ASYMMETRICAL CONTROL LIMITS FOR

INDIVIDUAL MEASUREMENT X

AND MOVING RANGE (n = 2)

mR CONTROL CHARTS

BY

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1994

Submitted to the Faculty of the
Graduate College of
Oklahoma State University
in partial fulfillment of
the requirements for
the Degree of
MASTER OF SCIENCE
December, 1997

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ACKNOWLEDGMENTS

I wish to extend my deepest appreciation to my major advisor and Chairman of my Master committee, Dr. Kenneth Case for his understanding, support, and friendship during my studies at Oklahoma State University. Your constructive guidance and inspiration were instrumental in completing my Master degree. I would also like to extend my sincere appreciation to my other committee members Dr. David Pratt and Dr. Wayne Turner for their assistance during this research. I wish to express a special thanks to Dr. Turner for allowing me to pursue opportunities in my other areas of interest.

A very special recognition goes to my wife, Cindy. Your love, support, patience and understanding provided me with the strength and encouragement to complete my studies. This would not have been possible without you.

Finally, I would like to thank the Department of Industrial Engineering and Management for the support over these two years of study.

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CHAPTER 1

THE PROBLEM AND ITS SETTING

INTRODUCTION

Dr. Walter A. Shewhart introduced the concept of control charts in the 1920's. The control charts were developed as tools for generating a picture of a process. The basis of these charts was that there are two types of variation: controlled (common cause) variation that is stable and consistent over time and uncontrolled (special cause) variation which changes over time. Dr. Shewhart made the following conclusions based on process variations; limits can be set, based on the natural variations of a process (common cause), so that as long as there are fluctuations between these limits only controlled (common cause) variation is present and fluctuations outside these limits indicate uncontrolled (special cause) variation. If the process is influenced by only common cause variation then it is in a state of statistical control (SOSC) and can be used as a predictor of future occurrences; if influenced by special cause variations then it is not in a state of statistical control. Dr. Shewhart stated the following as concerned with statistical control: "A phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to behave in the future (4, p. 6)."

There are many different types of control charts used to study processes. The most commonly used control charts are $\overline{X}$ and R charts which require measurable quality characteristics. The data used in these charts are made up of subgroups (typically consisting of about four or five pieces of data) collected from the process in a rational manner. The $\overline{X}$ and R values are plotted in series on their respective graphs to build control charts. These charts are utilized to monitor the process for changes in both location and dispersion. The $\overline{X}$ control chart monitors the location of the process by plotting the process average between subgroups. The R control chart monitors the dispersion of the data within the subgroups by plotting the range of data points within each subgroup.

The $\overline{X}$ chart is a very robust tool although its statistical foundation is based on the normal distribution. The robustness of this control charts is best explained by the central limit theorem which states that for a random sample of size n, if n is significantly large, the sample averages have approximately a normal distribution. The assumption of normality can be made even when the process's underlying distribution is non-normal.

The R chart can also be used to monitor variations in process spread when the underlying distribution is non-normal. The robustness of the R chart cannot, however, be explained by the central limit theorem. In fact, as sample sizes increase, the distribution of the subgroup ranges become more dissimilar from normal. Although the probabilities of type I errors for non-normal distributions fall short of those for the normal distribution, "...both the Average Chart and the Range Chart can be said to be robust to those departures from normality which are likely to be encountered in practice. They can be used with confidence. They will work and they will work well, even when 'the measurements are not normally distributed' (4, p.76)."

The $\overline{X}$ and R control charts are not suitable for all situations. Sometimes there are special circumstances in a process that make subgroups impractical. Natural subgroups may not be feasible if there are long periods of time between measurements, a single measurement represents one batch, measurements are too time consuming to obtain, or measurements are too expensive to obtain. In cases such as these, where n=1, $\overline{X}$ and R control charts are not applicable. Individual measurement X and moving range n=2 mR control charts are commonly applied when only a single measurement is taken at a time. An individual measurement X control chart is generated by plotting the individual measurements on a graph to evaluate the process's location. The moving range n=2 mR chart is generated by plotting the successive differences between the individual values.

The individual measurement X and moving range n=2 mR control charts do not possess the robustness of the $\overline{X}$ and R control charts. The underlying assumption of normality is much more critical when there are no subgroups. Since the central limit theorem does not apply to individual measurements, the quality characteristic measurements must be approximately normally distributed to easily and accurately generate existing individual measurement X and moving range n=2 mR control charts.

In practice, all events cannot be explained by the normal distribution. There are many instances where processes represent asymmetrical distributions. According to Irving Burr (1953), "...causes of non-normality is that the distribution may be unable to go beyond a certain point, such as zero. ...measurement has a physical limitation at zero

(5, p.80)." When the underlying distribution is asymmetrical the Pearson type III family of distributions can be used to approximate the data (5, p.67).

Despite the limitations, individual measurement X and moving range n=2 mR control charts are used in applications with non-normal distributions. As stated by Schilling and Nelson (1976), "In many applications the chart is applied without knowledge of the shape of the underlying distribution of individuals (3, p.183)." Conversely, Duncan (1986) states, "Control charts for individuals must be very carefully interpreted if the process shows evidence of marked departure from normality. In such cases, the multiples of σ used to set control limits might be better derived from other distributions for which the percentage points have been computed (6, p.400). " There is only limited research concerned with the use of individual measurement X and moving range n=2 mR control charts in industry when the underlying process distribution is non-normal.

GENERAL STATEMENT OF THE RESEARCH PROBLEM

The problem of this research is to create and validate a mathematical model for determining the location of upper and lower control limits on individual measurement X and moving range n=2 mR control charts for asymmetrical distributions.

The sub-objectives of this study are as follows:

(1) Develop mathematical models representative of the upper and lower control limits for asymmetrical distributions based on the shape parameter (α) and the scale parameter (β) from the Pearson Type III family of distributions with location parameter c=0 (gamma distributions).

(2) Evaluate the performance of the individual measurement X and moving range n=2 mR control charts, based on average run lengths (ARL) and variation of run length (VRL) using the Pearson type III family of distributions with location parameter c=0 (gamma distribution) control limits determined from objective 1. The performance will be evaluated against a level that is acceptable for practical application in industry and compared with methods having symmetrical control limits. A level that is acceptable for practical application in industry means that the average run length (ARL) for both the individual measurement X and moving range n=2 mR control charts is a minimum of 100, which is equivalent to a 1% chance of a type I error, when the process is in a state of statistical control.

(3) Compare the power of the individual measurement X and moving range n=2 mR control charts using the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) asymmetrical control limits with those methods having symmetrical control limits. The power in this case refers to the ability of the control charts to detect shifts in process location of 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0 σ_X units.

THE DELIMITATIONS

The following limitations pertain to this research:

- This study is limited to the evaluation of control limits for individual measurement X and moving range n=2 mR control charts which apply to non-normal distributions generated by the Pearson type III family of distribution where c=0 (gamma distribution).

- The Pearson type III family of distributions with location parameter c=0 have a range of values from (0, +∞); therefore, values of X (quality characteristic) cannot take on negative values.
- The Pearson type III family of distributions with location parameter c=0 will only be evaluated where the shape parameter alpha (α) is greater than or equal to the value of 1.
- Type I (α') errors are evaluated for points outside the upper and lower control limits. Runs rules are not used in the evaluation of these errors. The notation (α') is used to distinguish type I error fom the gamma distribution shape parameter (α).
- The evaluation of average run lengths (ARL) do not consider shifts in the process standard deviation. Only shifts in the mean are considered.

DEFINITION OF TERMS

Average run length(ARL) - The average number of subgroups taken before an out-of-control condition is given on the control chart.

Central limit theorem - Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with mean "μ" and standard deviation "σ". Then, if "n" is sufficiently large, the sample average has approximately a normal distribution with mean "μ" and standard deviation "$\sigma/\sqrt{n}$". The larger the value of "n" the better the approximation.

Control chart - A graphical chart with control limits and plotted values of some statistical measure for a series of samples or individual values. Control charts are tools used to detect the presence of uncontrolled variation in a process in order to indicate when predictions regarding the future can be made.

Control limits - Limits on a control chart based on the data or standards given which are used as criteria for action or for judging the significance of variations between samples or individual values.

Individual measurement X control chart - A control chart used to evaluate the process level in terms of a single observation per sample. These charts are usually used when rational subgrouping is not appropriate.

Moving range - The successive absolute differences between individual values.

Moving range n=2 mR control chart - A control chart for evaluating the variability within a process in terms of the range of the latest two observations in which the current observation has replaced the oldest of the previous two observations.

Pearson Type III family of distributions - A family of distributions that, according to Burr (5, p. 67), may be used as a second approximation of the curve shape of the distribution if much asymmetry is present. The Pearson Type III family of distributions with location parameter c=0 are gamma distributions which go from bell shaped curves with range $(0, +\infty)$ to J-shaped curves with range $(0, +\infty)$.

Process - The set of individuals, items, or data from which a statistical sample is taken, usually in time order.

Random sample - A sample that contains independent observations selected from the same population or universe.

Range - The distance between the largest and smallest values in a subgroup. The range is used as a measure of dispersion.

Run length - The number of subgroups taken before an out-of-control condition is given on the control chart.

Type I error (α') - The probability of demonstrating that a process is out-of-control when it is in control. It is the probability of getting a false alarm.

ABREVIATIONS AND NOTATIONS

Symbol	Term	Definition
ARL	Average run length	1/P or 1/P'
α'	Type I error	
α	Shape parameter for the gamma distribution	
β'	Type II error	
β	Scale parameter for the gamma distribution	
c	Location parameter for the Pearson Type III distribution	
CL_{mR}	Center line for moving range control charts	$\overline{mR}$

CL_X	Center line for individual control charts	$\overline{X}$
d_2	Bias correction factor	$\overline{R}/\sigma$ or $\overline{mR}/\sigma$
d_3	Bias correction factor	σ_R/σ
D_4	Control chart constant	$1+3\, d_3/d_2$
E_2	Control chart constant	$3/d_2$
k	Number of subgroups	
k'	Number of subgroups used to set control chart limits.	
LCL_X	Lower control limit for individual measurement X control charts	$\overline{X} - 3\,\overline{mR}/d_2$
μ	Mean of theoretical probability distribution	
mR	Moving range	$\lvert X_{i+1}-X_i \rvert$
$\overline{mR}$	Average moving range	$\sum mR/(N-1)$
mR chart	Moving range n=2 control chart	
n	Number of items in a subgroup	
N	Number samples or subgroups	
P	Probability of detection on an X chart	Probability(UCL_X<X or X<LCL_X)
P'	probability of detection on an mR chart	Probability(mR>UCL_{mR})

R	Range of a set of data	X_{max}-X_{min}
$\overline{R}$	Average range	$\sum_i R_i$ /N
s	Sample standard deviation for a set of data	$\sum_i \left(X_i - \overline{X}\right)^2$ /(n-1)
σ_R	Standard deviation of the theoretical distribution of ranges	$d_3\sigma$
σ_X	Process standard deviation	
SOSC	State of statistical control	
t	Multiple of σ units the control chart limits are from the center line.	
UCL_{mR}	Upper control limit for moving range control charts	$D_4\overline{mR}$
UCL_X	Upper control limits for individuals control charts	$\overline{X} + 3\overline{mR}/d_2$
VRL	Variance of run length expressed as multiples of standard deviations.	
X	An individual measurement	
$\overline{X}$	Average of a set of data	$\sum_i X_i$ /n
X chart	Individual measurement control chart	
X(α, β)	Gamma distribution with parameters α and β	

THE ASSUMPTIONS

The following assumptions pertain to this research:

- The use of individual measurement X and moving range n=2 mR control charts will continue to have widespread use in industry in the future.
- The individual measurements are not correlated.
- The acceptable minimum average run length (ARL) in industry for the combination of control charts (X and mR) is 100 when the process is in a state of statistical control (SOSC). An ARL of 100 is equivalent to a 1% risk of having a type I (α') error.

THE IMPORTANCE OF THE STUDY

The purpose of this research is to create a method of determining control limits for non-normal distributions which will support the widespread use of individual measurement X and moving range n=2 mR control charts in industry.

CHAPTER 2

REVIEW OF RELATED LITERATURE

HISTORY

Throughout history, quality has been built into products. The early colonists and immigrants in the United States followed the concepts of craftsmanship that were practiced in their countries of origin. At an early age, a boy would become an apprentice and learn a skilled trade from a master. One of the lessons learned from the master was to control the quality of the product through inspection before sale. The quality of the products was essential because the craftsman had a large stake in meeting customer needs. Product quality was a reflection of the craftsman's skill.

The industrial revolution, which began in Europe, brought changes to controlling the quality of products. The factory system of manufacturing products was becoming increasingly popular. The trades that the craftsman practiced were divided into many specialized tasks that could be performed by semiskilled or unskilled workers. The skilled craftsman were no longer needed and the ability of a person to self-inspect a product's quality throughout its entire manufacture was lost. To maintain quality under the factory system, full time inspectors would report to departmental production supervisors. Product was either "good" or "bad" based on specification limits.

In the 1920's, Dr. Walter A. Shewhart introduced the concept of statistical quality control to American industry. According to Dr. Shewhart, statistical tools could be applied in a manufacturing setting to control the quality of manufactured product. One of the tools of statistical quality control was the Shewhart control chart. The purpose of

Shewhart's control charts was to determine if a sequence of data may be used for predictions of what will occur in the future and to warn of instability. These control charts develop a picture of the process which aids in the evaluation of the process's performance. The history of quality can be found in part or in full in numerous texts such as Burr 1953 (5), Duncan 1986 (6), Joiner 1994 (11), and Juran 1995 (11).

SHEWHART CONTROL CHARTS

The basis of the Shewhart control charts is variation. There are two types of variation that can affect a process; chance cause (common cause) variation and assignable cause (special cause) variation. Chance cause variation, also referred to as controlled or common cause, is present in the process all the time. It is characterized by a stable and consistent pattern of variation over time. Assignable cause variation, also referred to as uncontrolled or special cause, is not always present in the process. This variation changes over time and comes from outside the process. References for process variation and the basis of Dr. Shewhart's control charts can be found in many texts including Burr 1953 (5), Duncan 1986 (6), Wheeler 1992 (4), Deming 1993 (9), and Joiner 1994 (11).

Dr. Shewhart made the following conclusion based on process variations: "Limits can be set, based on the natural variations of a process, so that as long as there are fluctuations between these limits only controlled variation is present, and fluctuations outside these limits indicate uncontrolled (special cause) variation. If the process is influenced by only common cause variation then it is in a state of statistical control

(SOSC) and can be used as a predictor of future occurrences, if influenced by special cause variations then it is not in a state of statistical control. Dr. Shewhart stated the following as concerned with statistical control: "A phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to behave in the future (4, p. 6)."

$\overline{X}$ AND R CONTROL CHARTS

There are many different types of control charts used in industry. The most commonly used control charts are the $\overline{X}$ and R control charts. According to Juran, "Where the characteristic under study can be measured along a scale of measurement, the $\overline{X}$ and R charts have proved to be of great value and should be used in place of p and c charts (7, p.389)." There are two requirements for using $\overline{X}$ and R charts. First, the quality characteristic must be measurable, and second, these control charts require that data be collected in subgroups. The subgroups should be collected in a rational manner. In other words, the subgroups should be such that if special causes are present they will show up in the differences between subgroups instead of within the subgroups.

The $\overline{X}$ control charts are used to monitor variation between subgroups. This is accomplished by monitoring the differences between subgroup averages. According to ANSI/ASQC Standard A1- 1978, "Averages are generally used for the purpose of determining whether there are differences between subgroup levels (12, p.3)."

The $\overline{X}$ chart has a center line and control limits. The center line of the $\overline{X}$ control chart is set at:

$$CL_{\bar{X}} = \bar{\bar{X}}$$

where $\bar{\bar{X}}$ is the average $\bar{X}$ of all the data (or the average of the subgroup averages).

The control limits $UCL_{\bar{X}}$ and $LCL_{\bar{X}}$ are set +/- 3 $\sigma_{\bar{X}}$ units away from $\bar{\bar{X}}$.

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3\hat{\sigma}_{\bar{X}}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - 3\hat{\sigma}_{\bar{X}}$$

where $\hat{\sigma}_{\bar{X}}$ is an estimate of $\sigma_{\bar{X}}$ derived from the data.

The estimate, $\hat{\sigma}_{\bar{X}}$,depends on the subgroup size, n, and is calculated as follows:

$$\hat{\sigma}_{\bar{X}} = \hat{\sigma}_X / \sqrt{n}$$

where $\hat{\sigma}_X$ is an estimate of the process standard deviation σ_X derived from the data.

The range (R) control charts monitor the variation within subgroups. This is accomplished by monitoring the range of data points that are collected for each subgroup. According to ANSI/ASQC Standard A1-1978, "Ranges of the individual observation within the subgroup or sample are used to estimate the variability from chance cause within short time intervals and ordinarily should not include assignable causes. These ranges serve to estimate the inherent variability within an essentially unchanging process (12, p.3)." Although standard deviation is a more common measure of variability in most applications, ranges are used because they are easier to compute. The range should not be used, however, for subgroup sizes greater than 10 (n>10).

The range control chart has a center line and control limits based solely on subgroup ranges. The center line of the R control chart is set at:

$$CL_R = \overline{R}$$

where $\overline{R}$ is the average of all the subgroup ranges.

The control limits UCL_R and LCL_R are set +/- 3 σ_R units away from $\overline{R}$.

$$UCL_R = \overline{R} + 3\hat{\sigma}_R$$

$$LCL_R = \overline{R} - 3\hat{\sigma}_R$$

where $\hat{\sigma}_R$ is an estimate of the range standard deviation σ_R derived from the data.

The $\overline{X}$ and R control charts are considered very robust tools although their statistical foundation is based on the normal distribution. The robustness of the $\overline{X}$ control chart is best explained by the central limit theorem. The central limit theorem states: "Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with mean "μ" and standard deviation "σ". Then, if "n" is sufficiently large, the sample average has approximately a normal distribution with mean "μ" and standard deviation "$\sigma/\sqrt{n}$". The larger the value of "n" the better the approximation." From the above definition of the central limit theorem, the $\overline{X}$ chart can be used without having concern about the underlying distribution of the process as long as the subgroup size is sufficiently large. According to Dr. Shewhart, "Such evidence...leads us to believe that in almost all cases in practice we may establish sampling limits for averages of samples of four or more on the basis of normal law theory (13)."

The R chart can also be used to monitor variations in process spread when the underlying distribution is non-normal. The robustness of the R chart cannot, however, be explained by the central limit theorem. In fact, as sample sizes increase, the distribution of the subgroup ranges may become more dissimilar to the parent distribution. In a study performed by Wheeler and Chambers (4, 1992), the subgroup ranges of five non-normal distributions were evaluated for type I errors using the common limits of $3\sigma_R$ units from the center line. The evaluation was performed for sample sizes of n= 2, 4, and 10. The resulting probabilities of a type I error for a highly skewed distribution were 0.026, 0.026, and 0.04, respectively. Although the probabilities fall short of the 0.01, 0.005, and 0.005 probability of a type I error for the normal distribution, "...both the Average Chart and the Range Chart can be said to be robust to those departures from normality which are likely to be encountered in practice. They can be used with confidence. They will work and they will work well, even when 'the measurements are not normally distributed' (4, p.76)."

INDIVIDUAL MEASUREMENT X AND MOVING RANGE n=2 mR CONTROL CHARTS

The $\overline{X}$ and R control charts are not suitable for all industrial situations. Sometimes there are special circumstances in a process that make subgroups impractical. Natural subgroups may not be feasible if there are long periods of time between measurements, a single measurement represents one batch, measurements are too time consuming to obtain, or measurements are too expensive to obtain. In cases such as these, where n=1, $\overline{X}$ and R control charts are not applicable. Individual measurement X and moving range n=2 mR control charts are commonly applied when only a single measurement is taken at a time. According to Wadsworth, et al., "Their use is generally reserved for process and product characteristics for which it is impractical or unreasonable to replicate observations and to form subgroups of observations to aid the study of process variation (14, p.143)."

The individual measurement X control chart monitors the process level. This control chart has a center line and control limits based on the individual value X. The center line on this control chart is set at

$$CL_X = \overline{X}$$

where $\overline{X}$ is the average of all the individual measures.

The control limits UCL_X and LCL_X are set at +/- $t\sigma_X$ from $\overline{X}$ (22, p. 275-7)

$$UCL_X = \overline{X} + t\hat{\sigma}_X$$

$$LCL_X = \overline{X} - t\hat{\sigma}_X$$

where $\hat{\sigma}_X$ is an estimate of the process standard deviation σ_X derived from the individual measurements X.

The common form of the individual measurement X control chart has an underlying process distribution that is normal. In common form, the multiple of standard deviations, t, the limits are from the mean is equal to 3 (4, p. 60). The control limits UCL_X and LCL_X become

$$UCL_X = \overline{X} + 3\hat{\sigma}_X$$

$$LCL_X = \overline{X} - 3\hat{\sigma}_X$$

The moving range n=2 mR control chart monitors the variation within the process. This control chart has a center line and control limits based on the range between the two latest individual measurements X. The center line on this control chart is set at

$$CL_R = \overline{mR}$$

where $\overline{mR}$ is the average of k-1 moving ranges formed from consecutive n=2 observations.

The upper control limit UCL_{mR} is set at $t\sigma_{mR}$ above the center line $\overline{mR}$ (22, p. 275-7)

$$UCL_{mR} = \overline{mR} + t\hat{\sigma}_{mR}$$

The common form of the moving range n=2 mR control chart has an underlying process distribution that is normal. In the common form, the multiple of standard deviations, t, the limit is away from the mean is equal to 3 (4, p. 60). The control limit UCL_{mR} becomes

$$UCL_{mR} = \overline{mR} + 3\hat{\sigma}_{mR}$$

The individual measurement X and moving range n=2 mR control charts do not possess the robustness of the $\overline{X}$ and R control charts. The underlying assumption of normality is much more critical when there are no subgroups (5, p. 266-7). Since the central limit theorem does not apply to individual measurements, because n=1, the quality characteristic measurements must be approximately normally distributed to easily and accurately generate existing individual measurement X and moving range n=2 mR control charts. When the process distribution is not approximately normally distributed, the value of t=3 may not produce control limits that are acceptable for use in industry.

APPLICATION OF X & mR CONTROL CHARTS

In practice, all events cannot be explained by the normal distribution. Many of the distributions encountered in every day experiences are non-normal. Economical, physical, chemical, and biological factors typically have distributions that are skewed. According to Irving Burr "...cause of non-normality is that the distribution may be unable to go beyond a certain point, such as zero. ...measurement has a physical limitation at zero (5, p.80)." When the underlying distribution is asymmetrical, the Pearson type III family of distributions can be used to approximate the data (5, p.67).

Despite the limitations, individual measurement X and moving range n=2 mR control charts based on the normal distribution are used in applications with non-normal distributions. As stated by Schilling and Nelson, "In many applications the chart is applied without knowledge of the shape of the underlying distribution of individuals (3, p.183)." Conversely, Duncan states, "Control charts for individuals must be very carefully interpreted if the process shows evidence of marked departure from normality. In such cases, the multiples of σ used to set control limits might be better derived from other distributions for which the percentage points have been computed (6, p.400). " By Duncan's statement above, the value of "t" used in setting control limits on individual measurement X and moving range n=2 mR control charts for skewed distributions should be based on a distribution more accurately representing the process. As stated in the previous paragraph, the Pearson type III family of distributions with location parameter c=0 can be used to approximate asymmetrical distributions.

Individual measurement X and moving range n=2 mR control charts are commonly used in industry. Unfortunately, they may produce inaccurate representations of the process if the underlying process distribution is non-normal. To use individual measurement X and moving range n=2 mR control charts appropriately there must be a method for setting control limits that more accurately predict the stability of the process. Research concerned with these control limits has been limited, although the need is justified.

CURRENT RESEARCH

The only research found that addresses non-normal individual measurement X and moving range n=2 mR control chart limits was performed by Jose Oyon, 1995. Oyon (8), in an unpublished master of science thesis, studied the effect of non-normality on individual measurement X and moving range n=2 mR control charts. In this thesis, Oyon did the following:

1. Evaluated the performance of the individual measurement X and moving range n=2 mR control charts using the constants d_2, d_3, and D_4 under the assumption of normality when the underlying distribution was Pearson type III family of distributions with location parameter c=0.
2. Determined empirical functions for the control chart constants d_2, d_3, and D_4 when the process distribution was approximated by the above distribution.

3. Compared the performance of the individual measurement X and moving range n=2 mR control charts with control limits based on the normal distribution to those based on the Pearson type III family of distributions with location parameter c=0.

Oyon made the following conclusion from his research:

1. The individual measurement X and moving range n=2 mR control charts based on the normal distribution do not work well when the underlying process distribution shows a marked departure from normality.
2. Control chart constants based on the gamma distribution perform better than those based on the normal distribution when the process distribution is non-normal and perform approximately the same when the process distribution is normal.

Although the gamma control chart constants perform better than the normal control chart constants, the false alarm rate produced from the gamma control chart constants does not meet industry standard of 1% when the process is in SOSC. One possible reason for the high false alarm rates is that the gamma control chart constants are used to produce symmetrical control limits for process distributions that are asymmetrical (skewed). It may be possible to improve the performance of individual measurement X and moving range n=2 mR control charts for skewed distributions if asymmetrical control limits are developed.

No other work was found that addresses the effects of non-normality on individual measurement X and moving range n=2 mR control charts.

CHAPTER 3

THE RESEARCH METHOD

Section 1: INTRODUCTION

The following sections of Chapter Three explain the methodology for performing this research. The sections of this chapter are outlined below:

1. Introduction

2. General Data

    2.1 The Data

    2.2 Criteria for Admissibility

    2.3 The Research Methodology

3. Specific Treatment of the Data for Each Sub-objective

    3.1 Sub-objective One

        3.1.1 Individual Measurement X Control Chart Limits

            3.1.1.1 The Upper Control Limit

            3.1.1.2 The Lower Control Limit

        3.1.2 Moving Range n=2 mR Control Chart Limits

    3.2 Sub-objective Two

    3.3 Sub-objective Three

Section one of Chapter Three is intended to clarify the methodology of this research. Section two is intended to characterize the data that is used to develop the asymmetrical control limits. *The Data* describes the primary source of the data used to develop the control limits. The *Criteria for Admissibility* defines the established limits and standards that the data must meet to be admitted into this research. *The Research Methodology* classifies the methodology of this research.

Section three of this chapter explains the specific steps for each sub-objective of this research. The flowchart on the following page (Figure 3-1) is included as a guide for the research methodology. Section three is broken into three main sub-sections; sub-objective one, sub-objective two, and sub-objective three. The statement of the sub-objectives is found in their respective sub-sections. The following is an overview of the main sub-sections:

<u>Sub-section 3.1 overview</u>

Sub-section 3.1 develops mathematical models representative of the upper and lower control limits for asymmetrical distributions based on the shape parameter (α) and the scale parameter (β) from the Pearson Type III family of distributions with location parameter c=0 (gamma distributions). These mathematical models are for the multiple of standard deviations the control limits are from the mean (t values). The mathematical models are generated in two different sections. One section is for the generation of the mathematical models for the individual measurement X control chart (t_1 and t_2) and the other for the moving range n=2 mR control chart (t_3).

Section 3.1.1 develops the mathematical models for the upper and lower control limits of the individual measurement X chart. To develop these mathematical models an upper control limit is found which leaves 0.00135 of the area under the Pearson type III (c=0) distribution beyond the upper control limit and a lower control limit is found which

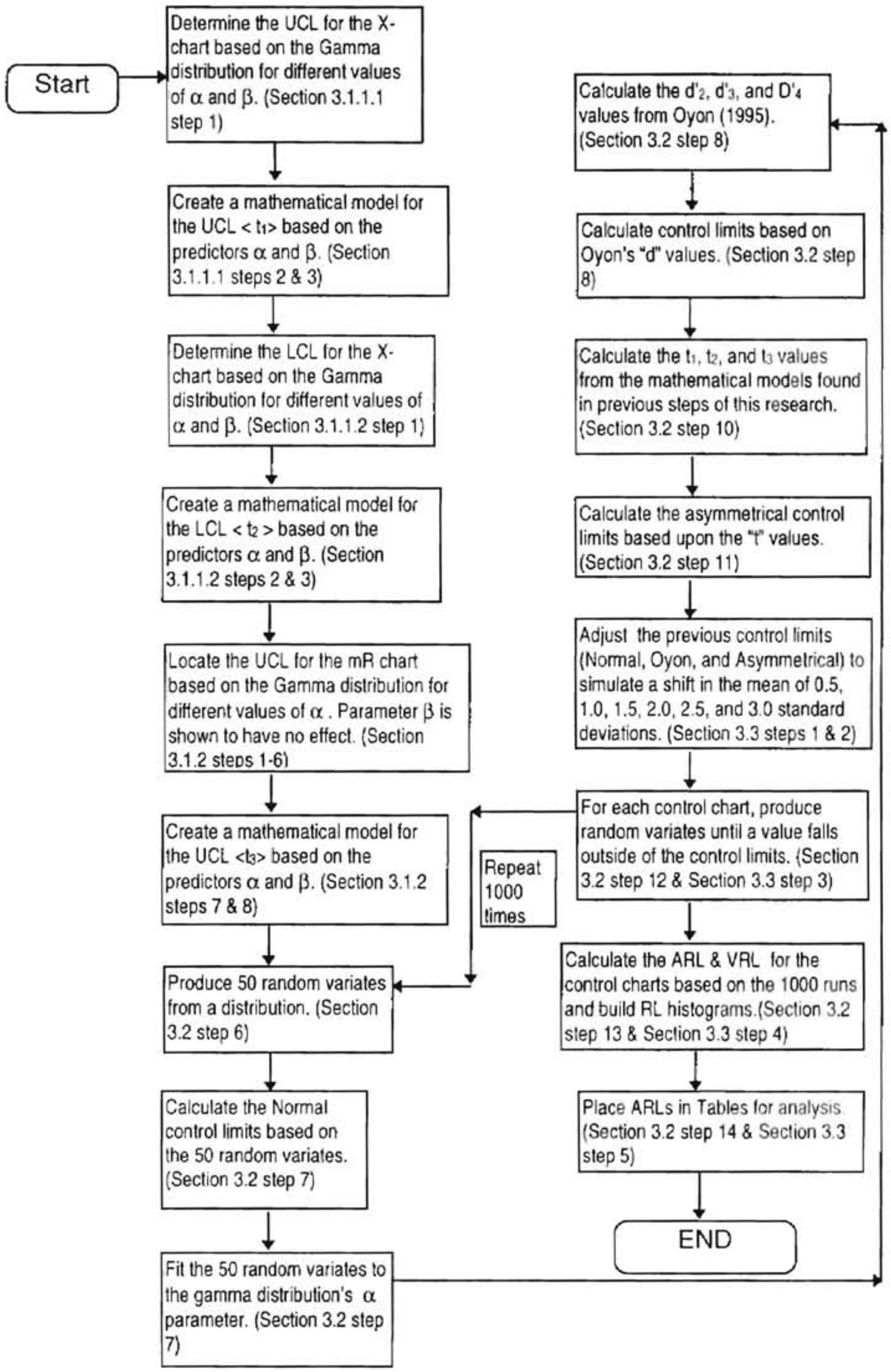

Figure 3-1: Research Methodology Flow Chart

leaves 0.00135 of the area below the lower control limit. In this section the area is found by integration. Control limits are located for different combinations of the shape parameter (α) and the scale parameter (β) of the Pearson type III (c=0) distribution. The control limits are expressed as multiples of the standard deviation from the mean. The t_1 value represents the multiple of standard deviations for the upper control limit and the t_2 value represents the multiple of standard deviations for the lower control limit.

The next step in developing the mathematical models for t_1 and t_2 is to use the "t" values (from the different α's and β's) to develop the actual mathematical expressions. Multiple regression models are developed which predict the "t" values using the (α) and (β) parameters as the predictors. There are different mathematical models which can represent the behavior of the "t" values, so, by trial and error, models are found which do a good job of predicting t_1 and t_2 but may not be the only models that can be used. A global F test is used to test the validity of the multiple regression models selected.

Section 3.1.2 develops the mathematical model for the upper control limit of the moving range n=2 mR control chart. In this section of the research, two streams of random numbers are generated from the Pearson type III (c=0) distribution and the range for the corresponding values of those streams are found. The ranges for subgroups n=2 are used instead of moving range values for two reasons:

1. There is correlation between the moving range values.
2. Current methods for setting control limits on the moving range charts are based on the range of n=2.

The next step is to find an upper control limit which leaves 0.0027 of the ranges beyond the limit. The area of 0.0027 is used because it is consistent with the probabilities of the individual measurement X chart when the subgroup size is less than seven (since only an upper control limit exists on the range chart). Control limits are found for different shape parameters (α) of the Pearson type III (c=0) distribution. Previous analysis of the individual measurement X control charts indicate that β does not have an effect on the control limit of the moving range chart. Appendix A demonstrates that β has little or no effect on the control limits; therefore, β is not included in the development of the moving range n=2 mR control limit. The control limit is stated as a multiple (t_3) of the standard deviation of the individual ranges. A mathematical model for t_3 is found in the same manner as for the individual measurement X control limits.

Sub-section 3.2 overview

Sub-section 3.2 evaluates the performance of the individual measurement X and moving range n=2 mR control charts based on average run lengths (ARL). Despite limitations, individual measurement X and moving range n=2 mR control charts based on the normal distribution are used in applications with underlying process distributions that are non-normal. As stated by Schilling and Nelson, "In many applications the chart is applied without knowledge of the shape of the underlying distribution of individuals (3, p.183)." The idea of this sub-section is to evaluate the performance of individual measurement X and moving range n=2 mR control charts having asymmetrical control limits based on the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) to those having symmetrical control limits based on the same distribution. The asymmetrical control limits are also compared to those based on the normal distribution since control chart limits based on normality are commonly used in industry.

The evaluation is performed by generating random variates from a parent distribution which is assumed to be unknown. Control limits are calculated for Normal Shewhart limits, symmetrical control limits based on the Pearson type III (c=0) distribution (Oyon 1995), and the asymmetrical control limits based on the Pearson type III (c=0) distribution. In order to calculate the latter two sets of control limits, the randomly generated variates are fit to the Pearson type III (c=0) distribution. The method of fit used in this research generates (α) and (β) values which are used in the mathematical models for calculating the "d" values (needed for Oyon's limits) and the "t" values from this research.

Next, random variates are generated from the same parent distribution until an out-of-control signal is detected on each of the three individual measurement X and three moving range control charts. A run length (RL) is recorded for each control chart (both X and mR) and the above steps (setting control limits and determining RL's) are repeated 1000 times. An average run length (ARL) and variance of run length (VRL) is found for each control chart (6 total) and recorded for analysis in Chapter Four. The run lengths are stored and presented as a histogram. The chart in Appendix J demonstrates the logic used in the evaluation of sub-objectives 3.2 and 3.3.

Sub-section 3.3 overview

Sub-section 3.3 compares the power of the individual measurement X and moving range n=2 mR control charts using the Pearson Type III c=0 asymmetrical control limits with those methods having symmetrical control limits (Normal and Oyon). The power, in this case, refers to the ability of the control charts to detect shifts in process location of 0.5, 1.0, 1.5, 2.0, 2.5, and $3.0\sigma_X$ units. A type II error (β') is the probability of concluding a process is in-control when it is actually out-of-control. The power of a control chart is a function of a type II error. The power is equal to $1 - \beta$ and is the probability of detecting an out-of-control condition. Generally, type I and type II errors are negatively correlated. As the type I errors are reduced, the type II errors increase which in turn decreases the power of the control charts. Discussions of these types of errors can be found in many texts including Hayes (17), Savage (18), Hair (19), and Miller (20). Previous sub-objectives of this research attempt to find control limits which have smaller type I errors than existing methods, therefore, it is important to evaluate the effect of asymmetrical limits on the power of the individual measurement X control charts.

Section 2: GENERAL DATA

2.1: The Data

The primary source of data used to develop asymmetrical control limits consist of values generated from the Pearson Type III family of distributions with location parameter c=0 (gamma distributions). Random variates are generated from the normal distribution, the log-normal distribution, and the gamma distribution to evaluate the performance of the asymmetrical control limits developed from the Pearson Type III family of distributions.

2.2: The Criteria for the Admissibility of the Data

The criteria for the admissibility of the data used for this research is as follows:

- Only values generated from the Pearson Type III family of distributions with location parameter c=0 (gamma distributions) are utilized in the development of the mathematical models for t_1, t_2, and t_3.
- Only α (shape parameter) values greater than or equal to the value of 1.0 are applied to the Pearson Type III family of distributions with location parameter c=0 (gamma distributions).
- Only β (scale parameter) values equal to 1, 2, and 5 are applied to the Pearson Type III family of distributions with location parameter c=0 (gamma distributions).

2.3: The Research Methodology

The method of research used in this study is based on numerical data. Since the data are numeric, quantitative methodology is utilized to conduct this research.

Section 3: SPECIFIC TREATMENT OF THE DATA FOR EACH SUB-OBJECTIVE

3.1: Sub-objective one:

Statement of the Sub-Objective: Develop mathematical models representative of the upper and lower control limits for asymmetrical distributions based on the shape parameter (α) and the scale parameter (β) from the Pearson Type III family of distributions with location parameter c=0 (gamma distributions) so often encountered in industry.

The Data Needed: The data needed for this sub-objective consist of values generated from the Pearson Type III family of distributions with location parameter c=0 (gamma distributions). The values generated from this distribution include individual measurements X, as taken from integrating the distribution, and range values, as produced from randomly generated observations.

The Location of the Data: The Pearson Type III family of distributions with location parameter c=0 (gamma distributions) have the following probability density function (pdf):

$$f(\mathrm{x}) = \frac{\left(\frac{1}{\beta}\right)^{\alpha}}{\Gamma(\alpha)} * \mathrm{x}^{\alpha-1} * \mathrm{e}^{-\mathrm{x}\left(\frac{1}{\beta}\right)}$$

All data needed in generating mathematical models representative of the upper and lower control limits for individual measurement X and moving range n=2 mR control charts are produced from this function.

The Means of Obtaining the Data: The control limits required for sub-objective one are obtained by integrating the above function (the specific treatment of the function is explained in the steps below). Mathcad for windows release 4.02 is utilized to perform the necessary integration of the Pearson Type III family of distributions with location parameter c=0 (gamma distributions) and Minitab for Windows release 10.5 is used to generate random variates.

Treatment of the Data: The treatment of the data is explained separately for the individual measurement X control limits and the moving range n=2 mR control limits. The explanations are as follows on sub-sections 3.1.1 and 3.1.2.

3.1.1: *Individual measurement X control chart limits :*

Individual measurement X control charts based on the normal distribution have upper and lower control limits set at +/- $3\sigma_X$ units above and below the average of a set of data. When these control limits are applied to a process having a normal distribution, there is a probability of approximately 0.00135 that a point will fall beyond the upper control limit and a probability of 0.00135 that a point will fall below the lower control limit. To stay consistent with normal probability theory of statistical process control, the Pearson Type III family of distributions with location parameter c=0 (gamma distributions) is evaluated against the same probabilities of a point falling outside control limits. This evaluation is described in the following paragraphs.

3.1.1.1: *The upper control limit*

The following steps describe the methodology for generating a mathematical model for the asymmetrical upper control limit on the individual measurement X control chart.

1) The value of the upper control limit for the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) is located by integrating the distribution on Mathcad. An upper control limit (UCL) is generated which leaves a tail area of 0.00135 beyond the limit. The UCL for the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) is denoted by (UCL) in

the equations below. The limit is evaluated in this manner for all combinations of α (shape parameter) = 1, 5(5)135 and β (scale parameter) = 1, 2, and 5. As demonstrated in Appendix I, the α values represent a range of skewed distributions from exponential to approximately normal (since the gamma distribution cannot generate an exact normal distribution). The UCL's are expressed as a multiple of σ_X units (t_l) to the right of the mean of the distribution. The following equations are used to generate the limits:

$$-.00135 = \left[\int_0^{UCL} \frac{\left(\frac{1}{\beta}\right)^{\alpha}}{\Gamma(\alpha)} * x^{\alpha-1} * e^{-x\left(\frac{1}{\beta}\right)} dx \right] - 1 \text{; (given } \alpha \text{ and } \beta \text{, find UCL)} \quad \text{(eq. 3-1)}$$

$$\sigma_x = \sqrt{\alpha * \beta^2} \quad \text{(eq. 3-2)}$$

$$\overline{X}(mean) = \alpha * \beta \quad \text{(eq. 3-3)}$$

$$t_l = \text{multiple of } \sigma_X \text{ units from the mean} = \frac{\text{UCL} - \alpha * \beta}{\sqrt{\alpha * \beta^2}} \quad \text{(eq. 3-4)}$$

The multiple of σ_X units from the mean (t_l) generated in this step are paired with their associated α and β values and recorded as demonstrated in the table (Table 3-1) on the following page.

2) A statistical software package (Minitab for Windows release 10.5), which features regression software, is used to generate different multiple regression models for predicting the t_l value with the predictors α and β. There are different mathematical models which can predict the t_l values, so, by trial and error, a model is found which does a good job of predicting t_l but may not be the only model that can be used. A model is chosen that has a high adjusted multiple coefficient of determination R^2.

Table 3-1: σ_X Units From the Mean (t_l)

	σ_X units from the mean (t_l)		
α	$\beta = 1$	$\beta = 2$	$\beta = 5$
1	5.6080	5.6080	5.6080
5	4.2005	4.2005	4.2005
. . .	. . .	. . .	. . .
135	3.2305	3.2305	3.2305

The adjusted multiple coefficient of determination (R^2) is a sample statistic that demonstrates how well the mathematical model fits the data; therefore, it represents a measure of adequacy of the model. The R^2 is defined as:

$$R^2 = 1 - \left[\sum (y_i - \hat{y}_i)^2 / \sum (y_i - \bar{y})^2\right] = 1 - SSE / SSyy$$

3) A global F test is used to test the validity of the multiple regression model selected. The null hypothesis of this F test is:

$$Ho : \lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_k = 0$$

where λ_n is the distance from the integrated t_l value to the corresponding t_l value calculated from the multiple regression equation. The n = 1, 2, 3, ..., k represent the λ for the respective $\alpha = 1, 5(5)135$.

The null hypothesis is tested against the alternative hypothesis

Ha : at least one of the λ parameters does not equal zero

The test statistic is defined by

$$F = (R^2/k)/\{(1-R^2)/[n-(k+1)]\}$$

and the rejection region by

$$F > F_{\alpha', (k, n-(k+1))}$$

where

k is the number of λ parameters in the multiple regression model excluding the constant term λ_0.

n is the number of integrated t_l values used to generate the multiple regression model.

λ_i's are the distances from the integrated t_1 value to the corresponding t_1 values calculated from the multiple regression equation.

α' is the significance level.

3.1.1.2: The lower control limit

The following steps describe the methodology for generating a mathematical model for the asymmetrical lower control limit on the individual measurement X control chart.

1) The value of the lower control limit for the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) is located by integrating the distribution on Mathcad. A lower control limit (LCL) is generated which leaves a tail area of 0.00135 below the limit. The LCL of the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) is denoted by (LCL) in the equations below. The limit is evaluated in this manner for all combinations of α (shape parameter) = 1,5(5)135 and β (scale parameter) = 1, 2, and 5. The LCL's are expressed as a multiple of σ_X units (t_2) to the left of the mean of the distribution. The following equations are used to generate the limits:

$$.00135 = \left[\int_0^{LCL} \frac{\left(\frac{1}{\beta}\right)^2}{\Gamma(\alpha)} * x^{\alpha-1} * e^{-x\left(\frac{1}{\beta}\right)} dx \right] \text{; (given } \alpha \text{ and } \beta \text{, find LCL)} \qquad \text{(eq. 3-5)}$$

$$\sigma_x = \sqrt{\alpha * \beta^2} \qquad \text{(eq. 3-2)}$$

$$mean = \alpha * \beta \qquad \text{(eq. 3-3)}$$

$$t_2 = \sigma_X \text{ units away from the mean} = \frac{\alpha * \beta - LCL}{\sqrt{\alpha * \beta^2}} \quad \text{(eq. 3-6)}$$

The t_2 values are placed in a table similar to Table 3-1 in step 1 of section 3.1.1.

Steps 2) and 3) are the same for the lower control limit as stated earlier for the upper control limit.

3.1.2: *Moving range n = 2 mR upper control chart limits:*

The moving range n=2 mR control charts are commonly used in industry. Unfortunately, they may produce inaccurate representations of the process if the underlying process distribution is non-normal. As seen from previous research by Oyon (1995), moving range n = 2 mR control charts fall well short of achieving ARLs of 100 (the assumed ARL for industry acceptance in the research) for moving ranges of skewed distributions. The result of the poor performance of these charts is the appearance of many false out-of-control signals. To use the moving range n=2 mR control charts appropriately, there must be a method for setting control limits that more accurately predicts the stability of the process. This portion of the research sets control limits based on the location (t_3) of the upper control limits as a multiple of the standard deviation of the ranges. The (t_3) values for the moving range n=2 mR control charts for skewed distributions are evaluated as follows:

1) The value of the upper control limit for the moving range n = 2 mR control charts based on the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) is located by simulating values of the distribution from Minitab. Two columns of k = 60,000 randomly generated observations are produced for all values of α (shape parameter) = 1, 5, 10, 15, 20,..., 135. The selection of the

number of subgroups, k = 60,000, is found in Appendix B. The scale parameter is not evaluated in generating this mathematical model for the ranges because it does not affect the value (t_3) as demonstrated in Appendix A.

The upper control limit for the mR chart is based on ranges of subgroup size two as is common with Shewhart's mR control charts. Ranges of subgroup size n=2 can be used instead of mR values. This is demonstrated in Appendix H.

2) The observations are grouped in the following manner:

$$(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), \ldots, (X_k, Y_k)$$

Where X_i represents the first column of k = 60,000 observations and Y_i represents the second column of k = 60,000 observations.

3) The range for each pair of data is found using the following equation:

$$R = |X_i - Y_i|$$

4) The average range, $\overline{R}$, is found with the equation:

$$\overline{R} = \frac{\sum_i R_i}{k}$$

where k = 60,000. The value of $\overline{R}$ is found for each value of α (shape parameter) = 1, 5(5)135.

5) The standard deviation of the ranges, σ_R, is found with the following equation:

$$\sigma_R = \sqrt{\frac{\sum \left(R_i - \overline{R}\right)^2}{k - 1}}$$

where k = 60,000. The value of σ_R is found for each value of α (shape parameter) = 1, 5(5)135.

6) An upper control limit is generated for each α which leaves a tail area of 0.0027 (see introduction) outside the limit. In order to accomplish this step, the following equation is used:

$$UCL_R = \overline{R} + t_3(\hat{\sigma}_R) \quad \text{(eq. 3-7)}$$

To locate the upper control limit, the values from steps 4 and 5 are applied to this equation and an appropriate (t_3) value is found. This is accomplished by increasing the value of (t_3) by 0.0001 until 0.27% of the ranges are outside the control limits.

The limit is evaluated in this manner for all values of α (shape parameter) = 1, 5, 10, 15, 20,..., 135. The results are expressed as a multiple of σ_R units (t_3) to the right of the mean range of the distribution. The multiple of σ_R units from the mean (t_3) generated in this step are paired with their associated α values as demonstrated in the table (Table 3-2) on the following page.

Steps 7) and 8) are the same as steps 2) and 3) for the upper and lower control limits of the individual measurement X control chart.

3.2: Sub-objective two:

Statement of the Sub-Objective: Evaluate the performance of the individual measurement X and moving range n=2 mR control charts, based on the average run length (ARL) using the Pearson type III family of distributions with location parameter c=0 (gamma distribution) control limits determined from sub-objective 1. The control charts are evaluated against an ARL that is acceptable for practical application in industry and compared with methods having symmetrical control limits. An ARL that is acceptable for practical application in industry means that the average run length (ARL)

Table 3-2: σ_X Units From the Mean (t_3) for the UCL of the Range Chart

σ_X units from the mean (t_3) for the UCL of the range chart	
α	$\beta = 1$
1	4.9826
5	4.1126
10	3.9821
. . .	. . .
135	3.6919

for each control chart is a minimum of 100 observations. An ARL of 100 is equivalent to a 1% chance of a type I error when the process is in a state of statistical control.

The Data Needed: The data for sub-objective two consist of randomly generated variates from the normal, log-normal, and gamma distributions. Individual measurement X and moving range n=2 mR control limits are also needed for the normal Shewhart, Oyon's symmetrical Pearson type III (c=0), and asymmetrical Pearson type III (c=0) control charts.

The Location of the Data: The location of the data for sub-objective two is as follows:

- A random variate generator is utilized to generate values from the normal, log-normal, and gamma distributions.
- Symmetrical individual measurement X and moving range n=2 mR control limit equations based on the normal distribution produced by Dr. Shewhart are found in various quality control texts including Wheeler and Chambers (1992), Burr (1953), and Duncan (1986). These equations can be found in step 6 below.
- Symmetrical individual measurement X and moving range n=2 mR control limit equations are produced using the d'_2, d'_3, and D'_4 values approximated by the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) from previous research by Jose Oyon (1995). These equations can be found in step 8 below.
- Asymmetrical individual measurement X and moving range n=2 mR control limit equations are produced using the mathematical models generated for the "t" values

approximated by the Pearson Type III family of distributions with location parameter c=0 (gamma distribution). The mathematical models are produced in sub-objective one above.

Means of Obtaining the Data: The Individual measurement X and moving range n=2 mR control limits are obtained through the calculation of the symmetrical and asymmetrical control limit equations. Equations 3-11 through 3-13 are for the normal control limits, equations 3-18 through 3-20 are for the Oyon control limits, and equations 3-21 through 3-23 are for the asymmetrical control limits.

Treatment of the Data: The following is a detailed procedure to achieve sub-objective two:

1) Five process distributions are selected to represent unknown parent distributions. The distributions are chosen to represent a variety of process distributions that occur in industry. The five process distributions selected are as follows:

- Normal $(40, 10^2)$
- Log-normal $(0, 1^2)$
- Gamma ($\alpha = 1.5$, $\beta = 1$)
- Chi-square (df = 4)
- Exponential ($\beta = 1$)

2) One set of k' = 50 observations is generated from one of the five distributions selected in the previous step.

3) The average ($\overline{X}$) is calculated for the 50 observations (the 50 observations for which the control limits are calculated are referred to as k'). The average is obtained using the following equation:

$$\overline{X} = \frac{\left(\sum X_i\right)}{k'} \qquad \text{(eq. 3-8)}$$

4) The moving range n=2 is calculated for the k' = 50 observations. The moving range n=2 is calculated by grouping the observations into subgroups of two consecutive measurements and then applying those subgroups to the following equation:

$$mR_j = \left|X_{i+1} - X_i\right| \qquad \text{(eq. 3-9)}$$

5) The average moving range is calculated from the 49 moving ranges calculated in step 4 for the k' = 50 observations using the following equation:

$$\overline{mR} = \frac{\sum_{y=1}^{k-1} mR_y}{(k'-1)} \qquad \text{(eq. 3-10)}$$

6) Control limits based on the normal distribution are calculated. The individual measurement X and moving range n=2 mR control chart limits are calculated with the following equations:

$$UCL_x = \overline{X} + 2.66\left(\overline{mR}\right) \qquad \text{(eq. 3-11)}$$

$$LCL_x = \overline{X} - 2.66\left(\overline{mR}\right) \qquad \text{(eq. 3-12)}$$

$$UCL_{mR} = 3.268\left(\overline{mR}\right) \qquad \text{(eq. 3-13)}$$

7) In order to calculate the control chart constants based on the Pearson Type III family of distributions with location parameter c=0 (gamma distribution), the shape parameter

(α) has to first be estimated. The following was written by Jose Oyon (1995) in regards to estimating the parameters α and β for the Pearson Type III family of distributions with location parameter c=0 (gamma distribution):

"In order to get the Pearson type III with c=0 (gamma) control chart constants d_2, d_3, and D_4 to be used in setting control limits for each run of (k') observations, Pearson type III parameters α and β have to be estimated from the (k') observations generated.

"Since the process distribution is supposedly unknown, the idea is to fit the data with a Pearson type III with c=0 distribution by estimating the parameters α and β from the (k') data values (gamma distribution assumption as the underlying process distribution). According to Fisher (1, p.332), the method of moments is inefficient to estimate parameters of a gamma distribution, except for a distribution closely resembling the normal distribution. Kendall and Stuart (2, p.38) show that the efficiency of the estimated shape parameter α of a gamma distribution by the method of moments may be as low 22 percent. Therefore, Fisher (1, p.332) and Law and Kelton (15, p.331) recommend the method of maximum likelihood estimation (MLE) in order to estimate the parameters α and β of type III from the data.

"The difficulty in applying the method of maximum likelihood estimation to estimate the parameters α and β of the gamma distribution is that closed expressions for the maximum likelihood estimators $\hat{\alpha}$ and

$\hat{\beta}$ cannot be obtained analytically. Therefore, numerical methods must be used to estimate the parameters α and β of the gamma distribution.

"Choi and Wette (9, p.683) developed a numerical technique of the maximum likelihood method to estimate the parameters of the gamma distribution. This method is recommended by Law and Kelton (15, p. 331) to estimate α and β. Therefore, this method is the one to be used in this (sub-objective two) to estimate α and β from the data in order to fit a Pearson type III distribution with location parameter c=0 (gamma distribution)."

The maximum likelihood method stated above utilizes a T statistic to estimate the parameters α and β (15, p.331). The T statistic is obtained with the following equation as given by Law and Kelton (15, p.410):

$$T = \left[\ln \overline{X} - \frac{\sum \ln X_i}{k} \right]^{-1} \qquad \text{(eq. 3-14)}$$

Using the T statistic from the above equation, the estimator $\hat{\alpha}$ can be obtained using Table 6.19 in Law and Kelton (15, p. 411). A reproduction of this table is included in Appendix C of this research.

8) The control chart constants d'_2, d'_3, and D'_4 based on the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) are calculated for the k' = 50 observations. The following mathematical models (as produced by research from Jose Oyon (1995)) are used to generate the constants:

$$d'_2 = 0.64282 + 0.09775\left(1 - e^{-0.5\alpha}\right) + 0.35736\left(1 - e^{-2\alpha}\right) + 0.02483\left(1 - e^{-0.1\alpha}\right) \quad \text{(eq. 3-15)}$$

$$d'_3 = 0.859457 + 0.2964\left(e^{-\alpha}\right) + 0.29099\left(e^{-0.5\alpha}\right) + 0.4758\left(e^{-2\alpha}\right) \quad \text{(eq. 3-16)}$$

$$D'_4 = 3.28976 + 1.87067\left(e^{-\alpha}\right) + 0.13663\left(e^{-0.1\alpha}\right) \quad \text{(eq. 3-17)}$$

9) Symmetrical control limits based on the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) are calculated. The individual measurement X and moving range n=2 mR control chart limits are calculated using the following equations:

$$UCL_x = \overline{X} + \left(\frac{3}{d'_2}\right)\left(\overline{mR}\right) \quad \text{(eq. 3-18)}$$

$$LCL_x = \overline{X} - \left(\frac{3}{d'_2}\right)\left(\overline{mR}\right) \quad \text{(eq. 3-19)}$$

$$UCL_{mR} = D'_4\left(\overline{mR}\right) \quad \text{(eq. 3-20)}$$

where $D_4 = (1 + 3\left(\frac{d_3}{d_2}\right))$.

10) The t_1, t_2, and t_3 values are calculated for the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) for the k' = 50 observations. The parameter (α) designated from step 7 of sub-objective two is used to estimate the t_1, t_2, and t_3 values using the mathematical models generated from sub-objective one of this research.

11) Asymmetrical control limits based on the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) are calculated. The individual measurement X and moving range n=2 mR control chart limits are calculated with the following equations:

$$UCL_x = \overline{X} + \left(\frac{t_1}{d'_2}\right)\left(\overline{mR}\right) \qquad \text{(eq. 3-21)}$$

$$LCL_x = \overline{X} - \left(\frac{t_2}{d'_2}\right)\left(\overline{mR}\right) \qquad \text{(eq. 3-22)}$$

$$UCL_{mR} = D'_4\left(\overline{mR}\right) \qquad \text{(eq. 3-23)}$$

where $D'_4 = (1 + t_3\left(\frac{d'_3}{d'_2}\right))$.

12) For the three sets of control limits (normal, Oyon, and asymmetrical), random variates are generated until a value falls outside each set control limits. A run length (number of values generated before an OOC signal) is recorded for each control chart.

13) Steps 2) through 12) are repeated 1,000 times for each of the five parent distributions stated in step one of this sub-objective. An average run length (ARL) for each of the five distributions is calculated using the following equation:

$$ARL = \frac{\sum(RL)}{1{,}000} \qquad \text{(eq. 3-24)}$$

14) A variance of the run length (VRL) for each of the five distributions is calculated with the following equation:

$$VRL = \sqrt{\frac{\sum_{i=1}^{1000}(RL_i - ARL)^2}{999}} \quad \text{(eq. 3-25)}$$

15) The 1000 run lengths are stored and presented on a histogram for each of the five parent distributions. The data from steps 13) and 14) of sub-objective two are grouped according to the parent distributions of the random variates and placed in a table for easy reference. The table (Table 3-3) is illustrated on the following page.

3.3: Sub-objective three:

Statement of the Sub-Objective: Compare the power of the individual measurement X and moving range n=2 mR control charts using the Pearson Type III c=0 asymmetrical control limits with those methods having symmetrical control limits. The power, in this case, refers to the ability of the control charts to detect shifts in process location of 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0σ_X units.

The Data Needed: The data for sub-objective two consist of randomly generated variates from the normal, log-normal, and gamma distributions. Individual measurement X and moving range n=2 mR control limits are also needed for the normal Shewhart, symmetrical Pearson type III (c=0), and asymmetrical Pearson type III (c=0) control charts.

The Location of the Data: The location of the data for sub-objective two is as follows:

- A random variate generator is utilized to generate values from the normal, log-normal, and gamma distributions.

Table 3-3: Control Chart ARLs and VRLs

Parent Distribution		Individual Measurement X: Shewhart	Individual Measurement X: Symmetrical	Individual Measurement X: Asymmetrical	Moving Range: Shewhart	Moving Range: Symmetrical	Moving Range: Asymmetrical
Normal $(40, 10^2)$	ARL						
	VRL as st. dev.						
Log-normal $(0, 1^2)$	ARL						
	VRL as st. dev.						
Gamma (1.5, 1)	ARL						
	VRL as st. dev.						
Chi-square (df = 4)	ARL						
	VRL as st. dev.						
Exponential (1)	ARL						
	VRL as st. dev.						

- Symmetrical individual measurement X and moving range n=2 mR control limit equations based on the normal distribution produced by Dr. Shewhart are found in various quality control texts including Wheeler and Chambers (1992), Burr (1953), and Duncan (1986). These equations can be found in step 6 of section 3.2.
- Symmetrical individual measurement X and moving range n=2 mR control limit equations are produced using the d'_2, d'_3, and D'_4 values approximated by the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) from previous research by Jose Oyon (1995). These equations can be found in step 8 of section 3.2.
- Asymmetrical individual measurement X and moving range n=2 mR control limit equations are produced using the mathematical models generated for the t_1, t_2, and t_3 values approximated by the Pearson Type III family of distributions with location parameter c=0 (gamma distribution). The mathematical models are produced in sub-objective one above.

Means of Obtaining the Data: All data used for this sub-objective are obtained from the data generated in sub-objective two. The normal, Oyon, and asymmetrical control limits calculated in sub-objective two are adjusted to represent a mean shift in sub-objective three. The random variates generated in sub-objective two are used to evaluate the adjusted control limits.

Treatment of the Data: This sub-objective evaluates the ability of the control charts to detect shifts in the mean on the individual measurement X control charts. The following is a detailed procedure to achieve sub-objective three:

1) The theoretical standard deviation is found for each of the five parent distributions listed in step one of sub-objective two. The following equations are used to find the standard deviations:

Exponential, Gamma, and Chi-square:

$$\sigma_x = \sqrt{\alpha * \beta^2} \qquad \text{(eq. 3-26)}$$

Log-normal:

$$\sigma_x = \sqrt{e^{2\alpha+\beta^2}\left(e^{\beta^2} - 1\right)} \qquad \text{(eq. 3-27)}$$

Normal: The standard deviation for the normal distribution is taken from the definition of the distribution's parameters. The normal distribution used in this research is a $N(40, 10^2)$; therefore, the mean is 40 and the standard deviation is 10.

2) Shifts in the process mean of +/- 0.5, 1.0, 1.5, 2.0, 2.5, and $3.0\sigma_X$ are simulated by adjusting the individual measurement X control chart limits generated in sub-objective two. The control limits are adjusted as follows:

$$\text{UCL(adjusted)} = \text{UCL} - \Delta * (\sigma_X) \qquad \text{(eq. 3-28)}$$

$$\text{LCL(adjusted)} = \text{LCL} - \Delta * (\sigma_X) \qquad \text{(eq. 3-29)}$$

where Δ is the process mean shift as a multiple of σ_X.

3) For the set of ten adjusted control limits, random variates are generated until a value falls outside each set of control limits. The number of values generated before the value fall outside the limits (run length (RL)) is recorded.

4) Steps 2) and 3) are repeated 1,000 times for each of the five parent distributions stated in step one of sub-objective two. An average run length (ARL) for each of the shifts in the five distributions is calculated using the following equation:

$$\mathrm{ARL} = \frac{\sum(\mathrm{RL})}{1{,}000} \qquad \text{(eq. 3-30)}$$

5) The data from step 4 of sub-objective three are grouped according to the parent distributions of the random variables and placed in a table for easy reference. The table (Table 3-4) is illustrated on the following page.

Table 3-4: ARLs/VRLs for Shifts in the Process Mean

		ARLs for Shifts in the Process Mean											
Parent Distribution		+0.5	+1.0	+1.5	+2.0	+2.5	+3.0	-0.5	-1.0	-1.5	-2.0	-2.5	-3.0
Normal (40, 10^2)	Normal												
	Symmetrical												
	Asymmetrical												
Log-normal (0. 1^2)	Normal												
	Symmetrical												
	Asymmetrical												
Gamma (1.5, 1)	Normal												
	Symmetrical												
	Asymmetrical												
Chi-square (df = 4)	Normal												
	Symmetrical												
	Asymmetrical												
Exponential (1)	Normal												
	Symmetrical												
	Asymmetrical												

CHAPTER 4

RESULTS AND ANALYSIS

The results of this thesis research are presented following the three sub-objectives described in chapter 1.

Section 1: SUB-OBJECTIVE ONE

The first sub-objective is to develop mathematical models representative of the upper and lower control limits for asymmetrical distributions based on the shape parameter (α) and the scale parameter (β) from the Pearson Type III family of distributions with location parameter c=0 (gamma distributions) so often encountered in industry.

1.1: <u>Individual Measurement X Control Chart Limits:</u>

Following the steps in Section 3.1.1.1 and 3.1.1.2, described in detail in Chapter 3 (pg. 3-8), the values of the individual measurement X upper and lower control limits for the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) are located by integrating the distribution on MathCad for Windows release 4.02. The value of the upper control limit is expressed as a multiple of σ_X units from the mean. An upper and lower control limit is generated which leaves a tail area of 0.00135 beyond each limit. The limit is evaluated in this manner for all combinations of α (shape parameter) = 1, 5(5)135 and β (scale parameter) = 1, 2, and 5.

The table on the following page, Table 4.1: Gamma Distribution Upper & Lower Control Limits, demonstrates the results of integrating the gamma distribution. As can be seen from Table 4.1, the values for β appear to have little or no effect on the upper

Table 4-1: Gamma Distribution Upper & Lower Control Limits

			Sigma units from average on the skew (upper) tail for 0.00135			Sigma units from average on the non-skew (lower) tail for 0.00135			Total sigma spread for 0.0027
Alpha	Sigma		$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 1$	$\beta = 2$	$\beta = 5$	
135	11.6190		3.2305	*	*	2.7718	*	*	6.0023
130	11.4018		3.2350	*	*	2.7675	2.7675	*	6.0025
125	11.1803		3.2395	*	*	2.7630	2.7630	*	6.0025
120	10.9545		3.2445	3.2445	*	2.7582	2.7582	*	6.0027
115	10.7238		3.2497	3.2497	*	2.7530	2.7530	2.7530	6.0027
110	10.4881		3.2555	3.2555	*	2.7475	2.7475	2.7475	6.0030
105	10.2470		3.2615	3.2615	3.2615	2.7416	2.7416	2.7416	6.0031
100	10.0000		3.2680	3.2680	3.2680	2.7354	2.7354	2.7354	6.0034
95	9.7468		3.2750	3.2750	3.2750	2.7285	2.7285	2.7285	6.0035
90	9.4868		3.2825	3.2825	3.2825	2.7211	2.7211	2.7211	6.0036
85	9.2195		3.2908	3.2908	3.2908	2.7132	2.7132	2.7132	6.0040
80	8.9443		3.2997	3.2997	3.2997	2.7045	2.7045	2.7045	6.0042
75	8.6603		3.3095	3.3095	3.3095	2.6949	2.6949	2.6949	6.0044
70	8.3666		3.3205	3.3205	3.3205	2.6843	2.6843	2.6843	6.0048
65	8.0623		3.3328	3.3328	3.3328	2.6725	2.6725	2.6725	6.0053
60	7.7460		3.3464	3.3464	3.3464	2.6594	2.6594	2.6594	6.0058
55	7.4162		3.3617	3.3617	3.3617	2.6444	2.6444	2.6444	6.0061
50	7.0711		3.3795	3.3795	3.3795	2.6273	2.6273	2.6273	6.0068
45	6.7082		3.4000	3.4000	3.4000	2.6075	2.6075	2.6075	6.0075
40	6.3246		3.4245	3.4245	3.4245	2.5840	2.5840	2.5840	6.0085
35	5.9161		3.4540	3.4540	3.4540	2.5559	2.5559	2.5559	6.0099
30	5.4772		3.4905	3.4905	3.4905	2.5211	2.5211	2.5211	6.0116
25	5.0000		3.5375	3.5375	3.5375	2.4765	2.4765	2.4765	6.0140
20	4.4721		3.6010	3.6010	3.6010	2.4166	2.4166	2.4166	6.0176
15	3.8730		3.6940	3.6940	3.6940	2.3297	2.3297	2.3297	6.0237
10	3.1623		3.8505	3.8505	3.8505	2.1870	2.1870	2.1870	6.0375
5	2.2361		4.2005	4.2005	4.2005	1.8820	1.8820	1.8820	6.0825
1	1.0000		5.6080	5.6080	5.6080	0.9986	0.9986	0.9986	6.6066

and lower control limits for the gamma distribution when expressed as a multiple of σ_X units. Following step 2 in 3.1.1.1 and 3.1.1.2, regression models are generated in Minitab for Windows release 10.5. This statistical software package is used to generate different multiple regression models for predicting the t_1 and t_2 values with predictors α (shape parameter) and β (scale parameter). There are different mathematical models which can predict the t_1 and t_2 values. By trial and error, a model is found which does a good job of predicting t_1 and t_2. The models found may not be the only models that can be used.

The output from Minitab can be found in Appendix D: Regression Output For Control Limits. The best t_1 and t_2 regression models, based on R^2, for the upper and lower control limits are as follows:

$$t_1 = 3.23 + 3.19*e^{(-\alpha)} + 0.852*e^{(-0.1\alpha)} + 0.442*e^{(-0.25\alpha)}$$

$$t_2 = 2.77 - 1.81*e^{(-\alpha)} - 0.751*e^{(-0.1\alpha)} - 0.438*e^{(-0.025\alpha)}$$

A global F test is used to test the validity of the multiple regression models. The F tests for the upper and lower control limits are shown in sections 1.1.1 and 1.1.2.

1.1.1: *The upper control limit*

The model for t_1 has a multiple coefficient of determination R^2 of 99.9%. The global F test is used to test the validity of the upper control (t_1) limit multiple regression model as indicated in Chapter 3 section 3.1.1.1 step 3.

From the Minitab output (Appendix D - Regression Output For Control Limits) , the value for the test statistic F is:

$$F = 6935.38$$

Using a significance level $\alpha' = 0.01$, the rejection region for the test is defined by the critical value $Fc_{\alpha'(k, n-(k+1))}$. From an F table, this critical value is:

$$Fc_{0.01(3, 23)} = 4.765$$

Clearly the null hypothesis Ho: $\lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_k = 0$ is rejected since the value of the F statistic is greater than the critical value Fc:

$$F > Fc_{\alpha'(k, n-(k+1))}$$

$$6935.38 > 4.765$$

Therefore, it is concluded that one can be very confident that this model is useful in predicting t_1.

1.1.2: *The lower control limit*

The model for t_2 has a multiple coefficient of determination R^2 of 99.8%. The global F test is used to test the validity of the lower control (t_2) limit multiple regression model as indicated in Chapter 3, section 3.1.1.2, step 3.

From the Minitab output (Appendix D - Regression Output For Control Limits), the value for the test statistic F is:

$$F = 5465.84$$

Using a significance level $\alpha' = 0.01$, the rejection region for the test is defined by the critical value $Fc_{\alpha'(k, n-(k+1))}$. From an F table, this critical value is:

$$Fc_{0.01(3, 23)} = 4.765$$

Clearly the null hypothesis Ho: $\lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_k = 0$ is rejected since the value of the F statistic is greater than the critical value Fc:

$$F > Fc_{\alpha'(k,\, n-(k+1))}$$

$$5465.84 > 4.765$$

Therefore, it is concluded that one can be very confident that this model is useful in predicting t_2.

1.2: Moving Range n = 2 Upper Control Chart Limits:

Following the steps in Section 3.1.2, described in detail in Chapter 3 (pg. 3-15), the values of the moving range n = 2 upper control chart limit for the Pearson Type III family of distributions with location parameter c=0 (gamma distribution) are located by simulating values in MathCad for Windows release 4.02. The value of the upper control limit is expressed as a multiple of σ_R units from the average range. An upper control limit is generated which leaves a tail area of 0.0027 beyond the upper limit. The limit is evaluated in this manner for all combinations of α (shape parameter) = 1, 5(5)135. As demonstrated in Appendix A, the values for β appear to have little or no effect on the upper control limit for the gamma distribution when expressed as a multiple of σ_R units. The table on the following page, Table 4.2: Gamma Distribution Upper Control Limits For Moving Range, demonstrates the results of simulating the gamma distribution.

Following step 7 in 3.1.2, a regression model is generated in Minitab for Windows release 10.5. This statistical software package is used to generate different multiple regression models for predicting the t_3 values with the predictor α (shape parameter). There are different mathematical models which can predict the t_3 values, so, by trial and error, a model is found which does a good job of predicting t_3 but may not be the only model that can be used.

Table 4-2: Gamma Distribution Upper Control Limits For Moving Range

α	t_3 @ $\beta = 1$
1	4.9826
5	4.1126
10	3.9821
15	3.8825
20	3.8271
25	3.7939
30	3.7458
35	3.7430
40	3.7119
45	3.7168
50	3.7105
55	3.7103
60	3.6884
65	3.7213
70	3.7091
75	3.7065
80	3.7155
85	3.6922
90	3.6955
95	3.6603
100	3.7005
105	3.7107
110	3.6967
115	3.7065
120	3.6903
125	3.6927
130	3.6614
135	3.6919

The output from Minitab can be found in Appendix D: Regression Output For Control Limits. The best t_3 regression model, based on R^2, for the upper control limits is as follows:

$$t_3 = 3.68 + 1.88*e^{(-\alpha)} + 0.564*e^{(-0.1\alpha)} + 0.0969*e^{(-0.025\alpha)}$$

A global F test is used to test the validity of the multiple regression models. The F tests for the upper and lower control limits are as follows:

The model for t_3 has a multiple coefficient of determination R^2 of 99.7%. The global F test is used to test the validity of the upper control (t_3) limit multiple regression model as indicated in Chapter 3 section 3.1.2, step 8.

From the Minitab output (Appendix D - Regression Output For Control Limits) , the value for the test statistic F is:

$$F = 2892.98$$

Using a significance level $\alpha' = 0.01$, the rejection region for the test is defined by the critical value $Fc_{\alpha'(k, n-(k+1))}$. From an F table, this critical value is:

$$Fc_{0.01(3, 23)} = 4.765$$

Clearly the null hypothesis Ho: $\lambda_1 = \lambda_2 = \lambda_3 = \ldots = \lambda_k = 0$ is rejected since the value of the F statistic is greater than the critical value Fc:

$$F > Fc_{\alpha'(k, n-(k+1))}$$

$$2892.98 > 4.765$$

Therefore, it is concluded that one can be very confident that this model is useful in predicting t_3.

Section 2: SUB-OBJECTIVE TWO

The second sub-objective is to evaluate the performance of the individual measurement X and moving range n=2 mR control charts, based on the average run length (ARL) using the Pearson type III family of distributions with location parameter c=0 (gamma distribution) control limits determined from sub-objective 1. The control charts are evaluated against an ARL that is acceptable for practical application in industry and compared with methods having symmetrical control limits. An ARL that is acceptable for practical application in industry means that the average run length (ARL) for each control chart is a minimum of 100 observations. An ARL of 100 is equivalent to a 1% chance of a type I error when the process is in a state of statistical control.

Following the steps in section 3.2, five process distributions were selected to represent unknown parent distributions. The distributions were chosen to represent a variety of process distributions that occur in industry. The five process distributions selected are as follows:

- Normal $(40, 10^2)$
- Log-normal $(0, 1^2)$
- Gamma ($\alpha = 1.5$, $\beta = 1$)
- Chi-square (df = 4)
- Exponential ($\beta = 1$)

A Turbo Pascal (version 6.0) program was written to perform steps 2 through 13 of section 3.2 (Chapter 3). The Turbo Pascal program for the Chi-square (df = 4) distribution can be found in Appendix E. To generate random variates from each of the five parent distributions, random variates were first generated from the uniform distribution. The random uniform variates were generated according to Marse and Roberts' random number generator found in Appendix F. Based on the numbers generated from the uniform distribution, random variates for each of the parent distributions were then generated according to the following algorithms, as recommended by Law and Kelton (15, p. 484-93):

2.1: <u>Normal (40, 10^2) Algorithm:</u>

The algorithm used to generate Normal (40, 10^2) random variates is known as the polar method.

Algorithm:

2.1.1 Generate U_1 and U_2 as IID U(0,1), let $V_i = 2U_i - 1$ for i = 1,2,... and let W = $V_1^2 = V_2^2$.

2.1.2 If W > 1, go back to step 1. Otherwise, let Y = $\sqrt{(-2 \ln W / W)}$, X_1 = V_1Y, and $X_2 = V_2Y$. Then X_1 and X_2 are IID N(0,1) random variates.

2.1.3 Given that Y ~N(0,1), X~N(μ,σ^2) can be obtained by using $X = \mu + \sigma Y$.

2.2: <u>Log-normal (0, 1^2) Algorithm:</u>

A special property of the log-normal distribution is that if Y~N(μ,σ^2) then e^Y~LN(μ,σ^2). Therefore, Log-normal variates can be generated based on Normal variates from the algorithm above (Chapter 4, section 2.1).

Algorithm:

2.2.1 Generate Y $\sim N(\mu,\sigma^2)$.

2.2.2 Return $X = e^Y$.

2.3: Gamma (α = 1.5, β = 1) Algorithm:

Random Gamma variates are typically generated according to three cases: $0 < \alpha < 1$; $\alpha = 1$; and $\alpha > 1$. Since $\alpha = 1.5$, the case for $\alpha > 1$ will be used. According to Law and Kelton (5, p. 489), "There are several good algorithms for the case $\alpha > 1$." However, they recommend a method due to Cheng (22) referred to as the GB algorithm.

Algorithm:

2.3.1 Generate U_1 and U_2 as IID U(0,1).

2.3.2 Let $V = a \ln [U_1/(1-U_1)]$, $Y = \alpha e^V$, $Z = U_1^2U_2$, and $W = b + qV - Y$.

2.3.3 If $W + d - \theta Z >= 0$, return X = Y. Otherwise, proceed to step 4.

2.3.4 If $W >= \ln Z$, return X = Y. Otherwise, go back to step 1.

where:

$a = 1/\sqrt{(2a - 1)}$

$b = \alpha - \ln 4$

$q = \alpha + 1/a$

$\theta = 4.5$

$d = 1 + \ln \theta$

2.4: Chi-square (df = 4) Algorithm:

The Chi-square distribution is a Gamma distribution with shape parameter α = df/2 and scale parameter $\beta = 2$. Therefore, the algorithm used to generate Gamma (α =

2, $\beta = 2$) will be used to generate the Chi-square distribution. The algorithm in section 4.2.3 for the case $\alpha > 1$ will be the one used for Chi-square (df = 4).

2.5: Exponential ($\beta = 1$) Algorithm:

The Gamma distribution with shape parameter $\alpha = 1$ and scale parameter β is an exponential distribution with mean β. The algorithm used to generate Gamma variates ($\alpha = 1, \beta$) is based on the inverse transform method.

Algorithm:

2.5.1 Generate $U \sim U(0,1)$.

2.5.2 Return $X = -\beta \ln (U)$.

Based on the algorithms above and steps 2 through 13 from section 3.2 (Chapter 3), 1000 run lengths were generated for each of the five parent distributions. The program output for the Normal distributions can be found in Appendix G. The output consists of 1000 run lengths based on the individual measurement X and moving range n=2 mR control charts for Shewhart, Oyon's symmetrical, and Ankney's asymmetrical control limits.

Average run lengths (ARLs) and variance of run lengths (VRLs) were calculated for each of the distributions. The ARLs and VRLs can be found in Table 4.3: Control Chart ARLs and VRLs on the following page. The 1000 run lengths are also presented on a histogram for each of the five parent distributions according to their relative control limits. These histograms, figures 4-1 through 4-15, are on the following pages.

Table 4-3: Control Chart ARLs and VRLs for No Mean Shift

Ideal ARL = ∞ *Acceptable ARL =* *100*		Individual Measurement X			Moving Range		
Parent Distribution		Shewhart	Oyon	Asymmetrical	Shewhart	Oyon	Asymmetrical
Normal (40, 10^2)	ARL	1118.9	1427.2	141.9	213.4	277.0	4011.7
	VRL as st. dev.	4693.9	6783.5	224.0	495.9	817.4	33429.5
Log-normal (0. 1^2)	ARL	33.0	40.7	161.0	33.9	56.0	148.2
	VRL as st. dev.	45.0	58.6	285.8	48.1	87.2	273.2
Gamma (1.5, 1)	ARL	58.3	80.1	1885.7	57.0	130.1	1418.9
	VRL as st. dev.	83.3	126.2	12566.5	78.4	284.7	5256.1
Chi-square (df = 4)	ARL	72.2	99.0	2079.1	70.1	144.1	1550.9
	VRL as st. dev.	108.2	192.2	9075.6	108.0	327.3	7198.2
Exponential (1)	ARL	49.7	77.8	1758.0	49.3	143.0	1713.9
	VRL as st. dev.	75.7	136.5	8897.2	73.8	318.1	6846.8

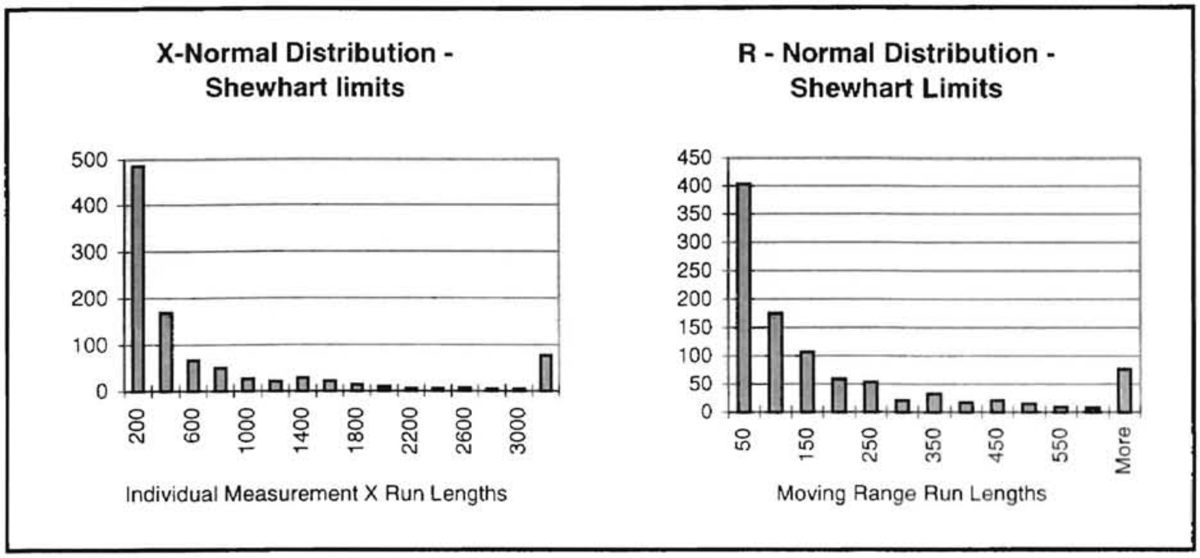

Figure 4-1: Normal Distribution Run Lengths - Shewhart Control Limits

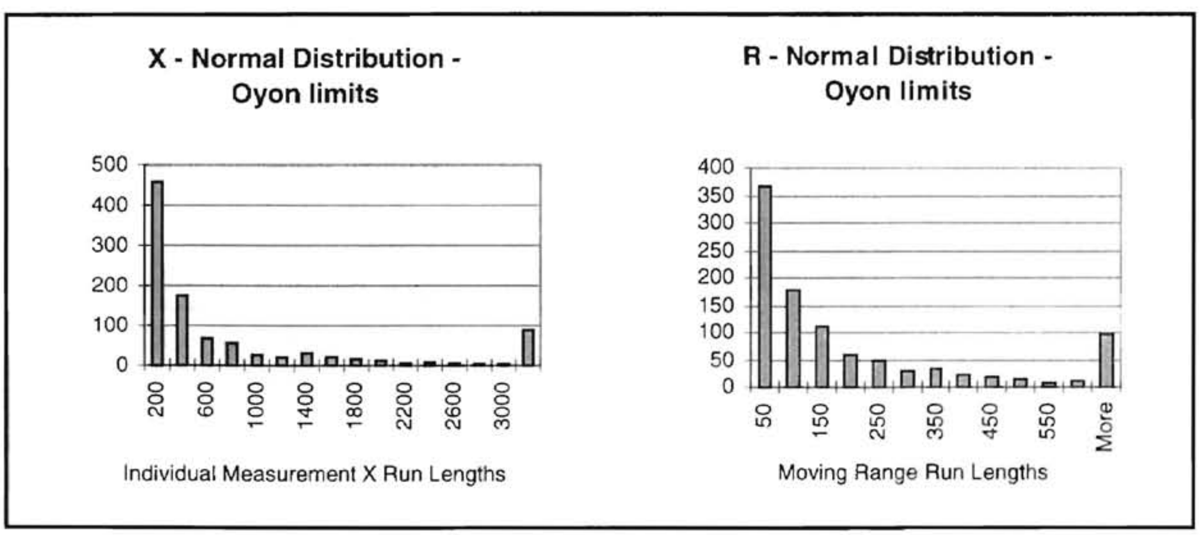

Figure 4-2: Normal Distribution Run Lengths - Oyon Control Limits

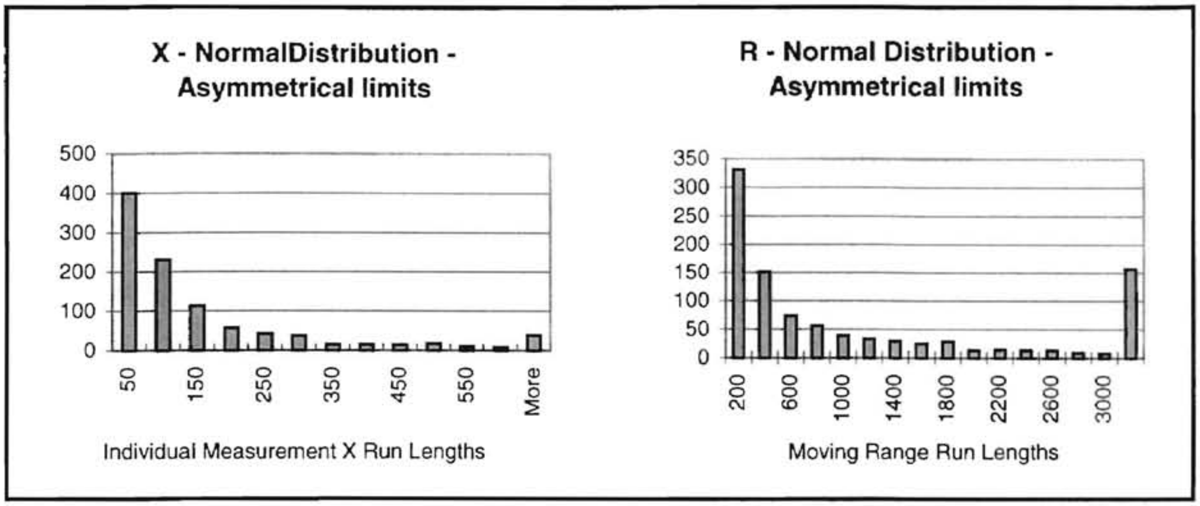

Figure 4-3: Normal Distribution Run Lengths - Asymmetrical Control Limits

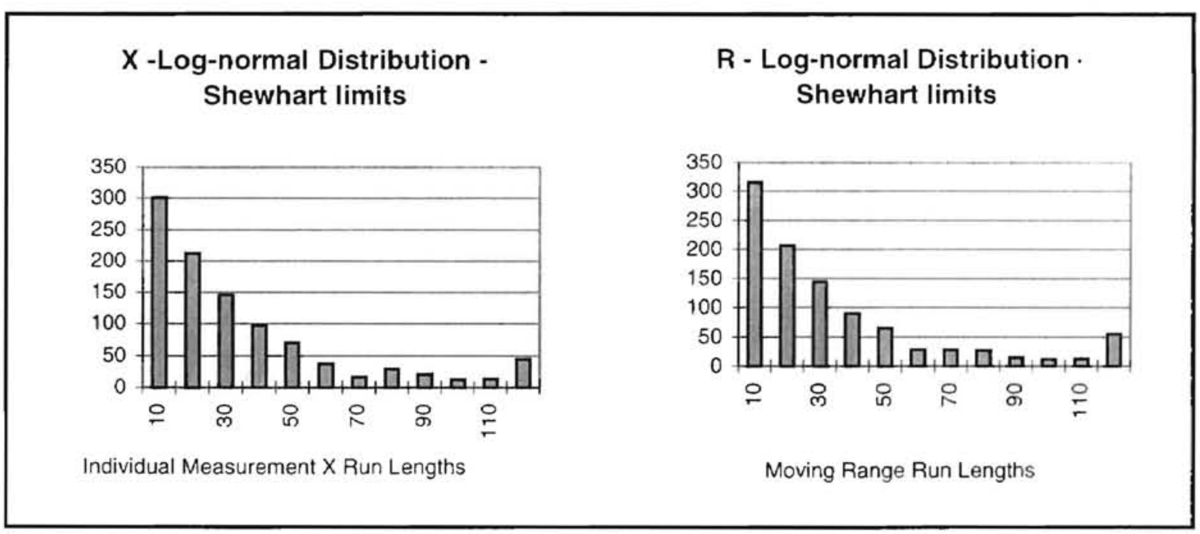

Figure 4-4: Log-normal Distribution Run Lengths - Shewhart Control Limits

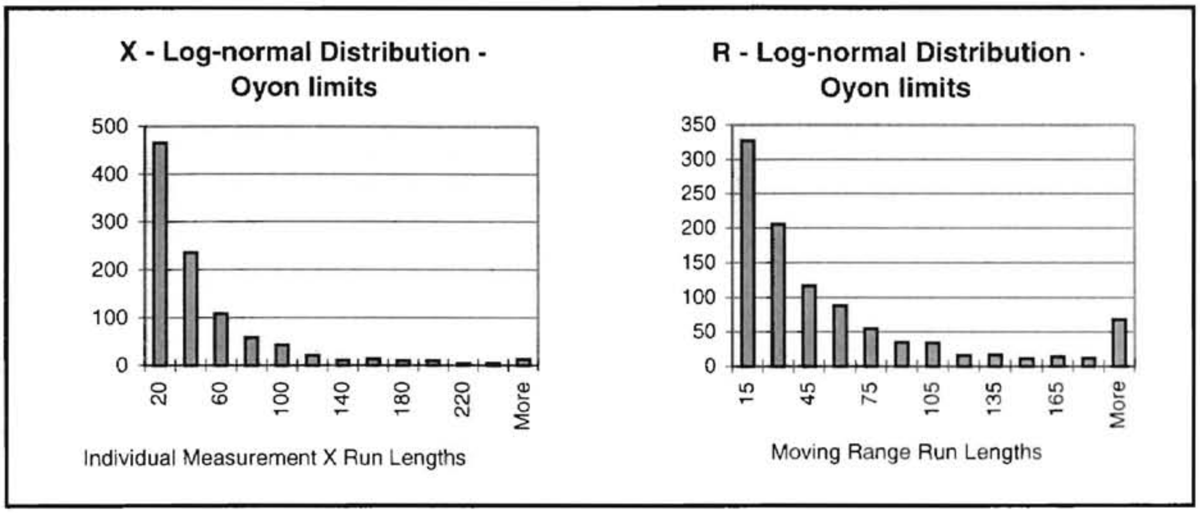

Figure 4-5: Log-normal Distribution Run Lengths - Oyon Control Limits

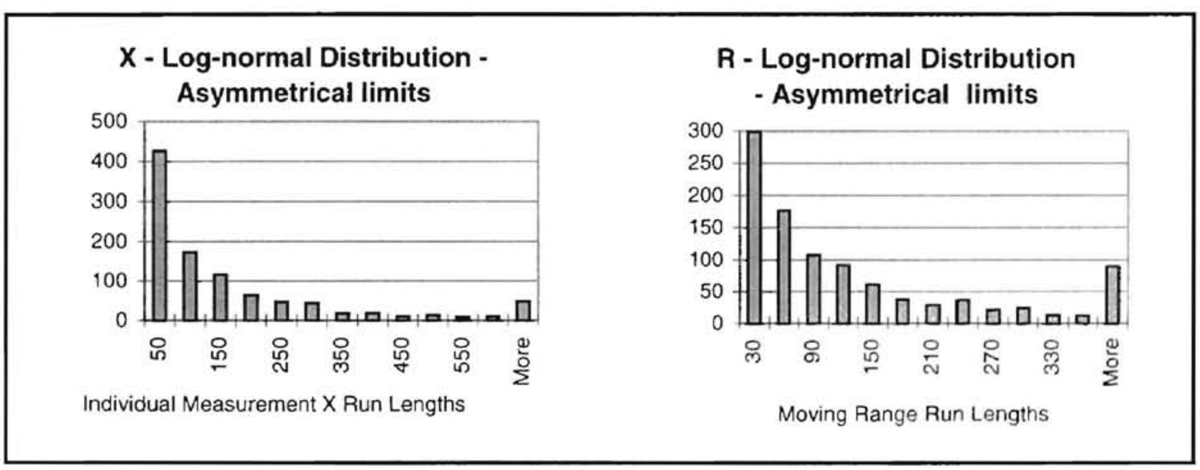

Figure 4-6: Log-normal Distribution Run Lengths - Asymmetrical Control Limits

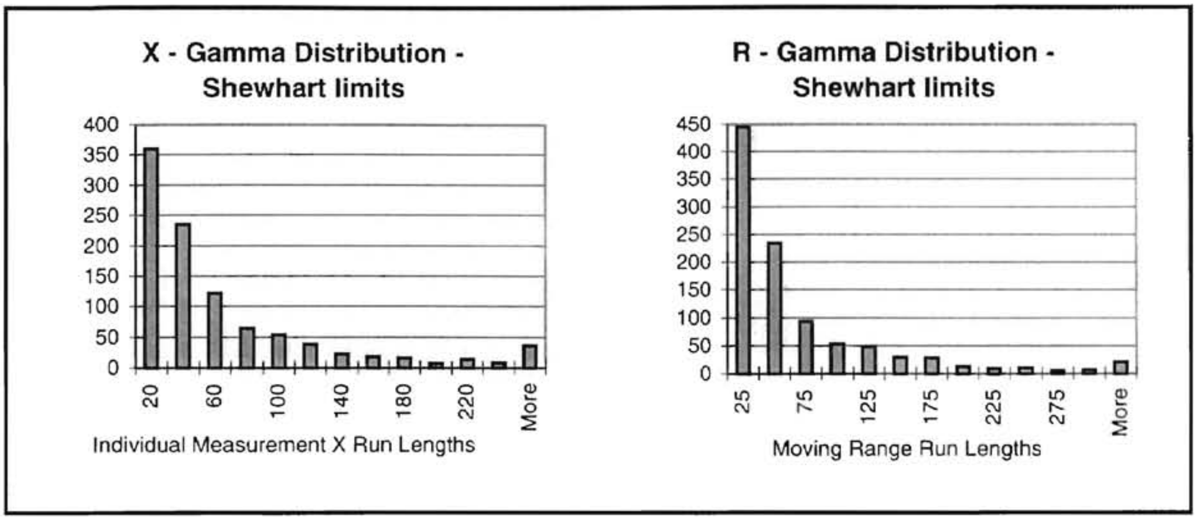

Figure 4-7: Gamma Distribution Run Lengths - Shewhart Control Limits

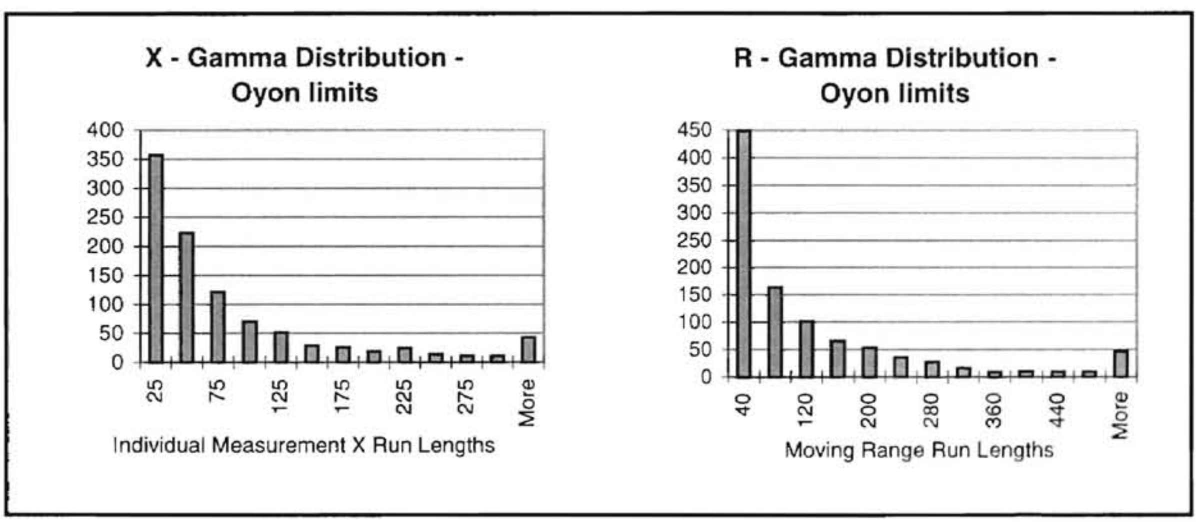

Figure 4-8: Gamma Distribution Run Lengths - Oyon Control Limits

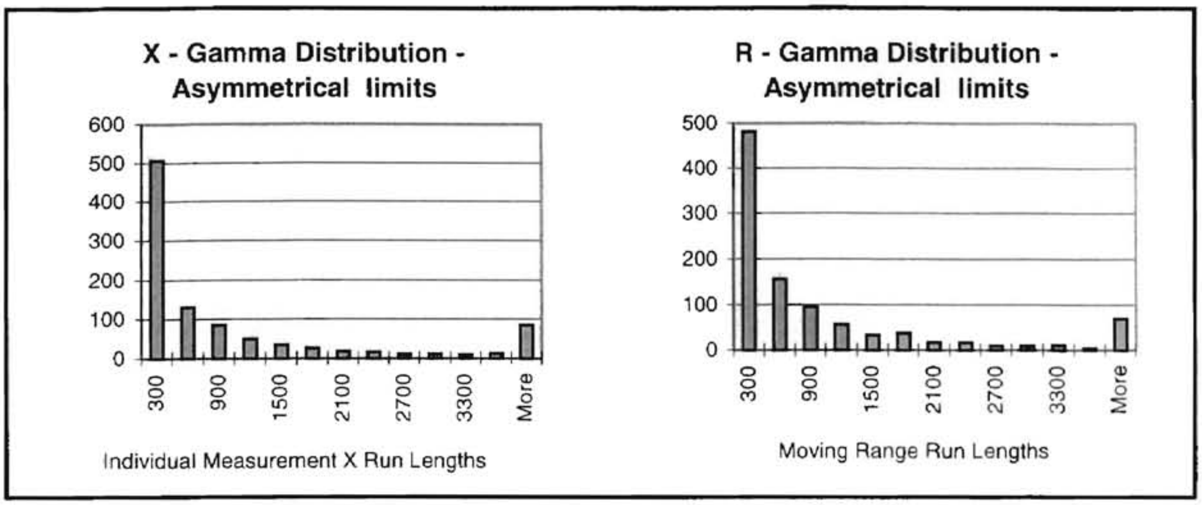

Figure 4-9: Gamma Distribution Run Lengths - Asymmetrical Control Limits

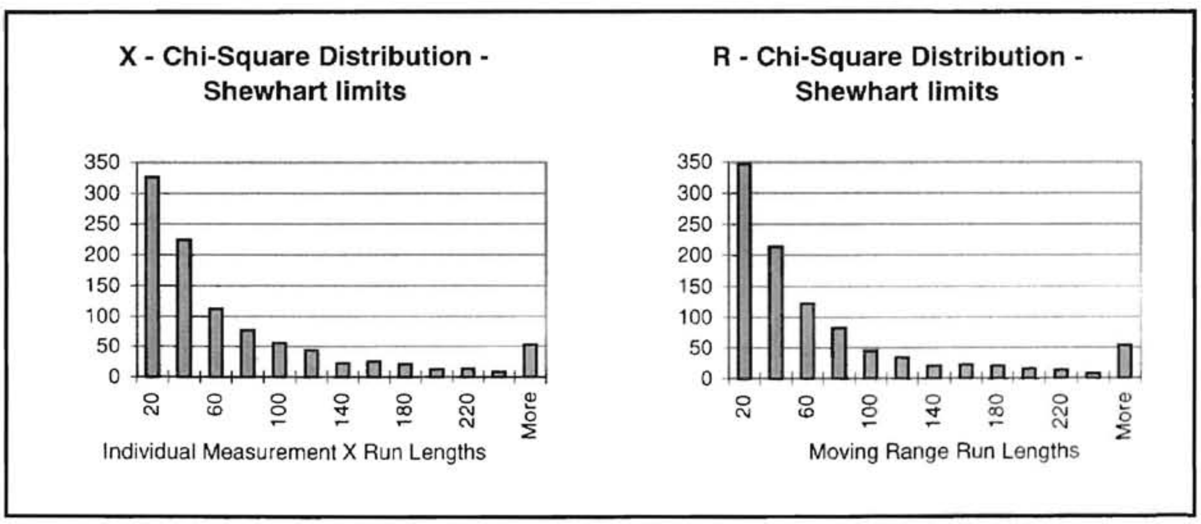

Figure 4-10: Chi-square Distribution Run Lengths - Shewhart Control Limits

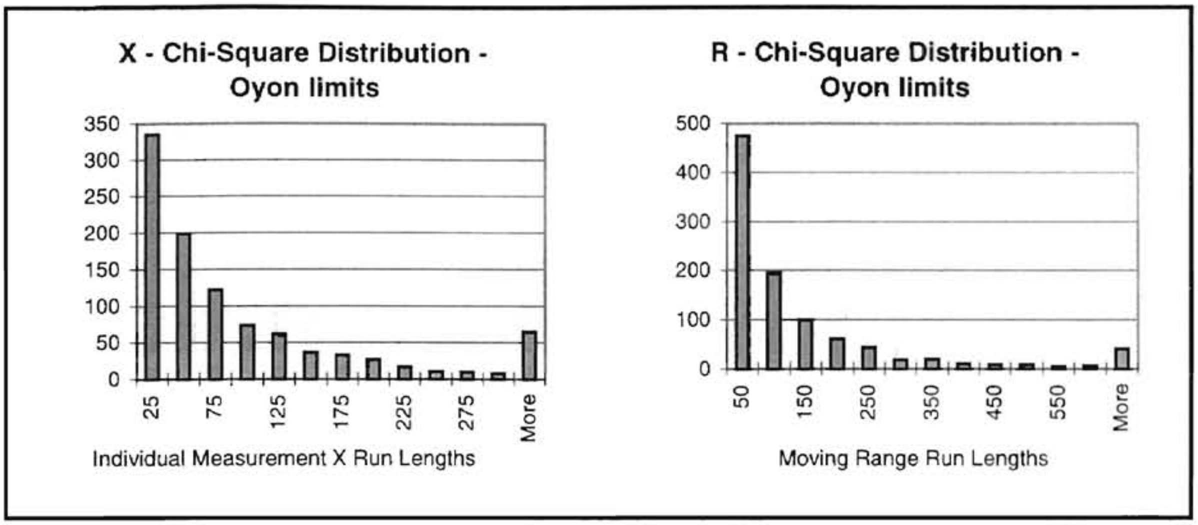

Figure 4-11: Chi-square Distribution Run Lengths - Oyon Control Limits

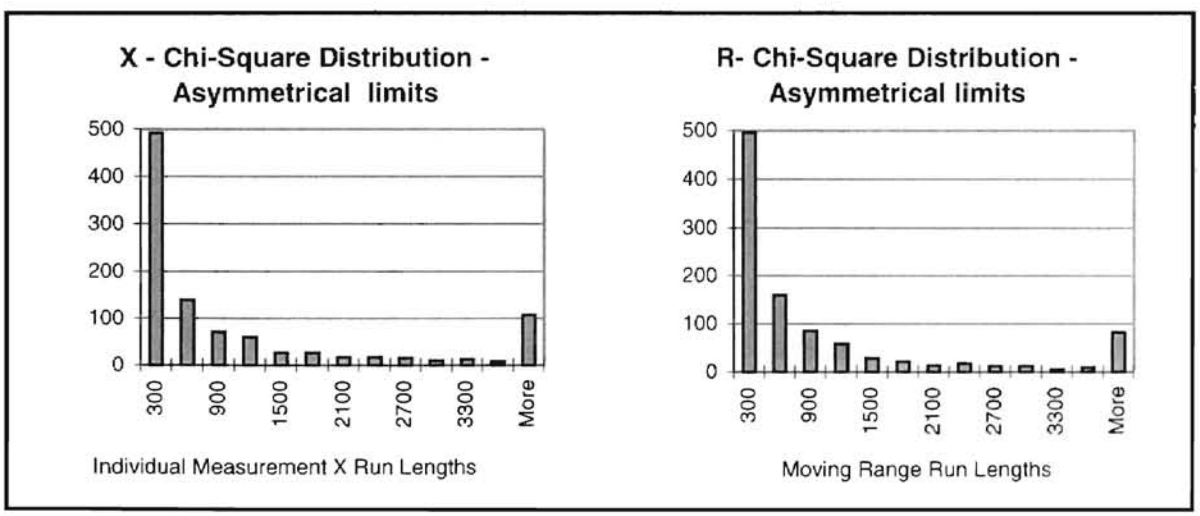

Figure 4-12: Chi-square Distribution Run Lengths - Asymmetrical Control Limits

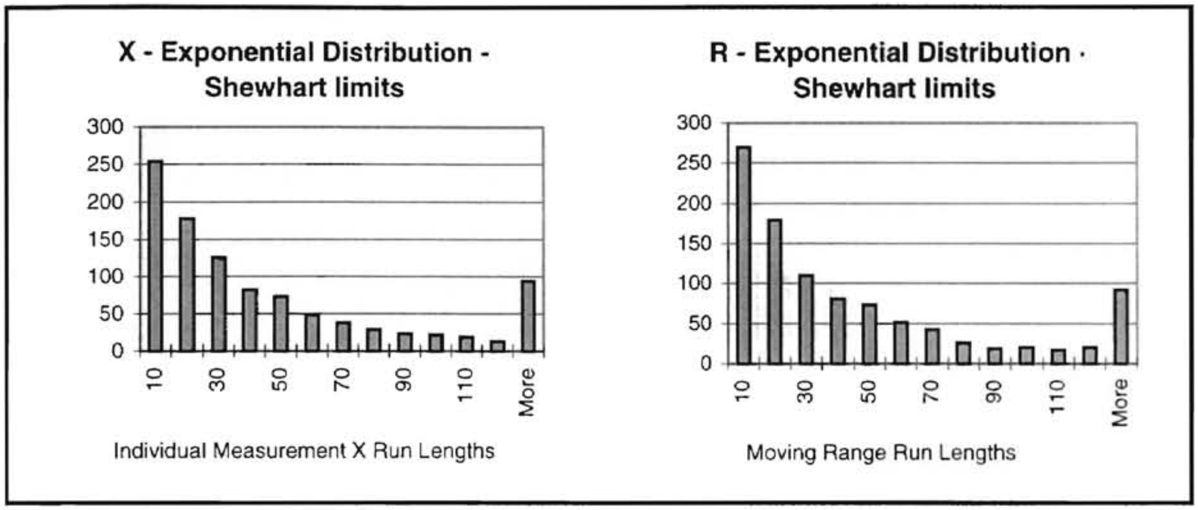

Figure 4-13: Exponential Distribution Run Lengths - Shewhart Control Limits

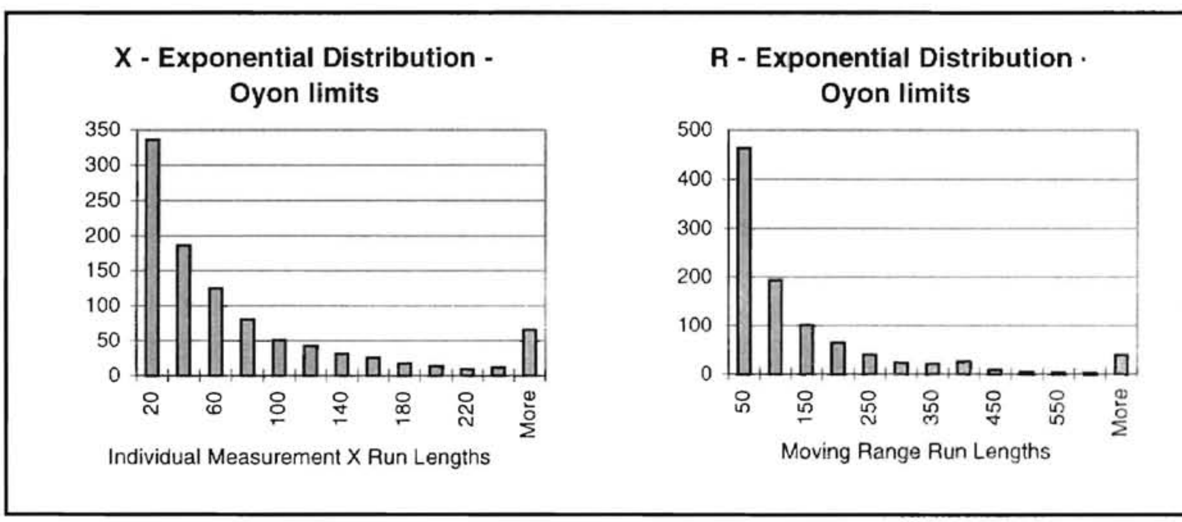

Figure 4-14: Exponential Distribution Run Lengths - Oyon Control Limits

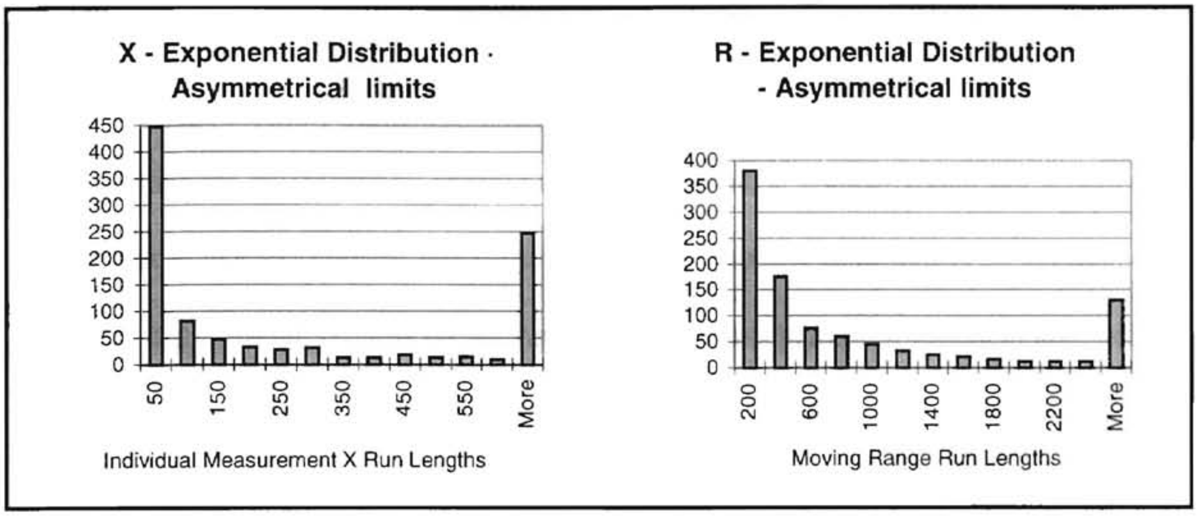

Figure 4-15: Exponential Distribution Run Lengths - Asymmetrical Control Limits

2.6: Analysis of the Normal Distribution; No Mean Shift:

Individual measurement X control chart limits:

The data in Table 4.3: Control Chart ARLs and VRLs for No Mean Shift indicate that the asymmetrical individual measurement X control limits are acceptable for practical use in industry when the underlying process distribution is normal and there is no shift in the process mean. An ARL that is acceptable for practical application in industry means that the average run length for the control chart is a minimum of 100 observations. Although the asymmetrical control limits are acceptable, they do not work as well as individual measurement X control limits produced by Shewhart or Oyon. When the underlying process distribution is normal and there is no shift in the process mean, the ARLs for Shewhart and Oyon individual measurement X control limits are approximately 1119 and 1427 observations, respectively. The asymmetrical control limits have an ARL of 141.9 observations.

Based on the histograms in Figures 4.1, 4.2, and 4.3 for individual measurement X control limits, the difference in the performance between the asymmetrical and symmetrical control limits is not so prevalent. The median run length for Shewhart limits is between 200 and 300 observations, Oyon is between 200 and 300 observations, and the asymmetrical limits are between 100 and 150. The differences in run lengths between the symmetrical and asymmetrical control charts are much smaller than when comparing ARLs. The median run length for the asymmetrical limits is only 150 to 200 observations less than that of the symmetrical limits. The symmetrical individual measurement X control limits perform better than those that are asymmetrical whether comparing ARLs or median RLs .

Moving range (n=2) mR control chart limits:

The data in Table 4.3 also indicate that the moving range (n=2) mR control chart limits based on the Pearson type III family of distributions work well when the underlying process distribution is Normal. The ARL for the asymmetrical control chart limits perform better than the Shewhart and Oyon limits. When the underlying process distribution is normal and there is no shift in the process mean, the ARLs for Shewhart and Oyon mR control limits are approximately 213 and 277 observations, respectively. The asymmetrical mR control limits have an ARL of 4012 observations.

The median RLs also indicate that the asymmetrical mR limits exceed the performance of the Shewhart and Oyon limits. From Figures 4.1, 4.2, and 4.3 for mR control limits, the median run length for Shewhart limits is between 75 and 125 observations, Oyon is approximately 100 observations, and the asymmetrical limits are between 500 and 700 observations. The differences in median run lengths between the symmetrical and asymmetrical control charts are smaller than when comparing ARLs. The median run length for the asymmetrical limits is 425 to 575 observations greater than that of the symmetrical limits. The asymmetrical mR control limits perform better than those that are symmetrical whether comparing ARLs or median RLs with no shift in the mean.

2.7: Analysis of Non-Normal Distributions; No Mean Shift:

The data in Table 4.3 for the log-normal, gamma, chi-square, and exponential distributions indicate that the asymmetrical control chart limits perform better than the Shewhart and Oyon limits when the underlying distribution is non-normal with no mean shift. The asymmetrical limits out-perform the other limits on both the individual measurement X and mR control charts. In all cases, the asymmetrical ARLs exceed 100 observations when there is no shift in the mean. For the highly skewed distributions, log-normal and exponential, the Shewhart and Oyon limits fall well short of 100 observations. In these cases, the Shewhart and Oyon individual measurement X limits have ARLs of 33 and 41 for the log-normal distribution and 50 and 78 observations for the exponential distribution.

The median run lengths follow the same pattern as the ARLs when there is no shift in the process mean. The median RLs for the asymmetrical limits exceed 100 observations in all but one instance. The median RL for the log-normal mR asymmetrical control limits falls between 75 and 105. The median run length comes very close to 100 but falls short. The median RLs for the Shewhart and Oyon limits are less than 30 observations when the underlying distribution is log-normal. The asymmetrical limits perform better than the symmetrical limits when there is no shift in the process mean, even though the median RL falls short of 100 observations.

Section 3: SUB-OBJECTIVE THREE

The third sub-objective is to compare the power of the individual measurement X and moving range n=2 mR control charts using the Pearson Type III c=0 asymmetrical control limits with those methods having symmetrical control limits. The power, in this case, refers to the ability of the control charts to detect shifts in process location of 0.5, 1.0, 1.5, 2.0, 2.5, and $3.0\sigma_X$ units.

To perform the third sub-objective, a Turbo Pascal (version 6.0) program was written to perform steps 1 through 5 of section 3.3 (Chapter 3). The Turbo Pascal program is the same program referred to in section 4.2, page 4-9. The program for the Chi-square (df = 4) distribution can be found in Appendix E and the program output for the Normal distribution is in Appendix G. The output consists of 1000 run lengths based on the individual measurement X and moving range n=2 mR control charts for Shewhart, Oyon's symmetrical, and Ankney's asymmetrical control limits at process mean shifts of 0.5, 1.0, 1.5, 2.0, 2.5, and $3.0\sigma_X$ units.

Average run lengths (ARLs) were calculated for each of the distributions according to the shift in the process mean. The ARLs can be found in Table 4.4: ARLs/VRLs for Shifts in th Process Mean on the following page.

Table 4-4: ARLs/VRLS for Shifts in the Process Mean

Ideal ARL = 1

Parent Distribution	Shift →	ARLs /VRLs for Shifts in the Process Mean +0.5	+1.0	+1.5	+2.0	+2.5	+3.0	-0.5	-1.0	-1.5	-2.0	-2.5	-3.0
Normal $(40, 10^2)$	Normal	437.5/ 1476.9	84.2/ 205.4	25.3/ 86.7	9.3/ 15.6	4.3/ 7.6	2.3/ 2.3	402.2/ 1490.9	91.1/ 230.1	25.4/ 63.9	8.3/ 12.9	4.1/ 5.6	2.2/ 2.1
	Symmetrical	485.9/ 1582.4	93.3/ 217.3	28.3/ 104.6	9.7/ 16.1	4.5/ 7.7	2.4/ 2.9	450.6/ 1571.6	98.0/ 239.0	28.4/ 72.4	9.9/ 31.8	4.3/ 6.1	2.3/ 3.0
	Asymmetrical	518.2/ 1059.6	1155.9/ 4846.4	764.0/ 6946.3	122.9/ 698.4	25.3/ 106.5	10.6/ 59.3	39.2/ 66.7	12.1/ 14.8	5.2/ 5.7	2.8/ 2.6	1.7/ 1.2	1.3/ 0.7
Log-normal $(0, 1^2)$	Normal	13.2/ 18.6	4.3/ 7.4	1.5/ 2.4	1.0/ 0.4	1.0/ 0.0	1.0/ 0.0	58.7/ 82.4	19.2/ 99.9	2.1/ 31.8	1.0/ 0.2	1.0/ 0.1	1.0/ 0.1
	Symmetrical	18.1/ 31.7	6.4/ 11.3	2.1/ 4.2	1.2/ 1.4	1.0/ 0.2	1.0/ 0.0	76.7/ 110.5	54.6/ 203.9	12.1/ 119.9	2.0/ 31.8	1.0/ 0.1	1.0/ 0.1
	Asymmetrical	96.5/ 214.6	54.3/ 170.8	26.3/ 94.0	12.7/ 51.0	5.5/ 28.9	2.8/ 21.7	1.2/ 0.6	1.1/ 0.2	1.0/ 0.2	1.0/ 0.1	1.0/ 0.1	1.0/ 0.0
Gamma (1.5, 1)	Normal	33.9/ 49.5	21.0/ 31.7	11.9/ 17.2	6.9/ 10.8	4.1/ 5.2	2.5/ 3.3	103.4/ 168.9	185.5/ 311.7	255.8/ 500.0	192.0/ 851.4	79.3/ 834.4	4.7/ 96.7
	Symmetrical	45.9/ 67.1	28.5/ 45.0	16.6/ 25.0	9.6/ 15.0	5.5/ 9.5	3.3/ 4.4	147.1/ 254.3	265.3/ 463.3	432.8/ 884.6	459.6/ 1291.4	362.9/ 2165.1	229.5/ 2470.2
	Asymmetrical	1059.0/ 3806.4	639.6/ 2649.8	349.0/ 1381.9	194.2/ 733.4	117.2/ 538.9	65.0/ 268.3	5.8/ 6.6	2.1/ 1.6	1.5/ 0.8	1.2/ 0.5	1.1/ 0.4	1.1/ 0.2
Chi-square (df = 4)	Normal	40.2/ 64.6	24.9/ 43.1	13.8/ 24.9	7.4/ 11.5	4.3/ 5.1	2.6/ 3.1	138.1/ 239.6	254.5/ 434.2	327.9/ 710.3	218.3/ 942.4	95.1/ 1282.51	1.7/ 1.4
	Symmetrical	52.5/ 86.4	30.1/ 49.1	18.2/ 33.2	10.2/ 19.8	5.6/ 8.2	3.4/ 4.4	183.4/ 310.7	322.7/ 542.5	545.3/ 1093.0	605.7/ 2146.5	413.0/ 2983.5	120.5/ 2633.2
	Asymmetrical	1237.8/ 7071.7	525.4/ 2225.3	309.3/ 1915.1	160.4/ 765.8	91.1/ 460.3	53.0/ 419.4	15.5/ 57.9	2.9/ 2.6	1.7/ 1.1	1.3/ 0.7	1.2/ 0.5	1.1/ 0.3
Exponential (1)	Normal	29.2/ 46.4	18.6/ 35.3	10.7/ 15.5	6.6/ 10.1	4.1/ 5.8	2.5/ 3.5	82.0/ 131.4	136.3/ 206.4	190.2/ 354.5	167.1/ 553.9	69.1/ 476.0	5.4/ 123.9
	Symmetrical	46.0/ 75.4	28.1/ 49.4	17.4/ 36.0	10.4/ 17.0	6.3/ 10.4	3.8/ 5.8	125.2/ 206.3	212.2/ 356.3	345.6/ 614.3	419.1/ 841.1	403.0/ 1290.1	226.0/ 1567.3
	Asymmetrical	1358.5/ 4947.8	872.5/ 3954.3	512.8/ 1503.8	297.4/ 796.3	192.0/ 602.2	122.8/ 400.1	2.6/ 2.1	1.6/ 1.0	1.3/ 0.6	1.2/ 0.4	1.1/ 0.4	1.1/ 0.2

3.1: Analysis of Negative shifts in the mean (shifts to the left):

The data in Table 4.4: ARLs/VRLs for Shifts in the Process Mean, indicate that the asymmetrical individual measurement X and mR control limits do a very good job of detecting negative shifts in the process mean. Even at a very small shift of -0.5 standard deviations, the asymmetrical control limits are very sensitive to the detection of shifts. Regardless of the underlying distribution, the asymmetrical limits detect a -0.5 standard deviation in less than 40 observations, a -1.0 shift in less than 13 observations, and a - 1.5 shift in less than 6 observations. For large shifts in the process mean of -2.5 and -3.0 standard deviations, the asymmetrical limits detect the shift within the first two observations. The asymmetrical limits are much more sensitive to negative shifts than the symmetrical limits. It can be concluded that the asymmetrical control limits do a good job of detecting negative shifts in the mean regardless of the underlying distribution.

3.2: Analysis of Positive shifts in the mean (shifts to the right):

The data in Table 4.4: ARLs/VRLs for Shifts in the Process Mean, indicate that the asymmetrical individual measurement X and mR control limits do not do a good job of detecting positive shifts in the process mean. The symmetrical control limits are more sensitive to detecting positive shifts. The only underlying distribution for which the asymmetrical limits appear to be effective in detecting positive shifts is the log-normal distribution. Although the asymmetrical limits appear to be moderately effective in this case, the symmetrical limits still perform better.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

Section 1: CONCLUSIONS & RECOMMENDATIONS

This section consists of conclusions and recommendations for this thesis research. Using the results and analysis generated in Chapter 4, the performance of the asymmetrical control limits are compared to that of Shewhart's and Oyon's control chart limits. The following conclusions are made based on the information in Tables 4.3, Table 4.4, and the analysis in Chapter 4, sections 2.6, 2.7, 3.1, and 3.2.

- The performance of the individual measurement X symmetrical control charts is much better than that of the asymmetrical charts when the underlying distribution is normal and there is no shift in the mean. This conclusion is supported in Chapter 3, section 2.6: *Individual measurement X control chart limits,* page 4-21.
- The performance of the moving range (n=2) asymmetrical control charts is much better than that of the symmetrical charts when the underlying distribution is normal and there is no shift in the mean. This conclusion is supported in Chapter 3, section 2.6: *Moving range (n=2) mR control chart limits,* page 4-22.
- The performance of the asymmetrical control charts is better than that of the symmetrical charts when the underlying distribution is non-normal and there is no shift in the mean. This conclusion is supported in Chapter 3, section 2.7, page 4-23.
- The performance of the asymmetrical control charts is better than that of the symmetrical charts when there is a negative shift in the mean, regardless of the

underlying distribution. This conclusion is supported in Chapter 3, section 3.1, page 4-26.

- The asymmetrical control charts do not do a good job of detecting positive shifts in the mean regardless of the underlying distribution. This conclusion is supported in Chapter 3, section 3.2, page 4-26.

The asymmetrical limits perform well when there is no mean shift and the underlying distribution is non-normal. The problem with the asymmetrical control charts is that they do not do a good job of detecting positive shifts in the process mean. In general, control charts for skewed distributions are most useful for detecting positive shifts in the mean. According Irving Burr (1953), "...causes of non-normality is that the distribution may be unable to go beyond a certain point, such as zero (5, p.80)..." As indicated by this statement, negative shifts in the mean will not occur because the inability to go beyond this point (zero in this research). Shifts in the mean will, in most cases, be positive. Based on the conclusion that the asymmetrical control limits do not do a good job of detecting positive shifts in the mean, the author recommends the asymmetrical control limits developed in this research not be used.

Section 2: RESEARCH CONTRIBUTIONS

- This thesis research provides empirical equations to calculate approximately the correct asymmetrical control chart constants t_1, t_2, and t_3 when the underlying process distribution is a gamma distribution with shape parameter α and scale parameter β.
- This thesis research provides empirical evidence that the asymmetrical gamma control charts (X and mR) perform better than the normal curve and symmetrical gamma control charts (X and mR) when the distribution has a marked departure from normality (represented in this research by skewed distributions) and there is no shift in the mean. However, more research is needed in this area since the asymmetrical control charts lack the power to detect positive shifts in the process mean. In this regard, this research opens avenues for future research providing improved methodology for setting control limits (X and mR) under skewed circumstances.

Section 3: FUTURE RESEARCH

The fact that the asymmetrical control charts lack the power to detect positive shifts in the process mean suggests that more research is needed in this area. It is the author's belief that the inability to detect positive shifts in the mean is due to the following three factors:

1. The use of 0.00135 of the observations falling outside the upper or lower control limits when setting those limits, regardless of the skew of the underlying distributions.

2. The empirical nature of the study (Number of observations).

3. The ability to accurately estimate the parameters α and β from the unknown underlying distributions.

Additional research is recommended in setting asymmetrical control limits (X and mR) based on the method for determining the location of the upper and lower control limits. The upper and lower control limits in this research are determined based on 0.00135 of the observations falling beyond each limit; regardless of the skew of the underlying distributions. The upper and lower control limits can be determined by varying the percent of outlying observations with the shape parameter α. When the underlying distribution is skewed, a higher percentage can be allotted to the upper control limit so that it is not set so far out on the tail; meanwhile, a lower percentage can be allotted to the lower control limit since the process will not produce values less than a specified lower value. For example, when the distribution is exponential, set the lower bound at 0.0000 and use all 0.0027 on the upper limit. The control chart will not be as sensitive to negative shifts in the process mean as the limits in this research but will become more sensitive to positive mean shifts.

This approach can be demonstrated with the theoretical run lengths for the exponential distribution under the cases where limits are one-sided and based on false alarm rates of 0.0027, 0.0050, and 0.0100 when there is no shift in the process mean. Table 5.1: Theoretical Run Lengths for Exponential Distribution on the following page demonstrates the theoretical ARLs of these limits. As seen Table 5.1, the one-sided control limits perform much better than the two-sided limits developed in this research. The one-sided limits have the power to detect positive shifts in the mean while maintaining an acceptable false alarm rate when no mean shift is present.

The one-sided asymmetrical control limits detect shifts in the process mean better than the two-sided asymmetrical control limits. The one-sided limits, however, do not detect shifts in the process mean as well as the symmetrical control limits. As can be seen from Tables 5.1 and 5.2, at a sigma shift of 3.0, the theoretical ARL for the one-sided asymmetrical control limits is 4.98 while the symmetrical control limits pick up the shift in 2.72. Although the symmetrical control limits perform better than the one-sided symmetrical control limits when the underlying distribution is exponential, both the symmetrical and asymmetrical control limits have good performance.

The one-sided asymmetrical control limits have a much better false alarm rate than the symmetrical limits. Based on the criteria defined in this research, the symmetrical control limits are not acceptable for practical use in industry because the run

Table 5-1: Theoretical Run Lengths for Exponential Distribution

Positive	Set Limits @ 0.00135			Set Upper Limits @ 0.0027	
Shift	Upper	Lower	ARL	Upper	ARL
0.0	0.001350	0.001350	370.37	0.002700	370.37
0.5	0.002225	0.000000	449.44	0.004452	224.62
1.0	0.003668	0.000000	272.63	0.007339	136.26
1.5	0.006048	0.000000	165.34	0.012101	82.64
2.0	0.009972	0.000000	100.28	0.019951	50.12
2.5	0.016441	0.000000	60.82	0.032893	30.40
3.0	0.027106	0.000000	36.89	0.054231	18.44

Positive	Set Upper Limit @ 0.005		Set Upper Limit @ 0.010	
Shift	Upper	ARL	Upper	ARL
0.0	0.005000	200.00	0.010000	100.00
0.5	0.008243	121.32	0.016487	60.65
1.0	0.013550	73.80	0.027182	36.79
1.5	0.022407	44.63	0.044816	22.31
2.0	0.036942	27.07	0.073888	13.53
2.5	0.060907	16.42	0.121821	8.21
3.0	0.100419	9.96	0.200849	4.98

lengths for the symmetrical limits are less than 100. Based on the theoretical ARL, the asymmetrical limits do have a run length of 100.

There is a tradeoff between the asymmetrical and symmetrical control limits. The tradeoff is a matter of economics. Compared to the symmetrical control limits, the asymmetrical control limits do not have as much power but do have a smaller false alarm rate. If the cost of defects is significantly larger than the cost of readjusting the process mean, than a higher false alarm rate would be more desirable than the inability to detect a shift in the mean. In this case the symmetrical control limits would be more desirable. If the cost of readjusting the process mean involves a much higher cost than the cost of defects, a lower false alarm rate would be more desirable than the power to detect a shift. In this case the one-sided asymmetrical control limits are more desirable. The selection and use of the control limits is dependent on the economics of the process.

The empirical nature of this research also affects the results. The control limits for this research are set on fifty observations per run. Using such a small number of observations creates variation in the control limits which generates ARLs that are not representative of those dictated by theory. This is apparent by comparing the theoretical results in Table 5.1 to the results shown in Table 4.4: ARLs for Shifts in the Process Mean. As can be seen, the run lengths generated in this research are much higher than what theory states.

Increasing the number of observations used in setting control limits improves the performance of the control charts under shifts in the process mean. Table 5.2: ARLs For Control Limits Set On Different Number Of Observations demonstrates the ability to produce control limits which are more representative of the underlying distribution by increasing the number of observations. Table 5.2 consists of ARLs for the exponential distribution when control limits are based on 50, 100, 500, and 1000 observations. By observation, it can be seen that increasing the number of observations greatly improves the performance of the control charts.

Table 5-2: ARLs For Control Limits Set On Different Number Of Observations

Shift >	0.0			+ 0.5		
Observations	Shewhart	Oyon	Asymmetrical	Shew	Oyon	Asymm
50	49.72	77.85	1758.01	29.24	46.00	1358.52
100	44.13	62.74	859.88	26.56	40.38	784.67
500	39.54	55.74	490.14	24.86	35.10	541.75
1000	40.06	56.78	422.14	23.14	35.10	509.04
theoretical	54.60	54.60	370.34	33.12	33.12	449.44

Shift >	+ 1.0			+ 1.5		
Observations	Shew	Oyon	Asymm	Shew	Oyon	Asymm
50	18.62	28.08	872.54	10.68	17.36	512.77
100	16.01	24.48	471.82	9.33	14.60	290.45
500	15.05	21.99	329.31	8.91	12.96	205.68
1000	14.65	20.95	307.01	8.91	12.98	187.19
theoretical	20.089	20.089	272.63	12.18	12.18	165.34

Shift >	+ 2.0			+ 2.5		
Observations	Shew	Oyon	Asymm	Shew	Oyon	Asymm
50	6.60	10.42	297.36	4.05	6.33	191.99
100	5.73	8.56	185.03	3.64	5.27	107.08
500	5.34	7.88	122.05	3.32	4.79	70.30
1000	5.28	7.76	114.83	3.14	4.65	67.83
theoretical	7.39	7.39	100.28	4.48	4.48	60.82

Shift >	+ 3.0		
Observations	Shew	Oyon	Asymm
50	2.52	3.79	122.77
100	2.19	3.23	67.81
500	1.96	2.96	43.13
1000	1.95	2.76	42.95
theoretical	2.72	2.72	36.89

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APPENDIX A

The Scale Parameter and the Range

THE SCALE PARAMETER AND THE RANGE

The scale parameter (β) for the Gamma distribution does not have an observable effect on the (t_3) value for the range n = 2 as concerned with this research. This is demonstrated in the chart below. To demonstrate this claim, upper control limits for the moving range n=2 mR control chart are generated at particular α (shape parameter) values for β (scale parameter) = 1, 2, and 5. This is accomplished by generating two rows of k = 10,000 observations. The ranges are calculated and the upper control limit is progressively increased by a value of 0.0001 until 0.0027 of the values are outside the upper control limit. This is done for each β value at a corresponding α and the random numbers are generated from the same generator base to demonstrate if β (scale parameter) has any effect on the ranges. The results are expressed as multiples of the standard deviation of the ranges from the mean (t). The results are in Table A-1 on the following page.

The data in Table A-1 demonstrates that β (scale parameter) has no observable effect on the control limit multiplier (t_3). The same random number generating base is used when testing different β values. Using the same base means that the uniform variates used to calculate the variates for the parent distribution are the same on all cases of β = 1, 2, and 5. When using the same random number generating base, the (t_3) value is the same regardless of the scale parameter.

Table A-1: Range Upper Control Limits as a Multiple (t_3) of the Standard Deviation

α (shape parameter)	Random # Base	Range Upper Control Limits as a Multiple (t_3) of the Standard Deviation		
		$\beta = 1$	$\beta = 2$	$\beta = 5$
1	2,500	5.1558	5.1558	5.1558
10	55,000	3.9308	3.9308	3.9308
25	100,000	3.9452	3.9452	3.9452
50	13,250	3.6883	3.6883	3.6883
75	20,000	3.524	3.524	3.524
100	81,096	3.8628	3.8628	3.8628

APPENDIX B

Sample Size Estimation

SAMPLE SIZE ESTIMATION

In order to create a mathematical model for t_3, random numbers are generated and a point found which leaves 0.0027 of the values outside the upper control limit. When evaluating data in this manner it is important that an appropriate sample size be used which provides adequate confidence of representative data values at the extremes of the tail of the distribution.

The parent distributions are those for the range n = 2 of the gamma distribution. Numbers randomly generated from this distribution result in a number of discrete points that can be approximated by the binomial distribution. According to Miller et al. (p. 274), the sample size is approximated by the following equation:

$$n = p(1-p)[\frac{z_{\alpha/2}}{E}]^2$$

where n is the sample size, p = binomial parameter, and E = maximum error of the estimate.

This formula cannot be used without a value of p. Since no data is available concerning the p value, it will be assumed that p*(1-p) = 0.25. The value p*(1-p) = 0.25 is chosen because it is the largest possible value that p*(1-p) can take on because $0 <= p <= 1$. The distribution is also evaluated for only one tail, so $\alpha/2$ becomes α. The substitution yields the equation:

$$n = 0.25[\frac{z_{\alpha}}{E}]^2$$

The value (E) for this portion of the research is equal to 0.0027 and α is set at 0.90. The corresponding z-value for 0.90 is 1.282. The resulting sample size is as follows:

$$n = 0.25[\frac{1.282}{0.0027}]^2 = 56,362$$

The sample size is rounded up to n = 60,000 for use in this research.

APPENDIX C

T Statistic Table

TABLE 6.19
$\hat{\alpha}$ as a function of T, gamma distribution

T	$\hat{\alpha}$	T	$\hat{\alpha}$	T	$\hat{\alpha}$	T	$\hat{\alpha}$
0.01	0.010	1.40	0.827	5.00	2.655	13.00	6.662
0.02	0.019	1.50	0.879	5.20	2.755	13.50	6.912
0.03	0.027	1.60	0.931	5.40	2.856	14.00	7.163
0.04	0.036	1.70	0.983	5.60	2.956	14.50	7.413
0.05	0.044	1.80	1.035	5.80	3.057	15.00	7.663
0.06	0.052	1.90	1.086	6.00	3.157	15.50	7.913
0.07	0.060	2.00	1.138	6.20	3.257	16.00	8.163
0.08	0.068	2.10	1.189	6.40	3.357	16.50	8.413
0.09	0.076	2.20	1.240	6.60	3.458	17.00	8.663
0.10	0.083	2.30	1.291	6.80	3.558	17.50	8.913
0.11	0.090	2.40	1.342	7.00	3.658	18.00	9.163
0.12	0.098	2.50	1.393	7.20	3.759	18.50	9.414
0.13	0.105	2.60	1.444	7.40	3.859	19.00	9.664
0.14	0.112	2.70	1.495	7.60	3.959	19.50	9.914
0.15	0.119	2.80	1.546	7.80	4.059	20.00	10.164
0.16	0.126	2.90	1.596	8.00	4.159	20.50	10.414
0.17	0.133	3.00	1.647	8.20	4.260	21.00	10.664
0.18	0.140	3.10	1.698	8.40	4.360	21.50	10.914
0.19	0.147	3.20	1.748	8.60	4.460	22.00	11.164
0.20	0.153	3.30	1.799	8.80	4.560	22.50	11.414
0.30	0.218	3.40	1.849	9.00	4.660	23.00	11.664
0.40	0.279	3.50	1.900	9.20	4.760	23.50	11.914
0.50	0.338	3.60	1.950	9.40	4.860	24.00	12.164
0.60	0.396	3.70	2.001	9.60	4.961	24.50	12.414
0.70	0.452	3.80	2.051	9.80	5.061	25.00	12.664
0.80	0.507	3.90	2.101	10.00	5.161	30.00	15.165
0.90	0.562	4.00	2.152	10.50	5.411	35.00	17.665
1.00	0.616	4.20	2.253	11.00	5.661	40.00	20.165
1.10	0.669	4.40	2.353	11.50	5.912	45.00	22.665
1.20	0.722	4.60	2.454	12.00	6.162	50.00	25.166
1.30	0.775	4.80	2.554	12.50	6.412		

APPENDIX D

Regression Output for Control Limits

Regression models were generated for the upper and lower control limits on the individual measurement X control chart and the upper control limit for the moving range, n = 2, mR control chart. The regression models were generated from the statistical software package Minitab for Windows release 10.5.

The statistical software package was used to generate different multiple regression models for predicting t_1, t_2, and t_3 with predictors α (shape parameter) and β (scale parameter). The output from Minitab is on the following pages.

Upper Tail Regression for X-Chart

Regression Analysis

```
* NOTE *  Alpha^2 is highly correlated with other  predictor
variables
* NOTE *  Alpha^3 is highly correlated with other  predictor
variables

The regression equation is
SigmaHi = 4.81 - 0.0655 Alpha +0.000855 Alpha^2 -0.000003
Alpha^3

Predictor         Coef        Stdev    t-ratio       p
Constant        4.8136       0.1603      30.02   0.000
Alpha         -0.06546      0.01047      -6.25   0.000
Alpha^2      0.0008545    0.0001813       4.71   0.000
Alpha^3    -0.00000345   0.00000088      -3.92   0.001

s = 0.2354      R-sq = 77.7%     R-sq(adj) = 74.9%

Analysis of Variance

SOURCE        DF          SS          MS         F        p
Regression     3      4.6275      1.5425     27.82    0.000
Error         24      1.3305      0.0554
Total         27      5.9579

SOURCE        DF      SEQ SS
Alpha          1      2.3930
Alpha^2        1      1.3816
Alpha^3        1      0.8528

Unusual Observations
Obs.    Alpha    SigmaHi        Fit  Stdev.Fit   Residual
St.Resid
  1       135     3.2305     3.0647     0.1569     0.1658
0.94 X
 28         1     5.6080     4.7490     0.1518     0.8590
4.77R

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.
```

Regression Analysis

```
The regression equation is
SigmaHi = 4.42 - 0.0280 Alpha +0.000154 Alpha^2

Predictor         Coef        Stdev    t-ratio       p
```

Constant	4.4222	0.1575	28.08	0.000
Alpha	-0.028016	0.005402	-5.19	0.000
Alpha^2	0.00015354	0.00003860	3.98	0.001

s = 0.2955   R-sq = 63.4%   R-sq(adj) = 60.4%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	3.7746	1.8873	21.61	0.000
Error	25	2.1833	0.0873		
Total	27	5.9579			

SOURCE	DF	SEQ SS
Alpha	1	2.3930
Alpha^2	1	1.3816

Unusual Observations

Obs.	Alpha	SigmaHi	Fit	Stdev.Fit	Residual	St.Resid
28	1	5.6080	4.3943	0.1530	1.2137	4.80R

R denotes an obs. with a large st. resid.

Regression Analysis

The regression equation is
SigmaHi = 3.97 - 0.00725 Alpha

Predictor	Coef	Stdev	t-ratio	p
Constant	3.9697	0.1365	29.08	0.000
Alpha	-0.007249	0.001735	-4.18	0.000

s = 0.3703   R-sq = 40.2%   R-sq(adj) = 37.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	2.3930	2.3930	17.45	0.000
Error	26	3.5649	0.1371		
Total	27	5.9579			

Unusual Observations

Obs.	Alpha	SigmaHi	Fit	Stdev.Fit	Residual	St.Resid
28	1	5.6080	3.9624	0.1350	1.6456	4.77R

R denotes an obs. with a large st. resid.

Regression Analysis

The regression equation is
SigmaHi = 3.40 + 6.04 e^-1

Predictor	Coef	Stdev	t-ratio	p
Constant	3.39926	0.04135	82.20	0.000
e^-1	6.0420	0.5947	10.16	0.000

s = 0.2147 R-sq = 79.9% R-sq(adj) = 79.1%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	4.7591	4.7591	103.22	0.000
Error	26	1.1988	0.0461		
Total	27	5.9579			

Unusual Observations

Obs.	e^-1	SigmaHi	Fit	Stdev.Fit	Residual	St.Resid
26	0.000	3.8505	3.3995	0.0413	0.4510	2.14R
27	0.007	4.2005	3.4400	0.0408	0.7605	3.61R
28	0.368	5.6080	5.6220	0.2147	-0.0140	3.62RX

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.

Regression Analysis

The regression equation is
SigmaHi = 3.23 + 3.19 e^-1 + 0.852 e^-.1 + 0.442 e^-.025

Predictor	Coef	Stdev	t-ratio	p
Constant	3.23311	0.00576	561.20	0.000
e^-1	3.19102	0.08362	38.16	0.000
e^-.1	0.85168	0.04487	18.98	0.000
e^-.025	0.44197	0.02487	17.77	0.000

s = 0.01691 R-sq = 99.9% R-sq(adj) = 99.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	5.9511	1.9837	6935.38	0.000

Error	24	0.0069	0.0003
Total	27	5.9579	

SOURCE	DF	SEQ SS
e^-1	1	4.7591
e^-.1	1	1.1016
e^-.025	1	0.0903

Unusual Observations

Obs.	e^-1	SigmaHi	Fit	Stdev.Fit	Residual	St.Resid
25	0.000	3.69400	3.72691	0.00668	-0.03291	-2.12R
26	0.000	3.85050	3.89078	0.00836	-0.04028	-2.74R
27	0.007	4.20050	4.16122	0.01443	0.03928	4.45RX
28	0.368	5.60800	5.60871	0.01691	-0.00071	-4.46RX

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.

Lower Tail Regression for X-Chart

Regression Analysis

The regression equation is
SigmaLo = 2.77 - 1.81 e^-1 - 0.751 e^-.1 - 0.438 e^-.025

Predictor	Coef	Stdev	t-ratio	p
Constant	2.77031	0.00504	549.96	0.000
e^-1	-1.80838	0.07312	-24.73	0.000
e^-.1	-0.75091	0.03923	-19.14	0.000
e^-.025	-0.43839	0.02175	-20.16	0.000

s = 0.01479   R-sq = 99.9%   R-sq(adj) = 99.8%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	3	3.5856	1.1952	5465.84	0.000
Error	24	0.0052	0.0002		
Total	27	3.5908			

SOURCE	DF	SEQ SS
e^-1	1	2.5437
e^-.1	1	0.9531
e^-.025	1	0.0888

```
Unusual Observations
Obs.      e^-1       SigmaLo          Fit   Stdev.Fit    Residual
St.Resid
 25      0.000      2.32970     2.30147     0.00584     0.02823
2.08R
 26      0.000      2.18695     2.15257     0.00731     0.03438
2.67R
 27      0.007      1.88195     1.91580     0.01261    -0.03385
4.39RX
 28      0.368      0.99865     0.99803     0.01479     0.00062
4.40RX

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.
```

Upper Tail Regression for mR-Chart

```
Regression Analysis


The regression equation is
```

T = 3.68 + 1.88 exp(-a) + 0.564 exp(-.1a) + 0.0969 exp(-.025)

Predictor	Coef	Stdev	t-ratio	p
Constant	3.68490	0.00483	763.64	0.000
exp(-a)	1.88402	0.07004	26.90	0.000
exp(-.1a	0.56355	0.03758	14.99	0.000
exp(-.02	0.09686	0.02083	4.65	0.000

```
s = 0.01417      R-sq = 99.7%      R-sq(adj) = 99.7%

Analysis of Variance
```

SOURCE	DF	SS	MS	F	p
Regression	3	1.74153	0.58051	2892.98	0.000
Error	24	0.00482	0.00020		
Total	27	1.74635			

SOURCE	DF	SEQ SS
exp(-a)	1	1.49678
exp(-.1a	1	0.24041
exp(-.02	1	0.00434

```
Unusual Observations
Obs.  exp(-a)            T          Fit   Stdev.Fit    Residual
St.Resid
  1      0.368      4.98260     4.98238     0.01416     0.00022
1.66 X
  2      0.007      4.11260     4.12488     0.01208    -0.01228
1.66 X
```

```
 20     0.000    3.66030    3.69395    0.00350   -0.03365        -
2.45R
 27     0.000    3.66140    3.68865    0.00422   -0.02725        -
2.02R
```

R denotes an obs. with a large st. resid.
X denotes an obs. whose X value gives it large influence.

APPENDIX E

Chi-square Distribution Turbo Pascal (version 6.0) Program

A Turbo Pascal (version 6.0) program was written to perform steps 2 through 13 of section 3.2 and steps 1 through 5 of section 3.3. The program on the following pages is the one used to generate run lengths for the Chi-square (df = 4) distribution.

```
Program qctest(chisq);
uses crt,printer;
var
  w,d,t,z,aaa,bbb,u1,u2,q,v,theta, x, mr, xsum, mrsum, lnsum, sigma,
  prev, xbar, mrbar, tstat, alpha, uclxs, lclxs, uclrs, lclrs,
  uclxo, lclxo, uclro, uclxa, lclxa, uclra, uniform, aa, uni,
  earl, y, yy, pill, s: real;

  dtwo, dthree, dfour, tone, ttwo, tthree: real;

  chisq:text;

  sx0, sxc0, qx, count, sr0, src0, ox0, oxc0, or0, orc0, ax0, axc0, ar0, arc0,
  sx5r, sxc5r, ox5r, oxc5r, ax5r, axc5r,
  sx1r, sxc1r, ox1r, oxc1r, ax1r, axc1r,
  sx15r, sxc15r, ox15r, oxc15r, ax15r, axc15r,
  sx2r, sxc2r, ox2r, oxc2r, ax2r, axc2r,
  sx25r, sxc25r, ox25r, oxc25r, ax25r, axc25r,
  sx3r, sxc3r, ox3r, oxc3r, ax3r, axc3r,
  sx5l, sxc5l, ox5l, oxc5l, ax5l, axc5l,
  sx1l, sxc1l, ox1l, oxc1l, ax1l, axc1l,
  sx15l, sxc15l, ox15l, oxc15l, ax15l, axc15l,
  sx2l, sxc2l, ox2l, oxc2l, ax2l, axc2l,
  sx25l, sxc25l, ox25l, oxc25l, ax25l, axc25l,
  sx3l, sxc3l, ox3l, oxc3l, ax3l, axc3l,
  a, b,number, seed:longint;

  zrng: array[1..100] of longint;
  zset: array[1..100] of longint;

 function rand:real;

      const
       b2e15=32768;
       b2e16=65536;
       modlus=2147483647;
       mult1=24112;
       mult2=26143;

    var
      hi15, hi31, low15, lowprd, ovflow, zi: longint;

    begin {rand}
        {generate the next random number}

        zi:=seed;
        hi15:=zi div b2e16;
        lowprd:=(zi-hi15*b2e16)*mult1;
        low15:=lowprd div b2e16;
        hi31:=hi15*mult1+low15;
        ovflow:=hi31 div b2e15;
        zi:=(((lowprd-low15*b2e16)-modlus)+(hi31-ovflow*b2e15)*b2e16)+ovflow;
        if zi<0 then zi:=zi+modlus;
        hi15:=zi div b2e16;
        lowprd:=(zi-hi15*b2e16)*mult2;
```

```
        low15:=lowprd div b2e16;
        hi31:=hi15*mult2+low15;
        ovflow:=hi31 div b2e15;
        zi:=(((lowprd-low15*b2e16)-modlus)+(hi31-ovflow*b2e15)*b2e16)+ovflow;
        if zi<0 then zi:=zi+modlus;
        seed:=zi;
        rand:=(2*(zi div 256)+1)/ 16777216.0

    end; {rand}

{**************************************************************
**************************************************************
**************************************************************}
procedure chisquare;

     begin {chi square}
        u1:=rand;
        u2:=rand;
        aaa:=1/(sqrt(3));
        bbb:=2-(ln(4));
        q:=2+(1/aaa); {???????????}
        theta:=4.5;
        d:=1+ln(theta);
        v:=aaa*ln(u1/(1-u1));
        yy:=2*exp(v);
        z:=u1*u1*u2;
        w:=bbb+q*v-yy;
        if (w+d-(theta*z))>=0 then
          y:=2*yy
        else if w>=ln(z) then
          y:=2*yy
        else chisquare;
     end;

      {*********************************************}

   procedure Generate;

    begin {generate}
    chisquare;
    number:=number+1;

    if sx0=0 then    {check for ooc signal on shewhart limits x chart
                 if there is a signal sx0=1 and there it will
                 stop counting on sxc0. sxc0 will be at least one}
      begin {sxc0}
          sxc0:=sxc0+1;
          if y<lclxs then sx0:=1
          else if y>uclxs then sx0:=1;
       end; {sxc0}

    if number>1 then  {check for ooc signal on shewhart limits mr chart
                 if there is a signal sr0=1 and there it will
                 stop counting on srxc0. srxc0 will be at least one}
      if sr0=0 then
```

```
      begin {src0}
          src0:=src0+1;
          if abs(y-earl)>uclrs then
            sr0:=1
        end; {src0}

   if ox0=0 then    {check for ooc signal on Oyon limits x chart
                 if there is a signal ox0=1 and there it will
                 stop counting on oxc0. oxc0 will be at least one}
    begin {oxc0}
        oxc0:=oxc0+1;
        if y<lclxo then ox0:=1
        else if y>uclxo then ox0:=1;
     end; {oxc0}

  if number>1 then
    if or0=0 then
      begin {orc0}
          orc0:=orc0+1;
          if abs(y-earl)>uclro then
            or0:=1
        end; {orc0}

   if ax0=0 then
    begin {axc0}
        axc0:=axc0+1;
         if y<lclxa then ax0:=1
         else if y>uclxa then ax0:=1;
    end; {axc0}

  if number>1 then
    if ar0=0 then
      begin {arc0}
        arc0:=arc0+1;
          if abs(y-earl)>uclra then
            ar0:=1
       end; {arc0}
{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

  if sx5r=0 then
  begin {sxc5r}
        sxc5r:=sxc5r+1;
        if (y+0.5*s)<lclxs then sx5r:=1
        else if (y+0.5*s)>uclxs then sx5r:=1;
    end; {sxc5r}

 if ox5r=0 then
    begin {oxc5r}
        oxc5r:=oxc5r+1;
        if (y+0.5*s)<lclxo then ox5r:=1
        else if (y+0.5*s)>uclxo then ox5r:=1;
    end; {oxc5r}

  if ax5r=0 then
    begin {axc5r}
```

```
      axc5r:=axc5r+1;
        if (y+0.5*s)<lclxa then ax5r:=1
        else if (y+0.5*s)>uclxa then ax5r:=1;
   end; {axc5r}
{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

 if sx1r=0 then
 begin {sxc1r}
      sxc1r:=sxc1r+1;
      if (y+s)<lclxs then sx1r:=1
      else if (y+s)>uclxs then sx1r:=1;
    end; {sxc1r}

if ox1r=0 then
   begin {oxc1r}
      oxc1r:=oxc1r+1;
      if (y+s)<lclxo then ox1r:=1
      else if (y+s)>uclxo then ox1r:=1;
    end; {oxc1r}

 if ax1r=0 then
   begin {axc1r}
      axc1r:=axc1r+1;
        if (y+s)<lclxa then ax1r:=1
        else if (y+s)>uclxa then ax1r:=1;
   end; {axc1r}
   {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

 if sx15r=0 then
 begin {sxc15r}
      sxc15r:=sxc15r+1;
      if (y+1.5*s)<lclxs then sx15r:=1
      else if (y+1.5*s)>uclxs then sx15r:=1;
    end; {sxc15r}

if ox15r=0 then
   begin {oxc15r}
      oxc15r:=oxc15r+1;
      if (y+1.5*s)<lclxo then ox15r:=1
      else if (y+1.5*s)>uclxo then ox15r:=1;
    end; {oxc15r}

 if ax15r=0 then
   begin {axc15r}
      axc15r:=axc15r+1;
        if (y+1.5*s)<lclxa then ax15r:=1
        else if (y+1.5*s)>uclxa then ax15r:=1;
   end; {axc15r}
 {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

 if sx2r=0 then
 begin {sxc2r}
      sxc2r:=sxc2r+1;
      if (y+2*s)<lclxs then sx2r:=1
      else if (y+2*s)>uclxs then sx2r:=1;
```

```
    end; {sxc2r}

 if ox2r=0 then
    begin {oxc2r}
        oxc2r:=oxc2r+1;
        if (y+2*s)<lclxo then ox2r:=1
        else if (y+2*s)>uclxo then ox2r:=1;
     end; {oxc2r}

  if ax2r=0 then
    begin {axc2r}
        axc2r:=axc2r+1;
          if (y+2*s)<lclxa then ax2r:=1
          else if (y+2*s)>uclxa then ax2r:=1;
    end; {axc2r}
    {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

  if sx25r=0 then
  begin {sxc25r}
        sxc25r:=sxc25r+1;
        if (y+2.5*s)<lclxs then sx25r:=1
        else if (y+2.5*s)>uclxs then sx25r:=1;
     end; {sxc25r}

 if ox25r=0 then
    begin {oxc25r}
        oxc25r:=oxc25r+1;
        if (y+2.5*s)<lclxo then ox25r:=1
        else if (y+2.5*s)>uclxo then ox25r:=1;
     end; {oxc25r}

  if ax25r=0 then
    begin {axc25r}
        axc25r:=axc25r+1;
          if (y+2.5*s)<lclxa then ax25r:=1
          else if (y+2.5*s)>uclxa then ax25r:=1;
    end; {axc25r}
  {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

  if sx3r=0 then
  begin {sxc3r}
        sxc3r:=sxc3r+1;
        if (y+3*s)<lclxs then sx3r:=1
        else if (y+3*s)>uclxs then sx3r:=1;
     end; {sxc3r}

 if ox3r=0 then
    begin {oxc3r}
        oxc3r:=oxc3r+1;
        if (y+3*s)<lclxo then ox3r:=1
        else if (y+3*s)>uclxo then ox3r:=1;
     end; {oxc3r}

  if ax3r=0 then
    begin {axc3r}
```

```
        axc3r:=axc3r+1;
          if (y+3*s)<lclxa then ax3r:=1
          else if (y+3*s)>uclxa then ax3r:=1;
     end; {axc3r}
{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

 if sx5l=0 then
 begin {sxc5l}
        sxc5l:=sxc5l+1;
        if (y-0.5*s)<lclxs then sx5l:=1
        else if (y-0.5*s)>uclxs then sx5l:=1;
    end; {sxc5l}

if ox5l=0 then
   begin {oxc5l}
        oxc5l:=oxc5l+1;
        if (y-0.5*s)<lclxo then ox5l:=1
        else if (y-0.5*s)>uclxo then ox5l:=1;
    end; {oxc5l}

 if ax5l=0 then
   begin {axc5l}
        axc5l:=axc5l+1;
          if (y-0.5*s)<lclxa then ax5l:=1
          else if (y-0.5*s)>uclxa then ax5l:=1;
   end; {axc5l}
 {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

 if sx1l=0 then
 begin {sxc1l}
        sxc1l:=sxc1l+1;
        if (y-s)<lclxs then sx1l:=1
        else if (y-s)>uclxs then sx1l:=1;
    end; {sxc1l}

if ox1l=0 then
   begin {oxc1l}
        oxc1l:=oxc1l+1;
        if (y-s)<lclxo then ox1l:=1
        else if (y-s)>uclxo then ox1l:=1;
    end; {oxc1r}

 if ax1l=0 then
   begin {axc1l}
        axc1l:=axc1l+1;
          if (y-s)<lclxa then ax1l:=1
          else if (y-s)>uclxa then ax1l:=1;
   end; {axc1l}
     {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

 if sx15l=0 then
 begin {sxc15l}
        sxc15l:=sxc15l+1;
        if (y-1.5*s)<lclxs then sx15l:=1
        else if (y-1.5*s)>uclxs then sx15l:=1;
```

```
   end; {sxc15l}

if ox15l=0 then
   begin {oxc15l}
      oxc15l:=oxc15l+1;
      if (y-1.5*s)<lclxo then ox15l:=1
      else if (y-1.5*s)>uclxo then ox15l:=1;
   end; {oxc15l}

 if ax15l=0 then
   begin {axc15l}
      axc15l:=axc15l+1;
        if (y-1.5*s)<lclxa then ax15l:=1
        else if (y-1.5*s)>uclxa then ax15l:=1;
   end; {axc15l}
   {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

 if sx2l=0 then
 begin {sxc2l}
      sxc2l:=sxc2l+1;
      if (y-2*s)<lclxs then sx2l:=1
      else if (y-2*s)>uclxs then sx2l:=1;
   end; {sxc2l}

if ox2l=0 then
   begin {oxc2l}
      oxc2l:=oxc2l+1;
      if (y-2*s)<lclxo then ox2l:=1
      else if (y-2*s)>uclxo then ox2l:=1;
   end; {oxc2l}

 if ax2l=0 then
   begin {axc2l}
      axc2l:=axc2l+1;
        if (y-2*s)<lclxa then ax2l:=1
        else if (y-2*s)>uclxa then ax2l:=1;
   end; {axc2l}
      {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

 if sx25l=0 then
 begin {sxc25l}
      sxc25l:=sxc25l+1;
      if (y-2.5*s)<lclxs then sx25l:=1
      else if (y-2.5*s)>uclxs then sx25l:=1;
   end; {sxc25l}

if ox25l=0 then
   begin {oxc25l}
      oxc25l:=oxc25l+1;
      if (y-2.5*s)<lclxo then ox25l:=1
      else if (y-2.5*s)>uclxo then ox25l:=1;
   end; {oxc25l}

 if ax25l=0 then
   begin {axc25l}
```

```
        axc25l:=axc25l+1;
          if (y-2.5*s)<lclxa then ax25l:=1
          else if (y-2.5*s)>uclxa then ax25l:=1;
    end; {axc25l}
      {xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}

  if sx3l=0 then
  begin {sxc3l}
        sxc3l:=sxc3l+1;
        if (y-3*s)<lclxs then sx3l:=1
        else if (y-3*s)>uclxs then sx3l:=1;
     end; {sxc3l}

  if ox3l=0 then
     begin {oxc3l}
        oxc3l:=oxc3l+1;
        if (y-3*s)<lclxo then ox3l:=1
        else if (y-3*s)>uclxo then ox3l:=1;
     end; {oxc3l}

  if ax3l=0 then
     begin {axc3l}
        axc3l:=axc3l+1;
          if (y-3*s)<lclxa then ax3l:=1
          else if (y-3*s)>uclxa then ax3l:=1;
     end; {axc3l}

  earl:=y;
  end; {generate}


 {Run the entire test 1000 times to get 1000 run lengths for
  all of the control charts. The loop starts here. Each seperate
  loop will regenerate conrol limits.}

begin {program}
clrscr;
writeln('   Relax, the program is running');
writeln('   It will be done in about 5 minutes');
seed:=1;
assign(chisq, 'c:chi.dat');
rewrite(chisq);
s:=sqrt(8);
a:=0;
repeat
{begin repeat1}
    {reset the required count variables}
    sx0:=0;
    sx5r:=0;sx1r:=0; sx15r:=0; sx2r:=0;sx25r:=0; sx3r:=0;
    sx5l:=0;sx1l:=0; sx15l:=0; sx2l:=0;sx25l:=0; sx3l:=0;
    sxc0:=0;
    sxc5r:=0;sxc1r:=0; sxc15r:=0;sxc2r:=0; sxc25r:=0; sxc3r:=0;
    sxc5l:=0;sxc1l:=0; sxc15l:=0;sxc2l:=0; sxc25l:=0; sxc3l:=0;
    sr0:=0;
```

```
src0:=0;
ox0:=0;
ox5r:=0;ox1r:=0; ox15r:=0; ox2r:=0;ox25r:=0; ox3r:=0;
ox5l:=0;ox1l:=0; ox15l:=0; ox2l:=0;ox25l:=0; ox3l:=0;
oxc0:=0;
oxc5r:=0;oxc1r:=0; oxc15r:=0;oxc2r:=0; oxc25r:=0; oxc3r:=0;
oxc5l:=0;oxc1l:=0; oxc15l:=0;oxc2l:=0; oxc25l:=0; oxc3l:=0;
or0:=0;
orc0:=0;
ax0:=0;
ax5r:=0;ax1r:=0; ax15r:=0; ax2r:=0;ax25r:=0; ax3r:=0;
ax5l:=0;ax1l:=0; ax15l:=0; ax2l:=0;ax25l:=0; ax3l:=0;
axc0:=0;
axc5r:=0;axc1r:=0; axc15r:=0;axc2r:=0; axc25r:=0; axc3r:=0;
axc5l:=0;axc1l:=0; axc15l:=0;axc2l:=0; axc25l:=0; axc3l:=0;
ar0:=0;
arc0:=0;

X:=0;
qx:=0;
Count:=0;
mr:=0;
mrsum:=0;
xsum:=0;
lnsum:=0;
sigma:=1;
number:=0;

{Generate 50 random variable from the specified distribution in order
 to get control limits for each on the three types on control limnits.
 the three types of control limits include Shewhart, Oyon, and Ankney.}

repeat
{ begin repeat2}
    chisquare;
    x:=y;
    xsum:=xsum+x;
    count:=count+1;
    if count>1 then mr:=abs(x-prev);
    prev:=x;
    mrsum:=mrsum+mr;
    lnsum:=lnsum+ln(x)
{ end repeat2}
until count=50;
   {Calculate the three sets of control limits based on the
    fifty random variable just produced in the above loop.}

xbar:=xsum/50;            {average x}
mrbar:=mrsum/49;           {average mr}
pill:=ln(xbar)-lnsum/50;
tstat:=1/pill;             {t statistic}
alpha:=0.109+0.503*tstat;  {estimated shape parameter}
uclxs:=xbar+2.66*mrbar;    {shewhart upper control limit}
lclxs:=xbar-2.66*mrbar;    {shewhart lower control limit}
uclrs:=3.268*mrbar;        {shewhart mr upper control limit}
```

```
    dtwo:=(0.64282)+(0.09775*(1-exp(-0.5*alpha)))+(0.35736*(1-exp(-2*alpha)))+(0.02483*(1-
exp(-0.1*alpha)));
    dthree:=(0.859457)-(0.2964*exp(-1*alpha))+(0.29099*exp(-0.5*alpha))+(0.4758*exp(-
2*alpha));
    dfour:=(3.28976)+(1.87067*exp(-1*alpha))+(0.13663*exp(-0.1*alpha));
    tone:=(3.23311)+(3.19102*exp(-1*alpha))+(0.85168*exp(-0.1*alpha))+(0.44197*exp(-
0.025*alpha));
    ttwo:=(2.77031)-(1.80838*exp(-1*alpha))-(0.75091*exp(-0.1*alpha))-(0.43839*exp(-
0.025*alpha));
    tthree:=(3.68490)+(1.88402*exp(-1*alpha))+(0.56355*exp(-0.1*alpha))+(0.09686*exp(-
0.025*alpha));
    uclxo:=xbar+((3*mrbar)/dtwo);   {oyon upper control limit}
    lclxo:=xbar-((3*mrbar)/dtwo);   {oyon lower control limit}
    uclro:=dfour*mrbar;          {oyon mr upper control limit}
    uclxa:=xbar+((tone*mrbar)/dtwo);  {ankney upper control limit}
    lclxa:=xbar-((ttwo*mrbar)/dtwo);  {ankney lower control limit}
    uclra:=mrbar+((tthree*dthree*mrbar)/dtwo);  {oyon mr upper control limit}

    {Generate random variable from the parent distribution until
     a point goes out of control on each of the control charts.}

    repeat
      Generate
    until sx0+sr0+ox0+or0+ax0+ar0>5;
      writeln (chisq, sxc0:7, src0:7, oxc0:7, orc0:7, axc0:7, arc0:7,
            sxc5r:7, oxc5r:7, axc5r:7,
            sxc1r:7, oxc1r:7, axc1r:7,
            sxc15r:7, oxc15r:7, axc15r:7,
            sxc2r:7, oxc2r:7, axc2r:7,
            sxc25r:7, oxc25r:7, axc25r:7,
            sxc3r:7, oxc3r:7, axc3r:7,
            sxc5l:7, oxc5l:7, axc5l:7,
            sxc1l:7, oxc1l:7, axc1l:7,
            sxc15l:7, oxc15l:7, axc15l:7,
            sxc2l:7, oxc2l:7, axc2l:7,
            sxc25l:7, oxc25l:7, axc25l:7,
            sxc3l:7, oxc3l:7, axc3l:7);
      {writeln (uclxs:7, lclxs:7 ,uclxo:7, lclxo:7,uclxa:7,lclxa:7);}
      a:=a+1
      {end repeat1}
until a=1002;
end.
```

APPENDIX F

Marse and Roberts Random Number Generator

Random uniform variates were used to generate random variates from the five parent distribution discussed in section 4.2. These uniform variates were generated according to Marse and Roberts random number generator. The Pascal code for this generator is on the following page.

```
program rand_test;
uses crt,printer;
var  i,n : integer;
     seed : longint;
     x : real;
     outfile : text;

function RandUnif : real;

const
  B2E15 =    32768;
  B2E16 =    65536;
  Modulus = 2147473647;
  Mult1 =    24112;
  Mult2 =    26143;

var   Hi15, Hi31, Low15, Lowprd, Ovflow,Zi : longint;

begin
  Zi := Seed;
  Hi15 := Zi DIV B2E16;
  Lowprd := (Zi - Hi15 * B2E16) * Mult1;
  Low15 := Lowprd DIV B2E16;
  Hi31 := Hi15 * Mult1 + Low15;
  Ovflow := Hi31 DIV B2E15;
  Zi := (((Lowprd - Low15 * B2E16) - Modulus) +
         (Hi31 - Ovflow * B2E15) * B2E16) + Ovflow;
  IF Zi < 0 THEN Zi := Zi + Modulus;
  Hi15 := Zi DIV B2E16;
  Lowprd := (Zi - Hi15 * B2E16) * Mult2;
  Low15 := Lowprd DIV B2E16;
  Hi31 := Hi15 * Mult2 + Low15;
  Ovflow := Hi31 DIV B2E15;
  Zi := (((Lowprd - Low15 * B2E16) - Modulus) +
         (Hi31 - Ovflow * B2E15) * B2E16) + Ovflow;
  IF Zi < 0 THEN Zi := Zi + Modulus;
  Seed := Zi;
  RandUnif := (2* (Zi DIV 256) + 1) / 16777216.0;
end;

begin
  clrscr;
  write('  enter Seed and n: ');
  readln( Seed, n);
  writeln;
{  assign(outfile,'a:rand.dat');
   rewrite(outfile);                          }
  for i := 1 to n do
    begin
      x := RandUnif;
      writeln(i:5, Seed:15, x:15:10);
{       writeln(outfile,x:10:8);             }
    end;
{  close(outfile);                            }
end.
```

```
enter Seed and n: 1 20

    1
    2       630360016    0.2935342193
    3      1549035330    0.7213258147
    4       264620982    0.1232237220
    5       529512731    0.2465736270
    6      1896697821    0.8832187057
    7      2116530888    0.9855864644
    8      1923129168    0.8955268264
    9      1674201058    0.7796105742
   10       108088067    0.0503324866
   11       859154222    0.4000748992
   12      1946499387    0.9064094424
   13      1377890442    0.6416302323
   14      1382793310    0.6439133286
   15       768302678    0.3577688336
   16      1014576563    0.4724490047
   17       514017889    0.2393582463
   18      2050350098    0.9547687173
   19      1928578391    0.8980643153
   20       863848128    0.4022606015
            246801402    0.1149258018
```

APPENDIX G

Normal Distribution Program Output

Turbo Pascal programs were written for this research. The programs generated 1000 run lengths for each set of control limits based on each of the five parent distributions. The run lengths were also generated for each mean shift simulated in this research. The program output for the normal distribution is supplied in this appendix. The out put on the following pages is an excerpt from a larger spreadsheet.

Shewart		Oyon		Ankney		+ 0.5		
x-chart	range	x-chart	range	x-chart	range	Shew	Oyon	Ank
101	101	101	101	114	186	14	14	187
62	48	62	48	62	1064	82	82	62
325	325	325	325	98	325	326	326	325
237	66	237	66	38	1470	237	237	237
46	184	46	184	25	202	5	5	46
115	47	115	47	12	47	48	48	115
29	58	29	58	18	173	29	29	29
593	393	593	393	39	592	22	22	593
206	211	206	247	52	247	206	206	248
26	734	26	734	26	1016	851	851	26
171	31	319	88	110	171	21	40	171
112	17	112	17	112	355	159	159	112
1299	64	1299	64	64	2931	127	127	822
105	105	115	105	14	537	69	69	115
33	120	33	120	11	120	97	97	33
592	196	592	196	478	12596	474	474	592
1	1337	1	1337	1	5584	2689	2689	1
3916	1586	3916	1586	287	10322	2526	3916	2691
307	306	307	306	206	378	93	307	379
140	139	140	139	185	139	140	140	185
225	69	225	69	278	4243	225	225	283
1330	197	1330	197	360	2453	198	198	1330
335	209	335	209	248	209	58	58	335
1326	30	1326	152	30	1325	40	40	30
145	144	145	144	96	144	77	77	145
264	61	264	61	62	1021	264	264	264
945	331	945	331	13	3365	214	214	945
180	3	180	3	103	690	7	7	103
15086	46	41349	46	847	88288	47	47	1072
28	27	28	27	114	27	28	28	28
980	103	980	103	54	103	112	112	291
65	109	65	109	61	592	65	65	226
7	53	7	53	7	53	33	33	7
19	18	19	18	19	19	20	20	19
164	41	164	41	74	163	41	41	164
243	3	1139	3	4	242	243	243	4
64	2	64	2	42	11	12	12	64
41	40	41	40	23	134	41	41	41

240	118	240	118	186	1363	42	42
103	102	103	102	187	346	22	22
63	24	63	24	49	197	19	19
190	189	190	553	187	1056	47	47
154	7	154	7	31	31	141	141
2022	34	2022	3209	185	6791	3063	3063
240	239	371	239	66	1971	114	114
272	62	272	62	63	62	62	62
29245	1477	29245	1477	182	132078	9447	9447
7163	52	7163	52	245	30972	4845	4845
165	164	165	164	8	165	108	108
243	242	243	242	184	1255	208	208
90	90	90	90	63	90	85	85
1306	313	1306	313	163	1490	185	185
220	9	220	9	220	4962	585	585
5165	307	5165	307	1214	5724	206	206
306	40	306	94	342	472	114	114
52	15	52	52	52	121	2	15
286	434	286	434	84	485	486	486
183	73	183	377	377	377	89	89
59	33	365	33	34	203	18	18
158	157	158	157	39	157	22	78
47	44	47	44	85	45	31	31
65	7	65	65	58	65	66	66
39	1	39	1	96	38	39	39
11	10	11	10	30	10	11	11
51	36	51	36	33	51	37	37
45	45	45	45	8	232	32	32
422	124	1820	124	315	761	125	125
2	1	2	1	2	1	1	1
61	10	61	10	99	127	35	35
389	187	389	187	260	940	258	258
112	111	112	111	17	111	8	8
75	23	75	74	374	74	57	57

	Shewart		Oyon		Ankney			+ 0.5
	x-chart	range	x-chart	range	x-chart	range	Shew	Oyon
average=	1118.876	213.391	1427.191	277.033	141.883	4011.67	437.488	485.891
Variance=	22032892	245915.4	46016238	668066.3	50173.24	1.12E+09	2181138	2503836
stdev=	4693.921	495.8986	6783.527	817.3532	223.9938	33429.52	1476.868	1582.352

APPENDIX H

R vs. mR

The upper control limit for the mR chart is based on ranges of subgroup size n=2, as is common with Shewhart's mR control charts. Ranges of subgroup size n=2 can be used instead of mR values. This is demonstrated with the following test:

1. Random numbers were generated from Minitab for both the normal and exponential distribution.
2. Ranges (n=2) were figured from the random numbers based on their respective distributions.
3. Control limits were calculated based on the ranges.
4. The number of out of control signals were counted for both the normal and exponential distribution based on ranges n = 2
5. Steps 2 - 4 were repeated using mR values instead of ranges n=2. The mR values were calculated from the 20,000 variates produced in step 1.

The results on the following page demonstrate that there is no significant difference between the number of OOC signals for the R (n=2) and mR cases (based on their respective distributions). The control limits in both cases are also at approximately the same value. The slight differences result from the fact that only 10,001 of the random variates were used to generate 10,000 moving ranges while all 20,000 variates were needed produce 10,000 ranges (n=2).

Normal - Range (n=2)

Average X:		40.1378	Average:	3.657326
Average mR:		11.17772		
T=	28.5772			
D4 =	3.32196			
Alpha=	14.45332			
ucl =	D4*mR(bar)	37.13196		

Total OOC:	88

Normal - Moving Range

Average X:		40.08921	Average:	3.656582
Average mR:		11.06392		
T=	28.96429			
D4 =	3.321343			
Alpha=	14.64694			
ucl =	D4*mR(bar)	36.74708		

Total OOC:	82

Exponential - Range (n=2)

Average X:		1.007212	Average:	-0.57598
Average mR:		1.011283		
T=	1.714771			
D4 =	4.108128			
Alpha=	0.990681			
ucl =	D4*mR(bar)	4.15448		

Total OOC:	165

Exponential - Moving Range

Average X:		1.012581	Average:	-0.56984
Average mR:		1.012012		
T=	1.717193			
D4 =	4.107237			
Alpha=	0.991941			
ucl =	D4*mR(bar)	4.156575		

Total OOC:	174

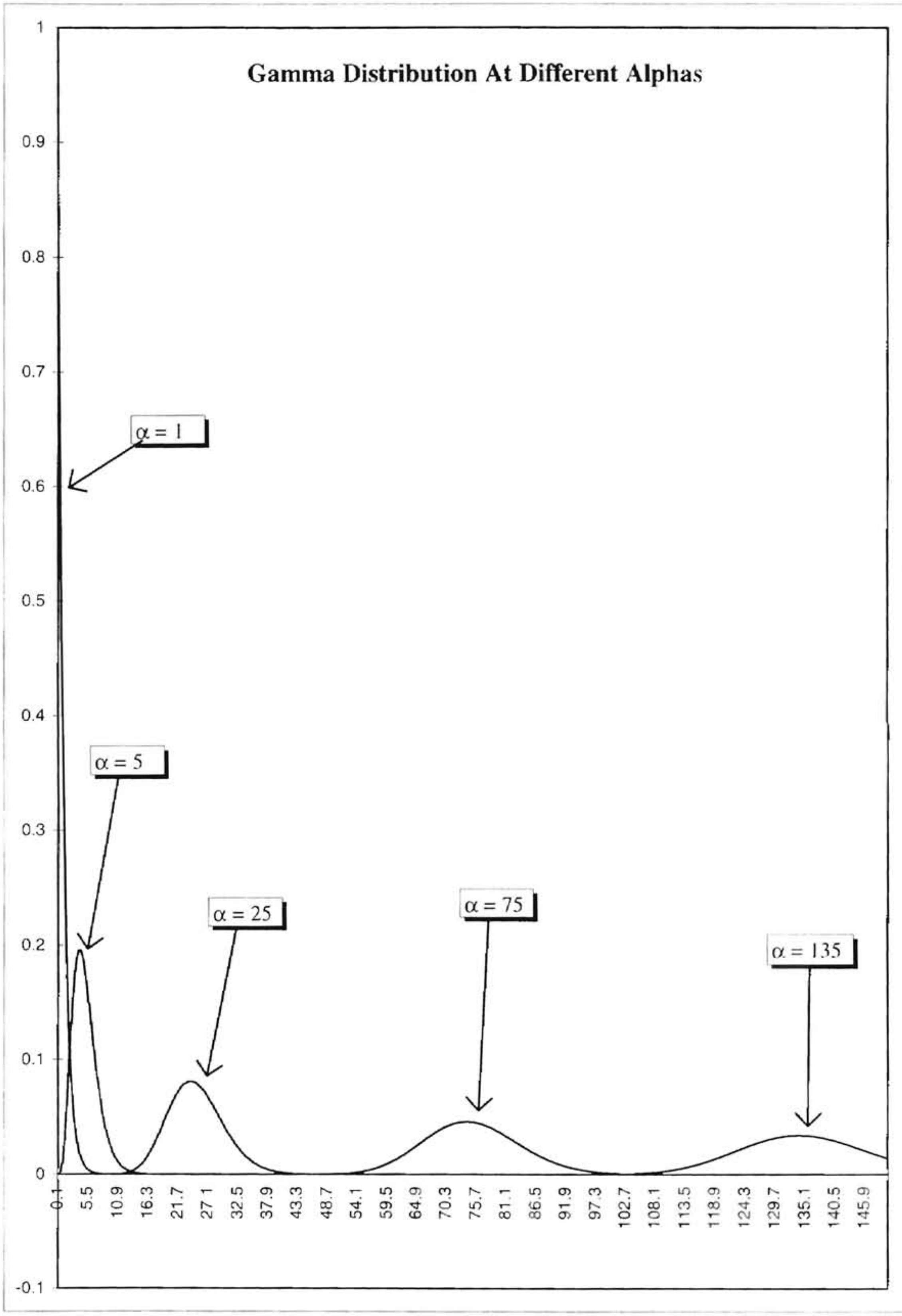
APPENDIX I

Gamma Distribution At Different Alphas

The graph on the following page represents the Gamma Distribution at different alpha (α) values. The plots were generated using the following probability density function:

$$f(x) = \frac{\left(\frac{1}{\beta}\right)^{\alpha}}{\Gamma(\alpha)} * x^{\alpha-1} * e^{-x\left(\frac{1}{\beta}\right)}$$

Alpha values of 1.0, 5.0, 25.0, 75.0, and 135.0 were chosen to demonstrate the effect of the shape parameter on the Gamma Distribution. The Beta value was held constant at a value of one (1) because it has no effect on the shape of the distribution.



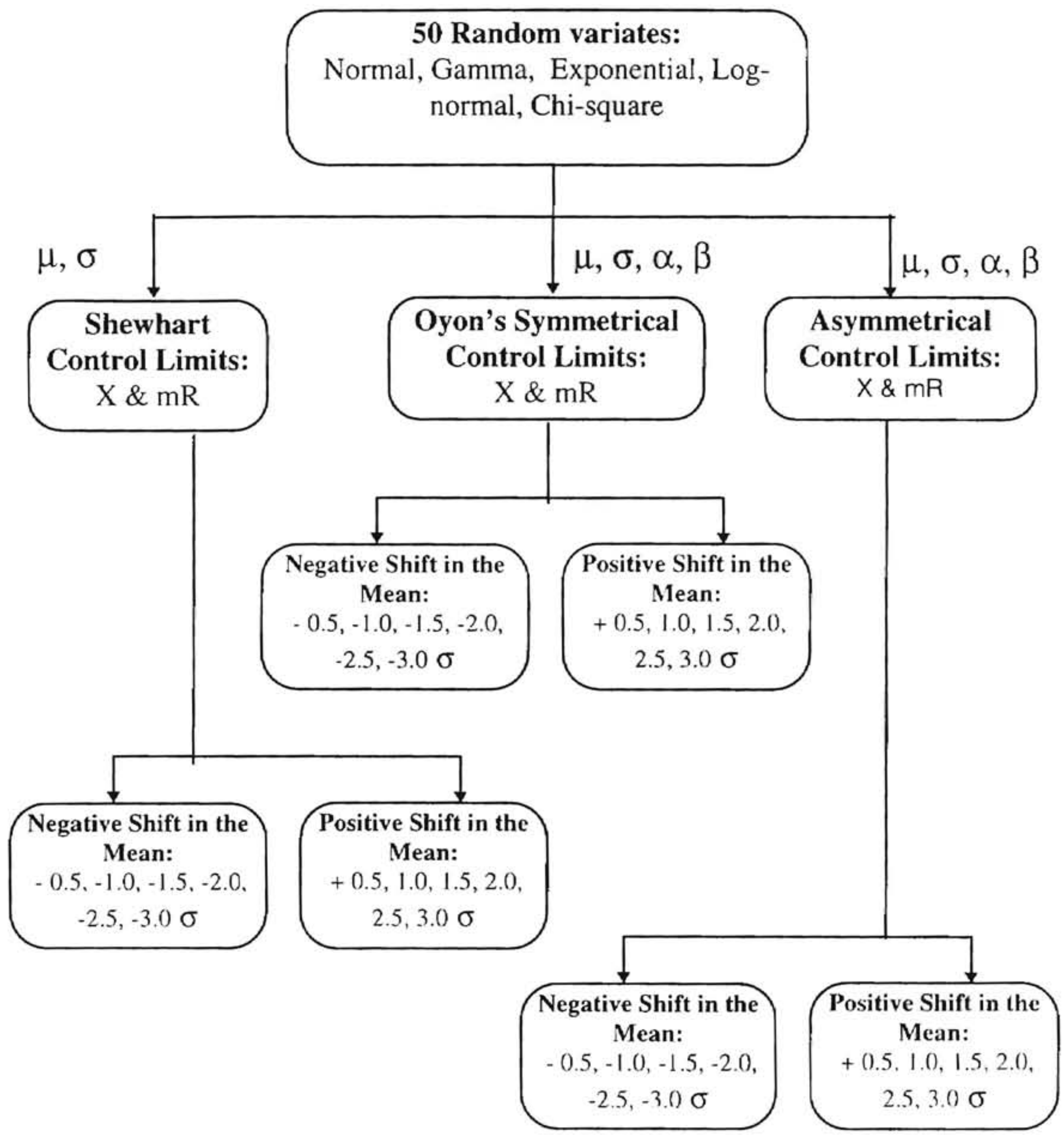
Gamma Distribution At Different Alphas
α = 1
α = 5
α = 25
α = 75
α = 135
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0.1
5.5
10.9
16.3
21.7
27.1
32.5
37.9
43.3
48.7
54.1
59.5
64.9
70.3
75.7
81.1
86.5
91.9
97.3
102.7
108.1
113.5
118.9
124.3
129.7
135.1
140.5
145.9

APPENDIX J

Sub-Objectives One and Two Program Logic

Turbo Pascal programs were written for this research. The programs generated 1000 run lengths for each set of control limits based on each of the five parent distributions. The run lengths were also generated for each mean shift simulated in this research. The program logic for generation of run lengths generated in sub-objective two and sub-objective three is supplied in the chart on the following page.

Figure I - 1: Sub-Objectives Two and Three Program Logic



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