# UNIVERSITY OF OKLAHOMA <br> GRADUATE COLLEGE 

THE BUSINESS CALCULUS GMTA:

# AN EXPLORATION OF TEACHING EXPERIENCES IN AN UNFAMILIAR COURSE 

A DISSERTATION SUBMITTED TO THE GRADUATE FACULTY in partial fulfillment of the requirements for the degree of Doctor of Philosophy

By
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## A DISSERTATION APPROVED FOR THE DEPARTMENT OF MATHEMATICS

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## Dedication

To my husband, Jeff, who supported me through it all, thank you! You never doubted my ability to complete this paper or my degree. I am so thankful to God for placing such an amazing man in my life.

To my daughter, Abigail, who was born during my PhD program. You are such a joy and pleasure to me. I have been so blessed to call myself your Mama.

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# Chapter 1: Introduction to Business Calculus 

## at the University of Oklahoma (OU)

Business calculus is commonly taught at universities nationwide. It is primarily used to fill a requirement in business departments at most schools. The twosemester business calculus sequence at the University of Oklahoma is no different in its purpose. "So, it's to provide business students with one of their mathematics requirements" (Administrator 4). More specifically,

It's a two semester sequence that is offered mostly to business students, some life sciences take it, some pre-med take it, a few other majors, but by the time they get to the second semester, it's almost all business students. And so the purpose is to give some exposure of calculus topics to business students. (Administrator 2)

The assumption of those in leadership roles in business calculus in the mathematics department is that the primary way in which the sequence serves the College of Business (CoB) is that it allows them to meet their accreditation requirements. "[Business calculus serves] to meet their accreditation requirements... I've heard anecdotally that the only place students see or use the calculus aspect of business calculus, is in business calculus" (Administrator 1).

I have never yet ... had any contact from the business college about their students. Absolutely the opposite is true with the engineering calculus and the engineering students. I am in regular contact. The slightest change we make there, we hear about it immediately whether they like it or whether they don't. The business college never contacts us about anything. (Administrator 4)

So, the Department of Mathematics administration assumes that either what they are doing is what the CoB wants them to be doing or that the CoB has no opinion about it whatsoever. (Data collection from administration sited in this chapter is discussed in Chapter 3.)

### 1.1 History of the Business Calculus Program at OU

The development of business calculus at OU into its current state has been an approximately 40 year process.

I can only assume that, maybe at one time there was a single calculus course and it became evident that students in business calculus were not quite as prepared as those in engineering for this kind of a calculus course. (Administrator 1)

Because of the differences in preparation, a separate calculus sequence was developed for those students in the CoB. "Also it became apparent that the business college didn't want their students taking a full four semester calculus course" (Administrator 1). Thus, the standard four-semester sequence of engineering calculus at OU was abbreviated to a two-semester sequence for business students. "[Business calculus across universities] is always a compact sequence. Ours is two semesters as opposed to four. Whatever the number of
semesters for [engineering] calculus, business calculus is almost always less" (Administrator 4).

Until 1998, business calculus at OU was a single variable calculus class that omitted trigonometric functions. "It was basically the kind of thing that they did with business calculus as long as there had been business calculus. It's essentially calculus light. It's very traditional, very traditional" (Administrator 2). Most topics standard to an engineering calculus class were also taught in this early version of business calculus, including topics such as integration by parts and volumes of solids of revolutions. "Their attempts to try and make it business related were just token gestures at best. You know, it's all very contrived in that sense. And that's what we did for a long time" (Administrator 2).

### 1.2 Instructors Assigned to Business Calculus

Business calculus at OU is not taught by the tenure or tenure-track faculty. Nor is it taught by post-doctoral faculty. This course has historically been taught by adjunct faculty, "and maybe some of the more experienced graduate teaching assistants" (Administrator 1). The course was typically an adjunct-dominated class until the graduate program started expanding. "The graduate program was expanding very rapidly and we've gone from, there were 25 graduate students [not too long ago and] there are 75 graduate students now. So, they do a lot more teaching than they ever used to" (Administrator 4). This increase led to a need for
more teaching assignments for the department to be able to continue offering assistantships to graduate students.

More specifically, business calculus is a two-semester sequence. "Business Calculus I is predominately graduate students with a few fairly experienced adjunct teachers. Business Calculus II is the mirror image. It is primarily experienced adjunct teachers with just a sprinkling of graduate students" (Administrator 4). Historically, there has been some concern in the department about putting Graduate Mathematics Teaching Assistants (GMTAs) in the Business Calculus II classroom.

Our feelings at the moment is, that it's very hard to ask someone to teach bus calc II if they haven't had enough time teaching Business Calculus I. So, this is why you'll see lots of adjunct teachers in bus calc II, because they have a large amount of experience in these courses. (Administrator 4)

As a result, Spring 2007 was the first semester that any graduate student was assigned to teach Business Calculus II.

### 1.3 Current Structure of the Business Calculus Program at OU

In 1998, the business calculus textbook that was being used changed editions. The department turned to a former GMTA who at the time was in her dissertation phase, for advice on selecting a new text (Matthews, 1998).

During the course of her dissertation she came across this book. It was around time for the department to choose a business calculus book $\ldots[T]$ hey asked her what she thought and she thought this was a strong book and a neat direction to go in and based a lot on her recommendation, they decided to take up this book. And, that decision changed everything. (Administrator 2)

Matthews (1998) was focused on technology in the classroom.
[She] was the local driving force in doing that, but what she was doing, I think was reflecting something that was going on nationally. I suppose it was just kind of a rethinking of how we should teach and equip these students with the kind of skills that might be useful to them in their careers. So, I think that the idea that the business calculus course should be less theoretically rigorous but more maybe model oriented, a more practical approach to the subject might be better integrated with what they do in their careers. (Administrator 1)

So, the first change made to the Mathematics Department business calculus sequence was the adoption of the textbook (LaTorre, Kenelly, Fetta, Carpenter and Harris, 1998) that was more data driven and followed a modeling approach. From the perspective of the administration, because of the book's unique approach it is necessary to go completely through the content at least twice to really get a full understanding of the mission of the textbook.

It really takes a person a couple of semesters or something to really see first of all what they're trying to accomplish, then to come to your terms with it. And try to do mostly what they are trying to do. (Administrator 2)

After the textbook change took place, instructors adjusted and began to make this new idea work in their own classrooms. "So the book was the first piece and then the other piece of it was the end of 2003" (Administrator 2). In the Fall
of 2003, the administration decided that it was time for business calculus to be coordinated. An instructor was then selected to coordinate the course.

The primary essence of that was just to have uniform exams ... We never thought of business calculus as being a particularly easy assignment. We tried to make the coordination of this much more than just uniformity of exams, by trying to make it mirror what goes on in precalculus, with study guide. (Administrator 4)

The Department administration wanted there to be uniformity across sections of business calculus. "We want to try to have as much uniformity as possible across those sections. So, I think the coordination, to me that's a really important thing" (Administrator 1).

### 1.4 The Textbook

The book used to teach business calculus at OU is Calculus Concepts: An Informal Approach to the Mathematics of Change (LaTorre, Kenelly, Reed, Harris and Carpenter, 2005). This book takes a rather unique approach to the topics in calculus. Calculus Concepts has a very different mission than previous texts.
[The] overall goal is to improve learning of basic calculus concepts by involving students with new material in a way that is different from traditional practice. The material involves many applications of real situations through its data-driven, technology-based modeling approach. It considers the ability to correctly interpret the mathematics of real-life situations of equal importance to the understanding of concepts of calculus in the context of change. Complete understanding of the concepts is enhanced by the continual use of the fourfold viewpoint: numeric, algebraic, verbal, and graphical. (LaTorre et al., p. xv)

In addition, the book assumes that students have a certain type of calculator and are able to use it affectively to be successful in the course.

Typically students use calculators for multiplication and division and we like to think they could do it without the calculator, given the time. There is no way they could do these best fit curves if we gave them all year, even if we gave them a straight line, not a single one of them would have the least idea of how to do it. So, the calculator's being used in a much, much more significant way. And apparently in a worse way because they have no idea how it's doing it. But by the end of the business calculus sequence that has all changed. In an ideal world, when we finish bus calc II, those students have now moved from using the calculator to do something they had no understanding of, to in principle, they could do it by hand if they were really pushed, probably if they were really, really pushed ...We have them fit a straight line to a relatively small data set. (Administrator 4)

There seems to be a lack of agreement among faculty about how those teaching this course felt about the new book at the time of the switch. "Certainly I never heard rumors of great upheaval or anything" (Administrator 4). Since the faculty at OU do not teach this course, they seemed to be impacted very little by the change.

I think when that change occurred, I recall we had some discussions about that in faculty meetings but they were mainly informational and really the faculty just really didn't care that much. And I think the adjuncts just kind of do what they're told. And if they're told, there's a new textbook to use for the course, they just make the adjustment. I don't recall that there was any brouhaha when this happened. People just accepted it and it worked out pretty smoothly. (Administrator 1)

However, one of those in administration remembers things a little differently.
Adjuncts, had problems with it. I mean [they] just picked it up and were immediately unhappy. I mean look at the subtitle, An Informal Approach to Mathematics of Change. And it's informal, and that's one of the first things that an instructor has to learn is the language, the fact that they're
just going to be less formal ... [In the first edition] there was no formal treatments of limits, but things have changed some in subsequent editions. I think a lot of those things have been addressed. Adjuncts were by and large unhappy because it was very different than they had done before, and [they] generally like to do as little work as possible and this meant reworking all lectures, and not just transposing section numbers because it was a different book with the same basic mission. It was a lot of work the first few times. But, [they] were skeptical. And I think that was basically everyone's feeling on it. I mean I don't think that any of [them] were like revved up, like cool, you know a new business calculus book. It won some of [them] over eventually, and I think some of [them] are still grappling with that. (Administrator 2)

Thus, not even the administrators seem to have a uniform idea about how this change affected those actually teaching the course.

### 1.5 Coordination Effects

According to the administrators interviewed, business calculus was coordinated primarily for two reasons, clearly of equal importance. First, business calculus was not an easy teaching assignment. There was a need for a framework set up to help allow GMTAs to be able to focus more on teaching and less on structuring an entire class. Secondly, there was a need for more uniformity between sections in terms of grade distributions and material being taught. Although, there was an interest in coordinating all multi-section courses for uniformity, there was not unanimous agreement among the faculty about this issue. Thus, the priority for attention became the courses that seemed to be causing the most problems from lack of uniformity: business calculus.

Before the sequence was coordinated, when a GMTA was assigned to teach the course, she had to decide which sections she would cover, establish a pace for the course, and write all her own exams. Teaching this course was viewed by some as a lot of work, especially in comparison to other teaching assignments.

It's much nicer if you can get away with just trying to teach one class at a time and not having to plan the whole class. Somebody's telling you, you need to cover these three sections over the next two weeks or whatever. You need to accept that and now you can focus on just doing that, instead of someone telling you, you need to cover 5 chapters in the semester and make sure your students understand significant parts of it. That requires you to do a lot of planning. (Administrator 4)

Having a coordinated class with reduced burden on individual instructors allowed the graduate program to continue to grow as more GMTAs could be comfortably placed into a business calculus classroom.

In addition, this shift to a coordinated sequence helped to alleviate concerns about equity between sections of business calculus. Previously, a disproportionate amount of the department chair's time was taken up by students having equity issues with business calculus.

We wanted to get some kind of consistency. [The department chair], a few years ago, noticed that there were strangely different grades from one section to another, and he felt that this wasn't a very good idea, that we needed more uniformity. That's when he came up with the idea of having someone who would coordinate it. (Administrator 3)

The equity issues were of particular concern to those in administrative roles because when they arose, "in my heart, I, though I was very rarely in a position to
agree with them and do what they wanted, I could see a lot of arguments in their favor" (Administrator 4). The concerns over equity made those in administration uneasy because they did not feel they could honestly support their faculty and still treat students with utmost fairness.
[The department chair] wanted someone to take control over that so there'd be less discrepancy between [grade] distributions from one section to another. He wanted a B in one section to mean almost the same thing as a B in another section. The way he put it, in a very practical sense, was he does some analysis at the end of the semester and before we became coordinated he would be very anxious that a student would show up and they would say my friend is in section 2, I'm in section 3 here's how I did, my work through the semester, and here's how she did her work through the semester, and I got a C and she got a B. This just doesn't seem right. And he knew in his heart that these were very valid complaints. (Administrator 2)

Thus, through coordination, it would happen that a particular grade would mean the same thing from section to section because, after all, all students would take the same exams.

Furthermore, coordination seemed to be of particular importance in the business calculus course because of who taught the course. "I think it's really essential given the number of sections we teach and the fact that the population is graduate teaching assistants and adjunct lecturers that's just a migratory population, changes over time" (Administrator 1). Since it is hard to even have consistency with who teaches the course from semester to semester, through coordination, consistency of content, pedagogy, and standards can be better achieved.

However, while the move to coordination seemed inevitable to most of the administration, they still recognized that there were some drawbacks to the course's being coordinated.

This has been required of some very experienced teachers that probably regret the lack of flexibility that they have now in assigning their grades, or the pace at which topics are covered. And in truth I sympathize with that too. (Administrator 4)

## Furthermore,

It doesn't exactly engender creativity and originality in terms of how you do the material. So, to an experienced instructor in business calculus, one who's gone through it over a couple of years, it's possible it might be a negative effect in terms of how they perceive teaching the course and their level of interest in teaching the course in terms of trying to do different things they see might result in better learning for the students. (Administrator 1)

However, when it came down to making the decision to coordinate the class, the benefits vastly outweighed the drawbacks in the opinion of those involved in making the decision. It was felt by administrators that this move would be best for the students enrolled, the graduate students assigned to teach the course, and the department as a whole.

### 1.6 Differences in Business Calculus and Engineering Calculus

Business calculus and engineering calculus at OU differ in many different ways. It is very important to those in administrative roles that these courses really are distinctly different. "It's very much got its own identity, I think ... [the
department] wants them to be separate and [business calculus to] still have substance" (Administrator 2).

The first way in which the courses differ is the initial setup for the calculus in the course.
[In engineering calculus] we start with functions and use the calculus techniques ... [However, in business calculus] it's all data based. A huge amount of work goes into guiding [students] from finite sets of data to functions by using the calculator before they can even start using calculus techniques and there is just none of that present in traditional calculus. (Administrator 4)

The entire makeup of business calculus is based on a modeling approach.
Students learn how to find the best-fitting curve to a set of data before calculus concepts are introduced.

Because of this modeling approach, technology (primarily the TI-83, TI83 Plus, and TI-84 calculators) is used in very different ways than they might be used in an engineering calculus class. "You'll find some technology in some engineering calculus classes, but sometimes you won't find any" (Administrator 2). Absence of technology is not the case in business calculus. Moving beyond the material in the first two weeks of the first course requires substantial incorporation of the calculator in the business calculus classroom. "We use technology in a more uniform way" (Administrator 2). Students use calculators in ways not common to all calculus courses. Some primary uses are curve-fitting and
other calculus related topics such as taking derivatives at a numerical value, finding definite integrals, and solving equations with matrices.

Once students arrive at the true calculus component of the course (week 6), there are also variations between business calculus and engineering calculus. "It is a little less mathematically rigorous than the engineering calculus" (Administrator 1). There are certain topics that are not covered in the business calculus course that would be covered in its equivalent engineering calculus course, such as epsilon-delta limit definitions, integration by parts, and infinite series. One of the bigger omissions seems to be the lack of time spent with trigonometric functions. "There's a lot less trigonometry in the business calc sequence" (Administrator 2). "They don't quite make as intensive use of some of the trigonometric functions that we do in the regular calculus course" (Administrator 1). Historically, trigonometry had been omitted entirely in business calculus, much to the chagrin of those teaching it.

To me it was always ridiculous to omit anything in life that's periodic, that's often biological, but also actually quite often financial. To pretend the stock market doesn't show some kind of periodicity is just amazing to me. But I never understood why it was set up the way it was. (Administrator 4)

Currently Business Calculus I does include a small section with trigonometry, but no integration of this information is made into Business Calculus II.

In addition, in business calculus much time is spent on units and interpretation of answers. The language involved is vital from the perspective of
those who have taught the class for a long time. "Pretty much, there's a context for every problem. It's math with an extra layer on top of it and you've got this front end of modeling" (Administrator 2). The semantics allow the students to express clearly whether they understand the material, which is important to the nature of the course as well. But it all boils down to this,

So, basically, you've got this core of traditional stuff that has sort of this extra interpretation, kind of verbal thing, thrown on it and then you've got some technology with the modeling that leads you to the place you can talk about calculus. It's, it's very much got its own identity, I think. (Administrator 2)

Finally, the biggest way in which business calculus differs from engineering calculus is the classroom environment itself. Engineering calculus is taught primarily by tenured/tenure-track faculty in large lecture format with three GMTAs assigned to each course to serve as discussion section leaders. There are also occasionally smaller evening sections taught by GMTAs or adjunct lecturers. However, business calculus is only taught by GMTAs and adjunct lecturers with the exception of the online version of the course. Furthermore, except for one large lecture class in each of Business Calculus I and Business Calculus II, there are fewer than 40 students enrolled in each class. Because of the multiple sections, business calculus is coordinated. "The main difference is that it's coordinated" (Administrator 2). Coordination is necessary because so many instructors are teaching a course with the same title. "I am actually myself of the mind that anytime there are multi-section courses there ought to be as complete
uniformity as we can possibly have" (Administrator 4). Thus, the content covered and tests administered from section to section are uniform in business calculus, whereas they may differ some between sections of engineering calculus.

### 1.7 Opinions about Business Calculus

Opinions about business calculus are as varied between groups of people as they are between individuals in the Department of Mathematics at OU. Furthermore, it can be difficult to gauge actual opinions about this sequence. Students tend to be less than forthcoming with their opinions. "I read all the evaluations, but there are rarely comments about the course" (Administrator 4). Furthermore, GMTAs and adjuncts have not been in positions to affect decisions so they also tend not to volunteer opinions. Faculty have not been involved in the courses and thus lack a framework on which to base opinions. As a result, there seem to be discrepancies even among administrators as to the opinions held by students, GMTAs, and faculty, as well as administrators' individual differences about the course.

Some administrators feel that students find the course very difficult, while others feel that students find the course relatively easy. "My guess is... they find it pretty hard. Whether they find it useful or not I don't know" (Administrator 4). In addition, there is a question of simply whether or not they like the course.

I don't think the students like the course. It's a class that they're required to take and they're not particularly happy about taking it in terms of the requirement. And I would say that the average student would probably rather be someplace else. (Administrator 1)

On the other hand, one administrator has a rather different viewpoint.
My perception is that they get put through the ringer to such an extent by the school of business that business calculus is not one of their higher priorities... I don't think it's a highly demanding class for most of them, for the average student. And grade-wise, I don't think that they, they stress about it, but I think mostly because they need the A to offset the C they get in Managerial Accounting. (Administrator 2)

In the CoB students are used to low averages and strange grading curves.
"They're also made to feel inadequate" (Administrator 2). However, in business calculus, the Department of Mathematics is up-front with students, and grades tend to look more the way students expect them to look in terms of average grades, numbers of As, Bs, and so forth. Thus, students are less anxious during the semester than they are in some of their other business courses.

And I think it's kind of a double-edged sword. On the one hand we're being up front with them by saying there isn't going to be a curve, but we're going to be realistic about you know our expectations, you know reasonable. But on the other hand, they end up expecting a curve anyway sometimes because that's the other thing they're used to. (Administrator 2)

However, it seems that primarily the feelings of those enrolled in these courses are somewhat unknown even to those in the administration.

The perceived opinions about GMTAs opinions of business calculus are a bit more uniform. Primarily graduate students do not appear to particularly like teaching this course.

It probably is not their favorite teaching assignment. My perception is that the average graduate teaching assistant would rather teach an engineering calculus course than a business calculus course. And part of that might be that in their experience with the students, they might think that the engineering calculus students are more motivated and serious than business calculus students. (Administrator 1)

As this administrator noted, one probable reason for the dislike of the course is the type of students enrolled in business calculus verses engineering calculus.
"There's this stigma of having to teach business students rather than teaching engineering students or science students or math students" (Administrator 2). Another possibility for this distaste for the class is the content itself or its use of the graphing calculator.

A few will come and complain to me that they really didn't want to have to teach this. Occasionally I will get some who will tell me that they deplore what they perceive is the lack of mathematics, even the lack of calculus in business calculus. In truth I'm not particularly sympathetic to those comments. I think they miss the point, certainly in recent times I've had a graduate student come and tell me they didn't want to teach the class because there was an over reliance on the calculator. (Administrator 4)

Furthermore, business calculus tends to be a more time consuming class for GMTAs to teach in terms of the time it takes in preparation. "I think it's also perceived as being more work, maybe, and it is, than teaching [precalculus] or teaching discussion sections [for engineering calculus]" (Administrator 2). When one is teaching precalculus at OU , the lesson plans are already created by the course coordinator and little work must be spent in preparation to teach a class.

And when teaching discussion sections for engineering calculus, one must only prepare to teach one hour a week, in comparison to 3 hours in business calculus.

The administration is consistent within themselves about how they believe Mathematics faculty members view the course but at different degrees. Some believe the faculty members never even think about the course, but when they do it is slightly negative. They view the course as calculus with rather less mathematics involved.

I think that the faculty would say that it's sort of a second tier calculus course. Again, most of them probably don't think about it, but on those rare occasions when they do think about, they probably just regard it as a calculus light that we teach to students in the college of business because we have to. But I'm sure that they don't put it at the same level as the engineering calculus. (Administrator 1)

Some comments are a bit stronger in the sense that Mathematics faculty members do not view the course very positively. These differences are due to the students served by the course. "They're just happy that they don't have to teach it... It's just too problematic. And it's not the material, by in large it's the students, when you [teach business calculus] it's just a different type of student" (Administrator 4). Finally, there are some who view the dislike of the course a bit stronger.

I think it makes them feel dirty. I think it's something that they feel like we do it because we have to do it. It's a service and we do it. It's something we provide to the university, and it makes us a team player in some sense because we're providing this service to the business college... If they think of it at all, and they don't, but if they do, they think of it as being, yeah, we don't want to talk about it. It makes them want to go take a shower. (Administrator 2)

So, it is clear, at least in the opinion of the administration, that faculty members do not like business calculus, but it is less clear to what extent they dislike it.

Finally, the opinions of administrators themselves vary. One perspective involves non-opinion to some extent. "I know very little about what's in the course. I know practically nothing. I only know that it's a lot different. It's taught in a different way. They use different books. The aim is much different"
(Administrator 3). Others more or less accept the course as a kind of necessary burden. "I would say that it's a necessary course and I understand and appreciate why we teach it the way we teach it. But at the same time it's not as mathematically rigorous or comprehensive as the engineering calculus"
(Administrator 1). Some have come to truly embrace the course.
I, by now, I quite like it. To me, well I'm very keen that business calculus be very different than engineering calculus... it seems appropriate to me. To the best of my knowledge, it serves all the needs of the business college." (Administrator 4)

Then there are some who are still unsure about how they feel about the course.
It's always a conflict for me. I think it's good, but I think it's alright. It's doing what it needs to do. It being coordinated really, really had to happen. It was just chaos before that. I mean the semester before [it was coordinated], there were some grad students that taught it, and it was just like throwing them to the wolves. They were eaten alive by their classes. It just wasn't pretty. I think that it's interesting that it does have this reform aspect to it. It makes it more challenging to teach, but it, yeah, I think it's alright. (Administrator 2)

### 1.8 What Makes Business Calculus so Hard to Teach?

Business calculus is taught to students who typically are not very excited about mathematics. Furthermore, they might be less prepared than the average engineering calculus student.

These students will quite frequently not have strong mathematics backgrounds or rather worse would not have done much mathematics in high school in recent years, so some of them are even sort of refuges from mathematics. (Administrator 4)

These issues can create some frustration for instructors because their students are simply not like them. "[GMTAs] have this burning, burning love of mathematics inside them still because they're young and they haven't had that worked out yet. So, they're the ones that really want to pass that [passion] on" (Administrator 2). However, the students in this course may not allow for this transfer of passion.

Another reason for difficulty when teaching this class is that every problem has a context. Students generally arrive in this class with a negative view of word problems and they are immediately hit with a book in which every problem is a word problem. In addition, it has an interpretation component with which students are not familiar. All answers have units and, for much of what students derive, they are asked to interpret.

The type of context in the word problems can also engender concern among instructors. The problems are business related. They are centered in a realm in which the average mathematician may have had very little experience.

The instructor must become comfortable with concepts such as cost, average cost, revenue, marginal revenue, profit, consumer expenditure, producer surplus, present value, and future value, just to name a few. This experience can generate a jarring feeling to the first-time instructor. Many of the business students are more familiar with these terms than the instructor may be.

The use of the calculator is also unusual for many GMTAs. Most have been taught in a non-technological environment where many of them actually prefer to do mathematics by hand. They would rather not use the calculator if they did not have to. However, in this course, the calculator is indispensable. The entire first two chapters (out of the five covered) in the course are centered around the idea of using the calculator to find a best-fit model for a collection of data points. So, initially, there is concern over using the calculator period. However, simply accepting that students have the aid of the calculator is not enough. The instructor must learn how to use the calculator to find these quantities as well. This topic is not typically covered in an undergraduate mathematics degree program. So, the GMTA teaching business calculus for the first time must come to grips with learning how to use the calculator in an initially unfamiliar way.

The culminating effect of these challenges is that GMTAs assigned to teach business calculus believe that this particular teaching assignment is "perceived as being more work, maybe, and it is, than teaching [precalculus] or teaching [engineering calculus] discussion sections" (Administrator 2). Instructors
must prepare differently. They must make their own lesson plans. They must come to understand the philosophy of the book. "It really takes a person a couple of semesters to really see first of all what they're trying to accomplish, then to come to your terms with it. And try to do mostly what they are trying to do" (Administrator 2). These issues are not what GMTAs are accustomed to. With other classes, they can draw upon their own learning experience of the material and even the first time teaching the class is not as foreign to them as the first time teaching business calculus.

### 1.9 Research Questions

Because of its unique nature there are many things to explore when a GMTA is assigned to teach business calculus for the first time. How do they approach teaching a class they never took? After all, they were in pure mathematics classes as an undergraduate student. They have not experienced a class relating mathematics to business. So, how is it that they approach this teaching assignment? What teaching methods will they employ? Will those methods be different from or similar to those they have used in other teaching assignments?

Beliefs play a role in everything one does. But, how do a GMTA's beliefs about teaching, learning, and mathematics affect their teaching of business calculus? It is clear that they will be teaching different content than they've taught
before, but does their teaching have to actually change to accommodate this difference? They are also now teaching students who are definitely not mathematics majors, so will this affect their teaching? Furthermore, GMTAs' views of mathematics may be challenged, because the content in the class looks so different than the classes the GMTA has taught (or taken) before. Will this difference affect their teaching or their attitude toward their teaching?

Perhaps more difficult to observe, but certainly a valuable question relates to attitudes. How do the attitudes of the instructor regarding the class affect the attitudes of the students regarding the class? Will students sense the positive or negative attitudes of their instructor regarding the course itself? If so, how will their attitudes be affected?

While the primary resource most GMTAs use for preparing to teach is their own prior knowledge, this may not be true for this course. Because of the differences in content and delivery of the content, what types of resources will the GMTA use to prepare for class? Because of the unique nature of the class, how can we better prepare GMTAs to teach this class? What resources can we provide? What support is needed? How can we make this experience the best learning experience for the students and the best teaching experience possible for the GMTA?

Finally, as in many universities the overall perception of this course in the Mathematics Department at OU is negative. How can we change this perception?

In what ways can we begin to change the perception of this course among GMTAs and then among the department as a whole? How can we better develop a correct idea of what business calculus is and the value it has for those to whom it is taught? How can we help GMTAs to believe teaching this course will positively affect their future? How can we get the department to positively support this program?

In this study, current literature regarding GMTAs, undergraduate teaching, and beliefs are examined and an overview of the research methodology is presented. Beliefs and classroom experiences of both GMTAs involved in the study are examined and compared. Finally results of the effects of beliefs and classroom experiences are examined and presented along with recommendations for further study and improvement of the GMTA business calculus teaching assignment.

## Chapter 2: Literature

Research on graduate mathematics teaching assistants (GMTAs) is a relatively new field of study.

At present little is known about the characteristics of graduate students who are mathematics teaching assistants. Equally unexamined are the factors that shape and facilitate development and change in those characteristics. GMTAs, however, play significant roles in the instruction of undergraduate mathematics students. (Speer, Gutmann, \& Murphy, 2005, p. 1)

While research on GMTAs is relatively new, the study of $\mathrm{K}-12$ teachers is well documented. Some of the research that has been performed at the K-12 level is applicable and useful at the collegiate level. However, different aspects must also be considered when one studies the GMTA experience. The youth of the field and its as-yet imprecise relationship to the K-12 literature has led to the use of case studies to discover and document areas of current and future interest.

One of the initial areas of interest involves GMTAs' beliefs, and how their beliefs influence the decisions they make during class. In addition, there are questions concerning the expectations teachers have of students based on their own beliefs. There is also interest in how GMTAs structure their classrooms around the beliefs they hold regarding problem solving.

Other influences also interact with a GMTA's beliefs as they teach. These include their limited teaching experience, lack of pedagogical content knowledge,
their support network, the availability of concrete resources, the expectations of the department, the available amount of preparation time, and the class pace. In addition, lack of experience can contribute to a GMTA's inability to make sound decision when situations arise.

Another topic of interest is a GMTA's ability to alter their teaching methods. This need can arise when a GMTA is expected to use methods with which they are less familiar. Making this transition is a complex process for GMTAs, involving changes to their beliefs as well as to their classroom practices. It is often necessary for GMTAs to make these types of shifts in their teaching styles when a less-traditional method is adopted.

Other concerns come from differences between GMTAs' experiences as students and the experiences their students have had. First, a student in a lowerlevel undergraduate classroom may not be interested in mathematics with the same passion that the GMTA was as an undergraduate. In addition, it is often difficult for GMTAs to make the transition to more applications-based classes from the traditional mathematics classes they encountered. It is also a challenge for GMTAs to incorporate technology they personally did not use when learning mathematics.

Finally, because of the need for quality GMTA teaching, aspects of TA (Teaching Assistant) training programs are examined. Recommendations are presented to improve the current TA training methods, and quality aspects of the
current TA training programs in the United States are highlighted and presented for dissemination to other countries.

### 2.1 Case Studies

"A good case study brings a phenomenon to life for readers and helps them understand its meaning" (Gall, Gall, \& Borg, 2007, p. 446). A phenomenon is an item of interest to the researcher. It could be "a process, event, or person or some other item... A case is a particular instance of the phenomenon" (p. 447). Case study research is in depth, is in its real-life context, considering at least one instance of the phenomenon, and reflects the perspective of the subject(s) being studied. In this type of research, data is collected over a period of time, often with more than one method of data collection. This data is often in the form of "words, images, or personal objects" (p. 448).

According to Gall et al. (2007), the purpose of a case study is either "to produce detailed descriptions of a phenomenon, to develop possible explanations of it, or to evaluate the phenomenon" (p.451). If its purpose is to describe the phenomenon, the case study will include statements that re-create the situation within its context including meanings and intentions pertinent to the situation. If the purpose is to explain the phenomenon, this explanation is called a pattern, "meaning one type of variation observed in a case study is systematically related to another observed variation" (p. 452). If the purpose is to evaluate the
phenomenon, there are several qualitative approaches possible. Each would include a detailed description and several forms of evaluation including identifying themes and patterns in the phenomenon.

When a case study is chosen, there may be many reasons why it is a valuable and appropriate choice. Case studies are ideal for researching unusual or previously unstudied phenomenon. They allow the researcher to gain insight into a specific phenomenon, and the focus of the research can be altered as new information comes to light. However, it is a time- and labor-intensive research method. In addition, highly developed verbal skills are needed to make the case come alive for the reader and to identify informative themes and patterns in the verbal data collected. However, there may be limited generalizability of the findings to other situations (Gall et al., 2007, p. 485). However,

The case study researcher, through the process of thick descriptions can bring a case to life in a way that is not possible using the statistical methods of quantitative research. Thus, readers of case study reports may have a better basis for developing theories, designing educational interventions, or taking some other action than they would have from reading only quantitative research reports. (p. 484)

For the purposes of looking into how GMTAs learn to teach a course they have never taken, a case study was a natural choice. GMTA research is a very young field and the phenomena are not yet fully established. A case study will help to develop a thick description of the phenomenon which would not be
attained using other research design methods (more about the choice of a case study for this project is discussed in chapter 3).

### 2.2 Applicable K-12 Research

Research about K-12 teaching is more developed than research about GMTAs. This research has been occurring for longer and the phenomena are well established. Therefore, $\mathrm{K}-12$ teaching research is an appropriate starting point when one begins looking at GMTA teaching. It is possible to draw many conclusions from the research that has taken place at the K-12 level. This research has extensions to teaching at the undergraduate level.

## How Teachers Make Decisions

In a case study by Thompson (1984), three junior high school teachers were each observed and interviewed daily for four weeks. These teachers had experience teaching at the same school and same grade level for at least three years. The purpose of the study was to see how these teachers integrated their knowledge of mathematics into practice.

The teachers varied in many ways including the "integratedness of their conceptions of mathematics, their awareness of the relationship between their beliefs and their practice, the affect of their actions on the students, and the difficulties and subtleties of the subject matter" (Thompson, 1984, p. 123).

Because of these differences Thompson believed that "teachers' conceptions are not related in a simple way to their instructional decisions and behavior. Instead, the relationship is a complex one" (p. 124). Many factors interacted with their teaching, affecting their decisions and behavior, including non-mathematically specific beliefs.

Patterns of behavior emerged from the teachers' instructional practices. Thompson suggested that these patterns may come from consciously held beliefs shaping their behavior. On the other hand, they may stem from unconsciously held beliefs arising out of their experiences (Thompson, 1984, p. 105).

## The Influence of Prior Teaching

In another study at the K-12 level, Borko and Putnam (1996) found that "experienced teachers attempts to learn to teach in new ways are highly influenced by what they already know and believe about teaching, learning, and learners" (p. 684-685). Thus, when teachers are hesitant to change aspects of their teaching, this reaction may be due to the beliefs they have already established during their teaching career about how students learn and about the effectiveness of the methods they currently use. If they do not believe in certain methods of delivery, or if they have been successful with certain teaching methods they may not be willing to try something new. In addition, if instructors hold certain beliefs
about learning that conflict with the way in which a new method guides students, they may be less apt to adjust.

## Levels of Beliefs

In a contemporaneous study of teacher beliefs at the K-12 level, by Franke, Fennema, and Carpenter (1997), 21 elementary teachers were followed for four years. Participants were thought to develop their "problem/strategy frameworks based on the development of children's mathematical thinking and adjust, build, and expand them based on the knowledge they gain(ed) as they interact(ed) with their students and the mathematics" (p. 260).

The researchers identified four levels of beliefs regarding problem solving. At level one, a teacher believed that students could only solve problems if they were explicitly taught how. Hence, these teachers used only direct teaching methods such as lecture in their classrooms. Level two teachers began to believe that students could solve problems without being shown a specific strategy. These instructors let students try to solve problems on their own before being given a specific strategy for solving them. Teachers at level three believed that students could solve problems without a strategy provided for them and allowed more opportunities for students to solve problems themselves. Teachers operating at level four believed that "they can and should use what they learn about their students' mathematical thinking to drive their instruction. They believe[d] that
knowing the individual children [was] of utmost importance" (Franke et al, 1997, p. 269). These instructors actively used their knowledge about their students to guide them in making instructional decisions.

### 2.3 Extensions from K-12 teachers to GMTAs

Much research in teaching can be applied across different levels of education. However, teaching college and teaching high school are different, each with their own issues and concerns. So, while some research findings of K-12 teaching literature can and should be applied to teaching college, there are some nuances to teaching college that must also be addressed in research at the undergraduate level.

The first issue related to a GMTA's teaching experience involves the training that GMTAs receive. Although many GMTAs participate in an orientation program, most do not participate in "compulsory, extensive, teacher preparation programs" (Speer, Gutmann, \& Murphy, 2005, p. 4). Furthermore, for many, their first GMTA position is the only time they will take part in any professional development about teaching. This situation is in stark contrast to the experience of a K-12 educator, who completes many college classes specifically designed to help them understand different teaching methods and pedagogies. Because college faculty are unlikely to receive further guidance in their teaching, "the practices they develop as GMTAs may shape their teaching for the rest of
their careers" ( p. 4). Thus, it is very important that this experience provides "rich opportunities to support and shape emerging instructional practices" of GMTAs (p. 2).

While the only formal training that GMTAs may receive is at the very beginning of their career, they will continue to develop their teaching practices for many years. Therefore, many types of interaction will shape how these practices evolve, including beliefs about teaching, pedagogical knowledge, and content knowledge. Because professional development opportunities tend to be minimal, the primary means for GMTAs to be shaped is through interaction with their students (Speer, 2001, p. 271). As with K-12 teachers, these interactions give GMTAs an indication of how students learn and what the role of the instructor is in the classroom.

Another difference affecting GMTAs is enculturation. According to Speer et al, "Graduate students must learn to function in their departments" (p. 4). In order to be successful, many will feel the need to adopt the attitudes of the faculty. "Pressures to become part of the existing culture are strong" (p. 4). Thus, even if they enter their GMTA experience with certain beliefs, they may alter them in order to fit in with the beliefs in their department. In addition, GMTAs can become isolated with no opportunity for growth through interactions with other GMTAs or faculty, in part because of the time demands placed upon the GMTA as they juggle their dual roles of student and teacher. They may disengage
with other GMTAs or faculty as they try to manage their time by eliminating what they may view as less important, including talking with colleagues about teaching.

### 2.4 Issues Encountered by GMTAs

## The Need for Fluid Knowledge

As students, GMTAs are often very good at acquiring static knowledge. However, Borko and Putnam (1996), found that teachers' "knowledge is often not sufficient or appropriate for supporting teaching that emphasizes student understanding and flexible use of knowledge" (p. 698). Static understanding can make applications in new areas difficult for a GMTA, as can teaching mathematics with new technologies or teaching strategies. Because of the lack of understanding of how to apply concepts, this static knowledge may not be enough to allow GMTAs to expand their own ideas to teach the material in a new context.

## Subject Material and Applications Change

In addition to new teaching methods, new subject material and application of mathematics may have developed since GMTAs were taught the mathematical concepts they are responsible for teaching in their classrooms. In a summary on case studies, Romberg (1997) found that often GMTAs "ha(ve) personally studied
mathematics as isolated areas of content with reasonable success" but when it comes to integrating mathematics in a new way they struggle. They have never "really done mathematics in the manner approached" by their current course content (p. 369). This discrepancy can cause problems for GMTAs when the curriculum demands the integration of this knowledge. A GMTA may have understood the material when it was presented in its own right, but may not understand how the material can actually be used in different areas or in different ways.

## Mathematics Came Easier for the GMTA

Speer (2001) also believes that a GMTAs' own experiences with mathematics may make them unprepared for how their students will understand mathematical ideas. Many GMTAs fail to realize that understanding mathematics does not come as easily to others as it may have come to them. People tend to think that others can and should be able to learn in the same way and with the same amount of effort they themselves did. Yet this belief is not always the case, especially in the types of classes that GMTAs are often asked to teach. As a result, there is a need for reform, a call for "new teaching practices, different from those that the teachers themselves experienced or were taught" (p. 272).

## The Benefits of Cases

Another unique problem that new GMTAs tend to encounter is that they have not seen a wide range of student responses to teaching. Thus, they do not yet have an idea of what works and what does not (Friedburg, 2005). To complicate the issue further, GMTAs have limited access to students who differ from themselves, "students who study mathematics for different reasons than they did or, even more strongly, students who find mathematics frightening or uninteresting" (p. 843). Friedburg suggests using situational readings, called cases, to prepare GMTAs better for these kinds of encounters. Cases give the reader and colleagues an opportunity to discuss an example that may occur in their classrooms. The purpose is to allow GMTAs to begin to construct ideas about how they would handle certain events if they were to occur in their classrooms.

### 2.5 Factors Influencing GMTAs

## Social Influences

The powerful influence of the social context resulting from the expectations of others can be a cause for concern among GMTAs. These expectations can come from students, peers, and supervisors. In addition, there is the institution itself constraining a GMTA's actions by way of an adopted text and
the system of assessment. These constraints often occur through coordinated classes.

These sources lead the teacher to internalize a powerful set of constraints affecting the enactment of the models of teaching and learning mathematics. The socialization effect of the context is so powerful that despite having differing beliefs about mathematics and its teaching, teachers in the same school are often observed to adopt similar classroom practices (Ernest, 1988).

Because of these constraints, GMTAs often find themselves uncertain about deviating from the norm, even if their beliefs or other experiences indicate that they should.

## The Influence of Beliefs

Another factor strongly influencing GMTAs is their beliefs. In a study by Speer (2001), the teaching methods and beliefs of two GMTAs were captured over the course of a semester through videoclips and traditional interviews. Speer videotaped class sessions and selected portions of each videotape (a video clip) to view with the instructor. Then she and the instructor viewed the video portions together and discussed what was happening in the video clip. In some sessions, she asked a series of questions about the taped session itself; for other sessions, she had the GMTA describe what was happening in the video clip.

Speer's work was a case study that examined the beliefs versus the classroom practices of the participants. Her work was based on research about
beliefs of K-12 teachers. Some research seems to suggest that beliefs and practices do not always coincide. Speer's study examined this phenomenon. Both GMTAs interviewed were discussion leaders for reformed-based calculus classes. From interviews addressing the beliefs of these GMTAs, it appeared they had very similar teaching styles. The language used by each participant when discussing his beliefs was very similar. However, from classroom observations it was clear that the students in each class were experiencing very different learning opportunities, and the learning outcomes differed substantially. Speer concluded that much of the difference between these two GMTAs lay in their beliefs regarding what constituted a student's understanding a concept. "(T)eacher's beliefs are a factor in how teachers teach. Beliefs also shape the way in which teaching practices develop initially and change in response to professional development and reform" (p. 259). The way they described their beliefs and the ways in which they were acting them out were not necessarily inconsistent. Rather there seemed to be a lack of consistency with what the GMTAs meant by the words they used when they were describing their beliefs and the ideas conjured in the mind of the researcher.

One specific difference between the two GMTAs in the study was that they had differing ideas of what it meant to learn mathematics and how an instructor can determine if a student understands the material they are studying. "In particular, the rationales for their decisions made frequent reference to what
they believed learning was and how they believed learning occurred" (Speer, 2001, p. 262). Thus, they evaluated their students in different ways based on what they felt learning mathematics meant.

Furthermore, in contrast to the idea that GMTAs may not know how to teach well when they enter their job as a GMTA, Speer found that "GMTAs bring well-formed beliefs about teaching, learning, students, and mathematics to their early teaching assignments" (p. 269). As a result, it is particularly important that there be training before and during the GMTA experience.

After examining these two GMTAs, Speer (2001) found that "general descriptions of teachers' beliefs and practices fail to do justice to the subtle complexities of teaching interactions" (p.259). She concluded that more specific ways of describing teaching and beliefs about teaching were needed. These ways would allow the researcher to address the differences between what a GMTA says they believe and what they mean by the vocabulary they are using to express their beliefs.

## External Influences

Like Speer, Belnap (2005) studied factors influencing GMTAs. In a set of case studies, Belnap studied eight different GMTAs at different levels of teaching experience and over different time frames by way of videotapes, interviews, and class observations. By examining the GMTA's beliefs, teaching practices, and the
external effects on both of these, he found a variety of factors influencing GMTAs' teaching. Specifically, Belnap noted that some factors influencing one GMTA strongly did not influence another GMTA at all. These facts prompted Belnap to investigate further the factors influencing the GMTA's teaching and beliefs.

According to Belnap (2005), one influencing factor was the current teaching context in which the GMTA was involved. This factor was the "most prevalent source of influence on GMTAs' teaching" (p. 175). The environment, students, and other current time commitments facing the GMTA influenced teaching significantly.

However, "pedagogical knowledge was a factor with varied effect" (p. 182). This knowledge of different teaching methods and ways of learning affected some GMTAs greatly and others not at all. For one GMTA it impacted how he prepared for class. Two other GMTAs wanted to try new pedagogies but felt limited in their knowledge and unsure how to incorporate them into the curriculum.

Colleagues, typically fellow GMTAs impacted GMTAs by serving as a support network and serving as a resource in their development as teachers (p. 183). These colleagues were significant networks of support for the GMTAs, allowing them to have resources from which they could grow in their development as teachers.

Course materials also served as a resource when GMTAs relied upon the textbook, syllabus, and instructor's guide. These resources saved them time in class preparation while still allowing them to incorporate group work in their classroom - which was a pedagogy emphasized by the department. They also allowed the GMTA to utilize other methods they might not have been willing to implement if left on their own.

Despite a variety of support resources, limited amounts of instructional time often kept the GMTAs using lecture, even when they wanted to try other methodologies because lecture was easier to prepare. Using lecture for this reason left the GMTAs unsatisfied with their teaching experience.

Finally, the department-outlined course pace seemed to dominate the need to continue with new material in spite of holes in student understanding. One GMTA admitted that sometimes the course pace held more influence than all other factors. The need to cover all the intended material often meant that even when GMTAs wanted to cement student understanding by a more in-depth focus on a particular topic, they opted instead to plow ahead so as not to fall behind the set schedule.

### 2.6 TA Training

## The Need for Change

In a review of the reform of mathematics teaching, Goldsmith and Shifter (1997) describe traditional mathematics instruction as the giving of "clear, comprehensible, and correct information about mathematical procedures" (p. 22). Furthermore, in traditional classrooms, the teacher and textbook are the authorities in determining right and wrong with the classroom being centered around the transfer of knowledge from teacher to students. This alleged transfer occurred through routine drill and practice by relying heavily on rote memorization. Unfortunately, traditional instruction "reinforce(s) the perception that mathematics is this mysterious and conceptually inaccessible" object (p.20). While this pedagogy is typically what a GMTA experienced, it is not the current direction in teaching.

As Goldsmith and Shifter (1997) describe traditional methods and the shift in mathematics to more interactive classrooms, they recognize that it is one thing for teachers to accept the current beliefs about teaching mathematics and something else entirely to structure a classroom "that actually stimulates mathematics learning in such ways" (p.25). Often teachers can pinpoint what they should change but do not know how to do so or simply may feel uncomfortable doing so.

Furthermore, not only do teachers need to acquire new strategies and techniques, but they also need the opportunity to reflect upon their beliefs about teaching, learning, and mathematics. Through reflection, teachers may make changes to their teaching techniques on their own, adapting their beliefs as they see positive results among their students. In order for this reflection to occur, teachers need compelling opportunities to observe their students and examine their own teaching practices. Therefore, merely learning about new teaching methods or realizing how they may help their students is not enough. Instructors also need time to think about their beliefs and their students in order to enact these kinds of changes in their classroom.

While teachers may be reluctant to make changes from their comfortable lecture-style instruction, "the motivation for helping teachers develop new forms of practice is high, but the means by which teachers actually do so are currently not well understood" (Goldsmith \& Shifter, 1997, p. 20). At both the K-12 and postsecondary levels, some schools are more helpful and/or forceful in wanting the change to more interactive classrooms to be made. They may encourage and actually assist change to happen or they may force the change to be made. However, other schools do not make this change a priority.

## Recommendations for Improvement of TA Training

Focusing on the improvements needed for TA training, Luo, Grady, and Bellows (2001) conducted a questionnaire-based quantitative research study with 304 TAs in 45 academic disciplines. Data were collected through mailings to study TAs' perceptions of various instructional issues. The study concluded with many different suggestions to help TA training developers better prepare TAs for their responsibilities in the classroom.

One identified need was for TAs to examine their beliefs about what constitutes good teaching. In addition, TAs needed to assess their "attitudes toward the value of communicating with students and using a more interactive teaching style" as well as to consider what their expectations were for the students in their classroom (Luo et al., 2001, p. 224). These activities all require focused time and might need to be directed activities - which has implications for TA training programs.

A second area of concern for Luo et al. (2001) was that TAs develop a better understanding of their students. "Effective learning requires that instructors have a good understanding of students. Only when instructors understand students can they be effective in helping students learn" (p. 225). The authors claim that by understanding students better, TAs will be better able to identify the expectations students have, the needs they might encounter, and the different learning styles that may appear within their classroom. By knowing the needs of their students,

TAs will be able to "enhance classroom communication" (p. 225). Considering the needs of individual students and the class as a whole can help teachers assist students in understanding mathematics to their fullest potential.

Finally, the study found that "TAs have identified good supervision as the most powerful influence on their career decision of being college faculty. In fact, TAs look up to supervisors as role models" (p. 225). When TAs are supervised by those who value teaching, a strong and lasting impact on TAs who may work in academia upon graduation is likely. On the other hand, lack of sufficient supervision gives TAs the impression that teaching is of little importance, and the value they should place on being an effective teacher is not very high.

## Applications for TAs in Other Countries

In an article summarizing major findings from the development of TA training programs across North America, Park (2004) makes many recommendations for those preparing TA training programs in the United Kingdom (UK). One early remark about the current training programs is that they have been "based on the premise that teaching can be learned, practiced, and continually improved" (p.351). Based on this belief, TA programs have been developed to improve the teaching of TAs.

Some of the aspects of a good TA training program that Park (2004) identified include the following. He saw that a good TA program evolves over
time, keeping the program fresh and up to date. If a training program stays static, it is not adjusting to current research in how students learn. He also found that, because all teaching has some aspects in common, TA programs should include general teaching information. Yet, because departments differ as well, TA programs should include departmental specific components to help TAs become better informed about the aspects of good teaching within their respective disciplines. Good programs also included supervision and peer mentoring. These aspects provide TAs with role models and peer support. Therefore, Park echos Lou et al. (2001) in the need for supervision of TAs, but he goes one step further to suggest that TAs also need peer mentoring, which was also recognized by Belnap (2005) as important to GMTAs.

As Park (2004) summarized suggestions to the UK for developing effective TA training, he strongly emphasized the need for TAs to use reflective practices to increase self-awareness of their teaching. He stated that TAs "should be encouraged to evaluate the difference between their actual and theoretical teaching styles, using appropriate reflective activities" and "to continually redefine their personal goals in the context of department-imposed conditions" (p. 357). In this respect, Park echos Goldsmith \& Shifter (1997) as he calls for TAs to reflect upon their teaching.

## More Suggestions to Assist GMTAs

One of the primary needs for GMTAs is a support structure. Because GMTAs experience a variety of challenges associated with their teaching, like policy issues, administration concerns, and interactions with students, a support structure is helpful for GMTAs. It can help them get ideas and cope with different situations. This structure could be very formalized by way of required attendance at meetings or a formal peer partner with whom the GMTA is to work, or it could be less formal but merely a departmentally guided activity. "(T)he focus of the department and the training programs would be better spent in building up a support system by helping GMTAs network with their peers and with knowledgeable faculty supervisors" (Belnap, 2005, p. 212). These support systems might also include a faculty member the GMTA would feel comfortable approaching for advice.

According to Belnap (2005), a second thing GMTAs need is more wellinformed pedagogical knowledge. Inexperience and lack of educational training limits many GMTAs in their teaching practices. "Preparation programs should help GMTAs develop knowledge about pedagogy, about curriculum, and how to integrate them" (Belnap, 2005, p. 213-214). These types of opportunities are very limited for GMTAs and would serve them well as they become the next generation of faculty.

In addition, including motivation as a component of GMTA training would enable GMTAs to see how they impact the department and encourage them to take a vested interest in the goals associated with educating undergraduate students. There should be reasons that a GMTA would want to exert an effort to maximize their teaching potential, perhaps rewards or recognition.

Finally, GMTAs need access to a wide variety of resources. These resources should include more than the textbook and syllabus. GMTAs also need access to experienced instructors, who should be carefully chosen, so as to support the program's goals without causing a time burden on the these instructors (Belnap, 2005). A wider range of resources would help to keep GMTAs from becoming stagnant in their development as effective instructors.

### 2.7 Conclusion

For several reasons, there is a need for more attention to GMTAs as teachers of undergraduates. First, most lower-level undergraduate mathematics classes at research universities are taught (or TA-d) by GMTAs. In addition, if GMTAs have an increased understanding of teaching and learning, they are likely to be more effective as teachers. Finally, because GMTAs have tended to be placed in the classroom with little or no formal educational experience, there is a need for training to support GMTAs as they make this transition from student to teacher. This training is likely to be most effective if it is based on a research-
driven understanding of the professional development of GMTAs as instructors. In particular, it is clear that GMTAs beliefs upon entering graduate school affect their teaching.

A specific area that has been examined is how GMTAs make pedagogical decisions about teaching methods. Decision making is a complex process and not all GMTAs make their decisions based on the same factors. Research that already exists from K-12 studies can be applied with some limitations to the GMTA experience, but it is important to realize the differences between TAs and inservice teachers. The few studies that have been completed seem to suggest that GMTAs simply do not have the knowledge of either different pedagogical options or how to apply different methods in the classroom.

Friedburg (2005) notes that there was a time not long ago when "teaching skills were of little importance to many institutions" (p. 842). However, this situation has changed and TA training programs have been developed and have become more common. GMTAs are a particularly important population because

If we cannot succeed at the teaching of mathematics to undergraduates, then the pressure to have others do so in our place will increase. In the long run, then, mathematics will do better if the next generation of mathematicians on university faculties are excellent teachers. (p. 842)

In other words, there is a need for attention to GMTAs as teachers both because they are in immediate contact with undergraduates and they are the pool from which the future professoriate will emerge.

## Chapter 3: Methods

In order to better understand how a GMTA develops their own teaching techniques we could ask them, we could watch them teach, or we could talk to them about a class we have watched them teach. However, each one of these methods is only a piece of the puzzle. In each method we would be missing something. We may have miscommunications in the vocabulary we use, or we may have perceptual differences in what we experience in the classroom.

It is quite plausible that there are situations where teachers state beliefs that are (intentionally or unintentionally) inconsistent with what they carry out in their classrooms. It is possible, however, that perceived discrepancies are sometimes the result of incomplete or inaccurate understanding of terms and descriptions used by teachers and researchers...This disconnect, or lack of shared understanding, between teachers and researchers means that resulting data may not accurately represent teachers' beliefs or practices and may shape findings and conclusions in significant ways. (Speer, 2005, p. 371)

Thus, we have a need for something more detailed and specific to allow us to better explain the teaching practices and developments GMTAs make in their teaching.

### 3.1 Reasons for a Case Study

GMTA research is such a young field that the phenomena are not yet welldefined. Furthermore, to date, no research has been performed on the teaching of
business calculus by GMTAs. In order to be able to develop theories about this experience, it is necessary first to create a detailed description of the phenomenon. Accordingly, a case study research design is a natural choice. By using a case study design we hope to achieve the type of detailed description that will lead to a formulation of possible patterns and themes.

As with all case study research, appropriate cases needed to be identified according to relevant criteria that contribute to explanatory power. One decision for this particular study was to involve two GMTAs who are currently teaching business calculus, for a comparison opportunity. To understand how teaching strategies develop in such a context, the GMTAs selected were in their first experience teaching Business Calculus I. To eliminate certain variables, the cases were selected to be fairly experienced instructors. Because they had already taught several semesters of other courses, we were able to examine how this teaching experience was different for them. In addition, because they were experienced, we were able to distinguish between issues related to learning to teach in general and issues related to learning to teach business calculus specifically. The number of candidates who met these criteria was predictably small. Typically, only a handful of GMTAs are new to teaching business calculus each semester; some semesters there have not been any new GMTAs teaching the course. Both GMTAs who met the criteria during the semester that data was collected were invited to participate and both agreed.

### 3.2 Justification for Videoclips

Videoclips offer a semi-structured way in which researchers can bring context into an interview in order to create shared understanding between the researcher and the GMTA. There are two ways this shared understanding is accomplished. The first is for the researcher to videotape a class session of the GMTA. Then the researcher chooses portions of the videotape on which to focus during the interview. These portions, referred to as videoclips, may be selected for a variety of reasons pertinent to the issues of interest. During the interview, the researcher and the GMTA will view the videoclips, providing a context for the interview and concrete examples upon which both the researcher and instructor may draw. Videoclips allow for a basis of commonality for the discussion which will ensue.

Using videoclips is a relatively new method created to develop a more connected picture of how the beliefs and teaching practices of GMTAs interact, bridging the previous gap in the understanding between teachers and researchers. In a case study of one teacher's beliefs and practices, Wood, Cobb, \& Yackel (1991) used videotaped recordings in order to identify changes in the teaching methods of the instructor. This study was one of the earliest employing videoclip data. It allowed for Wood et al. to begin understanding how changes in teaching methods take place.

As referenced in Chapter 2, the case study by Thompson (1984) involving three junior high school teachers also included observation, videotaping and interview. Furthermore, videoclips were used, "followed by systematic analysis and stimulated recall in informal interview settings...in order to gain access to the thoughts and mental processes that accompany the teachers' actions" (p. 126).

In addition, Frederickson, Sipusic, Sherin, \& Wolfe (1998) attempted to create and evaluate a "systematically valid" performance assessment by way of a "Video Portfolio" which consists of video recordings and classroom descriptions. In addition, one must have self-assessment from the teacher himself. This technique is similar to what others have referred to as videoclips.

Moreover, as the frontrunner of GMTA research, Speer (2001) relied heavily upon videoclips. In her study, Speer explained that "videoclip interviews improve shared understanding by utilizing a shared artifact that grounds the conversation in examples of the teacher's practice" (p. 75). She further explained that without this shared artifact, what the researcher and the GMTA mean by a certain phrase could be quite different, creating a lack of shared understanding. While the teacher and the researcher could still view a videoclip differently, videoclips give the interview a concrete example upon which to build the conversation. Through the use of videoclips Speer was able to see that the GMTAs in her particular study were using similar language to describe very different activities occurring in their respective classrooms.

Speer (2005) further stated that through videoclips "it is possible to obtain information beyond what is possible in traditional, de-contextualized interviews or in a combination of interviews and observations" (p. 377). Obtaining such information is possible because videoclips allow the researcher to collect data on beliefs specifically tied to the practices of the teacher, allowing for a more accurate description of the beliefs which could be attributed to the GMTA. Thus, it is possible for the researcher to better understand both how the GMTA believes he is teaching and how he is actually teaching from the viewpoint of an observer.

### 3.3 Participant Selection

Because this study focused specifically on Business Calculus I (MATH 1743), it was necessary that the GMTAs asked to participate in this study be teaching this particular course during the semester of data collection. It was clear that those who have not had prior experience teaching this course tended to misunderstand the nuances associated with teaching the course. As noted above, I did not want to focus on the difficulties associated with inexperience in the classroom, but rather the difficulties associated specifically with the business calculus teaching assignment. Yet I also did not want to have someone who had become comfortable with the course to such an extent that they had forgotten the difficulties associated with teaching it for the first time. Thus, my specific search criteria was for those I studied to be both experienced at teaching and teaching
this particular course for the first time. In order to identify such participants, I spoke with the coordinator of the course and found out that there were two such instructors assigned to the course during the semester of data collection, Terry and Jessica (pseudonyms used to protect confidentiality).

Both Terry and Jessica had been teaching at OU for at least 3 years, thus had established their own teaching practice and beliefs. However, neither one had taught business calculus before. It also turned out that each had their own unique reaction both to the teaching assignment and to the teaching experience. Upon explaining the purpose and plan for the research to both Terry and Jessica, including the time commitment on their part, each agreed to participate in the study.

It is also worth noting that Terry and Jessica had additional similar attributes, thus contributing to the reduction of extraneous variables. First, both were PhD students planning to pursue careers in academia. Hence, they knew the importance of teaching and planned to continue teaching in their future. In addition, both were considered successful in their teaching. In general their students responded positively to their teaching styles and had no major issues with them as instructors. Furthermore, Terry and Jessica both enjoyed teaching. It was not merely something that they did because the department required it of them; it was actually a positive part of their GMTA-ship.

### 3.4 GMTA Data Collection

Basing much of the research design on Speer (2001), I developed a basic protocol for an initial interview (Appendix A) with each participant. GMTAs were given the interview protocol prior to our first interview. I followed the same protocol at the final interview. Both the initial and final interviews were semistructured. The interview protocol was used as a reference and as topics came up the order in which the topics were addressed differed both between participants and between the initial and final interviews. Furthermore, other topics of interest were also noted and explored as they arose during the interview. All interviews were audio-recorded for the purpose of later transcription.

The initial interview served as a reference and guide during the videotaping and subsequent interviews, allowing me to better understand the background and professed beliefs of each participant. The final interview provided an opportunity to investigate whether any of the beliefs or teaching practices of the two participants had changed during the span of the study. It was not meant to directly point to the study as the reason for the change but to identify changes (if there were any), how they were made, and to discover the reason for the change.

The questions asked in both the initial and the final interviews addressed the GMTA's beliefs and their teaching practices. The conversations began with questions regarding their philosophies of teaching and learning as well as their
descriptions of what constitutes mathematics. We continued by discussing their perception of students in general and the students in their specific classes that semester. They described what they felt business calculus was and why it existed. I asked about their reaction to the teaching assignment and their current feelings regarding teaching business calculus. I also asked them to compare business calculus to engineering calculus in three different areas: the material taught, the students enrolled, and the overall classroom environment. Next, we turned our attention to their teaching practices by talking about their current and previous teaching methods, including any differences in these methods, how they prepared for a typical class session, and what a typical class session was like. We concluded by discussing what effect teaching this class might have on their future.

Before the initial interview, I observed the participants in their classroom to decide the best location for videotaping and to get a feel for the classroom environments. In addition, each class was informed about the research study so that the students would not be taken by surprise when the videotaping began. Between the initial and final interviews, I observed and videotaped three class sessions. After each class session, I reviewed the videotape and selected portions to be used as videoclips in the follow-up interview. During each follow up interview, the GMTA and I watched a videoclip together and then discussed what we observed, following a basic outline (Appendix B). After each interview, the audio-recordings were transcribed for the purpose of analysis.

The types of issues discussed during debriefing varied. The direction of the conversation was largely dictated by the videoclips themselves. However, many other types of issues were addressed. We discussed their current feelings regarding teaching business calculus as well as their attitude about the course on the day of the videoclip taping. We talked about their perception of the students' understanding of the material being covered that day and the types of teaching methods they chose to convey the information. They assessed their choice of teaching method and discussed any difficulties they experienced while teaching the course. We discussed how they prepared for the class session and any difficulties they may have experienced during their preparation, including their initial reaction to the material they were to cover. Finally, I asked them to talk about what they would do the same or differently if they were to teach the class session again.

All data collection took place over the course of approximately two months during one semester. Each participant met with me for a total of five interviews, one initial interview, three videoclip interviews, and one final interview. All audio recordings were transcribed and analyzed after the interviews. The following is an approximate schedule for how the interviewing/taping took place. The week number indicates the week of the study, not the week of the semester.

Week 1: Visit the class to inform them of the study
Week 2: Initial interview with the GMTA
Week 3: First videotaping and follow up interview
Week 4: Second videotaping and follow up interview
Week 5: Third videotaping and follow up interview
Week 6: No formal contact
Week 7: Final Interview

### 3.5 Analysis of GMTA Data

After all interviews were finished and transcription was complete, I began to analyze the information collected. The initial and final interviews established the professed beliefs and classroom practices of the GMTAs. The two participating GMTAs differed in many different ways, and I began to focus my attention on these differences. As noted above, prior to data collection, it appeared that the two participating GMTAs were very much alike, but data analysis indicated that their beliefs and practices were actually quite different. I created a matrix of their responses in the following areas:

- Teaching and Learning Philosophies
- Beliefs about Mathematics
- Thoughts on Business Calculus
- Beliefs about Students
- Teaching Methods
- Classroom Structure and Housekeeping
- Typical Class Session
- Effect on the Future

Then I examined, in detail, their beliefs and perspectives in each of these areas.
Next, I turned my focus to the videoclip data. I viewed and summarized each videoclip. Then I focused on the areas of interest in each interview pertaining to the videoclip. I included the perspective of the GMTA on the videoclip and related that to their professed beliefs. Finally, I compared the GMTAs in regard to their professed beliefs, classroom practices, and course outcomes.

### 3.6 Program History Data Collection

Once I began data collection, it became evident that the history and development of the business calculus program were more involved than I initially envisioned. Therefore, it seemed necessary to interview those in administrative positions within the Business Calculus program. I identified six administrators, past and present, who had knowledge of the history and structure of the program and who were still in the department. Each has a distinctive role that they have held or currently hold within the Business Calculus program. The types of roles held by these six people include the following: the instigator of the change to our
current text, the instigator of the coordination of the program, the Chair of the Department of Mathematics, the Associate Chair, the previous Department Chair, the instructors of the online version of the course, the coordinator of business calculus, the coordinator of the business pre-calculus course, and the faculty member in charge of scheduling the teaching of classes. Some of the administrators held more than one of the above roles. I asked all six for time to interview them regarding the history and development of the Business Calculus program and four agreed to meet with me.

I developed an interview protocol for administrators (Appendix C) and met with each participating administrator to discuss the protocol, which they received beforehand. Each of the sessions was audio-taped and transcribed. We began with questions concerning the purpose of business calculus and how it differed from engineering calculus. We talked about the development of the course into its current state including the choice of textbook and the decisions surrounding the coordination of the class and its benefits and drawbacks. We also discussed the components involved in determining teaching assignments for GMTAs. In addition, the perceptions of business calculus by different populations were explored, including the average business calculus student, the GMTA, and the faculty. The information presented in Chapter 1 is a summary of the information obtained through these interviews.

### 3.7 Analysis of Program History Data

After interviewing the four participating administrators, I compared their answers. In most cases, their answers were similar, but there were also cases in which they were not. I explored the following areas:

- History of the Business Calculus Program
- Instructors Assigned to Business Calculus
- The Textbook
- Coordination Effects
- Differences in Business Calculus and Engineering Calculus
- Opinions about Business Calculus
- What Makes Teaching Business Calculus so Hard?

I considered the viewpoints of all four administrators to obtain an accurate view of the business calculus program and its history. None of those interviewed was able to give input in every area, but among the four of them they were able to provide a holistic view of the way in which the business calculus program developed and their beliefs regarding the program.

### 3.8 Conclusion

As a result of data collection by way of videoclips in a case study, an accurate depiction of Terry's and Jessica's beliefs was obtained and verified. I
was able to establish the beliefs by which they conducted their classrooms, and I saw their beliefs in action. Furthermore, I was able to compare the two cases to establish ideas about how the mathematics department could better prepare GMTAs to teach business calculus in the future.

## Chapter 4: Terry's Beliefs

As noted in Chapter 2, it is well documented that beliefs play a major role in a teacher's practices. In this chapter, we take a closer look at Terry's beliefs, which had an impact on his actions and attitudes in the classroom. His philosophy of teaching and beliefs about how students learn affected what he perceived his role as an instructor to be, which in turn dictated how he structured his teaching. In addition, his beliefs about what constitutes mathematics allowed him to form a scaffolding of how business calculus fits into mathematics as a whole. Furthermore, his beliefs about business calculus and its usefulness to those taking it influenced his attitude about business calculus in the classroom. Beliefs that Terry held about students had an impact on his own interactions with his students and his attitude toward them. Specifically, his beliefs about business calculus students were called upon during the interviews.

The teaching methods that Terry employed in the classroom contributed to the overall business calculus course experience both for Terry and for his students. The ways in which Terry chose to evaluate his students created a classroom structure that led to how his students demonstrated their knowledge. In addition, how he chose to interact with his students in non-mathematical conversations created a distinct atmosphere in his classroom. Moreover, the
evolution of a typical class session under Terry's direction allowed students to see a certain view of both Terry and of mathematics. Finally, Terry's perception of how this teaching experience might affect his future was also a factor contributing to his attitude about the class and his students' success in the class.

### 4.1 Philosophy of Teaching and Learning

Terry professed a belief that students learn by practice, meaning that learning does not primarily occur in the classroom, but instead outside the classroom when students are studying, doing homework, and comparing notes. "In mathematics, [students] learn by practice and they can't get that much practice in the classroom" (Initial Interview). Under this belief that most learning does not occur in the classroom, the job of the teacher is to get students to do their homework so that learning can occur. Terry perceived a variety of ways that a teacher might accomplish this task: scare tactics may be used, threats about grades, or simply the likelihood of lower-than-hoped-for test grades.

However, according to Terry, in the classroom itself, the teacher should be a guide for the students, helping them to understand the material. He should show different ways of understanding the material and different ways of coming to a result.

The purpose I believe is more than just simply teaching a particular subject. The purpose of the subject is to guide them to the knowledge that they need to have for that material as best we can. Not necessarily make it too simple, but be flexible in explaining it different ways. Different people learn differently. (Final Interview)

Terry referred here to different ways of learning and implied that teachers should take these differences into account by being flexible in their methods of explanation and appealing to different learning styles. For example, Terry believed it could be helpful for some students to see both pictures and equations, as well as to hear verbal explanations and see written ones.

In addition, Terry asserted that it is extremely helpful to students to be consistent with the textbook being used. Students can become very frustrated when their teacher operates in a manner that is inconsistent with their text.

Whatever textbook is being used, I like to be consistent with it, as to not cause confusion between what the book and what I'm doing. Although, sometimes I'm not 100 percent consistent, but I'd like to think I'm close to 90 to 80 percent, around that neighborhood. (Final Interview)

This inclination to want students to have a resource for outside work is consistent with Terry's belief that most learning takes place outside the classroom when students are practicing.

### 4.2 Beliefs about Mathematics

Terry thought of mathematics as the study of patterns. To learn mathematics, students needed to develop and understand the patterns being used;
furthermore, patterns allow students to discover for themselves how mathematics works. To this end, Terry tried to help students learn to see mathematics in this way. "[I] show them the pattern, they recognize the rigor involved and the jargon that they have to learn in order to communicate the patterns effectively" (Initial Interview). According to Terry, in a calculus class students could see the patterns and still not see the grander purpose of what they are doing. While Terry thought it would be nice for students to understand the bigger picture, he also "want[ed] them to be able to have tools they can use in the future" (Initial Interview). He believed that understanding the underlying patterns would assist students in this goal. As students began to understand mathematics in this framework, they would not see it as an unfathomable mystery to get through, but rather a puzzle that they could solve, a set of patterns that they were capable of recognizing.

Because patterns were the cornerstone of mathematics for Terry, he said, "it's similar to learning a different language but not quite the same. Similar, because the rigor that's involved" (Initial Interview). Mathematics has its own terminology and syntax, hence its own language to some degree. When one is first learning the terminology everything seems foreign and complicated, but as one begins to learn the basics, they are able to communicate in a much more developed way. The rigor involved and the way mathematics is taught resemble the learning of another language because it begins with very simple ideas and then, building upon prior knowledge, leads to powerful results. He also thought
mathematics teaches students mental discipline in a similar way that learning another language does. Learning mathematics requires mental discipline, to develop ideas into useful arguments. He felt arithmetic may be where students start, but it does not cover the whole picture just as knowing the names of certain objects does not allow one to be fluent in another language.

### 4.3 Thoughts on Business Calculus

When Terry was assigned to teach business calculus, his reaction was nondescript, neither positive nor negative; it was simply his role to play that semester. He had not specifically requested to teach business calculus, but he was not avoiding it either. "I know that most people dislike it, but I'm fine with it" (Initial Interview). He knew business calculus would be different than the other classes he had taught. In addition, he believed that another new teaching assignment would give him more breadth in his experiences, which in turn would better prepare him for the future as a faculty member at a university. "I was content with it. It wasn't the one I applied for, but I'd never taught it and since I'd never taught it, it will look good on a resume" (Final Interview). However, he did not desire to teach the course again because he wanted to have a variety of teaching experiences.

When considering theory versus application, Terry admitted that theory was really more interesting to him. "I have a tendency to like theory better. Given
the opportunity to choose between the two, I would choose engineering [calculus]. But I don't have a hate for bus calc" (Initial Interview). In fact, he liked the type of applications he saw presented in business calculus and felt that the way in which business calculus was applied was interesting and appropriate for how these students would be using mathematics in their future. While the real world problems were not an issue for him, he missed the theory he so loved in engineering calculus. "There is some rigor involved in proof that can be beneficial, just thought process for the bus calc students that they may be missing. They're not seeing the depth or the beauty of the concepts" (Initial Interview). As a result, he occasionally tried to formalize portions of his lectures to help alleviate this concern, without deviating from the mission of the course or consistency with the textbook.

The use of the calculator as such an indispensable tool in business calculus surprised Terry, but he did not have any issues with it. He learned what he needed to learn to help his students master the material. What was most unexpected for Terry was the purpose for which the calculator was being used. It was the modeling aspect of the course that led to this usage of the calculator.

I didn't expect [business calculus] to be so model driven. I expected it to be business calculus problems where the function is already given and not necessarily them generating the model. But I understand the importance of it, because in real life they do have to go find the function if they're starting with a data set. (Initial Interview)

Thus, Terry appreciated that modeling allowed for actual data driven, real life examples and for those enrolled in the class he felt that this was a valuable component of their learning experience. Because the calculator was being used for modeling and not just basic arithmetic, the alleged reliance of students on their calculators did not cause any concern for Terry.

Terry believed the class was accurately titled as business calculus. He agreed that it was calculus, but not the same as traditional calculus. "It's calculus, some could argue about how much calculus. I don't think it's that much really, but it is. And I think very well applied to business" (Final Interview). For the particular students who are required to take business calculus, Terry believed that the applications were more important than the ideas from engineering calculus that were missing. From his experiences as a student, he understood the usefulness of curve fitting and regression, and he believed that business calculus was a good calculus course for those with non-mathematics-based career aspirations. While he believed that business calculus was not a pure mathematics class, he liked the applied problems for this group of students.

One particular aspect Terry liked about this course was the emphasis on interpretation. "Sometimes I have engineering calculus students who can do the number crunching, the formula crunching problems, but they are incapable of interpreting their answer and that makes for a pretty useless engineer" (Final Interview). As indicated in Chapter 1, in business calculus, before students learn
many of the concepts, they learn how to interpret what they will be finding. Terry believed that, if he were to get an opportunity to teach engineering calculus again, he would incorporate more applications and be more careful about specifics such as units and interpretations of the answers. He felt that in the past he had glossed over that component of the course and "left that to the physics people" to teach the students (Initial Interview). "So, the interpretation in business calculus, has helped me realize the importance of [interpretation of answers] and that [interpretation of answers] needs to appear a little more in engineering calculus" (Initial Interview). Terry believed that placing more emphasis on interpretations in engineering calculus would alleviate his concerns over his observation that some engineering students could number-crunch but were unable to use what they had found.

### 4.4 Beliefs about Students

Terry's primary belief about students was that they deserved respect. "They should be treated with respect as much as I expect to be treated with respect" (Final Interview). Terry asserted that if he wanted his students to treat him respectfully then they deserved the same courtesy from him. He applied this philosophy to his interactions with his students in class, office hours, via email, and even when speaking of his students to others.

Terry also believed it was important to know his students by name. So, he made the effort beginning on the first day of class to get to know his students' names. In doing so, he established that he cared about his students and they were not going to be able to hide in his classroom.

At the beginning of the semester I print off their pictures and make sure I start calling them by name. And I intentionally learn the names of the people in the back row first, because they're the ones that are back there hiding. I'm trying to let them know that they are not hiding. (Final Interview)

Because business calculus classes are smaller than engineering calculus classes (approximately 35 compared to 120 ), Terry believed that it was harder for a business calculus student to get lost in the shuffle than it was for an engineering calculus student. However, he also wanted his students to be aware that he knew who they were and noticed when they were not there.

Furthermore, he believed that $90 \%$ of students were capable of doing mathematics. He did not mean to imply that $90 \%$ would pass, only that they were capable of passing because it was their decision to apply themselves. He was also aware that the majority of the students coming into his business calculus class were not overly fond of mathematics, and he did not intend to try to turn them into mathematics majors. He continued to make efforts throughout the semester to remind himself that these students were not like him. They did not necessarily have a strong love for mathematics, and he might not be able to develop that passion in his students over the course of a single semester. However, he did want
them to leave his classroom having learned something useful. He hoped that when they finished his course they would have learned something that they could put into action in their lives beyond the classroom. "I see them as non-math majors that I hope by the end of the semester have developed an appreciation for the subject" (Initial Interview). So, perhaps a love for mathematics would not come about, but Terry was optimistic that he could help his students learn to appreciate mathematics.

However, Terry also noted that in order to achieve this end he must foster in his students accountability to the course. "I believe that they should be held responsible for their actions" (Final Interview). By holding them responsible for things like time spent on homework, the exam scores they received, and their preparation for class, Terry believed they would have a greater respect for mathematics and how it could be used. He did not believe it served them well to give them an elevated sense of their abilities by inflating grades, but rather that they were best served by giving them an accurate view of their strengths and weaknesses. "I consider myself a fair tough grader... I feel like I'm doing the right thing and anything less would be unethical" (Initial Interview).

Terry felt that while the majority of his students might not love mathematics or even be exceptionally good at it, there was not that much of a disparity between the students of the two courses. According to Administrator 2, engineering calculus instructors would not be willing to make the concessions
necessary to accommodate business students; however, Terry commented, "I think that bus calc is a little bit easier but not much, it's just a much different way of approaching the material" (Final Interview). For Terry, the classes were really difficult to compare in this way because he felt the way in which calculus was approached was so very different. However, he believed that an engineering student might perform a little bit better in a business calculus class than in the corresponding engineering calculus class. Also, he believed that the A students in business calculus might drop a letter grade if they were taking engineering calculus. But "an A student in business calculus certainly wouldn't flunk engineering calculus" (Final Interview). Terry thought that the most explicit way in which the students of each course differed was in their prior knowledge, specifically their skill level with basic algebraic manipulation. This lack of skill was the reason he believed that the typical business calculus student would probably struggle a bit more in engineering calculus than the engineering or mathematics major. The question of how successful business calculus students were in calculus had less to do with their aptitude for the material and more to do with their previous mathematical experiences.

### 4.5 Teaching Methods

Terry's teaching centered mostly around lecture. He used the chalkboard as his primary means of communication prior to teaching business calculus but
began to use the overhead projector more during the course of business calculus with conflicting opinions about its use. In the past, Terry had used the chalkboard as a default. It required less preparation and kept the students actively taking notes during the class session. Terry began to use transparencies when teaching business calculus because of the kinds of problems students were asked to complete.

I did [use the overhead with transparencies] so that I could present the models, which are long problems, quickly, and get to the actual math behind the models. Instead of writing two paragraphs on the chalkboard and using 10 minutes just to do that. (Initial Interview)

He wanted to make sure that he was able to cover all of the material. By using transparencies and posting them to the internet before class, his students would be able to think about the problem at hand instead of copying large amounts of writing. This strategy allowed Terry to cover more material and allowed his students to focus on the mathematics they were learning instead of focusing on taking notes consisting mostly of large amounts of text.

However, Terry did not like using overhead transparencies exclusively, "because people's eyes glaze over and I find them boring, and I know they do too at times" (Initial Interview). He was concerned about overusing transparencies because he felt that when the students already had the notes printed out they were less likely to stay involved in his lecture. However, if they were taking notes, they were at least involved to some extent. Unless there was modeling, with large
amounts of writing or data tables involved, he tended to gravitate toward the chalkboard instead of the overhead projector because he felt it better engaged the students.

Although lecture was his primary teaching method, Terry tried to incorporate discovery activities through group work when time permitted and the content lent itself to this type of learning.

I like the discovery method of group work. It reminds me, if anything, that things are not as easy as they are now [for me]. Taking the derivative is easy for me now, but it reminds me that when I first learned it, I struggled. It just reminds of things when I need to slow down. (Final Interview)

Terry wanted his students to have experiences with discovery learning and group learning in his classroom; however, "group work for me is a strictly when time permits kind of thing" (Final Interview). So, there was not necessarily an intentional effort on his part to incorporate this teaching method on a regular basis. He employed it when he felt he had the time, and when he did not have the time, he simply lectured instead. When time did permit, Terry liked to give students activities covering material on which he had not lectured a great deal to see what they would be able to come up with on their own. This strategy allowed them time to develop their own mathematical intuition and learn from one another. "For example, drawing the derivative graph. I had given them enough knowledge to draw a derivative graph, but I had never actually drawn one" (Initial Interview). He felt this time gave the students a chance to "fight through it and
discover things on their own" (Initial Interview). While he remarks that in this particular example no one came up with exactly the right answer, some of them were close and that success seemed to build their confidence in themselves.

Another type of teaching method present in Terry's classes was student board work. Occasionally, he would send students to the board to work out their solutions for the class. 'I volunteer them, so I select them... like, today's class period, I went around and saw the people that had the problems that were right or really, really close to it and sent them to the board" (Initial Interview). This type of activity would generally follow group work or an individual classwork session during which Terry observed his students at work. In the past, he sent students to the board who were completely wrong, but only if he perceived they could handle being told publicly that they were wrong. He also encountered students who preferred not to go to the board, because of personality or language issues, and he respected this preference as well. If a student refused to go to the board, he did not press the issue. In fact he noted that the few students who ever resisted going to the board eventually volunteered. However, Terry's primary selections were students who were correct or whose error was a common mistake. This latter strategy allowed him the opportunity to point out the mistake the student made and hopefully prevent others from making the same type of mistake in the future. In addition, the process of peer approval for a correct answer boosted the
confidence of the student at the board. "Hopefully going to the board and getting one right, establishes a little more confidence in the students" (Initial Interview).

### 4.6 Classroom Structure and Housekeeping

Terry's preparation for class primarily came from materials provided to him by the course coordinator. He used a provided CD for the creation of many of his transparencies to avoid having to create everything from scratch.

I didn't think [preparing to teach] was difficult as long as I followed [the coordinator's] CD, but I still got nervous about when exam time came. Did I cover everything? Did I present it the right way? Did I emphasize the right thing? (Final Interview)

He used the provided materials as a guide for what to focus on, to help clarify the main ideas that his students were expected to master. Terry felt that drawing on the materials created by the course coordinator (who writes the exams) would prepare his students better than materials he created on his own.

Terry also used the homework list in these provided materials to assign homework problems for his students. After glancing through the homework assignment, he chose other problems from the textbook that would help students complete the homework assignment successfully. These problems formed the remainder of the material he would cover during class, typically in a lecture format. While Terry did assign homework, he did not collect it. Instead he chose to have quizzes, typically unannounced, during class time. The quizzes were
intended to help students prepare for exams. He often used old exam questions as quiz questions. "That way I know I'm at least similar to what their tests are going to be like and can say study your quizzes. They have good reason to study their quiz come the next exam time" (Initial Interview). Quizzes created in this way allowed the students to have more materials at their disposal to study during exam time. It also allowed Terry to feel that he was measuring the same abilities that the test would measure, so as to give his students accurate feedback on their level of understanding.

Terry believed that the purpose of homework and quizzes was to give students "the opportunity to fix their mistakes before the exam" (Initial Interview). Terry also graded all problems in a complete way, giving feedback on small mistakes, even if the final answer was correct. "Just because they got the right answer, doesn't mean it's right... I grade how they communicate. They have to communicate their answer in the right way" (Initial Interview). Grading in this way meant that if they were deficient in their notation or in their level of understanding, they received feedback as well as on the manipulative process they used to find the numerical portion of the answer. Because of Terry's attention to detail, he felt his students often thought he was a tough grader. However, Terry believed this grading procedure was the only way he could be fair to his students. "I feel like I'm doing the right thing, conveying to them where they made a mistake and they can learn from it" (Initial Interview).

### 4.7 Typical Class Session

When Terry entered the classroom, he liked to start the class by strengthening his rapport with the students. He made small talk about things he believed the students might be interested in. These conversations may have been about current events, the weather, school-related activities such as sports, or community events. Sometimes he was even able to get the students involved in the chatter.

I try to get them to understand that I am really a person; tell them how my weekend went or if I watched a basketball game or something, something non-mathematical to begin the class period. It's usually mostly me talking, but sometimes I get them talking. (Initial Interview)

While he was making small talk, he was also getting ready for the class session by organizing papers, getting the overhead projector set up and preparing the calculator for the day.

Once Terry was set up and the chatter ceased, class began. Terry prepared most classes to be lectures, and he seldom deviated from his original plan. He occasionally spent some time reviewing the previous lesson before he began new material. When Terry first started teaching business calculus, he began class sessions with a question and answer time, but soon realized the questions they were asking were not relevant to the material. So, he decided it would be better to address individual questions in the tutoring center run by the Department or one
on one after class. Thus, at the time of this study he was no longer holding a question and answer time.

### 4.8 Anticipated Affect on Terry's Future

Terry identified several benefits he gained from having taught business calculus. He believed one benefit was becoming more involved with pertinent applications of calculus. He felt that in the future he would incorporate more word problems and strongly emphasize labeling answers with units.

In addition, his method of delivery might change a little bit as a result of teaching business calculus. He found transparencies to be a useful tool for word problems in business calculus and felt they would also be helpful in other classes. "If I was doing engineering calc, and I was presenting word problems I would want to use transparencies for the exact same reason" (Final Interview). The benefit of being able to quickly present a word problem by using a transparency outweighed the drawback of possibly disengaging the students. Thus, Terry felt that he would probably use the overhead projector in the same way for these types of problems in future classes.

In addition, Terry felt that having taught business calculus would "look good on a resume" (Final Interview). While he does not wish to teach business calculus again, he also does not wish to teach engineering calculus again. "I'm still fine with [teaching business calculus]. I don't think I'd want to do it again,
but I don't really want to teach engineering calculus I again. I want something I haven't done" (Initial Interview). Terry wants to experience as many different courses as possible during his graduate career and does not want to get comfortable with teaching the same class repeatedly. While many GMTAs believe teaching the same class repeatedly cuts down on their work requirements, Terry looks at repeated teaching assignments as a negative because it limits his exposure to new materials. This limitation might make him less attractive to employers when he looks for a job after graduation.

### 4.9 Conclusion

After examining Terry's beliefs, it is possible to speculate what one might expect to happen in his classroom. His beliefs also allow us insight into how he approached teaching business calculus, the teaching methods he employed, and the way in which he prepared to teach this course.

Having established Terry's beliefs, after visiting his classroom we hope to be able to learn more about the interplay between his beliefs and his classroom experiences. Finally, after exploring the classroom setting, we may be able to propose ways in which this teaching experience can be improved upon for both the students and the GMTA.

## Chapter 5: Jessica's Beliefs

Jessica's beliefs and actions in eight different areas impacted her effectiveness in the classroom and her ability to adapt to teaching business calculus. These are the same eight categories discussed in Chapter 4 related to Terry's Beliefs and discussed below.

Jessica's philosophy of teaching and her beliefs about how students learn mathematics affected her concept of what constitutes a good teacher. In addition, her beliefs about the role of mathematics in society limited her desire to fit business calculus into her framework of mathematics as a whole. Thus, she was dissatisfied with her teaching experience. Furthermore, her beliefs about business calculus and how it differed from engineering calculus in regard to the content covered, students enrolled, and overall usefulness to those taking it, influenced her attitude about business calculus in the classroom. Moreover, beliefs that Jessica held about students in general impacted her attitude toward them.

Jessica's teaching methods contributed to the experiences of her students in the classroom as well as to her own experience teaching the class. The way she chose to evaluate student work led to certain types of interactions between Jessica and her students. In addition, how she interacted with students in a nonmathematical way impacted the atmosphere of her classroom. Moreover, the way
in which Jessica constructed her class sessions suggested to students her beliefs about teaching, learning, and mathematics in addition to her perception of business calculus. Finally, Jessica's lack of belief that teaching business calculus would positively affect her future employment affected her teaching experience.

### 5.1 Philosophy of Teaching and Learning

Jessica believed that students learn by trial and error, and while she agreed that it is not always the most efficient way, she thought trial and error learning produced the best understanding. Thus, it was necessary to create an environment in which students try and fail, but then learn from their mistakes.

You try things and you find out whether or not you're right or wrong. And in kind of a perverse way, it is better if you are wrong a few times so that you can find out what's really right. (Initial Interview)

Jessica believed that often students think they understand the material because they get the right answers, but their depth of understanding will not hold up in all circumstances. "One of the problems that the students have is always being right, and then they never learn what they're doing right. They just keep doing it until it's wrong" (Initial Interview). Jessica thought these kinds of misconceptions could create extreme frustration for students who are not used to being wrong or for students who did not know how to go about finding their own mistakes.

In addition, Jessica believed that learning required personal effort and that effort was illustrated in practice. She thought that without practice, what was
learned conceptually may not become part of a student's permanent knowledge base. Thus, it was necessary to practice mathematical concepts to make them your own. She also believed that in mathematics practice was easier than in many other subjects because problems could be decomposed into smaller parts that could be easily practiced.

Actually, I think mathematics is one of the few things that is really designed in small [pieces] to actually practice. In English, the only way to practice a 12 page paper is to write a 12 page paper. But, mathematical concepts, even if it's a big problem, the parts of the problem can be broken down very easily to practice that particular aspect several times. (Initial Interview)

As a result, for Jessica, mathematics presented an opportunity to learn through practice. By doing mathematics and confirming correctness Jessica believed that students could verify if they understood the material.

Therefore, Jessica thought that a teacher's role was to facilitate student learning. Jessica did not think she was supposed to do all the work for the students, but that she was to supply them with information. She believed that her role was to get the students to practice. While the textbook was to be the primary resource, Jessica perceived herself as a supplemental resource. Jessica thought that the teacher should direct students to what they already have access to in their textbook and to be instrumental in allowing the students to learn on their own.

Jessica believed that a dedicated student could learn in spite of the whether or not she had a good teacher; however, "the teacher can have a great affect on
how much is learned or not learned by how effectively they do their teaching job" (Initial Interview). She also thought that ideally, teaching would occur in small classes with lots of interaction; however, in practice "teaching is more of disseminating information and hoping that people are absorbing it" (Final Interview). Thus, there was a dichotomy of how teaching should happen and how it actually did happen. So, a teacher should be this guiding force for students, but in reality teaching was more of an information outpour to students, hoping they could sort through what was presented both in class and in the text to make meaningful connections.

### 5.2 Beliefs about Mathematics

Jessica believed that mathematics was the language of the universe. "Mathematics is a way of encoding everything we do in life and finding logical outcomes" (Initial Interview). However, she also agreed that because things in life do not always result in logical outcomes, mathematics as a way of understanding our world is not always sufficient. However, she admitted it was the best predictor because mathematics was the logic behind life. Thus, she felt an instructor should teach logic.

Perhaps more importantly, to Jessica, mathematics was cross-cultural. It was the way we could all communicate and understand one another. The understanding of mathematics was uniform from the United States to Japan. "I
can write down a quadratic equation in any country I want, and people are going to understand the intent" (Initial Interview). Mathematics was "the one universal language that all peoples and all nations and all aliens can understand" (Final Interview). As a result, Jessica believed mathematics was accessible to all people and people could communicate mathematically even if they did not speak the same language.

However, she noted that in the midst of the conceptual role that mathematics played in all of life, it was also computational. When teaching mathematics, Jessica did not believe it was her job to teach someone logic skills, but rather how to find computational answers. Thus, teaching interpretation to answers was not what mathematics was about. She said that business calculus was not her favorite thing to teach because "I think I feel more like I'm teaching thinking skills and logic skills and semantics, more than I'm teaching mathematical, computational methods" (Initial Interview). So, while mathematics may be the logic behind the universe, she perceived her role as a mathematics instructor to be teaching computational methods, not the language and semantics associated with the mathematics.

### 5.3 Thoughts on Business Calculus

When Jessica saw her teaching assignment of Business Calculus I, she thought "I've been dodging this bullet for a long time. I knew it was going to
come" (Initial Interview). She commented "I was not necessarily surprised, but definitely not in joy either" (Initial Interview). She was unhappy from the first sight of the assignment, and her reaction changed very little during the course of the semester. She hoped that by teaching business calculus during this semester, she would avoid having to teach it again. She was well aware that her point of view was not uncommon among GMTAs. Thus, when she first learned of the assignment she wanted to get it over with as efficiently and as quickly as possible.

Jessica felt that business calculus called on her to teach non-mathematical concepts, and because of her love for pure mathematics, this material was not a welcomed opportunity. "I don't feel like I'm teaching math most of the time" (Initial Interview). While she agreed that computational skills and logic/semantics are not necessarily unrelated, she felt there was an overuse of time in the area of logic/semantics that she really did not like. She wanted to be teaching students to love mathematics, to enjoy the purity of it, and to see it the way she did. However, what she found was resistance among her students to these ideas. Instead, the majority of her preparation time was spent in understanding the semantics she was teaching rather than brushing up on her mathematical skills. "I myself am a math for math's sake kind of person. I never cared about application and never wanted application" (Initial Interview). So, the application-based approach taken by the book did not sit well with Jessica. She wanted the students to see the beauty of the mathematics, not the details of how to apply it in diverse contexts.

The use of calculators did not appeal to Jessica either. "I am not a big fan of the calculator. I don't think it's evil or anything, I just think it'd be more fun, personally, to just do the math" (Initial Interview). As a result, there were times in the course when Jessica would revert back to doing problems by hand after she and her students had spent time learning how to use the calculator for certain procedures. This lack of calculator use did not settle well with her students. Because of her inclination towards doing mathematics by hand, she avoided the calculator more than she used it. This avoidance caused some concern among her students during times when it was vital that they be proficient in their use of the calculator.

In addition, the modeling approach taken by the textbook did not excite Jessica. Because the functions students derived came from real life data, the coefficients in models were often not particularly nice looking numbers. Most often they were coefficients rounded to three decimal places. She felt that the time would have been better spent on the mathematical concepts of calculus if the coefficients were simpler to deal with. "I think the problems involve such large numbers and large decimals, that it was very hard" (Final Interview). Jessica felt that these kinds of numbers made simple concepts, like taking derivatives, unnecessarily complicated for students. Furthermore, modeling created this "pseudo-real world aspect to it, which is nice, but I don't think students understand the math well enough to understand they're really using math to solve
any of these problems" (Final Interview). So, while the intent of the modeling was to make the mathematics more applicable to students' lives outside the classroom, she felt students were not fluent enough in actual manipulations of calculus to do much with the real life data they would encounter in the future. She commented that it seemed they were not learning mathematics, but rather learning how to hide mathematics (mathematical computations) by using their calculators.

Jessica also had particular difficulties with the specific types of optimization problems her students were expected to be able to complete. From her experience, optimization problems were geometric and concrete. However, in business calculus students were also expected to optimize quantities such as profits and revenues. "That's artificially, in my mind, artificially geometric. They've taken it and made it geometric so they can use the methods, but it wasn't naturally geometric" (Initial Interview). This type of application made Jessica uneasy and caused her to omit many examples when she taught this section of material.

One aspect that caused Jessica frustration was that she did not "really look forward to teaching this class like I have other classes" (Initial Interview). She felt that the atmosphere was more threatening, intimidating, and somber than any other course she had ever taught. Business calculus was "more focused, more intense as a class, and I think for me it was more pressure packed too" (Initial Interview). Because of her lack of experience with the material, she did not feel
confident teaching the class. Furthermore, she felt her students did not embrace her love for mathematics, which meant that she was limited in how much enthusiasm she could reveal in class.

Jessica did not believe business calculus constituted a mathematics class. "It's a class in mostly semantics with, with not enough math to make it math. It's a lot of form without function" (Final Interview). She understood the need for a class to satisfy the business college requirements but felt that students would be better served if they were just required to take the engineering calculus course. Her ideal would have been to have one calculus sequence for all students, where students learned the same material, and this "pseudo-real world aspect" did not arise.

### 5.4 Beliefs about Students

Jessica believed that her students had good intentions. She thought they wanted to do well and they wanted to succeed, but sometimes they had unrealistic expectations of their abilities. She noted that they seemed to believe that they were better students, at least mathematically, than they really were. She thought they felt that doing well in high school mathematics guaranteed they would do well in college mathematics. Another common misconception she found was that they believed they could do it all: "Yeah I can work 20 hours a week and take 18 credit hours and be just fine. Lots of unrealistic expectations" (Initial Interview).

Another problem was that some students felt they were paying the university for a degree, rather than paying the university for an education. The students believed that grades should be given to them instead of earned by them. "Many students have a sense of entitlement, like, I'm here, I'm paying money, so I should get an A" (Initial Interview). She thought they expected that whatever effort they put forth should be enough for the grade they felt they deserved. "I think most students have gotten away from the idea of actually working and doing something for a grade. I think most students believe that if they show up and try their best then that's good enough" (Final Interview). She believed it was hard for students to accept that sometimes even their best really was not good enough to get the grade they wanted. In addition, sometimes she felt students did not know what their best was. They thought they were giving it their best, but really they were only putting a base level of effort into the task at hand instead of the maximum effort they could have been putting into the task.

Jessica believed that differences existed between the types of students enrolled in the different calculus sequences. "I think bus calc is more accessible to your average non-mathematical student" (Initial Interview). She felt that in a business calculus classroom the number of students who liked mathematics was much lower. "I would say that the biggest difference is that on average liking math as a subject on its own is much lower percentage-wise in the business calculus than it in the [engineering] calc classes" (Initial Interview). The lack of
enthusiasm for mathematics among her students also contributed to some of Jessica's dislike for this teaching assignment. It was hard for her to try to develop in her students enjoyment of mathematics and also hard for her to accept that they did not like mathematics.

Having taught engineering calculus, Jessica felt that the engineering calculus students had a bigger picture of what mathematics was about than students in business calculus. In addition, she thought that the good students in engineering calculus were better than the good students in business calculus but the bad students were about the same. Predominately, though, she felt students in engineering calculus seemed to have a better grasp of their own mathematical abilities. "[Among engineering calculus students] there seem to be more realistic expectations of how good at math I am and how good I'm going to do in this class. There weren't too many people who were really shocked" (Final Interview). However in Jessica's business calculus class, she found many students who thought they were going to do really well and did not.

The attitude among business calculus students in regard to their grades was also very heated. "They fight for every 0.2 percent they can get on a test. They fight for it" (Initial Interview). The students were also frustrated with the low scores they received on their quizzes. Jessica felt that her students thought she was mean and picky in her grading, something she had not experienced with her engineering calculus students. Initially she also had issues with students believing
she was denying them equity in their grades. Jessica believed that she was being fair and was trying to prepare them for the way they would be graded on their exams.

Before the first exam I know they hated me. Oh, they just hated me. I could feel hate waves coming off of them. And then the first test actually happened, and they hated me a little less because I think they realized that I was not asking them to do things that were really that different from what the exam was going to ask. (Initial Interview)

While Jessica felt students had good intensions, she also thought their expectations were unrealistic and that they operated under a system of entitlement. She also felt that they did not work as hard as they were capable of working. Furthermore, in comparing business and engineering students she saw differences between the students in regard to expectations and grading issues.

### 5.5 Teaching Methods

Jessica's primary teaching method for business calculus was lecture with the use of an overhead projector. "Most of the time I use the overhead projector and have the notes written out. That's mostly just out of necessity because that gets me prepped at the same time as preparing" (Initial Interview). The overhead projector provided Jessica with a sense of security. By having her lessons actually written out in advance, she was less likely to experience panic over presenting something she had not thoroughly worked through prior to class.

However, in the past, she preferred the chalkboard where "things happen in real time instead of my own time before" (Initial Interview). She also used the chalkboard in business calculus; primarily for exam review days. "I've had the slides, and I've just bagged them and gone up to the board where I feel much more comfortable" (Initial Interview). The chalkboard forced her to slow down, develop the calculus concepts, or review as needed. In addition, working at the chalkboard on a review day was less intimidating to Jessica than on other days because she had more time to become comfortable with the semantics during the weeks leading up to the exam.

In the past, Jessica employed problem solving and board work as methods of student learning in her engineering calculus and precalculus classes.

I would assign very specific problems and have the students come and do them on the board at the beginning of the next class period, or as soon as it was appropriate and have them do those problems in front of everybody. (Initial Interview)

She noted that some of the students were more excited about board work than others, but she felt it was a good learning opportunity for them. It allowed them the opportunity to receive peer feedback and to present their abilities to others. She noted that while she made mistakes herself on the board, they tended not to be the same mistakes that students made. "I tell them please feel free to be wrong because your mistakes are most likely the mistakes of other students" (Initial Interview). She also thought her acceptance of mistakes relieved some of the
anxiety that putting answers on the board tended to create in some students.
Furthermore, she did not want her classroom to be entirely teacher-driven. She felt by having students take the lead on working out some examples, all students would feel more comfortable and realize that they were capable of doing the mathematics they were learning.

However, in business calculus, Jessica did not employ this teaching method at all, in part because her class was shorter, 50 minutes as opposed to 75 minutes. While the time was one factor, a more important one contributing to Jessica's decision not to use board work was the type of problems presented in business calculus.

A problem [in engineering calculus] could be done in 5 minutes quite easily on the board, even some of the more difficult ones could be done in 10 , and they required a little bit less scope to be done, and so two or three people could be putting up problems at the same time and then we could just discuss them. (Initial Interview)

In business calculus the majority of the work is done on the calculator. Jessica felt that problems could not be put on the board in a meaningful way. "If people know how to use their calculators, they're going to get that answer too. So, I just don't see it as a time effective kind of option for me right now" (Initial Interview). She felt that it would take too long for students to present problems and that seeing the problems worked out by one another would not necessarily be beneficial.

Another teaching method that Jessica tried in business calculus, but encountered problems with, was allowing a question/answer time for the students.

I did try to do question and answer a lot when I gave them back quizzes or problems, but the thing that I found most disturbing was that most of the problems in business calculus take so long to do that answering one question could take the whole class period if you let it. So, that was kind of discouraging. (Final Interview)

Because of the time constraints, Jessica stopped encouraging questions when she handed work back and focused instead on continuing to move ahead in her lecture material.

### 5.6 Classroom Structure and Housekeeping

When assigned to teach business calculus, Jessica obtained a copy of notes from another GMTA who previously taught business calculus. She said, "I go through those notes carefully, do those examples, make sure that I can do them and understand how to do those examples and look at a few problems that might come up from the homework" (Initial Interview). In order to prepare herself to teach the material, she carefully copied all the notes onto transparencies to make sure she understood what she was going to be teaching.

As far as preparation time goes, it's much more difficult for me to walk into a class and be able to feel like I know everything that I need to say to them. You know if it was [an engineering] calc I or a calc II class, even without prep, on mathematical knowledge alone, I could get my way through, if I had to, and this class, I can't. If I don't know what I'm talking about, I'm dead in the water. And I need to spend intensive time finding out. It's not because I don't have the math skills. It's because I don't have the semantics and the other things that are involved in this class. (Initial Interview)

So, the majority of Jessica's preparation time was spent familiarizing herself with the terminology of the material. She felt that mathematically she was well versed in the material she would be presenting, but that alone was not sufficient for this class. She wanted to be better prepared with the language she used when presenting the material so that she would not make mistakes and hurt her credibility with her students.

Jessica assigned the homework problems suggested by the coordinator. However, she did not collect the homework she assigned. Instead, she chose to give her students quizzes. Her students did not particularly like her quizzes, and because of the low scores they were receiving on the quizzes, they temporarily believed that Jessica was not treating them fairly. However, Jessica was taking points off for rounding and units, which they soon learned was consistent with the grading the course coordinator established for the exams. Once the first exam was over, the issue of equity was resolved. "Lots of them were expecting 8 s and 10 s and they were getting 6 s and 7 s and some 5 s " (Initial Interview). Jessica created her own quizzes, some from homework questions she assigned, some from other textbook questions and some from her own imagination. Thus, sometimes she was getting questions similar to exam style questions and sometimes her questions were not as close to exam questions as would have been helpful to her students. Thus, quizzes were not necessarily preparing her students for what to expect on
exams in terms of questions that would be asked or the difficulty they should expect.

### 5.7 Typical Class Session

To avoid silence within the classroom, Jessica began each session by conversing with her students about topics not necessarily related to mathematics. Mostly for Jessica opening with conversation was a chance to break the ice and start the class out with at least some interaction on the part of the students. However, once class began, Jessica turned her focus to mathematics. She liked for each class session to have a definite start and finish. "I try to make each class have a defined beginning and end. I don't like to stop in the middle if I can help it. I like things to be encapsulated in one class" (Initial Interview). Thus, she worked to stay on track. She knew her purpose in the classroom was to teach mathematics, and she intended to spend the class period doing just that. She tried to give motivation for the topic and its usefulness but her main focus was examples. There was not much formalizing of topics or conceptual development of ideas in her classroom. Instead, she felt that the more examples the students saw, the more likely they would be to replicate the processes she was teaching on their own.

In addition, Jessica noted that if she used the overhead class felt very routine, very planned, and if she used the chalkboard she felt the students
interacted with her more. But as the "supplier of information" (Initial Interview), she knew "mostly it was me talking and them hopefully listening" (Final Interview). So, her style seemed to be disseminating information, hoping that her students would absorb what she was teaching and be able to duplicate what she showed them.

### 5.8 Anticipated Affect on Jessica's Future

Jessica acknowledged a few positive aspects that having taught this class might have on her future. On one hand, she did not particularly appreciate the applications of business calculus. On the other hand, she recognized that they provided her a way to answer the question, 'When will I use this in real life?' Thus, she felt that she would have more examples of real life applications of calculus. "It's exposed me to different kinds of things that I had never been exposed to before. So, it's added to my arsenal of real-life uses for this stuff, which is always a good thing" (Initial Interview). In addition, having taught business calculus made her more mindful of units and the interpretation of data.

I think, whereas in the past in [engineering] calc I, calc II, my attitude has been pretty much, let the physics people teach them the physics end; let the whatever people teach them the whatever specific application. My job is to teach them how to do it. I think I will pay more attention to things like units, where I haven't really cared too much in the past to pay attention to those things. (Final Interview)

One specific type of problem addressed in the course was the construction of slope graphs from the graph of a function. Jessica had never seen this topic taught in an engineering calculus class and thought it to be a useful idea. "I probably would never ask a test question over it, but I think it's a useful tool for understand what this new equation means" (Final Interview). Thus, she felt she would probably incorporate this idea in other calculus courses as a means to help students understand what the derivative of a function is.

When asked to summarize ways this class would positively affect her future teaching, she commented

The more I might be forced to [teach business calculus], I might start to enjoy it more, and get a little bit more enjoyment out of it. So it's not that bad, and it's forced me to start making more applications, not that I never made applications to other things in engineering calculus, but it's exposed me to different kinds of things that I had never been exposed to before, which is always a good thing. (Initial Interview)

Jessica's beliefs about exposure to new ideas were not voiced frequently. While she may have commented that this experience was a positive effect on her future, she also noted negative effects that she felt carried more weight for her future.

The primary opinion Jessica left the course with was the wish never to teach business calculus again.

Even though it is not my favorite thing to do and it probably will never be my favorite thing to do, having done it, I find it is not so horrible. But hope I never have to teach it again. (Initial Interview)

She felt this way for many reasons. One reason was because she did not enjoy teaching the course. Jessica was accustomed to enjoying the material in the courses she taught. However, she left this teaching experience feeling discouraged rather than joyful. "I didn't come out with any of the enjoyment that I usually come out of teaching with. Instead of seeing people learn math, I saw people continue to not learn math all the time" (Final Interview). Students not learning mathematics frustrated Jessica and caused her to dislike teaching business calculus. In addition, she also felt that it would not be valuable to her from a purely marketable standpoint. "On a personal side, for the kind of employment I'm after, teaching this class does nothing for me. There will be other classes that will be much better for me personally and on a resume" (Final Interview). Thus, she felt teaching business calculus cost her an opportunity to teach a class that could have been useful in her future career.

Jessica's beliefs about the effect teaching this class would have on her future were varied. She believed that the types of applications would add to her list of examples where calculus is useful, and even though she did not particularly like the heavy focus on units, she thought she would be more careful to include them in her future teaching. Furthermore, given more time teaching this class she might even come to enjoy teaching it. However, the negatives dominated her experience. She hated that her students seemed to continue to not learn
mathematics because of the content presented in the course. She also felt that this class would not benefit her on the job market.

### 5.9 Conclusion

Jessica's beliefs about teaching, learning, mathematics, and students set the stage for how she would behave in the classroom. Furthermore, her choices regarding the structure of the class, how she would evaluate her students, and her teaching methods caused certain results in her classroom. Her beliefs about business calculus were also cause for frustration for Jessica. Finally, as she focused on the future and how this class would help her she had somewhat mixed thoughts. She felt there might be a few benefits gleaned from this teaching experience. However, overall she was discouraged and felt that business calculus was not a helpful addition to her resume. Jessica's attitude, by her own admission, hindered her teaching of the course and perpetuated frustration as she taught it.

## Chapter 6: Terry's Classroom Experiences

There are mixed research findings about how well a teacher's professed beliefs agree with their practices as instructors in the classroom. Because these findings are inconclusive, we compare the beliefs GMTAs proclaim to have against how those beliefs are displayed in their classrooms. In Chapter 4, we looked at Terry's professed beliefs; in this chapter, we look at his experiences.

In Chapter 4 we found the following to be Terry's basic beliefs.

- Students learn by practice
- A teacher is a guide for students
- A teacher should be consistent with the textbook
- Mathematics is the study of patterns
- Learning mathematics is similar to learning another language
- Preferred theory to application
- Tended to formalize some mathematical concepts
- Liked the emphasis on interpretation and units
- Students deserve respect
- $90 \%$ of students are capable of mathematics
- Teaching method is primarily lecture with heavy use of the overhead projector
- Some group work and some board work when time permits
- Used coordinator's materials to prepare for class
- Chose to evaluate students with quizzes
- Began class with conversation to establish rapport
- More applications for future use
- This class will look good on a resume

All quotes used in the chapter are from observed classroom dialogue or from the debriefing interviews regarding the videoclips. Videoclips were chosen based on the length of the class and the material presented. Shorter class periods were viewed in their entirety. For longer class periods, portions were omitted from viewing when they strongly resembled other portions.

### 6.1 First Classroom Observation

I first visited Terry's class in week ten of the semester. Terry began this class session by entering the room before the official start time and chatting with his students about a recent OU basketball game while also setting up the overhead calculator display. At the official start time, he administered a quiz, which took about 20 minutes for the students to complete. Then he had students retrieve a
worksheet they had been given in a previous class session. He asked them to try a specific set of problems on the worksheets. He instructed them to work either jointly or independently. The worksheet consisted of functions for which the students had been asked to take the derivative. This class session followed a lecture regarding taking derivatives using the chain rule and the product rule as stated by Terry during the class session. The particular problems assigned at this point in class required the students to find the first and second derivative of a function.

While Terry's students were working, he walked around the classroom, observing. He would occasionally look at the answers they were getting and redirect their efforts when necessary. He was also attentive when students were having trouble on specific questions. The students all appeared to be working on the assignment. Some were working collaboratively with others and some were working independently. After the students had worked for about ten minutes, Terry reconvened the class and worked the problems on the chalkboard while students followed along.

As he began the discussion, he chose the function $f(x)=e^{2 x} x^{4}$ and had students help find the first derivative by calling out steps of the problem. He wrote on the board, but before he began he asked the students to tell him which derivative rules they were using and how he should proceed. He also warned students at this point that if they did not get the first derivative correct during their
individual/ small group work, then they needed to get caught up on the material. The next part of this question was to find the second derivative. Terry asked his students what this answer meant and a student told him, "the derivative of the derivative" at which Terry replied, "that's a good way to think about it."

While working on the second derivative he again asked students what rules they were using as they verbally guided him. During this activity, he assured students that there were applications for second derivatives and that they were important. After Terry completed the derivative a few students asked questions, which he answered. Then, he changed the sign on one of the terms in the first derivative and asked students how that adjustment would change the subsequent derivative. After a student correctly answered Terry's question, he paused for several seconds, asking if everyone understood.

Next, Terry decided to work another problem, $f(x)=5 x \ln x$, from the worksheet on the board. He later commented during the debriefing that he specifically chose a question for which simplifying the first derivative made taking the second derivative much easier. He also drew another extension for students by asking what they would do if they were required to find the third derivative, which is not something they had ever been asked to do before. His students correctly answered his question.

Terry then moved to the overhead projector, where he displayed a transparency that had another function, $f(x)=\left(\ln \left(1+x^{4}\right)\right)^{22}$. He asked the students how they would take the derivative. This example involved using the chain rule twice. He discussed with his students how the derivative would be found but did not complete the problem. Instead he continued to the second example on his transparency, which involved the model $C(t)=\frac{79.294}{1+.122 e^{.211 t}}$ dollars in an investment t years after 1970. He worked out the derivative with the verbal help of a student. After he obtained the answer, he reviewed what they had done and asked for questions, waiting a few moments before he continued. Then he had the students explain how to interpret the results with units. They correctly identified that the output units were dollars per year. Then he entered the original function into the graphing screen on his calculator to display for the class. He instructed his students to press the mathematics button, choose nDeriv, enter the function using appropriate calculator notation, and have the calculator find the derivative at the specified value of 2 .

He concluded this class session by noting that this calculator method would give them the same answer as if they entered the derivative formula itself into the calculator and evaluated it at the value of 2 . He also assured students as he dismissed them that if they did not absorb this new calculator feature they would be working on it more during the next class period.

### 6.2 Debriefing of First Classroom Observation

The class session began with a product rule quiz which was a modification of a previously administered chain rule quiz. Terry combined two functions into one function with multiplication. He commented that if the students had taken the time to look at the solutions from the previous quiz online this quiz would have been very easy for them. Terry said, "Hopefully stuff like that also helps them realize that if they're not getting something, they're not getting it and they need to. Give them something to study come test time. Realize that this 'isn't as easy as I thought'."

During our debriefing Terry noted that, up to this point, he had looked primarily at derivatives of functions and had avoided having his students take derivatives of models. Models in business calculus always include context and units, whereas functions generally do not. In regard to the classroom environment, he noted that this day was a good working day in class and that about $90 \%$ of the conversation was actually about taking derivatives. Terry commented on his reason for using this time for students to practice:

I can tell if they can do these problems really, really fast without putting any thought into them because they have so much experience with them. This slows them down and lets me get feedback from them and catch the ones that really need it, one on one as opposed to me just standing there putting it on the board and watching the ones that really need it, eyes glaze over and it just seems to, at least that day engage them a little bit more.

This activity enacted Terry's beliefs about students' learning through practice and it showed practice happening in his classroom. He noted that he used this type of activity about once a week (the class met three days a week) or a little bit less frequently.

At one point during the time his students were working, a student asked what second derivatives were used for. Terry said that applications are often a concern for this student, whom Terry described as a good student. "He wants to know when things are applicable." Terry answered the student's question about applications for second derivatives later in the class session when he was working at the chalkboard. He did not describe anything specifically, but he assured his students that they would be working on applications during the next class session. Terry felt that not answering this question was the best use of his time, because the point of this class session was not applications of the second derivative, so he chose to defer the question to another day.

In regard to changing the sign during his first example on the board he noted that sometimes he would make mistakes on purpose to see if his students would catch him. He also read body language when trying to gauge student understanding, specifically asking them to nod their head if they understood or shake their head "no" if they did not understand. "If I see the majority of them nodding their heads, I go on, but I want to see it moving somehow. So, that's just a quick way to get a response from the class."

His choice to get students to find his mistakes supports his claim that he wants to foster self-confidence in his students. Finding a teacher's mistake on the board, even if the mistake is intentional, can be one way that a student might gain confidence in their mathematical abilities. In addition, encouraging his students to respond to him and assure him they understand the material backs up Terry's claim that he does not want to let students hide in his classroom.

Terry chose to end the class session by showing students how to use their calculator to evaluate a derivative at a specific value without finding the derivative formula. He agreed that the first presentation of this method was quick and the students might not have caught all the details. "I didn't really expect anybody to get what I did there at the end, but I kind of like ending the class with a little bit of a punch. This is what's coming up kind of thing."

Terry's choice of introducing the calculator's way of evaluating a derivative supports his belief that he should be consistent with the textbook, since the textbook supports this method. While he knew that he did not have time to go into great detail, he felt that this preview would give them an idea of what they would be covering in the next class, which would also enable students to preview the material they were about to learn.

### 6.3 Second Classroom Observation

On week 12, Terry came into class and his overhead projector screen was gone. He assumed it had finally broken, but went looking for another one. When he was unable to find one, he decided to project onto the chalkboard instead. As he set up his calculator display, he made small talk about the situation.

As class got underway, Terry began by reviewing the material they had been working on during the pervious class period. He drew a function on the chalkboard and asked his students to identify the local extrema. Students correctly identified the x -values of the local extrema. As each solution was given, Terry agreed with the students' responses. Then he asked about absolute extrema. One student responded that the absolute maximum was infinity. Terry affirmed the student's idea but corrected the language he used to express that the function was unbounded. Then he changed the graph so that it was defined on an interval and would thus have a finite absolute maximum. In this case, students correctly identified that point.

Next Terry began to explain what it meant for a point to be a critical value.
He tried to elicit the right response from his students but was not completely successful. His students had some correct ideas but seemed to have difficulty expressing them in the right language. Terry corrected their choice of words so that their answers were expressed in the correct manner. He then asked them to identify the critical values, and they were able to do so. After this segment, he
created a new function, a vertical translation of $f(x)=x^{3}$, and asked his students for the local extrema and absolute extrema, which they easily identified as not existing because the function was unbounded. Then he asked about critical points and they were also able to state that there was a critical value.

The third example he presented in a different way. He used language to describe how the slope of the function changed and he asked his students to identify the critical values and local extrema without the graph being given. They were again successful. Terry commended their work and concluded the review time.

Next he turned his attention to a bit of housekeeping and talked about collecting a take-home quiz. He decided to give his students more time on the quiz because, as he admitted to them, he himself had a take-home test to complete over that weekend and would not have time to grade. At this point his students began to have questions. One asked him how far they had gotten in the book. Another asked him what a second derivative was, which he had already covered two weeks ago, during the first class session observation. A third student asked if they could get a hint on a quiz question. Terry respectfully answered the first two questions, and remarked to the third student that he had already done two of the quiz questions for them. The quiz was actually the worksheet he had used during the class observed previously, which consisted of functions for which the derivatives involved the chain rule and product rule. Terry looked at the quiz and
gave the student the hint they asked for, which was a reiteration of the rule they should be using. He also reminded them that they were welcome to go to the Department's tutoring center to get more help on the questions because he was running out of time in class to get things covered.

Then he passed back a stack of quizzes and began lecturing with the overhead projector. He indicated to students that the first example was the one he ended with the previous class period. He reminded his students that critical points will also occur when the derivative does not exist and reviewed ways in which the derivative can fail to exist. He then presented a slide of the First Derivative Test, which was not presented in the textbook in such a formal fashion.

For his next example, he had his students put a function into their graphing screen on their calculators. The example was a cubic equation, and he asked his students before they looked at the graph how many extrema they expected. A student answered three to which he responded, "Good guess, but it's actually one less." He then explained that the number of extrema would come from the number of roots of the derivative, which would be two in the case of his example. Then he showed his students how to locate local extrema using the calculate function on the calculator.

As a follow up, he asked his students to use the nDeriv command in their calculator to find the graph of the derivative. With both graphs, the original function and its derivative, on the same axes, he asked students about the
relationship between the relative extrema of the original function and the corresponding point on the derivative graph. Some students expressed that this relationship was pretty cool and seemed interested in this result. So, Terry went on to show them that they could also find the location of this zero on the derivative graph by using the Math Solver function on their calculator. He reminded students of the key strokes necessary to retrieve their answer. Some students had questions about the calculator and their classmates helped them in small groups.

Next, Terry created a table of intervals on the board indicating how the derivative and the function were changing between the intervals. His students were confused at the beginning but eventually began to respond with the correct answers. Once the table was complete Terry showed his students how they could use this table to find the local extrema of the function.

After this example, Terry asked about inflection points. A student said, "I bet you can find it on your calculator." Laughter ensued and Terry said, "I bet you're right." He then showed the students how to put the second derivative in their calculator and told them that the inflection point was where the second derivative equaled zero. He also drew upon their prior experience with concavity to show them how the second derivative graph corresponded to the concavity of the original function. Class concluded with Terry promising to work more with inflection points the next time they met.

### 6.4 Debriefing of Second Classroom Observation

Terry focused particularly on the language he used this day. He specifically kept using the phrase "critical x-value", at times he used the phrase "critical point" and would correct himself. "It seemed like [the coordinator] sent out an email about making that distinction. So, that's what I was trying to do." This focus resonates with Terry's desire to be consistent, in this case with the coordinator of the course.

In the first two problems, Terry believed that his students were following along and were fairly comfortable with the terminology he was using. He believed that the homework would assist them in making the appropriate connections as well. However, as he presented the third example, he wondered "Can they distinguish between the derivative and the original problem and relate them back and forth?" He was not sure how they were doing on this particular type of problem, but anticipated grading a quiz later that day, which he expected would help him to determine their progress. He again made the comment that he picked quiz questions from old exams to encourage the students to study their quizzes come exam time. Once again Terry was emphasizing his desire to see his students succeed.

Next Terry presented the First Derivative Test. While he agreed that it was not presented with this label in the textbook, he felt that this approach helped "to give them a language to communicate what they've already been doing." He
knew this formality was "mathematical jargon" but pointed out that he did not emphasize the language. However, based on his engineering calculus teaching experiences, he felt that it was "just too standard, it shouldn't be ignored." This reaction relates to his beliefs that business calculus could benefit from mimicking engineering calculus by using a little more formality.

During the interview, Terry remarked that most of his students were beginning to like the nDeriv calculator feature. He also felt that this command provided a good opportunity for his students to remember other calculator features that they had used previously in the semester, such as Math Solver. He remarked that at this point in the semester, he had become more technology savvy.

I think stuff like that, though, causes these students to... [think] this guy doesn't know what he's doing here. So, maybe I earned that back a little bit. You wish you could avoid things like that, but it just happens for everybody.

However, he also thought that incompleteness in instructor knowledge could help students not to feel so alone when they make mistakes using their calculators.

In regard to the nDeriv feature, Terry felt that, depending on what one wanted to emphasize, it could be a very appropriate tool. In fact he "wouldn't mind nDeriv in engineering calculus being used. Only after you taught them the other stuff," which further validates Terry's claims that engineering calculus could be improved upon with components from business calculus. He also felt
that business calculus students were not going to have to calculate a derivative by hand in the real world, so this method was important for them to be able to use.

### 6.5 Third Classroom Observation

Terry began class on week 14 with conversation and turned on the overhead for his lesson. As he began his lecture, Terry went back over the beginning of the example he was working on at the end of the previous class session. He had added a question to the slide from the previous class period as well. He asked his students how they would find a relative maximum value. A student correctly answered his question by stating to graph it and use the minimum feature on the calculator. Terry decided to go through it again with his class on the overhead calculator display. After he obtained the answer, he asked his students to interpret what it meant. He had an $x$ - and $y$-value, but no units. A student quickly responded with the appropriate units.

The next part of the question asked for the inflection point, so Terry asked the students how they would find it using the calculator. They correctly responded that they would graph the derivative. When he used the nDeriv feature, no graph displayed on the screen, so he asked his student what they should do next. They responded to zoomout or zoomfit. Then Terry had them use zoomfit to fit both graphs in the same window on their own calculators. The graph of the derivative was very hard to see in this particular calculator window, so he instructed his
students to manipulate the window by rescaling the $y$-axis. Terry found the relative maximum on the derivative graph, which also gave him the $x$-value of the inflection point. He wrote down the $x$ - and $y$-value of the relative maximum and asked his students what they had found. They correctly identified that x was the year, and $y$ was the rate of change of emissions, but they still had to go back and find the amount of emissions at this x-value. Again after each answer he had his students label the answer with units. To answer a student's question about units, Terry went back to the graph of both equations and discussed the relationship between the graphs to show why each answer had the units it did.

After this example, Terry changed the direction of the discussion, moving onto optimization. His first example was a house with a garden and 60 feet of fencing; the question was to optimize the garden's area. Terry drew several different pictures of how one could create this garden. He also reviewed the formulas for area and perimeter. One student seemed very confused about why the area would be different if the perimeter remained the same. Terry referred to the formula he created for area to demonstrate how this relationship worked. Then he showed his students how to find the derivative by hand, set it equal to zero, and find the maximum area of the garden and its dimensions. He also agreed with a student that they would get the same thing if they used the Maximum feature on their calculator. During this class session Terry used the chalkboard exclusively.

His next example was to minimize a distance that someone must travel to deliver supplies. He reminded his students that in order to maximize or minimize a function they would need to eliminate a variable. Once he eliminated a variable he showed his students how they could simplify their formula. He told them they could use the Minimum feature on the calculator or do it "the old fashioned way", as he showed them. After Terry got the answer, he asked, "x equals three what?" The students correctly gave him the unit "miles". Then the students found the corresponding $y$-value, indicating where the travelers must land. Using their distance formula they found the cost equation. Students were confused because they used their calculator to find the $y$-value, which for them was the value of the area, not the $y$-value of the location. Terry had trouble seeing where the confusion was coming from and finally told the students to see him after class. Immediately after class the camera caught Terry realizing the error the students were making.

The last example Terry had for his students involved minimizing storage and ordering costs for packages of Styrofoam plates. His students got the first answer (about storage) quite easily, but the next part (about ordering costs) caused them problems. Terry gave them a more specific question, which helped them to answer the second part. The third part asked the students to find the total cost function and a student gave him a wrong answer. However, he capitalized on the one portion of the answer that was correct. Another student got confused and incorrectly combined the answers, which Terry acknowledged as well. On the
fourth part, another student was wrong, but again Terry encouraged him, "It's not as complicated as you're making it, but you've got the right idea." The final part seemed to come more easily for students but they ran out of time to finish the problem in class. Terry let his students know they would finish the example the next class period.

### 6.6 Debriefing of Third Classroom Observation

As Terry presented the first example, he focused his students on using the calculator to graph the derivative and find the relative maximum. At this point he had also shown them how to find extrema by taking the derivative, setting it equal to zero, and solving. However, he had not emphasized it. He still preferred to solve problems like this one by hand and noted during the interview, "I'm better at the standard calculus way, but seeing the graph real quickly, using your calculator and looking at the max is a good way of checking your answer from the calculus way." Thus, he felt like the calculator was a good tool for finding this answer as well.

During this class period he tried to cover a lot of material for an upcoming exam. He felt that he was short on time because he had spent a little longer than the allotted time working on taking derivatives with the chain rule and product rule. Terry suspected that he would have planned his pace a bit differently if he were organizing the class alone instead of at the coordinated pace. Because of the
extra time he spent on derivatives, Terry was still covering new material the day before the exam. He kept emphasizing that day in class that they needed to do their homework for practice.

According to Terry, his students did not do their homework over the optimization section of material. Because they did not follow his directions, he was not pleased with their performance on that part of the exam. He also decided to spend more time on the subject of optimization the day after the exam. Again, as was consistent with his beliefs about learning, he continued to emphasize the need for students to practice. He warned them they would not do well if they failed to practice. "If they do their homework they'll do great. If they don't do their homework, they'll at least realize, 'I should have done my homework'," Terry said in reference to the optimization question on the exam the students took.

Terry explained that he chose the garden example because it was a very standard and easy optimization problem to begin with. Furthermore, pictures of what was happening were straightforward to draw and were helpful as his students began to see what was happening in optimization problems. The second problem involving minimizing distance was more difficult because the context of the problem was more complicated, as was the function, but Terry still felt it was very standard and easy to visualize. Terry concluded with a problem that was not geometric, but that was clearly a business-type problem. Although his students struggled some, he felt it would lead well into the rest of the business-related
problems he intended to present during the next class. Terry commented that this lesson would have worked just as well in an engineering calculus course as in a business calculus course, consistent with his recognition that the two courses had similarities. He felt that he was able to find all of the relative extrema by hand and would present the problems in that way even more prominently if he were to teach this material again. He was pleased that business calculus students were expected to do these types of problems by hand. This reaction seemed natural considering he enjoyed working these problems by hand himself.

Throughout this lesson Terry was very consistent about how he responded to incorrect answers. Many times a student would give him an answer that was partially right and when this happened he commended the correct portion. Even when a student was completely incorrect, Terry redirected the student, using statements that indicated there was correctness in their ideas about the problem. He was encouraging and supportive, which was consistent with his belief that his students were capable of what he was teaching them.

### 6.7 Conclusion

Terry's classroom experiences strongly echoed his professed beliefs. He did focus on attempting to get his students to practice on the problems, primarily at home but also some in class. He appeared to emphasize at different points whether an answer was reasonable and reminded students to include the units for
all answers. While primarily using interactive lecture, he incorporated a smattering of seat work and board work. His focus on being consistent with the textbook and the coordinator was also evident. Terry's desire to relate to his students and to see them succeed was apparent in each class period through his interactions with students and in the way he created his quizzes. His descriptions of his teaching methods and the format of a typical class session were accurate as well. Overall, Terry's professed beliefs were very close to his classroom practices.

## Chapter 7: Jessica's Classroom Experiences

Jessica's classroom was fairly consistent with the beliefs and opinions she held about business calculus, her students, and teaching. Her presentation was consistent with her profession of how her class is organized, but her students' interaction with her varied from day to day. She maintained a professional atmosphere and attempted to make class lighthearted even though she felt it was stifling at times.

In Chapter 5 we looked at the following beliefs of Jessica. In this chapter we will compare them to her classroom practices.

- Learning occurs through trial and error
- A teacher's role is to facilitate learning,
- A teacher is the supplier of information
- A teacher is a secondary resource to the textbook
- Mathematics is the language of the universe and it is logic
- Mathematics is cross-cultural and computational
- Disappointed with business calculus teaching assignment
- Did not like the modeling approach taken by the book
- Did not believe business calculus constituted a mathematics course
- Students have good intentions, but unrealistic expectations
- Students have a sense of entitlement
- Used lecture with the overhead and the chalkboard
- Used board work in the past, but not in this course
- Used a fellow GMTA's notes in their entirety
- Preparation involved mostly vocabulary preparation
- Assigned homework but evaluated with quizzes
- Opened class with conversation to eliminate silence
- Lectured exclusively with lots of examples
- Added to her applications of calculus
- Did not enjoy teaching like she normally did
- Hoped she never has to teach it again
- Did not think it will help her resume.


### 7.1 First Classroom Observation

On week 11 Jessica entered the classroom by creating small talk with her students. She began lecturing on the chalkboard. The topic Jessica discussed initially was creating the graph of an original function when given the graph of a derivative function. As she worked through her first example, her students were taking notes but were not very responsive as Jessica asked questions. During the
second example, the class was a little bit more responsive, but Jessica still did most of the talking and the work of creating the graph. According the debriefing afterward, this part of her lecture was in response to a question asked by a student the previous class period.

After she completed this material she asked her students to turn their focus back to the example she had not completed the previous day. She then wrote the answers they had previously obtained on the board. She explained that, during the previous class, she had lost track of what they were finding while they were working. Therefore, Jessica said she wanted to review what they had found in each step and how. The previous class period Jessica and her students had derived a model, which was cubic, and found the first and second derivatives. She completed these calculations by hand and set the second derivative equal to zero in the previous class. Because this graph was cubic, the second derivative was linear. According to Jessica during debriefing, setting the second derivative equal to zero and solving was a relatively straightforward task for her students because the function was cubic. So she had her students complete the next part of the example individually, which was finding the inflection point of the model. She gave them time to work and they computed the correct answer.

After they found the inflection point's $x$-value, she discussed how the graphs of the original function, first derivative, and second derivative would be related, harkening back to the work she had done with graphs at the beginning of
class. Then she had her students find the corresponding y-value. A student responded with an answer and Jessica said, "That seems right." Then she verbally verified that the rest of the class found a similar answer.

Next she discussed with her students how to correctly set the window in their graphing calculator to see the inflection point in order to determine if it was a point of most- or least-rapid increase. She had her students graph the original function to determine the correct answer to the question. Her students found two different graphs on their calculators. Jessica was not graphing the function herself and was not sure which graph was correct as she viewed their screens. She made an educated guess as to which one was correct and drew it on the board. Her students began chatting among themselves, trying to find the error in the entry of the function on their calculators. She then talked with her students about the answer to the problem.

After this example, she directed their attention to a textbox in their book that she thought would be helpful in identifying how a function changes at the inflection point. At this point Jessica realized she had not written units on one of the answers on the board then proceeded to include units. However, every other answer was still missing units which she either did not notice or did not decide to include at this point. Then she had her students find the rate of change (derivative) of the model at the inflection point. When they found the correct answer, she asked them what the units were and they correctly stated the answer.

About half-way through the class period she wrote another model from the textbook on the board. She elected to have her students use the nDeriv command in their calculator to construct the graph of the derivative and use it to find the graph of the second derivative. She answered questions her students asked about this feature.

Jessica asked her students to graph all three graphs at the same time. Her students asked questions regarding the appropriate window and Jessica again chose an appropriate viewing window using properties of the variables which would lend themselves naturally to certain constraints; for example, the natural constraints on temperature in degrees Farenheight would be 0 to 212 in most situations. Once her students began creating graphs, she had them alter the window they were using. Jessica did not use any of the Zoom features of the calculator to determine the appropriate window. She continued to view students' calculators as she moved around the front of the room. She had her students alter the window once more by changing the constraints on $x$ and $y$. Because she did not see the graphs appear on the screen herself, she guessed which graph was the original and which were the first and second derivatives, commenting on the fact that she was guessing.

Toward the end of class, she had her students find the inflection point using Math Solver on their calculator to determine where the second derivative was zero. Students got very different answers and Jessica said, "Okay,
something's wrong." Again she did not know which answer, if any, was correct, so she decided to conclude class at this point and come back to the question the next class period.

### 7.2 Debriefing of First Classroom Observation

The first two examples that Jessica performed on the board were chosen from among four different options she considered when preparing for this class period. This class session was the first time she presented the idea of starting with the graph of a derivative function and retrieving the graph of the original function.

Jessica decided to use the chalkboard instead of the overhead projector. This activity supported Jessica's statements about primarily using lecture, but preferring to use the chalkboard over the overhead when possible.

When asked why she chose to present material which was a bit beyond the scope of the class and whether she expected her student to replicate what she showed them that day on the board, she commented

Yes, but not probably after just one try. We're going to be doing at least one more problem with that. I don't think it's one of the objectives, but I think understanding how the backwards relationship goes helps them with the forward relationships.

She remarked that she knew this concept was not really one of the objectives, but believed it would help them be able to draw the graph of the derivative if they were asked, which was one of the objectives of the course.

When asked about whether she thought the class was following along, Jessica was hesitant to reply. She found this question difficult to answer and finally said

I think some of them were. I guess this is probably true of every class, but the most interactive group kind of sits at the front. So, I know if I don't have them along, I know I don't have anybody along for the ride. And when I do have them along, then I kind of just hope that the rest of the class is with us. I try to check them out, but they're pretty much blankfaced all the time.

So Jessica chose to gauge the class response by one particular group who were often responsive enough for her to know whether or not it was safe to continue. She further commented, "It's probably not the best gauge for everybody's understanding but it's a pretty good gauge of nobody's understanding."

As she turned her focus to the modeling questions, she commented that these particular questions took so long to complete that during the previous class period she had run into time management problems. "By the time we put it all in the calculator, and got the graphs on and found the inflection point nobody could remember... what the question was." For the first problem she intentionally had students find the inflection point by hand "from the beginning just to get a feel for doing that once." Again, Jessica's emphasis on having students solve things by hand reflects upon her desire to do mathematics by hand herself. When asked if everyone was working, Jessica said she did not know. In addition, her lack of
attention to her students actively finding the answer seemed not to be a concern for her when we discussed this instance of it.

Next we considered the second modeling problem where Jessica had her students focus on using the calculator to find the inflection point. When asked about the fact that she did not use the overhead calculator screen, she said that she liked to use it, but this particular day the overhead projector was acting up. But she commented that, "We have lots of calculator-type discussions. I encourage collaboration," which is exactly what was happening in the observed class. Some students were trying to help one another figure out why their graphs looked different.

One of the reasons that Jessica chose to focus on adjusting the window of the calculator screen herself and displaying it for students was because it gave her a chance to help her students see what would be reasonable inputs and outputs for the functions. This particular problem was one that Jessica did not look at prior to presenting it in class. She selected it on the spot while she was lecturing. Since she had not checked the assigned homework list, she was not sure if it was an assigned homework problem, but she was hoping that it was.

Jessica chose to use the calculator for this problem. "I looked at the clock and I erroneously decided it would be faster to do it by calculator than by hand." Thus, her choice to use technology was based on her perceived lack of time to do otherwise. This semester was the first semester she had used the nDeriv command
and she had mixed feelings about it. For these particular students she felt it was okay because they were not interested in the mathematics behind the derivative model. However, she personally preferred to take the derivative by hand.

Jessica chose to manipulate the window by adjusting the variable ranges because, "I have bad feelings about ZoomIn and ZoomOut, especially when I'm doing it with a class because it depends on where you start. It's the same with ZoomFit." However, this set of decisions seemed to complicate rather than simplify the situation. Jessica did not feel confident in using a combination of ZoomFit and manipulation of the x -values in a productive and time-saving way to construct good windows for the graphs. All of these decisions can be seen to stem from the fact that Jessica lacked interest in using calculators. She preferred to do mathematics by hand.

In addition, Jessica was using her language very loosely during this particular class session. She used the words increasing and decreasing without referring to what was increasing or decreasing. In these particular problems she was referring to the slopes increasing or decreasing but students did not seem at all clear about this distinction. They may have been thinking that $y$-values (ie: the graph) were increasing or decreasing.

### 7.3 Second Classroom Observation

Jessica began this class period on week 13 by writing on the board a list of old exam questions that she thought her students should study for their next exam. She also advised them of where they could find the old exams online. Then she encouraged the students to come back the next class period with any questions they had over these problems so that she could help them understand the answers.

Jessica chose to have her students review a worksheet over how to take derivatives using the chain rule and the product rule. This worksheet was the same worksheet that Terry used in his classroom, but at this point in Jessica's class, her students had received their grades for the worksheet. She selected one problem from each page that she felt many students had trouble with and worked out the derivative. Then she let the students ask one or two questions from that page. She also chose to work exclusively on the chalkboard during this class period.

The first example she chose was $f(x)=\ln \left(6^{x}\right)$. Before she attempted the problem, she had her students state the basic rules for calculating the derivative. She also reviewed some basic exponential properties, such as $\frac{1}{x^{n}}=x^{-n}$. As she listed these rules on the board, the students were quite involved as they recited the properties. With help from her students she produced $f^{\prime}(x)=\ln 6$. Then to probe further, she asked what the second derivative of the function would be. Her students fumbled with a couple of different, incorrect ideas which she ignored.

However, after she showed them what the answer should be, a student correctly stated why this answer was correct and Jessica commended the student. As this problem concluded, a student asked when it was necessary to simplify his answer and Jessica reviewed the guidelines for simplification common to all business calculus classes.

The next problem, chosen by the students, was $f(x)=e^{0.4 x}$. The students correctly identified that this problem would involve the chain rule. Furthermore, many students correctly stated what the next step should be and in fact arrived at the right conclusion. The second problem chosen by the students was $f(x)=\left(5^{x}\right)^{2}$. Students again correctly assisted in finding the derivative of this function.

Jessica then turned to the second page of the worksheet and chose the next problem herself. Her example was $f(x)=2^{x} \ln x$, which she pointed out to her students involved the product rule. Her students were able to help her with many portions of this problem. Jessica also presented this problem as the compilation of two derivatives. By this she meant that she had them find the derivative of each portion and then take another step to assemble the derivatives correctly using the product rule. The next question chosen by her students was $k(x)=e^{x} \pi^{x}$. Her students correctly identified that this problem would use the product rule but not the chain rule. She again used her compilation method to assemble the derivative.

On the third page Jessica chose $f(x)=3 x \sqrt{4 x+2}$. She asked her students what rules they had for radicals and several students suggested changing this equation into exponential form. She agreed and asked students which rule they should use and they correctly answered that this problem would use both the product rule and the chain rule. The students were able to assist Jessica in finishing this problem as well. She then asked the students which problem they would like to see. They chose $h(x)=\frac{50}{1+9 e^{-x}}$ and because the quotient rule is not taught in business calculus, Jessica asked her students how this function could be rewritten and they correctly stated $h(x)=50\left(1+9 e^{-x}\right)^{-1}$. She asked students which rule they should use, and they again gave the correct answer, the chain rule. One particular student assisted Jessica in finding the derivative.

Moving on to the next page, Jessica decided to look at the function $f(x)=\sqrt[4]{3 x^{4}+e^{2}-6 \ln x}$. Students rewrote this function in a fashion that made it easier for them to find the derivative. Next the students requested to see $f(x)=e^{\ln \left(2 x^{3}\right)}$. Jessica wrote the derivative of this function on the board with her students' help. The final question requested by students was $w(x)=x^{4} 4^{x^{2}}$. This example caused students some trouble as they had to identify it as a chain rule and a product rule.

She ended this portion of the class and had her students take a quiz over optimization, which they had covered during the previous class period. She also warned her students that the quiz is "probably easier than what would show up on an exam." As students completed the quiz, they left, all finishing before class was officially over.

### 7.4 Debriefing of Second Classroom Observation

Jessica graded the worksheet she was using as a reference in class that day. She felt that about half of her students were proficient with the chain rule and the product rule but the other half were really struggling. This section of material also caused Jessica some concerns. She felt torn about its purpose in the business calculus course. "As far as the goals of the class go this is one of those gray areas for me. Are we teaching them math? Or are we teaching them business?" However, because the nature of this material was, in Jessica's mind, mathematically driven, she enjoyed teaching it. However, she did not like the limited amount of time designated for this material. "I would've preferred much, much more time on this."

In reference to the worksheet, Jessica said her students tended to have a lot of problems with the questions involving the exponential rule $f(x)=b^{x}$. She also felt that many students were confused about when it was appropriate to simplify and when it was not necessary.

In regard to Jessica treating product rule derivatives as the assembly of many smaller problems, she felt that some students were missing problems unnecessarily. When a problem only involved the product rule, they were able to correctly find the derivative. However, when the problem involved both the product rule and the chain rule, they were making mistakes. In Jessica's opinion, these mistakes would have been less common if the students treated each part of the product as a separate function, took the derivative of each part, and then assembled the derivative of the product rule when they were finished. Jessica was extremely focused on which derivative rule the students were using for each step. This focus seemed to echo her sentiments that mathematics was logical and easy to practice in smaller pieces than other subjects.

Jessica chose a basic garden-with-a-fence type of optimization problem for her quiz. She noted that students either got full credit or virtually no credit. However, she also admitted that, when teaching this section of material, she presented only the geometric problems and avoided the business-related problems.

It was probably a naïve hope. I knew there were problems that were in the homework, so it was my naïve hope that they would do the homework and then ask a question about it if they didn't get it. But that was a fool's dream. So, I think I'd better throw some more of those in if I get the chance again.

Thus, Jessica felt that her students probably did not do well on this portion of the exam because it was business-related rather than a geometric problem.

This class session was very interactive. Many students were involved in either asking or answering questions. Students knew a test was impending and were interested in making sure they understood their mistakes. Jessica helped students gain insight into what to prepare for in terms of the types of questions asked and the types of simplification required. There was a strong connection between the content of this section and Jessica's preference for content that she viewed as mathematically-based rather than business-based. This material allowed her to enjoy what she was teaching and be more natural in her classroom with her students.

### 7.5 Third Classroom Observation

On week 15 Jessica started the class session with information about her finals week office hours. She noted that the Department's tutoring center was closed during finals week and offered to have a couple of office hours (which was not required of her by the Department). Then she reminded her students of homework due the following week.

Next Jessica turned her attention to finishing the final sections of new material for the semester, cyclic functions. As she began she referred to the material she had begun the previous class period and questioned her students about the appropriate uses for cyclic models. They correctly identify several uses for cyclic models. Using the chalkboard, she sketched one and a half periods of a
sine function. She also denoted on the graph the values of interest, namely the x intercepts, x -value at the maximum, and x -value at the minimum. Next she discussed the concavity of the graph by identifying intervals on which the graph was concave up or concave down.

Then she asked her students to consider the graph of the derivative of the sine function. She created the derivative graph by using the relative extrema and inflection points for reference. She had her students help with drawing the graph by identifying how the slopes of the sine curve were changing. Several students were able to correctly identify the majority of the information she asked. After drawing the curve, she asked her students, "What kind of a curve is this?" to which they responded, "A cosine curve." She agreed with them and reminded them that the cosine function is merely a horizontal shift of a sine function. She asked her students what general shape they would expect if they took the derivative of a cosine function. They guessed a sine function. She agreed but then stated it would be a negative sine function and she referenced the cyclic nature of the sine/cosine relationship between successive derivatives.

Next Jessica switched from the chalkboard to the overhead projector with prepared transparencies. The first thing written on the overhead was the information she had just derived about the derivatives of cyclic functions. Then she reminded her students that taking the second derivative was taking the derivative of the derivative.

The first example was $f(x)=x^{5}$. The students were directed to take the first and second derivative. There was little interaction among her students, so Jessica gave the answers to these questions and continued on to a sine function, $f(x)=\sin x^{5}$, which she admitted during the debriefing that she created on the spur of the moment. She had her students take the derivative and one student began to help her by responding with parts of the derivative. Her students identified that the second derivative would require the use of the chain rule and the product rule. She took derivatives of portions of the function and then assembled the answer. A student asked a question about simplifying the results, to which Jessica responded that it would only be necessary if they were asked to take another derivative of the function they had just found.

Next Jessica transitioned back to her prepared slides, and had her students find the derivative of three examples on their own commenting that they would discuss the examples afterward. Most students appeared to be working but only independently. After a short time, she began working the first example, $f(x)=\cos (3 x)+56$, making note of the fact that she would have to use the chain rule. As she began the second example, $f(r)=5.2 \cos (0.45 r+\pi)+80 r-6.34$, she advised students that they needed to remember to treat $\pi$ as a constant. She gave her students another portion of time to work. Then she reconvened and had her students help fill in the details of the derivative function. A student had a question
about the process and Jessica answered his question with an alternate way of writing the answer. For her final example of this type, $f(x)=6.9 \sin (-12 x+7)-4.2 \cos (8.6 x+13)$, she had her students find the first and second derivative. She gave them a little bit longer to work this time and then worked the example for them. Her students appeared to be actively involved in finding the derivatives when she gave them time to do so. She also simplified the derivatives as she worked so that she would be able to take the second derivative more easily. She asked her students if they were okay with this process. A couple of students had quick questions that she answered. She then took the second derivative without input from the class.

Next she reminded her students of a few formulas they could expect to see in the homework questions over this material. These were formulas that they had not seen in a while: formulas for change, percent change, and average rate of change. She commented that, "The hardest thing to figure out is which one of these they want you to find." She also reminded them that a question referring to the rate of change could have different meanings. If it gave them one $x$-value it was the instantaneous rate of change and if it gave them two $x$-values it was the average rate of change. The students echoed the correct answers to these questions as she explained.

Then the students were asked to find the relative extrema of the sine model $T(x)=1.204 \sin (0.469 x-2.293)+6.352$ percent of profit of an investment,
x years after 1998. She also asked her students why it was necessary that she specify over what interval they were to find the relative extrema. They correctly responded that there were infinitely many relative extrema for a sine curve. Jessica already had the calculator window details written out on the transparency, but she did discuss with her class why the values should seem reasonable. Next she described details about how this model would change the graph of the standard sine curve, in terms of the period, amplitude, and phase shift. This topic had not been discussed in her lectures. She chose to discuss this topic to emphasize the reasonableness of the window she chose.

For this particular example, Jessica elected to enter the information on the calculator. She asked her students for the number of maximum and minimum values. They were arriving at responses that matched the graph Jessica had on her calculator. However, she again did not use the overhead calculator screen to display her graph. Next she reviewed the three methods her students had for finding the relative extrema: using the calculator with the original function, using the calculator with the derivative function and Math Solver, and using the calculator and finding the zeros on the graphing screen. She had her students choose their favorite method. A student had a question about reasonable guesses with Math Solver and Jessica reminded her about the Trace feature on the calculator. As she advised her students to get the answers on their own, she also gave them the answers, with units, because she was out of class time. She went
over the details of the answer before dismissing the students. Other questions arose from her students about the methods for finding these extrema with Math Solver. She stayed with these students to explain further how to work through the problem using this feature.

### 7.6 Debriefing of Third Classroom Observation

Jessica chose to draw on her students' knowledge of how to draw a slope graph to explain that the derivative of sine is cosine. She was pleased with the fact that they had done well matching functions to their derivative graphs on their second exam. "It might be a factor of how much I've really talked about it. On the second exam I think I only had three students who didn't get perfect scores on their graphs." So this method was a natural way for Jessica to justify the derivative of the sine and cosine functions.

At one point one of Jessica's students answered a question incorrectly. While Jessica did hear the incorrect answer, she chose to ignore it. She thought the student was making a mistake in her choice of language, not that she was actually mistaken in her idea. A moment later another student answered the question correctly. This action seems to correspond to Jessica's beliefs that students have good intentions and want to do well. She believed her students were trying to arrive at the right answer.

Jessica's choice for her first trigonometric derivative was a spur of the moment decision. She did not intend for it to be the hardest derivative they would take, but felt that it worked out okay. She felt that her students did very well on the exam questions involving the chain rule and the product rule, so she was again drawing upon their prior knowledge, in particular something she felt they understood well. If she were teaching the lesson again, though, she felt this example would have been better as the final example.

Jessica was also allowing varying amounts of time for students to complete the problems. She had a few students in particular that she used to gauge how long to allow for a problem to be worked on before moving on. The students she used when considering her pace were what she considered her students of average mathematical skill. They were students who were in the low C to D range of grades. About one student in particular she commented, "If he's done writing, he's probably gotten it correct or given up on ever being able to do it." So when she observed that he, along with a few others had stopped writing, Jessica felt it was time to move on.

As Jessica reminded her students of the different formulas for change such as change, percent change and average rate of change, she herself seemed to give her students an excuse to be confused. Her explanations were not clear and she commented that deciding which formula to use was the most confusing part of these types of problems. Her reasons for choosing a specific rate of change
formula were not completely accurate. She appeared to have holes in her own understanding as she conversed with her students.

The overhead calculator screen was not used in class this particular day. Jessica agreed that she used it more in the beginning of the course but had not been using it lately. She would have only used it for this one problem and felt it was unnecessary to carry it into the classroom and set it up.

For the most part, my students that are trying hard, are able to put stuff in their calculator correctly... Although, I kind of wish I had it for this one because of the min and the max and the next point could have been a minimum on the window, and so it would have been nice to show them that.

Jessica failed to look ahead to the kind of issues that might arise, so she was unable to counteract these distractions efficiently. She realized that the overhead calculator screen would have helped if she had brought and used it.

At this point in the semester, Jessica was almost finished. She had about two weeks left of class and hoped she would never have to teach the course again. When asked if it was the type of material covered or the type of students she was teaching, she responded,

A lot of my students I really do like... Some of them I don't. I think, though, it's mostly a material issue...I like a lot of my students. But I do think it's more material driven, than it is class driven.

### 7.7 Conclusion

Jessica's classroom experiences were mostly in line with her beliefs. She lectured almost exclusively with lots of examples for them to work along with her, because she was responsible for disseminating information. She tried to give her students opportunities in the midst of her lectures to practice, and definitely encouraged them, through the use of grades, to do their homework. She did seem much more comfortable teaching components of the course more common to any calculus class as opposed to specific to business calculus and much less comfortable with the business related concepts. She also believed her students were trying and working hard in the class, and her beliefs about students having unrealistic expectations were not readily noticeable in the classroom. However, although Jessica claimed dislike of the business related curriculum, this distaste for the material was not obvious.

Her lack of enthusiasm concerning the semantics was apparent at times, however. Jessica's descriptions of her class and her beliefs coincided quite well with the observations of her classroom. She concealed from her class her negative beliefs about the course and about students.

## Chapter 8: Comparison Discussion and

## Implications

The research questions this study attempted to answer revolve around teaching a class one did not take as a student. Because this teaching experience is unique and GMTA research is such a young field, studies to date have not yet explored this idea. Thus, the research questions I wanted to explore were:

- How do GMTAs approach teaching a class they never took?
- What teaching methods do GMTAs employ?
- How do beliefs about teaching and learning affect the teaching of business calculus?
- How do beliefs about mathematics affect the teaching of business calculus?
- How does the different population of students affect GMTA teaching?
- How does the GMTA's attitude about the class affect their teaching of business calculus?
- What resources do GMTAs use in preparation for teaching this class?
- How can we better prepare GMTA to teach business calculus?
- How can we adjust the perception of this course among the Department as a whole?

This study was interview driven. An initial interview discussing beliefs and classroom practices took place followed by videoclip observations and debriefing interviews. Primary data collection concluded with a final interview with each participant. (Interviews with administration were secondary and are not discussed in this chapter.) This chapter will draw explicit comparisons between Terry and Jessica and will inform the community about GMTA beliefs and practices regarding teaching business calculus.

The similarity of the external factors surrounding Terry and Jessica when they were assigned to teach business calculus made them ideal candidates to compare for the purposes of this study. On the surface, they seemed to be a lot alike. However, it quickly became apparent during the respective initial interviews that many of their beliefs about teaching, learning, mathematics, students, and business calculus were quite different. In addition, while their teaching methods may have been similar in the past, there were apparent differences in their teaching experiences. Even when similarities in beliefs or practices existed, they often existed for very different reasons.

During class observations and debriefing interviews over the class sessions, it was evident that the students and the instructors in each of these sections were having a very different experience with this course. For the
instructors, their attitudes about business calculus, classroom practices, and beliefs about the effect this experience would have on their future contributed to the differences in their students' experiences. This study offers insight into how GMTAs approach teaching business calculus, the methods they choose for teaching, and why they might choose different methods than they previously have. Based upon the teaching methods employed by these GMTAs and how they prepared for teaching in this course, I offer suggestions about preparing GMTAs for this teaching assignment and supporting them during their first experience teaching this course. These suggestions are intended to help make this course a more positive experience for the GMTAs and for their students. Furthermore, the research exposed a wide-spread lack of respect for business calculus and this teaching assignment; these results led to thoughts about improving the reputation of business calculus among members of the mathematics community.

### 8.1 External Characteristics

Terry and Jessica had a lot in common. They were both GMTAs for the Department of Mathematics at the University of Oklahoma and were both graduate students seeking PhDs in mathematics. Because mathematics was their field of study, they were both taking graduate classes in mathematics and teaching undergraduate mathematics classes for the Department. They both had a love of
mathematics, and both wanted to share this love of mathematics with their students.

In addition, Terry and Jessica were both experienced GMTAs when they were given the business calculus teaching assignments. They had each taught mathematics at the college level for over four years at the time of the study. They had developed philosophies of teaching. They had established their preferred methods, and their beliefs were fairly clear about what constituted teaching and learning. They had also taught many of the same courses in the past including precalculus and engineering calculus.

While both had been teaching courses for the Department of Mathematics for several semesters and had taught a variety of classes, the semester of data collection was the first time they had been assigned to teach this course. This semester was their first exposure to business calculus in any form because, as was to be expected, neither one had taken such a course as an undergraduate. Furthermore, neither one had done any tutoring in the subject. In addition, when they were assigned to teach it, it was not a course either had an active desire to teach, nor had either specifically requested to teach.

Not only had both GMTAs taught a number of classes before this particular semester, but also both had been successful at the teaching component of their assistantship. They had each had positive experiences in the classroom, and their students commented positively about them in student evaluations. For
them, being a GMTA was more than just a job. It was a job that they were good at. The Department had rewarded both GMTAs for their classroom success with teaching awards. In addition, both GMTAs had also taught their own sections of engineering calculus prior to this teaching experience.

In addition to being good at teaching, they both enjoyed teaching. It was not something they were doing because they had to, but because they wanted to. In fact, each of them viewed being a GMTA as good training for their futures careers because both were planning to look for faculty positions upon completion of their doctorates. They did not view their GMTA appointments as the only time in their career when they would be teaching, but as the beginning of their roles as professors. Thus, they were good at, valued, and enjoyed teaching.

### 8.2 Philosophy of Teaching and Learning

In this section, we compare the philosophies of teaching and learning professed by the two participating GMTAs.

Table 1: Philosophy of Teaching and Learning

|  | Terry believed | Jessica believed |
| :--- | :--- | :--- |
| Students learn by... | practice | trial and error |
| Learning... | occurs primarily outside <br> the classroom | requires personal effort |
| The role of a teacher is... | a guide | to facilitate learning and <br> supply information |
| A teacher should be... | consistent with the <br> textbook | a secondary resource to <br> the textbook |

Terry and Jessica both believed that students learn by doing, and the role of the instructor is to get students to practice and use their textbook. However, in the classroom, Terry incorporated practice to a much greater extent than Jessica did. Jessica did not believe that within the business calculus course structure there was enough time for students to practice much during class. Thus, she served primarily to disseminate information. While Terry also used lecture primarily, he promoted practice within his classroom.

Furthermore, while Jessica believed that learning should occur through trial and error, this did not occur during the observed class sessions. She explained during the debriefings that she had forsaken trial and error because of time constraints. However, she did have students work examples along with her
during her lectures. Terry felt there was a lack of time in class as well, but as he deemed viable he still incorporated hands-on learning activities.

Both made reference to the textbook during interviews. Jessica viewed the book as the primary resource for her students, whereas Terry only felt a need to be consistent with the book. Thus, it seemed that in Jessica's class she perceived herself to be a secondary resource and the book a primary one, but in Terry's class, he mentioned only feeling the need to be consistent with the textbook. Thus, on the surface Terry's and Jessica's beliefs about teaching and learning were quite similar, but in practice they were somewhat different.

### 8.3 Beliefs about Mathematics

Next we explore the differences between Terry's and Jessica's beliefs about mathematics.

Table 2: Beliefs about Mathematics

|  | Terry believed | Jessica believed |
| :--- | :--- | :--- |
| Learning mathematics <br> is like learning... | a foreign language <br> because of the rigor | a universal language <br> because it is cross- <br> cultural |
| Mathematics is... | the study of patterns | logical and <br> computational |

Both GMTAs believed that mathematics was a language that one should learn. However, Terry believed it was like learning a foreign language because it was a difficult task that requires practice while Jessica believed it was a universal language because there was something about it that was innate. These are quite different viewpoints. From Terry's perspective, learning mathematics would appear to be fairly challenging. However, from Jessica's perspective, learning mathematics should be very natural.

In Jessica's case, her view of mathematics made it difficult for her to accept business calculus as mathematics. While she claimed to believe that mathematics was logic, she did not feel that it was her role as a mathematics teacher to teach logic, at least not as it related to this particular class. Her role was to teach computations, those in particular that she had learned as a calculus student herself and was proficient at teaching to her engineering calculus students. Jessica felt that the strong verbal component of interpretation of the course did not constitute mathematics. The teaching of semantics, namely teaching students to verbalize their answers and to answer questions with vocabulary atypical to an engineering calculus class (such as with percent change, percent rate of change, profit, revenue, marginal revenue, average cost, etc.), kept this course from being mathematics in Jessica's mind. Because Jessica viewed herself as a mathematics teacher, and in her eye this course was not mathematics, she did not particularly enjoy teaching business calculus even though she typically enjoyed teaching.

In Terry's case, he believed that mathematics was about understanding patterns. Because of his beliefs about mathematics, he was much more able to accept business calculus as a mathematics class. He did not view it as a pure mathematics class, but rather as applied mathematics. The verbal component did not pose nearly as much of a problem for Terry as it did for Jessica. He simply viewed this component as another part of the patterns that he was responsible for teaching his students. He felt he was a mathematics instructor teaching a mathematics class, so there was not the same sense of displeasure that Jessica experienced.

Terry and Jessica differed quite a bit in their beliefs about mathematics. The way in which they viewed the language of mathematics was similar initially but ultimately very different. In addition, because of what they individually believed about mathematics as a subject, they were able to incorporate business calculus into their definition of mathematics to differing extents.

### 8.4 Thoughts on Business Calculus

Turning our attention to specific beliefs about the business calculus course, we see that Terry's and Jessica's beliefs differ again.

Table 3: Thought on Business Calculus

|  | Terry | Jessica |
| :--- | :--- | :--- |
| Initial reaction to <br> teaching this class was... | nondescript | quiet discontent |
| Thought modeling with <br> the calculator... | was useful to the <br> students | detracted from them <br> learning calculus |
| Thought the focus on <br> interpretation was.. | useful to students | overemphasized |
| The title of business <br> calculus is ... | accurate, it is business <br> and calculus | inaccurate, it is not <br> really mathematics |

The initial reactions of these two GMTAs toward teaching business calculus were very different. While neither asked to teach the course, Terry was okay with the assignment, while Jessica really was not. Terry was neither excited nor upset about the teaching assignment, but Jessica expressed quiet discontent. Even though both knew they could not avoid this teaching assignment, Terry embraced the challenge and Jessica resisted it. She even commented that it was probably her own negative attitude that created the majority of her difficulties in teaching the class.

In terms of the content taught, Terry understood the usefulness of the business related content. He bought into the mission of the textbook and understood how a modeling approach would be useful to his students. He felt that for these students the use of the calculator and the emphasis on application was
appropriate. Therefore, Terry was able to teach this course believing that he was teaching his students mathematics that they could use. Jessica felt that the applications were a distraction from the calculus and that her students would not be able to apply calculus concepts because of the overemphasis on applications, units, and semantics. Jessica, unlike Terry, struggled because she did not agree with the applications-based view of the textbook.

Technology tends to cause many divisions among instructors, and the graphing calculator used as a part of this course was an instance of that difference for these two GMTAs. Terry accepted the calculator as a part of the way business calculus was taught. He learned how to use it and made every effort to make it as accessible to his students by using an overhead display. Jessica, on the other hand, resisted using the calculator. She learned only what was absolutely vital. She used the overhead display on occasions, but generally opted to work things out algebraically in spite of the calculator methods available to her and used by the textbook. Because she disliked using the calculator, she did not give her students the kind of practice within the classroom that Terry did.

Jessica's lack of full embrace of the textbook and its mission made teaching business calculus a much more difficult experience for both her and her students than for Terry and his students. While Jessica claimed to believe the textbook was the primary resource and she a secondary resource, she also felt that her students would learn from the book the types of problems she did not cover
during class. Terry's willingness to follow the textbook, accept the course content, and teach it as the coordinator and experienced instructors did allowed his students to have an experience more in line with the design of the course than Jessica's students had. Jessica's students were at a disadvantage in the course because of her resistance to the calculator and because of her avoidance of certain types of applications.

### 8.5 Beliefs about Students

Beliefs the GMTAs held about students had implications in both Terry's and Jessica's classrooms. In this area their views were very different.

Table 4: Beliefs about Students

|  | Terry | Jessica |
| :---: | :---: | :---: |
| Business calculus students... | deserve respect <br> $90 \%$ are capable <br> are not like him (did not love math) <br> must be held responsible for their actions | have good intentions have unrealistic expectations <br> have a sense of entitlement <br> do not know what their best is |
| Engineering students... | are not that much different from business calculus students | have a better grasp of their own mathematical ability |

Terry and Jessica differed greatly on their view of students. While Terry felt that most students were capable of doing mathematics, Jessica felt that most students had an inflated view of their mathematical abilities. Terry did not believe that his business calculus students were lacking mathematical ability but rather lacked manipulative ability. However, for Jessica mathematics was computational, so the student's lack of manipulative ability translated to a lack of mathematical ability.

Jessica and Terry also had very different views of business students in particular. They both felt that the engineering students were a bit better at the computational mathematics. For Terry, this belief simply came from business calculus students' lack of mathematical background, but it did not mean that they were not going to be successful in business calculus. For Jessica, because she felt that mathematics was primarily computational, even those who did well in business calculus had not demonstrated as much mathematical aptitude as those who did well in engineering calculus. Thus, while they both felt there was not a huge disparity between business calculus and engineering calculus students, they gave different reasons for the differences they saw.

As a result, Jessica's students were not as successful as those in the average business calculus section, while Terry claimed his section obtained scores which were average when compared to other business calculus classes. We know from prior research that students live up (or down) to a teacher's expectations
(Zeichner et al., 1998) and this relationship seems to have had an impact on the performance of the students in these two sections. The majority of Terry's students were successful and a large percentage of Jessica's students were not. Whether intentional or not, Terry and Jessica did in fact prove themselves right in their beliefs about their students.

### 8.6 Teaching Methods

The teaching methods employed by Terry and Jessica were a lot alike in the past and fairly similar in this class as well.

## Table 5: Teaching Methods

$\left.\begin{array}{|l|l|l|}\hline & \text { Terry } & \text { Jessica } \\ \hline \text { In the past... } & \begin{array}{l}\text { used lecture, discovery } \\ \text { learning activities, and } \\ \text { boardwork } \\ \text { primarily used the } \\ \text { chalkboard }\end{array} & \begin{array}{l}\text { used lecture and } \\ \text { boardwork }\end{array} \\ \hline \text { In business calculus... } & \begin{array}{l}\text { primarily used lecture } \\ \text { chalkboard }\end{array} \\ \begin{array}{l}\text { used boardwork and } \\ \text { discovery learning }\end{array} & \begin{array}{l}\text { exclusively used lecture } \\ \text { allowed time for } \\ \text { students to work along } \\ \text { used the chalkboard and } \\ \text { overhead }\end{array} \\ \text { used the chalkboard and } \\ \text { overhead }\end{array}\right\}$

On the surface it seemed that Terry and Jessica operated the majority of the class sessions in the same way. Each primarily lectured with the use of the overhead projector and occasionally the chalkboard. However, their reasons for doing so were quite different. Terry felt that using transparencies benefited his students because they would not have to write as much, particularly because he posted the transparencies online and his students could print them off prior to the class session. Jessica used the overhead because it felt safer. She felt that her students would be less likely to perceive her discomfort with the material she was presenting. Terry's reasons for choosing to use the overhead were student focused, while Jessica's reasons were instructor focused.

From their interview comments, it seemed that Terry and Jessica both had their students use board work to some degree in their engineering calculus classes. However, only Terry carried this method into business calculus. Terry felt there were good places where this method could be useful, while Jessica could not seem to incorporate this practice into business calculus. Both sited many of the same benefits of the incorporation of board work, but only Terry enabled his student's access to those benefits.

Both GMTAs were concerned about covering the prescribed material. So, both used lecture primarily because they felt they could move through the material more quickly. However, when time permitted, Terry chose other methods of teaching such as discovery learning and board work. In contrast, Jessica's lack
of comfort with the material simply did not let her attempt other teaching methods.

While the actual classroom practices on the days of observation in these two business calculus classes were very similar, it was for very different reasons that each GMTA chose the teaching methods that they did. Terry was focused on student learning, while Jessica was focused on instructor comfort.

### 8.7 Classroom Structure and Housekeeping

The evaluation methods used by each GMTA were similar, but the way in which each prepared for class was different.

Table 6: Classroom Structure and Housekeeping

|  | Terry | Jessica |
| :--- | :--- | :--- |
| Assigned... | coordinator suggested <br> homework but did not <br> collect | coordinator suggested <br> homework but did not <br> collect |
| Evaluated by... | quizzes created from old <br> exams | quizzes created from <br> textbook and <br> imagination |
| Prepared using... | notes from the <br> coordinator | notes from a fellow <br> GMTA |
| Prepared by... | created his own lessons <br> from the provided <br> material | using the material as she <br> received it |

Again, at a cursory glance Terry and Jessica seemed to be doing most of the same things. Both acquired the materials from which they prepared their lessons from others. However, Terry merely used them as a guide and created his own lectures, while Jessica copied notes that another instructor had made with few changes and little input from herself. They also used different materials to guide them. Terry followed the coordinator's materials while Jessica followed the material created by another GMTA. Jessica commented on the need to spend extra time in preparation with business-related vocabulary, but Terry made no such comment.

Both Jessica and Terry assigned the homework suggested by the course coordinator. Neither collected the homework, both preferring to assess student learning through in-class quizzes. Even though Jessica believed her grading was consistent with how the tests would be graded, she met with opposition from her students who felt she was unfair. Jessica created her quizzes, with questions from the textbook, the homework assignments, and sometimes her own imagination. Thus, sometimes Jessica's quizzes were similar to test questions and other times they were not, as was the case with her quiz over optimization. In contrast, Terry used old exams when creating his quizzes.

### 8.8 Typical Class Session

The casual observer would see few differences if they compared these classes. However, the motives behind the instructional decisions were different.

Table 7: Typical Class Session

|  | Terry | Jessica |
| :--- | :--- | :--- |
| Opened class with <br> conversation to... | establish rapport | alleviate silence |
| Lectured... | primarily | exclusively |

Here again Terry and Jessica's classrooms looked very similar. Each lectured using the overhead projector. Both had the tendency to chat with their students before class began. However, their reasons for conversation were quite different. Terry wanted to relate to his students. He wanted them to see him as a peer in some respects; after all, he too was a member of the University. Jessica, however, chatted to avoid silence. It was for her own comfort, to ease the uneasiness she felt before she taught. Terry's reasons for his actions were more student-oriented, while Jessica's actions were more instructor-oriented.

### 8.9 Anticipated Affect on the Future

Terry and Jessica viewed the effect this teaching assignment would have on their respective futures in very different ways.

Table 8: Anticipated Affect on the Future

|  | Terry believed | Jessica believed |
| :--- | :--- | :--- |
| More applications... | were beneficial | added to her arsenal of <br> real life data |
| The interpretation <br> aspect... | was useful | was useful |
| On his/her resume, <br> teaching business <br> calculus... | would look good | would have no effect |

Terry and Jessica both appreciated the added dimension that business calculus gave them in terms of different types of applications and the interpretation of answers with units specified. However, Terry noted these positive aspects to a much stronger degree. Terry believed he would actively use these components if he were to teach engineering calculus again. Jessica commented on it much more loosely. She believed it would be another way to answer the question, "When will I use this in real life?" rather than actively teaching with a focus on any of these areas. Furthermore, while she felt that some of the applications were okay, in general she felt there were too many
applications. The applications did not allow the students enough time to learn the calculus concepts well enough to apply to the applications in a productive way. In regard to resumes, Terry felt that having taught business calculus would help his resume while Jessica felt no future employer would care that she had taught it.

### 8.10 Research Questions

As established in Chapter 1, the research questions I attempted to answer through the course of this study were the following.

## How do GMTAs approach teaching a class they never took?

This study offers insight into the ways that a GMTA might think about and approach a business calculus teaching assignment. For example, When a GMTA is assigned to teach this course, it may not be at their request. In fact, it appears that business calculus is not a popular course to teach. Neither GMTA in this study expressed excitement when they received this assignment. Their attitudes ranged from indifference to displeasure. Both knew they could not avoid the assignment, but that did not mean they had to embrace it.

In regard to the fact that this class was one they never took, both GMTAs expressed concern over the vocabulary used. Both had to learn new ways of thinking about calculus concepts and they had to learn some business components related to the course, such as profit, revenue, cost, and average cost.

## What teaching methods do GMTAs employ?

In both classrooms, the GMTAs primarily used lecture. The overhead projector was also a big component of their lectures. While Jessica forsook all other teaching methods she used before, Terry still employed some group work and board work activities as time permitted. Both GMTAs made mention of time constraints, which in their minds justified their use of lecture as a time-effective way to cover all the material they were expected to teach. Thus, because of timerestraints and comfort levels with lecture, both GMTAs' teaching styles were grounded primarily in lecture.

## How do beliefs about teaching and learning affect the teaching of business

 calculus?Beliefs about teaching and learning affected Terry and Jessica in their teaching of business calculus. Terry's beliefs that students learn by practice, and that the role of the instructor is to get students to practice, were adaptable to business calculus. Terry's beliefs allowed him to teach business calculus in way similar to his previous classes. Jessica's ideas of students practicing did not come to fruition during class. Her students were in fact expected to absorb the information on which she lectured, then to practice on their own after class.

## How do beliefs about mathematics affect the teaching of business calculus?

Beliefs about mathematics played a decisive role for Terry and Jessica in their business calculus teaching experience. Terry believed mathematics was the study of patterns, and he could see patterns in business calculus. Therefore, Terry felt business calculus was in fact a mathematics course. Jessica, however, believed that mathematics was primarily computational. To her, business calculus was not a mathematics class, which led to a lot of frustration. There was too much time spent on interpretation to have enough computation to make this course truly a mathematics course. Thinking about one's beliefs about mathematics in particular seems to be a valuable activity. If one cannot accept business calculus as mathematics, the experience of teaching it will be negative.

## How does the different population of students affect GMTA teaching?

Ideas about students affected these GMTAs' classrooms. Terry's belief that his students were capable of learning mathematics caused him to teach in a way that was conducive to the success of his students. His classroom was centered on his students and ways in which they could succeed. He chose to use group work and board work to increase self-confidence as well as using his quizzes as an additional way to prepare his students for their exams. Jessica's belief that her students were out of touch with their actual mathematical abilities kept her from actively looking for ways to help her students succeed. She believed that they had
unrealistic expectations of their own abilities. She did not have students participate during class in activities she claimed would assist in building mathematical confidence. The expectations of these teachers were quite different, and students tend to live up (or down) to a teacher's expectation (Zeichner et al., 1998).

## How does the GMTA's attitude about the class affect their teaching of business calculus?

Attitudes of the GMTAs regarding the class seemed to help (or hinder) the GMTA in teaching the course. For Terry, teaching business calculus was a positive experience. He noted primarily positive aspects and maintained a positive attitude during the experience. Jessica, however, admitted that her attitude was quite negative and probably the thing most inhibiting her success in teaching the course. Jessica felt the experience was just about as bad as it could have been.

## What resources do GMTAs use in preparation for teaching this class?

An examination of how these GMTAs prepared to teach a business calculus lesson reveals that they did not use their own prior knowledge. Because of the difference in content from engineering calculus, both relied upon materials provided to them by others who had taught the course, as well as the text itself. Jessica even talked in depth about her need to learn the semantics of the course to
be able to teach her students in the manner of the text. Thus, GMTAs were less apt to start their lessons from scratch, using the textbook as their primary guide in preparation. Instead, they tended to rely upon the work already done by those who had taught the class.

## How can we better prepare GMTA to teach business calculus?

There are several ways in which GMTAs could be better prepared to teach this course. The evidence suggests that GMTAs new to teaching business calculus would benefit from interaction with those who have had experience teaching the course. GMTAs need a support structure (Belnap, 2005). To provide this needed support, one possibility is for a GMTA new to teaching business calculus to be paired with a GMTA who has become proficient at teaching the course. This pairing could include having both GMTAs sit in on one another's classes. This type of support might be most helpful if the contact were frequent and regularly planned. Some benefits might include:

- The new GMTA feeling less threatened by the course material
- The experienced GMTA assisting in a concrete way
- The new GMTA receiving feedback to improve teaching

Another helpful type of support could come from the Department. GMTAs would benefit from the wisdom of experienced faculty supervision. In addition, the GMTA experience may be the only time the next generation of
university faculty receive feedback from colleagues and supervisors about teaching (Speer, 2001). Feedback allows GMTAs to make changes they might not otherwise consider. It allows them to become more aware of their teaching from someone else's viewpoint.

## How can we adjust the perception of this course among the Department as a

 whole?Changing the perception of business calculus among members of the Mathematics Department has to happen from the bottom up because instructors are the people most closely associated with the course. The full-time faculty do not teach this course, which may or may not contribute to overall negative attitude within the Department. However, the need for change must begin with those who are teaching the course because those not involved have no reason to sense a need for change.

Because of the need for support, GMTAs could be afforded help in a variety of ways. In addition to one-on-one peer support, group support for those assigned to teach business calculus would benefit GMTAs. One way support could happen is first for an orientation meeting at the beginning of each semester for the new business calculus GMTAs and a few experienced business calculus instructors. This type of meeting would be for the primary purpose of talking about what business calculus is, its benefits, and its potential problems if one does
not take care when teaching it. Afterward, regular meetings of this cohort could occur frequently so that new instructors are kept aware of what is coming up. This time could provide the opportunity for a forum discussion on how experienced GMTAs have approached teaching the material in the past. These types of meetings may assist in a more accurate perception of business calculus and its value to the average business student.

Additional support and information could change the perception of business calculus in the Department, at least among those directly associated with the course. As discussed above, it might also be beneficial to GMTAs for a faculty member to provide feedback on their teaching. While allowing GMTAs to receive feedback on their teaching methods and delivery, faculty might become more aware of what business calculus looks like and its unique purpose in the university, hopefully assisting in the creation of an overall positive departmental attitude regarding the course.

### 8.11 Further Directions for Research

Much more needs to be examined in the area of GMTA research and specifically in the area of business calculus GMTA research. More case study data would be useful to validate the information obtained in this case study and to identify additional valuable information.

Another study that would be extremely useful would be to make some or all of the suggested changes and see if in fact the attitudes of GMTAs and of the Department in general can be altered. The first study I would suggest is to implement pairing new and experienced GMTAs. Then by using a pretest/posttest, perhaps by interviewing, one could determine if this intervention did in fact result in a more positive opinion of business calculus among those teaching the course. Other suggestions could then be made to improve upon this pairing. Observations by faculty with pre/post interviews could also be implemented in another study, as could an experiment with regularly held meetings of the business calculus instructors.

Some are not sure that assisting GMTAs in improving their teaching is a valid use of time, however, as Friedburg (2005) comments, "In the long run, then, mathematics will do better if the next generation of mathematicians on university faculties are excellent teachers" (p. 842).

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# Initial/Final Interview Outline (FORM A) 

Teaching Philosophy
Description of mathematics
How one learns
Overall perception of students
Description of MATH 1743
Initial reaction to this teaching assignment
Current feelings regarding this teaching assignment
Compare/Contrast MATH 1743 to MATH 1823 in terms of
Material taught
Students enrolled
General Classroom experiences/environments
Types of teaching methods employed in MATH 1743
Types of teaching methods employed in the past
Differences in the above? Why?
Preparation for a typical class session
A typical class session is like ...
Difficulties experienced thus far in teaching this course
What effect (if any) will teaching this course have on you/your teaching/etc. in the future

# Observation Debriefing Interview Outline (FORM B) 

Current feelings regarding this teaching assignment Overall attitude toward students on this particular day

Perception of students on this particular day
Your impression of the student's perception of the material on this particular day
Initial reaction to topic being presented
Types of teaching methods this day
Assessment of your teaching method for this topic
Difficulties experienced while teaching this lesson
Types of explanations provided to students
Assessment of your answers to students' questions
Preparation for this class session
Difficulties experienced while preparing for this lesson
What changes would be made if you could do it all over again?
What would you keep the same, that worked particularly well?

## Questions for Administration of Business Calculus. (FORM C)

What is the purpose of business calculus?
How does it differ from traditional calculus?
Why is it necessary?
How does it help the business department?
How did the business calculus class develop into what it is today?
Why was there a shift to a reform book?
How did faculty/staff react to the change to a reform book?
Who teaches business calculus?
What all goes in to the process of the scheduling of classes to instructors?
Why is the class coordinated?
What are the benefits/drawbacks to the class being coordinated?
What is the overall opinion of business calculus among students in the course?
What is the overall opinion/perception of business calculus among mathematics graduate teaching assistants?

What is the overall opinion/perception of business calculus among mathematics faculty?

What is your opinion/perception of business calculus?
What are some possible reasons for differences in the above?

