GAUGE COUPLING UNIFICATION

IN THEORIES WITH LARGE

EXTRA DIMENSIONS

By

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CHAPTER 1

INTRODUCTION

Extra compact dimensions beyond our usual four dimensional space-time appear naturally in string theory. In fact, consistent string theories can exist only in a 10-dimensional space-time. Since these extra dimensions were not detected by current accelerators they must be curled up into a small compact space (for example a torus, a sphere or any other closed manifold). As a result, the coordinates associated with these extra dimensions are necessarily periodic (unlike the usual 3+1 dimensions which are unconstrained). This mechanism is often called *compactification* and the extra dimensions are said to be *compactified*. An immediate consequence of compactification can be understood in terms of elementary quantum mechanics. If a spatial dimension is periodic the momentum in that direction is quantized, $p = n/R$, $n = 0, 1, 2, ...$ where R is the compactification radius. As a result a particle living in the higher dimensional space develops so-called infinite Kaluza-Klein towers of momentum states (KK modes or excitations for short), one for each extra dimension. The spacing between these KK modes depends on the size of the extra dimension as $1/R$ and vanishes in the decompactified limit $R \rightarrow \infty$.

The sizes of these extra dimensions are not generally fixed by the string dynamics.

These may be close to the inverse of the Plank scale in which case they will have very little direct phenomenological implications. However, recent developments in string theory allow the possibility that sizes of these extra dimensions may be very large [1, 2], such as the inverse of a TeV [2]-[5], or even in the sub-millimeter range [6]. This has generated the exciting possibility for their direct phenomenological implications, such as the modification of the Newton's law of gravity in the sub-millimeter range [6], effects in low energy astrophysical phenomena [7], and in the high energy collider physics [6, 8]. Some of the Standard Model (SM) gauge and Higgs bosons and their supersymmetric (SUSY) partners may live in a D-brane containing some of these few TeV^{-1} compact dimensions. Then, the effect of their low-lying Kaluza-Klein (KK) excitations should be observed in the forthcoming high energy colliders either through the direct production of some of these KK states or through their indirect off-shell effects.

The question of gauge coupling unification was raised soon after the discovery of the standard model. It was pointed out that embedding the SU(3) $\times$ SU(2) $\times$ U(1) model into a higher local symmetry would lead to two distinct conceptual advantages: (i) it may provide quark-lepton unification [9, 10], thus providing a unified understanding of the apriori separate interactions of the two different types of matter and (ii) it can lead to a description of different forces in terms of a single gauge coupling constant [10, 11]. Using the renormalization group equations known at that time, it was shown that the gauge couplings of the SM can indeed unify at a very high scale of order 10^{15} GeV. However, in GUT theories, obliteration of the quark-lepton distinction leads to baryon instability such as proton decay whose rate is proportional to the 4th power of the unification scale. The minimal GUT model based on SU(5) symmetry led to a prediction for proton lifetime τ_p between 1.6×10^{30} yrs. to 2.5×10^{25} yrs. Attempts to observe the proton decay at this level failed, ruling out the unification within the nonsupersymmetric SU(5). Also, further investigation showed that SM unification is not consistent with low energy experimental data.

Supersymmetry seems to be the cure. There are several advantages of supersymmetric GUT theories. First, a theoretical understanding of the large hierarchy between the week scale and the GUT scale is possible. The Minimal Supersymmetric Standard Model (MSSM) GUT scale is about 3×10^{16} GeV, in agreement with current bounds on proton lifetime. Supersymmetric GUT's also have the potential to explain the quantization of the electric charge as well as the cosmological baryon/anti-baryon asymmetry. Unfortunately, the unification scale is too high to allow direct probes of GUT physics in foreseeable collider experiments.

About a year ago, it was pointed out that if the SM particles propagates into these extra dimensions, then the contribution of their KK excitations give additional contributions to the beta functions above the compactification scale, μ_0. This modifies the running of the gauge couplings from the usual logarithmic running to an approximate power law running [12]. Depending on the choice of μ_0, this can lead to the unification of gauge couplings at a scale much smaller than the usual GUT scale. Typically, the unification occurs at a scale of $\approx 1.5\mu_0$ to $\approx 20\mu_0$ depending on the number of extra dimensions, and regardless of the number of fermion families contributing. This gives the possibility of having the unification scale as low as few TeV, depending on the choice of μ_0. This is very exciting, because it not only eliminates the usual gauge hierarchy problem but it also allows the prospect of observing GUT physics at the forthcoming colliders, such as LHC. However, more detailed study (including the two loop contributions below μ_0) shows that such an unification does not occur [14]. Using the accurately measured values of $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$ to determine the unification scale, one finds the values of $\alpha_3(M_Z)$ much higher than the experimentally measured range [14], unless the scale of compactification is very high, such as 10^{12} GeV. Subsequent investigation showed that the unification with low scale μ_0 can be achieved if one alters [15]-[17] the MSSM spectrum in the extended $4+\delta$-dimensional space, or extend the gauge group with an intermediate scale [18]. In theories with extra dimensions, the effect of higher dimensional operators (induced by the quantum gravitational effects) on the gauge coupling unification as well as the possibility of TeV scale unification have also been investigated [19].

In all of these works, it was implicitly assumed that the supersymmetry is exact at the higher dimensional theory, and it breaks after the compactification to the four dimensions. Thus the compactification scale, μ_0, was always taken to be higher than the SUSY breaking scale, μ_{SUSY}.

The object of this work is to make a detailed study of the gauge coupling unification within MSSM with large extra dimensions. Our analysis include several scenarios not previously considered (but allowed by string theory). We do not extend the particle content (other than those required by the extra dimensions) or the gauge group. In addition to the case $\mu_{SUSY} < \mu_0$, our investigation includes scenario in which the SUSY is broken at the higher dimension (before compactification), so that the SUSY breaking scale is larger than the compactification scale. We are particularly interested in the cases in which both the compactification scale as well as the SUSY breaking scale are in the few or few tens of a TeV scale. We find that for this scenario, ($\mu_0 < \mu_{SUSY}$), the unification of the gauge couplings can be achieved with $\alpha_3(M_Z)$ lying within 1σ of the experimentally measured range and with both μ_0 and μ_{SUSY} in the few TeV scale. Such a scenario can be tested at the LHC. We also study the unification for the cases where only the gluons or the W, Z, H and/or the matter contribute above μ_0 and find that unification does not take place in these cases. Finally, we analyze the scenario in which there are two scales of compactification, μ_{10} and μ_{20}. Here we find two cases which give rise to unification with both μ_{SUSY} and μ_{10} in the few TeV range.

The thesis is organized as follows. A brief review of the literature on the subject of gauge coupling unification with extra dimensions is presented in Chapter 2. In Chapter 3 we discuss the formalism, the relevant equations and the method used for the numerical analysis. In Chapter 4 we consider the case of a single compactification scale with $\mu_{SUSY} < \mu_0$. Here we compare our results with those obtained in [14]. Chapter 5 contains our most interesting results. Here we give the results for the case $\mu_{SUSY} > \mu_0$. In Chapter 6 we discuss the results for the various cases with two scale compactification. Chapter 7 contains our conclusions.

CHAPTER 2

PREVIOUS WORKS

This chapter is devoted to a brief review of the literature available on the subject of higher dimensions. An average rate of about four papers per week in the last year dealing with various issues in theories with low scale extra dimensions shows that the prospect of contemplating physics beyond four dimensions is a very exciting one. In what follows we restrict ourselves to those articles that are most closely related to the present work, namely the unification of gauge couplings in the presence of extra dimensions. The main ideas along with a brief description of the numerical analysis are included.

Dienes, Dudas and Gherghetta [12] began this kind of analysis for the minimal supersymmetric standard model. They used power law unification, noted originally by Taylor and Veneziano [13] to argue that MSSM leads to approximate unification in the presence of extra compactified dimensions with arbitrary compactification radii between TeV^{-1} and the inverse of the GUT scale. The analysis uses the experimentally measured values of $\alpha_1(M_Z)$, $\alpha_2(M_Z)$, $\alpha_3(M_Z)$ and the evolution of the couplings is computed at one loop level. It is found that *approximate* unification can be achieved at scales as low as 10^6 GeV. Unification in a non-supersymmetric context is also investigated and it is found that the SM spectrum alone does not sustain unification. Proton decay constraints are also discussed and a mechanism is proposed in which compactification on a Z_2 orbifold ensures that interactions responsible for proton decay vanish at the orbifold fixed points.

The analysis of Dienes et al. was shortly followed by a more refined one by D. Ghilencea and G. Ross [14]. Instead of running all three couplings from the Z-mass up to the (approximate) unification point, the authors use the accurately measured values of $\alpha_1(M_Z)$ and $\alpha_2(M_Z)$ and calculate the prediction for $\alpha_3(M_Z)$ after imposing unification at a scale $\Lambda > M_Z$. A two-loop calculation including the $\overline{\mathrm{MS}} \to \overline{\mathrm{DR}}$ conversion factors is employed below the scale of the additional space-time dimensions. Above this scale the full gauge and Higgs sectors of MSSM along with η generations of matter fields (minimal scenario) contribute (through their KK excitations) to the gauge coupling evolution at one loop level. The prediction for $\alpha_3(M_Z)$ is calculated in this framework and the results are compared with those obtained without extra dimensions. It is found that the value of $\alpha_3(M_Z)$ is systematically increased compared to the two-loop Minimal Supersymmetric Standard Model prediction, while the unification scale is decreased. However, for very low values of the decompactification scale, the prediction is unacceptable.

Subsequent works brought some improvements by altering the MSSM spectrum above the compactification scale. This was done in two ways, either by considering that only a subset of the MSSM gauge and Higgs sectors develop KK excitations in the $4 + \delta$ dimensional space or by adding extra matter multiplets above the compactification scale. In all these models, the string constraint that bulk matter may only transform under bulk gauge groups was taken into account.

Three non-minimal scenarios are considered in [15] where only a subset of the MSSM spectrum is allowed to feel the extra dimensions. The choice for bulk MSSM fields in these scenarios are: (i) SU(3), SU(2), U(1), 3E, 3L (the gauge fields and the leptons live in the bulk); (ii) SU(3), U(1), U, D 3E (the SU(3) and U(1) gauge bosons, the three generations of right-handed leptons and one generation of right-handed up and down quarks live in the bulk); (iii) SU(2), U(1), H, 3L, E with two exotic SU(5) $\mathbf{5} + \overline{\mathbf{5}}$ pairs in which only the leptons live in the bulk. Assuming a supersymmetric spectrum at the top quark mass m_t, the three couplings are evolved from M_Z up to an *approximate* unification scale. This procedure is iterated with trial values of $\alpha_3(M_Z)$ until a suitable three coupling unification is achieved. Although the departure from minimal scenario brings some improvement on the prediction for $\alpha_3(M_Z)$ it is found that this is unacceptable for low values of μ_0.

A detailed analysis can also be found in [16]. It is shown that with enlarged extra dimensions, unification of the gauge couplings can be maintained in the supersymmetric case by including certain extra states above the compactification scale μ_0. These are identified by examining systematically all of the SU(5) irreducible representations up to the **75**. Unification is also demonstrated in the non-supersymmetric case provided that extra matter is also included above μ_0. The compactification and unification scales are rather high, typically above 10^9 GeV for good agreement with low energy data.

A general class of models that extend the MSSM spectrum are also discussed in [17], including non-canonical hypercharge models and a $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

string model that breaks to $SU(3)_C \times SU(2)_L \times U(1)_Y$ at Λ. For these scenarios unification is studied in 4D and the effect of δ extra dimension is computed by imposing unification with the 4D 1-loop prediction for $\alpha_3(M_Z)$.

In [18] A. Perez-Lorenzano and R.N. Mohapatra present a nice analysis of unification with extra dimensions. Novel scenarios discussed here include (i) the minimal supersymmetric left-right symmetric model with the gauge fields in the bulk and (ii) models with non-canonical normalization of gauge couplings.

Higher loop corrections to gauge coupling renormalization in the context of extra dimensions are discussed in [20] using both field theoretical arguments and string perturbation techniques. It is found that with N=1 compactification the 2-loop corrections are subleading. This is due to the fact that at the heavy KK levels the spectrum as well as the interactions are N=2 supersymmetric.

CHAPTER 3

THE FORMALISM

In this section we write down the relevant equations and present the details of how we perform our calculations leading to the results discussed in Chapters 4, 5, 6. The running of the gauge couplings, α_i, up to two loops, is given by:

$$\mu \frac{d\alpha_i(\mu)}{d\mu} = \frac{b_i}{(2\pi)}\,\alpha_i^2(\mu) + \sum_{j=1}^{3} \frac{b_{ij}}{(8\pi^2)}\,\alpha_i^2(\mu)\,\alpha_j(\mu) \tag{3.1}$$

where b_i and b_{ij}'s are the one and two loop β-function coefficients. Eq. 3.1 can be integrated iteratively by using the 1-loop approximation for the α_j's in the second term,

$$\alpha_j^{-1}(\mu) = \alpha_j^{-1}(\mu') - \frac{b_j}{(2\pi)} \ln\frac{\mu}{\mu'} \ . \tag{3.2}$$

The resulting equations give the couplings at a higher scale μ_2 in terms of the couplings at a lower scale $\mu_1 \le \mu_2$:

$$\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i}{(2\pi)} \ln\frac{\mu_2}{\mu_1} + \frac{1}{(4\pi)} \sum_{j=1}^{3} \frac{b_{ij}}{b_j} \ln\left(1 - \frac{b_j}{(2\pi)}\,\alpha_j(\mu) \ln\frac{\mu_2}{\mu_1}\right) . \tag{3.3}$$

Using Eq. 3.3, we start the running of the couplings at the Z-mass, including the thresholds at m_t and μ_{SUSY} (for the case $\mu_{SUSY} < \mu_0$), and using the appropriate values of the coefficients b_i and b_{ij}'s. The $\overline{\mathrm{MS}} \rightarrow \overline{\mathrm{DR}}$ conversion factors

$$\Delta_i^{conversion} = -\frac{C_2\left(G_i\right)}{12\pi}$$

are included above μ_{SUSY}. Beyond the compactification scale, μ_0, the effect of the extra dimensions on the running of the gauge couplings was first computed in [13]. The particles living in the $4+\delta$ dimensional space develop Kaluza-Klein excitations due to momentum quantization in the compactified dimensions. These KK excitations circulate in the one-loop vacuum polarization diagrams, thus modifying the scale dependence of the couplings. As a result the couplings exibit approximate power law evolution which, at the one loop level, is given by [12]:

$$\alpha_i^{-1}\left(\mu_0\right) = \alpha_i^{-1}\left(\Lambda\right) + \frac{b_i - \tilde{b}_i}{(2\pi)} \ln\frac{\Lambda}{\mu_0} + \frac{\tilde{b}_i}{(2\pi)} \frac{X_\delta}{\delta} \left[\left(\frac{\Lambda}{\mu_0}\right)^\delta - 1\right]. \tag{3.4}$$

The coefficients $\tilde{b}_i \equiv (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$ are the appropriate beta function coefficients including the contributions of the excited KK modes of all the particles living in the $4+\delta$-dimensional space (see Table 3.1 for contributions due to various MSSM particles), and $\Lambda > \mu_0$. Λ can be identified with the GUT scale. X_δ is the volume of a δ-dimensional unit sphere, given by:

$$X_\delta = \frac{2\pi^{\delta/2}}{\delta\,\Gamma\left(\delta/2\right)}$$

where Γ is the Euler gamma function.

In the running process we use Eq. 3.3 and 3.4, with the following input parameters:

$$m_t = 175\,\mathrm{GeV}$$

Particle	$(b_1,\ b_2,\ b_3)$
gauge	(0,-4,-6)
H	(3/5,3,2)
$Q+\overline{Q}$	$(1/5,3,2)\eta$
$U+\overline{U}$	$(8/5,1)\eta$
$D+\overline{D}$	$(2/5,0,1)\eta$
$L+\overline{L}$	$(3/5,1,0)\eta$
$E+\overline{E}$	$(6/5,0,0)\eta$

Table 3.1: β-function contribution from MSSM particles

$$\begin{aligned} M_Z &= 91.187\,\mathrm{GeV} \\ \alpha_1^{-1}(M_Z) &= 58.9946 \\ \alpha_2^{-1}(M_Z) &= 29.571\,. \end{aligned}$$

The value of $\alpha_3(M_Z) \equiv x$ was treated as a variable to be solved for, along with $\Lambda/\mu_0 \equiv y$ and $\alpha_{GUT} \equiv z$. Thus, we have three equations for $\alpha_1(\mu)$, $\alpha_2(\mu)$ and $\alpha_3(\mu)$ (obtained by matching Eq. 3.3 and Eq. 3.4 at μ_0), and three unknowns, x, y and z. These were solved for numerically, using the unification condition:

$$\alpha_1(\Lambda) = \alpha_2(\Lambda) = \alpha_3(\Lambda) = \alpha_{GUT} \tag{3.5}$$

For the case of $\mu_0 < \mu_{SUSY}$, and also for the two scale compactification (μ_{10} and μ_{20}), the evolution equations and the beta function coefficients were adjusted appropriately. The values of the coefficients are given for each case in Sec. 4, 5 and 6. The output of our calculations consists of $\alpha_3(M_Z)$, Λ and α_{GUT}. This method has the advantage that one can easily consider various possibilities for μ_{SUSY} and μ_0 (or μ_{10} and μ_{20}). As a general rule, the combinations that lead to the value of $\alpha_3(M_Z)$ outside the the 1σ range of the experimental value (0.1191 ± 0.0018) are discarded. So are combinations that lead to the unified coupling outside the perturbative range.

CHAPTER 4

ONE SCALE SCENARIO WITH $\mu_{SUSY} \leq \mu_0$

As a first example we consider the minimal scenario of Dienes, Dudas and Gherghetta [12]. The β-function coefficients are:

$$b_i^{MSSM} = (33/5,\ 1,\ -3)$$

for the supersymmetric four dimensional running and:

$$\tilde{b}_i^{MSSM} = (3/5,\ -3,\ -6)$$

in the presence of extra dimensions above the compactification scale. For the numerical analysis we vary μ_{SUSY} from 1 TeV up to 2×10^3 TeV and search for compactification scales $\mu_0 \geq \mu_{SUSY}$ that lead to acceptable predictions for $\alpha_3\,(M_Z)$. Results are discarded if the prediction is off by more than 1σ. Our numerical results (see Table 4.1) for the case $\delta = 1$, $\eta = 0$ indicate that the lowest SUSY breaking scale for which unification can occur is $\mu_{SUSY} = 1.48\,\text{TeV}$, in which case the compactification scale must be $\mu_0 = 3.27 \times 10^{12}$ TeV, leading to unification at $\Lambda = 6.25 \times 10^{12}$ TeV. Increasing the number of extra dimensions has the effect of slightly increasing these lower bounds on μ_{SUSY} and μ_0. For this case, our results are in agreement with [14].

In Fig. 4.1 we plot the ratio $R = \log_{10}(\mu_{SUSY}/\mu_0)$ against μ_{SUSY}. The vertical and horizontal spreads in the figure represent the ranges for which we get solution at the 1σ range of $\alpha_3(M_Z)$. As a general feature, as the SUSY breaking scale increases, the compactification scale needed for unification decreases, a ratio of approximately 1 being obtained around $\mu_{SUSY} \approx 1 \times 10^3$ TeV. This corresponds to the situation in which supersymmetry is broken as soon as the extra dimensions compactify. Same result is shown in Fig. 4.2 where μ_0 is plotted against μ_{SUSY}. The bands correspond to the regions in the plane for which unification is achieved within 1σ range of $\alpha_3(M_Z)$. It is interesting to note that for the unification band μ_0 is approximately proportional to μ_{SUSY}^{-3}. Fig. 4.3 gives a plot of the unification scale against the compactification scale. The results indicate that the unification scale Λ is approximately proportional to the compactification scale μ_0, with a proportionality constant strongly dependent on the number of extra dimensions. Therefore we obtain two mass relations required by the unification:

$$\begin{aligned} \mu_0 &\sim (\mu_{SUSY})^{-3} \\ \Lambda &= k(\delta)\,\mu_0 \end{aligned}$$

where $k(\delta)$ is about 10 for $\delta = 1$ and of order unity for $\delta = 6$.

It can be concluded that there are no solutions leading to both μ_{SUSY} and μ_0 in the 100 TeV or less range. Therefore this scenario is not of interest for near future collider experiments. Allowing $\eta \geq 1$ generations of matter fields to live in the $4+\delta$-dimensional space drives the unified coupling α_{GUT} towards higher values while preserving unification (in agreement with previous works).

δ	η	μ_{SUSY}	μ_0	$\alpha_3(M_Z)$	Λ/μ_0	Λ	α_{GUT}
1	0	1.48×10^3	3.27×10^{15}	0.1208	1.91	6.25×10^{15}	0.0384
1	0	5.32×10^3	2.29×10^{13}	0.1208	5.14	1.18×10^{14}	0.0330
6	0	1.78×10^3	3.92×10^{15}	0.1206	1.20	4.71×10^{15}	0.0379
6	0	5.32×10^3	3.66×10^{14}	0.1193	1.37	5.00×10^{14}	0.0345
1	1	5.32×10^3	2.29×10^{13}	0.1208	5.14	1.18×10^{14}	0.0383
1	2	5.32×10^3	2.29×10^{13}	0.1208	5.14	1.18×10^{14}	0.0457
1	3	5.32×10^3	2.29×10^{13}	0.1208	5.14	1.18×10^{14}	0.0567

Table 4.1: A few relevant numerical results for a one threshold scenario with $\mu_{SUSY} < \mu_0$. The behavior under changes of δ and η is shown. All the mass scales are in GeV units. Relevant plots are presented in Fig. 4.1 and 4.2.

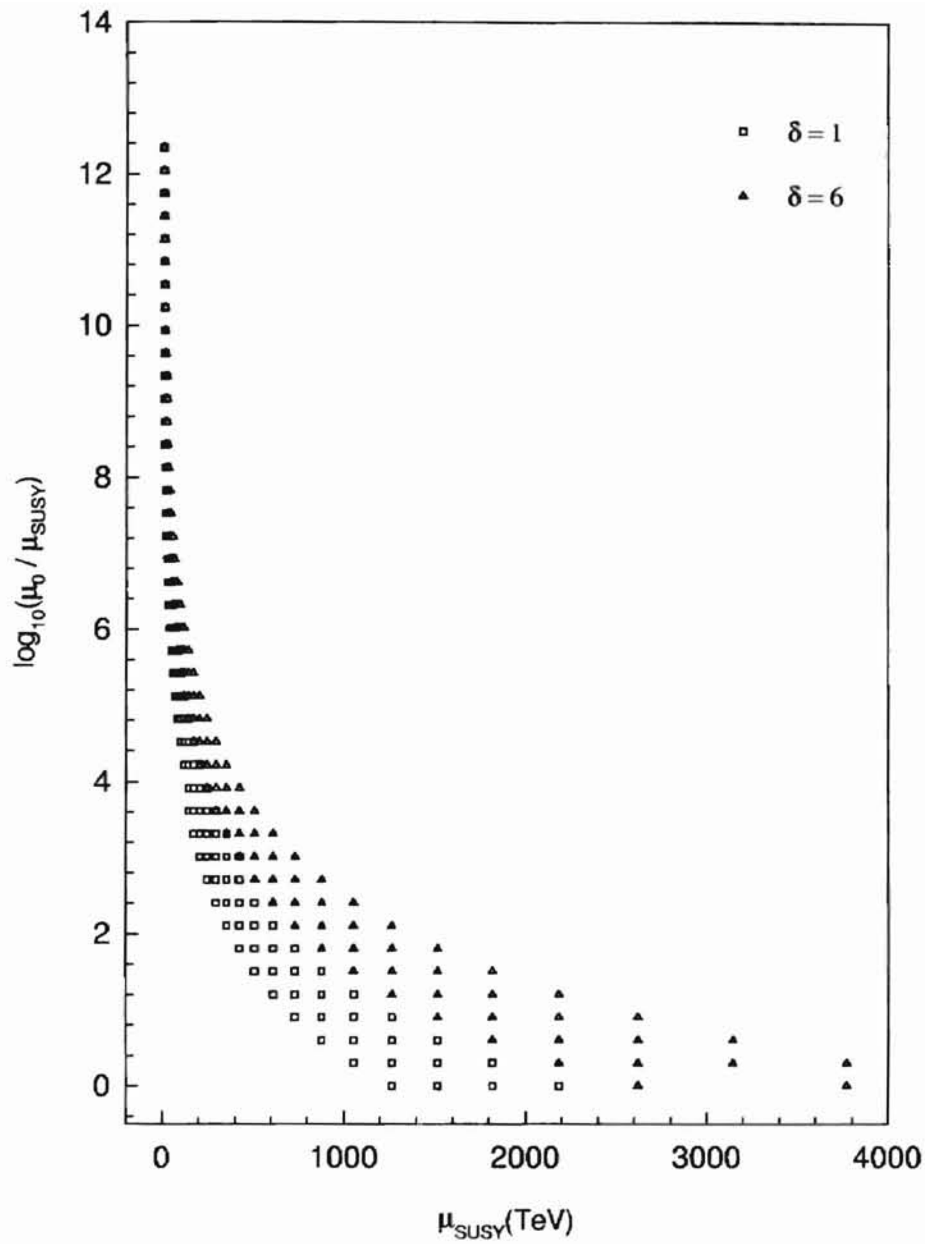

Figure 4.1: The ratio μ_0/μ_{SUSY} plotted against various SUSY breaking scales, μ_{SUSY}. Only results within 1σ of $\alpha_3\,(M_Z)$ are presented. Unification is spoiled for points lying outside the corresponding bands.

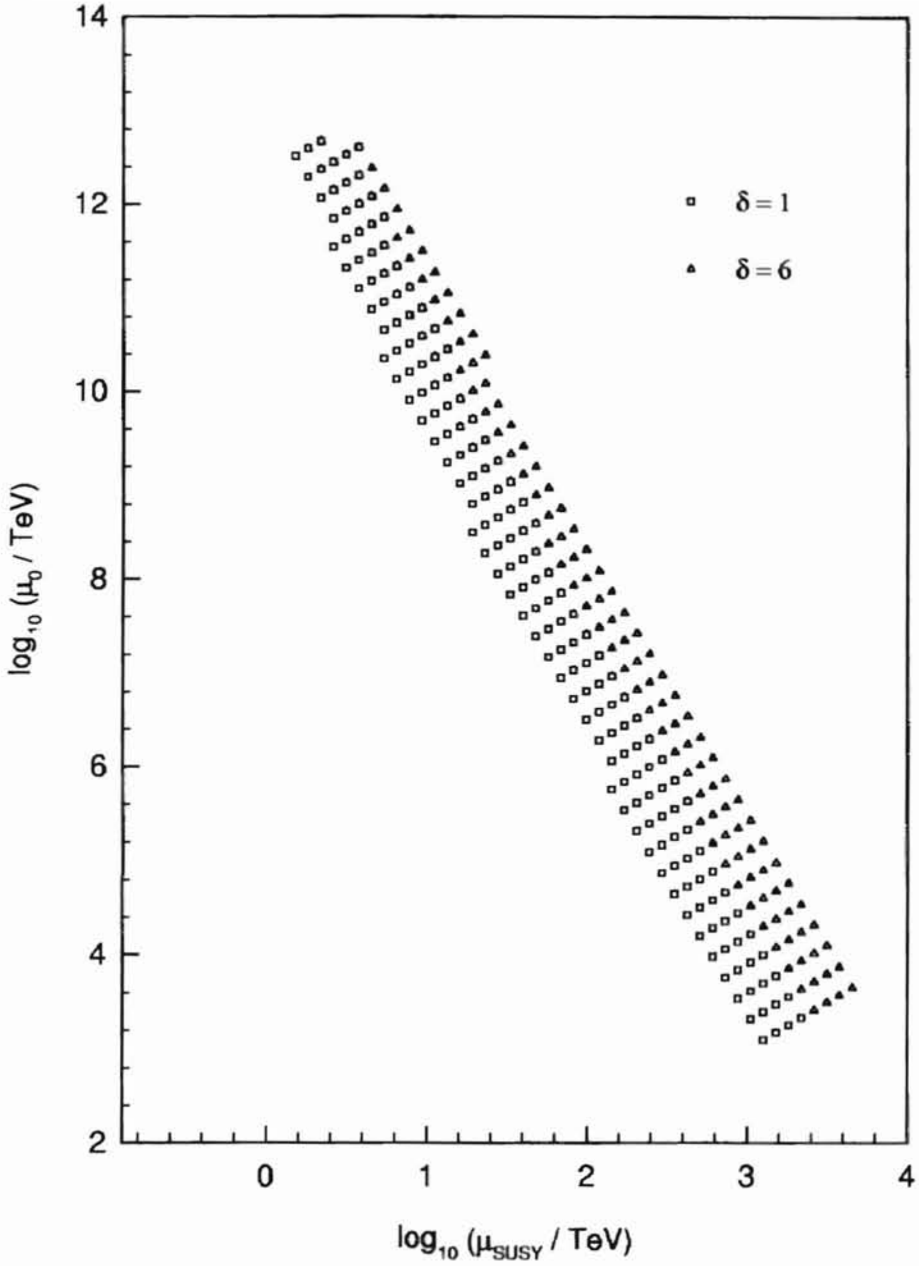

Figure 4.2: Scattered plot of the allowed compactification scales, μ_0, for various SUSY breaking scales, μ_{SUSY}. Only results within 1σ of $\alpha_3(M_Z)$ are presented. The same set of points as for the previous plot was used. Unification is spoiled for points lying outside the corresponding bands.

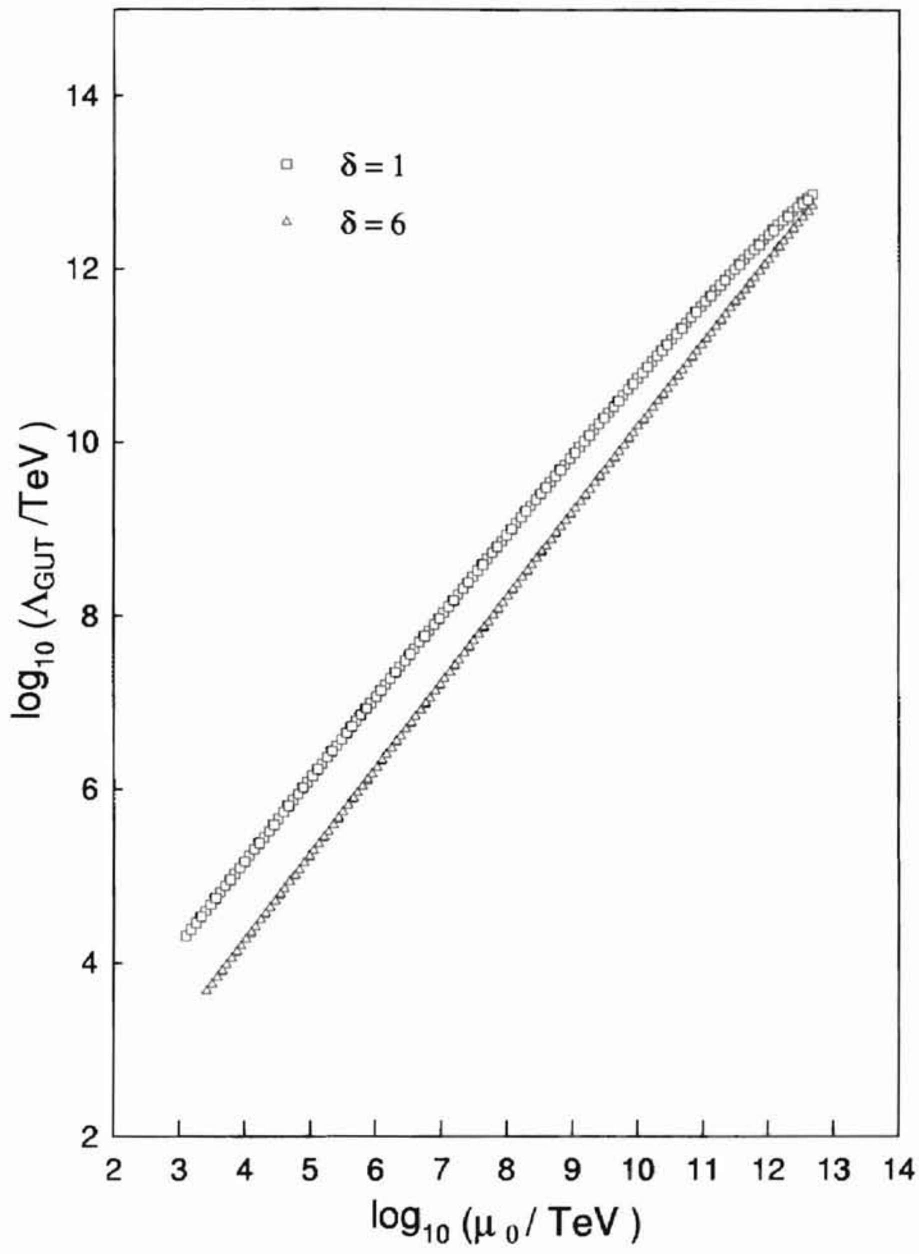

Figure 4.3: The unification scale plotted against the compactification scale. The results show a linear dependence of Λ on μ_0. Slightly lower unification scales can be obtained if one increases the number of extra dimensions from 1 to 6.

CHAPTER 5

ONE SCALE SCENARIO WITH $\mu_{SUSY} \geq \mu_0$

In this section we consider the posibility that the supersymmetry breaking occurs at a scale higher than the compactification scale, $\mu_{SUSY} \geq \mu_0$. For energies in the range $\mu_0 \leq \mu \leq \mu_{SUSY}$ the theory is non-supersymmetric but the gauge and Higgs sectors of SM along with η generations of matter fields exhibit KK excitations. The corresponding contributions to the running are given by

$$\tilde{b}_i^{SM} = (1/10, -41/6, -21/2) + \eta\,(8/3, 8/3, 8/3)\,.$$

At μ_{SUSY} the theory becomes supersymmetric and additional KK excitations of the sparticles lead to

$$\tilde{b}_i^{MSSM} = (3/5, -3, -6) + \eta\,(4, 4, 4)\,.$$

For the numerical analysis we choose various compactification scales μ_0 (starting in the TeV range) and search for SUSY breaking scales that lead to acceptable predictions for $\alpha_3\,(M_Z)$ (within 1σ of the central experimental value).

For the simplest case, $\eta = 0$, the results are shown in Fig. 5.1 where the allowed values of μ_{SUSY} are plotted against the corresponding compactification scale μ_0, for $\delta = 1$ and $\delta = 6$. Relevant numerical results are presented in Table 5.1.

As a generic feature, to each compactification scale it corresponds a specific range of μ_{SUSY} that are needed for unification and are consistent with low-energy experimental data. The length of these intervals is, of course, determined by our requirement of 1σ (or 3σ) agreement with experimental value of $\alpha_3(M_Z)$ but it is found to increase with μ_0. The fact that the upper bound of these ranges is finite shows that, within this model, supersymmetry is in fact needed for unification. Unification cannot occur within the SM spectrum. This was also noticed in [12] for the case $\mu_{SUSY} < \mu_0$.

This scenario is particularly appealing from the experimental point of view. Ignoring possible constrains on μ_0 we consider a compactification scale as low as $\mu_0 = 1\,\text{TeV}$ which enforces $\mu_{SUSY} = 4.5$ TeV and $\mu_{SUSY} = 1.46$ TeV for $\delta = 1$ and $\delta = 6$ respectively. This leads to unification at $\Lambda = 75.2$ TeV for $\delta = 1$ and $\Lambda = 2.68$ TeV for $\delta = 6$. A more realistic case would be $\mu_0 = 3\,\text{TeV}$, $\mu_{SUSY} = 11.9\,\text{TeV}$ with unification at $\Lambda = 198$ TeV for $\delta = 1$ or $\mu_{SUSY} = 4.3$ TeV with unification at $\Lambda = 7.86$ TeV for $\delta = 6$. Needless to say, these cases are well within the LHC reach and can be investigated at future experiments. The case $\eta = 3$, not present in Table 5.1, led to negative unified coupling for the range of μ_0 shown in Fig. 5.1.

We conclude this section with a few remarks. It was suggested in the literature [12] that the compactification scale could be identified with the SUSY breaking scale. Our results in this section and Sec. 4 indicate that, if this is the case, then this common scale cannot be lower than 10^6 GeV (around 10^6 GeV the ratio μ_{SUSY}/μ_0 required for unification approaches 1 in both scenarios). Also, it was pointed out in [12] that the unified coupling α_{GUT} is nonperturbative unless the unification scale is

$\Lambda \geq 10^5\, GeV$ for $\eta = 2$ and $\Lambda \geq 3 \times 10^{10} GeV$ for $\eta = 3$. This lower bound is no longer required in this scenario since α_{GUT} remains perturbative for any combination of μ_0 and μ_{SUSY} allowed by low-energy experimental data.

One question need to be addressed here. Does string theory allow a scenario in which the compactification scale is lower than the SUSY breaking scale ? In this case, SUSY has to be broken in higher dimension before compactification. There are several possibilities for that to happen. One possibility is a string solution in which SUSY is broken at the string level. In general, non-SUSY string solutions are unstable. String theory prefers vacua which are supersymmetric. Dilaton and other modulii tend to run away to infinity, and restore SUSY. However, given the reach complexities and possibilities in string theory, such a scenario can not be ruled out. A second possibility is the gaugino condensation in higher dimensional gauge theory. The gauge coupling could be of order unity, causing gaugino condensation and breaking $N = 2$ (or even $N = 1$) SUSY, before compactification to four dimensions. Yet another possibility is that the SM particles (plus their SUSY partners) live in a non-BPS brane which is stable but does not preserve supersymmetry at all [21]. Thus, we conclude that a scenario with $\mu_{SUSY} > \mu_0$ is not totally crazy.

δ	η	μ_0	μ_{SUSY}/μ_0	μ_{SUSY}	$\alpha_3\,(M_Z)$	Λ/μ_0	Λ	α_{GUT}
1	0	1×10^3	4.5	4.5×10^3	0.1187	75.2	7.52×10^4	0.0197
1	0	2×10^3	4.2	8.3×10^3	0.1190	69.3	1.39×10^5	0.0199
1	0	3×10^3	4.0	1.19×10^4	0.1190	66.0	1.98×10^5	0.0200
1	0	4×10^3	3.8	1.53×10^4	0.1191	63.6	2.55×10^5	0.0200
1	0	5×10^3	3.8	1.86×10^4	0.1191	61.8	3.09×10^5	0.0201
1	0	7×10^3	3.6	2.50×10^4	0.1191	59.2	4.14×10^5	0.0201
1	0	9×10^3	3.5	3.11×10^4	0.1191	57.1	5.14×10^5	0.0202
6	0	1×10^3	1.46	1.46×10^3	0.1201	2.68	2.68×10^3	0.0187
6	0	2×10^3	1.45	2.89×10^3	0.1194	2.64	5.29×10^3	0.0188
6	0	3×10^3	1.43	4.3×10^3	0.1194	2.62	7.86×10^3	0.0189
6	0	4×10^3	1.43	5.71×10^3	0.1190	2.61	1.04×10^4	0.0190
6	0	5×10^3	1.42	7.10×10^3	0.1191	2.59	1.29×10^4	0.0190
6	0	7×10^3	1.41	9.86×10^3	0.1192	2.57	1.80×10^4	0.0191
6	0	9×10^3	1.40	1.26×10^4	0.1191	2.56	2.3×10^4	0.0192
1	1	1×10^3	4.5	4.5×10^3	0.1187	75.2	7.52×10^4	0.0332
1	2	1×10^3	4.5	4.5×10^3	0.1187	75.2	7.52×10^4	0.1040
6	1	1×10^3	1.46	1.46×10^3	0.1201	2.68	2.68×10^3	0.0327
6	2	1×10^3	1.46	1.46×10^3	0.1201	2.68	2.68×10^3	0.1300

Table 5.1: Numerical results for the 1-scale scenario with $\mu_{SUSY} \geq \mu_0$. The compactification scales μ_0 were taken as input and the allowed values of SUSY breaking scales were determined numerically. The behaviour under changes of δ and η is shown. All the mass scales are in GeV. See Fig. 5.1 for a relevant plot.

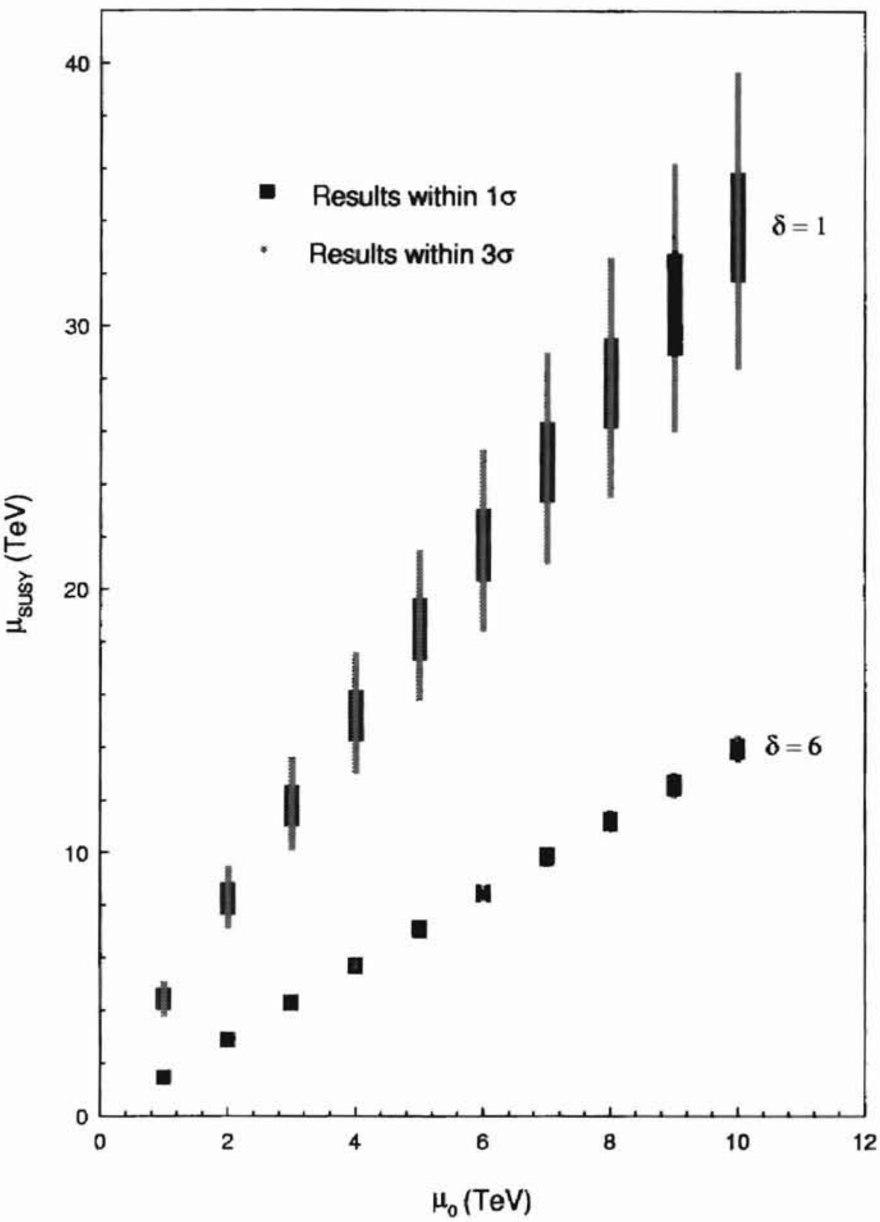

Figure 5.1: Allowed values of SUSY breaking scale, μ_{SUSY}, for various choices of μ_0 in a scenario with $\mu_0 < \mu_{SUSY}$. Results within 1σ and 3σ of $\alpha_3\,(M_Z)$ are presented, for $\delta = 1$ and $\delta = 6$. Unification is spoiled if μ_{SUSY} lies outside the corresponding vertical spreads shown in the plot.

CHAPTER 6

TWO SCALE SCENARIOS

In the analysis of Sec. 4 and 5 we assumed that the compactification of the extra dimensions takes place at a single mass scale, μ_0. However, possibility exists that the different extra dimensions compactify at different mass scales. Also, particles with different gauge quantum numbers may belong to different D-branes associated with different compactification scales. This section is devoted to numerical analyses of such scenarios with two different mass scales, μ_{10} and μ_{20} with $\mu_{10} < \mu_{20}$. In these models the MSSM spectrum (or only a subset of it) is split up into two parts, with the first part developing KK excitations at the first compactification scale μ_{10} and with the remainder contributing only after the second scale μ_{20} is crossed. When constructing these models the string constraint that bulk matter may only transform under bulk gauge groups is taken into account.

In all the subsequent cases SUSY breaking scale is assumed to be lower than μ_{10}. For practical purposes we restrict ourselves to compactification scales μ_{10} that are within the LHC reach and to $\delta = 1$ extra dimensions. Only results that lead to predictions of $\alpha_3(M_Z)$ within 1σ of the central experimental value are presented.

In what follows we consider several scenarios in which the splitting of the MSSM gauge sector is based on color. Relevant numerical results for these models are presented in Table 6.2 and the β-function coefficients corresponding to the two compactification scales for the cases A, B, C, D presented below, are given by:

$$\begin{aligned} \tilde{b}_i^{(10)} &= (0,0,-6) \\ \tilde{b}_i^{(20)} &= (0,-4,-6) + \eta\,(4,4,4,) \end{aligned} \qquad (6.1)$$

with the appropriate choice of η.

Case A)

$\mu_{10} \to$ SU(3)

$\mu_{20} \to$ SU(3)$\otimes$SU(2)$\otimes$U(1)

The notation is that only the gluons (along with their SUSY partners) develop KK excitations at μ_{10} while the full MSSM gauge sector contribute above μ_{20}. The β-function coefficients are given by Eq. 6.1 with $\eta = 0$. For SUSY breaking scales in the TeV range and μ_{10} within the reach of LHC ($\leq 14\,$TeV), a ratio μ_{20}/μ_{10} of about 7 is needed in order to achieve unification (with the prediction for $\alpha_3(M_Z)$ within 1σ of the central experimental value). The unification scale is as low as $4\times10^2\,$TeV. Note that for this scenario the value of the couplings at the unification scale ($\alpha_{GUT} \approx 0.015$) is significantly smaller than $\alpha_3\,(M_Z)$ and well within the perturbative regime. As a general feature, attempts to bring the compactification scale μ_{10} down to μ_{SUSY} (at fixed μ_{20}/μ_{10}) tend to drive the unified coupling towards higher values.

Case B)

$\mu_{10} \rightarrow$ SU(3)

$\mu_{20} \rightarrow$ SU(3)⊗SU(2)⊗U(1) ⊕ 1 generation of matter fields

($\eta = 1$ in Eq. 6.1). The addition of $\eta = 1$ generation of matter fields at μ_{10} preserves unification while increasing the coupling at the unification scale ($\alpha_{GUT} \approx 0.032$). This case shares all the features of the previous one.

Case C)

$\mu_{10} \rightarrow$ SU(3)

$\mu_{20} \rightarrow$ SU(3)⊗SU(2)⊗U(1) ⊕ 2 generations of matter fields

($\eta = 2$ in Eq. 6.1). With an MSSM spectrum at the TeV scale we found that this scenario does not lead to unification for μ_{10} within the LHC reach (although a *mathematical* unification is achieved, either the unified coupling α_{GUT} has unphysical values or the prediction for $\alpha_3(M_Z)$ is outside 3σ of the experimental value). However, extending the range of μ_{10} beyond the reach of LHC we found that unification can be achieved for $\mu_{10} \geq 5 \times 10^2$ TeV and only for $\mu_{20}/\mu_{10} \approx 5.5$. The unification scale can be as low as $\Lambda = 7.8 \times 10^4$ TeV and the unified coupling is in the perturbative regime.

Case D)

$\mu_{10} \rightarrow$ SU(3)

$\mu_{20} \rightarrow$ SU(3)⊗SU(2)⊗U(1) ⊕ 3 generations of matter fields

($\eta = 3$ in Eq. 6.1). This case is similar to Case C). A minimum compactification scale of $\mu_{10} \approx 7 \times 10^7$ TeV and a ratio $\mu_{10}/\mu_{20} \approx 3.4$ are required for unification. Consequently, the unification scale is pushed towards about $\Lambda = 3.1 \times 10^9$ TeV.

In Table 6.1 we list several other cases that were investigated but found NOT to give results of interest for future experiments at LHC.

Several conclusions can be drawn from the results above. Most importantly, the 2-scale scenarios allow for very low compactification scales (in the TeV range) even for the case in which the SUSY braking scale is lower than the compactification scale. This was not possible in 1-scale scenarios. Moreover, results with $\mu_{SUSY} = \mu_{10} \approx$ few TeV are obtained, which encourages the identification of SUSY breaking scale with the compactification scale. Specification of μ_{SUSY} along with the requirement that the first threshold is within the LHC reach, completely determined the second threshold as well as the unification scale (of course, with small variations determined by the error bar on the experimental value of $\alpha_3 (M_Z)$).

μ_{10}	SU(3)
μ_{20}	SU(3)⊗SU(2)⊗U(1) ⊕ H
μ_{10}	SU(3)
μ_{20}	SU(3)⊗SU(2)⊗U(1) ⊕ 3(L, E)
μ_{10}	SU(3)
μ_{20}	SU(3)⊗SU(2)⊗U(1) ⊕ 3(L, Q)
μ_{10}	SU(3) ⊗ U(1) ⊕ 3(U, D)
μ_{20}	SU(3)⊗SU(2)⊗U(1) ⊕ 3(Q, U, D, L, E)
μ_{10}	SU(3) ⊗ U(1) ⊕ 3(U, D)
μ_{20}	SU(3)⊗SU(2)⊗U(1) ⊕ 3(Q, U, D, L, E) ⊕ H
μ_{10}	SU(2)
μ_{20}	SU(3)⊗SU(2)⊗U(1)
μ_{10}	SU(2) ⊗ U(1) ⊕ 3(L, E) ⊕ H
μ_{20}	SU(3)⊗SU(2)⊗U(1) ⊕ 3(Q, U, D, L, E) ⊕ H

Table 6.1: Two compactification scale scenarios which DO NOT lead to unification with both μ_{SUSY} and μ_{10} within the reach of LHC.

η	μ_{SUSY}	μ_{10}	R	μ_{20}	$\alpha_3(M_Z)$	Λ/μ_{10}	Λ	α_{GUT}
0	1×10^3	2×10^3	7.2	1.44×10^4	0.1189	212	4.23×10^5	0.0156
0	2×10^3	2×10^3	7.2	1.44×10^4	0.1176	211	4.21×10^5	0.0155
0	2×10^3	4×10^3	7.2	2.88×10^4	0.1196	207	8.28×10^5	0.0157
0	2×10^3	6×10^3	7.2	4.32×10^4	0.1209	205	1.23×10^6	0.0158
0	3×10^3	3×10^3	7.2	2.16×10^4	0.1179	208	6.23×10^5	0.0156
0	3×10^3	5×10^3	7.2	3.60×10^4	0.1195	205	1.03×10^6	0.0157
0	3×10^3	7×10^3	7.2	5.04×10^4	0.1205	204	1.43×10^6	0.0158
0	5×10^3	5×10^3	7.2	3.60×10^4	0.1184	205	1.02×10^6	0.0157
0	5×10^3	7×10^3	7.2	5.04×10^4	0.1194	203	1.42×10^6	0.0158
0	5×10^3	9×10^3	7.2	6.48×10^4	0.1202	202	1.81×10^6	0.0158
1	1×10^3	2×10^3	7.2	1.44×10^4	0.1189	212	4.23×10^5	0.0332
1	2×10^3	2×10^3	7.2	1.44×10^4	0.1176	211	4.21×10^5	0.0327
1	2×10^3	4×10^3	7.2	2.88×10^4	0.1196	207	8.28×10^5	0.0329
1	2×10^3	6×10^3	7.2	4.32×10^4	0.1209	205	1.23×10^6	0.0329
1	3×10^3	3×10^3	7.2	2.16×10^4	0.1179	208	6.23×10^5	0.0325
1	3×10^3	5×10^3	7.2	3.60×10^4	0.1195	205	1.03×10^6	0.0326
1	3×10^3	7×10^3	7.2	5.04×10^4	0.1205	204	1.43×10^6	0.0327
1	5×10^3	5×10^3	7.2	3.60×10^4	0.1184	205	1.02×10^6	0.0322
1	5×10^3	7×10^3	7.2	5.04×10^4	0.1194	203	1.42×10^6	0.0323
1	5×10^3	9×10^3	7.2	6.48×10^4	0.1202	202	1.81×10^6	0.0324
2	3×10^3	3.1×10^7	5.1	1.55×10^8	0.1174	99	3.04×10^9	0.1364
2	3×10^3	5.3×10^7	5.1	2.69×10^8	0.1190	98	5.15×10^9	0.1302
2	3×10^3	9.1×10^7	5.1	4.64×10^8	0.1206	96	8.73×10^9	0.1244
2	3×10^3	5.5×10^5	6	3.31×10^6	0.1175	142	7.81×10^7	0.3285
2	3×10^3	9.5×10^5	6	5.72×10^6	0.1191	139	1.33×10^8	0.2935
2	3×10^3	1.7×10^6	6	9.89×10^6	0.1208	137	2.26×10^8	0.2652
3	3×10^3	4.8×10^{11}	3	1.43×10^{12}	0.1177	30	1.4×10^{13}	0.1386
3	3×10^3	8.2×10^{11}	3	2.27×10^{12}	0.1193	29	2.4×10^{13}	0.1248
3	3×10^3	1.4×10^{12}	3	4.27×10^{12}	0.1209	28	3.9×10^{13}	0.1135
3	3×10^3	7.7×10^{10}	3.4	2.62×10^{11}	0.1177	40	3.1×10^{12}	0.3127
3	3×10^3	9.2×10^{10}	3.4	3.14×10^{11}	0.1183	40	3.7×10^{12}	0.2884
3	3×10^3	1.3×10^{11}	3.4	4.53×10^{11}	0.1193	39	5.2×10^{12}	0.2497

Table 6.2: Numerical results for two scale compactification scenarios. The cases $\eta = 0, 1, 2, 3$ correspond to cases A,B,C,D respectively and $R = \mu_{20}/\mu_{10}$. The number of extra dimensions is $\delta = 1$ and the mass scales are in GeV.

CHAPTER 7

CONCLUSIONS

In this work we have made a detailed investigation for the unification of the gauge couplings in MSSM with extra dimensions. We do not extend the gauge group or the field content (except for those required by the higher dimensions). In the previous studies, it was implicitly assumed that supersymmetry breaks at four dimensions before the compactification, and thus the scale of SUSY breaking, μ_{SUSY} is lower than the compactification scale, μ_0. In this case, it was observed that the three gauge couplings do not unify (satisfying the experimental range of $\alpha_3\,(M_Z)$) with both μ_{SUSY} and μ_0 less than few tens of a TeV. We have investigated several new scenarios for which the couplings unify with both μ_{SUSY} and μ_0 in the few TeV scale. One particularly interesting scenario is when SUSY is broken at higher dimension (either through string dynamics or via gaugino condensation or in a non-BPS brane) before decompactification, so that $\mu_{SUSY} > \mu_0$. In this case we obtained gauge coupling unification with both μ_{SUSY} and μ_0 in the few TeV scale. This is very exciting, since for this scenario, LHC ($\sqrt{s}$ = 14 TeV) will be able to probe experimentally the existence of these compact dimensions. The direct experimental test will be the observation of the low lying KK resonance of SM particles, or the off shell effect of these particles via the usual SM processes. A family of two scale compactification scenarios in which the MSSM gauge sector is split into its colored and uncolored subsets was also considered. It was found that with $\eta = 0, 1$ matter generations contributing above the second scale μ_{20} the unification can be achieved with both μ_{SUSY} and μ_{10} in the few TeV scale. In all cases unification can be achieved only for a specific narrow range of the ratio μ_{20}/μ_{10}.

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