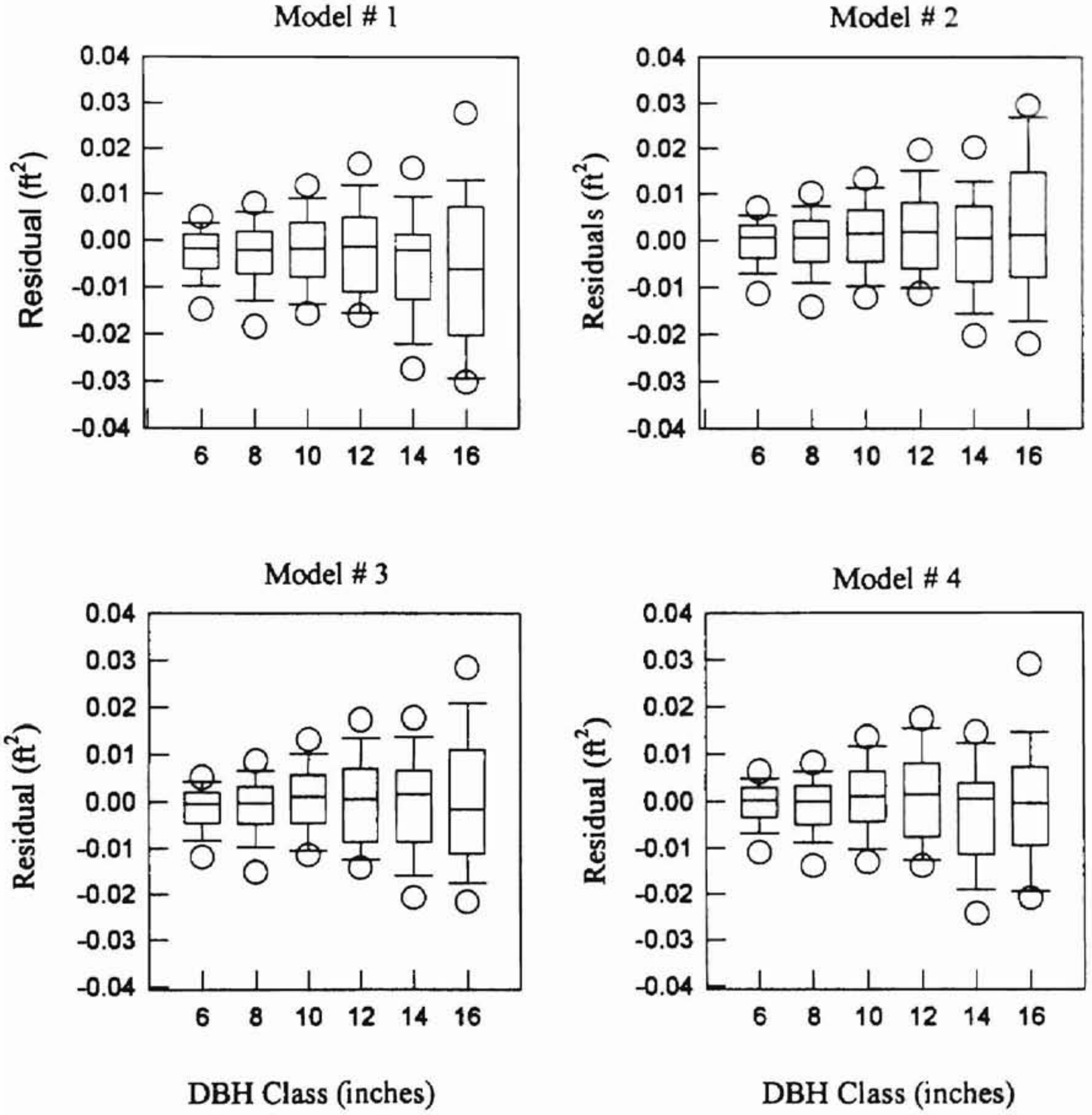


Figure 2. Plot of Residuals vs. DBH Class for all Models When Evaluated with the Independent Dataset

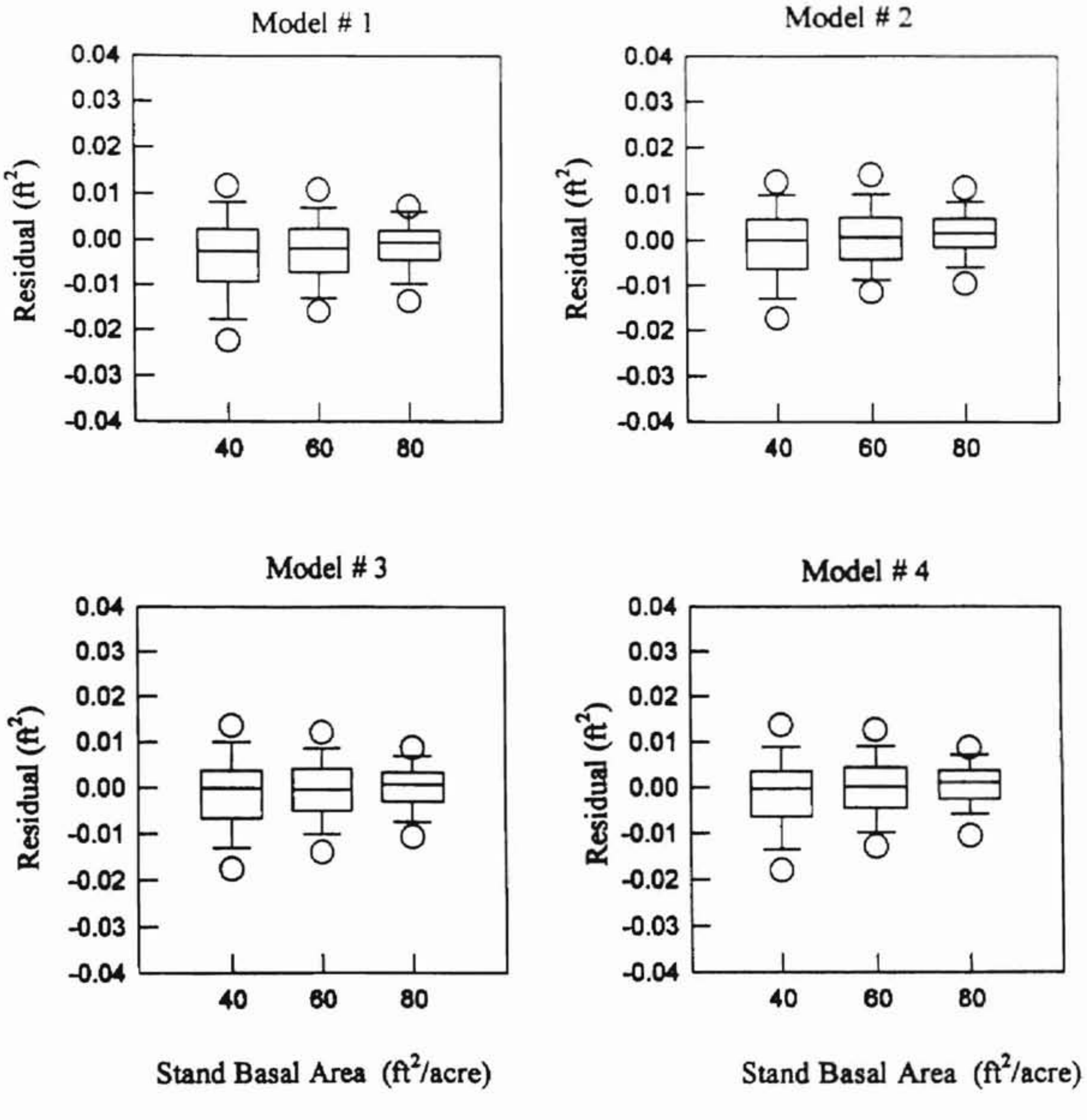
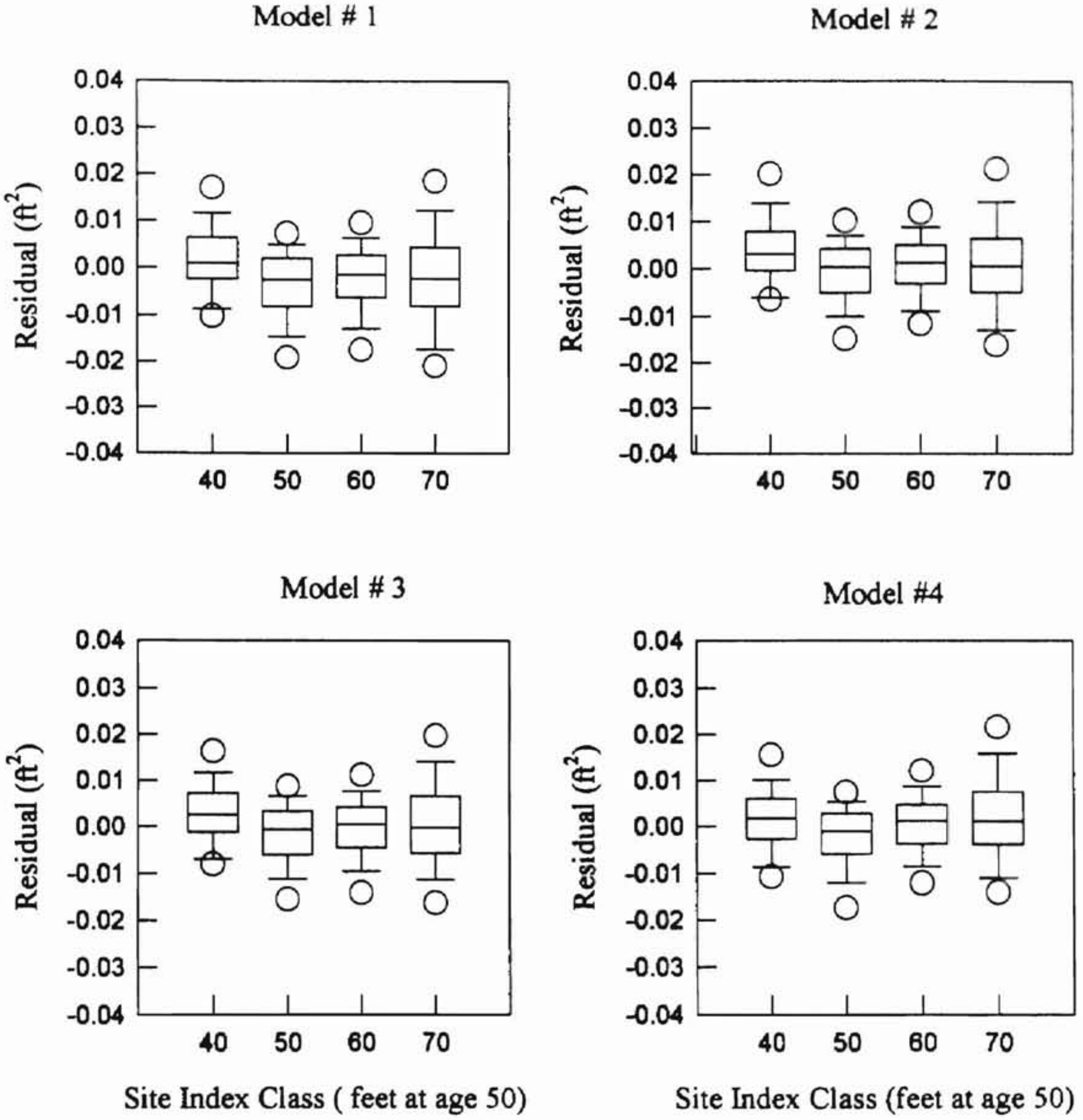


Figure 3. Plot of Residuals vs. Stand Basal Area for all Models When Evaluated With the Validation Dataset



Model # 1

Model # 2

Model # 3

Model #4

Figure 4. Plot of Residuals vs. Site index for all Model When Evaluated With the Validation Dataset

CHAPTER V

SUMMARY AND CONCLUSIONS

Based on the results of the validation tests in combination with other criteria such as model structure and simplicity, independent variables, and growth theories, Model 2 was retained as the best individual tree basal area growth model for uneven-aged stands of the Interior Highlands of Arkansas and Oklahoma. Model 1 was not selected because it performed poorly based on all criteria considered relative to all other models. It had the lowest fit index and was biased with respect to individual tree diameter at breast height (dbh) and site index. Even though Model 1 had the simplest form, another major drawback is that it included two highly correlated independent variables: the natural logarithm of dbh and individual tree basal area.

Models 2, 3, and 4 were of the form: Potential * Modifier where Model 2 used a potential described by Shifley (1987), Model 3 used a potential growth of open grown trees described by Smith and others (1992), and Model 4 used a potential growth described by Hahn and Leary (1979). Model 3 was not selected because it had lower fit index than Model 2; otherwise Model 2 and Model 3 performed similarly based on mean square errors, average deviations and absolute deviations. Model 3 had also an inconvenience of using a potential based on metric units and requires extensive computations to transform English units to metric units.

Model 4 performed well based on mean squares error, average deviations and absolute deviations, comparably to Model 2. But, Model 4 had lower fit index than Model 2 and had several nonsignificant parameters; dbh c3 as already described. The nonsignificance of c3 (p>0.6) suggested that there was no difference between crown position of dominant and codominant trees in the model and that their effect on individual tree basal area growth could be combined in one variable. The effect of crown position of intermediate and overtopped trees was significant (p=0.0001). This was expected since dominant and codominant trees do not compete heavily against each other while intermediate and overtopped trees do not get direct sunlight for photosynthetic activities. It should be noted that when dummy variables for crown position were added, the model improved the fit index from 0.4575 to 0.4714.

The choice of Model 2 over Model 4 also was a matter of structure and simplicity of the potential growth model used. Model 4 was not selected because the potential model had 5 parameters, but also used site index as independent variable, a variable which is not easily determined for ordinary uneven-aged stands. Also, Model 4 has a lower fit index, 0.4714 vs. 0.4814. In addition, Model 2 proved to perform well without crown position, an already questionable variable in uneven-aged stands since future values of crown position are difficult to predict. In uneven-aged stands, trees that are overtopped at young ages may become dominants and codominants at later ages.

Model 2 has an advantage of simplicity; only two parameters need to be estimated for the potential growth function. This potential, which is a variation of the Chapman-Richards function, has normally three parameters. However, specification of maximum tree size according to the procedure of Shifley and Brand (1984) fixes one of the parameters. This parameter represents the maximum size a tree can achieve and, therefore, is mathematically the maximum asymptote, where growth equals zero. Model 2 performed well for all criteria considered although it tends to underestimate the growth of trees in the 16-inch and larger diameter classes. This underestimation may be explained by the fact that there were few trees in these dbh classes in the dataset. For normal uneven-aged management, trees approaching larger classes are regularly cut under the selection system. The average deviations by dbh class are almost zero for most dbh classes with a slight increase where fewer trees were present.

Model 2 was refitted to the combined calibration and validation datasets so that the following was obtained:

$$\mathrm{aabag} = \frac{b_1 \cdot \mathrm{Ba}^{b_2} - b_1 / M^{(1 - b_2)} \cdot \mathrm{Ba}}{1 + \exp(b_3 + b_4\mathrm{SBA} + b_5\mathrm{BAL} + b_6\mathrm{SI} + b_7\mathrm{DBH})} \tag{24}$$

where aabag = average annual basal area growth (ft^2 /year),
Ba = individual tree basal area at midpoint (ft^2),
SBA = stand basal area at midpoint (ft^2/acre),
BAL = proportion of basal area of all trees per acre of all trees as large or larger than the subject tree (ft^2/acre),
SI = site index (base age 50),
M = 7.068384.

The model explained 50.6 percent of the variation in individual tree basal area growth for the combined dataset with a mean square error of 0.0000853. Parameter estimates are reported in Table XVII.

Table XVII

PARAMETER ESTIMATES FOR THE FINAL MODEL WHEN FITTED TO THE COMBINED DATASET

Parameter	Estimate	Standard Deviation
b_1	0.1793376	-
b_2	0.8619109	-
b_3	0.269814	0.31984929
b_4	0.018350	0.00159987
b_5	-0.022824	0.00348683
b_6	0.592277	0.14646376
b_7	-0.077601	0.01726682

For the final model fitted to the combined dataset, average deviations were calculated by dbh class and the results are shown in Figure 5. The model was not biased with respect to dbh in predicting annual average basal area growth of individual tree. For trees in larger diameter classes, the model tended to underpredict the annual average basal area growth but this underprediction might have been due to the fact that few trees were represented. Given that the study is undertaken on uneven-aged stand where selective harvesting in larger diameter classes is regular, and given the diameter frequency distribution theories in uneven aged stands, the underpresentation of the larger diameter classes was expected. In order to reduce the variance in larger dbh classes, trees of dbh larger than 15 inches were pooled together in one diameter class to produce Figure 5. Average deviations for the final model fitted to the entire dataset are relatively low in all diameter classes (Figure 5).

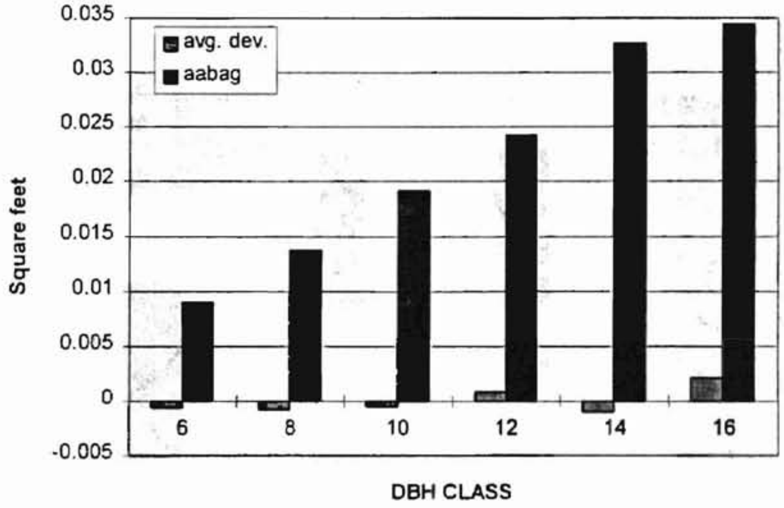


Figure 5. Average Deviations and Mean Average Annual Individual Tree Basal Area Growth by DBH Class for the Final Model When Fitted to the Entire Dataset

Figure 5 reveals that the dataset contained no trees where basal area growth had culminated, implying that trees in this system were harvested when basal area growth is still at a high rate. The maximum tree diameter observed in the data of study was less than 21 inches, whereas trees more than 25 inches of dbh were observed in Hitch's study in the same region. Therefore this model should not be used to predict growth of trees having sizes not represented in this study.

The model overpredicted basal area growth in site index 40 which might have been due to the fact that there were few trees (144) in that site index class (Figure 6). Average basal area growth deviations increases as the stand basal area increases. The model shows that individual tree basal area growth decreases as the stand basal area increases which is in agreement with forest growth theories (Figure 7).

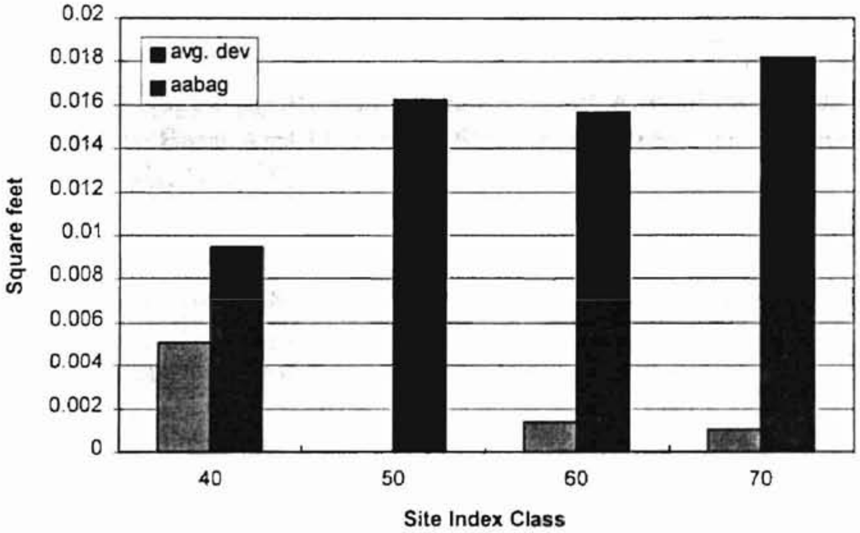


Figure 6. Average Deviations and Mean Average Annual Individual Tree Basal Area Growth by Site Index Class for the Final Model When Fitted to the Entire Dataset

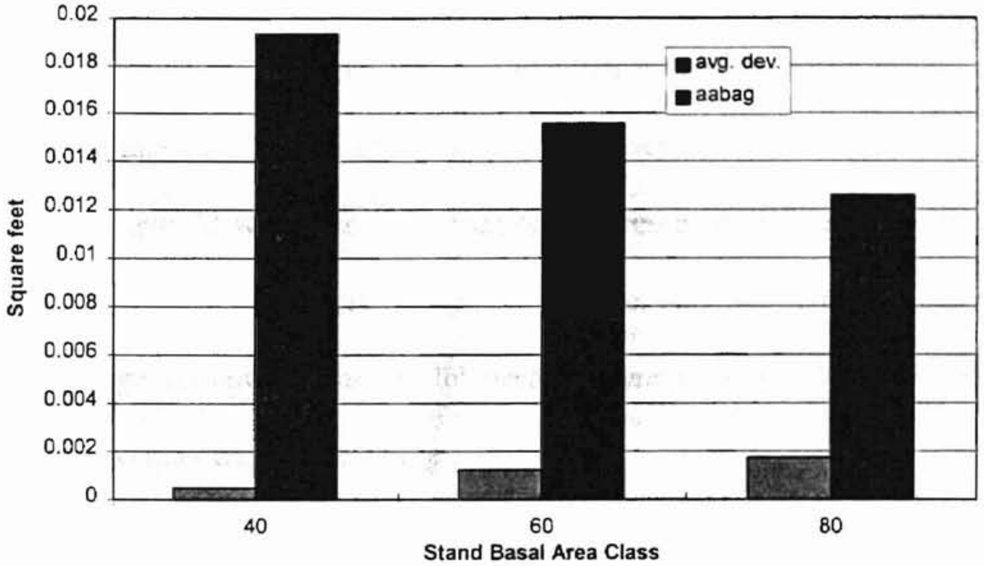


Figure 7. Average Deviations and Mean Average Annual Individual Tree Basal Area Growth by Stand Basal Area Class for the Final Model When Fitted to the Entire Dataset

Forest managers dealing with uneven-aged stands of shortleaf pine in the Ouachita Mountains are the primary potential users of the current model. This is the first individual tree basal area growth model for uneven-aged shortleaf stands in the region, and should be a useful tool to predict future growth to provide information for management decisions. The model can be incorporated with other functions such as survival and ingrowth models to predict uneven-aged shortleaf pine growth dynamics in the Ouachita Mountains. The model should also be restricted to the region represented by the data used for the model development. Therefore users are encouraged to compare their stand characteristics to the data described in this study in order to discern whether their conditions are similar to the study area.

The data used for this study were from ordinal inventory permanent plots as opposed to research plots. The data therefore may not have represented equally all the possible stand conditions. Large dbh classes (>16") were not well represented. Site index classes 40 and 70 were also not adequately represented. The user of the model should take into account these shortcomings. The model may not perform correctly for stands having large trees over 16-inches dbh because there were few plots that contained in the larger dbh classes trees in the dataset.

The equation presented here could be improved by collecting more data on sites with site index of less than 45 and and by refitting the model to well-balanced data, especially from permanent research plots.

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VITA

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Master of Science

Thesis: A DISTANCE INDEPENDENT INDIVIDUAL TREE BASAL AREA GROWTH MODEL FOR NATURAL UNEVEN-AGED STANDS OF SHORTLEAF PINE (*Pinus echinata* Mill.) IN THE OUACHITA MOUNTAINS

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