HYPERPLANE DESIGN IN OBSERVER-BASED DISCRETE SLIDING MODE CONTROL

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PREFACE

The design of digital control systems for practical applications demands the designer to spend a great amount of time and effort in trial-and-error procedures and computer simulations. The reason for this is that only a few works exist in the literature that address all the issues relevant to practical situations, like the effects of computational time delays, presence of disturbances and parametric uncertainties, and the use of state estimators. This is especially true in the case of Sliding Mode Control. This paper presents a general method for the design of a key parameter in Observer-Based Discrete Sliding Mode Control: the sliding hyperplane. Two ways of selecting hyperplane coefficients are developed and tested by simulation.

I must express my thankfulness to my graduate advisor, Dr. Eduardo Misawa, for hisconstant support and supervision of the present work, which is ultimately based on a digital control strategy he designed.

I wish also to thank my wife Francisca for her unconditional support and help during graduate school, and to my parents Alfonso and Ana Maria, which have always encouraged any academic undertaking in many ways.

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Chapter 1

Introduction

Many powerful controller design methodologies that reduce design complexity are available for continuous time systems, but the same is not true for digital control of dynamic systems, especially when significant modeling errors and unknown disturbances exist. They often involve a significant amount of guesswork and trial-and-error procedures. Sliding Mode Control is a technique originally conceived for continuous systems that is remarkably good in rejecting certain disturbances and parameter variations. Direct digital implementation of the technique suffers from severe limitations, so several design schemes have been developed that specifically address the issues relevant to digital control. Among those techniques, only a few have the characteristics that allow them to have practical importance, like the observer based discrete sliding mode control (OBDSMC). This technique also involves a great deal of trial-and-error procedures when modeling errors exist, due to the sensitivity to the selection of coefficients in the definition of sliding surface. The only design guideline available for choosing the sliding coefficients is a stability criterion which has the twofold disadvantage of being just a sufficient condition and involving a complex matrix structure when a boundary layer and observer are used.

1.1 Objective of the Research

The research effort started as an attempt to develop a general procedure for assigning arbitrary eigenvalues to the sliding equivalent matrix that appears in OBDSMC systems when motion occurs inside the boundary layer. Investigation shows that the equivalent matrix and system dynamics can be decomposed into two subsystems, having one of them the same structure as a continuous sliding system. The influence of the sliding gain and boundary layer thickness in the system's motion inside the boundary layer is linked to an eigenvalue of the equivalent matrix. The attempt is succesful in solving the eigenvalue assignment problem for arbitrary controllable systems. Furthermore, the reduction of the problem to a continuous time case, for which several results already exist, allows to develop and test an LQ-optimal criterion for discrete sliding surface design.

1.2 Major Contributions of the Research

The eigenvalue assignment problem is solved for arbitrary controllable discrete systems and a LQ-optimal design method is proposed and tested, giving superior performance when compared to manual selection of eigenvalues. The theoretical results also leave the doors open to further refinements of the technique, namely LQR design (which is already available for the continuous-time case), and LQG-LTR design, which is at least conceptually feasible due to the linear nature of the system dynamics inside the boundary layer.

1.3 Limitations

The version of OBDSMC upon which this work is based relies on an assumption which may not be rigurously true in a mathematical sense, but has practical relevance. The present work relies on the same assumption just as the OBDSMC does. Specifically, the assumption states that, since observer dynamics are independent of tracking error dynamics, observer error must decay to zero after a "short" transient and therefore does not affect tracking error dynamics (further discussion about this assumption is available in Section 3.1).

Chapter 2

Overview of Variable-Structure and Sliding Mode Control

2.1 Variable Structure Control Systems

One of the main achievements in the research of uncertain systems has been the formal development of the Variable Structure Control (VSC) approach. As evidenced by their name, VSC systems constitute a class of nonlinear systems in which the control law, or control structure, is qualitatively changed during the control process to attain improved overall characteristics in the controlled system. As we shall see, in the Sliding Mode Control case the major improvement corresponds to insensitivity to parameter variations and external disturbances. Historically, VSC systems are characterized by a control structure which is switched as the system state crosses specified discontinuity surfaces in the state-space, and the sliding mode describes the particular case when, following a preliminary motion onto the switching surfaces, the system state is constrained to lie upon the surfaces [1]. A familiar example that falls into the category of VSC is On-Off temperature control using relays. The discontinuity surface is defined by the desired temperature, and different control actions are produced each time the controlled temperature crosses the discontinuity surface. Figure 2.1 illustrates the concept. As pointed out in [1], the major practical disadvantage, and at the same time, the greatest limiting factor to the applicability of this approach is the need for a discontinuous control structure. To attain the performance prescribed by theory, the existence of physical elements capable of realizing "continuous switching action", or instantaneous switching, is required. Since no such elements can be ever built, theory has been developed for VSC systems that either rely on continuous approximations to discontinuous elements, for example in Burton and Zinober [2], or that specifically address the presence of boundary layers and dead zones (Slotine [3]). In the next section, we shall focus our attention into a class of VSC systems known as Sliding Mode Control.

2.2 Sliding Mode Control in Continuous Time

A large amount of work related to Continuous-Time Sliding Mode Control (CTSMC) can be found in the literature. The essentials of the technique can be found in Slotine [4], Itkis [5], Utkin [6] and many others. The origin of the theory is linked to Russian literature. The simplest way to introduce CTSMC is to view it as the joint action of two control inputs, one corresponding to the familiar concept of feedback linearization and the other corresponding to a switching action. The first control

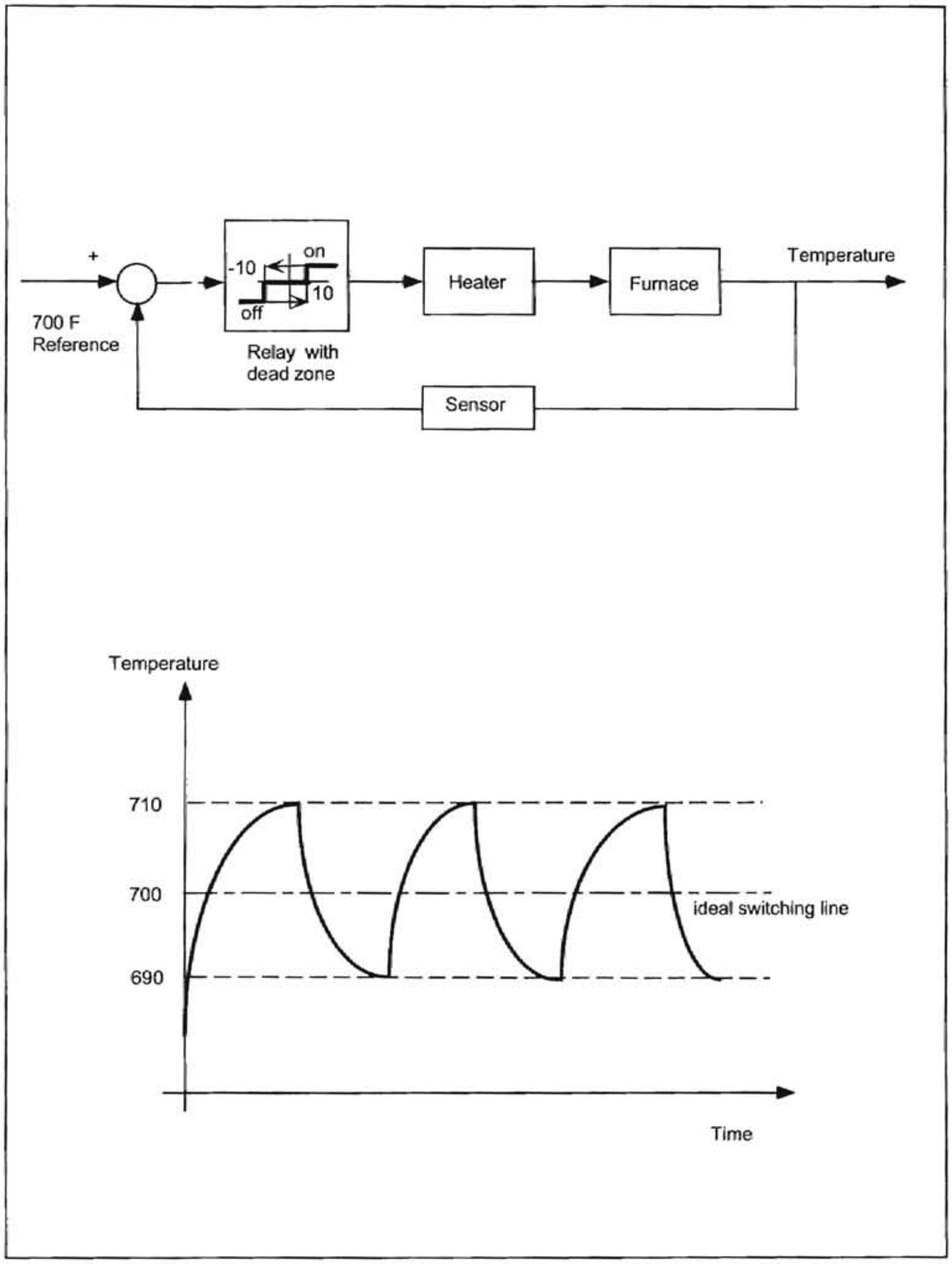

Figure 2.1: A simple example of Variable Structure Control : Temperature Control of a Furnace

input replaces system dynamics by desired, easy-to-control dynamics, as in feedback linearization, and the switching action guarantees robustness against unmodeled dynamics, external disturbances and parametric uncertainties, in the same way as an On-Off temperature control is able to maintain the temperature within close limits, regardless of external thermal influences [1]. As stated in the previous section, the major drawback that prevents CTSMC from being a universal solution to control problems is the need for a true instantaneous switching device. The inability of real components to switch instantaneously causes a phenomenon known as chattering, which consists of high frequency, finite amplitude oscillations of the device between equilibrium positions. This phenomenon not only seriously limits expected performance but has a destructive physical effect on the switching component and other physical system elements. To introduce the idea of CTSMC let us consider the single-input, single-output second-order nonlinear dynamic system:

$$\ddot{y} + f(y, \dot{y}) = u + d$$

where u is the control input and d represents an input disturbance, while $f(y, \dot{y})$ is an arbitary nonlinear function of the two states. Define the "sliding surface" s as a linear combination of the states such that $s = 0$ defines the desired system dynamic behaviour:

$$s = \dot{y} + \lambda y$$

In this case it is clear that $s = 0$ yields an asymptotically stable solution for y as long as λ is chosen to be positive. Moreover, the rate of decay can be freely chosen. If we

[1] As long as there is enough power to compensate for them

can guarantee that arbitrary initial conditions in the state space result in trajectories that approach the sliding surface, and that such attractiveness is maintained even when disturbances and modeling errors exist, the sliding surface becomes an invariant subspace of the phase plane, thus the system exhibits the desired behavior. Conditions for the existence of a sliding mode for a more general case can be found in the literature, (e.g. Itkis [5]). In our example, we can choose the control input u so that s^2 becomes a Lyapunov function, which can be interpreted as a square measure of the distance of the state to the sliding surface. If s^2 is a Lyapunov function, then its time derivative needs to be negative, yielding the condition[2] :

$$s\dot{s} < 0 \tag{2.1}$$

A more explicit way to approach the design of the control u is to specify the dynamics of s when $s \neq 0$, for example, we can choose u such that :

$$s\dot{s} = -\eta|s|$$

or:

$$\dot{s} = -\eta\,\mathrm{sgn}(s) \tag{2.2}$$

where η is an arbitrary positive number. In this case it can be easily shown [4] that arbitrary initial conditions will result in a trajectory that reaches $s = 0$ in finite time and that the sliding surface is indeed an invariant subspace, thus the system motion approaches the origin according to prescribed dynamics. Note that the presence of the signum function in Eq. 2.2 indicates that the control u will contain

[2] Actually, condition 2.1 is a general condition for linear sliding surfaces.

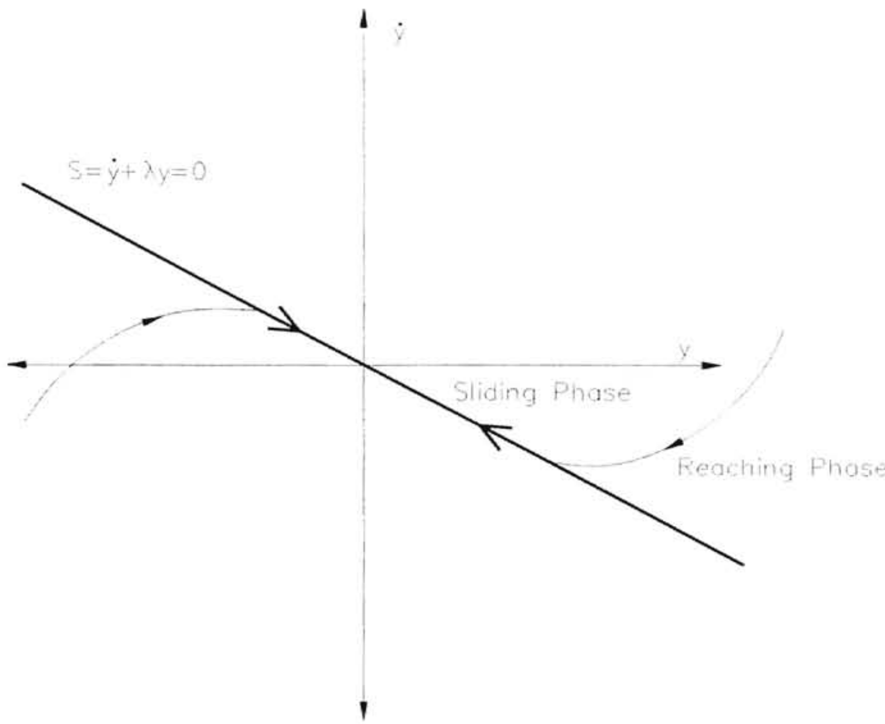

Figure 2.2: Sliding motion in the state space

a discontinuous element. A formal description of the motion in sliding regime when such a discontinuous control law is used was developed by Filippov [7]. Filippov obtains a general construction method for the so-called "equivalent control", that is, the control input when in sliding mode, as a linear combination of the control inputs at both sides of the sliding surface, namely $s > 0$ and $s < 0$. Continuing with our example, and assuming that the disturbance is bounded, that is,

$$|d| < \delta$$

for some positive number δ, and also assumming that there is perfect knowledge about the nonlinear function $f(y, \dot{y})$, it is easy to verify that the control law

$$u = -\lambda\dot{y} + f(y, \dot{y}) - \eta\,\mathrm{sgn}(s)$$

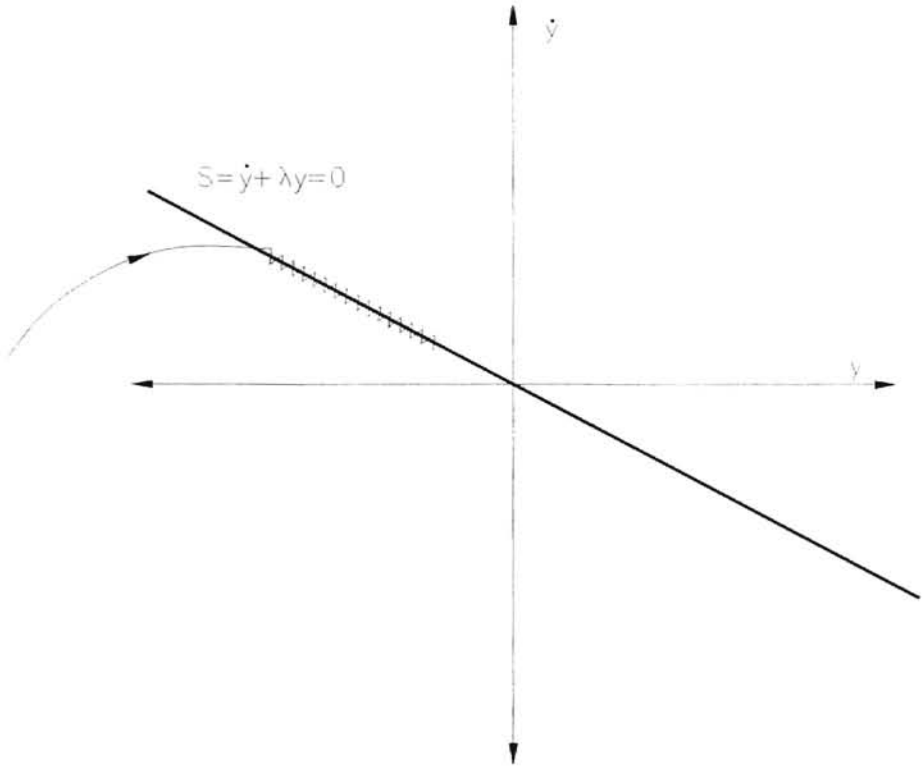

Figure 2.3: Chattering as a result of imperfect switching

where $\eta \geq \delta$ guarantees that $s\dot{s} < 0$, which is the sliding condition. It becomes evident now that there is a requirement for a switching element in the control signal, an element capable of physically realizing the signum function. When using a relay as an approximation to this, the problem of chattering becomes clear. During the time interval it takes for the relay to switch from one position to another, the disturbance will drive the state outside the sliding surface, but since the surface is always attractive, the trajectory will return to it. This phenomenon is known as chattering and its ocurrence is not limited to physical realization with relays, but also in digital control systems, in which the computational delay is responsible for its ocurrence. Many approaches exist to overcome the chattering effect. In one of these approaches, the discontinuous signum function is replaced by the saturation function, which is continuous. The effect produced is to allow the phase trajectory to oscillate within a neighborhood of the sliding surface known as "boundary layer" without switching action. This method assigns a low pass filter structure to the local dynamics of the variable s, thus eliminating chattering. Conditions for boundary layer attractiveness are given in [8]. In our example, replacing the control signal with

$$u = -\lambda\dot{y} + f(y, \dot{y}) - \eta\,\mathrm{sat}(\frac{s}{\phi})$$

where ϕ is the boundary layer thickness at each side of the sliding surface, and choosing $\eta \geq \delta$ as before[3] the boundary layer attractiveness is guaranteed and chattering is eliminated, under the assumption of perfect model knowledge. Among other formal developments and advances we can mention the works of Hashimoto and Konno [9], Zinober [10], who investigated methods for designing the sliding surface; Slotine and Coetsee [11] developed an adaptive version of CTSMC in which uncertain parameters are estimated on-line; Kachroo and Tomizuka [12] analyzed several continuous control approximations inside the boundary layer; Sira-Ramirez [13] investigated CTSMC in a rigorous differential algebra framework. Numerous applications exist for this technique, including automobile fuel-injection control [14]; magnetic levitation [15]; torque control of DC electrical machinery [16], control of DC-to-DC power converters [17] and many others.

[3]This condition holds for a constant boundary layer thickness; an appropriate condition can be stated for time-varying ϕ.

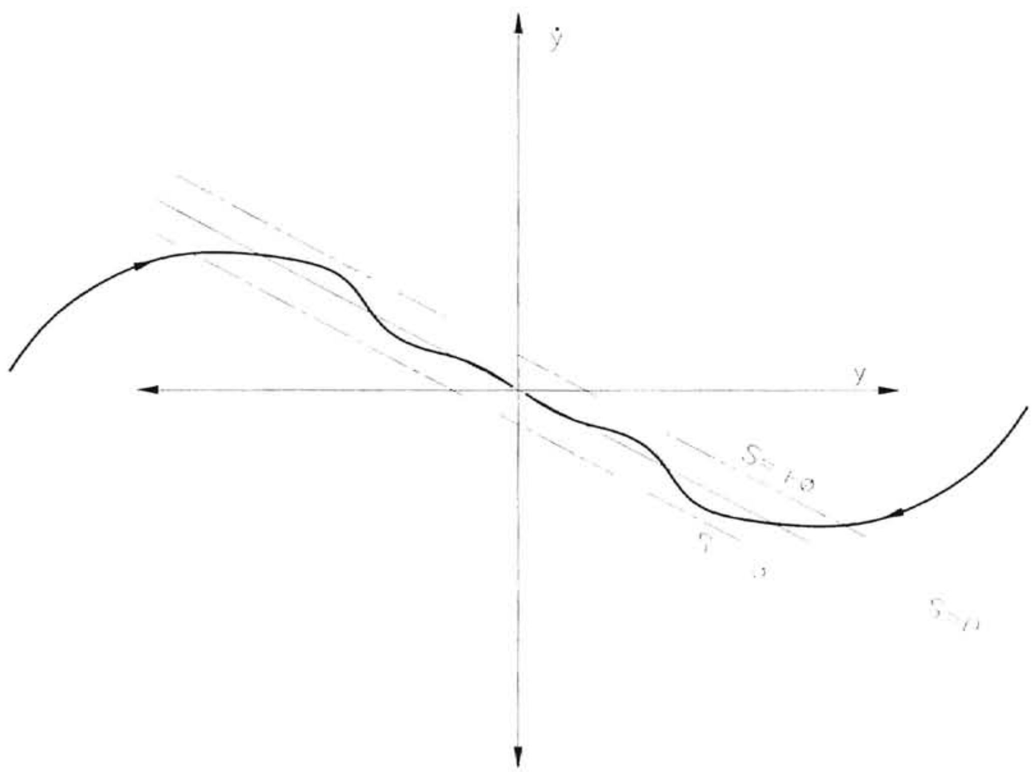

Figure 2.4: The boundary layer eliminates chattering

2.3 Overview of Discrete-Time Sliding Mode Control

Control systems that were conceptually developed for the continuous time may not perform well - or may even become unstable - when direct digital implementation is attempted. This widely accepted fact is particularly true for sliding systems. Thus, many researchers have either addressed the limitations when direct implementation is done or have proposed designs which take the sampling process into account. Milosavljevic [18] was among the first researchers to formally state that the sampling process limits the existence of a true sliding mode; Drakunov and Utkin [19], Furuta [20], and others investigated the effects of sampling in sliding systems; Sarptürk [21], Spurgeon [22] and Kotta [23] specifically addressed the stability issue. Several designs have been proposed (for example, Misawa [24]) for nonlinear plants when unmatched uncertainties are allowed; Su and co-workers [25] developed a similar design where uncertainties have to be matched, Pieper and Goheen [26] attempted to obtain a controller based on input-output models; Paden and Tomizuka [27] designed a discrete time sliding mode controller for position control; Misawa [28] proposed an observer-based discrete sliding controller for linear plants which uses a Luenberger observer and that takes into account the computational time delay, paper upon which this thesis is based; and Sira-Ramirez and co-workers [30] obtained a design for linear systems that uses a sliding structure in the observer. A typical characteristic of discrete sliding mode control systems is that the system trajectories are no longer constrained to lie upon the sliding surface, but are al-

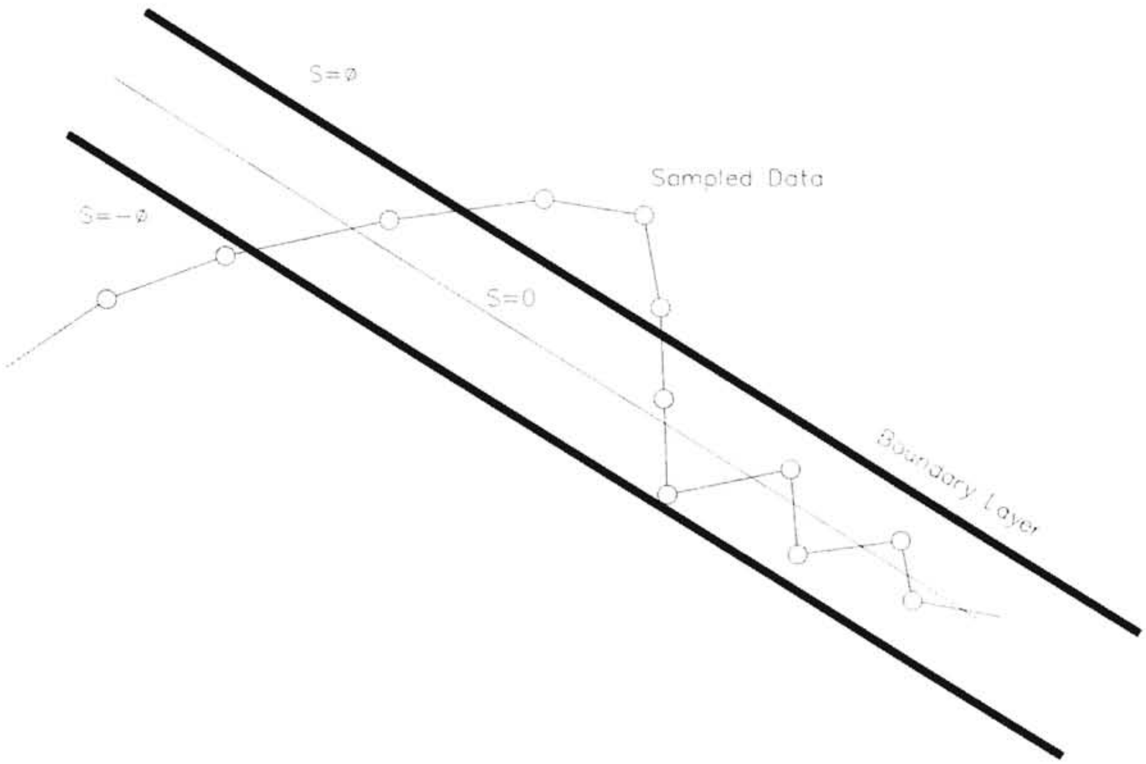

Figure 2.5: Quasi-sliding motion in the state-space

lowed to remain in a well defined neighborhood of the surface, called the boundary layer. Research efforts were aimed at obtaining a condition for the existence of a quasi sliding mode, in which the boundary layer is made attractive. A discrete-time counterpart [21] of the continuous sliding condition 2.1 was proposed as:

$$|s_i(k+1)| < |s_i(k)|,\ i = 1, 2, ..., m \tag{2.3}$$

Spurgeon [22] later proved that condition 2.3 was sufficient but not necessary. A large amount of applications of VSC have been reported in the literature for the control of robots. Also, developments exist for electrohydraulic and pneumatic actuators, ([31], [32], [33], [27]). Some applications to electromechanical positioning devices have been reported, being a representative work the application of VSC to head positioning in disk drives. [37]. Besides the difficulties associated with the sampling process, practical implementations of sliding mode control systems involves the use of state estimators and the issue of disturbances and parameter uncertainties becomes exremely important. Only a few works exist in the literature that address all of the issues mentioned above, so the designer must expect a large amount of trial-and-error and simulation procedures. It is the purpose of this work to provide a technique to design discrete sliding mode control systems so that the amount of guesswork is reduced, while attainable performance is increased.

Chapter 3

Discrete Sliding Surface Design

3.1 A version of Observer-Based Discrete Sliding Mode Control

In Chapter 2, an overview of the research work in discrete sliding mode systems was given. Practical control engineering applications generally demand the use of a state estimator when the control strategy involves state measurements. Sliding Mode Control, continuous or discrete, is not input-output based, but relies on the availability of state measurements. Research activity on the combination of sliding systems and observers is not easily found in the literature, especially in the discrete case. Work on observer-based controllers for the continuous case was reported in [30] and, although not the paper's main topic, some discussion can be found in [35]. Misawa proposed a version of observer-based discrete sliding mode control (OBDSMC) reported in [29], upon which this work is mostly based. Following is a brief overview of the results reported in the mentioned article. Let a discrete-time, linear SISO system of dimension n be described by the state equations:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{3.1}$$

Matrices A, B and C are constant and assumed to be perfectly known, and such that (A, B) is controllable and (A, C) observable. A Luenberger observer is used to estimate the states, and is given as:

$$\hat{x}(k) = (A - HC)\hat{x}(k-1) + Bu(k-1) + Hy(k-1) \tag{3.2}$$

where $\hat{x}(k)$ is the state estimate and H is the observer gain. Define the tracking error $\tilde{x}(k)$ and the sliding surface $s(k)$ as:

$$\begin{aligned} \tilde{x}(k) &= x_d(k) - \hat{x}(k) \\ s(k) &= G\tilde{x}(k) \end{aligned}$$

where $x_d(k)$ is the desired state trajectory and G is a row vector that defines the sliding hyperplane. The control law is given by:

$$u(k) = (GB)^{-1}[G((I - A)\hat{x}(k) + \Delta x_d) + K \operatorname{sat}\left(\frac{s(k)}{\phi}\right)] \tag{3.3}$$

$$K = \gamma + 2\Delta t\epsilon \tag{3.4}$$

$$\phi \geq \gamma + \Delta t\epsilon \tag{3.5}$$

$$\Delta x_d(k) = x_d(k+1) - x_d(k) \tag{3.6}$$

$$\operatorname{sat}\left(\frac{s(k)}{\phi}\right) = \begin{cases} 1 & \text{, if } s > \phi \\ \frac{s}{\phi} & \text{, if } |s| \leq \phi \\ -1 & \text{, if } s < -\phi \end{cases}$$

where K is the sliding gain, ϕ is the boundary layer thickness, γ is a positive number that bounds disturbances, Δt is the sampling period, and ϵ is an arbitrary positive number. Although a rigorous proof of convergence (when an observer is used) is not available, the following arguments and practical experience with the controller support its validity. From the above equations, it is straightforward to deduce the following identities:

$$\begin{aligned} z(k+1) &= (A - HC)z(k) && (3.7) \\ \tilde{x}(k+1) &= [A - BG(GB)^{-1}(A - I)]\tilde{x}(k) \\ &\quad -B(GB)^{-1}[K \operatorname{sat}\left(\frac{s(k)}{\phi}\right) + G(I - A)z(k)] && (3.8) \\ s(k+1) &= s(k) - K \operatorname{sat}\left(\frac{s(k)}{\phi}\right) && (3.9) \end{aligned}$$

Eq.(3.7) shows that estimation error dynamics, $z = x - \hat{x}$, decay as in a linear system, that is, independently of the choice of control. This is obtained, however, assuming a perfect model and no disturbances. A key argument in the article is that $z(k)$ is almost zero after a "short" transient. With that assumption, it follows from equations (3.9) and (3.8) that s goes to zero asymptotically and that so does $\tilde{x}(k)$, provided the following condition due to Furuta [20] is met:

Condition 1. *The sliding surface G should be determined such that the eigenvalues of*

$$A - \frac{BG}{GB}(A - I)$$

lie inside the unit circle.

It is evident that the above condition is just a stability requirement, and no guidelines are given as to how to choose vector G. The main objective of the work presented in the following sections is to solve the eigenvalue assignment problem and address performance issues.

3.2 A Note on Model Following Control

The following arguments are valid for both continuous-time and discrete-time systems; we choose to adopt the discrete notation. Let a linear time-invariant, SISO discrete plant model be given by:

$$x(k+1) = Ax(k) + Bu(k)$$

and consider the model following problem that consists of finding the control u such that the plant state x tracks the desired state given by:

$$x_d(k+1) = Ax_d(k) + Bu_d(k)$$

where u_d is a convenient control input. In the OBDSMC framework, the second equation represents the *xd-generator* [24]. Subtracting the first equation from the second one gives the tracking error dynamics:

$$x_d(k+1) - x(k+1) = \tilde{x}(k+1) = A\tilde{x}(k) + B(u_d(k) - u(k)) = A\tilde{x}(k) + B\tilde{u}(k)$$

It is clear that the tracking error dynamics are caracterized by matrices A and B, so certain tracking control problems can be approached as regulation problems in error state space with the same matrices. While this observation is widely known, it is not obvious that the same situation is found when a linear observer is present and the objective is to have the *state estimate* track the desired state. As will be seen in the following section, OBDSMC generates linear tracking error dynamics that still verify this property.

3.3 Another Stability Condition

In this section, a slightly different condition for stability will be presented, that will be shown to include Condition 1. For this purpose, from Eq.(3.8), we obtain tracking error dynamics *inside the boundary layer* by taking

$$\text{sat}\left(\frac{s(k)}{\phi}\right) = \frac{s(k)}{\phi} = \frac{1}{\phi}G\tilde{x}$$

Rearranging, and noting that GB is a scalar, we can write :

$$\tilde{x}(k+1) = [A - \frac{BG}{GB}[A - (1 - \frac{K}{\phi})I]]\tilde{x}(k) + (I - \frac{BG}{GB})(x_d(k+1) - Ax_d(k)) - HCz(k) \tag{3.10}$$

From the previous section, we assume that the desired state trajectory is generated so that:

$$x_d(k+1) = Ax_d(k) + Bu_d(k)$$

From the above equation, it is easy to check that the second summand in the right-hand side of Eq.(3.10) is zero. Also, continuing with the assumption that the estimation error $z(k)$ has vanished, we can write:

$$\tilde{x}(k+1) = [A - \frac{BG}{GB}[A - (1 - \frac{K}{\phi})I]]\tilde{x}(k)$$

This equation represents linear tracking error dynamics, and it is evident that it has the general form of state feedback, closed-loop error dynamics $A - BF$. Therefore, strategies for selecting F such as LQ methods will be based on matrices A and B.

Condition 2. *The sliding surface G should be determined such that the eigenvalues of*

$$A - \frac{BG}{GB}[A - (1 - \frac{K}{\phi})I]$$

lie inside the unit circle.

This statement is equivalent to Condition 1, as will be shown in the following section.

3.4 Theoretical Results and Proofs

Theorem 1. *Let A be an n-by-n matrix and B an n-dimensional column vector such that the pair (A, B) is controllable. Let G be an n-dimensional row vector and γ a scalar. Define:*

$$A_s = (I - \frac{BG}{GB})A \tag{3.11}$$

$$A_{eq} = A - \frac{BG}{GB}(A - \gamma I) = A_s + \gamma\frac{BG}{GB} \tag{3.12}$$

Assume that A_{eq} is nonsingular and has at least $n - 1$ distinct eigenvalues. Then:

1. *A_s has at least one zero eigenvalue*

2. *The eigenvalues of A_{eq} are γ and the eigenvalues of A_s, excluding the zero eigenvalue mentioned in 1.*

The usefulness of the result lies in the fact that the A_s matrix is the equivalent matrix during actual sliding for continuous-time systems, for which the placement problem has already been solved. Also, formulations for robust pole placement exist when this structure is present, so techniques such as LQ optimal placement can be readily implemented in the discrete time case.

Theorem 2. *Condition 1 implies Condition 2.*

3.4.1 Proof of Theorem 1

Definition. A polynomial $m(\lambda)$ is called the *minimal polynomial* of the square matrix A if it is the polynomial of least degree such that $m(A) = 0$

Lemma 1. *Let A be an n-by-n matrix with at least $n-1$ distinct eigenvalues and let $h(\lambda)$ be the characteristic polynomial of A. If $\bar{g}(\lambda)$ is a polynomial of degree less than n such that $\bar{g}(A) = 0$, then $\bar{g}(\lambda)$ is a factor of $h(\lambda)$.*

Lemma 2. *If a square matrix X has rank one, then*

$$\det(I - X) = 1 - trace(X)$$

This is a standard result. See, for instance, Kailath, *Linear Systems*, p. 658

Lemma 3. *If A is invertible and $AB = 0$ then $B = 0$.*

Proof of 1. Since G and B are row and column vectors respectively, it is clear that $\frac{BG}{GB}$ has both rank and trace one. Then, by Lemma 2 $\det(I - \frac{BG}{GB}) = 1 - 1 = 0$. Then $\det(A_s) = \det(I - \frac{BG}{GB})\det(A) = 0$. It follows that A_s is singular and has at least one zero eigenvalue. (The determinant is the product of eigenvalues) □

Proof of 2. First, let us show that γ is an eigenvalue of A_{eq}:

$$\det(A_{eq} - \gamma I) = \det((I - \frac{BG}{GB})A + \gamma\frac{BG}{GB} - \gamma I) = \det(I - \frac{BG}{GB})\det(A - \gamma I) = 0$$

because the fist factor in the last expression is zero, as seen in the proof of 1.

Now let $g(\lambda)$ and $h(\lambda)$ be the characteristic polynomials of A_s and A_{eq}, respectively. By part 1, $g(\lambda) = \lambda\bar{g}(\lambda)$, where $\bar{g}(\lambda)$ has degree $n-1$. By substitution it is easy to check that $g(A_{eq}) = 0$. In fact,

$$g(\lambda) = \det(A_s - \lambda I)$$

is a polynomial in λ. Now substitute A_{eq} and use definitions 3.11 and 3.12 :

$$g(A_{eq}) = \det(A_s - A_{eq}) = \det(-\gamma\frac{BG}{GB}) = 0$$

so we have

$$g(A_{eq}) = A_{eq}\bar{g}(A_{eq}) = 0$$

that is, A_{eq} satisfies the characteristic polynomial of A_s and, by Lemma 3, $\bar{g}(A_{eq}) = 0$, since A_{eq} is assumed to be nonsingular. Now, using Lemma 1, we have a polynomial $\bar{g}$ of degree less than n such that $\bar{g}(A_{eq}) = 0$, with A_{eq} having at least $n-1$ distinct eigenvalues, by assumption. It follows that $\bar{g}(\lambda) = \frac{1}{\lambda}g(\lambda)$ is a factor of $h(\lambda)$. This proves that the $n-1$ possibly nonzero roots of $g(\lambda)$ (the characteristic polynomial of A_s) are also eigenvalues of A_{eq}. □

Proof of Lemma 1. It is well known from Linear Algebra that if an n-by-n matrix has at least $n-1$ distinct eigenvalues then the degree of the minimal polynomial is at least $n-1$, i.e, $deg(m(\lambda)) \geq n-1$. Now use the division algorithm:

$$h(\lambda) = \bar{g}(\lambda)q(\lambda) + r(\lambda)$$

with either $r = 0$ or $deg(r) < deg(\bar{g})$,where q is the quotient and r the remainder of the division. In our case, $deg(\bar{g}) \leq n - 1$ by hypothesis, so the remainder satisfies $r = 0$ or $deg(r) < n - 1$.

By the Cayley-Hamilton theorem $h(A) = 0$, so

$$h(A) = 0 = \bar{g}(A)q(A) + r(A)$$

this means $r(A) = 0$, since $\bar{g}(A) = 0$ by hypothesis. We would have $r(A) = 0$ with $deg(r) < n - 1$, which is less than the degree of the minimal polynomial, i.e, $deg(m) \geq n - 1$, so the only possibility is that $r = 0$, giving an exact division. It follows that $\bar{g}$ is a factor of h. □

Proof of Theorem 2. It is enough to show that the eigenvalues of the matrix defined in Condition 1 are 1 and the $n - 1$ eigenvalues of A_s which are not always zero. For this, just apply Theorem 1 using $\gamma = 1$. □

Justification of Assumptions

Matrix A_{eq} has the form $A - BF$, and being (A, B) a controllable pair, F can be always selected to give a nonsingular A_{eq}, which is desirable for control purposes. The assumption that A_{eq} has at least $n - 1$ distinct eigenvalues is explained as follows: As a consequence of the theorem, placing the eigenvalues of A_{eq} is done by directly specifying γ (which equals $1 - \frac{K}{\phi}$) and by placing the $n - 1$ assignable eigenvalues of A_s by standard methods. As it is well known, eigenvalues cannot be placed with multiplicity greater than one. [1] In the worst-case scenario, the independent γ eigenvalue could be selected to be equal to one of the $n-1$ eigenvalues of A_s, which would leave A_{eq} with $n-1$ distinct eigenvalues. Otherwise, it would have n distinct eigenvalues. This assumption is crucial because it allows to conclude that an annihilating polynomial of degree less than n is a factor of the characteristic polynomial. Consider the following counter-example:

$$A = \left(\begin{array}{ccc|c} 2 & 1 & & \\ & 2 & 1 & \\ & & 2 & \\ \hline & & & 1 \end{array}\right)$$

Here $h(\lambda) = (\lambda-2)^3(\lambda-1)$ is the characteristic polynomial of degree 4 and $\bar{g}(\lambda) = (\lambda-2)(\lambda-1)(\lambda-123456789)$ has degree 3<4 and verifies $\bar{g}(A) = 0$ (because $(\lambda-2)(\lambda-1)$ is the minimal polynomial), but we see that $\bar{g}$ is not a factor of $h(\lambda)$. The missing condition is that A should have at least 3 distinct eigenvalues.

3.5 Applications of Theorem 1

An inmediate consequence of Theorem 1 is that the eigenvalue assignment problem should be done on the simpler A_s matrix, which is the one that characterizes tracking error dynamics when $s = 0$, both for continuous and discrete time systems. This will place $n-1$ eigenvalues, and the remaining pole is selected using sliding gain K and

[1] In the general MIMO case, eigenvalues cannot be placed with multiplicity greater than the number of inputs

boundary layer thickness ϕ as $(1 - \frac{K}{\phi})$. For the sake of completeness, we verify that A_s is the equivalent matrix during sliding, following a development analogous to the one found in [10]. In fact, adopting the discrete notation, consider the system (3.1). If proper control is used, $s(k) = s(k+1) = 0$ for some instant. That means

$$0 = G\tilde{x}(k+1) = GA\tilde{x}(k) + GBu(k), \text{ or:}$$

$$GBu(k) = -GA\tilde{x}(k)$$

which can be rearranged to define the *equivalent control*

$$u_{eq}(k) = -(GB)^{-1}GA\tilde{x}(k)$$

Substituting this expression into the system equations produces A_s:

$$\tilde{x}(k+1) = (I - \frac{BG}{GB})A\tilde{x}(k) \text{ , if } s(k) = s(k+1) = 0$$

3.5.1 Eigenvalue Assignment

The reduction of the assignment problem to the continuous case allows us to follow existing procedures developed by Zinober, [10]. Given the discrete system matrices A and B with (A, B) controllable, let T be an invertible matrix such that

$$T^{-1}B = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 1 \end{bmatrix}$$

In the reference [10] an orthogonal matrix T is used, but further developments with this choice make it necessary to perform a QR decomposition, which is numerically inconvenient. In the multi-input case, the matrix T used is related to the Kalman canonical form. In our case, it is sufficient to use the controller canonical form, which is a particular case of the Kalman form. Apply the state transformation $x = Tw$ so that the similar system becomes:

$$w(k+1) = T^{-1}ATw(k) + T^{-1}Bu(k)$$

Now partition the new state $w(k)$ and the matrices as:

$$\left[\begin{array}{c} w_1(k+1) \\ \hline w_2(k+1) \end{array}\right] = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c} w_1(k) \\ \hline w_2(k) \end{array}\right] + \left[\begin{array}{c} 0 \\ \hline 1 \end{array}\right] u(k)$$

The sliding surface in the new coordinates become $s(k) = GTw(k)$ which we also partition as:

$$s(k) = \left[\begin{array}{c|c} G_{1T} & G_{2T} \end{array}\right] \left[\begin{array}{c} w_1(k) \\ \hline w_2(k) \end{array}\right]$$

If actual sliding happens, the dynamics of the $(n-1)$ dimensional subsystem associated with state w_1 become:

$$\begin{cases} w_1(k+1) = A_{11}w_1(k) + A_{12}w_2(k) \\ w_2(k) = -G_{2T}^{-1}G_{1T}w_1(k) \end{cases}$$

We see that state w_1 is unaffected by the control u and that the subsystem is indeed a constant state feedback system in which w_2 plays the role of control. We have the form $w_2(k) = -Fw_1(k)$, where $F = G_{2T}^{-1}G_{1T}$. In our single input case G_{2T} is just a scalar, and we can assume, without loss of generality, that $G_{2T} = 1$, so that

$$GT = \left[\begin{array}{c|c} F & 1 \end{array}\right], \text{ so:}$$

$$G = \left[\begin{array}{c|c} F & 1 \end{array} \right] T^{-1}$$

and F is selected so that $A_{11} - A_{12}F$ has desired eigenvalues. This is accomplished by any standard method such as Ackermann's formula. The Matlab function "place-g.m" performs all the necessary operations and is included at the appendix.

Example on eigenvalue assignment

Let :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 8 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$DP = \begin{bmatrix} 0.1 & 0.8 \end{bmatrix} \text{ (desired poles)}$$

$$K = 0.25$$

$$\phi = 1$$

The following Matlab commands illustrate the usage:

```
>> G=place_g(A,B,DP)


G =
```

```
    3.4200    0.9950 -1.4700


>> eig(A-B*G*(A-(1-k/phi)*eye(3))/(G*B))


ans =


    0.1000

    0.8000

    0.7500
```

The last pole is the one assigned by selection of K and ϕ, while the first two are assigned by G.

3.5.2 LQ-Optimal Sliding Surface

The problem can be stated as:

Find the sliding coefficients G that minimize the performance index

$$J = \sum_{k_s}^{\infty} \tilde{x}' Q \tilde{x}$$

where Q is positive-definite and symmetric and k_s is the sample index at which sliding begins. As justified before, the problem can be treated as the regulation case, with matrices A and B. The solution of the problem is analogous to [10], pag. 10-11 up to the formulation of the equivalent cost function. In fact, using the same partition as in the placement case in the previous section, and partitioning $T'QT$ accordingly, define:

$$\begin{aligned}\hat{Q} &= Q_{11} - Q_{12}Q_{22}^{-1}Q_{21} \\ \hat{A} &= A_{11} - A_{12}Q_{22}^{-1}Q_{21} \\ v(k) &= \tilde{w}_2(k) + Q_{22}^{-1}Q_{21}\tilde{w}_1(k)\end{aligned}$$

where $\tilde{w}$ is the tracking error $T^{-1}(x_d - \hat{x})$. Note that being Q symmetric implies $Q'_{22} = Q_{22}$ and $Q'_{21} = Q_{12}$. Also, since we deal with the SISO case, Q_{22} is a scalar. Using this definitions it is easy to check that the problem can be restated as:

Minimize

$$J = \sum_{k_s}^{\infty} \tilde{w}_1'\hat{Q}\tilde{w}_1 + v'Q_{22}v$$

subject to:

$$w_1(k+1) = \hat{A}w_1(k) + A_{12}v(k)$$

This corresponds to a standard LQR problem formulation in discrete time. The solution is given by the control law:

$$v(k) = -Kw_1(k), \text{ where:}$$

$$K = (Q_{22} + A'_{12}PA_{12})^{-1}A'_{12}P\hat{A}$$

and P is the unique positive definite solution to the algebraic discrete Riccati equation:

$$0 = P - \hat{A}'P\hat{A} + \hat{A}'PA_{12}(Q_{22} + A'_{12}PA_{12})^{-1}A'_{12}P\hat{A} - \hat{Q}$$

CAD packages such as Matlab's Control Toolbox are able to compute K. Using the definition of $v(k)$ we find:

$$w_2(k) = -(K + Q_{22}^{-1}Q_{21})w_1(k) = -Fw_1(k)$$

Here we meet the second-to-last step in the ordinary placement problem, i.e, we have determined F. Now we need to translate back to G:

$$G = \left[\begin{array}{c|c} F & 1 \end{array} \right] T^{-1}$$

The Matlab function "lqsmc.m" performs all necessary operations and is included at the appendix.

Example on LQ-Optimal Design

Keeping the same parameters as in the previous example, and choosing Q as:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

gives the following:

```
>> G=lqsmc(A,B,Q)


G =


    2.3035    0.6859   -0.8917


>> eig(A-B*G*(A-(1-k/phi)*eye(3))/(G*B))


ans =
```

```
-0.3846

 0.2081

 0.7500
```

In the following chapter, a numerical example relevant to controller design is offered.

3.6 Remarks

It is interesting to note that the form adopted by matrix A_s is totally independent from the choice of control. This fact follows from the derivations in Sec. 3.5. Therefore, it is possible to design the sliding surface even before a control law has been found that guarantees convergence into the boundary layer, and this makes the findings in this work to be valid for any control strategy that guarantees the convergence of $s(k)$ and $z(k)$. The Luenberger observer is a choice that ensures the decay of $z(k)$ regardless of the choice of control if no disturbances are present, but other controller/observer combinations might work as well. OBDSMC as presented in this paper, creates a boundary layer, i.e, a region in tracking error space where the system behaves linearly. Other control strategies may eliminate this linearity region, thus limiting the design to the assignment of the $n-1$ eigenvalues that arise when $s=0$ and the selection of appropriate parameters for the particular control being used.

Chapter 4

Application Example : Control of a Flexible Beam

4.1 Mathematical Model of "True" Plant

The subject of this example is taken from [34], pag. 597. A continuous state-space model for a slender beam with a poorly damped vibrational mode at 6 Hz is given as:

$$\begin{aligned}\dot{x} &= A_{true}x + B_{true}u \\ y &= C_{true}x\end{aligned}$$

with

$$
A_{true} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - (\frac{w_{rp}}{w_{rz}})^2 & 2\zeta_r(\frac{w_{rp}}{w_{rz}} - (\frac{w_{rp}}{w_{rz}})^2) \\ 0 & 0 & 0 & w_{rp} \\ 0 & 0 & -w_{rp} & 2\zeta_r w_{rp} \end{bmatrix}
$$

$$
B_{true} = \begin{bmatrix} 0 \\ a(\frac{w_{rp}}{w_{rz}})^2 \\ a w_{rp} \end{bmatrix}
$$

$$
C_{true} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}
$$

$w_{rp} = 2\pi f_{rp}$ and $w_{rz} = 2\pi f_{rz}$ are the pole and zero resonant frequencies expressed in rad/sec, and ζ_r is the damping ratio, assumed to be equal for both resonance poles and zeros. Values for these parameters are taken as:

$$
\begin{aligned}
f_{rp} &= 5.8 \text{ Hz.} \\
f_{rz} &= 6.0 \text{ Hz.} \\
\zeta_r &= 0.002 \\
a &= 1.2
\end{aligned}
$$

The discrete-time, zero-order hold equivalent model with a sampling period of $T_s =$ 0.02 sec. is obtained as:

$$
\begin{aligned}
x(k+1) &= \Phi x(k) + \Gamma u(k) \\
y(k) &= C x(k)
\end{aligned}
$$

with

$$\Phi = \begin{bmatrix} 1.0 & 0.02 & 0.0 & 0.0 \\ 0 & 1.0 & 0.0012 & 0.00005 \\ 0 & 0 & 0.7457 & 0.6669 \\ 0 & 0 & -0.667 & 0.7484 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 0.0002 \\ 0.0225 \\ 0.3052 \\ 0.8003 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

Fig.(4.1) shows the continuous-time magnitude plot of the true plant. In the reference, a nonlinear technique called "extended proximate time-optimal servomechanism" (XPTOS) is used. The purpose of the following sections is to show how can OBDSMC be designed to match the performance obtained using XPTOS.

4.2 OBDSMC design using full plant model

As in the reference, the resonance characteristics of the plant are included in the design model, both for the observer and the sliding mode controller itself.

4.2.1 Observer design

The observer gain, H, was taken as the Kalman filter gain, yielding eigenvalues of $\Phi - HC$ at $0.744 \pm 0.667i$ and $0.985 \pm 0.015i$

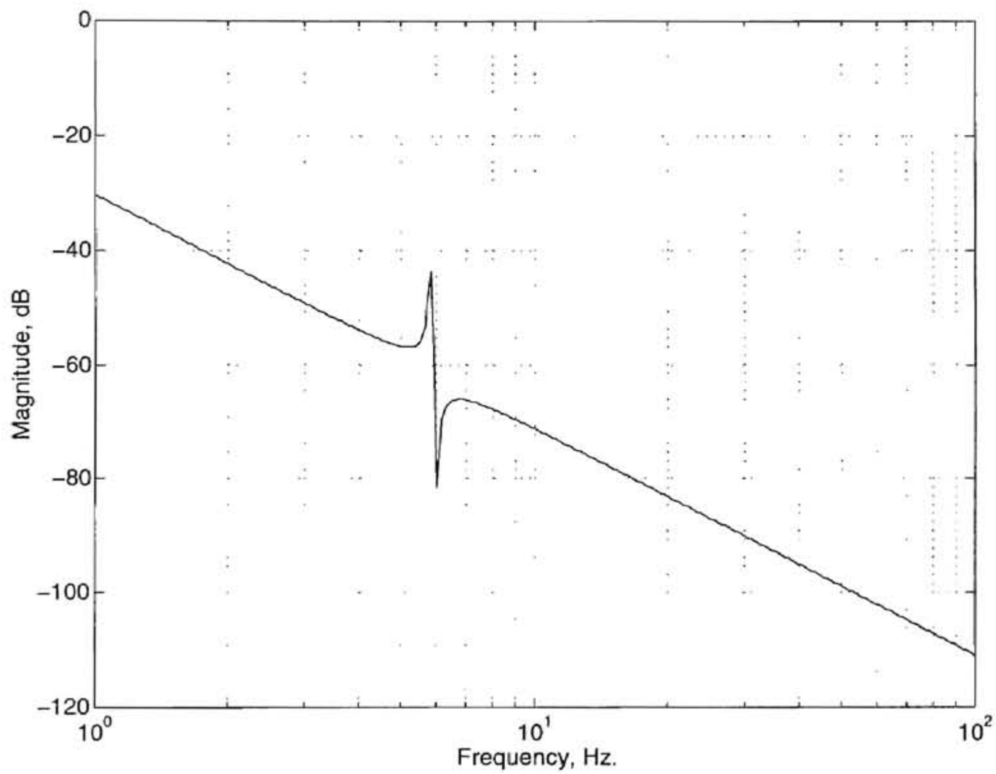

Figure 4.1: Frequency response of slender beam with flexible mode at 6 Hz.

4.2.2 Desired state trajectories (Xd-generator)

As discussed in previous sections, tracking design is viewed as a model following problem. In this case the desired state trajectories are obtained by simulating the fourth order discrete plant under LQR control, so that the settling time corresponding to the desired states is about 1 sec. A unit step input is applied to the Xd-generator.

4.2.3 Sliding surface design using desired eigenvalues

Three eigenvalues for Φ_{eq} are arbitrarily chosen from inside the unit circle as $0.5, 0.6$ and 0.7. Also, $K = 0.3$, $\phi = 1$, placing the remaining Φ_{eq} eigenvalue at 0.7. The *lqsmc* routine was used to calculate the corresponding sliding coefficients G.

4.2.4 Sliding surface design using LQ approach

One simulation is done using $Q = I_{4x4}$, yielding Φ_{eq} eigenvalues at $0.448, 0.983 \pm 0.0098$, and 0.75 (K and ϕ are 0.25 and 1). Another simulation is done using $Q = \text{diag}(7000, 250, 1, 1)$. (See results below for details)

4.2.5 Results

Two sets of simulations are performed: an "exact" implementation where computational time delay is not included, and another in which delays are included that represent a more realistic situation. Fig.(4.2) schematizes the implementation in block form.

a. No computational time delay

Figure 4.3 shows the results when desired eigenvalues are specified. As seen, response time matches the XPTOS, and design is somewhat simpler, because no numerical optimization procedure is used and no trial-and-error was used in obtaining control parameters. Also shown in the figure are the 4 desired state trajectories plotted along with the actual states, the control input, and the output estimate. If the LQ approach is used using $Q = I_{4x4}$, results are almost identical. This will not be true when modeling errors -such as delays- are present.

b. Including delays of one sample period

Figure 4.4 shows the results when desired eigenvalues are specified. Even when observer poles and Φ_{eq} are inside the unit circle, the system becomes unstable. The observer, however, is correctly estimating the plant states. The LQ approach produces a better selection of eigenvalues. Using $Q = I_{4x4}$ stabilizes the plant, and Fig.(4.5) shows the results when $Q = \mathrm{diag}(7000, 250, 1, 1)$, maintaining K and ϕ as before. This choice not only stabilizes the plant, but significantly improves tracking characteristics.

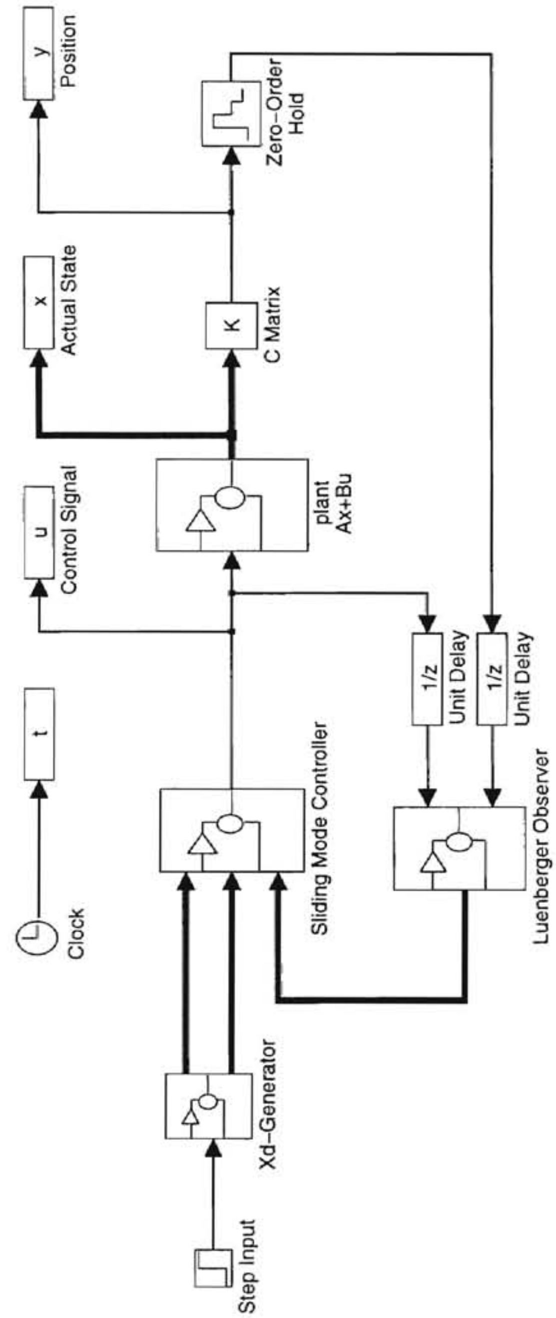

Figure 4.2: Simulink simulation diagram including delays

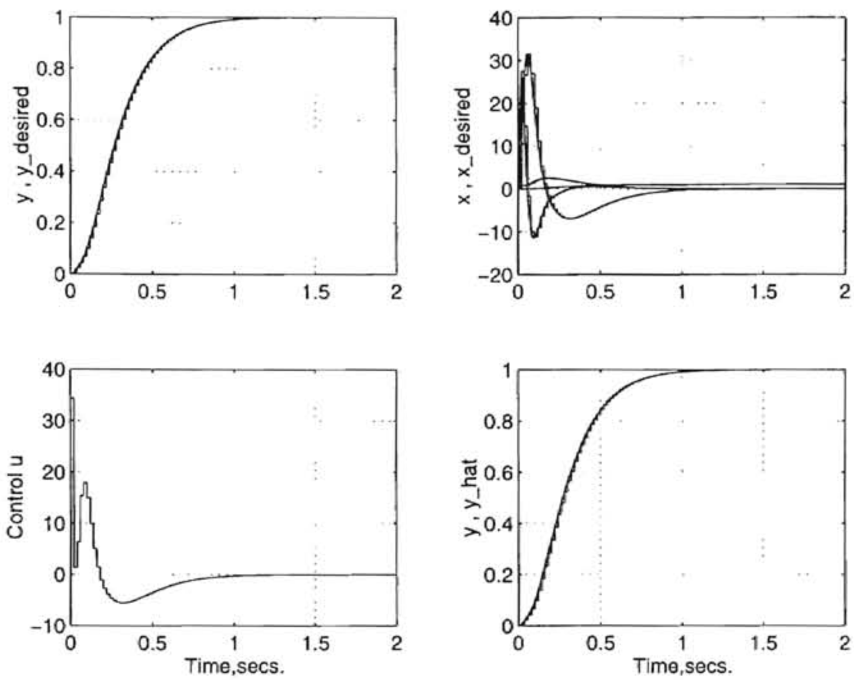

Figure 4.3: OBDSMC response with no delays, manual eigenvalue selection.

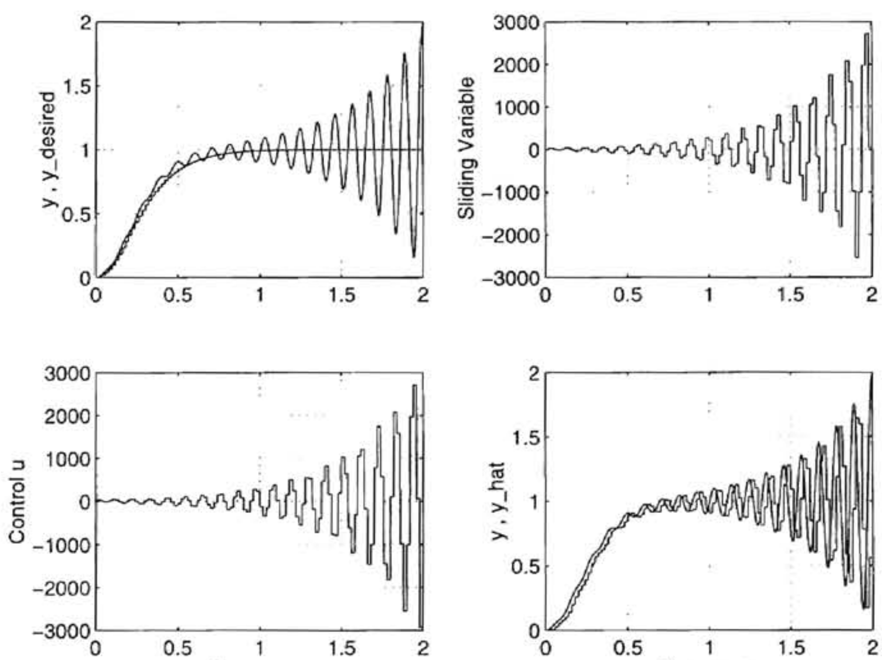

Figure 4.4: OBDSMC response with delays, manual eigenvalue selection.

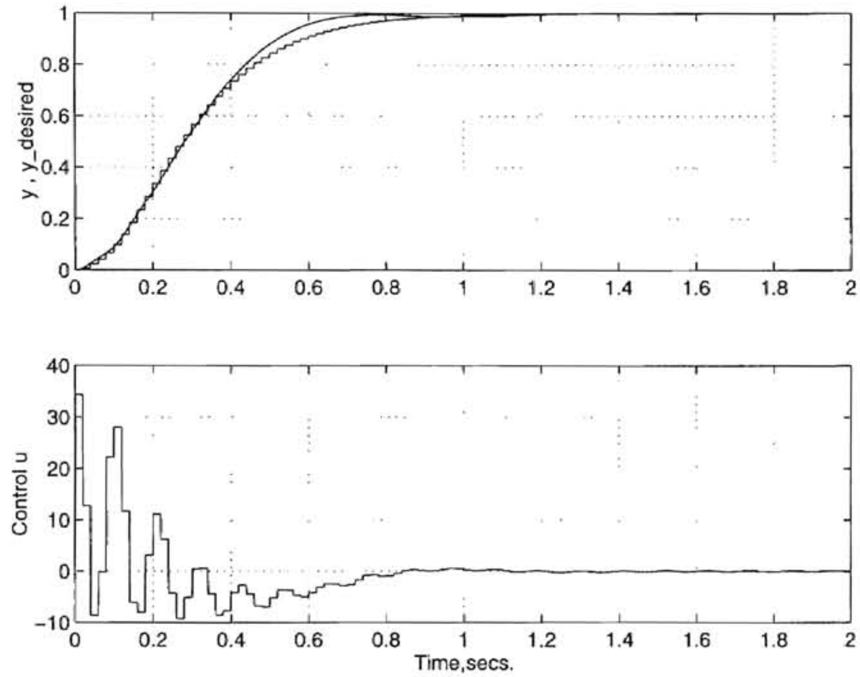

Figure 4.5: OBDSMC response with delays, LQ approach.

4.3 OBDSMC Design Using a Double Integrator Model

In the reference, the full 4th order model is used in the observer for control purposes. In this section, design is attempted using a simple double integrator plant model. The discrete-time plant model is reduced to :

$$\Phi_2 = \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} 0.02 \\ 0.0002 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0 & 1.12133 \end{bmatrix}$$

4.3.1 Observer Design

The observer gain, H, was taken as the Kalman filter gain, yielding eigenvalues of $A - HC$ at $0.506 \pm 0.739i$

4.3.2 Desired State Trajectories (Xd-generator)

In this case the desired state trajectories are obtained by simulating the second order discrete plant under LQR control, so that the settling time corresponding to the desired states is about 1 sec. A unit step input is applied to the Xd-generator.

4.3.3 Sliding Surface Design

The LQ-optimal approach is used, with matrix Q selected as $Q = \text{diag}(5, 100)$. This produces an eigenvalue of A_{eq} at 0.9144. Also $K = 0.25$ and $\phi = 1$, placing the remaining A_{eq} eigenvalue at 0.75.

4.3.4 Results

The same set of parameters was used for both exact and delayed simulations. Figure 4.6 shows the results when no delays are present. Tracking is accurate, and residual vibrations due to resonance are effectively attenuated, though not completely, making the approach useful in not very demanding applications, where controller size is critical. Figure 4.7 shows the results when delays of one sample period are included at observer input. Tracking accuracy is maintained only at low frequencies, but the level of vibration supression is not altered.

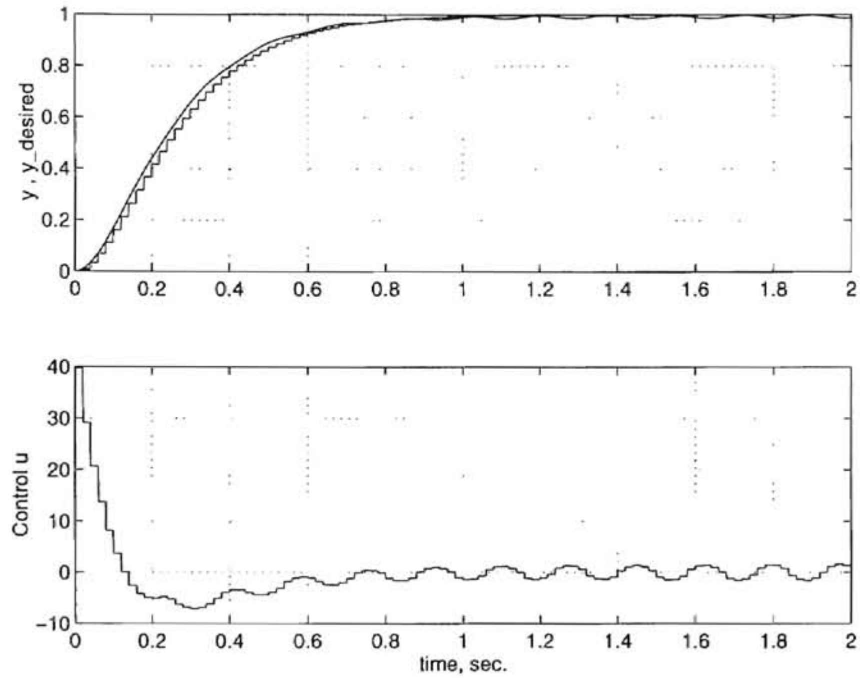

Figure 4.6: Double integrator-based OBDSMC response with no delays.

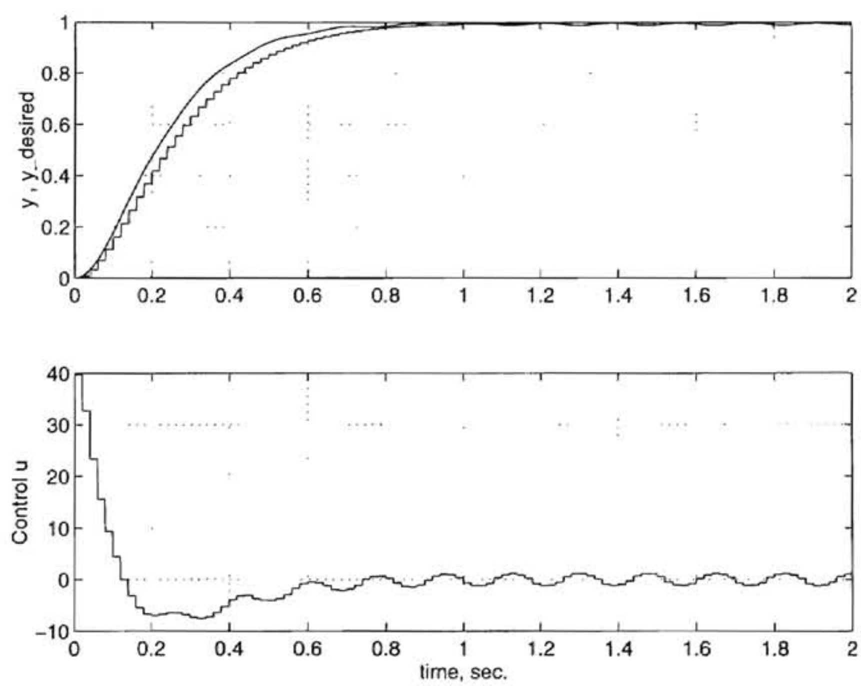

Figure 4.7: Double integrator-based OBDSMC response with delays.

Chapter 5

Conclusions and Future Research

The technique for assigning eigenvalues of the equivalent matrix in OBDSMC systems presented in this work can be applied to any controllable system, given its state space description. Furthermore, the technique is applied in the development of two methods for designing the sliding surface. The LQ-optimal design offers improved stability robustness against neglected dynamics such as computational time delays, as verified by simulation. The technique also considerably reduces the amount of trial-and-error in the design process. When modeling errors exist, the stability conditions for OBDSMC systems are not limited to the location of observer and equivalent matrix eigenvalues inside the unit circle, as seen in a simulation. Future research effort in the design of OBDSMC systems should be directed towards finding more stringent conditions for stability when certain classes of modeling error exist. It is also known from experience, that observer design considerably affects the performance of the controlled system. Research on the subject could include extending the LQ optimal design into an LQR approach, and then into LQG-LTR. This is at least conceptually feasible, because system dynamics inside the boundary layer are analogous to simple linear feedback. Results for LQR sliding surface design in continuous time are available in [36], where the R matrix is used to penalize equivalent control, which is what keeps the system sliding and behaving as a linear state feedback. The tractability of such LQR approach is limited, because the cost function includes the G matrix, which precisely is what is being sought. Another approach for which continuous-time results exist is the frequency-shaped LQ method, which has the drawback of requiring extended states both for design and actual implementation, but is effective in enhancing frequencies of interest.

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Appendix : Matlab Routines

```
%PLACE_G.M

function [G] = place_g(A,B,DP)  %DP: vector of desired pole locations of Aeq

Cx=ctrb(A,B);     %The first 7 steps compute the controllable form

                  %and the transformation matrix T

n=rank(Cx);  %Get system size (controllability assumed)

pol=real(poly(eig(A)));

Aw=[zeros(n-1,1),eye(n-1);fliplr(-pol(2:n+1))];

Bw=[zeros(n-1,1);1];

Cw=ctrb(Aw,Bw);

T=Cx*inv(Cw);

A11=Aw(1:n-1,1:n-1);  %Obtain partitioned matrices

A12=Aw(1:n-1,n);

F=acker(A11,A12,DP);

G=[F 1]*inv(T);        %Map back to G

------------------------------------------------------------------

%LQSMC.M

function [G] = lqsmc(A,B,Q);

n=rank(ctrb(A,B));  %Get system size (controllability assumed)

Cx=ctrb(A,B);

pol=real(poly(eig(A)));

Aw=[zeros(n-1,1) eye(n-1);fliplr(-pol(2:n+1))];
```

```
Bw=[zeros(n-1,1);1];

Cw=ctrb(Aw,Bw);

T=Cx*inv(Cw);

A11=Aw(1:n-1,1:n-1);     %Obtain partitioned matrices

A12=Aw(1:n-1,n);

Qw=T'*Q*T;

Q11=Qw(1:n-1,1:n-1);

Q12=Qw(1:n-1,n);

Q21=Q12';

Q22=Qw(n,n);

Qhat=Q11-Q12*inv(Q22)*Q21;

Ahat=A11-A12*inv(Q22)*Q21;

gain=dlqr(Ahat,A12,Qhat,Q22);

F=gain+inv(Q22)*Q21;

G=[F 1]*inv(T);          %Map back to G
```

VITA

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