A NON-INTEGRAL METHOD TO ESTIMATE THE MEAN, VARIANCE AND PROBABILITY DISTRIBUTION OF THE PRESENT WORTH OF A LUMP-SUM CASH FLOW

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CHAPTER 1

INTRODUCTION

The objective of this thesis is to present a non-integral method to estimate the mean, variance and probability distribution of the present worth of a lump-sum cash flow. Specifically, this thesis studies the situation where there is a lump-sum probabilistic cash flow that occurs at a probabilistic time at a probabilistic interest rate. Furthermore, this thesis assumes that all three of these probabilistic variables are described by a beta probability distribution.

Traditionally, when a decision-maker must evaluate an economic project based on the maximization of wealth, a common approach is to estimate the cash flows, the cash flow timing and the interest rate, and perform a net present worth analysis [40][14]. An economic project with a positive net present worth is then accepted as a good investment [40]. Unfortunately, some or all of the information may not be known with certainty.

An event is uncertain if the frequency distribution of the possible results of the event is unknown [10]. An event involves risk if the probability distribution of the possible results of the event is known [10]. This thesis assumes that the probability distribution of an uncertain event can be estimated. Therefore, this thesis uses the term risk to describe both situations.

If only the cash flow and/or the timing of the cash flow are under risk then researchers provide methods that allow a decision-maker to calculate the expected

present worth of an economic project [1, 4, 5, 8, 17, 19, 20, 28, 29, 26, 33, 34, 35, 36, 38, 40, 41]. However, if the interest rate is under risk, the decision-maker is directed to add a risk premium to the market risk-free interest rate. Unfortunately, the literature does not specify exactly how this risk premium should be derived.

This thesis proposes a method to assist a decision-maker in the case where all three variables are under risk and can be approximated using a beta probability distribution. To this end, and assuming that the independent variables are beta distributed, this thesis predicts the mean, variance and probability distribution for the present worth of a lump-sum cash flow. The goal is to give the decision-maker the ability to estimate risk of an economic project when the cash flow, cash flow timing and interest rate are probabilistic. Underlying this goal is the assumption that economic project risk is, in part, determined by the mean, variance and knowledge of the probability distribution of the expected present worth. This thesis does not attempt to quantify economic project risk. It does, however, present estimates that a decision-maker can use in conjunction with the firm's risk situation to assist in determining an economic project's risk.

This thesis adds to the knowledge base in engineering economics by showing that the beta probability distribution will estimate the present worth of a lump-sum cash flow when the cash flow, the cash flow timing and the interest rate are random beta probability variables. Additionally, this thesis shows that the mean and variance of the present worth under these conditions can be estimated without the use of integrals. This last point is useful, since the integrals for the mean and variance of the present worth become very complicated at even small values for the beta probability distribution parameters, α and β.

CHAPTER 2

LITERATURE REVIEW

The literature presents many studies concerning risky cash flows, cash flow timing and interest rates. These studies include investigations involving probabilistic single period and multi-period cash flows with probabilistic timing and time horizons. Additionally, the literature presents discussion concerning the use of a risk adjusted interest rate in conjunction with probabilistic cash flows and cash flow timing. However, the literature does not show the effect on the present worth when all of these variables are probabilistic.

The first section discusses general investigations dealing with uncertainty and risk. Sections two through four discuss research in the areas of uncertain/risky cash flow timing, cash flows and interest rate. The last section of this chapter summarizes the literature in these three areas and shows the need for this thesis' research.

Uncertainty and Risk

Zinn and Lesso [41] state that there are three relevant factors regarding the analysis of an economic project. The first two are the magnitude and the timing of the cash flows. The third is the uncertainty of these cash flows. The magnitude and timing analysis is generally performed using the net present worth method. However, there is no

universally accepted method to analyze the cash flow uncertainty. Zinn and Lesso generalize the approaches common in the literature by classifying them as either deterministic or probabilistic. The four deterministic methods normally in use at the present are the

1. Payback Period Method,
2. Risk Adjusted Discount Rate (RADR) Method,
3. Certainty Equivalent (CE) Method and the
4. Sensitivity Analysis Method.

The payback period is equal to the earliest time that the future cash flows (discounted at zero interest) will recover the initial investment. The underlying assumption for the Payback Period Method is that the shorter the payback period, the less risky the economic project. However, this method does not account for the time value of money and is considered an inadequate indicator of risk [41].

The risk-adjusted discount rate (RADR) is the risk-free discount rate plus an arbitrary amount (risk premium) that accounts for the risk of a specific economic project. The chief difficulty with this approach is deciding the correct value of the risk premium [41].

The Certainty Equivalent (CE) Method uses the risk-free discount rate to discount the period cash flows and accounts for uncertainty by multiplying the discounted period cash flows by a risk adjustment coefficient. As in the case of the RADR method, specifying this risk adjustment coefficient accurately is difficult [41].

Sensitivity analysis describes how sensitive the net present worth (or any measure of merit) is to changes in the key economic elements. For example, a manager might

choose to use the minimum, most likely or maximum values of each key element. Zinn and Lesso state that sensitivity analysis should not be used alone since it cannot adequately describe the overall risk of the investment.

The main disadvantage of the deterministic methods is their use of a single value or best estimate for future cash flows. Specifically, these single value estimates do not show the uncertainty that is inherent in future cash flows. They mask the variability of the investment data and completely avoid the problem of present worth risk [41].

The probabilistic approach overcomes the deficiencies encountered with the deterministic approach. It provides a decision-maker the ability to use a probability distribution instead of single value estimates for each key economic variable [41]. The most important advantage of this approach is that it provides management with an estimate of a projects probabilistic outcome. However, this approach has the disadvantage of providing only statistical estimates rather than exact values.

Bierman and Hausman [5] take a different approach regarding cash flow risk. They approach this problem by studying the resolution of uncertainty. Their argument is that a decision-maker will view an economic project, with regards to its risk, based on how quickly the risk is substantially removed over the project's life. Park and Thuesen [29] research this view in more depth in their 1979 paper. Smith [35] attempts to incorporate borrowing and lending decisions into economic project evaluation models as a means of further describing the resolution of uncertainty.

Park and Thuesen [29] also discuss four problems associated with using the coefficient of variation [29, p. 111] to evaluate a risky economic project and give the following equation.

$$CV_t = \frac{S_t}{E[NPW]} \qquad (1)$$

where,

CV_t = coefficient of variation,

S_t = square root of the variance of the NPW and

E[NPW] = expected net present worth.

The first problem caused by using the coefficient of variation is that it only measures the relative dispersion of a set of data. Basically, the coefficient of variation places the same level of importance on deviation above and below the expected value. The second problem is that the S_t and E[NPW] do not provide the shape of the cash flow pattern to the decision-maker. It can be important to the decision-maker to know the timing and magnitude of each cash flow. Third, there are no precisely defined decision rules for using the coefficient of variation to evaluate economic projects. Finally, the coefficient of variation does not indicate the magnitude relationship of the cash flow as compared to alternate cash flows [29, p. 112].

Saxena [33] presents an algorithm for obtaining the probability distribution of discrete random variables when the probability range constraints are known. The disadvantage of this approach is that it is difficult to estimate the probability range constraints when the number of constraints is even of moderate size (see Greer [16]).

Buck and Askin [8] propose using partial means as a measure of economic project risk. However, this approach is limited to the use of only one uncertain variable. For example, a decision-maker can either evaluate cash flows with uncertain amounts and fixed timing or cash flows with fixed amounts and uncertain timing.

Timing

Greer [17] discusses three methods that are used by industry decision-makers to cope with uncertain timing of an economic project. The first method treats the decision-maker's estimate of the most likely time of the cash flow as certain knowledge. With the second method, the decision-maker calculates the mean time from a set of probability estimates. The theory behind this approach is that through repetition, this method will produce a very nearly correct average of the true time. The third method allows the decision-maker to break the uncertain time ranges into intervals and assign subjective probability numbers to each interval. The probability numbers measure the likelihood that a specific interval is the correct interval.

Greer shows that the first method overestimates the present worth. The second method produces a present worth measurement that is incorrect due to an overly late time estimate. The third method is theoretically sound and decision-makers use it with confidence when the problem is simple, according to Greer. However, as the complexity of the problem increases, the number of intervals necessary to adequately describe the timing range estimate also increases. This increase in the number of intervals leads to an increase in difficulty for the decision-maker to assign a subjective probability number to each interval. Furthermore, the decision-maker must try to establish a relation between the adjacent interval probability numbers, to produce a smooth and realistic statistical distribution over the range estimate.

To reduce the effort necessary to analyze complex timing problems, Greer suggests substituting a continuous statistical distribution for the discrete interval probability numbers. He further suggests that the distribution selected should have the

four following features. First, the shape of the distribution should be flexible enough to allow for either a skewed distribution or a symmetric distribution. Second, the conditional probability of an event occurring during the next time interval, given that it has not occurred by time t, should be an increasing probability. Third, the distribution should have a finite range. Finally, Greer suggests that ease of use is the fourth feature [17].

Greer proposes that the Beta distribution [17, p.107] (Equation 2) has all four of the required features and should be the distribution of choice.

$$f(t) = (K)(t-a)^{\alpha}(b-t)^{\beta} \tag{2}$$

where,

K = constant (required to set the area under the curve equal to 1),

a, b = endpoints of the finite range of the Beta distribution and

α, β = Beta probability distribution parameters.

The beta distribution parameters, α and β, can be estimated using a system developed for the Project Evaluation Review Technique (PERT), with which many decision-makers are familiar [17]. In the development of a PERT analysis, the decision-maker provides only three subjective estimates (optimistic, most likely and pessimistic). These estimates represent the best reasonable outcome, most likely outcome and worst reasonable outcome that can be expected. In relation to a beta distributed cash flow, these outcomes relate to the beginning of the Beta distribution's range, a, the mode of the distribution, M, and the end of the Beta distribution's range, b.

To calculate the Beta distribution's parameters α and β. Greer assumes that the Beta distribution's standard deviation is equal to one-sixth of the range of the distribution (equation 3).

$$\sigma = \frac{b-a}{6} \tag{3}$$

Next, he standardizes the Beta distribution [17, p. 109] by letting

$$x = \frac{t-a}{b-a} \tag{4}$$

where,

x = the standardized variable.

The mode and the variance of the standardized Beta distribution [17, p. 109] are shown in equations 5 and 6.

$$m = \frac{M-a}{b-a} = \frac{\alpha}{\alpha+\beta} \tag{5}$$

$$Var(x) = \frac{(\alpha+1)(\beta+1)}{(\alpha+\beta+2)^2(\alpha+\beta+3)} \tag{6}$$

Greer assumes that the standard deviation of the Beta distribution is one-sixth the range and, since the range of the standardized beta distribution is equal to one, the standard deviation of the standardized Beta distribution is one-sixth. Therefore, the variance of the standardized Beta distribution is 1/36.

Equation 5 can be rewritten to solve for β [17, p. 109] as shown in equation 7.

$$\beta = \frac{\alpha}{m} - \alpha \tag{7}$$

Substituting equation 7 into equation 6 and rearranging gives the following cubic equation [17, p. 109] in terms of the standardized mode, m and the Beta parameter α.

$$\alpha^3 + (7m - 36m^2 + 36m^3)\alpha^2 - 20m^2\alpha - 24m^3 = 0 \tag{8}$$

Solving this equation for α and substituting the value of α into equation 7 gives the value for the Beta parameter β.

Young and Contreras [40] state that the "maximization of expected present worth is a widely accepted decision principle in engineering and economics [40, p. 257]." For example, if F represents an independent single lump-sum payment that is paid at some future random time N, at a continuous compound interest rate r, then the present worth $PW(N)$ of the cash flow can be written as equation 9 [40, p. 259].

$$PW(N) = Fe^{-rN} \tag{9}$$

However, since the present worth does not relate linearly to timing, Young and Contreras show that using expected time to calculate expected present worth [40, p. 259] (equation 10) does not equal the expected present worth (equation 11) [40, p. 259].

$$E[PW(N)] = F * e^{-rE[N]} \tag{10}$$

does not equal

$$E[PW(N)] = F * E[e^{-rN}]. \tag{11}$$

Young and Contreras also show that the same is true for an independent uniform cash flow A that has random start and end times N and is discounted at the continuous interest rate r [40. p. 264] (equations 12 and 13, respectively).

$$E[PW(N)] = \frac{A}{r}(1 - e^{-rE[N]}) \tag{12}$$

does not equal

$$E[PW(N)] = \frac{A}{r}(1 - E[e^{-rN}]). \tag{13}$$

Young and Contreras support their argument for the above two situations by computing the expected present worth with five different timing distributions. They are the exponential, uniform, normal, gamma and arbitrary distributions. In each case, they show that the expected present worth is not equal to the expected present worth using the expected time. Rosenthal [31], with corrections by McClintock and Chalmet [24], furthers Young and Contreras' work by presenting calculations for the variance for each of the scenarios.

Sivazlian [34] studies the expected present worth of uniform annual cash flows with probabilistic lives. His studies show that using the expected life in the expected present worth analysis overestimates the expected present worth.

Ancel and Griffiths [1] compare present worths of mutually exclusive economic projects that have different lives and different risks. This research describes risk in the following manner. If project A is riskier than project B then the interest rate for the

present worth analysis for project A will be greater than that for the present worth analysis for project B.

Cash Flow

Harvey and Cabot [19] present a model for an economic project with risky multi-period cash flows that does not require independent period cash flows. This allows the decision-maker to make period cash flow estimates based on the previous period's cash flows.

Hillier [20] proposes a method to present a series of normal, periodic cash flows with identical probability distributions as a single probability distribution. Hillier describes the risk of the resulting single cash flow using the mean, variance and a graph of the estimated probability distribution.

Perrakis and Henin [26] build on Hillier's work by allowing the period cash flows to occur at random times. Zinn and Lesso [41] follow Hillier's [20] approach and investigate cash flows with a probabilistic initial investment, economic life and salvage value.

Tufekci and Young [38] investigate economic projects that have random lump-sum payments, cash flow profiles and initial investments that occur at random times. The key finding of this research is that it shows that there is a correlation between the present worth of a lump sum payment and the present worth periodic cash flows if they are discounted through the same random time.

Park [28] shows that the Mellin transform can be used to evaluate the present worth of a cash flow profile that includes products and/or quotients of independent

random variables. Specifically, the Mellin transform provides a means of calculating the statistical moments of the present value distribution. However, cash flow timing and interest rate are exponents in the continuous compounding case and, therefore, the Mellin transform method only applies to cash flows.

Spahr [36] investigates normally distributed cash flows with normally distributed reinvestment rates. A key finding in this work is that if the same stochastic reinvestment rate is used for different economic projects, there will be a non-zero inter-project covariance.

Interest Rate

Benzion and Yagil [4] compare two discounting methods currently in use to evaluate identical and independent over time, multi-period stochastic cash flows. Specifically, they compare the single risk adjusted discount rate (SRADR) and the certainty-equivalent-discount rate (CEDR). The difference in these two discounting methods concerns the underlying assumption regarding the time pattern of the exchange rate between risky and riskless income streams. The SRADR assumes a declining exchange rate. The CEDR assumes a constant exchange rate.

Benzion and Yagil demonstrate that when the SRADR approach is used for a multi-period project with independent risky cash flows the present worth is a function of the first period risky interest rate and the risk-free interest rate. This result tends to bias in favor of short-term projects. However, the CEDR assumes that the interest rate between any two periods is the same as the one period interest rate. Benzion and Yagil's

conclusion to this study is that a combination of these two methods is probably a more realistic approach.

Booth [6] proposes that the actual RADR will depend on the correlation structure of the firm's revenues and costs and is an empirical, not a theoretical, problem. While, Celec and Pettaway [11] and Lewellen [22] argue about whether the RADR should be decreased or increased as the risk of a cash outflow increases. Robichek and Myers [30] argue that if uncertainty is expected to be resolved at a constant rate over time then a constant RADR does accurately describe the rate of income realization.

Gallagher and Zumwalt [16] state that the present value of an economic project is "hypervolatile" (16, pp 105) when the RADR becomes negative. Furthermore, they argue that the present value of an economic project is undefined when the discount rate is -1. As a solution to this problem, Gallagher and Zumwalt propose that the certainty equivalent (CE) approach should be used.

Haley [18] discusses whether using the constant RADR to evaluate an economic project implies that the risk of the project's cash is increasing with time. In answer to this query, Haley states that, yes, a constant RADR in a multi-period economic project does imply an increasing discount for risk over time. This increase in risk, however, is a result of three factors.

1. The underlying cash flow uncertainty.
2. The uncertainty regarding future changes in the expected cash flows.
3. The market price risk in each time period at and before the period in which the cash flow occurs.

In conclusion, Haley argues that the increasing risk may be due to future expectation uncertainty and not due to an increase in cash flow uncertainty over time.

Ariel [2] states, "risk adjusted discount rates in capital budgeting is firmly entrenched in business practice. There is agreement that the RADR method will yield the right result if used correctly, but significant disagreement remains over what is correct [2, p. 17]." The disagreement is centered on whether cash outflows (costs) and cash inflows (revenues) should be discounted at the same RADR. The incorrect view, as Ariel argues, is that cash outflows should be discounted at lower RADRs as they become more risky and cash inflows should be discounted a higher rates as they become more risky. Ariel shows that costs (negative cash flows) are no more or no less risky than revenues (positive cash flows) and therefore both should be discounted by the same RADR. Stated another way, the discount rate for risky cash flows is independent of whether the flow is a cost or revenue.

Hull [21] discusses the CE method and the RADR method. He argues that the RADR is a hyperbolic function of the product of a cash flow's coefficient of variation and its coefficient of correlation with the return on the market. Hull concludes his study stating that because of this hyperbolic relationship, it is very difficult to use subjective judgment to determine the correct RADR for a given economic project [21].

Ferson and Locke [15] discuss the sources of errors encountered when estimating the cost of capital through time. Their research shows that the errors in the estimation for the risk measure (beta) are relatively small when compared to the errors arising from the estimation of the risk premium. Ferson and Locke further state that their simulation

shows that a decision-maker can reduce the error in the risk premium by 40% if indicators about the current state of the economy are used instead of historical averages.

SUMMARY OF THE LITERATURE REVIEW

The preceding sections show that there are several different viewpoints regarding the correct method a decision-maker should use to evaluate an economic project. In some instances the literature presents contradictory information (see the interest rate section). This section highlights what this author believes is some of the essential findings of the literature.

Evaluating economic projects using deterministic methods does not satisfactorily describe the risk associated with uncertain key economic variables [41]. The probabilistic approach provides the decision-maker with information such as the mean, variance and probability distribution [20][26][41]. The distribution is important because it gives the decision maker knowledge about deviation above and below the mean not just overall deviation. Additionally, the probability distribution gives information about the magnitude of the cash flow. Greer [17] argues that the optimistic, most likely and pessimistic estimates developed for PERT analysis technique, coupled with the beta probability distribution, has an important place in real world engineering economic analysis.

The literature shows that the use of expected or most likely timing values to compute the expected present worth results in a biased estimate of the present worth [17][40]. Moreover, the use of the least likely (pessimistic) or most likely (optimistic) values as a measure of project risk is unrealistic [17]. Also, the coefficient of variation

does not provide a satisfactory measure of cash flow risk because it does not provide the degree of variance above (below) the expected value and it does not provide the decision-maker with any knowledge concerning the cash flow pattern [29].

Finally, the certainty equivalent method does not adequately account for risk. Interest rate risk is "accounted for" in a NPW analysis by adding a risk factor to a defined risk free interest rate. As is seen in the literature review, there are many opinions about the correct value for interest rate under risk.

FINAL COMMENTS CONCERNING THE CURRENT RESEARCH

As stated at the beginning of this chapter, the literature presents many studies concerning uncertain/risky cash flows, cash flow timing and time horizons, and interest rates. However, the literature does not provide studies encompassing the situation where all three of these variables are probabilistic. Furthermore, in the case of interest rate risk, the literature is void of probabilistic interest rate risk studies. Instead, researchers concentrate their efforts on discovering the "right" value.

Table 1 serves as an overview of the combinations of the current studies. As is seen, the literature restricts its studies to where, at most, only two of the three possible variables are probabilistic. However, the literature does provide the groundwork for the next phase of study. The following list summarizes this groundwork.

1. The optimistic, most likely and pessimistic estimates developed for the PERT analysis technique should be used to derive a beta probability distribution for the independent variables of the present worth function [17].

Table 1

Areas of Investigation Concerning Uncertain/Risky Cash Flows, Interest Rates, Timing and Time Horizons

Researcher(s)	Cash Flow - Uncertain/ Under Risk	Interest Rate - Uncertain/ Under Risk	Timing - Uncertain/ Under Risk
Ancel and Griffiths [1]	NO	YES	YES
Ariel [2]	NO	YES	NO
Benzion and Yagil [4]	YES	YES	NO
Bierman and Hausman [5]	NO	NO	YES
Booth [6]	NO	YES	NO
Buck and Askin [8]	YES	NO	YES
Celec and Pettaway [11]	NO	YES	NO
Fama [12]	NO	YES	NO
Ferson and Locke [15]	NO	YES	NO
Gallagher and Zumwalt [16]	NO	YES	NO
Greer [17]	NO	NO	YES
Haley [18]	NO	YES	NO
Harvey and Cabot [19]	YES	NO	NO
Hillier [20]	YES	NO	NO
Hull [21]	NO	YES	NO
Lewellen [22]	NO	YES	NO
Park [28]	YES	NO	NO
Park and Thuesen [29]	YES	NO	NO
Perrakis and Henin [26]	YES	NO	NO
Robichek and Myers [30]	NO	YES	NO
Saxena [33]	YES	NO	NO
Sivazlian [34]	NO	NO	YES
Smith [35]	YES	NO	NO
Spahr [36]	YES	YES	NO
Tufekci and Young [38]	YES	NO	YES
Young and Contreras [40]	NO	NO	YES
Zinn and Lesso [41]	YES	NO	YES

2. Interest rate risk is a highly debated topic of discussion in the literature. therefore a probabilistic approach would provide a decision-maker the ability to describe interest rate risk. The probabilistic approach does not require the decision-maker to assume certain knowledge about the risk premium.
3. The decision-maker needs knowledge of the mean, variance and probability distribution of the present worth to satisfactorily estimate the risk of an economic project [20][26][41].

From the above list the following questions arise. How is the expected present worth of an economic project affected when the cash flow, interest rate and cash flow timing follow beta probability distributions? How can a decision maker estimate the mean, variance and probability distribution of the present worth without resorting to integrating the difficult probability distribution?

CHAPTER 3

METHODOLOGY

Introduction

The literature shows several different situations regarding risky economic projects. There are studies concerning random lump sum cash flows, random uniform and non-uniform cash flows, random cash flow occurrences and random project lives [1][5][8][17][19][28][29][33][35][36][38][41]. These random events are modeled using uniform, exponential, normal, gamma or arbitrary distributions [20][24][26][31][34][40]. Risky interest rates are modeled by adding risk premiums to a risk free interest rate [2][4][6][11][15][16][18][21][22][30]. In addition, researchers suggest that the mean, variance and estimated probability distribution of the present value of an economic project are the foundation for determining the risk of an economic project [20][26][41].

However, the literature does not consider the situation where the value of a lump sum cash flow, the timing of this cash flow and the interest rate are all under risk and each is described by a beta probability distribution. If a decision-maker faces this situation, due to the lack of research in this area, the decision-maker must address this problem by one of the following methods.

1. The decision-maker assumes that expected values are known with certainty.

2. The decision-maker uses his/her intuition and/or company knowledge to address this issue.

This thesis investigates this deficiency in the literature and gives a decision maker the ability to predict the mean, variance and the probability distribution of the present worth of an economic alternative that has a probabilistic cash flow occurring at a probabilistic time and discounted at a probabilistic interest rate. Furthermore, these predictions are made using equations that do not require the decision maker to perform the daunting task of evaluating the exact integrals for the mean and variance.

Overview of Methodology

This thesis uses the following procedure for this investigation. First, a left skew, right skew and symmetric beta probability distribution is assumed for each of the independent variables [$B(\alpha_{CF}, \beta_{CF})$, $B(\alpha_R, \beta_R)$ and $B(\alpha_N, \beta_N)$], where, CF (lump sum cash flow), N (cash flow timing) and R (interest rate) are the independent variables for the respective distribution. The assumption of the distribution also includes certain knowledge of the mean and range values of the independent variables. Second, assuming that an economic alternative's present value is described by equation 14, this thesis predicts the mean and variance for each combination of the independent variables of the present worth of the economic alternative.

$$\text{Present Value} = \text{Cash Flow}\left(e^{-\text{Timing} * \text{Interest Rate}}\right) \tag{14}$$

This thesis compares the predicted means and theoretical means by reporting the percentage difference, the minimum difference and the maximum difference between the

two values. This thesis then compares the predicted variance values to the theoretical values using analysis of variance. Next, this investigation assumes that the resulting present worth is represented with a beta probability distribution and computes two sets of beta probability distribution parameters, α and β, using the theoretical and predicted values for the mean and variance. Additionally, in both cases the theoretical minimum and maximum values for range of the present worth are used.

This thesis simulates each combination of independent variables and generates a probability graph and a cumulative probability graph. Then, this investigation performs an analysis of variance on the sets of means and variances resulting from the simulations and the set of theoretical means and variances to check that the simulation is accurately modeling the present worth. Finally, this thesis compares the probability graph to the derived beta distributions (the two distributions derived using the theoretical and predicted means and variances) using a Chi-squared goodness-of-fit test.

Detailed Methodology

As Greer [16] states, the beta probability distribution is a good choice for modeling, since it is flexible enough to allow for either a skewed or symmetric distribution. In addition, it has an increasing conditional probability and a finite range. The fourth feature, that it is easy to use, is not true when applied to the timing and interest rate independent variables of the present worth equation. The difficulty is how to evaluate the mean and variance of the exponential term. Obviously, these values are calculable by integrating the distributions. However, these integrals are complex. This thesis gives

formulas to predict the mean and variance in this situation. Additionally, this thesis presents an estimate of the probability distribution.

To test these formulas, this thesis introduces three Beta distributions, $B(\alpha, \beta)$, for each of the three independent variables. The following constraints define the distributions.

1. This thesis investigates only lump sum cash flows.
2. The lump sum cash flow has a mean value of \$1000 and a range of \$1000.
3. The cash flow timing has a mean value of 4 years and a range of 4 years.
4. The interest rate has a mean value of 12% and a range of 12%.
5. The difference between the three sample sets for each uncertain variable is in the skew of the distribution. Specifically, the first sample set describes a left skew distribution ($\alpha > \beta$). The second sample set describes a symmetric distribution ($\alpha = \beta$). The third set describes a right skew distribution ($\alpha < \beta$).
6. The left skew Beta distribution parameters are $\alpha = 6$ and $\beta = 2$ ($\sigma^2 = 0.02083$). The symmetric beta distribution parameters are $\alpha = 5$ and $\beta = 5$ ($\sigma^2 = 0.02273$). The right skew beta distribution parameters are $\alpha = 2$ and $\beta = 6$ ($\sigma^2 = 0.02083$). These values of six and two are chosen because they can show the effect of skewness of each independent variable and they allow less cumbersome calculations of the integrals for the theoretical means and variances.
7. Each sample set is independent of all other sample sets.

Development of the Beta Distributions for the Lump-Sum Cash Flow, Cash Flow Timing and Interest Rate

Based on the above constraints, this thesis generates the following Beta probability distributions [27].

Left skew Beta distribution with parameter $\alpha = 6$ and $\beta = 2$, B(6, 2)

$$f(x) = \frac{\Gamma(a+\beta)}{\Gamma(a)\Gamma(\beta)} x^{a-1}(1-x)^{\beta-1} \quad 0 \le x \le 1, a > 0, \beta > 0 \tag{15}$$

$$f(x) = 42 * x^{5}(1-x) \quad 0 \le x \le 1 \tag{16}$$

Symmetric Beta distribution with parameter $\alpha = 5$ and $\beta = 5$, B(5, 5)

$$f(x) = \frac{\Gamma(a+\beta)}{\Gamma(a)\Gamma(\beta)} x^{a-1}(1-x)^{\beta-1} \quad 0 \le x \le 1, a > 0, \beta > 0 \tag{17}$$

$$f(x) = 630 * x^{4}(1-x)^{4} \quad 0 \le x \le 1 \tag{18}$$

Right skew Beta distribution with parameter $\alpha = 2$ and $\beta = 6$, B(2, 6)

$$f(x) = \frac{\Gamma(a+\beta)}{\Gamma(a)\Gamma(\beta)} x^{a-1}(1-x)^{\beta-1} \quad 0 \le x \le 1, a > 0, \beta > 0 \tag{19}$$

$$f(x) = 42 * x \ (1-x)^{5} \quad 0 \le x \le 1 \tag{20}$$

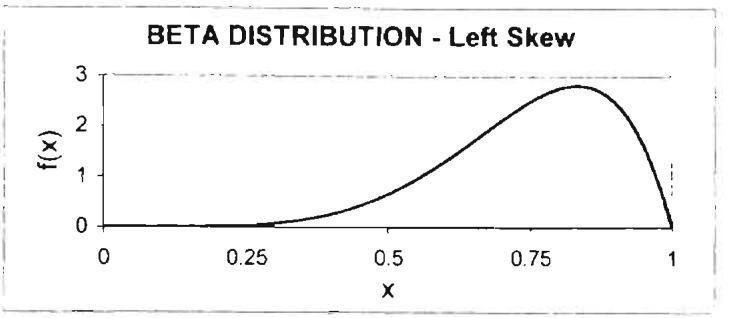

Figure 1. Graphical Representation of Equation 16

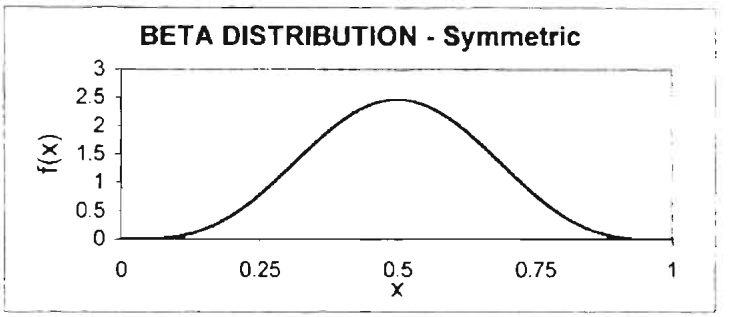

Figure 2. Graphical Representation of Equation 18

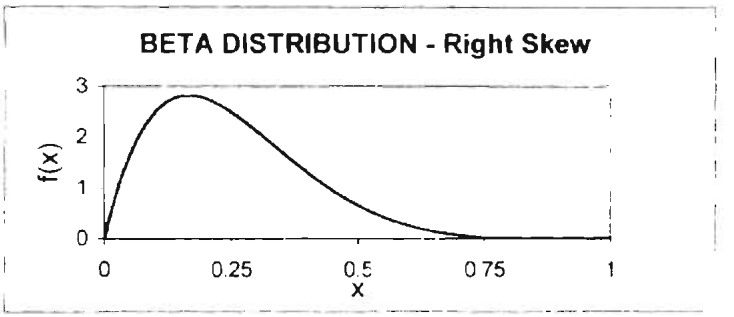

Figure 3. Graphical Representation of Equation 20

While the above equations satisfy the constraints for the beta distribution parameters, as is seen in figures 1 - 3, they do not satisfy the range and mean constraints. Transforming the independent variable satisfies the range and mean constraints.

Let,

$$y = L + (H - L)x \tag{21}$$

where,

L = the lower limit of the range and

H = the upper limit of the range.

Solving this equation for x gives equation 22.

$$x = \frac{y - L}{H - L} \tag{22}$$

Substituting equation 22 into equations 16, 18 and 20 results in equations 23, 24 and 25.

Left Skew:

$$f(y) = \frac{42}{(H - L)^7} * (y - L)^5 (H - y) \quad L \le y \le H \tag{23}$$

Symmetric:

$$f(y) = \frac{630}{(H - L)^9} * (y - L)^4 (H - y)^4 \quad L \le y \le H \tag{24}$$

Right Skew:

$$f(y) = \frac{42}{(H - L)^7} * (y - L)\ (H - y)^5 \quad L \le y \le H \tag{25}$$

Equations 23 – 25 now satisfy constraints (Figures 4 – 12, shown at the end of this section).

Formulas for the Predicted Mean and Variance for the Present Worth

This thesis now presents the formulas for predicting the mean and variance of the present worth. This presentation assumes that equation 26 describes the present worth of a lump-sum cash flow, under continuous compounding [27].

$$PW = CFe^{-NR} \tag{26}$$

where,

CF = lump-sum cash flow.

N = timing of the lump-sum cash flow and

R = the interest rate at which the lump-sum cash flow is discounted.

Assuming that CF, N and R are all under risk, this thesis estimates the mean and the variance of the present worth (equation 26) using a Taylor's series expansion about the means of the CF, N and R (equations 27 and 28) [27].

$$E[f(x)] \cong f(\mu) + \frac{1}{2}\sum_{k=1}^{m}\sigma_k^2\left(\frac{\partial^2 f}{\partial x_k^2}\right)_\mu + \sum_{k<j}\sigma_{kj}\left(\frac{\partial^2 f}{\partial x_k \partial x_j}\right)_\mu \tag{27}$$

$$Var[f(x)] \cong \sum_{k=1}^{m}\sigma_k^2\left[\left(\frac{\partial f}{\partial x_k}\right)_\mu\right]^2 + \sum_{k\neq j}\sigma_{kj}\left(\frac{\partial f}{\partial x_k}\right)_\mu\left(\frac{\partial f}{\partial x}\right)_\mu \tag{28}$$

The covariance terms are zero in this study, since the CF, N and R are assumed to be mutually independent. Equations 29 and 30 show the estimated mean and variance in terms of variables used in this thesis.

$$E[PW] \cong \mu_{CF} e^{-\mu_N \mu_R} + \frac{1}{2}[\sigma_N^2 \mu_R^2 \mu_{CF} e^{-\mu_N \mu_R} + \sigma_R^2 \mu_N^2 \mu_{CF} e^{-\mu_N \mu_R}] \tag{29}$$

$$Var[PW] \cong \sigma_{CF}^2 e^{-2\mu_N \mu_R} + \sigma_N^2 \mu_R^2 \mu_{CF}^2 e^{-2\mu_N \mu_R} + \sigma_R^2 \mu_N^2 \mu_{CF}^2 e^{-2\mu_N \mu_R} \tag{30}$$

where,

μ_{CF} = Cash flow mean,

σ_{CF}^2 = Cash flow variance,

μ_N = Timing mean,

σ_N^2 = Timing variance,

μ_R = Interest rate mean

and

σ_R^2 = Interest rate variance.

Park and Sharp-Bette [27] present the following equations for the mean and variance of a beta distribution.

$$\mu = Min_{range} + (Range)\frac{\alpha}{\alpha + \beta} \tag{31}$$

$$\sigma = \frac{(Range)^2 \alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \tag{32}$$

where,

α, β = the beta distribution parameters.

Range = the range of the beta distribution and

Min_{Range} = the minimum of the range.

The equations for the mean and variance of the cash flow, cash flow timing and the interest rate follow directly from equations 31 and 32.

μ_{CF} = Cash flow mean

$$= Min_{CF} + Range_{CF} \frac{\alpha_{CF}}{\alpha_{CF} + \beta_{CF}} \tag{33}$$

μ_N = Cash flow timing mean

$$= Min_N + Range_N \frac{\alpha_N}{\alpha_N + \beta_N} \tag{34}$$

μ_R = Interest rate mean

$$= Min_R + Range_R \frac{\alpha_R}{\alpha_R + \beta_R} \tag{35}$$

σ^2_{CF} = Cash flow variance

$$= \frac{(Range_{CF})^2 \alpha_{CF} \beta_{CF}}{(\alpha_{CF} + \beta_{CF})^2 (\alpha_{CF} + \beta_{CF} + 1)} \tag{36}$$

σ^2_N = Cash flow timing variance

$$= \frac{(Range_N)^2 \alpha_N \beta_N}{(\alpha_N + \beta_N)^2 (\alpha_N + \beta_N + 1)} \tag{37}$$

σ_R^2 = Interest rate variance

$$= \frac{(Range_R)^2 \alpha_R \beta_R}{(\alpha_R + \beta_R)^2 (\alpha_R + \beta_R + 1)} \tag{38}$$

where,

α_{CF} = Beta distribution parameter alpha for the cash flow,

β_{CF} = Beta distribution parameter beta for the cash flow,

α_N = Beta distribution parameter alpha for the cash flow timing,

β_N = Beta distribution parameter beta for the cash flow timing,

α_R = Beta distribution parameter alpha for the interest rate and

β_R = Beta distribution parameter beta for the interest rate.

Formulas for the Theoretical Mean and Variance for the Present Worth

This thesis calculates the theoretical values for the mean and variance of the present worth following the skew combination table (Table 2) and equations 39, 40 and 41 [3]. In addition, because the predictions for the mean and variance concern the exponential term of the present worth equation, this thesis also calculates the theoretical value for this term (Equations 42 - 44) [32].

$$E[PW] = \mu_{PW} = \int_{CF1}^{CF2} \int_{N1}^{N2} \int_{R1}^{R2} B(\alpha_{CF}, \beta_{CF}) B(\alpha_N, \beta_N) B(\alpha_R, \beta_R) e^{-RN} CF \, dRdNdCF \tag{39}$$

Table 2

Skew Combination Table

Distribution of Lump-Sum Cash Flow, CF	Distribution of Lump-Sum Cash Flow Timing, N	Distribution of Interest Rate, R	Present Worth, f(CF, N, R) - PWxyz
Left skew - B(6,2)	Left skew - B(6,2)	Left skew - B(6,2)	PW_{LLL}
Left skew - B(6,2)	Left skew - B(6,2)	Symmetric - B(5,5)	PW_{LLS}
Left skew - B(6,2)	Left skew - B(6,2)	Right skew - B(2,6)	PW_{LLR}
Left skew - B(6,2)	Symmetric - B(5,5)	Left skew - B(6,2)	PW_{LSL}
Left skew - B(6,2)	Symmetric - B(5,5)	Symmetric - B(5,5)	PW_{LSS}
Left skew - B(6,2)	Symmetric - B(5,5)	Right skew - B(2,6)	PW_{LSR}
Left skew - B(6,2)	Right skew - B(2,6)	Left skew - B(6,2)	PW_{LRL}
Left skew - B(6,2)	Right skew - B(2,6)	Symmetric - B(5,5)	PW_{LRS}
Left skew - B(6,2)	Right skew - B(2,6)	Right skew - B(2,6)	PW_{LRR}
Symmetric - B(5,5)	Left skew - B(6,2)	Left skew - B(6,2)	PW_{SLL}
Symmetric - B(5,5)	Left skew - B(6,2)	Symmetric - B(5,5)	PW_{SLS}
Symmetric - B(5,5)	Left skew - B(6,2)	Right skew - B(2,6)	PW_{SLR}
Symmetric - B(5,5)	Symmetric - B(5,5)	Left skew - B(6,2)	PW_{SSL}
Symmetric - B(5,5)	Symmetric - B(5,5)	Symmetric - B(5,5)	PW_{SSS}
Symmetric - B(5,5)	Symmetric - B(5,5)	Right skew - B(2,6)	PW_{SSR}
Symmetric - B(5,5)	Right skew - B(2,6)	Left skew - B(6,2)	PW_{SRL}
Symmetric - B(5,5)	Right skew - B(2,6)	Symmetric - B(5,5)	PW_{SRS}
Symmetric - B(5,5)	Right skew - B(2,6)	Right skew - B(2,6)	PW_{SRR}
Right skew - B(2,6)	Left skew - B(6,2)	Left skew - B(6,2)	PW_{RLL}
Right skew - B(2,6)	Left skew - B(6,2)	Symmetric - B(5,5)	PW_{RLS}
Right skew - B(2,6)	Left skew - B(6,2)	Right skew - B(2,6)	PW_{RLR}
Right skew - B(2,6)	Symmetric - B(5,5)	Left skew - B(6,2)	PW_{RSL}
Right skew - B(2,6)	Symmetric - B(5,5)	Symmetric - B(5,5)	PW_{RSS}
Right skew - B(2,6)	Symmetric - B(5,5)	Right skew - B(2,6)	PW_{RSR}
Right skew - B(2,6)	Right skew - B(2,6)	Left skew - B(6,2)	PW_{RRL}
Right skew - B(2,6)	Right skew - B(2,6)	Symmetric - B(5,5)	PW_{RRS}
Right skew - B(2,6)	Right skew - B(2,6)	Right skew - B(2,6)	PW_{RRR}

$$E[PW^2] = \int_{CF1}^{CF2}\int_{N1}^{N2}\int_{R1}^{R2} B(\alpha_{CF},\beta_{CF})B(\alpha_N,\beta_N)B(\alpha_R,\beta_R)(e^{-RN})^2 CF^2 \, dRdNdCF \qquad (40)$$

$$\sigma^2_{PW} = E[PW^2] - \mu^2_{PW} \qquad (41)$$

The integrating limits, CF1, CF2, N1, N2, R1 and R2, are the minimum and maximum values for the cash flow, cash flow timing and interest rate, respectively. This thesis calculates these values using equations 42-43 [27].

$$Min = \mu_X - (Range)\mu_S \text{ and} \tag{42}$$

$$Max = Min + Range \tag{43}$$

Where,

μ_X = the mean of the respective independent variable,

Range = the range of the respective independent variable and

μ_S = the mean of the standard beta distribution

$$= \frac{\alpha}{\alpha + \beta}$$

Comparison of the Theoretical Mean and Variance to the Predicted Mean and Variance

This thesis compares the theoretical values for the mean and variance to the predicted values by performing an analysis of variance test at a 0.05 level of significance.

Predicted Probability Distribution of the Present Worth

This thesis assumes that a beta distribution estimates the resulting distribution for the present worth. To calculate the beta probability distribution parameters α and β, this thesis uses the following method. First, this investigation calculates the standardized beta probability distribution mean and variance for the present worth (Equation 44 and 45)

[26]. Then, this thesis calculates the minimum and maximum values for each independent variable using equations 46 and 47 [27].

$$\mu_S = \frac{\mu - Min}{Range} \tag{44}$$

$$\sigma_S^2 = \frac{\sigma^2}{Range^2} \tag{45}$$

The mean, μ, and range are the assumed mean and range values for the respective independent variable.

$$Max = Cash\ Flow_{MAX} * e^{-Time_{MIN} * InterestRate_{MIN}} \tag{46}$$

$$Min = Cash\ Flow_{MIN} * e^{-Time_{MAX} * InterestRate_{MAX}} \tag{47}$$

Cash $Flow_{MAX}$, $Time_{MAX}$ and Interest $Rate_{MAX}$ are the maximum values for the cash flow, timing and interest rate, respectively. Likewise, Cash $Flow_{MIN}$, $Time_{MIN}$ and Interest $Rate_{MIN}$ are the minimum values for the cash flow, timing and interest rate.

Rearranging the equation for the mean for the standard beta probability distribution (equation 48), results in the solution β [27] (Equation 49).

$$\mu_S = \frac{\alpha}{\alpha + \beta} \tag{48}$$

$$\beta = \frac{1 - \mu_S}{\mu_S} * \alpha \tag{49}$$

Substituting this expression for β into the standard beta probability distribution equation for the variance (Equation 50, [27]) and rearranging to solve for α gives equation 51.

$$\sigma_S^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{50}$$

$$\alpha = \frac{K-(1+K)^2\sigma_S^2}{(1+K)^3\sigma_S^2} \tag{51}$$

where,

$$K = \frac{1-\mu_S}{\mu_S}. \tag{52}$$

Given this expression for α, β is now known. The beta probability distribution for the theoretical mean and variance values, and the predicted mean and variance values, both use the procedure shown above. The difference is the use of the theoretical values for the mean and variance of the present worth and the predicted values for the mean and variance worth.

Simulating the Present Worth

Finally, this thesis performs 20,000 simulation runs of each case and plots a probability graph and a cumulative probability graph. This simulation uses equation 53 [32] to generate random beta variates that follow the necessary beta distribution.

$$X = \frac{-\log\prod_{i=1}^{n}U_i}{-\log\prod_{i=1}^{n}U_i - \log\prod_{n+1}^{n+m}U_i} \tag{53}$$

where.

n = the value of the beta parameter alpha (integers only),

m = the value of the beta parameter beta (integers only) and

U = a uniform random deviate.

This investigation checks the accuracy of the simulation in two ways.

1. By comparing theoretical B(6,2), B(5,5) and B(2,6) distributions to the simulation results using a chi-squared goodness-of-fit test at an α level = .05.
2. By comparing the theoretical means and variances for each combination of independent variables are compared to the simulation means and variances using analysis of variance.

This thesis performs a chi-squared goodness-of-fit analysis for the simulation frequency data versus the derived beta probability distribution that is based on the theoretical mean and variance values for the present worth. Additionally, this thesis performs a chi-squared goodness-of-fit analysis comparing the simulation frequency data to the derived beta probability distribution that is based on the predicted mean and variance values for the present worth.

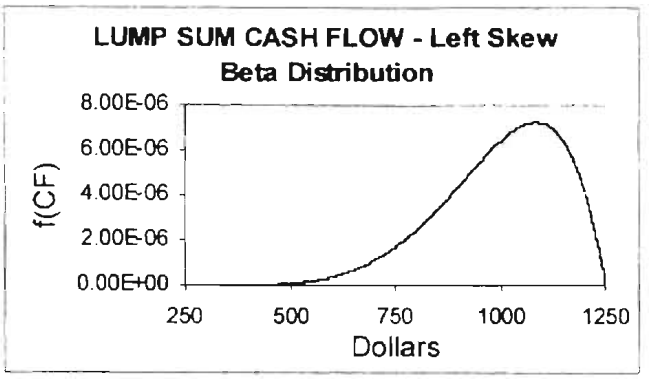

Figure 4. Left Skew Beta Density Distribution for a Lump-Sum Cash Flow

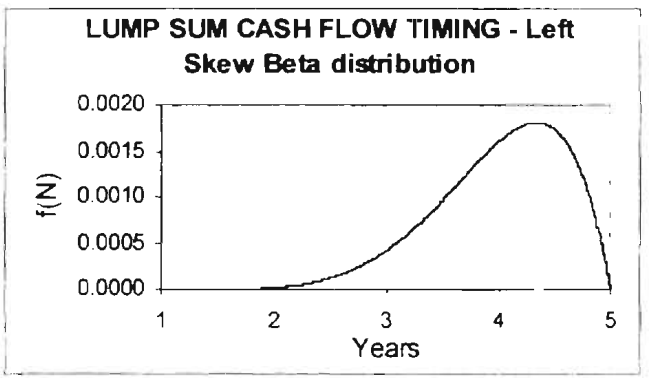

Figure 5. Left Skew Beta Density Distribution for Lump-Sum Cash Flow Timing

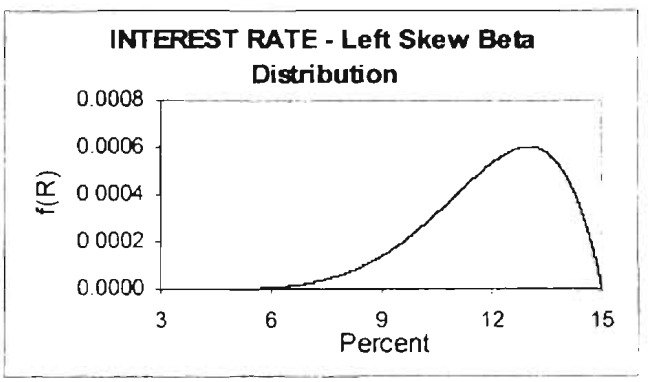

Figure 6. Left Skew Beta Density Distribution for a Lump-Sum Cash Flow Interest Rate

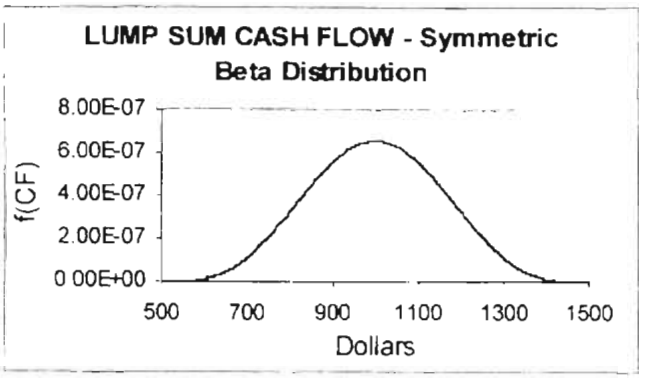

Figure 7. Symmetric Beta Density Distribution for a Lump-Sum Cash Flow

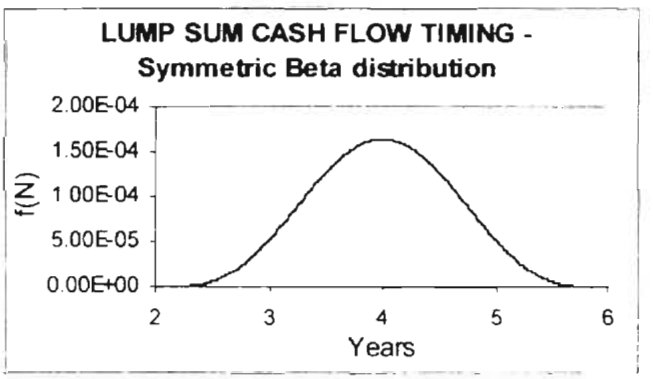

Figure 8. Symmetric Beta Density Distribution for Lump-Sum Cash Flow Timing

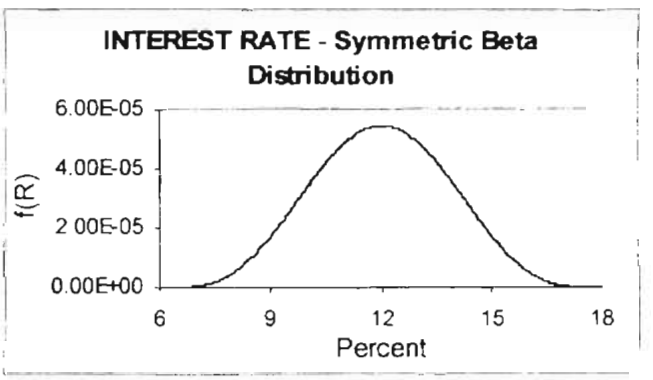

Figure 9. Symmetric Beta Density Distribution for a Lump-Sum Cash Flow Interest Rate

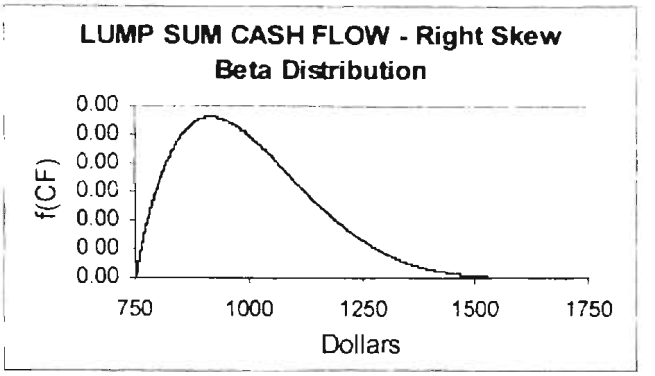

Figure 10. Right Skew Beta Density Distribution for a Lump-Sum Cash Flow

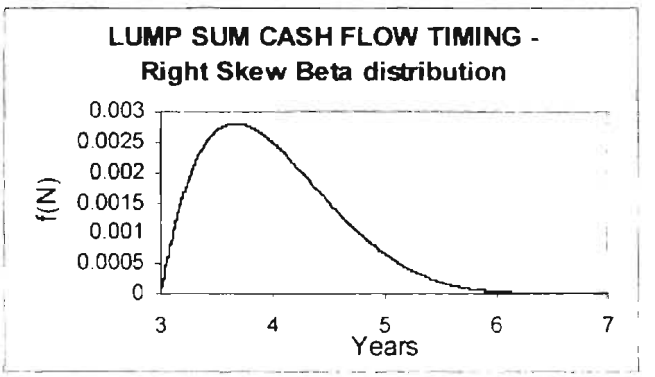

Figure 11. Right Skew Beta Density Distribution for lump Sum Cash Flow Timing

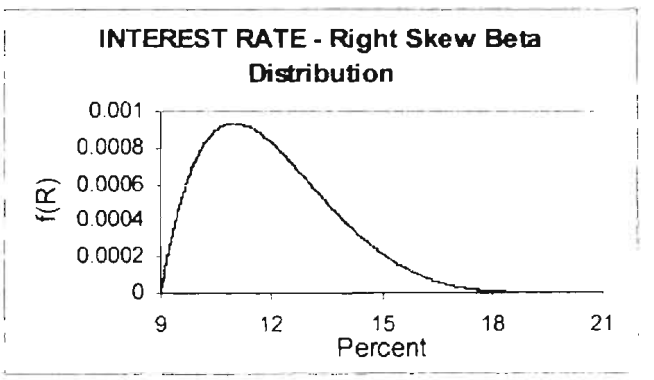

Figure 12. Right Skew Beta Density Distribution for a Lump-Sum Cash Flow Interest Rate

Example

This section gives an example of using a left skew beta probability distribution (α = 6, β = 2) for the cash flow (CF), the cash flow timing (N) and the interest rate (R), to calculate the predicted, and theoretical, mean and variance of the present worth. In addition, this section shows how to use the procedure in the last section to estimate the present worth using a beta probability distribution.

To predict the mean and variance of the present worth, first calculate the variance of the independent variables. The reader should note that the mean and range of the independent variables are known with certainty in this thesis. Therefore, the mean and the range of the cash flow are 1000 and 1000. The mean and the range of the cash flow timing are 4 and 4. And, the mean and the range of the interest rate are 0.12 and 0.12.

σ_{CF}^2 = Cash flow variance

$$= \frac{(Range_{CF})^2 \alpha_{CF} \beta_{CF}}{(\alpha_{CF} + \beta_{CF})^2 (\alpha_{CF} + \beta_{CF} + 1)} = \frac{1000^2 (6)(2)}{(6+2)^2 (6+2+1)} \tag{54}$$

$$= 20833.33$$

σ_N^2 = Cash flow timing variance

$$= \frac{(Range_N)^2 \alpha_N \beta_N}{(\alpha_N + \beta_N)^2 (\alpha_N + \beta_N + 1)} = \frac{4^2 (6)(2)}{(6+2)(6+2+1)} \tag{55}$$

$$= 0.33333$$

σ_R^2 = Interest rate variance

$$= \frac{(Range_R)^2 \alpha_R \beta_R}{(\alpha_R + \beta_R)^2 (\alpha_R + \beta_R + 1)} = \frac{0.12^2 (6)(2)}{(6+2)^2 (6+2+1)} \tag{56}$$

$$= 0.0003000$$

The predicted mean and variance of the present worth is equal to equations 57 and 58, respectively.

$$E[PW] \cong \mu_{CF} e^{-\mu_N \mu_R} + \frac{1}{2}[\sigma_N^2 \mu_R^2 \mu_{CF} e^{-\mu_N \mu_R} + \sigma_R^2 \mu_N^2 \mu_{CF} e^{-\mu_N \mu_R}] \tag{57}$$

$$E[PW] \cong 1000 e^{-4*0.12} + \frac{1}{2}[0.3333 * 0.12^2 * 1000 e^{-4*0.12} + 0.0003 * 4^2 * 1000 e^{-4*0.12}]$$

$$E[PW] \cong 621.75$$

$$Var[PW] \cong \sigma_{CF}^2 e^{-\mu_N \mu_R} + \sigma_N^2 \mu_R^2 \mu_{CF}^2 e^{-2\mu_N \mu_R} + \sigma_R^2 \mu_N^2 \mu_{CF}^2 e^{-2\mu_N \mu_R} \tag{58}$$

$$Var[PW] \cong 20833.3 e^{-4*0.12} + 0.3333 * 0.12^2 * 1000^2 e^{-2*4*0.12} + 0.0003 * 4^2 * 1000^2 e^{-2*4*0.12}$$

$$Var[PW] \cong 11653$$

This thesis calculates the theoretical mean and variance for the present worth using the following equations.

Mean:

$$E[PW] = \mu_{PW} = \int_{CF1}^{CF2}\int_{N1}^{N2}\int_{R1}^{R2} B(\alpha_{CF},\beta_{CF})B(\alpha_N,\beta_N)B(\alpha_R,\beta_R)e^{-RN}CF\ dRdNdCF \tag{59}$$

where,

$$B(\alpha_{CF},\beta_{CF}) = \frac{42}{(Range_{CF})^7} * (CF - Min_{CF})^5 (Max_{CF} - CF)$$

$$Min_{CF} \le CF \le Max_{CF} \tag{60}$$

$$Min_{CF} = CF1 = \mu_{CF} - (Range_{CF})\frac{\alpha_{CF}}{\alpha_{CF}+\beta_{CF}} = 1000 - 1000\frac{6}{6+2} = 250 \tag{61}$$

$$Max_{CF} = CF2 = Min_{CF} + Range_{CF} = 250 + 1000 = 1250 \tag{62}$$

$$B(\alpha_N,\beta_N) = \frac{42}{(Range_N)^7} * (N - Min_N)^5 (Max_N - N)$$

$$Min_N \le N \le Max_N \tag{63}$$

$$Min_N = N1 = \mu_N - (Range_N)\frac{\alpha_N}{\alpha_N+\beta_N} = 4 - 4\frac{6}{6+2} = 1 \tag{64}$$

$$Max_N = N2 = Min_N + Range_N = 1 + 4 = 5 \tag{65}$$

$$B(\alpha_R,\beta_R) = \frac{42}{(Range_R)^7} * (R - Min_R)^5 (Max_R - R)$$

$$Min_R \le R \le Max_R \tag{66}$$

$$Min_R = R1 = \mu_R - (Range_R)\frac{\alpha_R}{\alpha_R + \beta_R} = 0.12 - 0.12\frac{6}{6+2} = 0.03 \tag{67}$$

$$Max_R = R2 = Min_R + Range_R = 0.03 + 0.12 = 0.15 \tag{68}$$

$$E[PW] = 621.81.$$

Variance:

$$E[PW^2] = \int_{CF1}^{CF2}\int_{N1}^{N2}\int_{R1}^{R2} B(\alpha_{CF}, \beta_{CF})B(\alpha_N, \beta_N)B(\alpha_R, \beta_R)(e^{-RN})^2 CF^2 \, dRdNdCF \tag{69}$$

$$E[PW^2] = 398623$$

$$\sigma_{PW}^2 = E[PW^2] - \mu_{PV}^2 = 398623 - 621.81^2 = 11973.70 \tag{70}$$

To summarize, the results thus far are shown in table 3.

Table 3

Theoretical and Predicted Mean and Variance Values for the Present Worth and the Exponential Term of the Present Worth Equation

	Mean (Present Worth)	Variance (Present Worth)
Theoretical	621.81	11973
Predicted	621.75	11653

Next, this thesis calculates the beta probability distribution parameters α and β using the theoretical and predicted values. Equations 71 and 72 show the standardized beta probability distribution mean and variance for the present worth using the theoretical mean and variance for the present worth.

Theoretical:

$$\mu_S = \frac{\mu_{PW} - Min_{PW}}{Range_{PW}} = \frac{621.81 - 118.09}{1094.97} = 0.46 \tag{71}$$

$$\sigma_S^2 = \frac{\sigma_{PW}^2}{Range_{PW}^2} = \frac{11973.70}{1094.97^2} = 0.009987 \tag{72}$$

Equations 73 – 75 show the minimum, maximum and range values for each present worth.

$$Min_{PW} = Min_{CF} * e^{-Max_N * Max_R} = (250)e^{-5(0.15)} = 118.09 \tag{73}$$

$$Max_{PW} = Max_{CF} * e^{-Min_N * Min_R} = (1250)e^{-1(0.03)} = 1213.06 \tag{74}$$

$$Range_{PW} = Max_{PV} - Min_{PV} = 1213.06 - 118.09 = 1094.97 \tag{75}$$

Equations 76 and 77 show the standardized beta probability distribution mean and variance for the present worth using the theoretical mean and variance for the present worth.

Predicted:

$$\mu_S = \frac{\mu_{PW} - Min_{PW}}{Range_{PW}} = \frac{621.75 - 118.09}{1094.97} = 0.46 \tag{76}$$

$$\sigma_S^2 = \frac{\sigma_{PW}^2}{Range_{PW}^2} = \frac{11653}{1094.97^2} = 0.009719 \tag{77}$$

The minimum, maximum and range values are same as computed above.

The beta probability distribution parameters α and β, using the theoretical mean and variance of the present worth are calculated below.

$$K = \frac{1 - \mu_S}{\mu_S} = \frac{1 - 0.46}{0.46} = 1.17391 \tag{78}$$

$$\alpha = \frac{K - (1+K)^2 \sigma_S^2}{(1+K)^3 \sigma_S^2} = \frac{1.17391 - (1+1.17391)^2 0.009719}{(1+1.17391)^3 0.009719} = 11.3 \tag{79}$$

$$\beta = K * \alpha = 1.17391(11.3) = 13.26 \tag{80}$$

Equation 81 shows the resulting beta probability distribution.

$$B(10.982,12.891) = \frac{\Gamma(11.3+13.26)}{\Gamma(11.3)\Gamma(13.26)} PW^{10.3}(1-PW)^{12.26} \tag{81}$$

The beta probability distribution parameters α and β, using the predicted mean and variance of the present worth are calculated next.

$$K = \frac{1-\mu_S}{\mu_S} = \frac{1-0.457}{0.457} = 1.18818 \tag{82}$$

$$\alpha = \frac{K-(1+K)^2\sigma_S^2}{(1+K)^3\sigma_S^2} = \frac{1.18818-(1+1.18818)^2 0.0096916}{(1+1.18818)^3 0.0096916} = 11.244$$

(83)

$$\beta = K * \alpha = 1.18818(11.244) = 13.360 \tag{84}$$

Equation 85 shows the resulting beta probability distribution.

$$B(11.244,13.360) = \frac{\Gamma(11.244+13.360)}{\Gamma(11.244)\Gamma(13.360)} PW^{10.244}(1-PW)^{12.360} \tag{85}$$

Figures 13 and 14 show the beta probability and cumulative distribution derived from the theoretical mean and variance compared to a simulation of the present worth. Figures 15 and 16 shows the beta probability and cumulative distribution derived from the predicted mean and variance compared to a simulation.

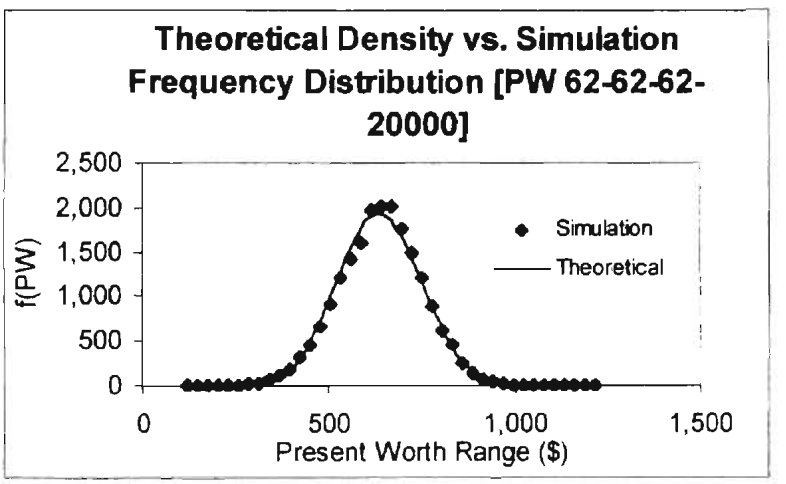

Figure 13. Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash

Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

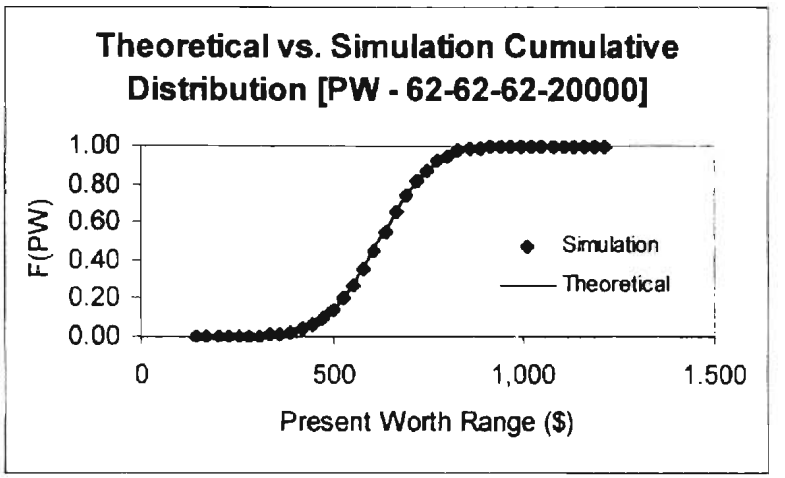

Figure 14. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

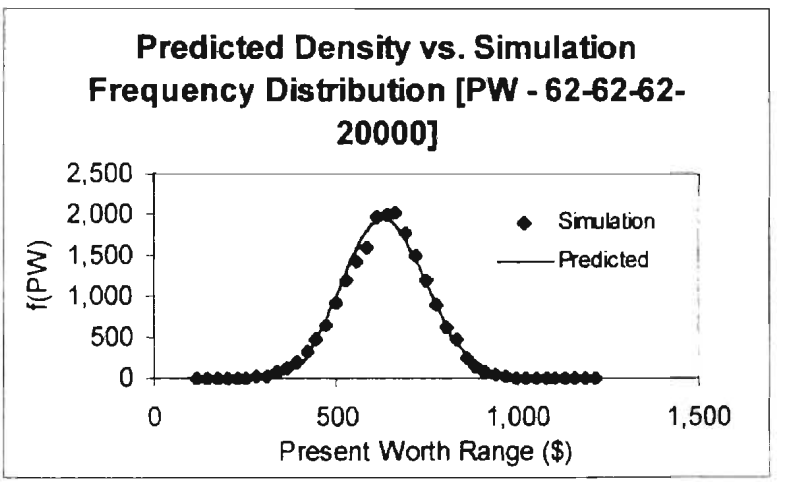

Figure 15. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

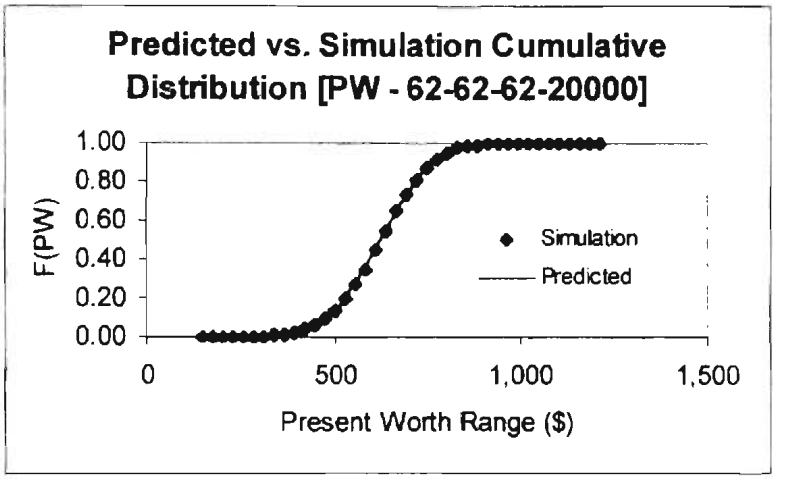

Figure 16. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

CHAPTER 4

RESULTS

Introduction

This chapter presents the results of this thesis' investigation and consists of six major sections. The first major section is this introduction. The second major section has five subsections and discusses the predicted and theoretical means and variances of the present worths of the 27 combinations shown on Table 2 in Chapter 3. Subsections 1 and 2 present the predicted and theoretical mean and variance values of the 27 combinations shown on Table 2 in Chapter 3. Subsections 3 and 4 compare these mean and variance values using ANOVA. Subsection 5 shows the beta distribution parameters for the assumed beta distributed present worth derived from the theoretical means and variances of the present worth and derived from the predicted means and variances of the present worth.

Major section three presents the results from testing the simulation program. The first subsection in the third major section performs three simulations for the B(6,2), B(5,5) and B(2,6) probability distributions defined in Chapter 3. This subsection then compares the results of these simulations to a theoretical B(6,2), B(5,5) and a B(2,6) probability distribution using a chi-squared goodness-of-fit test at an alpha level of 0.05. The second and final subsection compares the mean and variance of a simulation of each

of the 27 combinations shown in the skew combination table to the theoretical mean and variance using a chi-squared goodness-of-fit test at and alpha level of 0.05.

The fourth major section shows the results of comparing the simulation data to the theoretical and predicted data. The two subsections in major section four compares the simulation results for each of the above 27 combinations to the beta probability distribution derived from the theoretical and predicted present worth mean and variance values. Each subsection uses a chi-squared goodness-of-fit analysis at a 0.05 level of significance for the comparison. The last major section provides a discussion of the results shown in the preceding four major sections.

Mean and Variance Values for the Present Worth

Table 4 shows the predicted values for the present worth mean and variance for each of the 27 combinations. Table 5 shows the theoretical mean and variance values for the present worth mean and variance for each of the 27 combinations.

Table 6 shows that there is no significant difference between the predicted present worth mean and the theoretical present worth mean at a level of significance of 0.05 (p-value = 0.76). Table 7 shows the comparison between the theoretical variance values of the present worth and the predicted variance values of the present worth using ANOVA ($\alpha = 0.05$). The analysis of variance shows that there is no significant difference between the theoretical and predicted variance values at a 0.05 level of significance (p-value = 0.17).

Table 4

Predicted Mean and Variance for the Present Worth

Skew Combination	Predicted Mean	Predicted Variance
PW_{LLL}	621.75	11653
PW_{LLS}	621.88	11820
PW_{LLR}	621.74	11653
PW_{LSL}	621.89	11820
PW_{LSS}	622.02	11987
PW_{LSR}	621.89	11820
PW_{LRL}	621.75	11653
PW_{LRS}	621.89	11820
PW_{LRR}	621.75	11653
PW_{SLL}	621.75	12378
PW_{SLS}	621.89	12545
PW_{SLR}	621.75	12378
PW_{SSL}	621.89	12545
PW_{SSS}	622.02	12712
PW_{SSR}	621.89	12545
PW_{SRL}	621.75	12378
PW_{SRS}	621.89	12545
PW_{SRR}	621.75	12378
PW_{RLL}	621.75	11653
PW_{RLS}	621.89	11820
PW_{RLR}	621.75	11653
PW_{RSL}	621.89	11820
PW_{RSS}	622.02	11987
PW_{RSR}	621.89	11820
PW_{RRL}	621.75	11653
PW_{RRS}	621.89	11820
PW_{RRR}	621.75	11653

Table 5

Theoretical Mean and Variance for the Present Worth

Skew Combination	Theoretical Mean	Theoretical Variance
PW_{LLL}	621.81	11974
PW_{LLS}	621.92	12051
PW_{LLR}	621.76	11790
PW_{LSL}	621.92	12051
PW_{LSS}	622.03	12127
PW_{LSR}	621.87	11867
PW_{LRL}	621.76	11790
PW_{LRS}	621.87	11867
PW_{LRR}	621.71	11606
PW_{SLL}	621.81	12713
PW_{SLS}	621.92	12791
PW_{SLR}	621.76	12529
PW_{SSL}	621.92	12791
PW_{SSS}	622.03	12868
PW_{SSR}	621.87	12606
PW_{SRL}	621.76	12529
PW_{SRS}	621.87	12606
PW_{SRR}	621.71	12345
PW_{RLL}	621.81	11974
PW_{RLS}	621.92	12051
PW_{RLR}	621.76	11790
PW_{RSL}	621.92	12051
PW_{RSS}	622.03	12127
PW_{RSR}	621.87	11867
PW_{RRL}	621.76	11790
PW_{RRS}	621.87	11867
PW_{RRR}	621.71	11606

Table 6

Analysis of Variance Comparing the Predicted Present Worth Mean Values for the 27 Skew Combinations and the Theoretical Present Worth Mean Values for the Same Skew Combinations

Anova: Single Factor - alpha = 0.05				
SUMMARY				
Groups	Count	Sum	Average	Variance
Predicted Mean	27	16790	621.84	0.0085
Theoretical Mean	27	16790	621.85	0.0093

ANOVA						
Source of Variation	SS	Df	MS	F	P-value	F crit
Between Groups	0.0008	1	0.0008	0.10	0.76	4.03
Within Groups	0.4625	52	0.0089			
Total	0.4634	53				

Table 7

Analysis of Variance Comparing the Predicted Present Worth Variance Values for the 27 Skew Combinations and the Theoretical Present Worth Variance Values for the Same Skew Combinations

Anova: Single Factor - alpha = 0.05				
SUMMARY				
Groups	Count	Sum	Average	Variance
Predicted Variance	27	324157	12006	134247
Theoretical Variance	27	328018	12149	151123

ANOVA						
Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	276047	1	276047	1.93	0.17	4.03
Within Groups	7419621	52	142685			
Total	7695668	53				

Assumed Beta Probability Distribution Parameters

Columns 3 and 4 in Table 8 show the beta probability distribution parameters α and β for the assumed beta distribution when the predicted mean and variance values of the present worth are utilized. Columns 5 and 6 in Table 8 show the beta probability distribution parameters when the theoretical mean and variance values of the present worth are utilized.

Testing the Simulation

Chi-Squared Goodness of Fit Test for Theoretical vs. Simulation for a B(6, 2), B(5, 5), and a B(2, 6) Probability Distribution.

Three simulations were performed for each of the three beta distributions presented in Chapter 3 (B(6,2), B(5,5) and B(2,6)). Tables 9, 10 and 11 show the results of a chi-squared goodness-of-fit test, at a 0.05 level of significance, for the simulated distributions versus the theoretical distributions. All of the tests show that there is no significant difference between the simulated data and the theoretical distribution. Appendix B shows the chi-squared goodness-of-fit data sheets and the frequency/density graphs for this analysis.

Table 8

Beta Probability Distribution Parameters α and β for the Assumed Beta Distribution when the Predicted Mean and Variance Values of the Present Worth and the Theoretical Mean and Variance of the Present Worth are Utilized

Skew Combination	Predicted Alpha	Predicted Beta	Theoretical Alpha	Theoretical Beta
PW_{LLL}	11.30	13.26	11.10	13.01
PW_{LLS}	11.34	12.10	11.11	11.86
PW_{LLR}	11.58	11.29	11.44	11.15
PW_{LSL}	11.34	12.10	11.11	11.86
PW_{LSS}	10.92	9.89	10.78	9.77
PW_{LSR}	10.58	8.10	10.53	8.07
PW_{LRL}	11.58	11.29	11.44	11.15
PW_{LRS}	10.58	8.10	10.53	8.07
PW_{LRR}	9.50	5.60	9.54	5.62
PW_{SLL}	7.90	17.08	7.68	16.61
PW_{SLS}	8.79	16.60	8.61	16.27
PW_{SLR}	9.73	16.31	9.61	16.11
PW_{SSL}	8.79	16.60	8.61	16.27
PW_{SSS}	9.43	14.77	9.31	14.58
PW_{SSR}	10.00	13.15	9.95	13.08
PW_{SRL}	9.73	16.31	9.61	16.11
PW_{SRS}	10.00	13.15	9.95	13.08
PW_{SRR}	10.05	10.38	10.08	10.41
PW_{RLL}	4.72	18.99	4.59	18.46
PW_{RLS}	6.26	20.26	6.13	19.86
PW_{RLR}	7.83	21.31	7.74	21.06
PW_{RSL}	6.26	20.27	6.13	19.86
PW_{RSS}	7.79	19.72	7.69	19.48
PW_{RSR}	9.20	18.88	9.16	18.80
PW_{RRL}	7.83	21.31	7.74	21.06
PW_{RRS}	9.20	18.88	9.16	18.80
PW_{RRR}	10.25	16.29	10.29	16.36

Table 9

Chi-Squared Goodness-Of-Fit Results for the Simulated B(6,2) Distribution Versus a Theoretical B(6,2) Distribution.

Simulation Run #		
1	Test Statistic	15.7
	Critical Value (a = 0.05)	26.3
2	Test Statistic	9.9
	Critical Value (a = 0.05)	26.3
3	Test Statistic	20.6
	Critical Value (a = 0.05)	26.3

Table 10

Chi-Squared Goodness-Of-Fit Results for the Simulated B(5,5) Distribution Versus a Theoretical B(5,5) Distribution.

Simulation Run #		
1	Test Statistic	22.1
	Critical Value (a = 0.05)	26.3
2	Test Statistic	11.6
	Critical Value (a = 0.05)	26.3
3	Test Statistic	17.5
	Critical Value (a = 0.05)	26.3

Table 11

Chi-Squared Goodness-Of-Fit Results for the Simulated B(2,6) Distribution Versus a Theoretical B(2,6) Distribution

Simulation Run #		
1	Test Statistic	12.3
	Critical Value (a = 0.05)	26.3
2	Test Statistic	12.2
	Critical Value (a = 0.05)	26.3
3	Test Statistic	18.9
	Critical Value (a = 0.05)	26.3

Comparison of Simulation Mean and Variance to Theoretical Mean and Variance

Tables 12 –14 show the comparison of the mean and variance of a simulation of each of the 27 combinations shown in the skew combination table to the theoretical mean and variance using ANOVA at an alpha level of 0.05. The analysis of variance shows that there is no significant difference between the simulation results and the theoretical results. The p-value for the mean comparison is 0.28. The p-value for the variance comparison is 0.85.

Simulation Results

The input data for the simulation are the beta distribution parameters for the lump-sum cash flow, lump-sum cash flow timing and interest rate distributions and, the range and mean value of each variable. The values and assumptions for these variables are presented in Chapter 3 and are repeated below for convenience.

1. This thesis investigates only lump sum cash flows.
2. The lump sum cash flow has a mean value of $1000 and a range of $1000.
3. The cash flow timing has a mean value of 4 years and a range of 4 years.
4. The interest rate has a mean value of 12% and a range of 12%. The difference between the three sample sets for each uncertain variable is in the skew of the distribution. Specifically, the first sample set describes a left skew distribution ($\alpha > \beta$). The second sample set describes a symmetric distribution ($\alpha = \beta$). The third set describes a right skew distribution ($\alpha < \beta$).

Table 12

Simulation Mean and Variance Values and Theoretical Mean and Variance Values for the Present Worth of the 27 Combinations Shown in the Skew Combination Table

Skew Combination	Theoretical Mean	Simulation Mean	Theoretical Variance	Simulation Variance
PW_{LLL}	621.81	621.83	11974	12130
PW_{LLS}	621.92	622.72	12051	11932
PW_{LLR}	621.76	621.96	11790	11619
PW_{LSL}	621.92	620.94	12051	12132
PW_{LSS}	622.03	621.75	12127	12296
PW_{LSR}	621.87	622.23	11867	11759
PW_{LRL}	621.76	623.17	11790	11740
PW_{LRS}	621.87	621.08	11867	11879
PW_{LRR}	621.71	622.80	11606	11546
PW_{SLL}	621.81	622.16	12713	12719
PW_{SLS}	621.92	620.71	12791	12911
PW_{SLR}	621.76	622.09	12529	12422
PW_{SSL}	621.92	622.46	12791	12868
PW_{SSS}	622.03	623.09	12868	12862
PW_{SSR}	621.87	621.82	12606	12636
PW_{SRL}	621.76	621.32	12529	12509
PW_{SRS}	621.87	622.02	12606	12569
PW_{SRR}	621.71	621.87	12345	12293
PW_{RLL}	621.81	621.13	11974	11772
PW_{RLS}	621.92	621.96	12051	12067
PW_{RLR}	621.76	622.18	11790	11626
PW_{RSL}	621.92	622.62	12051	12014
PW_{RSS}	622.03	621.07	12127	12154
PW_{RSR}	621.87	622.49	11867	11832
PW_{RRL}	621.76	622.17	11790	11769
PW_{RRS}	621.87	622.50	11867	12026
PW_{RRR}	621.71	621.56	11606	11371

Table 13

Analysis of Variance Results for the Simulation Mean Values for the 27 Combinations Versus the Theoretical Mean Values for the Same Combinations

Anova: Single Factor - alpha = 0.05				
SUMMARY				
Groups	Count	Sum	Average	Variance
Theoretical Mean	27	16790	621.85	0.0093
Simulation Mean	27	16794	621.99	0.4228

ANOVA						
Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.26	1	0.26	1.21	0.28	4.03
Within Groups	11.24	52	0.22			
Total	11.50	53				

Table 14

Analysis of Variance Results for the Simulation Variance Values for the 27 Combinations Versus the Theoretical Variance Values for the Same Combinations

Anova: Single Factor - alpha = 0.05				
SUMMARY				
Groups	Count	Sum	Average	Variance
Theoretical Variance	27	328018	12149	151123
Theoretical Variance	27	327453	12128	191150

ANOVA						
Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	5913	1	5913	0.03	0.85	4.03
Within Groups	8899101	52	171137			
Total	8905014	53				

5. The left skew Beta distribution parameters are $\alpha = 6$ and $\beta = 2$ ($\sigma^2 = 0.02083$). The symmetric beta distribution parameters are $\alpha = 5$ and $\beta = 5$ ($\sigma^2 = 0.02273$). The right skew beta distribution parameters are $\alpha = 2$ and $\beta = 6$ ($\sigma^2 = 0.02083$). These values of six and two are chosen because they can show the effect of skewness of each independent variable and they allow the calculations of the integrals for the theoretical means and variances to be less cumbersome.
6. Each sample set is independent of all other sample sets.

Each simulation consists of 20,000 runs. The reported simulation mean and variance values are the average and sample variance of the 20,000 present worth values resulting from the 20,000 runs. Appendix C shows the data sheets for these tests, the associated frequency/density graphs and the associated cumulative distribution graphs.

Comparison of the Simulation Results to the Assumed Beta Distributions that are Derived from Theoretical Mean and Variance Values

Table 15 shows the comparison between the assumed beta distributions for the present worth of all 27 combinations shown on the skew combination table that are derived from the theoretical mean and variance values to the simulated data using a chi-squared goodness-of-fit test. Six of the 27 combinations show that there is no significant difference between the two data sets at a 0.05 level of significance. The term "Theoretical" shown in the column headings denotes the assumed beta distributions that are derived from the theoretical mean and variance values.

Table 15

Chi-Squared Goodness-Of-Fit Test Results Comparing the Simulation Frequency Distribution with the Frequency Distribution Derived from the Theoretical Mean and Variance Values of the Present Worth for all 27 Combinations Presented in the Skew Combination Table

Skew Combination	Theoretical Test Statistic	Chi-Squared Critical Value	
PW_{LLL}	263	37.65	
PW_{LLS}	156	36.42	
PW_{LLR}	169	36.42	
PW_{LSL}	203	37.65	
PW_{LSS}	145	40.11	
PW_{LSR}	64	38.89	
PW_{LRL}	179	36.42	
PW_{LRS}	100	37.65	
PW_{LRR}	35	40.11	*
PW_{SLL}	14	33.92	*
PW_{SLS}	37	35.17	
PW_{SLR}	26	33.92	*
PW_{SSL}	28	33.92	*
PW_{SSS}	29	35.17	*
PW_{SSR}	55	36.42	
PW_{SRL}	28	33.92	*
PW_{SRS}	47	35.17	
PW_{SRR}	115	37.65	
PW_{RLL}	115	30.14	
PW_{RLS}	277	31.41	
PW_{RLR}	311	32.67	
PW_{RSL}	189	30.14	
PW_{RSS}	366	32.67	
PW_{RSR}	384	32.67	
PW_{RRL}	338	30.14	
PW_{RRS}	440	32.67	
PW_{RRR}	613	35.17	

Note: * indicates this combination passed a chi-squared goodness-of-fit test.

Comparison of Simulation Results to the Assumed Beta Distributions that are Derived from Predicted Mean and Variance Values

Table 16 shows a comparison between the assumed beta distributions for the present worth of all 27 combinations shown on the skew combination table that are derived from the predicted mean and variance values to the simulated data using a chi-squared goodness-of-fit test. Five of the 27 combinations show that there is no significant difference between the two data sets at a 0.05 level of significance. The term "Predicted" shown in the column headings denotes the assumed beta distributions that are derived from the predicted mean and variance values.

Discussion of Results

The predicted and theoretical, mean and variance data sets are not significantly different at a level of significance of 0.05. All but three of the 27 predicted present worth variance values are less than the theoretical present worth variance values. The three predicted present worth variance values that are greater than the theoretical present worth variance values occur when the timing variable and the interest rate variable are both right skewed.

This phenomenon occurs because of the non-linearity of the exponential function. In the right skew situation the tail of the distribution is to the right of the mean. And even though the probability associated with the tail values is low, these values have an exponential effect on the present worth. In the case where the timing and interest rate are both left skew the tail values again have an exponential effect, but the effect exponentially decreases (not increases) as the values decreases.

Table 16

Chi-Squared Goodness-Of-Fit Test Results Comparing the Simulation Frequency Distribution with the Frequency Distribution Derived from the Predicted Mean and Variance Values of the Present Worth for all 27 Combinations Presented in the Skew Combination Table

Skew Combination	Predicted Test Statistic	Chi-Squared Critical Value	
PW_{LLL}	318	37.65	
PW_{LLS}	168	36.42	
PW_{LLR}	168	36.42	
PW_{LSL}	229	37.65	
PW_{LSS}	156	40.11	
PW_{LSR}	64	38.89	
PW_{LRL}	185	36.42	
PW_{LRS}	100	37.65	
PW_{LRR}	37	40.11	*
PW_{SLL}	22	33.92	*
PW_{SLS}	45	35.17	
PW_{SLR}	26	33.92	*
PW_{SSL}	35	33.92	
PW_{SSS}	29	35.17	*
PW_{SSR}	56	36.42	
PW_{SRL}	30	33.92	*
PW_{SRS}	47	35.17	
PW_{SRR}	115	37.65	
PW_{RLL}	105	30.14	
PW_{RLS}	283	31.41	
PW_{RLR}	311	32.67	
PW_{RSL}	186	30.14	
PW_{RSS}	371	32.67	
PW_{RSR}	385	32.67	
PW_{RRL}	340	30.14	
PW_{RRS}	442	32.67	
PW_{RRR}	610	35.17	

Note: * indicates this combination passed a chi-squared goodness-of-fit test.

A comparison of the simulation results with a beta distribution derived from the theoretical mean and variance value for each of the 27 combinations under study in this

thesis, shows that six of the estimated beta distributions passed a chi-squared goodness-of-fit test at a level of significance of 0.05. Five of the estimated beta distributions derived from the predicted mean and variance value for each of the 27 combinations under study in this thesis, passed a chi-squared goodness-of-fit test at a level of significance of 0.05. The conclusion from these results is that the proposed beta distribution does not fit the resulting present worth. Moreover, this is true not only for the proposed beta distribution derived from the predicted mean and variance but also for the proposed beta distribution derived from the theoretical mean and variance.

Park and Sharp-Bette [27] state that if the skew (equation 86) of a distribution is greater than zero, then the distribution is positively or right skewed.

$$Skew_{std} = \frac{E[X^3] - 3\mu E[X^2] + 2\mu^3}{\sqrt[3]{\sigma^2}} \tag{86}$$

If the skew is less than zero then the distribution is negatively or left skewed. And finally, if the skew is equal to zero, then the distribution is symmetric. Table 17 shows that if the cash flow is left skewed then the present worth is also left skewed. While not within the scope of this thesis, this researcher has experimented with several negatively skewed cash flows and found that this results holds for most cases. The exceptions occur when the cash flow is only slightly, negatively skewed.

Finally, table 17 shows that the kurtosis (equation 87, [27]) for the present worths studied in this thesis range from 2.72 to 3.51. Park and Sharp-Bette [27] state that the kurtosis of a normal distribution is equal to 3.

$$Kurtosis_{std} = \frac{E[X^4] - 4\mu E[X^3] + 6\mu E[X^2] - 3\mu^4}{\sqrt[4]{\sigma^2}} \quad (87)$$

Table 17

Skew and Kurtosis Values for the Present Worth Under Study in this Thesis

Skew Combination	Present Worth Skew	Present Worth Kurtosis
PW_{LLL}	-0.080	2.96
PW_{LLS}	-0.123	2.89
PW_{LLR}	-0.181	2.87
PW_{LSL}	-0.123	2.89
PW_{LSS}	-0.165	2.82
PW_{LSR}	-0.223	2.80
PW_{LRL}	-0.181	2.87
PW_{LRS}	-0.223	2.80
PW_{LRR}	-0.286	2.77
PW_{SLL}	0.306	2.95
PW_{SLS}	0.264	2.88
PW_{SLR}	0.221	2.84
PW_{SSL}	0.264	2.88
PW_{SSS}	0.223	2.81
PW_{SSR}	0.179	2.77
PW_{SRL}	0.221	2.84
PW_{SRS}	0.179	2.77
PW_{SRR}	0.132	2.72
PW_{RLL}	0.708	3.51
PW_{RLS}	0.658	3.43
PW_{RLR}	0.624	3.39
PW_{RSL}	0.658	3.44
PW_{RSS}	0.609	3.35
PW_{RSR}	0.575	3.30
PW_{RRL}	0.624	3.39
PW_{RRS}	0.575	3.30
PW_{RRR}	0.536	3.25

Therefore, even though two of the three distributions assumed for the lump-sum cash flow, the timing and the interest rate were distinctly not normal, the resulting present worth is tending toward a normal shape.

CHAPTER 5

CONCLUSION

Summary

The objective of this thesis is to present a non-integral method to estimate the mean, variance and the probability distribution of the present worth of an economic project. This research shows that within the bounds of this thesis, this thesis' estimators for the mean and variance of the present worth provide satisfactory results within a 0.05 level of significance. Also, this research shows that even when a study incorporates the theoretical mean and variance values for the present worth directly, the resulting beta distribution does not fit the simulated present worth distribution. However, the degree of fit using either the estimated mean and variance or the theoretical mean and variance is relatively the same (refer to Appendix C for the complete data sheets and graphs). Whether the predicted cumulative distribution fits the actual present worth cumulative distribution satisfactorily for practical purposes is left to the judgment of the decision-maker.

This research shows that if the lump-sum cash flow, cash flow timing and interest rate are distinctly not normal, the resulting present worth begins to approach a normal appearance. If multiple periodic and independent cash flows must be studied, this investigation tends to support the idea that the resulting present worth will more nearly approach a normal distribution. This researcher also believes that this same result will occur if the periodic cash flows are dependent.

Finally, Appendix D shows the results of two additional simulation runs. The first run simulates a lump-sum cash flow ~B(1,10) (mean = 1000. range = 750), lump-sum cash flow timing ~B(1,10) (mean = 4.5, range = 2) and interest rate ~B(1,10) (mean = 0.2, range = .015). The second run simulates a lump-sum cash flow ~B(1,10) (mean = 1000, range = 1250), lump-sum cash flow timing ~B(10,1) (mean = 5, range = 4) and interest rate ~B(13,2) (mean =0.15, range = 0.07). The results of theses two simulations, that are well outside the scope of this thesis, support the idea that the results of this thesis is applicable in general.

Contribution to Current Knowledge

This thesis adds to the knowledge base in engineering economic analysis by showing that the present worth of an economic project does not fit a beta distribution derived by direct use of the theoretical mean and variance of the present worth. However, this thesis shows that a relatively simple estimate of present worth distribution is not significantly different from a distribution that requires the integral calculations for the theoretical present worth mean and variance. This information is important to a decision-maker who is trying to develop a first estimate of the distribution of the present worth as easily as possible.

Additionally, this thesis adds to the knowledge base in engineering economics by showing that the mean and variance of the present worth under the scope of this thesis are predictable without the use of integrals. This last point is exceedingly useful, since the integrals for the mean and variance of the present worth become very complicated at even small values for the beta probability distribution parameters, α and β.

Topics for Further Research

This thesis opens several areas for further research. First, a study detailing what values are needed for the beta distribution parameter so that the resulting distribution fits the actual present worth distribution. These values then need to be compared to the theoretical mean and variance values to develop a reliable relationship.

Second, further research should be performed to expand the scope presented in this thesis. As stated in the previous section, Appendix D shows two additional simulation results that incorporate different mean and range values for the CF, N and R. However, a detailed study should be undertaken to verify that the results of this thesis are true in general. In addition, this study should include periodic independent and dependent cash flows.

The third area for further research concerns probabilistic interest rate. A detailed study should be performed investigating the variability of the interest rate and how the probabilistic approach would address the concerns given in the literature concerning interest rate under risk. Finally, further research should be conducted to improve the mean and variance estimators presented in this thesis.

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APPENDIXES

APPENDIX A – BETA DISTRIBUTION

The constant K in Equation 1 ($f(t) = (K)(t-a)^{\alpha}(b-t)^{\beta}$) is equal to

$$K = \frac{\Gamma(\alpha+\beta-2)}{\Gamma(\alpha-1)\Gamma(\beta-1)}. \tag{A-1}$$

The equation that this thesis will use for the beta distribution is shown in equation A-2. [26]

$$f(t) = (K)(t-a)^{\alpha-1}(b-t)^{\beta-1} \tag{A-2}$$

This necessitates that the constant K is equal to

$$K = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}. \tag{A-3}$$

The mean for equation A-2 is shown in equation A-4.

$$\mu = \frac{\alpha}{\alpha+\beta} \tag{A-4}$$

APPENDIX B – DATA SHEETS AND PROBABILITY GRAPHS USED TO TEST THE SIMULATION PROGRAM

Table 18

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for a B(6,2) Distribution – Run #1

Chi-squared Test on B(6,2) 16 Degrees of Freedom alpha = 0.05				Test Statistic		15.7
				Critical Value		26.3
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
4	7	0.0004	7.42	-0.42	0.18	0.02
5	25	0.0010	19.43	5.57	31.01	1.60
6	52	0.0024	48.96	3.04	9.24	0.19
7	107	0.0052	104.33	2.67	7.11	0.07
8	222	0.0098	196.68	25.32	641.00	3.26
9	320	0.0169	337.20	-17.20	295.84	0.88
10	557	0.0268	535.80	21.20	449.44	0.84
11	858	0.0399	798.40	59.60	3552.16	4.45
12	1093	0.0562	1124.20	-31.20	973.44	0.87
13	1502	0.0752	1503.40	-1.40	1.96	0.00
14	1898	0.0956	1912.40	-14.40	207.36	0.11
15	2278	0.1155	2310.60	-32.60	1062.76	0.46
16	2599	0.1318	2635.40	-36.40	1324.96	0.50
17	2806	0.1399	2797.40	8.60	73.96	0.03
18	2733	0.1337	2674.40	58.60	3433.96	1.28
19	2084	0.1053	2106.20	-22.20	492.84	0.23
20	859	0.0444	887.60	-28.60	817.96	0.92

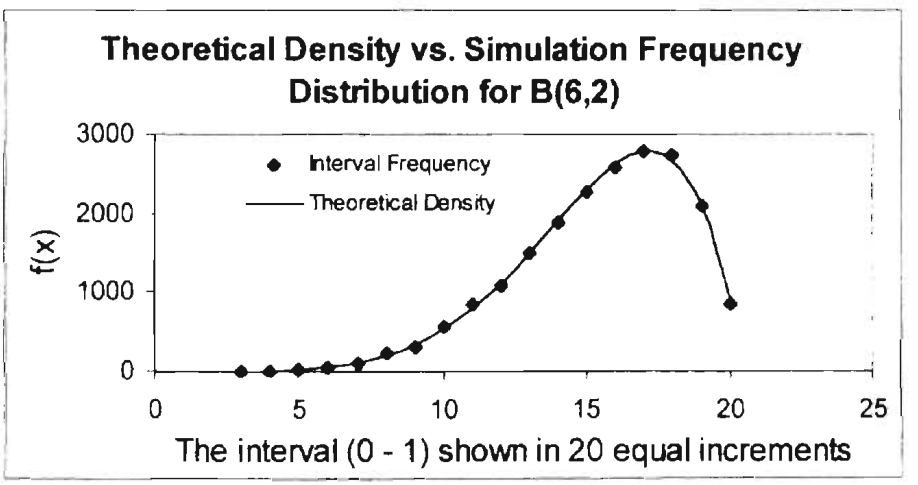

Figure 17: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for a B(6,2) Distribution – Run #1

Table 19

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for a B(6,2) Distribution – Run #2

Chi-squared Test on B(6,2) 17 Degrees of Freedom alpha = 0.05				Test Statistic		9.94
				Critical Value		26.30
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
4	7	0.0004	7.42	-0.42	0.18	0.02
5	22	0.0010	19.43	2.57	6.60	0.34
6	46	0.0024	48.96	-2.96	8.77	0.18
7	107	0.0052	104.33	2.67	7.11	0.07
8	196	0.0098	196.68	-0.68	0.47	0.00
9	352	0.0169	337.20	14.80	219.04	0.65
10	551	0.0268	535.80	15.20	231.04	0.43
11	832	0.0399	798.40	33.60	1128.96	1.41
12	1140	0.0562	1124.20	15.80	249.64	0.22
13	1501	0.0752	1503.40	-2.40	5.76	0.00
14	1938	0.0956	1912.40	25.60	655.36	0.34
15	2260	0.1155	2310.60	-50.60	2560.36	1.11
16	2562	0.1318	2635.40	-73.40	5387.56	2.04
17	2863	0.1399	2797.40	65.60	4303.36	1.54
18	2657	0.1337	2674.40	-17.40	302.76	0.11
19	2114	0.1053	2106.20	7.80	60.84	0.03
20	852	0.0444	887.60	-35.60	1267.36	1.43

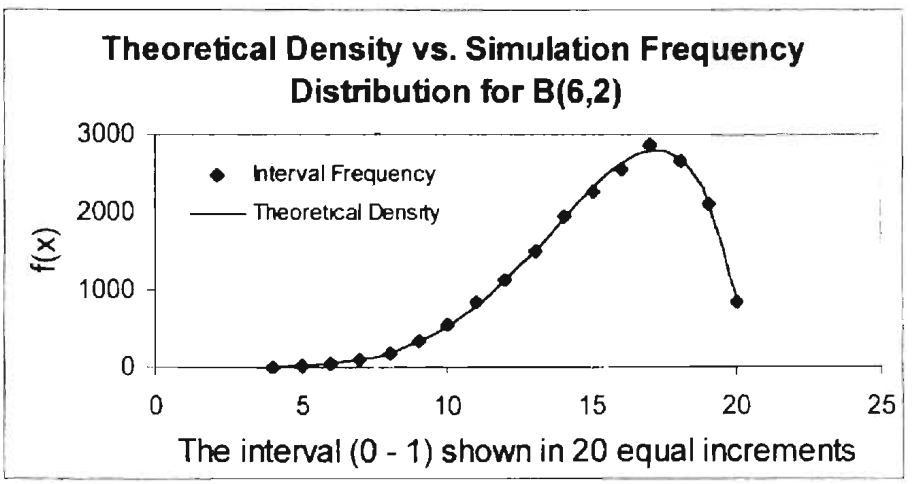

Figure 18: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for a B(6,2) Distribution – Run #2

Table 20

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for a B(6,2) Distribution – Run #3

Chi-squared Test on B(6,2) 17 Degrees of Freedom alpha = 0.05				Test Statistic Critical Value		20.55 26.30
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
4	14	0.0004	7.42	6.58	43.24	5.82
5	19	0.0010	19.43	-0.43	0.19	0.01
6	35	0.0024	48.96	-13.96	194.90	3.98
7	98	0.0052	104.33	-6.33	40.12	0.38
8	197	0.0098	196.68	0.32	0.10	0.00
9	365	0.0169	337.20	27.80	772.84	2.29
10	566	0.0268	535.80	30.20	912.04	1.70
11	805	0.0399	798.40	6.60	43.56	0.05
12	1172	0.0562	1124.20	47.80	2284.84	2.03
13	1476	0.0752	1503.40	-27.40	750.76	0.50
14	1890	0.0956	1912.40	-22.40	501.76	0.26
15	2315	0.1155	2310.60	4.40	19.36	0.01
16	2586	0.1318	2635.40	-49.40	2440.36	0.93
17	2817	0.1399	2797.40	19.60	384.16	0.14
18	2720	0.1337	2674.40	45.60	2079.36	0.78
19	2051	0.1053	2106.20	-55.20	3047.04	1.45
20	874	0.0444	887.60	-13.60	184.96	0.21

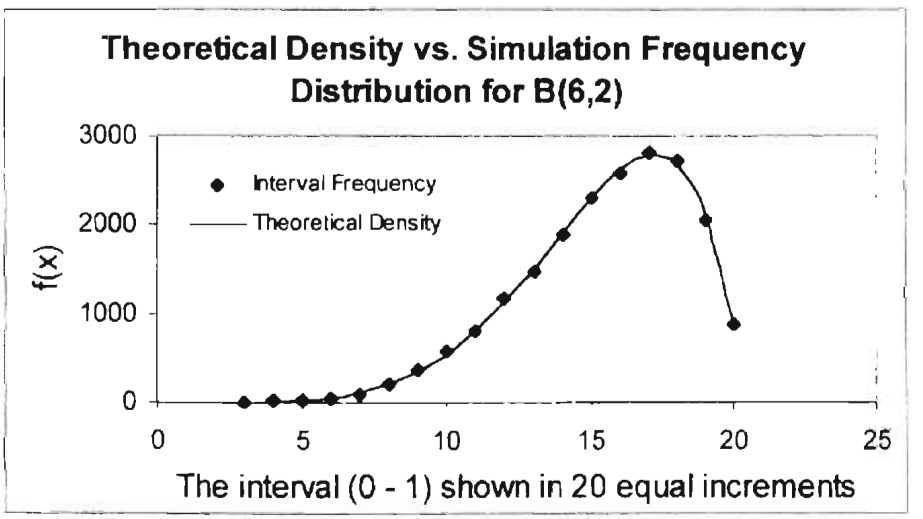

Figure 19: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for a B(6,2) Distribution – Run #3

Table 21

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for a B(5,5) Distribution – Run #1

Chi-squared Test on B(5,5) 16 Degrees of Freedom alpha = 0.05				Test Statistic		22.07
				Critical Value		26.30
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
2	23	0.0009	17.82	5.18	26.85	1.51
3	79	0.0047	94.75	-15.75	248.21	2.62
4	290	0.0140	279.00	11.00	121.00	0.43
5	571	0.0294	587.00	-16.00	256.00	0.44
6	925	0.0499	997.60	-72.60	5270.76	5.28
7	1441	0.0729	1458.20	-17.20	295.84	0.20
8	1851	0.0949	1897.00	-46.00	2116.00	1.12
9	2245	0.1120	2240.20	4.80	23.04	0.01
10	2454	0.1214	2428.40	25.60	655.36	0.27
11	2551	0.1214	2428.40	122.60	15030.76	6.19
12	2185	0.1120	2240.20	-55.20	3047.04	1.36
13	1907	0.0949	1897.00	10.00	100.00	0.05
14	1480	0.0729	1458.20	21.80	475.24	0.33
15	998	0.0499	997.60	0.40	0.16	0.00
16	581	0.0294	587.00	-6.00	36.00	0.06
17	298	0.0140	279.00	19.00	361.00	1.29
18	104	0.0047	94.75	9.25	85.47	0.90
19	17	0.0009	17.15	-0.15	0.02	0.00

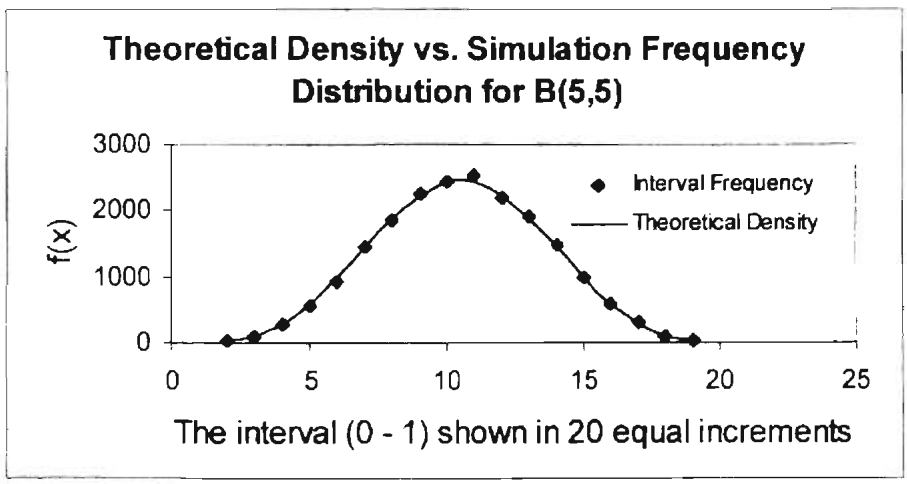

Figure 20: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for a B(5,5) Distribution – Run #1

Table 22

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for a B(5,5) Distribution – Run #2

Chi-squared Test on B(5,5) 16 Degrees of Freedom alpha = 0.05				Test Statistic		11.6
				Critical Value		26.3
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
2	16	0.0009	17.15	-1.15	1.33	0.08
3	107	0.0047	94.75	12.25	149.94	1.58
4	294	0.0140	279.00	15.00	225.00	0.81
5	554	0.0294	587.00	-33.00	1089.00	1.86
6	1028	0.0499	997.60	30.40	924.16	0.93
7	1502	0.0729	1458.20	43.80	1918.44	1.32
8	1918	0.0949	1897.00	21.00	441.00	0.23
9	2278	0.1120	2240.20	37.80	1428.84	0.64
10	2427	0.1214	2428.40	-1.40	1.96	0.00
11	2409	0.1214	2428.40	-19.40	376.36	0.15
12	2174	0.1120	2240.20	-66.20	4382.44	1.96
13	1872	0.0949	1897.00	-25.00	625.00	0.33
14	1457	0.0729	1458.20	-1.20	1.44	0.00
15	986	0.0499	997.60	-11.60	134.56	0.13
16	602	0.0294	587.00	15.00	225.00	0.38
17	266	0.0140	279.00	-13.00	169.00	0.61
18	96	0.0047	94.75	1.25	1.55	0.02
19	14	0.0009	17.15	-3.15	9.95	0.58

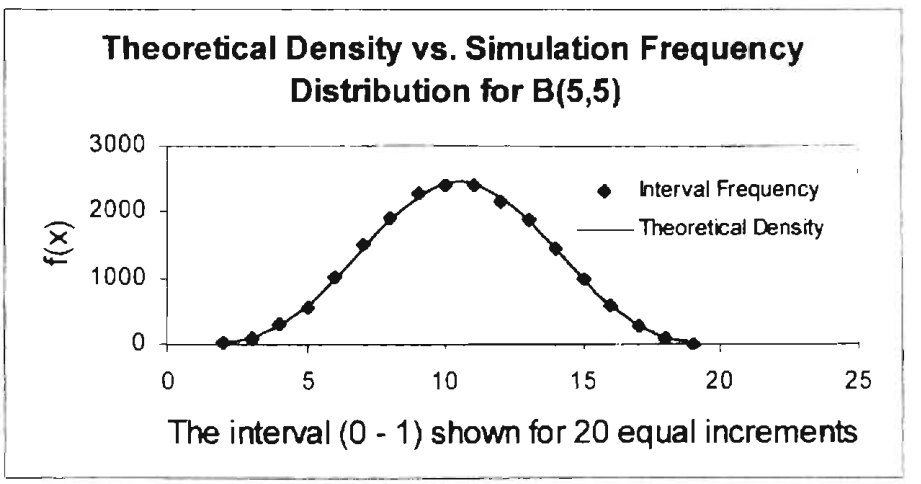

Figure 21: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for a B(5,5) Distribution – Run #2

Table 23

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for a B(5,5) Distribution – Run #3

Chi-squared Test on B(5,5) 16 Degrees of Freedom alpha = 0.05				Test Statistic / Critical Value		17.49 / 26.30
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
2	14	0.0009	17.15	-3.15	9.95	0.58
3	85	0.0047	94.75	-9.75	95.16	1.00
4	282	0.0140	279.00	3.00	9.00	0.03
5	568	0.0294	587.00	-19.00	361.00	0.61
6	996	0.0499	997.60	-1.60	2.56	0.00
7	1378	0.0729	1458.20	-80.20	6432.04	4.41
8	1904	0.0949	1897.00	7.00	49.00	0.03
9	2215	0.1120	2240.20	-25.20	635.04	0.28
10	2485	0.1214	2428.40	56.60	3203.56	1.32
11	2475	0.1214	2428.40	46.60	2171.56	0.89
12	2193	0.1120	2240.20	-47.20	2227.84	0.99
13	1922	0.0949	1897.00	25.00	625.00	0.33
14	1494	0.0729	1458.20	35.80	1281.64	0.88
15	987	0.0499	997.60	-10.60	112.36	0.11
16	569	0.0294	587.00	-18.00	324.00	0.55
17	302	0.0140	279.00	23.00	529.00	1.90
18	113	0.0047	94.75	18.25	332.89	3.51
19	18	0.0009	17.15	0.85	0.72	0.04

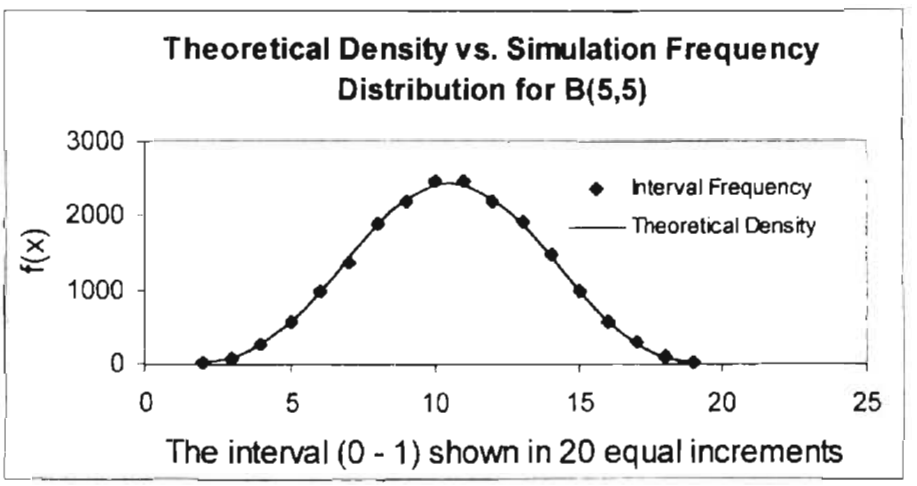

Figure 22: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for a B(5,5) Distribution – Run #3

Table 24

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Frequency Values for a B(2,6) Distribution – Run #1

Chi-squared Test on B(2,6) 16 Degrees of Freedom alpha = 0.05				Test Statistic		12.30
				Critical Value		26.30
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
1	834	0.0444	887.60	-53.60	2872.96	3.24
2	2095	0.1053	2106.20	-11.20	125.44	0.06
3	2700	0.1337	2674.40	25.60	655.36	0.25
4	2807	0.1399	2797.40	9.60	92.16	0.03
5	2669	0.1318	2635.40	33.60	1128.96	0.43
6	2253	0.1155	2310.60	-57.60	3317.76	1.44
7	1939	0.0956	1912.40	26.60	707.56	0.37
8	1556	0.0752	1503.40	52.60	2766.76	1.84
9	1137	0.0562	1124.20	12.80	163.84	0.15
10	804	0.0399	798.40	5.60	31.36	0.04
11	532	0.0268	535.80	-3.80	14.44	0.03
12	330	0.0169	337.20	-7.20	51.84	0.15
13	189	0.0098	196.68	-7.68	59.01	0.30
14	86	0.0052	104.33	-18.33	336.14	3.22
15	44	0.0024	48.96	-4.96	24.61	0.50
16	18	0.0010	19.43	-1.43	2.05	0.11
17	7	0.0004	6.03	0.97	0.93	0.15

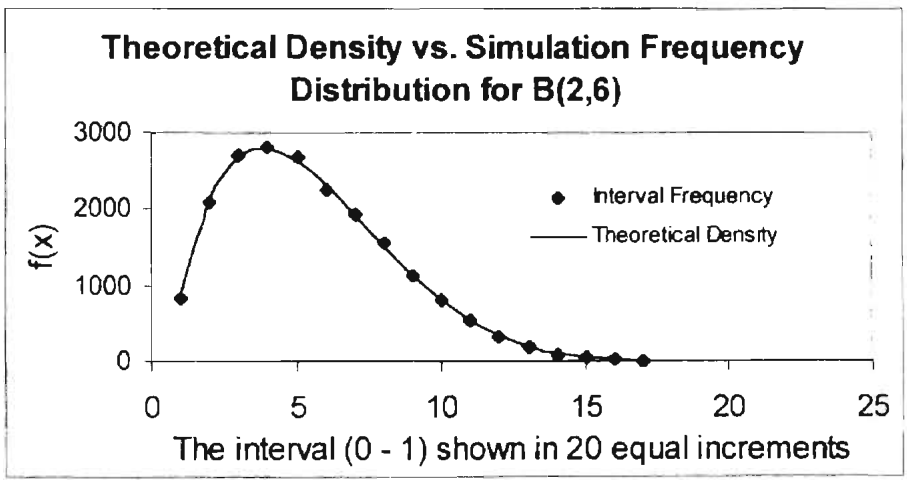

Figure 23: Graph Showing the Simulation Frequency Values versus the Theoretical density Values for a B(2,6) Distribution – Run #1

Table 25

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for a B(2,6) Distribution – Run #2

Chi-squared Test on B(2,6) 16 Degrees of Freedom, alpha = 0.05				Test Statistic		12.18
				Critical Value		26.30
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
1	874	0.0444	887.60	-13.60	184.96	0.21
2	2077	0.1053	2106.20	-29.20	852.64	0.40
3	2709	0.1337	2674.40	34.60	1197.16	0.45
4	2822	0.1399	2797.40	24.60	605.16	0.22
5	2640	0.1318	2635.40	4.60	21.16	0.01
6	2269	0.1155	2310.60	-41.60	1730.56	0.75
7	1984	0.0956	1912.40	71.60	5126.56	2.68
8	1453	0.0752	1503.40	-50.40	2540.16	1.69
9	1150	0.0562	1124.20	25.80	665.64	0.59
10	790	0.0399	798.40	-8.40	70.56	0.09
11	524	0.0268	535.80	-11.80	139.24	0.26
12	313	0.0169	337.20	-24.20	585.64	1.74
13	213	0.0098	196.68	16.32	266.28	1.35
14	112	0.0052	104.33	7.67	58.77	0.56
15	48	0.0024	48.96	-0.96	0.92	0.02
16	15	0.0010	19.43	-4.43	19.64	1.01
17	7	0.0004	6.03	0.97	0.93	0.15

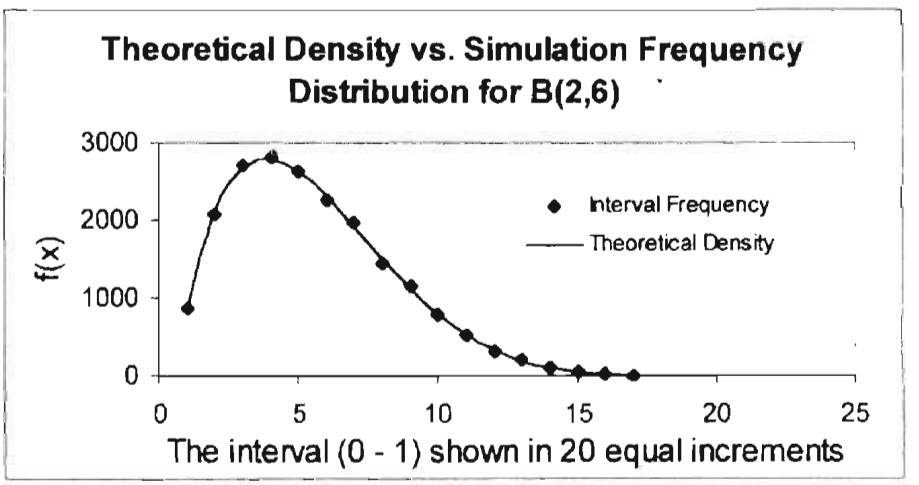

Figure 24: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for a B(2,6) Distribution – Run #2

Table 26

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for a B(2,6) Distribution – Run #3

Chi-squared Test on B(2,6) 16 Degrees of Freedom alpha = 0.05				Test Statistic Critical Value		18.90 26.30
	Interval Frequency	Theoretical	Theoretical, Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
1	876	0.0444	887.60	-11.60	134.56	0.15
2	2153	0.1053	2106.20	46.80	2190.24	1.04
3	2675	0.1337	2674.40	0.60	0.36	0.00
4	2683	0.1399	2797.40	-114.40	13087.36	4.68
5	2624	0.1318	2635.40	-11.40	129.96	0.05
6	2335	0.1155	2310.60	24.40	595.36	0.26
7	1939	0.0956	1912.40	26.60	707.56	0.37
8	1503	0.0752	1503.40	-0.40	0.16	0.00
9	1145	0.0562	1124.20	20.80	432.64	0.38
10	819	0.0399	798.40	20.60	424.36	0.53
11	561	0.0268	535.80	25.20	635.04	1.19
12	339	0.0169	337.20	1.80	3.24	0.01
13	173	0.0098	196.68	-23.68	560.84	2.85
14	103	0.0052	104.33	-1.33	1.78	0.02
15	52	0.0024	48.96	3.04	9.24	0.19
16	10	0.0010	19.43	-9.43	88.95	4.58
17	10	0.0004	6.03	3.97	15.73	2.61

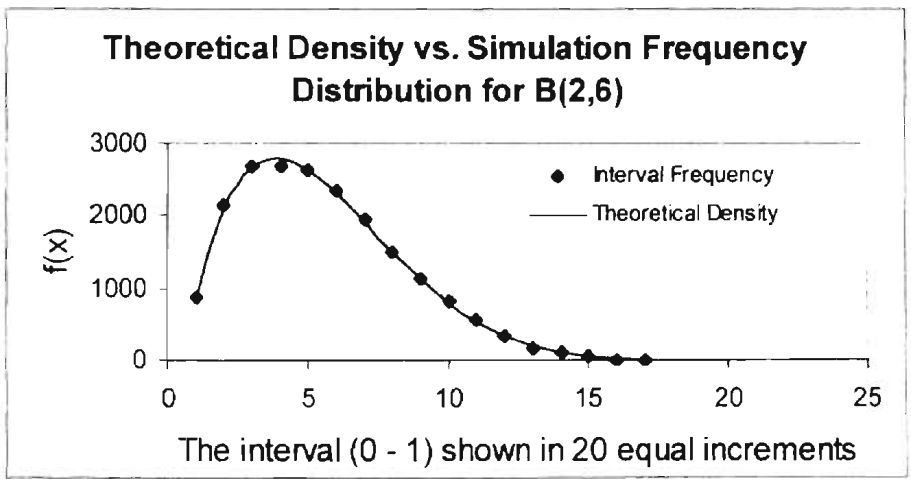

Figure 25: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for a B(2,6) Distribution – Run #3

APPENDIX C – DATA SHEETS, CHI-SQUARED TEST DATA SHEETS AND RESULTING GRAPHS FOR THE COMPARISON BETWEEN THE THEORETICAL AND PREDICTED BETA DISTRIBUTIONS VERSUS THE SIMULATION DATA.

Table 27

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
118.09	0	0	0
145.47	0	0	0
172.84	0	0	0
200.21	0	0	0
227.59	0	0	0
254.96	6	1	0
282.34	15	3	3
309.71	25	12	10
337.08	68	33	29
364.46	124	79	72
391.83	194	162	152
419.21	323	294	281
446.58	463	482	467
473.96	662	722	710
501.33	916	1,001	994
528.70	1,211	1,291	1,293
556.08	1,418	1,560	1,571
583.45	1,605	1,773	1,793
610.83	1,972	1,901	1,928
638.20	2,011	1,928	1,956
665.57	2,018	1,852	1,876
692.95	1,766	1,684	1,701
720.32	1,491	1,450	1,458
747.70	1,212	1,180	1,179
775.07	894	906	898
802.44	630	654	642
829.82	467	442	429
857.19	241	278	266
884.57	138	162	152
911.94	65	86	79
939.32	42	41	37
966.69	18	18	16
994.06	2	7	6
1,021.44	3	2	2
1,048.81	0	1	0
1,076.19	0	0	0
1,103.56	0	0	0
1,130.93	0	0	0
1,158.31	0	0	0
1,185.68	0	0	0
1,213.06	0	0	0

Table 28

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Chi-Squared Test - [PW - 62-62-62-20000]					
Theoretical Alpha			11.10	Test	
Theoretical Beta			13.01	Statistic	Critical Value
Number Of Data Points			20000	263.32	37.65
alpha = 0.05					
Degrees of Freedom			25		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
6	3.57E-05	0.71	5.29	27.94	39.15
15	1.59E-04	3.17	11.83	139.86	44.07
25	5.77E-04	11.54	13.46	181.16	15.70
68	1.65E-03	33.04	34.96	1,222.45	37.00
124	3.93E-03	78.64	45.36	2,057.42	26.16
194	8.08E-03	161.51	32.49	1,055.82	6.54
323	1.47E-02	293.60	29.40	864.36	2.94
463	2.41E-02	481.60	-18.60	345.96	0.72
662	3.61E-02	722.20	-60.20	3,624.04	5.02
916	5.00E-02	1,000.60	-84.60	7,157.16	7.15
1,211	6.45E-02	1,290.80	-79.80	6,368.04	4.93
1,418	7.80E-02	1,559.60	-141.60	20,050.56	12.86
1,605	8.86E-02	1,772.80	-167.80	28,156.84	15.88
1,972	9.51E-02	1,901.40	70.60	4,984.36	2.62
2,011	9.64E-02	1,928.40	82.60	6,822.76	3.54
2,018	9.26E-02	1,851.80	166.20	27,622.44	14.92
1,766	8.42E-02	1,683.80	82.20	6,756.84	4.01
1,491	7.25E-02	1,449.60	41.40	1,713.96	1.18
1,212	5.90E-02	1,179.60	32.40	1,049.76	0.89
894	4.53E-02	905.60	-11.60	134.56	0.15
630	3.27E-02	653.80	-23.80	566.44	0.87
467	2.21E-02	441.80	25.20	635.04	1.44
241	1.39E-02	278.00	-37.00	1,369.00	4.92
138	8.09E-03	161.74	-23.74	563.62	3.48
65	4.31E-03	86.10	-21.10	445.40	5.17
42	2.07E-03	41.43	0.57	0.32	0.01
18	8.86E-04	17.73	0.27	0.08	0.00
5	4.65E-04	9.30	-4.30	18.49	1.99

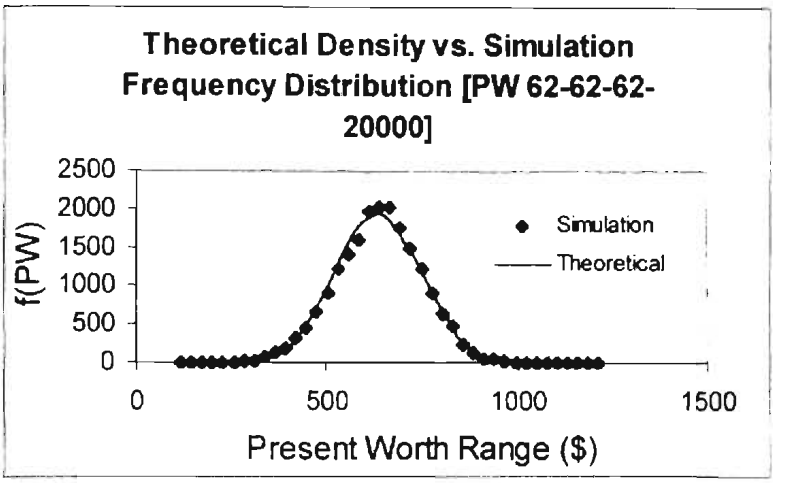

Figure 26. Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

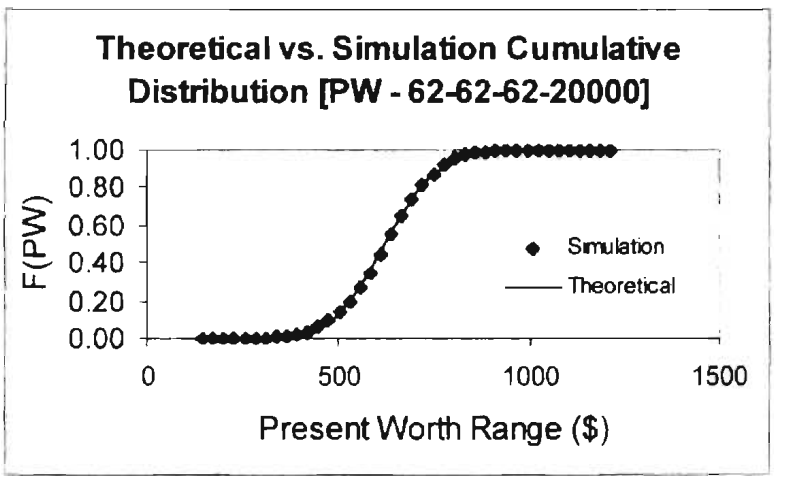

Figure 27. Graph Showing the Simulation versus Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Table 29

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Chi-Squared Test - [PW - 62-62-62-20000]					
Predicted Alpha			11.30	Test	
Predicted Beta			13.26	Statistic	Critical Value
Number Of Data Points			20000	317.78	37.65
alpha = 0.05					
Degrees of freedom			25		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
6	2.80E-05	0.56	5.44	29.59	52.83
15	1.32E-04	2.64	12.36	152.87	57.99
25	4.99E-04	9.97	15.03	225.79	22.64
68	1.47E-03	29.50	38.50	1,482.54	50.26
124	3.61E-03	72.13	51.87	2,690.21	37.30
194	7.58E-03	151.50	42.50	1,806.03	11.92
323	1.40E-02	280.60	42.40	1,797.76	6.41
463	2.34E-02	467.40	-4.40	19.36	0.04
662	3.55E-02	710.00	-48.00	2,304.00	3.25
916	4.97E-02	993.80	-77.80	6,052.84	6.09
1,211	6.46E-02	1,292.60	-81.60	6,658.56	5.15
1,418	7.86E-02	1,571.20	-153.20	23,470.24	14.94
1,605	8.97E-02	1,793.20	-188.20	35,419.24	19.75
1,972	9.64E-02	1,927.80	44.20	1,953.64	1.01
2,011	9.78E-02	1,956.40	54.60	2,981.16	1.52
2,018	9.38E-02	1,876.40	141.60	20,050.56	10.69
1,766	8.51E-02	1,701.40	64.60	4,173.16	2.45
1,491	7.29E-02	1,458.00	33.00	1,089.00	0.75
1,212	5.90E-02	1,179.00	33.00	1,089.00	0.92
894	4.49E-02	897.80	-3.80	14.44	0.02
630	3.21E-02	641.60	-11.60	134.56	0.21
467	2.14E-02	428.60	38.40	1,474.56	3.44
241	1.33E-02	265.80	-24.80	615.04	2.31
138	7.60E-03	152.07	-14.07	197.82	1.30
65	3.97E-03	79.40	-14.40	207.22	2.61
42	1.87E-03	37.35	4.65	21.59	0.58
18	7.78E-04	15.57	2.43	5.91	0.38
5	3.91E-04	7.82	-2.82	7.93	1.01

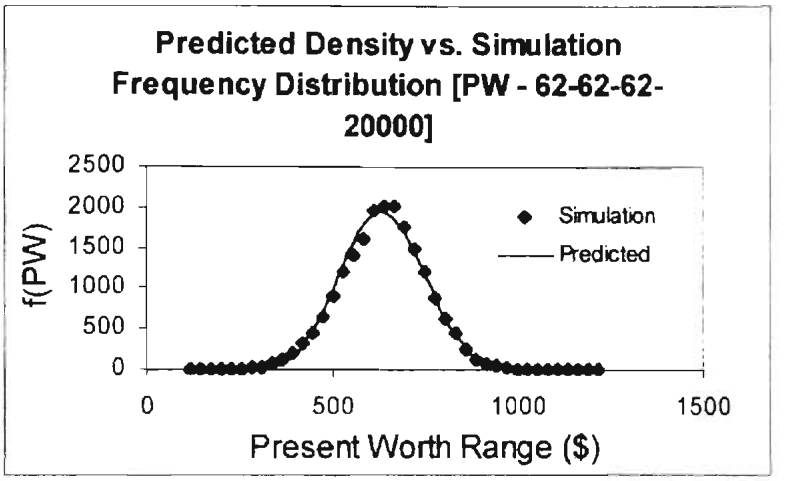

Figure 28. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

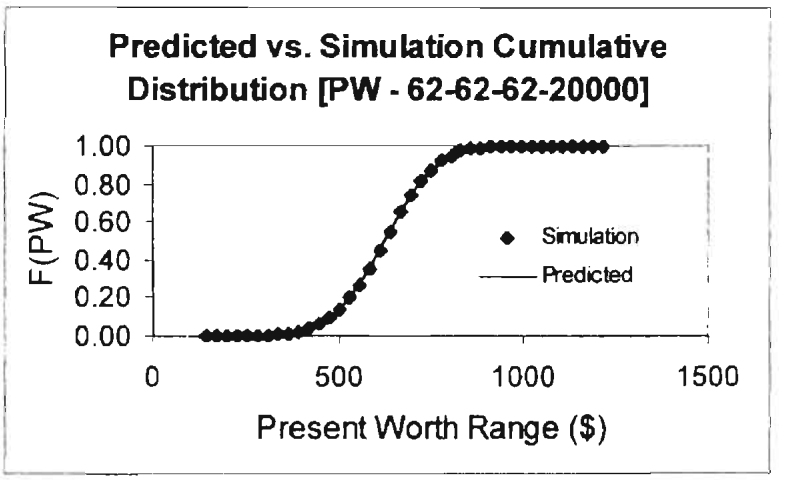

Figure 29. Graph Showing the Simulation versus Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Table 30

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
101.64	0	0	0
128.53	0	0	0
155.42	0	0	0
182.31	0	0	0
209.20	0	0	0
236.09	2	0	0
262.98	7	2	1
289.87	16	6	5
316.76	29	19	17
343.64	68	46	43
370.53	128	100	94
397.42	200	190	183
424.31	336	327	317
451.20	488	514	504
478.09	713	747	739
504.98	912	1,011	1,008
531.87	1,197	1,284	1,287
558.76	1,432	1,536	1,545
585.65	1,643	1,736	1,751
612.53	1,820	1,858	1,877
639.42	2,016	1,886	1,906
666.31	1,873	1,817	1,835
693.20	1,813	1,661	1,674
720.09	1,555	1,440	1,446
746.98	1,233	1,181	1,180
773.87	894	914	908
800.76	645	665	656
827.65	456	452	443
854.54	270	286	277
881.43	141	166	159
908.31	73	88	83
935.20	19	42	39
962.09	16	18	16
988.98	2	6	6
1,015.87	1	2	2
1,042.76	2	0	0
1,069.65	0	0	0
1,096.54	0	0	0
1,123.43	0	0	0
1,150.32	0	0	0
1,177.21	0	0	0

Table 31

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Chi-Squared Test - [PW - 62-62-55-20000]				Test Statistic	Critical Value
Theoretical Alpha			11.11		
Theoretical Beta			11.86		
Number Of Data Points			20000	156.15	36.42
alpha = 0.05					
Degrees of freedom			24		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
9	9.77E-05	1.95	7.05	49.64	25.40
16	3.09E-04	6.17	9.83	96.61	15.66
29	9.28E-04	18.55	10.45	109.17	5.88
68	2.32E-03	46.33	21.67	469.75	10.14
128	4.99E-03	99.74	28.26	798.50	8.01
200	9.50E-03	190.07	9.93	98.56	0.52
336	1.63E-02	326.80	9.20	84.64	0.26
488	2.57E-02	513.80	-25.80	665.64	1.30
713	3.73E-02	746.80	-33.80	1,142.44	1.53
912	5.06E-02	1,011.40	-99.40	9,880.36	9.77
1,197	6.42E-02	1,284.40	-87.40	7,638.76	5.95
1,432	7.68E-02	1,536.20	-104.20	10,857.64	7.07
1,643	8.68E-02	1,736.00	-93.00	8,649.00	4.98
1,820	9.29E-02	1,858.00	-38.00	1,444.00	0.78
2,016	9.43E-02	1,886.20	129.80	16,848.04	8.93
1,873	9.09E-02	1,817.40	55.60	3,091.36	1.70
1,813	8.31E-02	1,661.40	151.60	22,982.56	13.83
1,555	7.20E-02	1,439.80	115.20	13,271.04	9.22
1,233	5.90E-02	1,180.60	52.40	2,745.76	2.33
894	4.57E-02	913.60	-19.60	384.16	0.42
645	3.32E-02	664.60	-19.60	384.16	0.58
456	2.26E-02	452.20	3.80	14.44	0.03
270	1.43E-02	285.80	-15.80	249.64	0.87
141	8.32E-03	166.41	-25.41	645.68	3.88
73	4.41E-03	88.23	-15.23	231.91	2.63
19	2.10E-03	41.96	-22.96	526.99	12.56
19	1.31E-03	26.12	-7.12	50.72	1.94

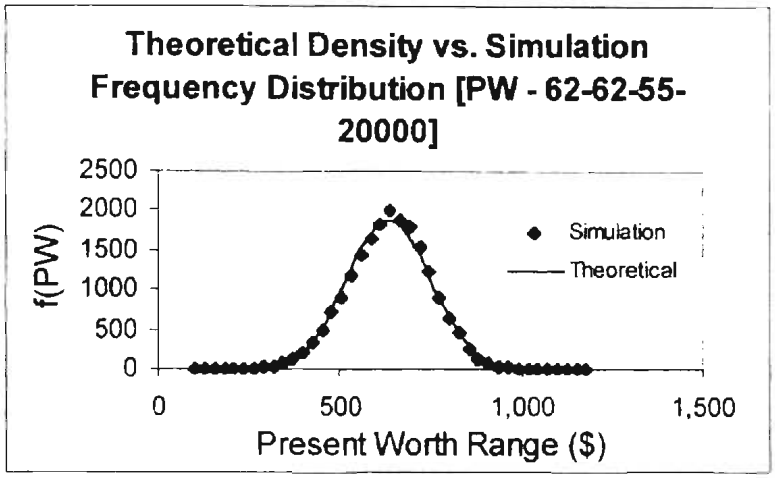

Figure 30:Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

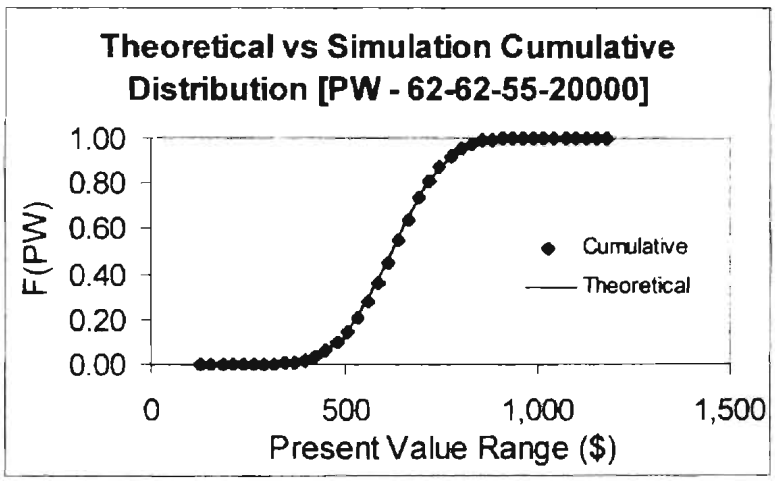

Figure 31:Graph Showing the Simulation versus Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Table 32

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Chi-Squared Test - [PW - 62-62-55-20000]				Test Statistic	Critical Value
Predicted Alpha			11.34		
Predicted Beta			12.10		
Number Of Data Points			20000	167.90	36.42
alpha = 0.05					
Degrees of freedom			24		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
9	8.39E-05	1.68	7.32	53.61	31.95
16	2.75E-04	5.49	10.51	110.37	20.09
29	8.46E-04	16.92	12.08	145.89	8.62
68	2.16E-03	43.12	24.89	619.26	14.36
128	4.72E-03	94.41	33.59	1,128.09	11.95
200	9.13E-03	182.50	17.50	306.25	1.68
336	1.59E-02	317.40	18.60	345.96	1.09
488	2.52E-02	504.20	-16.20	262.44	0.52
713	3.70E-02	739.00	-26.00	676.00	0.91
912	5.04E-02	1,007.80	-95.80	9,177.64	9.11
1,197	6.43E-02	1,286.60	-89.60	8,028.16	6.24
1,432	7.72E-02	1,544.80	-112.80	12,723.84	8.24
1,643	8.75E-02	1,750.60	-107.60	11,577.76	6.61
1,820	9.38E-02	1,876.60	-56.60	3,203.56	1.71
2,016	9.53E-02	1,905.60	110.40	12,188.16	6.40
1,873	9.17E-02	1,834.60	38.40	1,474.56	0.80
1,813	8.37E-02	1,673.80	139.20	19,376.64	11.58
1,555	7.23E-02	1,445.80	109.20	11,924.64	8.25
1,233	5.90E-02	1,180.40	52.60	2,766.76	2.34
894	4.54E-02	908.20	-14.20	201.64	0.22
645	3.28E-02	656.00	-11.00	121.00	0.18
456	2.21E-02	442.60	13.40	179.56	0.41
270	1.38E-02	276.80	-6.80	46.24	0.17
141	7.97E-03	159.30	-18.30	334.97	2.10
73	4.16E-03	83.28	-10.28	105.72	1.27
19	1.95E-03	38.96	-19.96	398.35	10.22
19	1.18E-03	23.55	-4.55	20.67	0.88

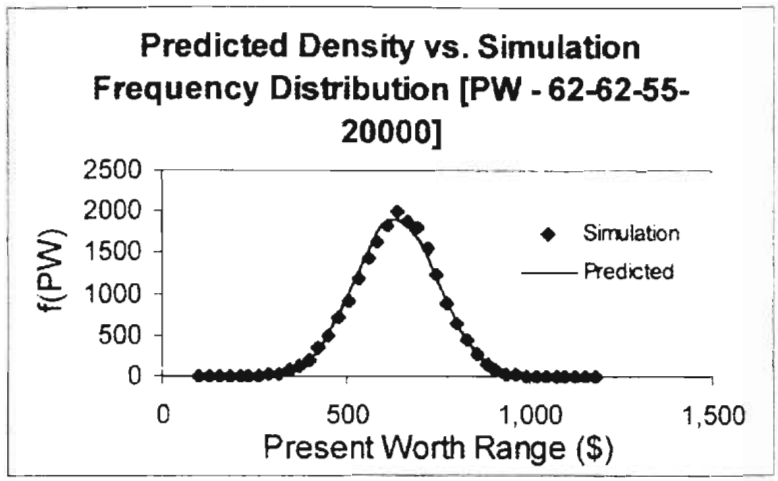

Figure 32:Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

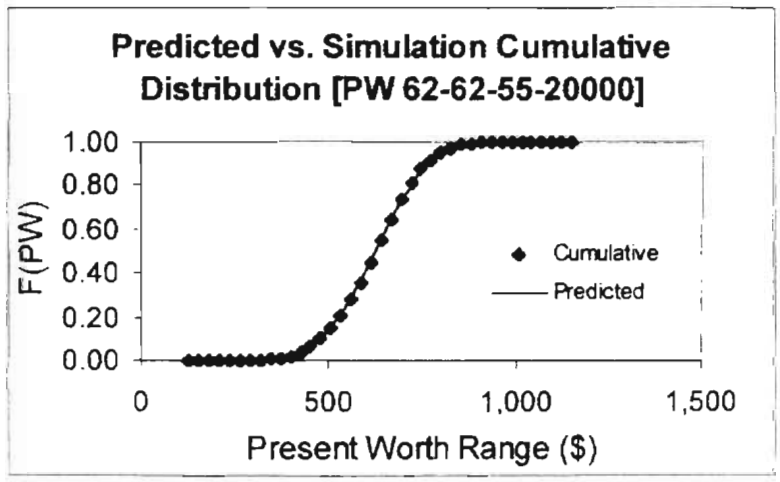

Figure 33:Graph Showing the Simulation versus Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Table 33

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
87.48	0	0	0
113.86	0	0	0
140.23	0	0	0
166.60	0	0	0
192.98	0	0	0
219.35	1	0	0
245.72	4	1	1
272.10	4	3	3
298.47	21	10	9
324.84	44	25	24
351.22	82	58	56
377.59	129	117	114
403.96	257	213	208
430.34	375	352	347
456.71	485	539	534
483.08	723	768	764
509.46	943	1,026	1,024
535.83	1,188	1,290	1,291
562.20	1,506	1,532	1,538
588.58	1,601	1,725	1,734
614.95	1,818	1,842	1,854
641.32	1,871	1,869	1,881
667.70	1,887	1,802	1,812
694.07	1,780	1,648	1,656
720.44	1,516	1,429	1,433
746.82	1,360	1,172	1,171
773.19	932	905	902
799.56	637	656	651
825.94	421	443	437
852.31	224	277	271
878.68	107	158	154
905.05	48	82	79
931.43	23	37	36
957.80	9	15	14
984.17	4	5	4
1,010.55	0	1	1
1,036.92	0	0	0
1,063.29	0	0	0
1,089.67	0	0	0
1,116.04	0	0	0
1,142.41	0	0	0

Table 34

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Chi-Squared Test - [PW - 62-62-26-20000]				Test Statistic	Critical Value
Theoretical Alpha			11.44		
Theoretical Beta			11.15		
Number Of Data Points			20000	169.02	36.42
alpha = 0.05					
Degrees of freedom			24		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
9	1.92E-04	3.84	5.16	26.58	6.92
21	4.79E-04	9.58	11.42	130.35	13.60
44	1.27E-03	25.47	18.53	343.41	13.48
82	2.91E-03	58.15	23.85	568.63	9.78
129	5.86E-03	117.21	11.79	139.01	1.19
257	1.06E-02	212.60	44.40	1,971.36	9.27
375	1.76E-02	352.40	22.60	510.76	1.45
485	2.70E-02	539.20	-54.20	2,937.64	5.45
723	3.84E-02	768.20	-45.20	2,043.04	2.66
943	5.13E-02	1,025.80	-82.80	6,855.84	6.68
1,188	6.45E-02	1,289.60	-101.60	10,322.56	8.00
1,506	7.66E-02	1,532.20	-26.20	686.44	0.45
1,601	8.62E-02	1,724.60	-123.60	15,276.96	8.86
1,818	9.21E-02	1,842.40	-24.40	595.36	0.32
1,871	9.35E-02	1,869.40	1.60	2.56	0.00
1,887	9.01E-02	1,801.60	85.40	7,293.16	4.05
1,780	8.24E-02	1,648.20	131.80	17,371.24	10.54
1,516	7.15E-02	1,429.20	86.80	7,534.24	5.27
1,360	5.86E-02	1,171.60	188.40	35,494.56	30.30
932	4.53E-02	905.20	26.80	718.24	0.79
637	3.28E-02	655.80	-18.80	353.44	0.54
421	2.22E-02	443.00	-22.00	484.00	1.09
224	1.38E-02	276.80	-52.80	2,787.84	10.07
107	7.91E-03	158.16	-51.16	2,616.96	16.55
48	4.08E-03	81.55	-33.55	1,125.35	13.80
23	1.86E-03	37.22	-14.22	202.11	5.43
13	1.05E-03	20.91	-7.91	62.59	2.99

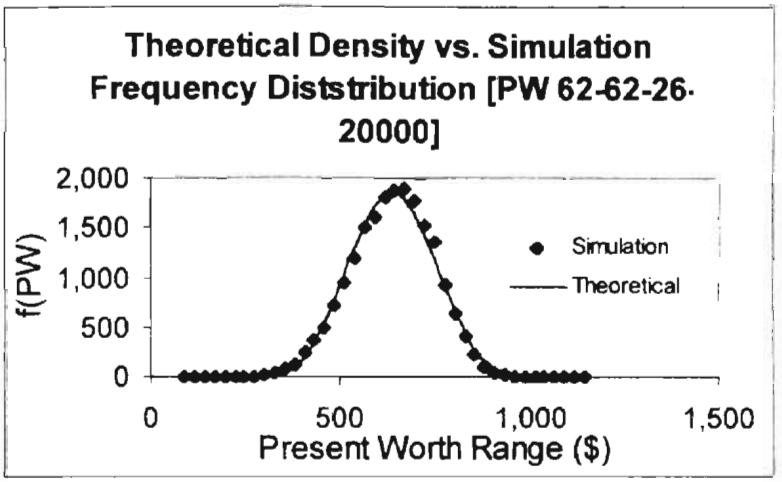

Figure 34: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

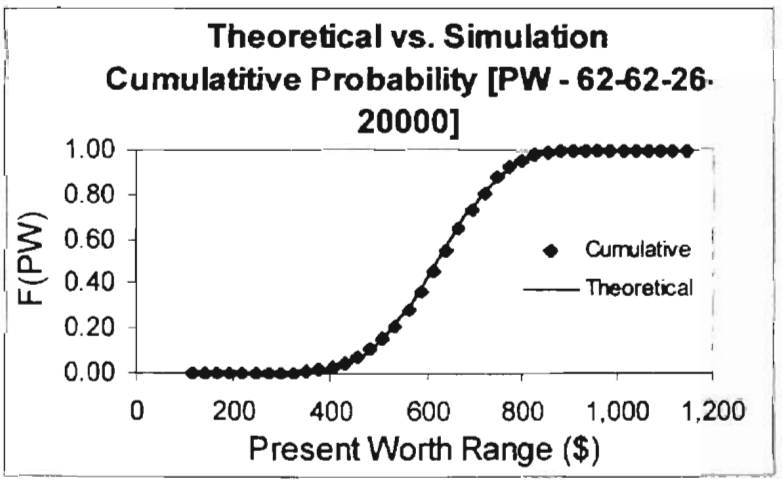

Figure 35: Graph Showing the Simulation versus Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Table 35

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Chi-Squared Test - [PW - 62-62-26-20000]				Test Statistic	Critical Value
Predicted Alpha			11.58		
Predicted Beta			11.29		
Number Of Data Points			20000	167.90	36.42
alpha = 0.05					
Degrees of freedom			24		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
9	1.77E-04	2.75	6.25	39.01	14.16
21	4.50E-04	8.99	12.01	144.19	16.03
44	1.21E-03	24.21	19.79	391.64	16.18
82	2.79E-03	55.89	26.11	681.85	12.20
129	5.68E-03	113.69	15.31	234.29	2.06
257	1.04E-02	208.00	49.00	2,401.00	11.54
375	1.73E-02	346.80	28.20	795.24	2.29
485	2.67E-02	533.80	-48.80	2,381.44	4.46
723	3.82E-02	764.00	-41.00	1,681.00	2.20
943	5.12E-02	1,024.00	-81.00	6,561.00	6.41
1,188	6.46E-02	1,291.40	-103.40	10,691.56	8.28
1,506	7.69E-02	1,537.80	-31.80	1,011.24	0.66
1,601	8.67E-02	1,733.60	-132.60	17,582.76	10.14
1,818	9.27E-02	1,853.60	-35.60	1,267.36	0.68
1,871	9.41E-02	1,881.00	-10.00	100.00	0.05
1,887	9.06E-02	1,811.80	75.20	5,655.04	3.12
1,780	8.28E-02	1,655.60	124.40	15,475.36	9.35
1,516	7.16E-02	1,432.60	83.40	6,955.56	4.86
1,360	5.86E-02	1,171.40	188.60	35,569.96	30.37
932	4.51E-02	901.80	30.20	912.04	1.01
637	3.25E-02	650.60	-13.60	184.96	0.28
421	2.19E-02	437.20	-16.20	262.44	0.60
224	1.36E-02	271.40	-47.40	2,246.76	8.28
107	7.70E-03	153.91	-46.91	2,200.74	14.30
48	3.93E-03	78.66	-30.66	939.83	11.95
23	1.78E-03	35.52	-12.52	156.83	4.41
13	9.80E-04	19.60	-6.60	43.50	2.22

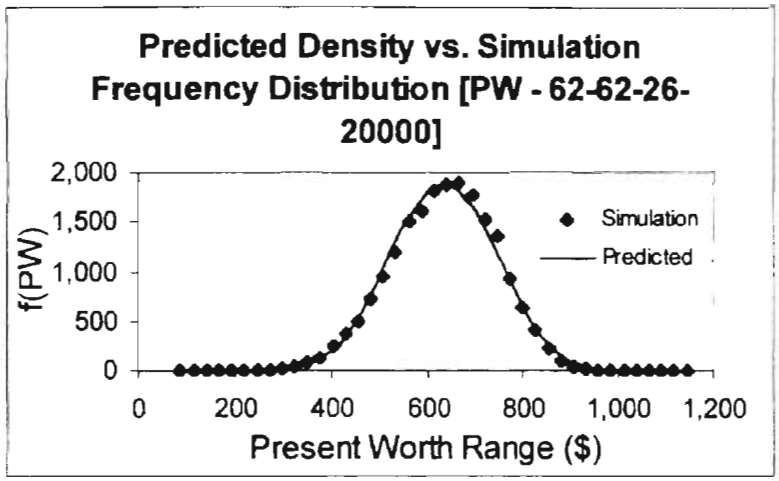

Figure 36: Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

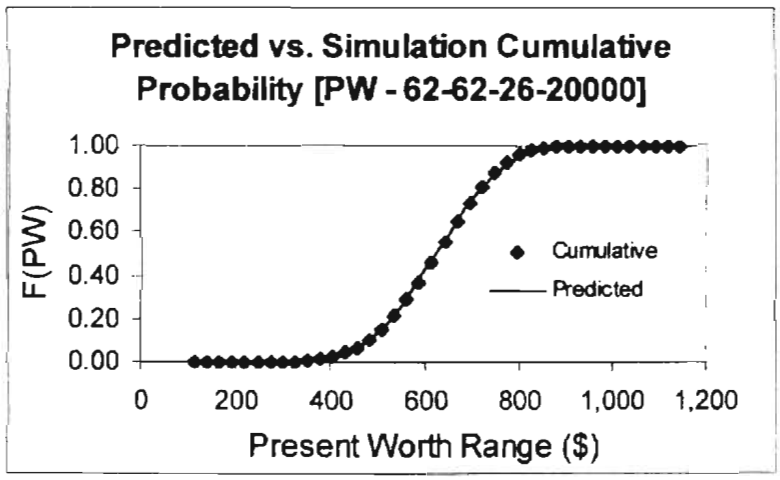

Figure 37: Graph Showing the Simulation versus Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Table 36

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
101.64	0	0	0
128.53	0	0	0
155.42	0	0	0
182.31	0	0	0
209.20	0	0	0
236.09	4	0	0
262.98	9	2	1
289.87	14	6	5
316.76	35	19	17
343.64	75	46	43
370.53	131	100	94
397.42	216	190	183
424.31	372	327	317
451.20	497	514	504
478.09	729	747	739
504.98	929	1,011	1,008
531.87	1,142	1,284	1,287
558.76	1,479	1,536	1,545
585.65	1,684	1,736	1,751
612.53	1,916	1,858	1,877
639.42	1,913	1,886	1,906
666.31	1,832	1,817	1,835
693.20	1,736	1,661	1,674
720.09	1,526	1,440	1,446
746.98	1,308	1,181	1,180
773.87	870	914	908
800.76	668	665	656
827.65	415	452	443
854.54	239	286	277
881.43	137	166	159
908.31	72	88	83
935.20	36	42	39
962.09	8	18	16
988.98	6	6	6
1,015.87	2	2	2
1,042.76	0	0	0
1,069.65	0	0	0
1,096.54	0	0	0
1,123.43	0	0	0
1,150.32	0	0	0
1,177.21	0	0	0

Table 37

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Chi-Squared Test - [PW - 62-55-62-20000]				Test Statistic	Critical Value
Theoretical Alpha			11.11		
Theoretical Beta			11.86		
Number Of Data Points			20000	203.07	37.65
alpha = 0.05					
Degrees of freedom			25		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
13	9.77E-05	1.95	11.05	122.00	62.42
14	3.09E-04	6.17	7.83	61.30	9.93
35	9.28E-04	18.55	16.45	270.55	14.58
75	2.32E-03	46.33	28.67	822.19	17.75
131	4.99E-03	99.74	31.26	977.05	9.80
216	9.50E-03	190.07	25.93	672.25	3.54
372	1.63E-02	326.80	45.20	2,043.04	6.25
497	2.57E-02	513.80	-16.80	282.24	0.55
729	3.73E-02	746.80	-17.80	316.84	0.42
929	5.06E-02	1,011.40	-82.40	6,789.76	6.71
1,142	6.42E-02	1,284.40	-142.40	20,277.76	15.79
1,479	7.68E-02	1,536.20	-57.20	3,271.84	2.13
1,684	8.68E-02	1,736.00	-52.00	2,704.00	1.56
1,916	9.29E-02	1,858.00	58.00	3,364.00	1.81
1,913	9.43E-02	1,886.20	26.80	718.24	0.38
1,832	9.09E-02	1,817.40	14.60	213.16	0.12
1,736	8.31E-02	1,661.40	74.60	5,565.16	3.35
1,526	7.20E-02	1,439.80	86.20	7,430.44	5.16
1,308	5.90E-02	1,180.60	127.40	16,230.76	13.75
870	4.57E-02	913.60	-43.60	1,900.96	2.08
668	3.32E-02	664.60	3.40	11.56	0.02
415	2.26E-02	452.20	-37.20	1,383.84	3.06
239	1.43E-02	285.80	-46.80	2,190.24	7.66
137	8.32E-03	166.41	-29.41	864.96	5.20
72	4.41E-03	88.23	-16.23	263.37	2.99
36	2.10E-03	41.96	-5.96	35.48	0.85
8	8.77E-04	17.53	-9.53	90.90	5.18
8	4.29E-04	8.59	-0.59	0.34	0.04

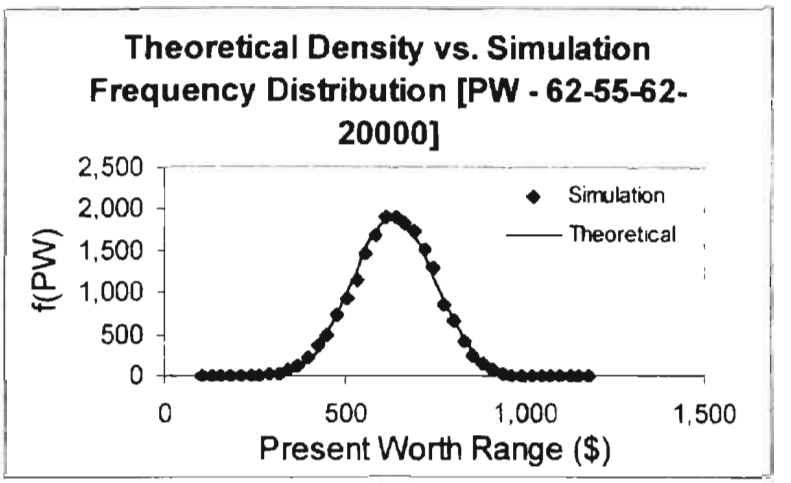

Figure 38: Graph Showing the Simulation Frequency Values versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

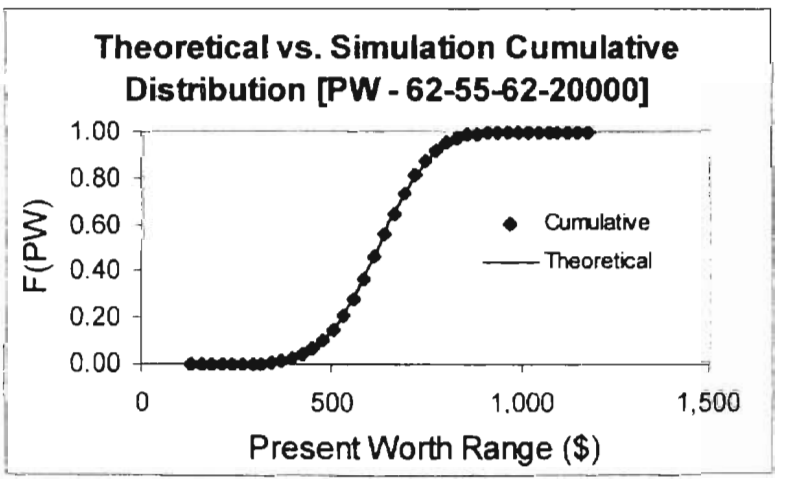

Figure 39: Graph Showing the Simulation versus Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Table 38

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Chi-Squared Test - [PW - 62-55-62-20000]				Test Statistic	Critical Value
Predicted Alpha			11.34		
Predicted Beta			12.10		
Number Of Data Points			20000	229.16	37.65
alpha = 0.05					
Degrees of freedom			25		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
13	8.39E-05	1.68	11.32	128.19	76.39
14	2.75E-04	5.49	8.51	72.35	13.17
35	8.46E-04	16.92	18.08	326.83	19.31
75	2.16E-03	43.12	31.89	1,016.65	23.58
131	4.72E-03	94.41	36.59	1,338.61	14.18
216	9.13E-03	182.50	33.50	1,122.25	6.15
372	1.59E-02	317.40	54.60	2,981.16	9.39
497	2.52E-02	504.20	-7.20	51.84	0.10
729	3.70E-02	739.00	-10.00	100.00	0.14
929	5.04E-02	1,007.80	-78.80	6,209.44	6.16
1,142	6.43E-02	1,286.60	-144.60	20,909.16	16.25
1,479	7.72E-02	1,544.80	-65.80	4,329.64	2.80
1,684	8.75E-02	1,750.60	-66.60	4,435.56	2.53
1,916	9.38E-02	1,876.60	39.40	1,552.36	0.83
1,913	9.53E-02	1,905.60	7.40	54.76	0.03
1,832	9.17E-02	1,834.60	-2.60	6.76	0.00
1,736	8.37E-02	1,673.80	62.20	3,868.84	2.31
1,526	7.23E-02	1,445.80	80.20	6,432.04	4.45
1,308	5.90E-02	1,180.40	127.60	16,281.76	13.79
870	4.54E-02	908.20	-38.20	1,459.24	1.61
668	3.28E-02	656.00	12.00	144.00	0.22
415	2.21E-02	442.60	-27.60	761.76	1.72
239	1.38E-02	276.80	-37.80	1,428.84	5.16
137	7.97E-03	159.30	-22.30	497.39	3.12
72	4.16E-03	83.28	-11.28	127.29	1.53
36	1.95E-03	38.96	-2.96	8.75	0.22
8	7.99E-04	15.97	-7.97	63.54	3.98
8	3.79E-04	7.58	0.42	0.18	0.02

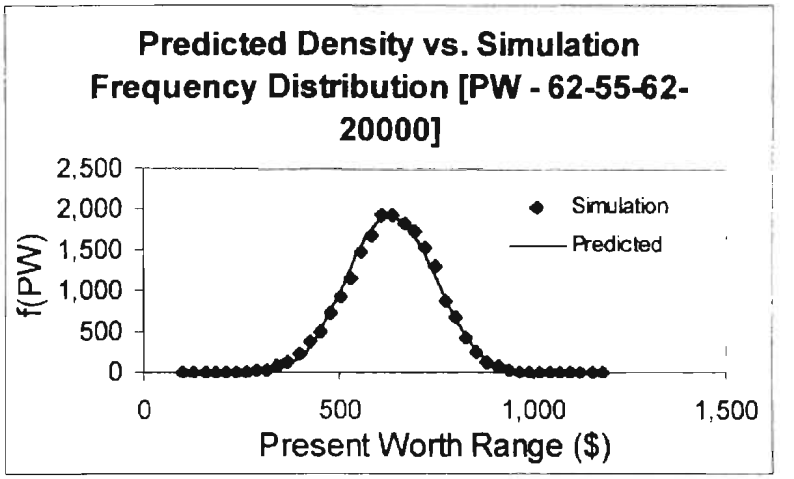

Figure 40: Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

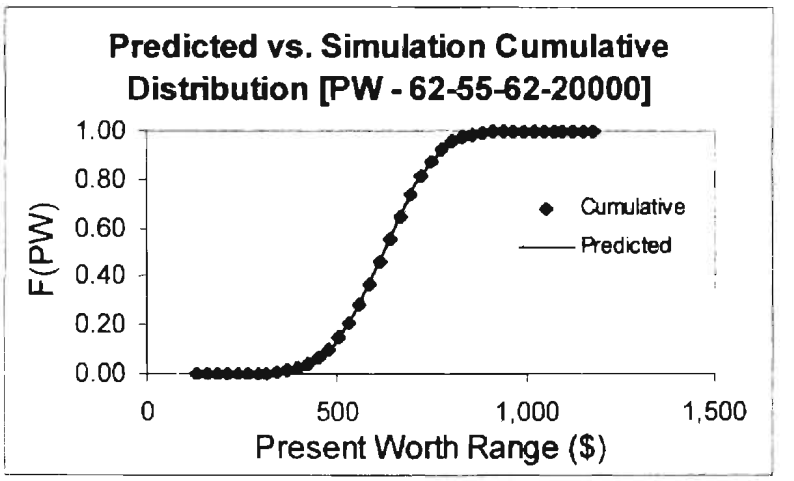

Figure 41: Graph Showing the Simulation Frequency versus Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Table 39

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
84.90	0	0	0
110.49	0	0	0
136.09	0	0	0
161.68	0	0	0
187.27	0	0	0
212.87	1	0	0
238.46	4	1	1
264.06	5	3	3
289.65	17	9	8
315.24	37	22	21
340.84	78	49	47
366.43	139	97	94
392.02	202	175	170
417.62	315	288	283
443.21	455	441	435
468.81	549	632	627
494.40	785	853	850
519.99	1,041	1,090	1,090
545.59	1,261	1,323	1,326
571.18	1,468	1,528	1,535
596.77	1,639	1,684	1,694
622.37	1,769	1,771	1,782
647.96	1,781	1,779	1,790
673.56	1,780	1,705	1,715
699.15	1,610	1,558	1,564
724.74	1,430	1,354	1,357
750.34	1,176	1,117	1,116
775.93	934	870	867
801.53	635	638	632
827.12	390	436	430
852.71	261	276	271
878.31	132	160	155
903.90	72	83	80
929.49	21	38	36
955.09	8	15	14
980.68	4	5	4
1,006.28	1	1	1
1,031.87	0	0	0
1,057.46	0	0	0
1,083.06	0	0	0
1,108.65	0	0	0

Table 40

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Chi-Squared Test - [PW - 62-55-55-20000]					
Theoretical Alpha			10.78	Test	Critical
Theoretical Beta			9.77	Statistic	Value
Number Of Data Points			20000	144.93	40.11
alpha = 0.05					
Degrees of freedom			27		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
5	4.46E-05	0.89	4.11	16.87	18.89
5	1.42E-04	2.84	2.16	4.68	1.65
17	4.34E-04	8.67	8.33	69.38	8.00
37	1.11E-03	22.18	14.82	219.52	9.90
78	2.46E-03	49.26	28.74	826.01	16.77
139	4.87E-03	97.38	41.62	1,732.27	17.79
202	8.73E-03	174.58	27.42	752.10	4.31
315	1.44E-02	287.80	27.20	739.84	2.57
455	2.20E-02	440.60	14.40	207.36	0.47
549	3.16E-02	631.60	-82.60	6,822.76	10.80
785	4.26E-02	852.80	-67.80	4,596.84	5.39
1,041	5.45E-02	1,090.00	-49.00	2,401.00	2.20
1,261	6.61E-02	1,322.80	-61.80	3,819.24	2.89
1,468	7.64E-02	1,528.40	-60.40	3,648.16	2.39
1,639	8.42E-02	1,684.20	-45.20	2,043.04	1.21
1,769	8.86E-02	1,771.40	-2.40	5.76	0.00
1,781	8.90E-02	1,779.00	2.00	4.00	0.00
1,780	8.53E-02	1,705.20	74.80	5,595.04	3.28
1,610	7.79E-02	1,558.00	52.00	2,704.00	1.74
1,430	6.77E-02	1,354.40	75.60	5,715.36	4.22
1,176	5.59E-02	1,117.00	59.00	3,481.00	3.12
934	4.35E-02	870.40	63.60	4,044.96	4.65
635	3.19E-02	637.80	-2.80	7.84	0.01
390	2.18E-02	436.20	-46.20	2,134.44	4.89
261	1.38E-02	276.00	-15.00	225.00	0.82
132	7.98E-03	159.61	-27.61	762.46	4.78
72	4.15E-03	82.97	-10.97	120.37	1.45
21	1.89E-03	37.90	-16.90	285.46	7.53
8	7.36E-04	14.72	-6.72	45.14	3.07
5	2.95E-04	5.91	-0.91	0.83	0.14

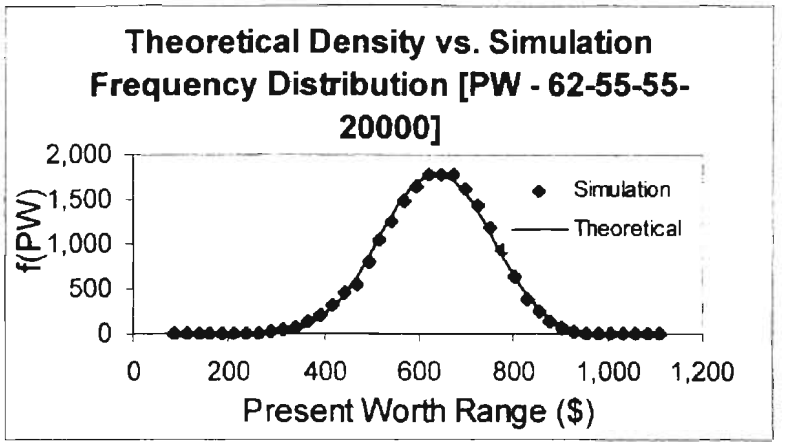

Figure 42. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

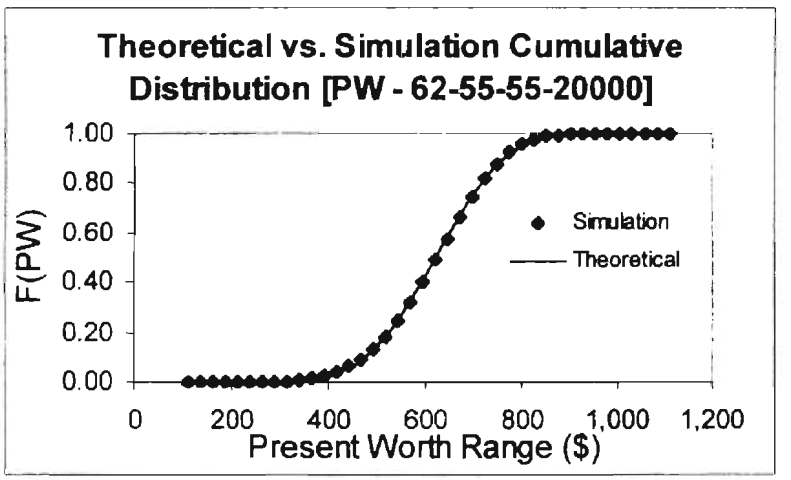

Figure 43. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Table 41

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Chi-Squared Test - [PW - 62-55-55-20000]					
Predicted Alpha			10.92	Test Statistic	Critical Value
Predicted Beta			9.89		
Number Of Data Points			20000	155.62	40.11
alpha = .05					
Degrees of freedom			27		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
5	4.03E-05	0.81	4.19	17.58	21.79
5	1.31E-04	2.62	2.38	5.66	2.16
17	4.07E-04	8.13	8.87	78.67	9.68
37	1.05E-03	21.06	15.94	253.99	12.06
78	2.36E-03	47.27	30.73	944.33	19.98
139	4.71E-03	94.30	44.70	1,998.25	21.19
202	8.52E-03	170.37	31.63	1,000.37	5.87
315	1.41E-02	282.60	32.40	1,049.76	3.71
455	2.18E-02	435.20	19.80	392.04	0.90
549	3.14E-02	627.00	-78.00	6,084.00	9.70
785	4.25E-02	850.00	-65.00	4,225.00	4.97
1,041	5.45E-02	1,089.80	-48.80	2,381.44	2.19
1,261	6.63E-02	1,326.00	-65.00	4,225.00	3.19
1,468	7.68E-02	1,535.00	-67.00	4,489.00	2.92
1,639	8.47E-02	1,693.60	-54.60	2,981.16	1.76
1,769	8.91E-02	1,782.40	-13.40	179.56	0.10
1,781	8.95E-02	1,790.00	-9.00	81.00	0.05
1,780	8.57E-02	1,714.60	65.40	4,277.16	2.49
1,610	7.82E-02	1,564.40	45.60	2,079.36	1.33
1,430	6.79E-02	1,357.20	72.80	5,299.84	3.90
1,176	5.58E-02	1,116.40	59.60	3,552.16	3.18
934	4.34E-02	867.00	67.00	4,489.00	5.18
635	3.16E-02	632.40	2.60	6.76	0.01
390	2.15E-02	430.40	-40.40	1,632.16	3.79
261	1.35E-02	270.60	-9.60	92.16	0.34
132	7.77E-03	155.31	-23.31	543.21	3.50
72	4.00E-03	80.01	-8.01	64.23	0.80
21	1.81E-03	36.16	-15.16	229.75	6.35
8	6.93E-04	13.86	-5.86	34.40	2.48
5	2.73E-04	5.45	-0.45	0.21	0.04

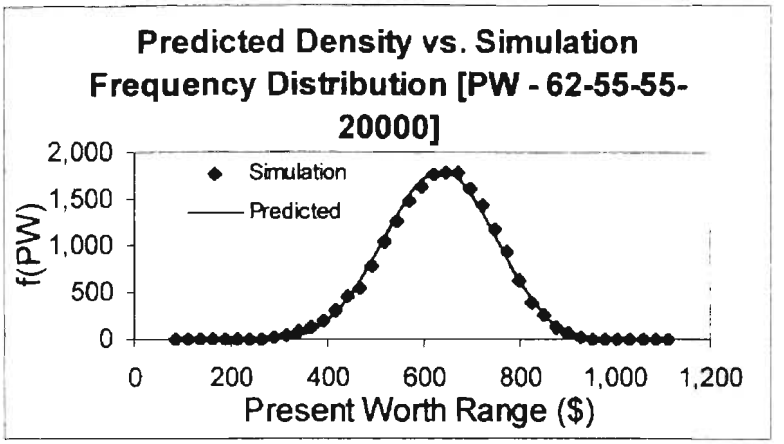

Figure 44. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

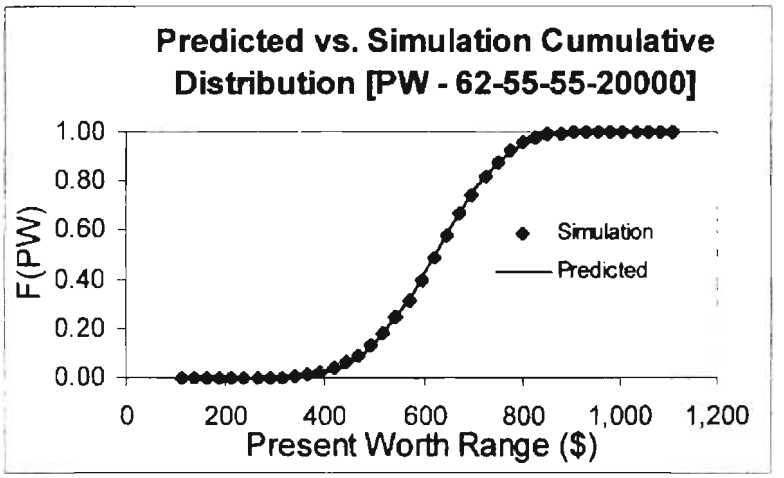

Figure 45. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Table 42

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
70.91	0	0	0
95.24	0	0	0
119.57	0	0	0
143.90	0	0	0
168.23	0	0	0
192.56	0	0	0
216.89	2	0	0
241.22	3	1	1
265.55	3	4	4
289.88	15	10	10
314.21	41	24	23
338.54	59	49	48
362.87	88	90	89
387.20	153	155	154
411.52	237	248	246
435.85	407	373	371
460.18	491	530	528
484.51	703	715	714
508.84	874	921	920
533.17	1,135	1,133	1,133
557.50	1,231	1,335	1,337
581.83	1,501	1,510	1,512
606.16	1,644	1,638	1,641
630.49	1,695	1,705	1,709
654.82	1,733	1,702	1,706
679.15	1,703	1,628	1,631
703.48	1,491	1,488	1,490
727.81	1,414	1,296	1,297
752.14	1,100	1,071	1,071
776.46	835	836	834
800.79	610	611	609
825.12	370	415	413
849.45	238	258	256
873.78	139	144	142
898.11	59	70	69
922.44	20	29	28
946.77	5	9	9
971.10	1	2	2
995.43	0	0	0
1,019.76	0	0	0
1,044.09	0	0	0

Table 43

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Chi-Squared Test - [PW - 62-55-26-20000]					
Theoretical Alpha			10.53	Test Statistic	Critical Value
Theoretical Beta			8.07		
Number Of Data Points			20000	64.35	38.89
alpha = 0.05					
Degrees of freedom			26		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
8	2.85E-04	5.69	2.31	5.33	0.94
15	5.23E-04	10.46	4.54	20.59	1.97
41	1.19E-03	23.87	17.14	293.61	12.30
59	2.43E-03	48.68	10.32	106.44	2.19
88	4.52E-03	90.44	-2.44	5.96	0.07
153	7.75E-03	155.09	-2.09	4.36	0.03
237	1.24E-02	248.00	-11.00	121.00	0.49
407	1.86E-02	372.80	34.20	1,169.64	3.14
491	2.65E-02	529.60	-38.60	1,489.96	2.81
703	3.58E-02	715.20	-12.20	148.84	0.21
874	4.60E-02	920.60	-46.60	2,171.56	2.36
1,135	5.66E-02	1,132.80	2.20	4.84	0.00
1,231	6.68E-02	1,335.40	-104.40	10,899.36	8.16
1,501	7.55E-02	1,509.80	-8.80	77.44	0.05
1,644	8.19E-02	1,637.80	6.20	38.44	0.02
1,695	8.53E-02	1,705.20	-10.20	104.04	0.06
1,733	8.51E-02	1,702.40	30.60	936.36	0.55
1,703	8.14E-02	1,627.80	75.20	5,655.04	3.47
1,491	7.44E-02	1,487.80	3.20	10.24	0.01
1,414	6.48E-02	1,295.80	118.20	13,971.24	10.78
1,100	5.36E-02	1,071.00	29.00	841.00	0.79
835	4.18E-02	835.60	-0.60	0.36	0.00
610	3.06E-02	611.00	-1.00	1.00	0.00
370	2.07E-02	414.60	-44.60	1,989.16	4.80
238	1.29E-02	257.60	-19.60	384.16	1.49
139	7.19E-03	143.88	-4.88	23.78	0.17
59	3.52E-03	70.35	-11.35	128.76	1.83
20	1.45E-03	28.94	-8.94	79.86	2.76
6	5.94E-04	11.88	-5.88	34.56	2.91

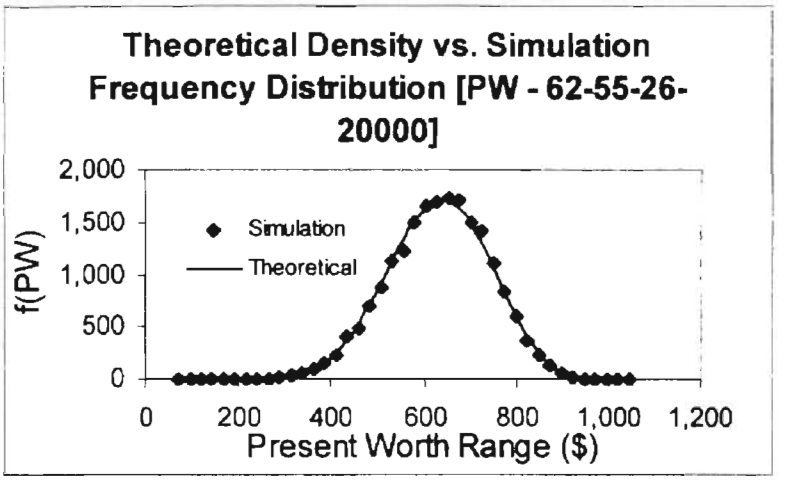

Figure 46. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

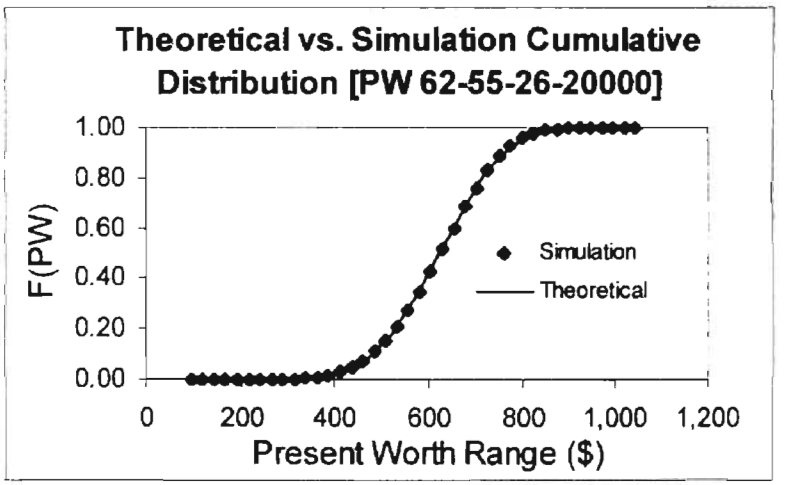

Figure 47. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Table 44

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Chi-Squared Test - [PW - 62-55-26-20000]				Test Statistic	Critical Value
Predicted Alpha			10.58		
Predicted Beta			8.10		
Number Of Data Points			20000	64.00	38.89
alpha = 0.05					
Degrees of freedom			26		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
8	2.77E-04	5.54	2.46	6.06	1.09
15	5.12E-04	10.24	4.76	22.61	2.21
41	1.17E-03	23.46	17.54	307.78	13.12
59	2.40E-03	48.01	10.99	120.86	2.52
88	4.47E-03	89.44	-1.44	2.06	0.02
153	7.69E-03	153.74	-0.74	0.55	0.00
237	1.23E-02	246.40	-9.40	88.36	0.36
407	1.86E-02	371.00	36.00	1,296.00	3.49
491	2.64E-02	528.00	-37.00	1,369.00	2.59
703	3.57E-02	713.80	-10.80	116.64	0.16
874	4.60E-02	920.00	-46.00	2,116.00	2.30
1,135	5.67E-02	1,133.20	1.80	3.24	0.00
1,231	6.68E-02	1,336.80	-105.80	11,193.64	8.37
1,501	7.56E-02	1,512.20	-11.20	125.44	0.08
1,644	8.21E-02	1,641.20	2.80	7.84	0.00
1,695	8.54E-02	1,708.80	-13.80	190.44	0.11
1,733	8.53E-02	1,706.00	27.00	729.00	0.43
1,703	8.15E-02	1,630.80	72.20	5,212.84	3.20
1,491	7.45E-02	1,489.80	1.20	1.44	0.00
1,414	6.48E-02	1,296.60	117.40	13,782.76	10.63
1,100	5.35E-02	1,070.80	29.20	852.64	0.80
835	4.17E-02	834.40	0.60	0.36	0.00
610	3.05E-02	609.20	0.80	0.64	0.00
370	2.06E-02	412.60	-42.60	1,814.76	4.40
238	1.28E-02	255.80	-17.80	316.84	1.24
139	7.12E-03	142.48	-3.48	12.12	0.09
59	3.47E-03	69.43	-10.43	108.82	1.57
20	1.42E-03	28.44	-8.44	71.26	2.51
6	5.80E-04	11.60	-5.60	31.32	2.70

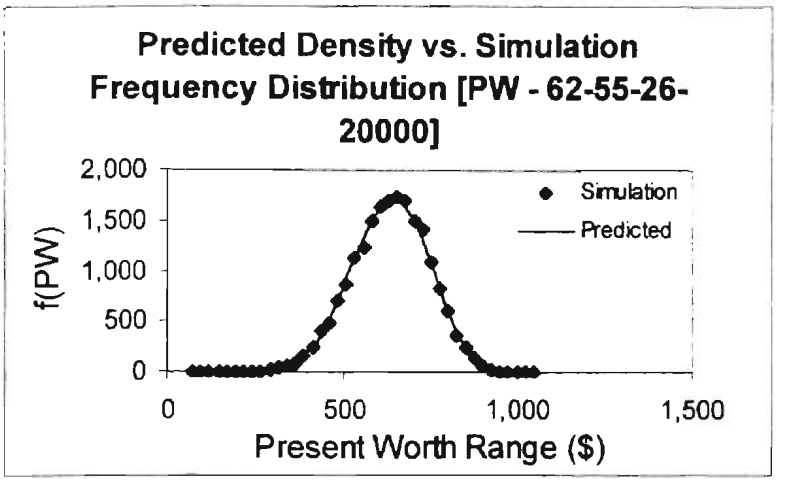

Figure 48. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

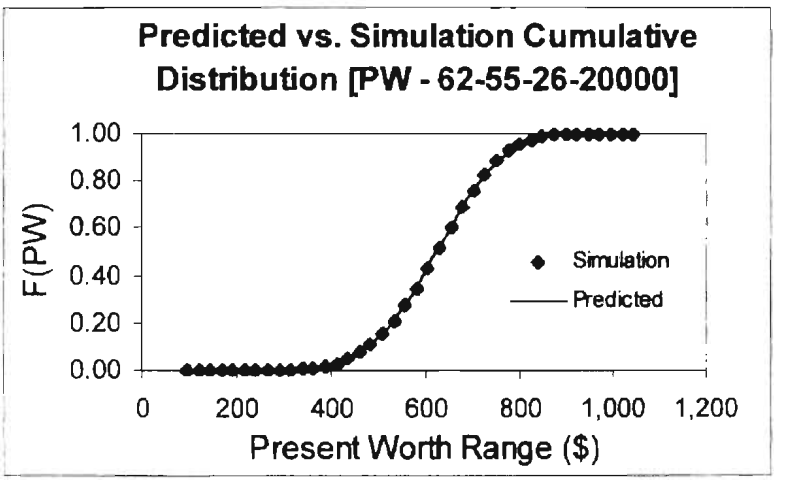

Figure 49. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Table 45

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
87.48	0	0	0
113.86	0	0	0
140.23	0	0	0
166.60	0	0	0
192.98	1	0	0
219.35	0	0	0
245.72	3	1	1
272.10	6	3	3
298.47	23	10	9
324.84	43	25	24
351.22	84	58	56
377.59	145	117	114
403.96	219	213	208
430.34	385	352	347
456.71	499	539	534
483.08	709	768	764
509.46	897	1,026	1,024
535.83	1,140	1,290	1,291
562.20	1,475	1,532	1,538
588.58	1,648	1,725	1,734
614.95	1,842	1,842	1,854
641.32	1,872	1,869	1,881
667.70	1,876	1,802	1,812
694.07	1,821	1,648	1,656
720.44	1,535	1,429	1,433
746.82	1,298	1,172	1,171
773.19	956	905	902
799.56	656	656	651
825.94	401	443	437
852.31	235	277	271
878.68	123	158	154
905.05	62	82	79
931.43	30	37	36
957.80	12	15	14
984.17	4	5	4
1,010.55	0	1	1
1,036.92	0	0	0
1,063.29	0	0	0
1,089.67	0	0	0
1,116.04	0	0	0
1,142.41	0	0	0

Table 46

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Chi-Squared Test - [PW - 62-26-62-20000]				Test Statistic	Critical Value
Theoretical Alpha			11.44		
Theoretical Beta			11.15		
Number Of Data Points			20000	179.48	36.42
alpha = 0.05					
Degrees of freedom			24		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
10.00	1.92E-04	3.84	6.16	37.89	9.86
23.00	4.79E-04	9.58	13.42	180.02	18.79
43.00	1.27E-03	25.47	17.53	307.35	12.07
84.00	2.91E-03	58.15	25.85	668.02	11.49
145.00	5.86E-03	117.21	27.79	772.31	6.59
219.00	1.06E-02	212.60	6.40	40.96	0.19
385.00	1.76E-02	352.40	32.60	1,062.76	3.02
499.00	2.70E-02	539.20	-40.20	1,616.04	3.00
709.00	3.84E-02	768.20	-59.20	3,504.64	4.56
897.00	5.13E-02	1,025.80	-128.80	16,589.44	16.17
1,140.00	6.45E-02	1,289.60	-149.60	22,380.16	17.35
1,475.00	7.66E-02	1,532.20	-57.20	3,271.84	2.14
1,648.00	8.62E-02	1,724.60	-76.60	5,867.56	3.40
1,842.00	9.21E-02	1,842.40	-0.40	0.16	0.00
1,872.00	9.35E-02	1,869.40	2.60	6.76	0.00
1,876.00	9.01E-02	1,801.60	74.40	5,535.36	3.07
1,821.00	8.24E-02	1,648.20	172.80	29,859.84	18.12
1,535.00	7.15E-02	1,429.20	105.80	11,193.64	7.83
1,298.00	5.86E-02	1,171.60	126.40	15,976.96	13.64
956.00	4.53E-02	905.20	50.80	2,580.64	2.85
656.00	3.28E-02	655.80	0.20	0.04	0.00
401.00	2.22E-02	443.00	-42.00	1,764.00	3.98
235.00	1.38E-02	276.80	-41.80	1,747.24	6.31
123.00	7.91E-03	158.16	-35.16	1,235.96	7.81
62.00	4.08E-03	81.55	-19.55	382.05	4.69
30.00	1.86E-03	37.22	-7.22	52.08	1.40
16.00	1.05E-03	20.91	-4.91	24.12	1.15

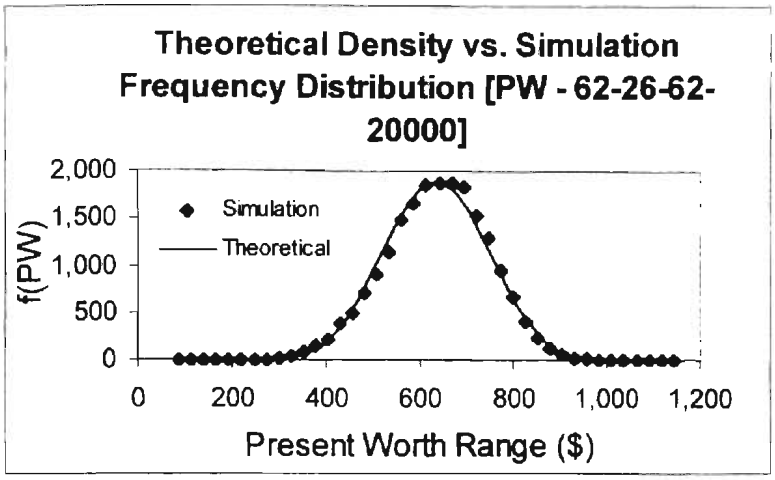

Figure 50. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

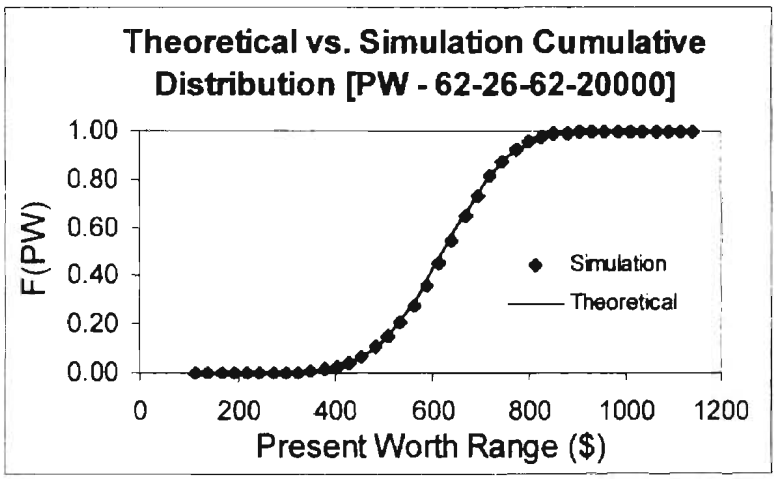

Figure 51. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Table 47

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Chi-Squared Test - [PW - 62-26-62-20000]					
Predicted Alpha			11.58	Test Statistic	Critical Value
Predicted Beta			11.29		
Number Of Data Points			20000	184.65	36.42
alpha = 0.05					
Degrees of freedom			24		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
10	1.77E-04	3.53	6.47	41.80	11.83
23	4.50E-04	8.99	14.01	196.22	21.82
43	1.21E-03	24.21	18.79	353.06	14.58
84	2.79E-03	55.89	28.11	790.30	14.14
145	5.68E-03	113.69	31.31	980.10	8.62
219	1.04E-02	208.00	11.00	121.00	0.58
385	1.73E-02	346.80	38.20	1,459.24	4.21
499	2.67E-02	533.80	-34.80	1,211.04	2.27
709	3.82E-02	764.00	-55.00	3,025.00	3.96
897	5.12E-02	1,024.00	-127.00	16,129.00	15.75
1,140	6.46E-02	1,291.40	-151.40	22,921.96	17.75
1,475	7.69E-02	1,537.80	-62.80	3,943.84	2.56
1,648	8.67E-02	1,733.60	-85.60	7,327.36	4.23
1,842	9.27E-02	1,853.60	-11.60	134.56	0.07
1,872	9.41E-02	1,881.00	-9.00	81.00	0.04
1,876	9.06E-02	1,811.80	64.20	4,121.64	2.27
1,821	8.28E-02	1,655.60	165.40	27,357.16	16.52
1,535	7.16E-02	1,432.60	102.40	10,485.76	7.32
1,298	5.86E-02	1,171.40	126.60	16,027.56	13.68
956	4.51E-02	901.80	54.20	2,937.64	3.26
656	3.25E-02	650.60	5.40	29.16	0.04
401	2.19E-02	437.20	-36.20	1,310.44	3.00
235	1.36E-02	271.40	-36.40	1,324.96	4.88
123	7.70E-03	153.91	-30.91	955.55	6.21
62	3.93E-03	78.66	-16.66	277.44	3.53
30	1.78E-03	35.52	-5.52	30.50	0.86
16	9.80E-04	19.60	-3.60	12.93	0.66

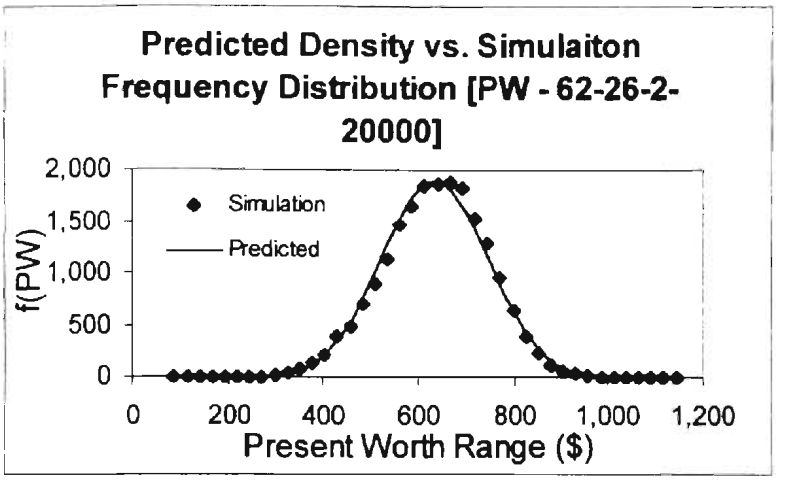

Figure 52. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

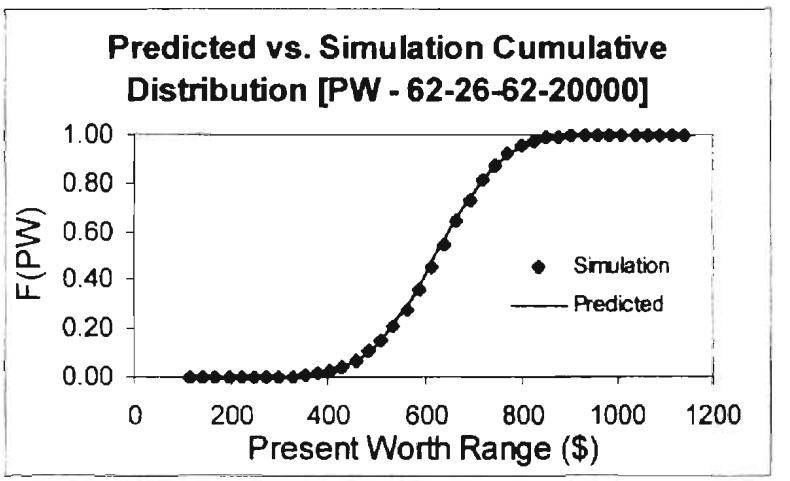

Figure 53. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Table 48

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
70.91	0	0	0
95.24	0	0	0
119.57	0	0	0
143.90	0	0	0
168.23	0	0	0
192.56	0	0	0
216.89	0	0	0
241.22	3	1	1
265.55	12	4	4
289.88	16	10	10
314.21	26	24	23
338.54	64	49	48
362.87	101	90	89
387.20	190	155	154
411.52	279	248	246
435.85	362	373	371
460.18	553	530	528
484.51	696	715	714
508.84	861	921	920
533.17	1,029	1,133	1,133
557.50	1,270	1,335	1,337
581.83	1,526	1,510	1,512
606.16	1,612	1,638	1,641
630.49	1,763	1,705	1,709
654.82	1,755	1,702	1,706
679.15	1,614	1,628	1,631
703.48	1,482	1,488	1,490
727.81	1,383	1,296	1,297
752.14	1,114	1,071	1,071
776.46	879	836	834
800.79	637	611	609
825.12	361	415	413
849.45	224	258	256
873.78	111	144	142
898.11	52	70	69
922.44	21	29	28
946.77	3	9	9
971.10	1	2	2
995.43	0	0	0
1,019.76	0	0	0
1,044.09	0	0	0

Table 49

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Chi-Squared Test - [PW - 62-26-55-20000]				Test Statistic	Critical Value
Theoretical Alpha			10.53		
Theoretical Beta			8.07		
Number Of Data Points			20000	99.56	37.65
alpha = 0.05					
Degrees of freedom			25		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
15	2.85E-04	5.69	9.31	86.64	15.22
16	5.23E-04	10.46	5.54	30.66	2.93
26	1.19E-03	23.87	2.14	4.56	0.19
64	2.43E-03	48.68	15.32	234.60	4.82
101	4.52E-03	90.44	10.56	111.48	1.23
190	7.75E-03	155.09	34.91	1,218.89	7.86
279	1.24E-02	248.00	31.00	961.00	3.88
362	1.86E-02	372.80	-10.80	116.64	0.31
553	2.65E-02	529.60	23.40	547.56	1.03
696	3.58E-02	715.20	-19.20	368.64	0.52
861	4.60E-02	920.60	-59.60	3,552.16	3.86
1,029	5.66E-02	1,132.80	-103.80	10,774.44	9.51
1,270	6.68E-02	1,335.40	-65.40	4,277.16	3.20
1,526	7.55E-02	1,509.80	16.20	262.44	0.17
1,612	8.19E-02	1,637.80	-25.80	665.64	0.41
1,763	8.53E-02	1,705.20	57.80	3,340.84	1.96
1,755	8.51E-02	1,702.40	52.60	2,766.76	1.63
1,614	8.14E-02	1,627.80	-13.80	190.44	0.12
1,482	7.44E-02	1,487.80	-5.80	33.64	0.02
1,383	6.48E-02	1,295.80	87.20	7,603.84	5.87
1,114	5.36E-02	1,071.00	43.00	1,849.00	1.73
879	4.18E-02	835.60	43.40	1,883.56	2.25
637	3.06E-02	611.00	26.00	676.00	1.11
361	2.07E-02	414.60	-53.60	2,872.96	6.93
224	1.29E-02	257.60	-33.60	1,128.96	4.38
111	7.19E-03	143.88	-32.88	1,080.88	7.51
52	3.52E-03	70.35	-18.35	336.63	4.79
25	2.04E-03	40.82	-15.82	250.14	6.13

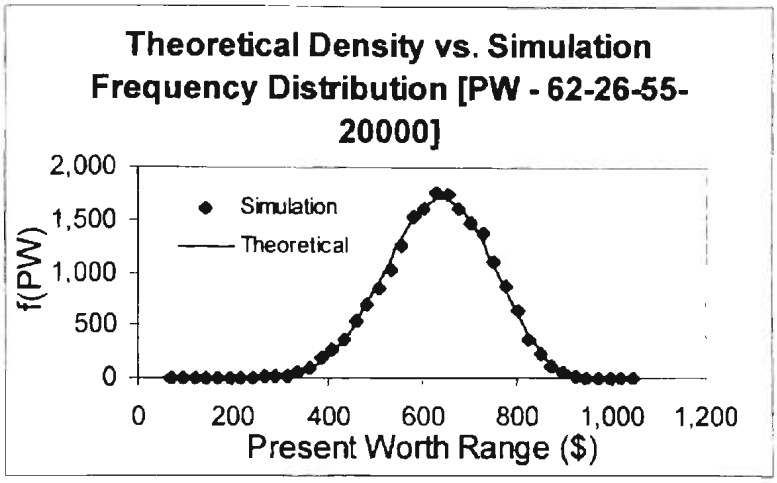

Figure 54. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

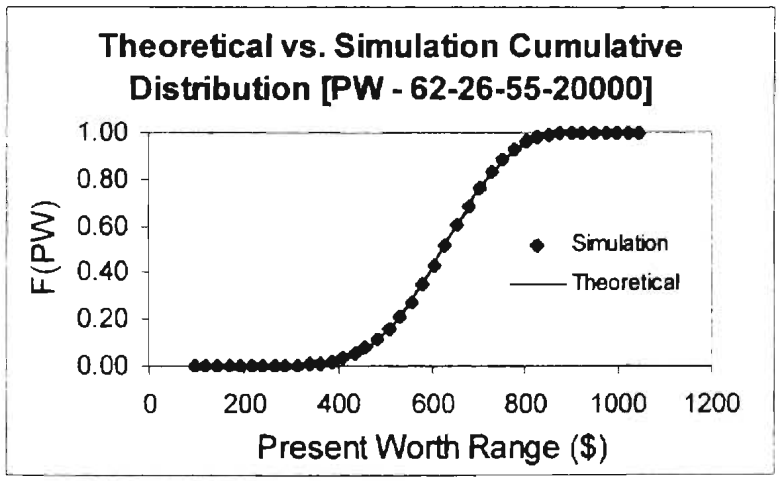

Figure 55. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Table 50

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Chi-Squared Test - [PW - 62-26-55-20000]				Test Statistic	Critical Value
Predicted Alpha			10.58		
Predicted Beta			8.10		
Number Of Data Points			20000	100.44	37.65
alpha = 0.05					
Degrees of freedom			25		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
15	2.77E-04	5.54	9.46	89.53	16.17
16	5.12E-04	10.24	5.76	33.12	3.23
26	1.17E-03	23.46	2.54	6.47	0.28
64	2.40E-03	48.01	15.99	255.80	5.33
101	4.47E-03	89.44	11.56	133.71	1.49
190	7.69E-03	153.74	36.26	1,314.56	8.55
279	1.23E-02	246.40	32.60	1,062.76	4.31
362	1.86E-02	371.00	-9.00	81.00	0.22
553	2.64E-02	528.00	25.00	625.00	1.18
696	3.57E-02	713.80	-17.80	316.84	0.44
861	4.60E-02	920.00	-59.00	3,481.00	3.78
1,029	5.67E-02	1,133.20	-104.20	10,857.64	9.58
1,270	6.68E-02	1,336.80	-66.80	4,462.24	3.34
1,526	7.56E-02	1,512.20	13.80	190.44	0.13
1,612	8.21E-02	1,641.20	-29.20	852.64	0.52
1,763	8.54E-02	1,708.80	54.20	2,937.64	1.72
1,755	8.53E-02	1,706.00	49.00	2,401.00	1.41
1,614	8.15E-02	1,630.80	-16.80	282.24	0.17
1,482	7.45E-02	1,489.80	-7.80	60.84	0.04
1,383	6.48E-02	1,296.60	86.40	7,464.96	5.76
1,114	5.35E-02	1,070.80	43.20	1,866.24	1.74
879	4.17E-02	834.40	44.60	1,989.16	2.38
637	3.05E-02	609.20	27.80	772.84	1.27
361	2.06E-02	412.60	-51.60	2,662.56	6.45
224	1.28E-02	255.80	-31.80	1,011.24	3.95
111	7.12E-03	142.48	-31.48	991.10	6.96
52	3.47E-03	69.43	-17.43	303.87	4.38
25	2.00E-03	40.04	-15.04	226.15	5.65

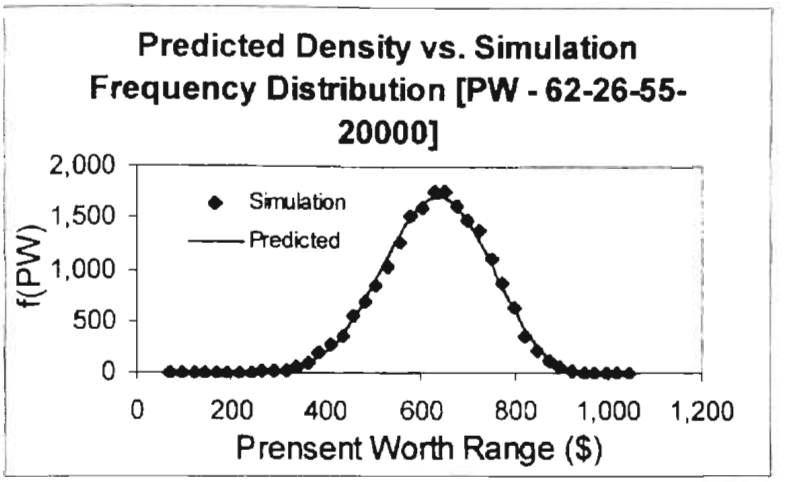

Figure 56. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

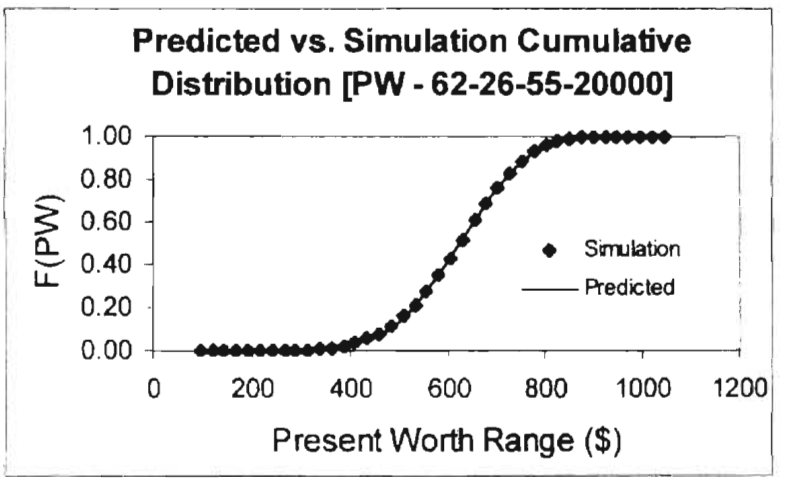

Figure 57. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Table 51

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
57.48	0	0	0
79.90	0	0	0
102.32	0	0	0
124.74	0	0	0
147.16	0	0	0
169.57	0	0	0
191.99	0	0	0
214.41	1	1	1
236.83	1	2	2
259.25	4	5	5
281.67	8	11	11
304.09	20	22	22
326.50	38	41	41
348.92	82	70	71
371.34	120	114	115
393.76	175	175	176
416.18	267	257	258
438.60	340	361	362
461.02	474	488	489
483.43	633	635	635
505.85	789	798	798
528.27	910	970	969
550.69	1,137	1,142	1,140
573.11	1,270	1,302	1,300
595.53	1,356	1,438	1,435
617.95	1,546	1,537	1,533
640.36	1,646	1,589	1,585
662.78	1,641	1,586	1,583
685.20	1,588	1,524	1,522
707.62	1,434	1,407	1,405
730.04	1,232	1,241	1,240
752.46	1,006	1,039	1,040
774.88	846	819	821
797.29	656	600	603
819.71	375	402	405
842.13	222	240	242
864.55	121	122	124
886.97	52	49	50
909.39	8	14	14
931.81	2	2	2
954.22	0	0	0

Table 52

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Chi-Squared Test - [PW - 62-26-26-20000]					
Theoretical Alpha			9.54	Test Statistic	Critical Value
Theoretical Beta			5.62		
Number Of Data Points			20000	35.02	40.11
alpha = 0.05					
Degrees of freedom			27		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
6	3.93E-04	7.86	-1.86	3.44	0.44
8	5.48E-04	10.96	-2.96	8.78	0.80
20	1.10E-03	22.00	-2.00	3.98	0.18
38	2.03E-03	40.65	-2.65	7.04	0.17
82	3.50E-03	70.08	11.92	142.08	2.03
120	5.69E-03	113.73	6.27	39.35	0.35
175	8.75E-03	175.00	0.00	0.00	0.00
267	1.28E-02	256.80	10.20	104.04	0.41
340	1.81E-02	361.00	-21.00	441.00	1.22
474	2.44E-02	487.60	-13.60	184.96	0.38
633	3.17E-02	634.60	-1.60	2.56	0.00
789	3.99E-02	798.00	-9.00	81.00	0.10
910	4.85E-02	970.20	-60.20	3,624.04	3.74
1,137	5.71E-02	1,142.00	-5.00	25.00	0.02
1,270	6.51E-02	1,302.00	-32.00	1,024.00	0.79
1,356	7.19E-02	1,437.60	-81.60	6,658.56	4.63
1,546	7.68E-02	1,536.80	9.20	84.64	0.06
1,646	7.94E-02	1,588.60	57.40	3,294.76	2.07
1,641	7.93E-02	1,585.60	55.40	3,069.16	1.94
1,588	7.62E-02	1,524.40	63.60	4,044.96	2.65
1,434	7.03E-02	1,406.80	27.20	739.84	0.53
1,232	6.20E-02	1,240.60	-8.60	73.96	0.06
1,006	5.19E-02	1,038.80	-32.80	1,075.84	1.04
846	4.10E-02	819.00	27.00	729.00	0.89
656	3.00E-02	600.40	55.60	3,091.36	5.15
375	2.01E-02	402.40	-27.40	750.76	1.87
222	1.20E-02	240.00	-18.00	324.00	1.35
121	6.10E-03	122.06	-1.06	1.13	0.01
52	2.46E-03	49.24	2.76	7.61	0.15
10	7.80E-04	15.59	-5.59	31.29	2.01

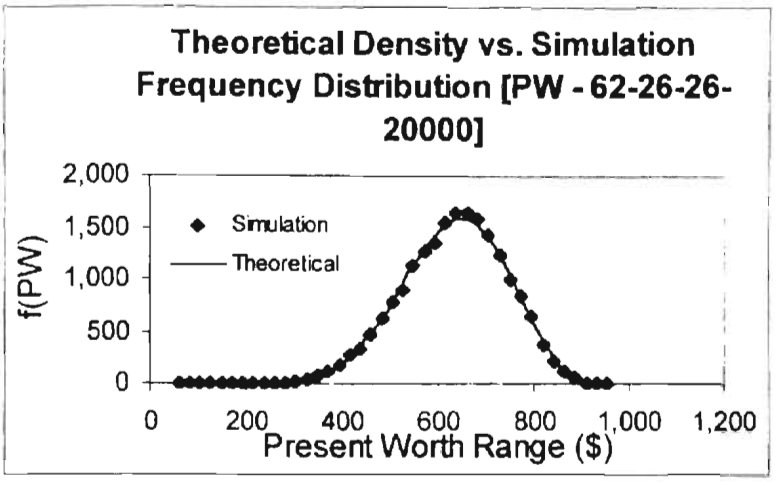

Figure 58. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

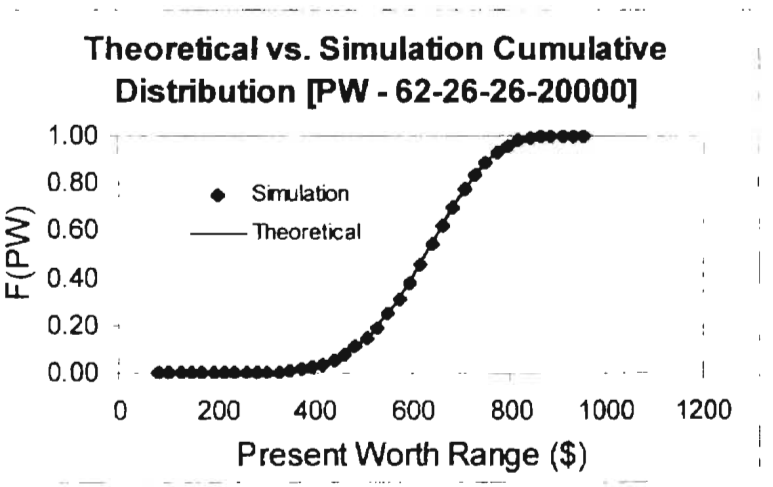

Figure 59. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Table 53

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Chi-Squared Test - [PW - 62-26-26-20000]				Test Statistic	Critical Value
Predicted Alpha			9.50		
Predicted Beta			5.60		
Number Of Data Points			20000	37.45	40.11
alpha = 0.05					
Degrees of freedom			27		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
6	4.03E-04	8.05	-2.05	4.22	0.52
8	5.59E-04	11.17	-3.17	10.08	0.90
20	1.12E-03	22.35	-2.35	5.51	0.25
38	2.06E-03	41.19	-3.19	10.17	0.25
82	3.54E-03	70.83	11.17	124.79	1.76
120	5.73E-03	114.69	5.31	28.18	0.25
175	8.81E-03	176.15	-1.15	1.31	0.01
267	1.29E-02	258.00	9.00	81.00	0.31
340	1.81E-02	362.20	-22.20	492.84	1.36
474	2.44E-02	488.60	-14.60	213.16	0.44
633	3.18E-02	635.20	-2.20	4.84	0.01
789	3.99E-02	798.00	-9.00	81.00	0.10
910	4.85E-02	969.40	-59.40	3,528.36	3.64
1,137	5.70E-02	1,140.40	-3.40	11.56	0.01
1,270	6.50E-02	1,299.60	-29.60	876.16	0.67
1,356	7.17E-02	1,434.60	-78.60	6,177.96	4.31
1,546	7.67E-02	1,533.20	12.80	163.84	0.11
1,646	7.93E-02	1,585.20	60.80	3,696.64	2.33
1,641	7.91E-02	1,582.60	58.40	3,410.56	2.16
1,588	7.61E-02	1,522.00	66.00	4,356.00	2.86
1,434	7.03E-02	1,405.40	28.60	817.96	0.58
1,232	6.20E-02	1,240.40	-8.40	70.56	0.06
1,006	5.20E-02	1,040.00	-34.00	1,156.00	1.11
846	4.11E-02	821.00	25.00	625.00	0.76
656	3.02E-02	603.00	53.00	2,809.00	4.66
375	2.03E-02	405.00	-30.00	900.00	2.22
222	1.21E-02	242.20	-20.20	408.04	1.68
121	6.19E-03	123.71	-2.71	7.33	0.06
52	2.51E-03	50.15	1.85	3.41	0.07
8	8.01E-04	16.01	-8.01	64.18	4.01

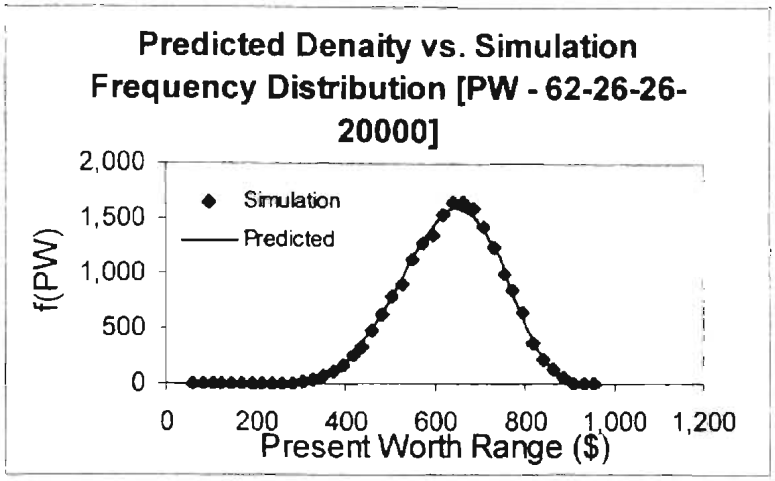

Figure 60. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

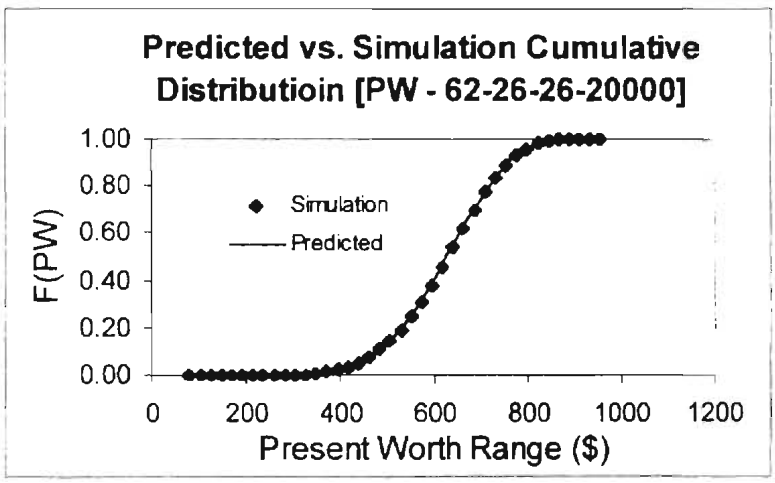

Figure 61. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(6,2), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Table 54

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
236.18	0	0	0
266.67	0	0	0
297.16	1	0	0
327.64	6	7	6
358.13	36	38	33
388.62	125	126	116
419.11	306	304	288
449.59	583	582	563
480.08	954	940	924
510.57	1,286	1,328	1,322
541.05	1,706	1,686	1,692
571.54	1,951	1,954	1,973
602.03	2,049	2,095	2,122
632.52	2,118	2,095	2,124
663.00	1,952	1,966	1,992
693.49	1,788	1,739	1,758
723.98	1,449	1,456	1,465
754.46	1,167	1,155	1,155
784.95	858	869	862
815.44	632	620	610
845.93	415	420	409
876.41	262	269	259
906.90	152	163	154
937.39	105	93	87
967.87	60	50	46
998.36	27	25	22
1,028.85	5	12	10
1,059.34	3	5	4
1,089.82	3	2	2
1,120.31	1	1	1
1,150.80	0	0	0
1,181.28	0	0	0
1,211.77	0	0	0
1,242.26	0	0	0
1,272.75	0	0	0
1,303.23	0	0	0
1,333.72	0	0	0
1,364.21	0	0	0
1,394.69	0	0	0
1,425.18	0	0	0
1,455.67	0	0	0

Table 55

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Chi-Squared Test - [PW - 55-62-62-20000]					
Theoretical Alpha			7.68	Test Statistic	Critical Value
Theoretical Beta			16.61		
Number Of Data Points			20000	13.73	33.92
alpha = 0.05					
Degrees of freedom			22		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	3.55E-04	7.09	-0.09	0.01	0.00
36	1.88E-03	37.70	-1.70	2.88	0.08
125	6.32E-03	126.46	-1.46	2.12	0.02
306	1.52E-02	304.40	1.60	2.56	0.01
583	2.91E-02	582.40	0.60	0.36	0.00
954	4.70E-02	939.80	14.20	201.64	0.21
1,286	6.64E-02	1,328.40	-42.40	1,797.76	1.35
1,706	8.43E-02	1,685.80	20.20	408.04	0.24
1,951	9.77E-02	1,954.40	-3.40	11.56	0.01
2,049	1.05E-01	2,095.00	-46.00	2,116.00	1.01
2,118	1.05E-01	2,094.60	23.40	547.56	0.26
1,952	9.83E-02	1,965.60	-13.60	184.96	0.09
1,788	8.70E-02	1,739.40	48.60	2,361.96	1.36
1,449	7.28E-02	1,455.80	-6.80	46.24	0.03
1,167	5.77E-02	1,154.60	12.40	153.76	0.13
858	4.34E-02	868.80	-10.80	116.64	0.13
632	3.10E-02	620.40	11.60	134.56	0.22
415	2.10E-02	420.00	-5.00	25.00	0.06
262	1.35E-02	269.20	-7.20	51.84	0.19
152	8.15E-03	163.09	-11.09	123.08	0.75
105	4.65E-03	93.07	11.93	142.38	1.53
60	2.49E-03	49.83	10.17	103.47	2.08
27	1.25E-03	24.90	2.10	4.40	0.18
5	5.77E-04	11.54	-6.54	42.78	3.71
7	3.88E-04	7.77	-0.77	0.59	0.08

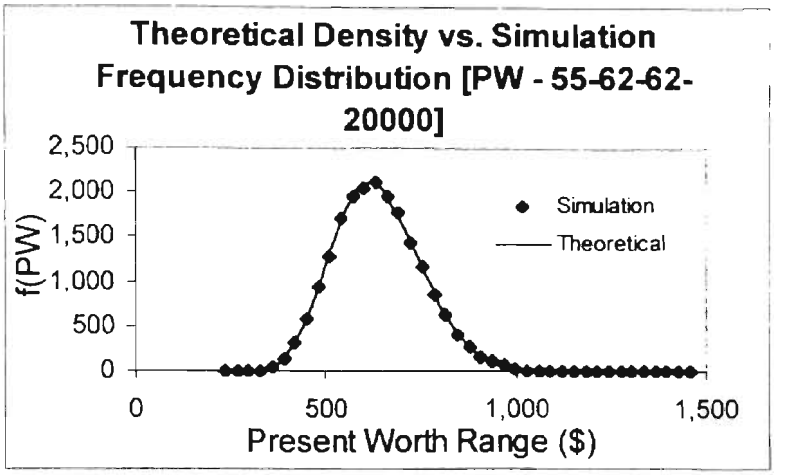

Figure 62. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

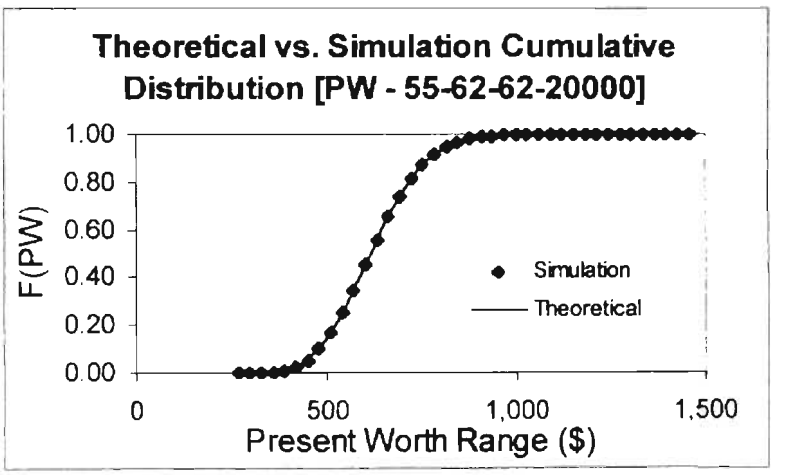

Figure 63. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Table 56

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Chi-Squared Test - [PW - 55-62-62-20000]					
Predicted Alpha			7.90	Test Statistic	Critical Value
Predicted Beta			17.08		
Number Of Data Points			20000	22.27	33.92
alpha = 0.05					
Degrees of freedom			22		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	2.96E-04	5.92	1.08	1.16	0.20
36	1.67E-03	33.38	2.62	6.88	0.21
125	5.82E-03	116.36	8.64	74.70	0.64
306	1.44E-02	288.20	17.80	316.84	1.10
583	2.82E-02	563.40	19.60	384.16	0.68
954	4.62E-02	924.00	30.00	900.00	0.97
1,286	6.61E-02	1,321.80	-35.80	1,281.64	0.97
1,706	8.46E-02	1,692.00	14.00	196.00	0.12
1,951	9.87E-02	1,973.00	-22.00	484.00	0.25
2,049	1.06E-01	2,122.20	-73.20	5,358.24	2.52
2,118	1.06E-01	2,124.20	-6.20	38.44	0.02
1,952	9.96E-02	1,992.00	-40.00	1,600.00	0.80
1,788	8.79E-02	1,758.00	30.00	900.00	0.51
1,449	7.33E-02	1,465.00	-16.00	256.00	0.17
1,167	5.78E-02	1,155.00	12.00	144.00	0.12
858	4.31E-02	862.40	-4.40	19.36	0.02
632	3.05E-02	610.00	22.00	484.00	0.79
415	2.04E-02	408.60	6.40	40.96	0.10
262	1.29E-02	258.60	3.40	11.56	0.04
152	7.72E-03	154.46	-2.46	6.07	0.04
105	4.34E-03	86.74	18.26	333.60	3.85
60	2.28E-03	45.61	14.39	207.12	4.54
27	1.12E-03	22.34	4.66	21.72	0.97
5	5.06E-04	10.12	-5.12	26.26	2.59
7	3.27E-04	6.55	0.45	0.20	0.03

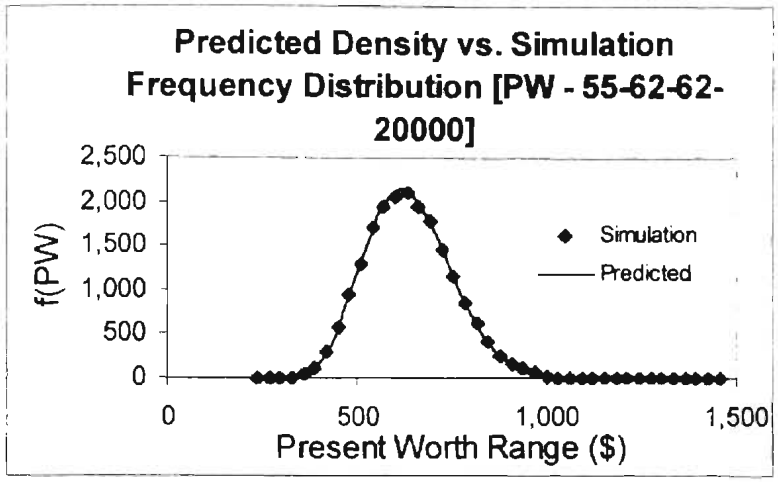

Figure 64. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

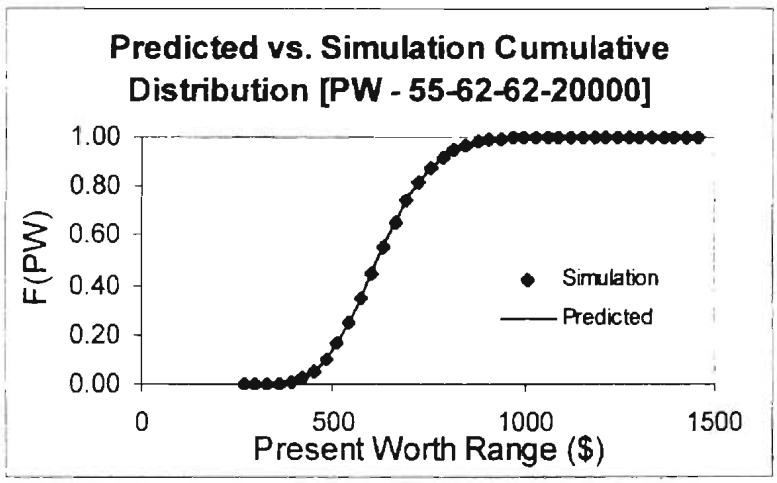

Figure 65. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Table 57

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	1	1	1
324.22	4	11	11
354.46	45	44	45
384.69	108	129	132
414.92	306	290	298
445.16	552	539	555
475.39	899	864	890
505.63	1,293	1,228	1,263
535.86	1,578	1,579	1,620
566.09	1,843	1,863	1,903
596.33	2,104	2,037	2,070
626.56	2,113	2,079	2,099
656.80	1,975	1,991	1,996
687.03	1,761	1,797	1,787
717.26	1,430	1,532	1,509
747.50	1,212	1,236	1,204
777.73	895	944	909
807.97	695	682	649
838.20	480	466	437
868.43	312	300	278
898.67	188	182	166
928.90	89	104	93
959.14	64	55	48
989.37	31	27	23
1,019.60	9	12	10
1,049.84	8	5	4
1,080.07	2	2	1
1,110.31	1	1	0
1,140.54	0	0	0
1,170.77	1	0	0
1,201.01	1	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 58

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Chi-Squared Test - [PW - 55-62-55-20000]				Test Statistic	Critical Value
Theoretical Alpha			8.61		
Theoretical Beta			16.27		
Number Of Data Points			20000	37.25	35.17
alpha = 0.05					
Degrees of freedom			23		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
5	5.97E-04	11.95	-6.95	48.28	4.04
45	2.22E-03	44.34	0.66	0.44	0.01
108	6.45E-03	128.99	-20.99	440.76	3.42
306	1.45E-02	290.00	16.00	256.00	0.88
552	2.70E-02	539.00	13.00	169.00	0.31
899	4.32E-02	863.80	35.20	1,239.04	1.43
1,293	6.14E-02	1,227.80	65.20	4,251.04	3.46
1,578	7.90E-02	1,579.00	-1.00	1.00	0.00
1,843	9.31E-02	1,862.80	-19.80	392.04	0.21
2,104	1.02E-01	2,036.60	67.40	4,542.76	2.23
2,113	1.04E-01	2,078.80	34.20	1,169.64	0.56
1,975	9.96E-02	1,991.40	-16.40	268.96	0.14
1,761	8.99E-02	1,797.40	-36.40	1,324.96	0.74
1,430	7.66E-02	1,532.40	-102.40	10,485.76	6.84
1,212	6.18E-02	1,236.00	-24.00	576.00	0.47
895	4.72E-02	943.80	-48.80	2,381.44	2.52
695	3.41E-02	682.00	13.00	169.00	0.25
480	2.33E-02	465.80	14.20	201.64	0.43
312	1.50E-02	300.20	11.80	139.24	0.46
188	9.11E-03	182.15	5.85	34.25	0.19
89	5.18E-03	103.59	-14.59	212.89	2.06
64	2.75E-03	54.97	9.03	81.61	1.48
31	1.35E-03	27.04	3.96	15.66	0.58
9	6.12E-04	12.24	-3.24	10.49	0.86
8	2.52E-04	5.05	2.95	8.73	1.73
5	1.35E-04	2.71	2.29	5.25	1.94

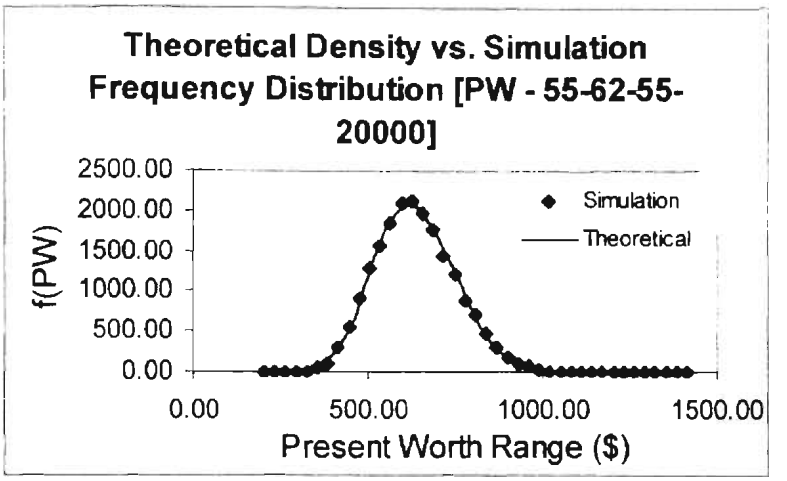

Figure 66. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

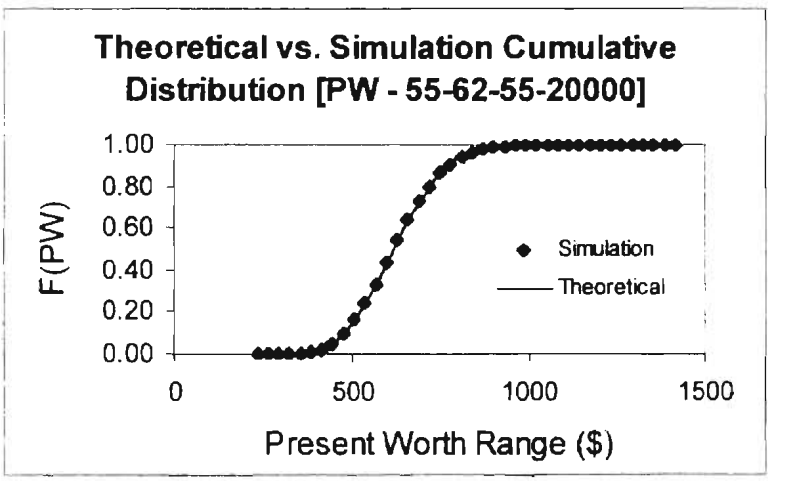

Figure 67. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Table 59

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Chi-Squared Test - [PW - 55-62-55-20000]					
Predicted Alpha			8.79	Test Statistic	Critical Value
Predicted Beta			16.60		
Number Of Data Points			20000	45.40	35.17
alpha = 0.05					
Degrees of freedom			23		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
5	5.30E-04	10.60	-5.60	31.37	2.96
45	2.04E-03	40.85	4.15	17.20	0.42
108	6.09E-03	121.79	-13.79	190.16	1.56
306	1.40E-02	279.00	27.00	729.00	2.61
552	2.63E-02	526.20	25.80	665.64	1.26
899	4.26E-02	852.60	46.40	2,152.96	2.53
1,293	6.11E-02	1,222.40	70.60	4,984.36	4.08
1,578	7.91E-02	1,582.00	-4.00	16.00	0.01
1,843	9.37E-02	1,874.80	-31.80	1,011.24	0.54
2,104	1.03E-01	2,055.20	48.80	2,381.44	1.16
2,113	1.05E-01	2,100.00	13.00	169.00	0.08
1,975	1.01E-01	2,011.20	-36.20	1,310.44	0.65
1,761	9.06E-02	1,812.40	-51.40	2,641.96	1.46
1,430	7.70E-02	1,540.80	-110.80	12,276.64	7.97
1,212	6.19E-02	1,237.60	-25.60	655.36	0.53
895	4.70E-02	940.00	-45.00	2,025.00	2.15
695	3.37E-02	674.80	20.20	408.04	0.60
480	2.29E-02	457.40	22.60	510.76	1.12
312	1.46E-02	292.20	19.80	392.04	1.34
188	8.77E-03	175.39	12.61	158.95	0.91
89	4.93E-03	98.58	-9.58	91.78	0.93
64	2.58E-03	51.62	12.38	153.34	2.97
31	1.25E-03	25.02	5.98	35.78	1.43
9	5.57E-04	11.14	-2.14	4.56	0.41
8	2.25E-04	4.51	3.49	12.21	2.71
5	1.17E-04	2.34	2.66	7.05	3.01

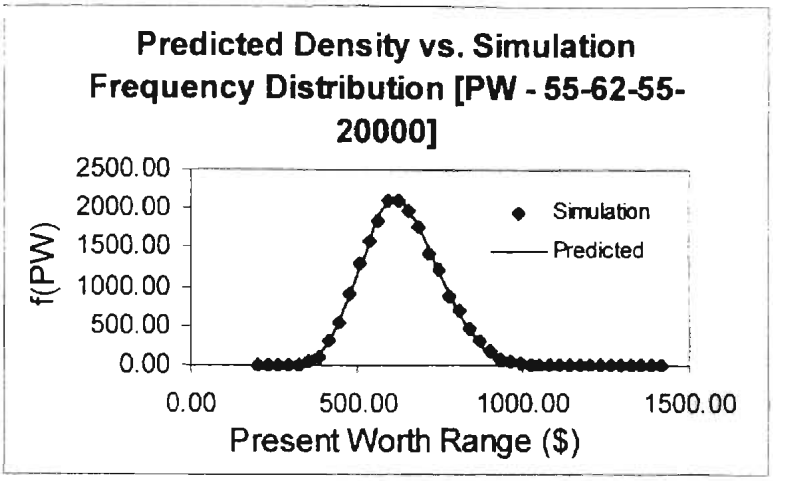

Figure 68. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

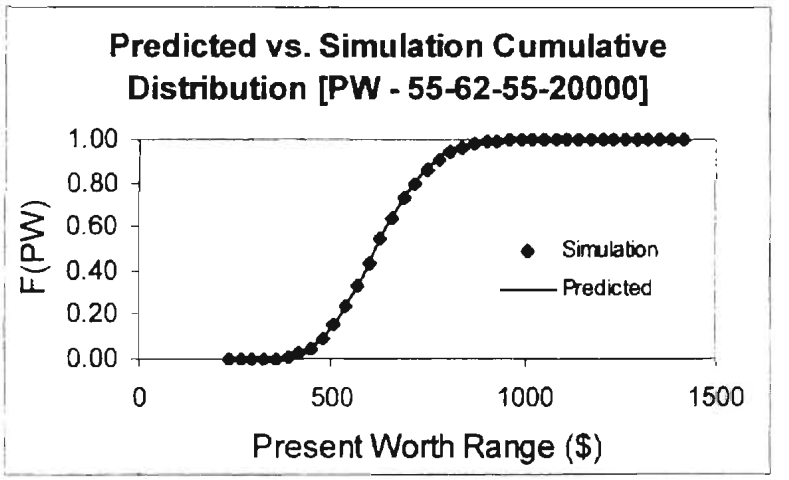

Figure 69. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Table 60

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
57.48	0	0	0
79.90	0	0	0
102.32	0	0	0
124.74	0	0	0
147.16	0	0	0
169.57	0	0	0
191.99	0	0	0
214.41	1	1	1
236.83	1	2	2
259.25	4	5	5
281.67	8	11	11
304.09	20	22	22
326.50	38	41	41
348.92	82	70	71
371.34	120	114	115
393.76	175	175	176
416.18	267	257	258
438.60	340	361	362
461.02	474	488	489
483.43	633	635	635
505.85	789	798	798
528.27	910	970	969
550.69	1,137	1,142	1,140
573.11	1,270	1,302	1,300
595.53	1,356	1,438	1,435
617.95	1,546	1,537	1,533
640.36	1,646	1,589	1,585
662.78	1,641	1,586	1,583
685.20	1,588	1,524	1,522
707.62	1,434	1,407	1,405
730.04	1,232	1,241	1,240
752.46	1,006	1,039	1,040
774.88	846	819	821
797.29	656	600	603
819.71	375	402	405
842.13	222	240	242
864.55	121	122	124
886.97	52	49	50
909.39	8	14	14
931.81	2	2	2
954.22	0	0	0

Table 61

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 55-62-26-20000]					
Theoretical Alpha			9.61	Test Statistic	Critical Value
Theoretical Beta			16.11		
Number Of Data Points			20000	26.06	33.92
alpha = 0.05					
Degrees of freedom			22		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
9	8.44E-04	16.89	-7.89	62.23	3.68
43	2.48E-03	49.57	-6.57	43.20	0.87
115	6.58E-03	131.58	-16.58	274.77	2.09
270	1.41E-02	282.40	-12.40	153.76	0.54
496	2.57E-02	514.00	-18.00	324.00	0.63
878	4.10E-02	819.20	58.80	3,457.44	4.22
1,166	5.84E-02	1,168.80	-2.80	7.84	0.01
1,533	7.58E-02	1,516.80	16.20	262.44	0.17
1,816	9.05E-02	1,810.80	5.20	27.04	0.01
1,961	1.00E-01	2,006.20	-45.20	2,043.04	1.02
2,037	1.04E-01	2,075.40	-38.40	1,474.56	0.71
2,036	1.01E-01	2,013.80	22.20	492.84	0.24
1,885	9.19E-02	1,838.80	46.20	2,134.44	1.16
1,574	7.92E-02	1,583.00	-9.00	81.00	0.05
1,318	6.43E-02	1,286.40	31.60	998.56	0.78
1,002	4.93E-02	986.60	15.40	237.16	0.24
708	3.57E-02	713.80	-5.80	33.64	0.05
480	2.43E-02	486.40	-6.40	40.96	0.08
280	1.56E-02	311.40	-31.40	985.96	3.17
175	9.33E-03	186.65	-11.65	135.61	0.73
102	5.21E-03	104.24	-2.24	5.00	0.05
65	2.70E-03	53.92	11.08	122.77	2.28
31	1.28E-03	25.64	5.36	28.75	1.12
10	5.55E-04	11.10	-1.10	1.20	0.11
10	3.19E-04	6.38	3.62	13.07	2.05

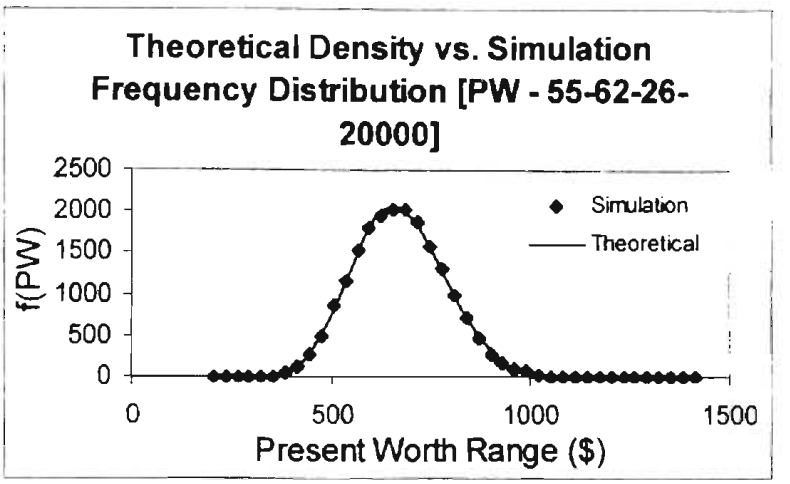

Figure 70. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

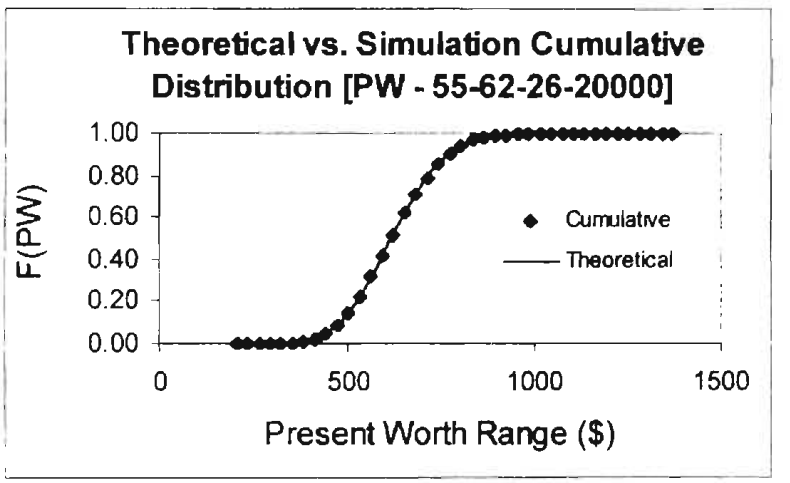

Figure 71. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Table 62

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 55-62-26-20000]				Test Statistic	Critical Value
Predicted Alpha			9.66		
Predicted Beta			16.37		
Number Of Data Points			20000	25.51	33.92
alpha = 0.05					
Degrees of freedom			22		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
9	7.87E-04	15.74	-6.74	45.47	2.89
43	2.36E-03	47.23	-4.23	17.93	0.38
115	6.36E-03	127.10	-12.10	146.52	1.15
270	1.38E-02	275.80	-5.80	33.64	0.12
496	2.53E-02	506.40	-10.40	108.16	0.21
878	4.06E-02	812.40	65.60	4,303.36	5.30
1,166	5.83E-02	1,165.40	0.60	0.36	0.00
1,533	7.59E-02	1,518.20	14.80	219.04	0.14
1,816	9.09E-02	1,817.60	-1.60	2.56	0.00
1,961	1.01E-01	2,017.20	-56.20	3,158.44	1.57
2,037	1.04E-01	2,088.40	-51.40	2,641.96	1.27
2,036	1.01E-01	2,026.40	9.60	92.16	0.05
1,885	9.24E-02	1,848.60	36.40	1,324.96	0.72
1,574	7.94E-02	1,588.80	-14.80	219.04	0.14
1,318	6.44E-02	1,287.80	30.20	912.04	0.71
1,002	4.92E-02	984.60	17.40	302.76	0.31
708	3.55E-02	709.40	-1.40	1.96	0.00
480	2.41E-02	481.00	-1.00	1.00	0.00
280	1.53E-02	306.20	-26.20	686.44	2.24
175	9.11E-03	182.30	-7.30	53.25	0.29
102	5.05E-03	101.04	0.96	0.93	0.01
65	2.59E-03	51.82	13.18	173.81	3.35
31	1.22E-03	24.40	6.60	43.58	1.79
10	5.22E-04	10.44	-0.44	0.20	0.02
10	2.95E-04	5.90	4.10	16.84	2.86

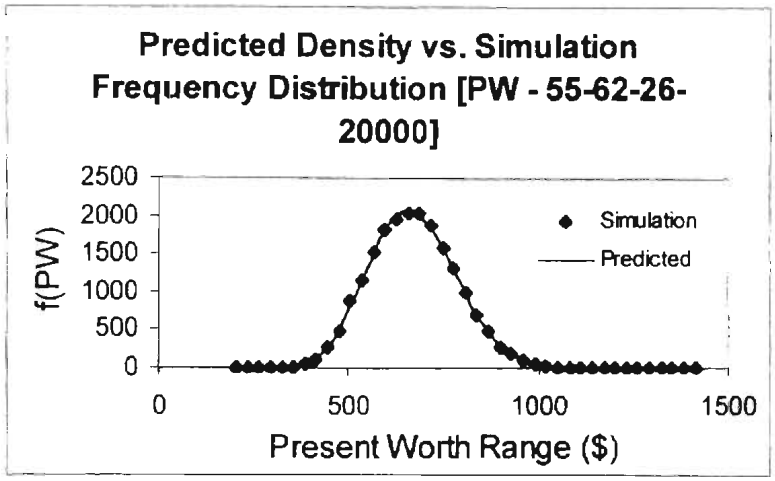

Figure 72. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

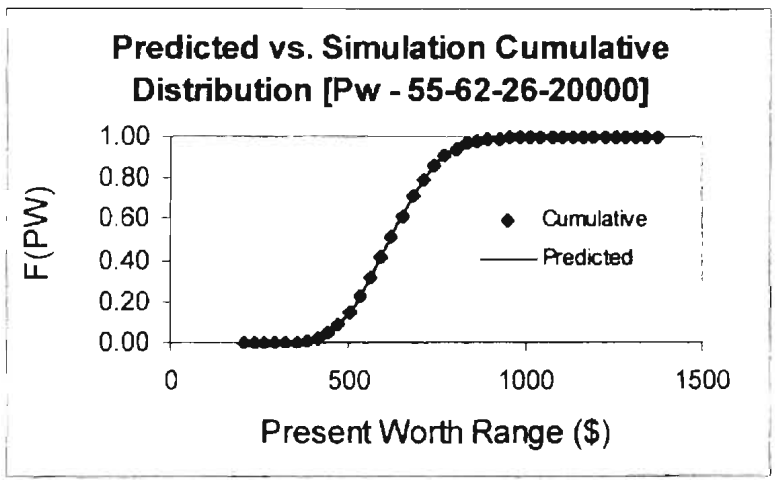

Figure 73. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Table 63

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	1	1
324.22	7	11	9
354.46	31	44	41
384.69	148	129	122
414.92	268	290	279
445.16	553	539	526
475.39	883	864	853
505.63	1,194	1,228	1,222
535.86	1,643	1,579	1,582
566.09	1,849	1,863	1,875
596.33	2,009	2,037	2,055
626.56	2,074	2,079	2,100
656.80	1,917	1,991	2,011
687.03	1,809	1,797	1,812
717.26	1,535	1,532	1,541
747.50	1,213	1,236	1,238
777.73	972	944	940
807.97	717	682	675
838.20	502	466	457
868.43	298	300	292
898.67	187	182	175
928.90	85	104	99
959.14	56	55	52
989.37	29	27	25
1,019.60	11	12	11
1,049.84	7	5	5
1,080.07	2	2	2
1,110.31	0	1	1
1,140.54	1	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 64

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 55-55-62-20000]				Test Statistic	Critical Value
Theoretical Alpha			8.61		
Theoretical Beta			16.27		
Number Of Data Points			20000	28.49	33.92
alpha = 0.05					
Degrees of freedom			22		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	5.97E-04	11.95	-4.95	24.48	2.05
31	2.22E-03	44.34	-13.34	177.83	4.01
148	6.45E-03	128.99	19.01	361.22	2.80
268	1.45E-02	290.00	-22.00	484.00	1.67
553	2.70E-02	539.00	14.00	196.00	0.36
883	4.32E-02	863.80	19.20	368.64	0.43
1,194	6.14E-02	1,227.80	-33.80	1,142.44	0.93
1,643	7.90E-02	1,579.00	64.00	4,096.00	2.59
1,849	9.31E-02	1,862.80	-13.80	190.44	0.10
2,009	1.02E-01	2,036.60	-27.60	761.76	0.37
2,074	1.04E-01	2,078.80	-4.80	23.04	0.01
1,917	9.96E-02	1,991.40	-74.40	5,535.36	2.78
1,809	8.99E-02	1,797.40	11.60	134.56	0.07
1,535	7.66E-02	1,532.40	2.60	6.76	0.00
1,213	6.18E-02	1,236.00	-23.00	529.00	0.43
972	4.72E-02	943.80	28.20	795.24	0.84
717	3.41E-02	682.00	35.00	1,225.00	1.80
502	2.33E-02	465.80	36.20	1,310.44	2.81
298	1.50E-02	300.20	-2.20	4.84	0.02
187	9.11E-03	182.15	4.85	23.55	0.13
85	5.18E-03	103.59	-18.59	345.62	3.34
56	2.75E-03	54.97	1.03	1.07	0.02
29	1.35E-03	27.04	1.96	3.83	0.14
11	6.12E-04	12.24	-1.24	1.54	0.13
10	3.88E-04	7.75	2.25	5.04	0.65

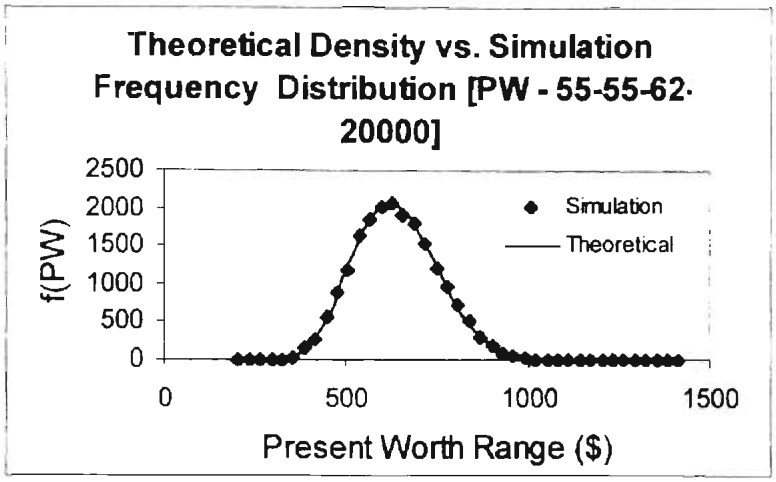

Figure 74. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

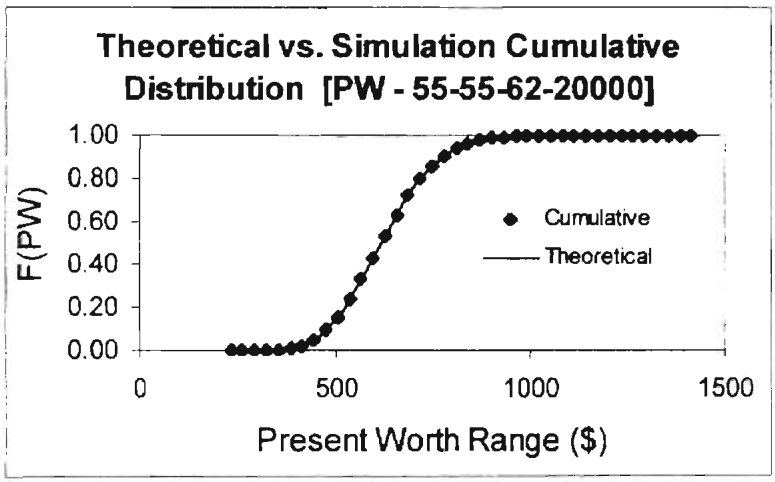

Figure 75. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Table 65

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 55-55-62-20000]					
Predicted Alpha			8.71	Test Statistic	Critical Value
Predicted Beta			16.64		
Number Of Data Points			20000	35.07	33.92
alpha = 0.05					
Degrees of freedom			22		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	5.30E-04	10.60	-3.60	12.97	1.22
31	2.04E-03	40.85	-9.85	97.07	2.38
148	6.09E-03	121.79	26.21	686.96	5.64
268	1.40E-02	279.00	-11.00	121.00	0.43
553	2.63E-02	526.20	26.80	718.24	1.36
883	4.26E-02	852.60	30.40	924.16	1.08
1,194	6.11E-02	1,222.40	-28.40	806.56	0.66
1,643	7.91E-02	1,582.00	61.00	3,721.00	2.35
1,849	9.37E-02	1,874.80	-25.80	665.64	0.36
2,009	1.03E-01	2,055.20	-46.20	2,134.44	1.04
2,074	1.05E-01	2,100.00	-26.00	676.00	0.32
1,917	1.01E-01	2,011.20	-94.20	8,873.64	4.41
1,809	9.06E-02	1,812.40	-3.40	11.56	0.01
1,535	7.70E-02	1,540.80	-5.80	33.64	0.02
1,213	6.19E-02	1,237.60	-24.60	605.16	0.49
972	4.70E-02	940.00	32.00	1,024.00	1.09
717	3.37E-02	674.80	42.20	1,780.84	2.64
502	2.29E-02	457.40	44.60	1,989.16	4.35
298	1.46E-02	292.20	5.80	33.64	0.12
187	8.77E-03	175.39	11.61	134.74	0.77
85	4.93E-03	98.58	-13.58	184.42	1.87
56	2.58E-03	51.62	4.38	19.21	0.37
29	1.25E-03	25.02	3.98	15.85	0.63
11	5.57E-04	11.14	-0.14	0.02	0.00
10	3.42E-04	6.85	3.15	9.92	1.45

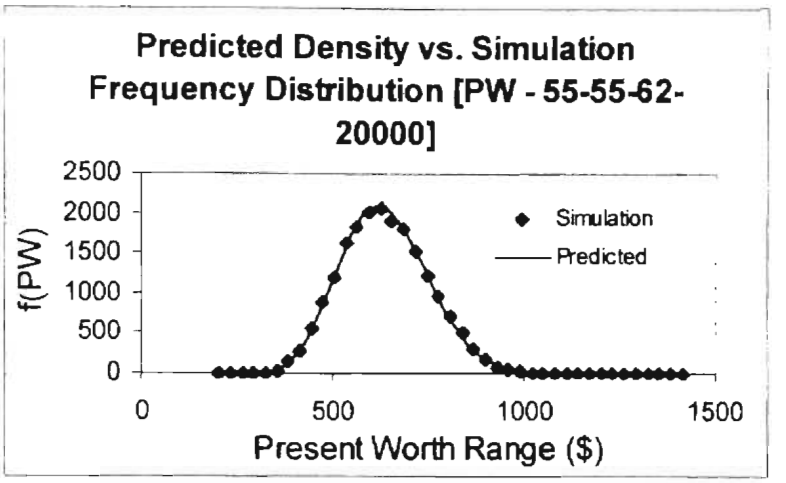

Figure 76. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

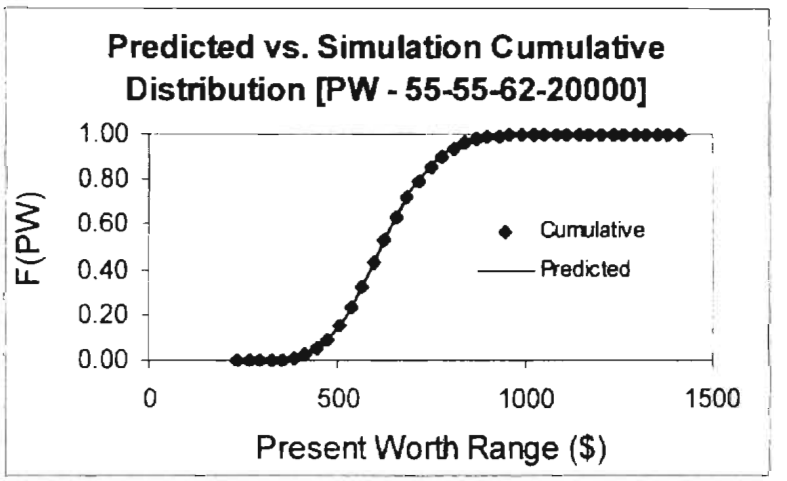

Figure 77. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Table 66

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	0	0
324.22	1	2	2
354.46	8	11	10
384.69	25	39	37
414.92	95	104	100
445.16	194	224	218
475.39	401	411	404
505.63	666	666	659
535.86	1,002	971	966
566.09	1,268	1,292	1,291
596.33	1,624	1,589	1,593
626.56	1,844	1,820	1,829
656.80	2,030	1,955	1,966
687.03	1,906	1,976	1,989
717.26	1,846	1,887	1,898
747.50	1,713	1,705	1,714
777.73	1,488	1,460	1,465
807.97	1,152	1,186	1,186
838.20	883	912	909
868.43	688	664	659
898.67	459	456	451
928.90	304	295	290
959.14	177	179	174
989.37	114	101	98
1,019.60	62	53	51
1,049.84	30	25	24
1,080.07	14	11	10
1,110.31	2	4	4
1,140.54	0	1	1
1,170.77	3	0	0
1,201.01	1	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 67

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 55-55-55-20000]					
Theoretical Alpha			9.31	Test Statistic	Critical Value
Theoretical Beta			14.58		
Number Of Data Points			20000	28.90	35.17
alpha = 0.05					
Degrees of freedom			23		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
9	6.82E-04	13.63	-4.63	21.47	1.57
25	1.96E-03	39.22	-14.22	202.28	5.16
95	5.19E-03	103.70	-8.70	75.69	0.73
194	1.12E-02	223.60	-29.60	876.16	3.92
401	2.06E-02	411.20	-10.20	104.04	0.25
666	3.33E-02	666.20	-0.20	0.04	0.00
1,002	4.85E-02	970.60	31.40	985.96	1.02
1,268	6.46E-02	1,292.00	-24.00	576.00	0.45
1,624	7.95E-02	1,589.00	35.00	1,225.00	0.77
1,844	9.10E-02	1,820.40	23.60	556.96	0.31
2,030	9.77E-02	1,954.60	75.40	5,685.16	2.91
1,906	9.88E-02	1,976.00	-70.00	4,900.00	2.48
1,846	9.44E-02	1,887.00	-41.00	1,681.00	0.89
1,713	8.53E-02	1,705.40	7.60	57.76	0.03
1,488	7.30E-02	1,460.40	27.60	761.76	0.52
1,152	5.93E-02	1,185.60	-33.60	1,128.96	0.95
883	4.56E-02	911.80	-28.80	829.44	0.91
688	3.32E-02	663.60	24.40	595.36	0.90
459	2.28E-02	455.80	3.20	10.24	0.02
304	1.47E-02	294.80	9.20	84.64	0.29
177	8.94E-03	178.77	-1.77	3.13	0.02
114	5.05E-03	101.07	12.93	167.07	1.65
62	2.65E-03	52.92	9.08	82.36	1.56
30	1.27E-03	25.44	4.56	20.78	0.82
14	5.55E-04	11.10	2.90	8.40	0.76
6	3.19E-04	6.39	-0.39	0.15	0.02

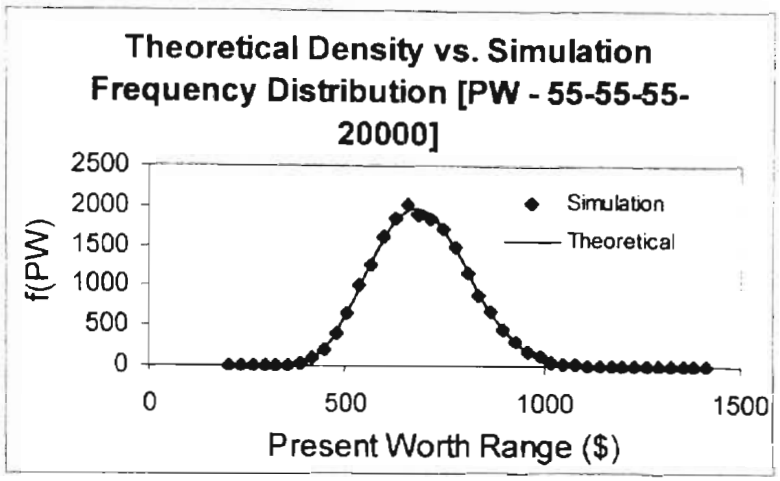

Figure 78. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

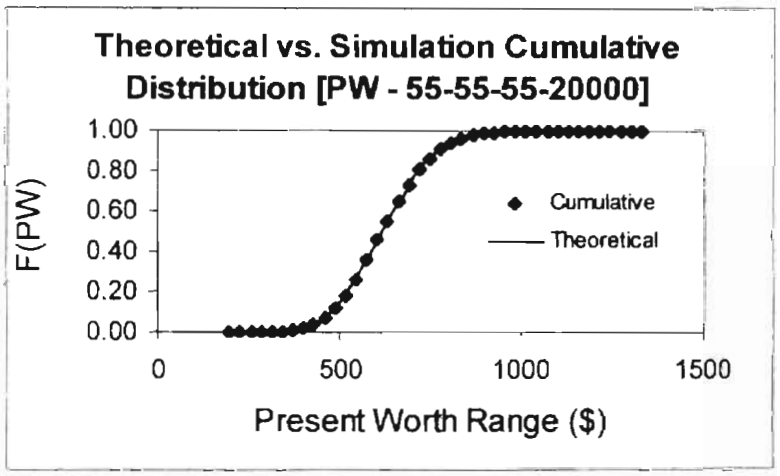

Figure 79. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Table 68

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 55-55-55-20000]					
Predicted Alpha			9.36	Test Statistic	Critical Value
Predicted Beta			14.83		
Number Of Data Points			20000	29.43	35.17
alpha = 0.05					
Degrees of freedom			23		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
9	6.33E-04	12.67	-3.67	13.47	1.06
25	1.86E-03	37.26	-12.26	150.36	4.04
95	4.99E-03	99.89	-4.89	23.87	0.24
194	1.09E-02	217.60	-23.60	556.96	2.56
401	2.02E-02	404.00	-3.00	9.00	0.02
666	3.30E-02	659.00	7.00	49.00	0.07
1,002	4.83E-02	965.80	36.20	1,310.44	1.36
1,268	6.46E-02	1,291.20	-23.20	538.24	0.42
1,624	7.96E-02	1,592.80	31.20	973.44	0.61
1,844	9.14E-02	1,828.80	15.20	231.04	0.13
2,030	9.83E-02	1,966.40	63.60	4,044.96	2.06
1,906	9.94E-02	1,988.80	-82.80	6,855.84	3.45
1,846	9.49E-02	1,898.40	-52.40	2,745.76	1.45
1,713	8.57E-02	1,713.80	-0.80	0.64	0.00
1,488	7.33E-02	1,465.00	23.00	529.00	0.36
1,152	5.93E-02	1,186.20	-34.20	1,169.64	0.99
883	4.55E-02	909.20	-26.20	686.44	0.75
688	3.30E-02	659.00	29.00	841.00	1.28
459	2.25E-02	450.60	8.40	70.56	0.16
304	1.45E-02	289.80	14.20	201.64	0.70
177	8.72E-03	174.50	2.50	6.27	0.04
114	4.90E-03	97.91	16.09	258.79	2.64
62	2.54E-03	50.83	11.17	124.79	2.46
30	1.21E-03	24.20	5.80	33.68	1.39
14	5.22E-04	10.44	3.56	12.66	1.21
6	2.95E-04	5.89	0.11	0.01	0.00

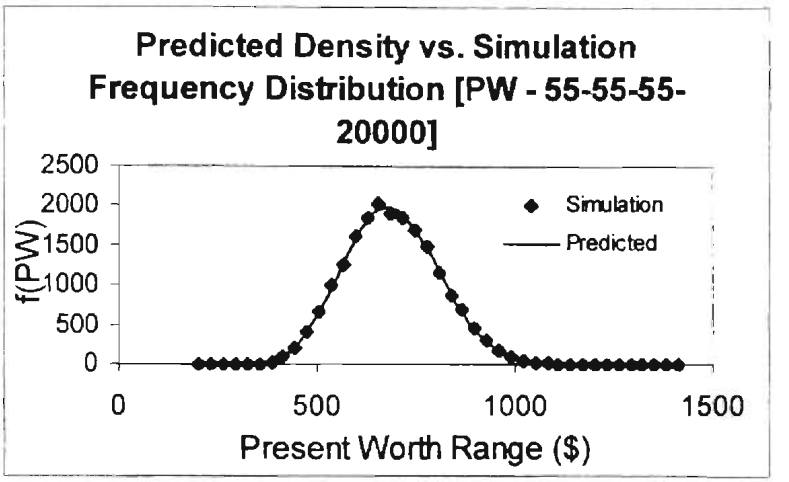

Figure 80. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

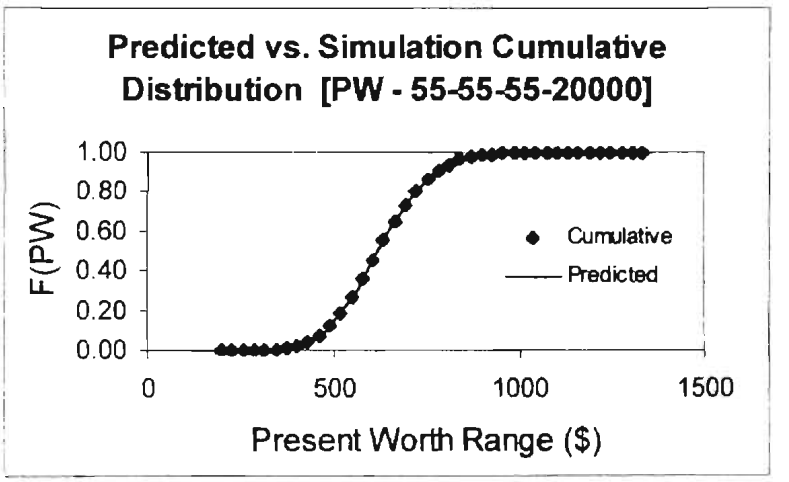

Figure 81. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Table 69

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	0	0
324.22	0	0	0
354.46	3	3	3
384.69	4	11	11
414.92	25	34	33
445.16	57	84	83
475.39	158	175	173
505.63	283	319	317
535.86	532	521	518
566.09	825	773	771
596.33	1,102	1,058	1,056
626.56	1,393	1,345	1,345
656.80	1,704	1,601	1,603
687.03	1,785	1,793	1,796
717.26	1,879	1,896	1,901
747.50	1,824	1,899	1,904
777.73	1,794	1,803	1,808
807.97	1,631	1,626	1,629
838.20	1,360	1,391	1,393
868.43	1,057	1,129	1,129
898.67	839	868	867
928.90	630	630	628
959.14	453	431	429
989.37	279	276	274
1,019.60	176	165	163
1,049.84	113	91	90
1,080.07	53	46	45
1,110.31	27	21	21
1,140.54	8	9	8
1,170.77	6	3	3
1,201.01	0	1	1
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 70

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW -55-55-26-20000]					
Theoretical Alpha			9.95	Test Statistic	Critical Value
Theoretical Beta			13.08		
Number Of Data Points			20000	54.97	36.42
alpha = 0.05					
Degrees of freedom			24		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	7.15E-04	14.30	-7.30	53.22	3.72
25	1.70E-03	34.06	-9.06	82.16	2.41
57	4.20E-03	83.97	-26.97	727.53	8.66
158	8.75E-03	175.09	-17.09	292.10	1.67
283	1.60E-02	319.20	-36.20	1,310.44	4.11
532	2.60E-02	520.80	11.20	125.44	0.24
825	3.87E-02	773.40	51.60	2,662.56	3.44
1,102	5.29E-02	1,057.60	44.40	1,971.36	1.86
1,393	6.73E-02	1,345.00	48.00	2,304.00	1.71
1,704	8.00E-02	1,600.80	103.20	10,650.24	6.65
1,785	8.96E-02	1,792.80	-7.80	60.84	0.03
1,879	9.48E-02	1,896.00	-17.00	289.00	0.15
1,824	9.49E-02	1,898.80	-74.80	5,595.04	2.95
1,794	9.02E-02	1,803.40	-9.40	88.36	0.05
1,631	8.13E-02	1,625.60	5.40	29.16	0.02
1,360	6.96E-02	1,391.00	-31.00	961.00	0.69
1,057	5.65E-02	1,129.00	-72.00	5,184.00	4.59
839	4.34E-02	867.60	-28.60	817.96	0.94
630	3.15E-02	630.00	0.00	0.00	0.00
453	2.15E-02	430.60	22.40	501.76	1.17
279	1.38E-02	275.80	3.20	10.24	0.04
176	8.23E-03	164.66	11.34	128.55	0.78
113	4.54E-03	90.87	22.13	489.52	5.39
53	2.30E-03	45.92	7.08	50.19	1.09
27	1.05E-03	20.97	6.03	36.38	1.73
8	4.25E-04	8.51	-0.51	0.26	0.03
6	2.07E-04	4.15	1.85	3.43	0.83

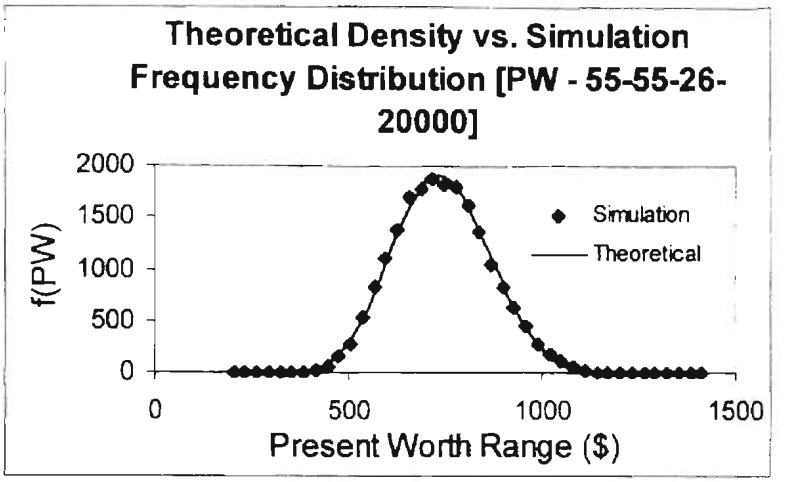

Figure 82. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

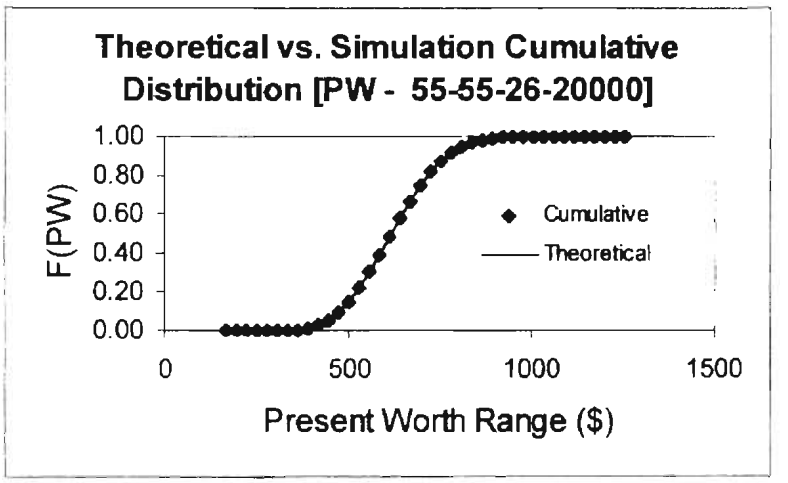

Figure 83. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Table 71

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 55-55-26-20000]					
Predicted Alpha			9.94	Test Statistic	Critical Value
Predicted Beta			13.21		
Number Of Data Points			20000	55.53	36.42
alpha = 0.05					
Degrees of freedom			24		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	6.94E-04	13.88	-6.88	47.32	3.41
25	1.67E-03	33.35	-8.35	69.76	2.09
57	4.13E-03	82.64	-25.64	657.51	7.96
158	8.65E-03	173.04	-15.04	226.35	1.31
283	1.58E-02	316.60	-33.60	1,128.96	3.57
532	2.59E-02	518.00	14.00	196.00	0.38
825	3.86E-02	771.00	54.00	2,916.00	3.78
1,102	5.28E-02	1,056.40	45.60	2,079.36	1.97
1,393	6.73E-02	1,345.20	47.80	2,284.84	1.70
1,704	8.02E-02	1,603.00	101.00	10,201.00	6.36
1,785	8.98E-02	1,796.40	-11.40	129.96	0.07
1,879	9.50E-02	1,900.80	-21.80	475.24	0.25
1,824	9.52E-02	1,903.60	-79.60	6,336.16	3.33
1,794	9.04E-02	1,807.60	-13.60	184.96	0.10
1,631	8.14E-02	1,628.80	2.20	4.84	0.00
1,360	6.96E-02	1,392.80	-32.80	1,075.84	0.77
1,057	5.65E-02	1,129.20	-72.20	5,212.84	4.62
839	4.33E-02	866.60	-27.60	761.76	0.88
630	3.14E-02	628.20	1.80	3.24	0.01
453	2.14E-02	428.60	24.40	595.36	1.39
279	1.37E-02	273.80	5.20	27.04	0.10
176	8.15E-03	163.02	12.98	168.37	1.03
113	4.48E-03	89.69	23.31	543.55	6.06
53	2.26E-03	45.15	7.85	61.63	1.36
27	1.03E-03	20.53	6.47	41.83	2.04
8	4.15E-04	8.29	-0.29	0.09	0.01
6	2.00E-04	4.01	1.99	3.96	0.99

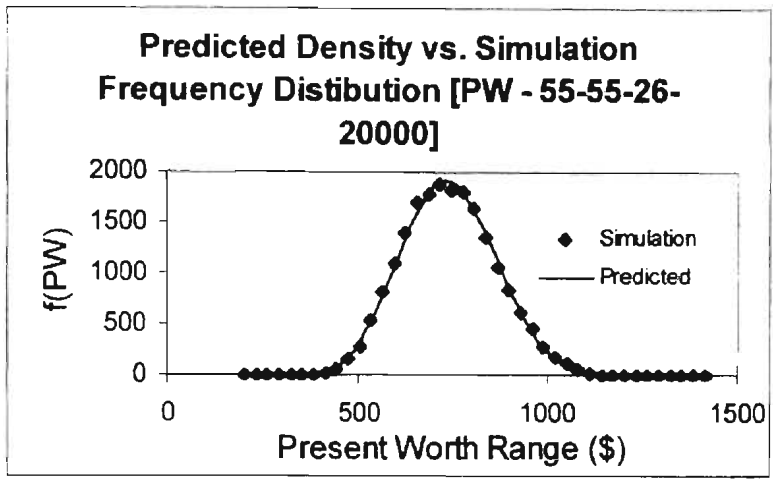

Figure 84. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

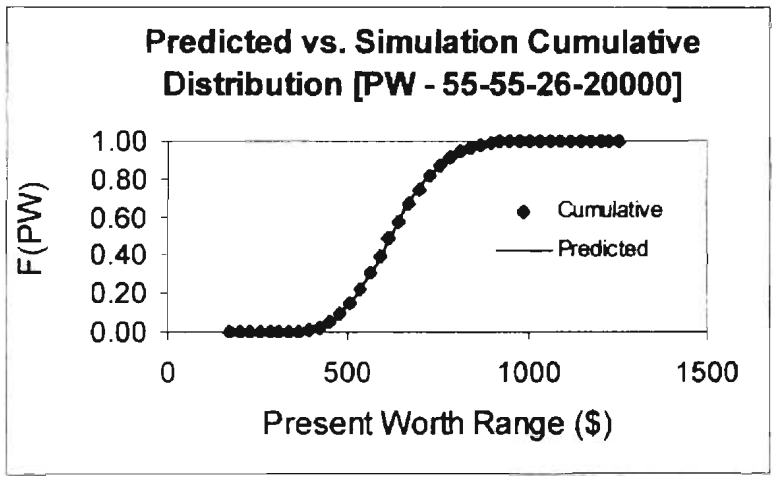

Figure 85. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Table 72

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	0	0
324.22	0	3	2
354.46	6	14	13
384.69	50	50	47
414.92	132	132	127
445.16	279	282	276
475.39	531	514	506
505.63	848	819	812
535.86	1,193	1,169	1,165
566.09	1,507	1,517	1,518
596.33	1,790	1,811	1,818
626.56	2,061	2,006	2,017
656.80	2,055	2,075	2,088
687.03	1,973	2,014	2,026
717.26	1,833	1,839	1,849
747.50	1,620	1,583	1,589
777.73	1,265	1,286	1,288
807.97	947	987	985
838.20	729	714	709
868.43	480	486	481
898.67	305	311	306
928.90	214	187	182
959.14	90	104	101
989.37	58	54	52
1,019.60	18	26	24
1,049.84	6	11	10
1,080.07	6	4	4
1,110.31	3	1	1
1,140.54	0	0	0
1,170.77	1	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 73

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Valu the Theoretical Density Values for the Present Worth when the Lump-Sum C Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Ra ~B(6,2)

Chi-squared Test - [PW - 55-26-62-20000]					
Theoretical Alpha		9.61		Test Statistic	Critical Value
Theoretical Beta		16.11			
Number Of Data Points		20000		28.26	33.92
alpha = 0.05					
Degrees of freedom		22			
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
6	8.44E-04	16.89	-10.89	118.56	7.02
50	2.48E-03	49.57	0.43	0.18	0.00
132	6.58E-03	131.58	0.42	0.18	0.00
279	1.41E-02	282.40	-3.40	11.56	0.04
531	2.57E-02	514.00	17.00	289.00	0.56
848	4.10E-02	819.20	28.80	829.44	1.01
1,193	5.84E-02	1,168.80	24.20	585.64	0.50
1,507	7.58E-02	1,516.80	-9.80	96.04	0.06
1,790	9.05E-02	1,810.80	-20.80	432.64	0.24
2,061	1.00E-01	2,006.20	54.80	3,003.04	1.50
2,055	1.04E-01	2,075.40	-20.40	416.16	0.20
1,973	1.01E-01	2,013.80	-40.80	1,664.64	0.83
1,833	9.19E-02	1,838.80	-5.80	33.64	0.02
1,620	7.92E-02	1,583.00	37.00	1,369.00	0.86
1,265	6.43E-02	1,286.40	-21.40	457.96	0.36
947	4.93E-02	986.60	-39.60	1,568.16	1.59
729	3.57E-02	713.80	15.20	231.04	0.32
480	2.43E-02	486.40	-6.40	40.96	0.08
305	1.56E-02	311.40	-6.40	40.96	0.13
214	9.33E-03	186.65	27.36	748.30	4.01
90	5.21E-03	104.24	-14.24	202.65	1.94
58	2.70E-03	53.92	4.08	16.65	0.31
18	1.28E-03	25.64	-7.64	58.35	2.28
6	5.55E-04	11.10	-5.10	25.98	2.34
10	3.19E-04	6.38	3.62	13.07	2.05

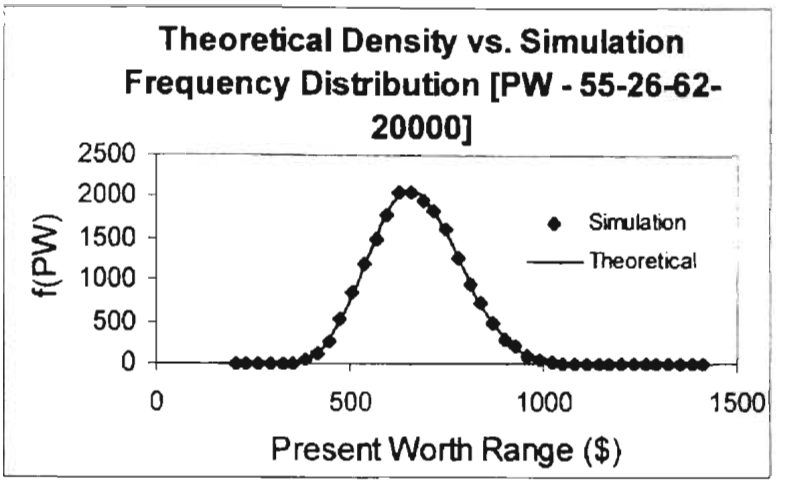

Figure 86. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

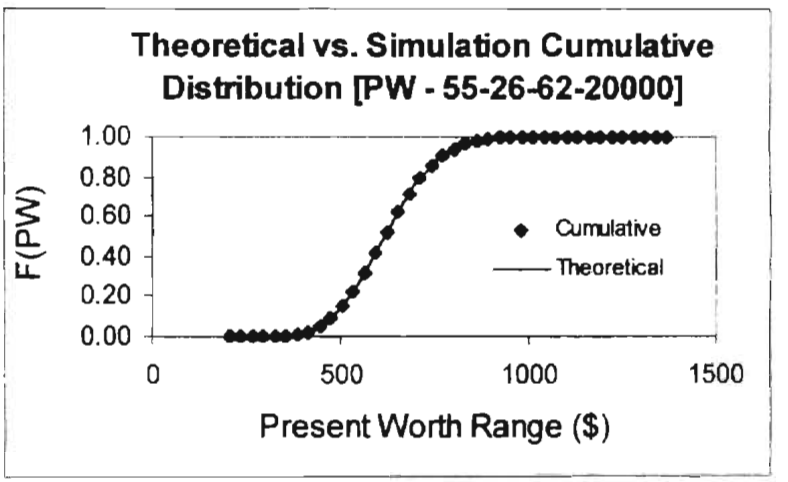

Figure 87. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Table 74

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 55-26-62-20000]					
Predicted Alpha			9.66	Test Statistic	Critical Value
Predicted Beta			16.37		
Number Of Data Points			20000	30.23	33.92
alpha = 0.05					
Degrees of freedom			22		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
6	7.87E-04	15.74	-9.74	94.93	6.03
50	2.36E-03	47.23	2.77	7.65	0.16
132	6.36E-03	127.10	4.90	23.97	0.19
279	1.38E-02	275.80	3.20	10.24	0.04
531	2.53E-02	506.40	24.60	605.16	1.20
848	4.06E-02	812.40	35.60	1,267.36	1.56
1,193	5.83E-02	1,165.40	27.60	761.76	0.65
1,507	7.59E-02	1,518.20	-11.20	125.44	0.08
1,790	9.09E-02	1,817.60	-27.60	761.76	0.42
2,061	1.01E-01	2,017.20	43.80	1,918.44	0.95
2,055	1.04E-01	2,088.40	-33.40	1,115.56	0.53
1,973	1.01E-01	2,026.40	-53.40	2,851.56	1.41
1,833	9.24E-02	1,848.60	-15.60	243.36	0.13
1,620	7.94E-02	1,588.80	31.20	973.44	0.61
1,265	6.44E-02	1,287.80	-22.80	519.84	0.40
947	4.92E-02	984.60	-37.60	1,413.76	1.44
729	3.55E-02	709.40	19.60	384.16	0.54
480	2.41E-02	481.00	-1.00	1.00	0.00
305	1.53E-02	306.20	-1.20	1.44	0.00
214	9.11E-03	182.30	31.70	1,005.04	5.51
90	5.05E-03	101.04	-11.04	121.81	1.21
58	2.59E-03	51.82	6.18	38.24	0.74
18	1.22E-03	24.40	-6.40	40.94	1.68
6	5.22E-04	10.44	-4.44	19.75	1.89
10	2.95E-04	5.90	4.10	16.84	2.86

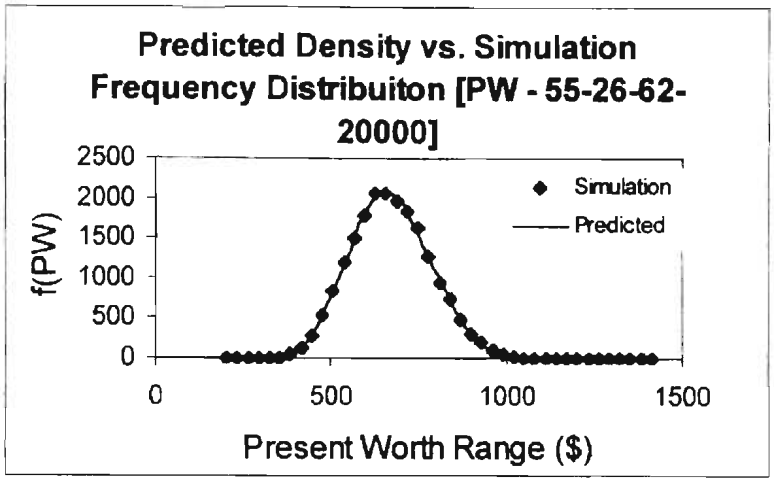

Figure 88. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

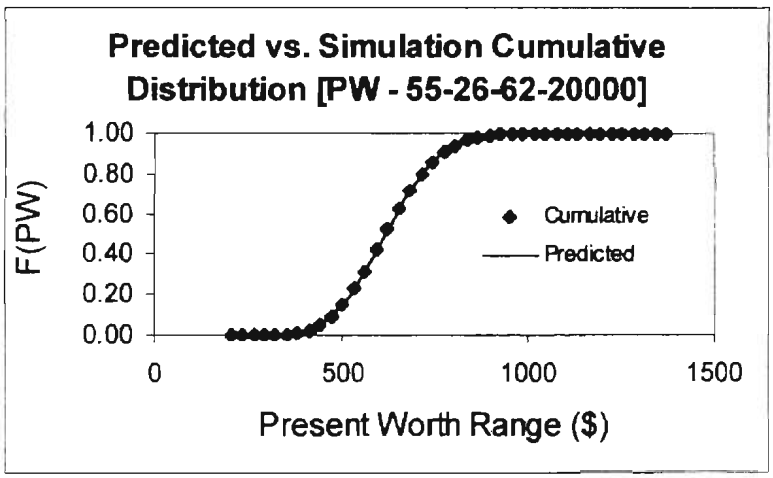

Figure 89. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Table 75

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	0	0
324.22	0	0	0
354.46	0	3	3
384.69	7	11	11
414.92	16	34	33
445.16	53	84	83
475.39	162	175	173
505.63	311	319	317
535.86	536	521	518
566.09	831	773	771
596.33	1,099	1,058	1,056
626.56	1,395	1,345	1,345
656.80	1,588	1,601	1,603
687.03	1,798	1,793	1,796
717.26	1,917	1,896	1,901
747.50	1,846	1,899	1,904
777.73	1,734	1,803	1,808
807.97	1,633	1,626	1,629
838.20	1,383	1,391	1,393
868.43	1,106	1,129	1,129
898.67	855	868	867
928.90	663	630	628
959.14	446	431	429
989.37	274	276	274
1,019.60	161	165	163
1,049.84	102	91	90
1,080.07	43	46	45
1,110.31	30	21	21
1,140.54	7	9	8
1,170.77	2	3	3
1,201.01	2	1	1
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 76

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 55-26-55-20000]					
Theoretical Alpha			9.95	Test Statistic	Critical Value
Theoretical Beta			13.08		
Number Of Data Points			20000	47.38	35.17
alpha = 0.05					
Degrees of freedom			23		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	7.15E-04	14.30	-7.30	53.22	3.72
16	1.70E-03	34.06	-18.06	326.32	9.58
53	4.20E-03	83.97	-30.97	959.31	11.42
162	8.75E-03	175.09	-13.09	171.37	0.98
311	1.60E-02	319.20	-8.20	67.24	0.21
536	2.60E-02	520.80	15.20	231.04	0.44
831	3.87E-02	773.40	57.60	3,317.76	4.29
1,099	5.29E-02	1,057.60	41.40	1,713.96	1.62
1,395	6.73E-02	1,345.00	50.00	2,500.00	1.86
1,588	8.00E-02	1,600.80	-12.80	163.84	0.10
1,798	8.96E-02	1,792.80	5.20	27.04	0.02
1,917	9.48E-02	1,896.00	21.00	441.00	0.23
1,846	9.49E-02	1,898.80	-52.80	2,787.84	1.47
1,734	9.02E-02	1,803.40	-69.40	4,816.36	2.67
1,633	8.13E-02	1,625.60	7.40	54.76	0.03
1,383	6.96E-02	1,391.00	-8.00	64.00	0.05
1,106	5.65E-02	1,129.00	-23.00	529.00	0.47
855	4.34E-02	867.60	-12.60	158.76	0.18
663	3.15E-02	630.00	33.00	1,089.00	1.73
446	2.15E-02	430.60	15.40	237.16	0.55
274	1.38E-02	275.80	-1.80	3.24	0.01
161	8.23E-03	164.66	-3.66	13.41	0.08
102	4.54E-03	90.87	11.13	123.77	1.36
43	2.30E-03	45.92	-2.92	8.50	0.19
30	1.05E-03	20.97	9.03	81.57	3.89
11	6.33E-04	12.66	-1.66	2.75	0.22

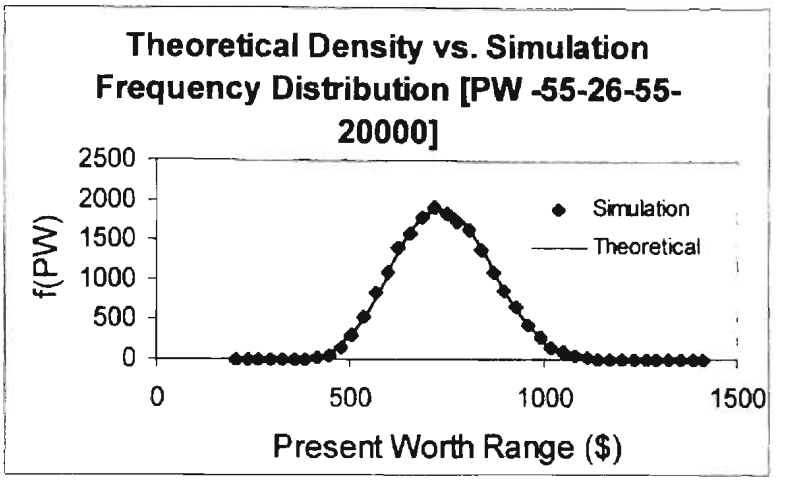

Figure 90. Graph Showing the Simulation Frequency versus the Theoretical Den Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lum Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

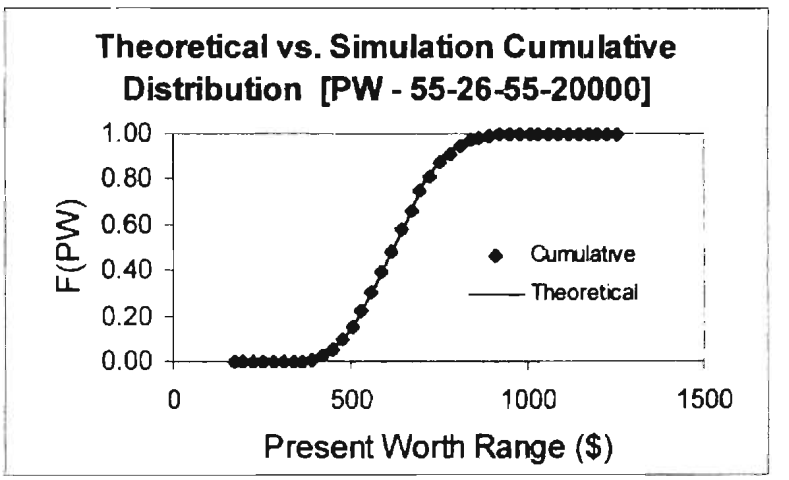

Figure 91. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Table 77

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Valı the Predicted Density Values for the Present Worth when the Lump-Sum Cash ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(

Chi-squared Test - [PW- 55-26-55-20000]				Test Statistic	Critical Value
Predicted Alpha			9.94		
Predicted Beta			13.21		
Number Of Data Points			20000	47.42	35.17
alpha = 0.05					
Degrees of freedom			23		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	6.94E-04	13.88	-6.88	47.32	3.41
16	1.67E-03	33.35	-17.35	301.11	9.03
53	4.13E-03	82.64	-29.64	878.65	10.63
162	8.65E-03	173.04	-11.04	121.99	0.70
311	1.58E-02	316.60	-5.60	31.36	0.10
536	2.59E-02	518.00	18.00	324.00	0.63
831	3.86E-02	771.00	60.00	3,600.00	4.67
1,099	5.28E-02	1,056.40	42.60	1,814.76	1.72
1,395	6.73E-02	1,345.20	49.80	2,480.04	1.84
1,588	8.02E-02	1,603.00	-15.00	225.00	0.14
1,798	8.98E-02	1,796.40	1.60	2.56	0.00
1,917	9.50E-02	1,900.80	16.20	262.44	0.14
1,846	9.52E-02	1,903.60	-57.60	3,317.76	1.74
1,734	9.04E-02	1,807.60	-73.60	5,416.96	3.00
1,633	8.14E-02	1,628.80	4.20	17.64	0.01
1,383	6.96E-02	1,392.80	-9.80	96.04	0.07
1,106	5.65E-02	1,129.20	-23.20	538.24	0.48
855	4.33E-02	866.60	-11.60	134.56	0.16
663	3.14E-02	628.20	34.80	1,211.04	1.93
446	2.14E-02	428.60	17.40	302.76	0.71
274	1.37E-02	273.80	0.20	0.04	0.00
161	8.15E-03	163.02	-2.02	4.10	0.03
102	4.48E-03	89.69	12.31	151.64	1.69
43	2.26E-03	45.15	-2.15	4.62	0.10
30	1.03E-03	20.53	9.47	89.64	4.37
11	6.15E-04	12.30	-1.30	1.69	0.14

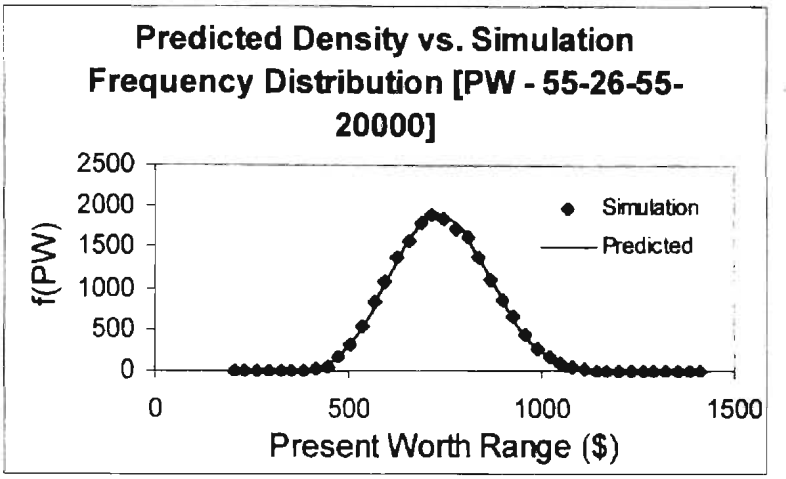

Figure 92. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

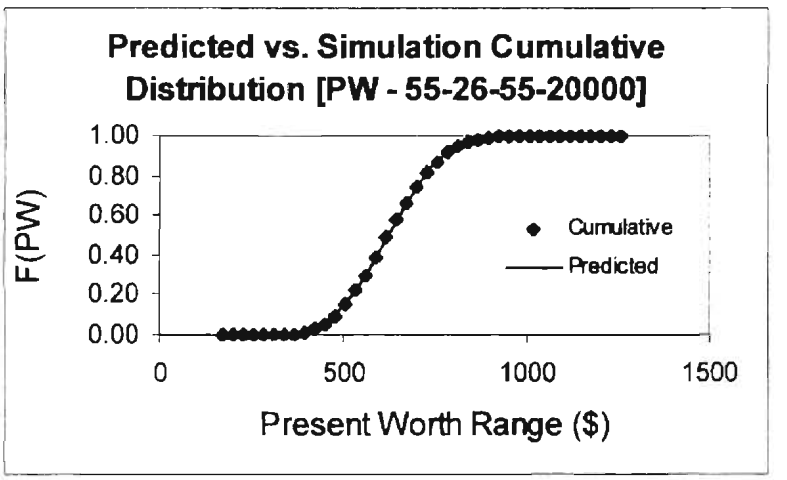

Figure 93. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Table 78

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	0	0
324.22	0	0	0
354.46	0	1	1
384.69	1	3	3
414.92	1	9	9
445.16	10	24	24
475.39	28	55	56
505.63	87	111	112
535.86	163	200	201
566.09	309	330	331
596.33	490	501	502
626.56	766	710	710
656.80	995	944	944
687.03	1,309	1,186	1,186
717.26	1,492	1,413	1,412
747.50	1,669	1,600	1,598
777.73	1,740	1,728	1,725
807.97	1,673	1,780	1,777
838.20	1,739	1,751	1,748
868.43	1,513	1,644	1,643
898.67	1,410	1,473	1,472
928.90	1,290	1,258	1,258
959.14	1,013	1,020	1,021
989.37	752	784	786
1,019.60	569	569	570
1,049.84	396	387	388
1,080.07	259	245	246
1,110.31	154	143	144
1,140.54	89	76	76
1,170.77	52	36	36
1,201.01	18	15	15
1,231.24	7	5	5
1,261.48	4	1	1
1,291.71	2	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 79

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 55-26-26-20000]				Test Statistic	Critical Value
Theoretical Alpha			10.08		
Theoretical Beta			10.41		
Number Of Data Points			20000	115.22	37.65
alpha = 0.05					
Degrees of freedom			25		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
12	1.80E-03	36.03	-24.03	577.52	16.03
28	2.75E-03	55.09	-27.09	733.76	13.32
87	5.55E-03	110.93	-23.93	572.79	5.16
163	1.00E-02	200.20	-37.20	1,383.84	6.91
309	1.65E-02	329.60	-20.60	424.36	1.29
490	2.50E-02	500.80	-10.80	116.64	0.23
766	3.55E-02	709.80	56.20	3,158.44	4.45
995	4.72E-02	944.20	50.80	2,580.64	2.73
1,309	5.93E-02	1,186.20	122.80	15,079.84	12.71
1,492	7.07E-02	1,413.00	79.00	6,241.00	4.42
1,669	8.00E-02	1,600.40	68.60	4,705.96	2.94
1,740	8.64E-02	1,727.60	12.40	153.76	0.09
1,673	8.90E-02	1,779.60	-106.60	11,363.56	6.39
1,739	8.75E-02	1,750.60	-11.60	134.56	0.08
1,513	8.22E-02	1,644.20	-131.20	17,213.44	10.47
1,410	7.37E-02	1,473.40	-63.40	4,019.56	2.73
1,290	6.29E-02	1,257.80	32.20	1,036.84	0.82
1,013	5.10E-02	1,020.40	-7.40	54.76	0.05
752	3.92E-02	784.20	-32.20	1,036.84	1.32
569	2.84E-02	568.60	0.40	0.16	0.00
396	1.93E-02	386.60	9.40	88.36	0.23
259	1.22E-02	244.60	14.40	207.36	0.85
154	7.13E-03	142.63	11.37	129.19	0.91
89	3.78E-03	75.65	13.35	178.19	2.36
52	1.79E-03	35.85	16.15	260.73	7.27
18	7.41E-04	14.81	3.19	10.14	0.68
7	2.58E-04	5.15	1.85	3.41	0.66
6	8.84E-05	1.77	4.23	17.90	10.12

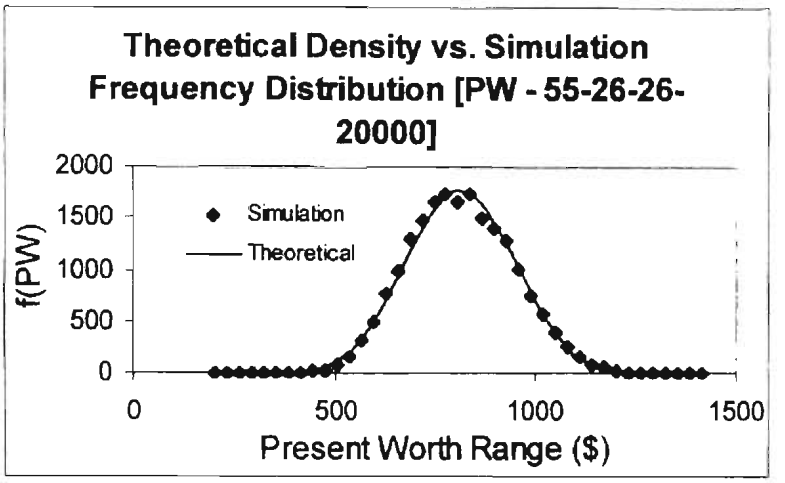

Figure 94. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

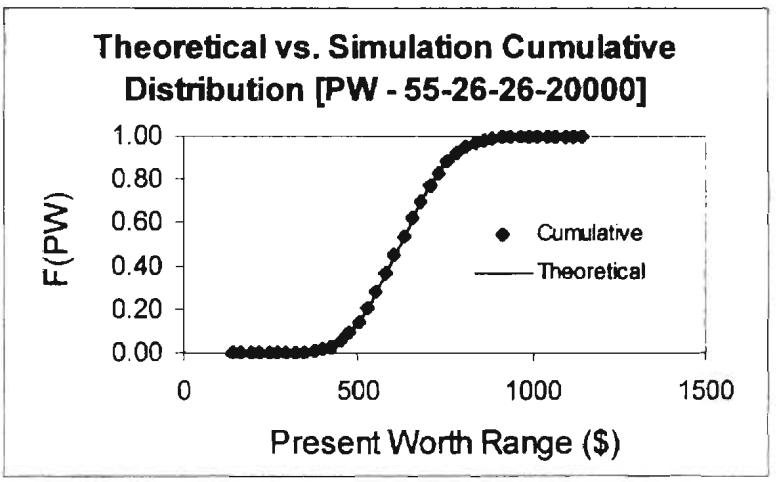

Figure 95. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Table 80

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 55-26-26-20000]					
Predicted Alpha			10.01	Test Statistic	Critical Value
Predicted Beta			10.46		
Number Of Data Points			20000	114.95	37.65
alpha = 0.05					
Degrees of freedom			25		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
12	1.82E-03	36.48	-24.48	599.08	16.42
28	2.78E-03	55.55	-27.55	759.11	13.66
87	5.58E-03	111.64	-24.64	606.99	5.44
163	1.01E-02	201.20	-38.20	1,459.24	7.25
309	1.65E-02	330.60	-21.60	466.56	1.41
490	2.51E-02	501.80	-11.80	139.24	0.28
766	3.55E-02	710.40	55.60	3,091.36	4.35
995	4.72E-02	944.40	50.60	2,560.36	2.71
1,309	5.93E-02	1,185.60	123.40	15,227.56	12.84
1,492	7.06E-02	1,411.60	80.40	6,464.16	4.58
1,669	7.99E-02	1,598.40	70.60	4,984.36	3.12
1,740	8.63E-02	1,725.20	14.80	219.04	0.13
1,673	8.89E-02	1,777.20	-104.20	10,857.64	6.11
1,739	8.74E-02	1,748.40	-9.40	88.36	0.05
1,513	8.21E-02	1,642.60	-129.60	16,796.16	10.23
1,410	7.36E-02	1,472.40	-62.40	3,893.76	2.64
1,290	6.29E-02	1,257.60	32.40	1,049.76	0.83
1,013	5.11E-02	1,021.20	-8.20	67.24	0.07
752	3.93E-02	785.60	-33.60	1,128.96	1.44
569	2.85E-02	570.00	-1.00	1.00	0.00
396	1.94E-02	388.00	8.00	64.00	0.16
259	1.23E-02	246.00	13.00	169.00	0.69
154	7.18E-03	143.66	10.34	106.82	0.74
89	3.82E-03	76.36	12.64	159.89	2.09
52	1.81E-03	36.27	15.73	247.31	6.82
18	7.52E-04	15.03	2.97	8.81	0.59
7	2.62E-04	5.25	1.75	3.07	0.59
6	9.05E-05	1.81	4.19	17.56	9.70

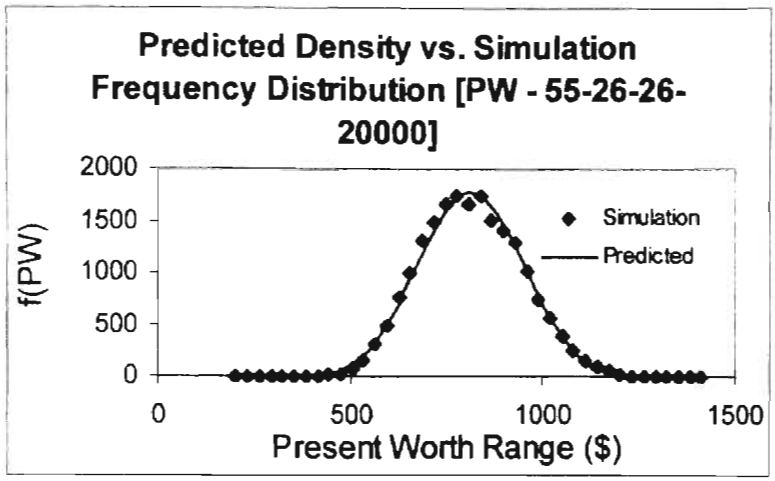

Figure 96. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

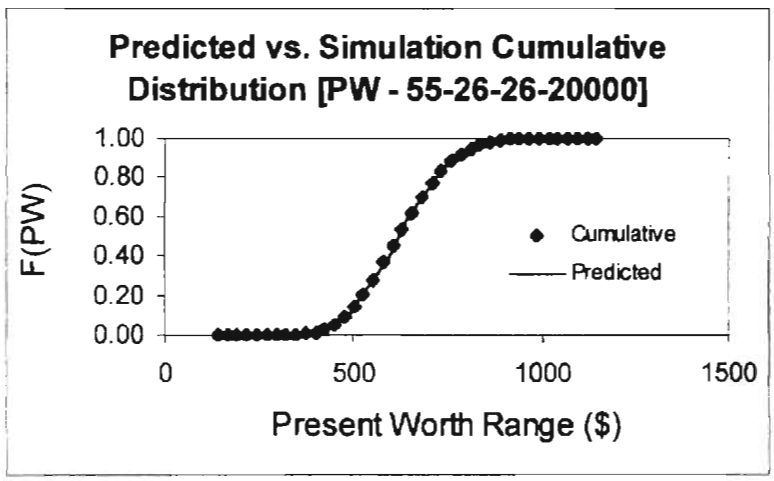

Figure 97. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(5,5), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Table 81

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	2	10	8
263.75	78	159	144
293.99	513	584	555
324.22	1,156	1,201	1,174
354.46	1,929	1,815	1,805
384.69	2,416	2,263	2,275
414.92	2,637	2,467	2,495
445.16	2,438	2,431	2,465
475.39	2,186	2,209	2,240
505.63	1,824	1,877	1,899
535.86	1,461	1,504	1,514
566.09	1,063	1,143	1,144
596.33	805	827	821
626.56	520	572	562
656.80	393	378	368
687.03	228	239	230
717.26	147	145	137
747.50	88	84	78
777.73	45	46	43
807.97	36	24	22
838.20	20	12	11
868.43	10	6	5
898.67	3	3	2
928.90	1	1	1
959.14	1	0	0
989.37	0	0	0
1,019.60	0	0	0
1,049.84	0	0	0
1,080.07	0	0	0
1,110.31	0	0	0
1,140.54	0	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 82

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 26-62-62-20000]					
Theoretical Alpha			4.59	Test Statistic	Critical Value
Theoretical Beta			18.46		
Number Of Data Points			20000	114.97	30.14
alpha = 0.05					
Degrees of freedom			19		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
80	8.44E-03	168.74	-88.74	7,874.29	46.67
513	2.92E-02	584.00	-71.00	5,041.00	8.63
1,156	6.00E-02	1,200.80	-44.80	2,007.04	1.67
1,929	9.07E-02	1,814.80	114.20	13,041.64	7.19
2,416	1.13E-01	2,262.60	153.40	23,531.56	10.40
2,637	1.23E-01	2,466.80	170.20	28,968.04	11.74
2,438	1.22E-01	2,430.60	7.40	54.76	0.02
2,186	1.10E-01	2,209.40	-23.40	547.56	0.25
1,824	9.39E-02	1,877.00	-53.00	2,809.00	1.50
1,461	7.52E-02	1,503.80	-42.80	1,831.84	1.22
1,063	5.71E-02	1,142.80	-79.80	6,368.04	5.57
805	4.14E-02	827.20	-22.20	492.84	0.60
520	2.86E-02	571.80	-51.80	2,683.24	4.69
393	1.89E-02	378.00	15.00	225.00	0.60
228	1.20E-02	239.20	-11.20	125.44	0.52
147	7.24E-03	144.89	2.11	4.46	0.03
88	4.20E-03	83.91	4.09	16.73	0.20
45	2.32E-03	46.41	-1.41	1.98	0.04
36	1.22E-03	24.46	11.54	133.10	5.44
20	6.13E-04	12.26	7.74	59.94	4.89
10	2.91E-04	5.82	4.18	17.48	3.00
5	2.18E-04	4.36	0.64	0.41	0.09

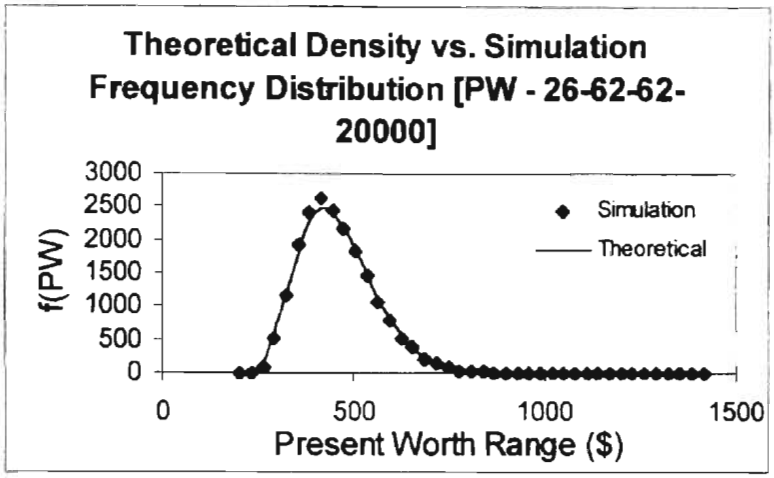

Figure 98. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

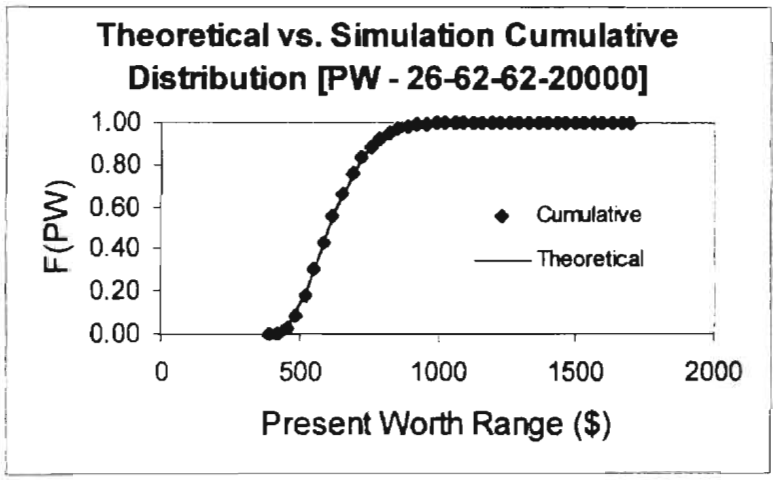

Figure 99. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Table 83

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 26-62-62-20000]					
Predicted Alpha	4.64			Test Statistic	Critical Value
Predicted Beta	18.93				
Number Of Data Points	20000			104.59	30.14
alpha = 0.05					
Degrees of freedom	19				
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
80	7.62E-03	152.44	-72.44	5,247.84	34.43
513	2.78E-02	555.20	-42.20	1,780.84	3.21
1,156	5.87E-02	1,173.60	-17.60	309.76	0.26
1,929	9.03E-02	1,805.00	124.00	15,376.00	8.52
2,416	1.14E-01	2,275.00	141.00	19,881.00	8.74
2,637	1.25E-01	2,495.40	141.60	20,050.56	8.04
2,438	1.23E-01	2,465.40	-27.40	750.76	0.30
2,186	1.12E-01	2,240.40	-54.40	2,959.36	1.32
1,824	9.49E-02	1,898.60	-74.60	5,565.16	2.93
1,461	7.57E-02	1,514.20	-53.20	2,830.24	1.87
1,063	5.72E-02	1,143.60	-80.60	6,496.36	5.68
805	4.11E-02	821.20	-16.20	262.44	0.32
520	2.81E-02	562.40	-42.40	1,797.76	3.20
393	1.84E-02	367.80	25.20	635.04	1.73
228	1.15E-02	229.80	-1.80	3.24	0.01
147	6.86E-03	137.28	9.72	94.39	0.69
88	3.92E-03	78.31	9.69	93.98	1.20
45	2.13E-03	42.59	2.41	5.79	0.14
36	1.10E-03	22.05	13.95	194.66	8.83
20	5.42E-04	10.83	9.17	84.06	7.76
10	2.52E-04	5.03	4.97	24.68	4.90
5	1.81E-04	3.62	1.38	1.90	0.52

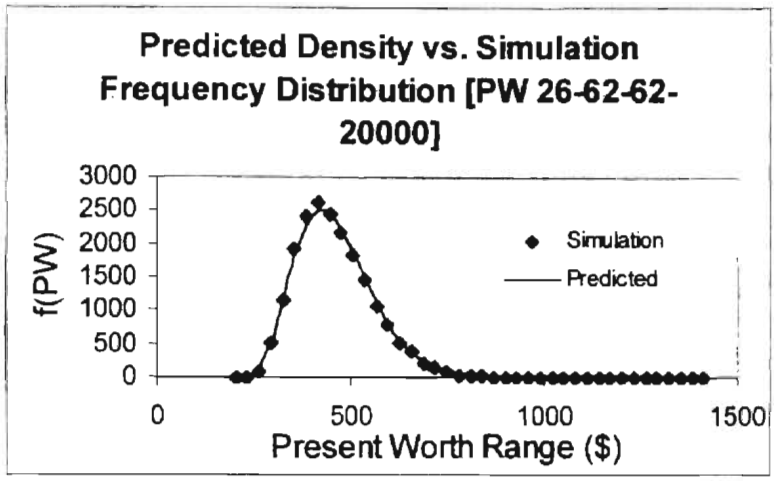

Figure 100. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

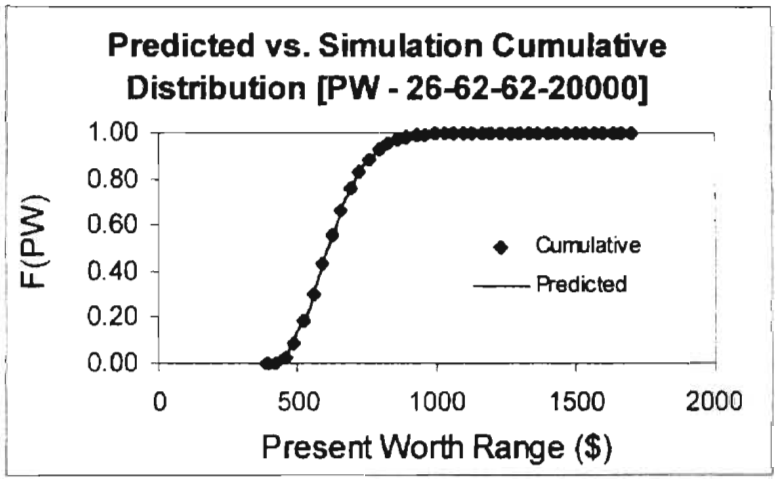

Figure 101. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(6,2)

Table 84

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	18	16
293.99	59	130	121
324.22	304	420	404
354.46	805	891	873
384.69	1,527	1,449	1,438
414.92	2,208	1,957	1,959
445.16	2,488	2,303	2,318
475.39	2,570	2,430	2,453
505.63	2,278	2,343	2,367
535.86	2,054	2,091	2,110
566.09	1,645	1,743	1,754
596.33	1,200	1,366	1,369
626.56	947	1,010	1,007
656.80	696	707	700
687.03	466	469	461
717.26	299	295	288
747.50	176	176	170
777.73	123	100	95
807.97	70	53	50
838.20	38	27	25
868.43	17	13	12
898.67	10	6	5
928.90	13	2	2
959.14	4	1	1
989.37	2	0	0
1,019.60	1	0	0
1,049.84	0	0	0
1,080.07	0	0	0
1,110.31	0	0	0
1,140.54	0	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 85

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 26-62-55-20000]					
Theoretical Alpha			6.13	Test Statistic	Critical Value
Theoretical Beta			19.86		
Number Of Data Points			20000	277.23	31.41
alpha = 0.05					
Degrees of freedom			20		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
59	7.43E-03	148.68	-89.68	8,042.55	54.09
304	2.10E-02	420.40	-116.40	13,548.96	32.23
805	4.46E-02	891.20	-86.20	7,430.44	8.34
1,527	7.24E-02	1,448.60	78.40	6,146.56	4.24
2,208	9.79E-02	1,957.20	250.80	62,900.64	32.14
2,488	1.15E-01	2,303.00	185.00	34,225.00	14.86
2,570	1.22E-01	2,430.00	140.00	19,600.00	8.07
2,278	1.17E-01	2,342.80	-64.80	4,199.04	1.79
2,054	1.05E-01	2,091.00	-37.00	1,369.00	0.65
1,645	8.72E-02	1,743.00	-98.00	9,604.00	5.51
1,200	6.83E-02	1,365.60	-165.60	27,423.36	20.08
947	5.05E-02	1,009.80	-62.80	3,943.84	3.91
696	3.53E-02	706.60	-10.60	112.36	0.16
466	2.35E-02	469.00	-3.00	9.00	0.02
299	1.48E-02	295.20	3.80	14.44	0.05
176	8.81E-03	176.24	-0.24	0.06	0.00
123	4.99E-03	99.72	23.28	542.08	5.44
70	2.67E-03	53.37	16.63	276.48	5.18
38	1.35E-03	26.96	11.04	121.98	4.53
17	6.40E-04	12.80	4.20	17.61	1.38
10	2.85E-04	5.69	4.31	18.54	3.25
13	1.18E-04	2.36	10.64	113.23	47.99
7	6.81E-05	1.36	5.64	31.78	23.33

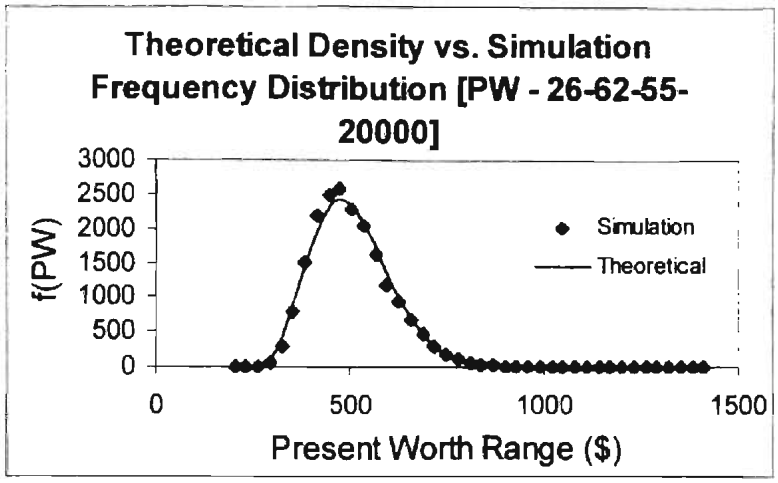

Figure 102. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

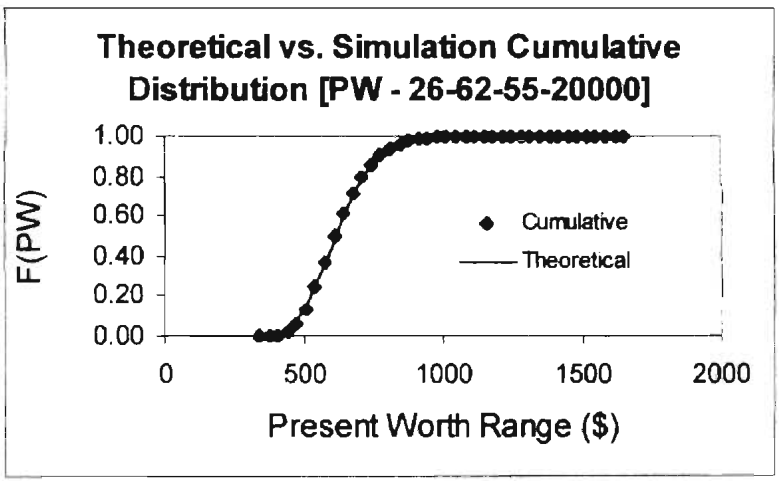

Figure 103. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Table 86

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 26-62-55-20000]					
Predicted Alpha			6.17	Test Statistic	Critical Value
Predicted Beta			20.23		
Number Of Data Points			20000	283.44	31.41
alpha = 0.05					
Degrees of freedom			20		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
59	6.91E-03	138.22	-79.22	6,275.24	45.40
304	2.02E-02	404.20	-100.20	10,040.04	24.84
805	4.37E-02	873.20	-68.20	4,651.24	5.33
1,527	7.19E-02	1,437.80	89.20	7,956.64	5.53
2,208	9.80E-02	1,959.40	248.60	61,801.96	31.54
2,488	1.16E-01	2,318.20	169.80	28,832.04	12.44
2,570	1.23E-01	2,452.80	117.20	13,735.84	5.60
2,278	1.18E-01	2,366.80	-88.80	7,885.44	3.33
2,054	1.06E-01	2,110.20	-56.20	3,158.44	1.50
1,645	8.77E-02	1,754.40	-109.40	11,968.36	6.82
1,200	6.85E-02	1,369.00	-169.00	28,561.00	20.86
947	5.03E-02	1,006.80	-59.80	3,576.04	3.55
696	3.50E-02	700.00	-4.00	16.00	0.02
466	2.31E-02	461.00	5.00	25.00	0.05
299	1.44E-02	287.60	11.40	129.96	0.45
176	8.50E-03	169.96	6.04	36.44	0.21
123	4.75E-03	95.08	27.92	779.34	8.20
70	2.51E-03	50.26	19.74	389.74	7.75
38	1.25E-03	25.04	12.96	168.09	6.71
17	5.86E-04	11.71	5.29	27.96	2.39
10	2.56E-04	5.12	4.88	23.77	4.64
13	1.04E-04	2.08	10.92	119.14	57.15
7	5.84E-05	1.17	5.83	34.01	29.11

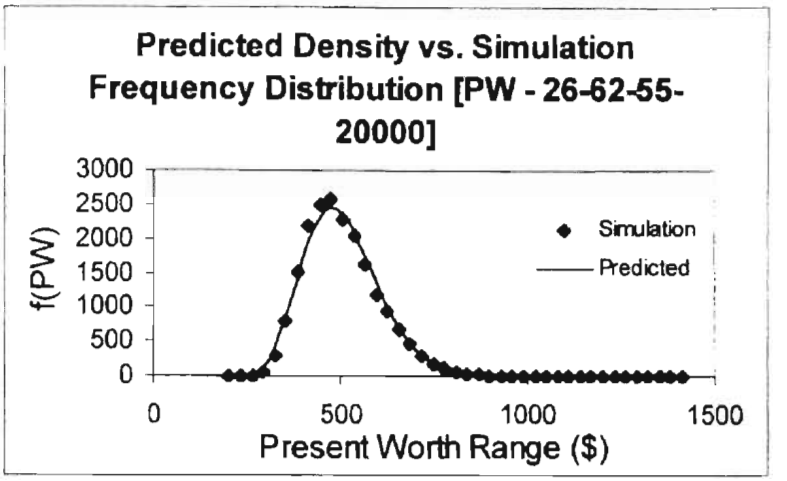

Figure 104. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(5,5)

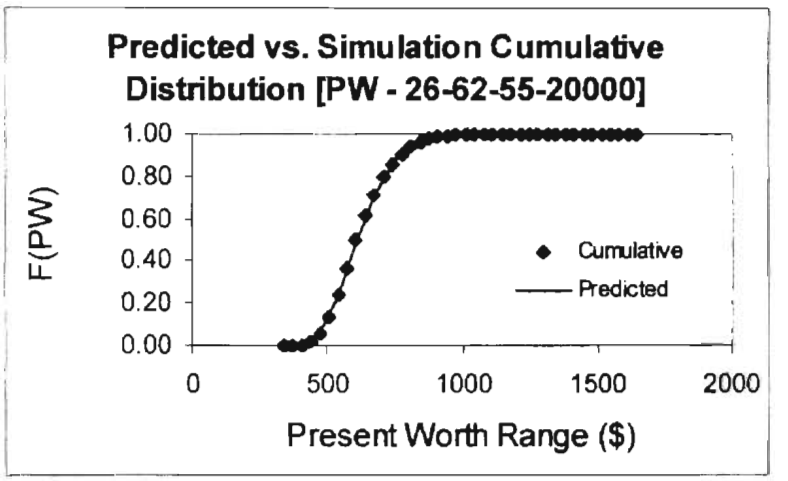

Figure 105. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Table 87

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	2	1
293.99	6	22	20
324.22	37	111	107
354.46	202	335	327
384.69	609	720	710
414.92	1,291	1,222	1,215
445.16	1,939	1,741	1,740
475.39	2,396	2,159	2,166
505.63	2,518	2,392	2,404
535.86	2,442	2,407	2,422
566.09	2,160	2,227	2,240
596.33	1,808	1,910	1,919
626.56	1,442	1,529	1,532
656.80	1,054	1,146	1,145
687.03	753	807	803
717.26	541	534	529
747.50	307	333	328
777.73	235	195	191
807.97	96	107	104
838.20	64	55	53
868.43	57	27	25
898.67	20	12	11
928.90	12	5	5
959.14	5	2	2
989.37	3	1	1
1,019.60	2	0	0
1,049.84	1	0	0
1,080.07	0	0	0
1,110.31	0	0	0
1,140.54	0	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 88

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate B(2,6)

Chi-squared Test - [PW - 26-62-26-20000]					
Theoretical Alpha			7.74	Test Statistic	Critical Value
Theoretical Beta			21.06		
Number Of Data Points			20000	310.94	32.67
alpha = 0.05					
Degrees of freedom			21		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
6	1.16E-03	23.15	-17.15	294.13	12.71
37	5.55E-03	111.08	-74.08	5,487.37	49.40
202	1.68E-02	335.00	-133.00	17,689.00	52.80
609	3.60E-02	720.20	-111.20	12,365.44	17.17
1,291	6.11E-02	1,222.40	68.60	4,705.96	3.85
1,939	8.70E-02	1,740.80	198.20	39,283.24	22.57
2,396	1.08E-01	2,159.20	236.80	56,074.24	25.97
2,518	1.20E-01	2,391.80	126.20	15,926.44	6.66
2,442	1.20E-01	2,407.00	35.00	1,225.00	0.51
2,160	1.11E-01	2,227.00	-67.00	4,489.00	2.02
1,808	9.55E-02	1,910.40	-102.40	10,485.76	5.49
1,442	7.64E-02	1,528.80	-86.80	7,534.24	4.93
1,054	5.73E-02	1,146.00	-92.00	8,464.00	7.39
753	4.03E-02	806.60	-53.60	2,872.96	3.56
541	2.67E-02	534.00	7.00	49.00	0.09
307	1.66E-02	332.80	-25.80	665.64	2.00
235	9.75E-03	194.95	40.05	1,604.24	8.23
96	5.36E-03	107.27	-11.27	127.01	1.18
64	2.77E-03	55.31	8.69	75.57	1.37
57	1.33E-03	26.63	30.37	922.17	34.63
20	5.96E-04	11.93	8.07	65.17	5.46
12	2.47E-04	4.94	7.06	49.84	10.09
5	9.40E-05	1.88	3.12	9.74	5.18
6	4.64E-05	0.93	5.07	25.72	27.70

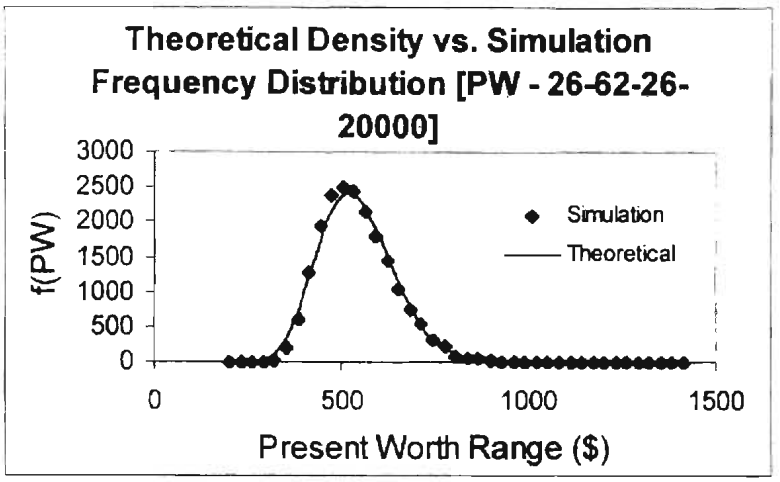

Figure 106. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

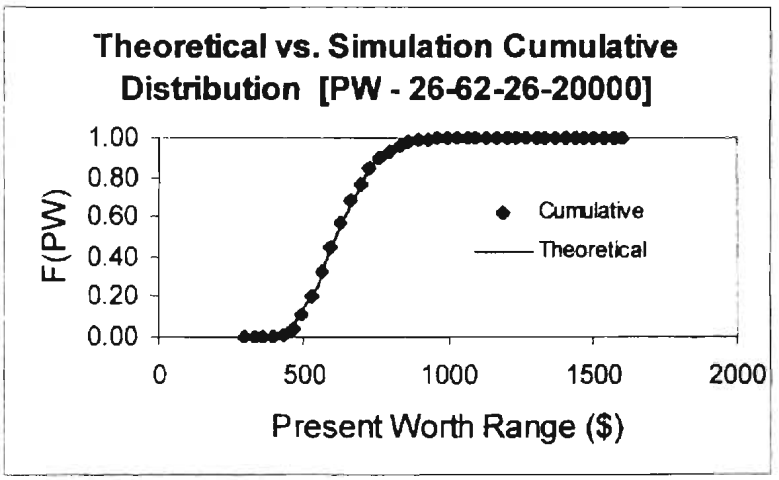

Figure 107. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Table 89

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 26-62-26-20000]					
Predicted Alpha			7.75	Test Statistic	Critical Value
Predicted Beta			21.32		
Number Of Data Points			20000	311.25	32.67
alpha = 0.05					
Degrees of freedom			21		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
6	1.08E-03	21.70	-15.70	246.35	11.35
37	5.33E-03	106.67	-69.67	4,853.27	45.50
202	1.63E-02	326.80	-124.80	15,575.04	47.66
609	3.55E-02	710.00	-101.00	10,201.00	14.37
1,291	6.08E-02	1,215.00	76.00	5,776.00	4.75
1,939	8.70E-02	1,739.80	199.20	39,680.64	22.81
2,396	1.08E-01	2,165.80	230.20	52,992.04	24.47
2,518	1.20E-01	2,404.40	113.60	12,904.96	5.37
2,442	1.21E-01	2,421.60	20.40	416.16	0.17
2,160	1.12E-01	2,239.80	-79.80	6,368.04	2.84
1,808	9.60E-02	1,919.00	-111.00	12,321.00	6.42
1,442	7.66E-02	1,532.20	-90.20	8,136.04	5.31
1,054	5.73E-02	1,145.00	-91.00	8,281.00	7.23
753	4.01E-02	802.80	-49.80	2,480.04	3.09
541	2.65E-02	529.00	12.00	144.00	0.27
307	1.64E-02	327.80	-20.80	432.64	1.32
235	9.54E-03	190.82	44.18	1,951.66	10.23
96	5.21E-03	104.25	-8.25	68.07	0.65
64	2.67E-03	53.32	10.68	114.03	2.14
57	1.27E-03	25.45	31.55	995.44	39.11
20	5.64E-04	11.29	8.71	75.94	6.73
12	2.31E-04	4.62	7.38	54.40	11.76
5	8.69E-05	1.74	3.26	10.64	6.12
6	4.21E-05	0.84	5.16	26.60	31.56

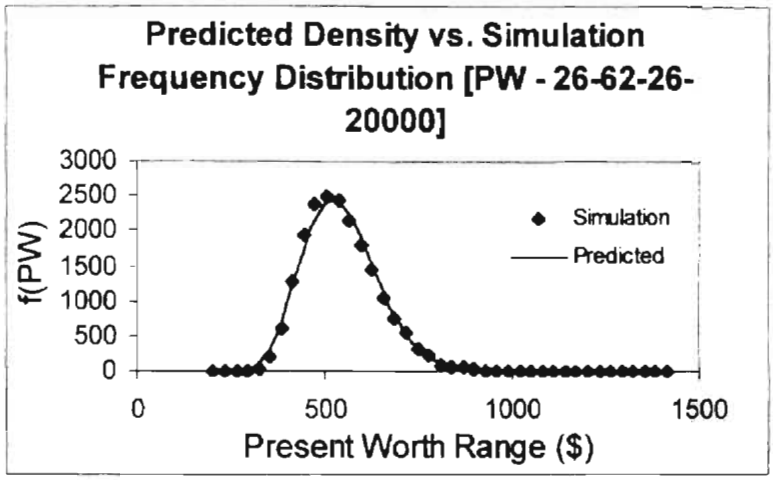

Figure 108. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

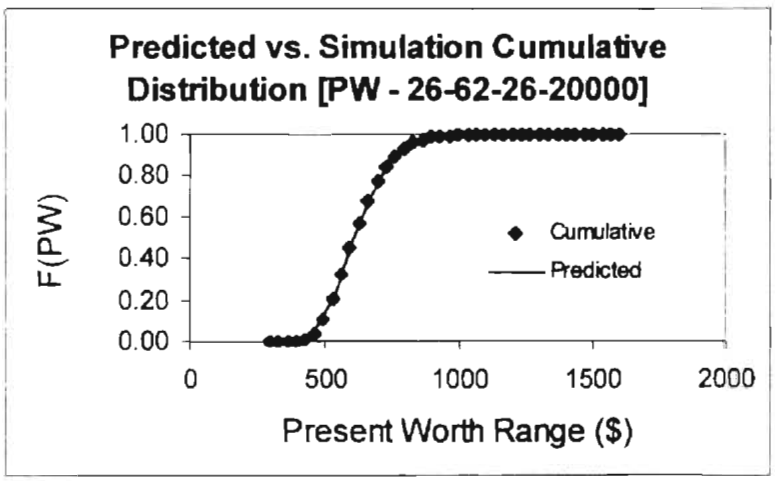

Figure 109. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(6,2) and the Interest Rate ~B(2,6)

Table 90

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	2	18	16
293.99	57	130	121
324.22	295	420	404
354.46	851	891	873
384.69	1,491	1,449	1,438
414.92	2,135	1,957	1,959
445.16	2,472	2,303	2,318
475.39	2,580	2,430	2,453
505.63	2,279	2,343	2,367
535.86	2,071	2,091	2,110
566.09	1,624	1,743	1,754
596.33	1,291	1,366	1,369
626.56	930	1,010	1,007
656.80	653	707	700
687.03	492	469	461
717.26	317	295	288
747.50	206	176	170
777.73	119	100	95
807.97	57	53	50
838.20	39	27	25
868.43	19	13	12
898.67	13	6	5
928.90	2	2	2
959.14	2	1	1
989.37	3	0	0
1,019.60	0	0	0
1,049.84	0	0	0
1,080.07	0	0	0
1,110.31	0	0	0
1,140.54	0	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 91

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 26-55-62-20000]				Test Statistic	Critical Value
Theoretical Alpha			6.13		
Theoretical Beta			19.86		
Number Of Data Points			20000	189.24	30.14
alpha = 0.05					
Degrees of freedom			19		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
59	7.43E-03	148.68	-89.68	8,042.55	54.09
295	2.10E-02	420.40	-125.40	15,725.16	37.41
851	4.46E-02	891.20	-40.20	1,616.04	1.81
1,491	7.24E-02	1,448.60	42.40	1,797.76	1.24
2,135	9.79E-02	1,957.20	177.80	31,612.84	16.15
2,472	1.15E-01	2,303.00	169.00	28,561.00	12.40
2,580	1.22E-01	2,430.00	150.00	22,500.00	9.26
2,279	1.17E-01	2,342.80	-63.80	4,070.44	1.74
2,071	1.05E-01	2,091.00	-20.00	400.00	0.19
1,624	8.72E-02	1,743.00	-119.00	14,161.00	8.12
1,291	6.83E-02	1,365.60	-74.60	5,565.16	4.08
930	5.05E-02	1,009.80	-79.80	6,368.04	6.31
653	3.53E-02	706.60	-53.60	2,872.96	4.07
492	2.35E-02	469.00	23.00	529.00	1.13
317	1.48E-02	295.20	21.80	475.24	1.61
206	8.81E-03	176.24	29.76	885.57	5.02
119	4.99E-03	99.72	19.28	371.82	3.73
57	2.67E-03	53.37	3.63	13.16	0.25
39	1.35E-03	26.96	12.04	145.07	5.38
19	6.40E-04	12.80	6.20	38.40	3.00
13	2.85E-04	5.69	7.31	53.37	9.37
7	1.86E-04	3.72	3.28	10.75	2.89

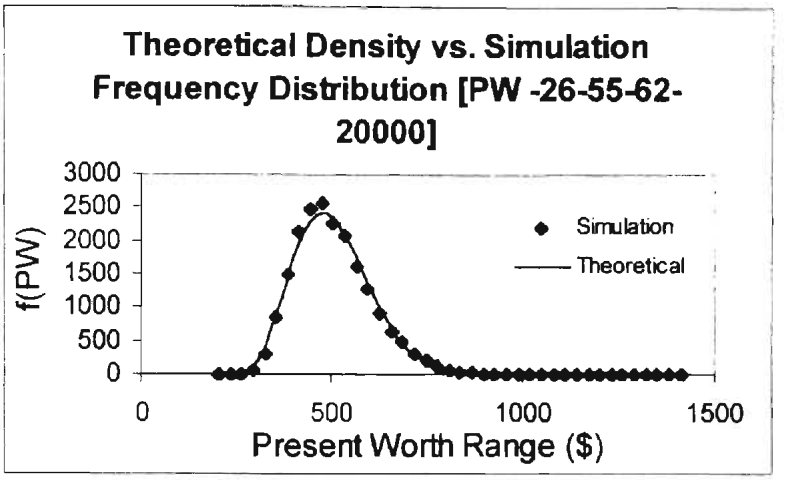

Figure 110. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

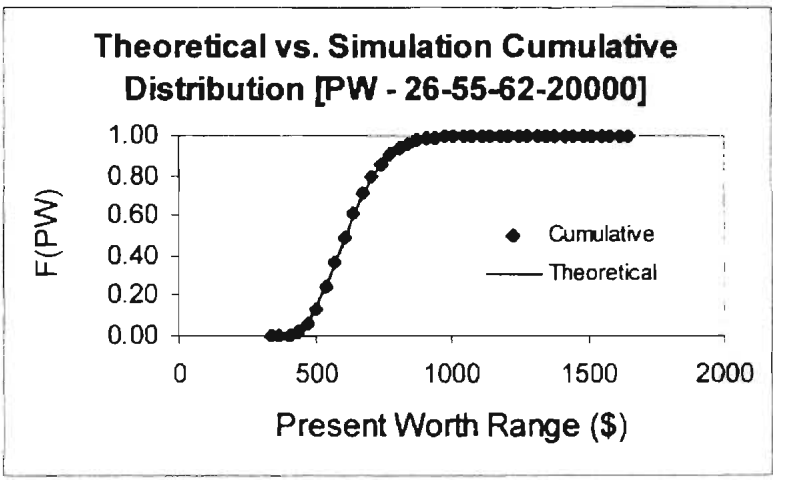

Figure 111. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Table 92

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 26-55-62-20000]				Test Statistic	Critical Value
Predicted Alpha			0		
Predicted Beta			0		
Number Of Data Points			20000	185.51	30.14
alpha = 0.05					
Degrees of freedom			19		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
59	6.91E-03	138.22	-79.22	6275.24	45.40
295	2.02E-02	404.20	-109.20	11924.64	29.50
851	4.37E-02	873.20	-22.20	492.84	0.56
1491	7.19E-02	1437.80	53.20	2830.24	1.97
2135	9.80E-02	1959.40	175.60	30835.36	15.74
2472	1.16E-01	2318.20	153.80	23654.44	10.20
2580	1.23E-01	2452.80	127.20	16179.84	6.60
2279	1.18E-01	2366.80	-87.80	7708.84	3.26
2071	1.06E-01	2110.20	-39.20	1536.64	0.73
1624	8.77E-02	1754.40	-130.40	17004.16	9.69
1291	6.85E-02	1369.00	-78.00	6084.00	4.44
930	5.03E-02	1006.80	-76.80	5898.24	5.86
653	3.50E-02	700.00	-47.00	2209.00	3.16
492	2.31E-02	461.00	31.00	961.00	2.08
317	1.44E-02	287.60	29.40	864.36	3.01
206	8.50E-03	169.96	36.04	1298.64	7.64
119	4.75E-03	95.08	23.92	572.00	6.02
57	2.51E-03	50.26	6.74	45.45	0.90
39	1.25E-03	25.04	13.96	195.02	7.79
19	5.86E-04	11.71	7.29	53.11	4.53
13	2.56E-04	5.12	7.88	62.03	12.11
7	1.63E-04	3.25	3.75	14.04	4.32

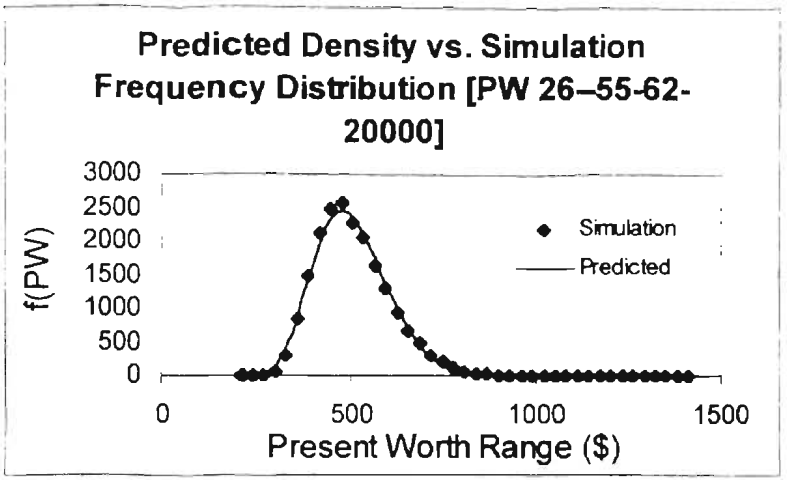

Figure 112. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

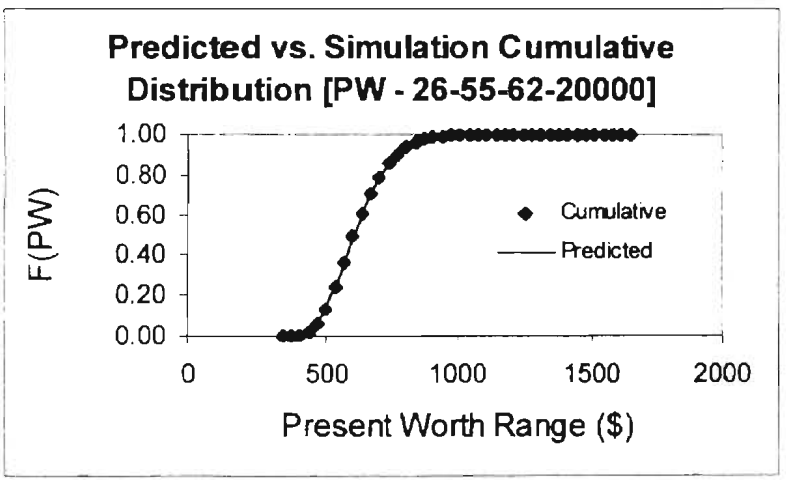

Figure 113. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(6,2)

Table 93

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	1	1
293.99	3	15	14
324.22	28	81	77
354.46	147	251	244
384.69	435	558	549
414.92	1,009	984	976
445.16	1,635	1,460	1,456
475.39	2,158	1,891	1,894
505.63	2,401	2,192	2,202
535.86	2,433	2,314	2,327
566.09	2,292	2,250	2,264
596.33	1,863	2,034	2,045
626.56	1,517	1,719	1,725
656.80	1,232	1,364	1,366
687.03	911	1,019	1,017
717.26	685	718	714
747.50	467	478	473
777.73	296	300	295
807.97	195	177	173
838.20	121	99	96
868.43	82	51	50
898.67	43	25	24
928.90	24	11	11
959.14	10	5	4
989.37	7	2	2
1,019.60	4	1	1
1,049.84	1	0	0
1,080.07	1	0	0
1,110.31	0	0	0
1,140.54	0	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 94

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 26-55-55-20000]					
Theoretical Alpha			7.69	Test Statistic	Critical Value
Theoretical Beta			19.48		
Number Of Data Points			20000	365.88	32.67
alpha = 0.05					
Degrees of freedom			21		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
31	4.84E-03	96.84	-65.84	4,334.45	44.76
147	1.25E-02	250.60	-103.60	10,732.96	42.83
435	2.79E-02	558.20	-123.20	15,178.24	27.19
1,009	4.92E-02	984.40	24.60	605.16	0.61
1,635	7.30E-02	1,460.00	175.00	30,625.00	20.98
2,158	9.46E-02	1,891.00	267.00	71,289.00	37.70
2,401	1.10E-01	2,192.00	209.00	43,681.00	19.93
2,433	1.16E-01	2,313.60	119.40	14,256.36	6.16
2,292	1.13E-01	2,250.20	41.80	1,747.24	0.78
1,863	1.02E-01	2,033.80	-170.80	29,172.64	14.34
1,517	8.59E-02	1,718.80	-201.80	40,723.24	23.69
1,232	6.82E-02	1,364.00	-132.00	17,424.00	12.77
911	5.10E-02	1,019.20	-108.20	11,707.24	11.49
685	3.59E-02	718.40	-33.40	1,115.56	1.55
467	2.39E-02	477.80	-10.80	116.64	0.24
296	1.50E-02	299.80	-3.80	14.44	0.05
195	8.87E-03	177.33	17.67	312.07	1.76
121	4.93E-03	98.65	22.35	499.52	5.06
82	2.57E-03	51.47	30.53	932.12	18.11
43	1.25E-03	25.09	17.91	320.81	12.79
24	5.68E-04	11.37	12.63	159.53	14.03
10	2.38E-04	4.76	5.24	27.46	5.77
7	9.13E-05	1.83	5.17	26.76	14.65
6	4.53E-05	0.91	5.09	25.95	28.63

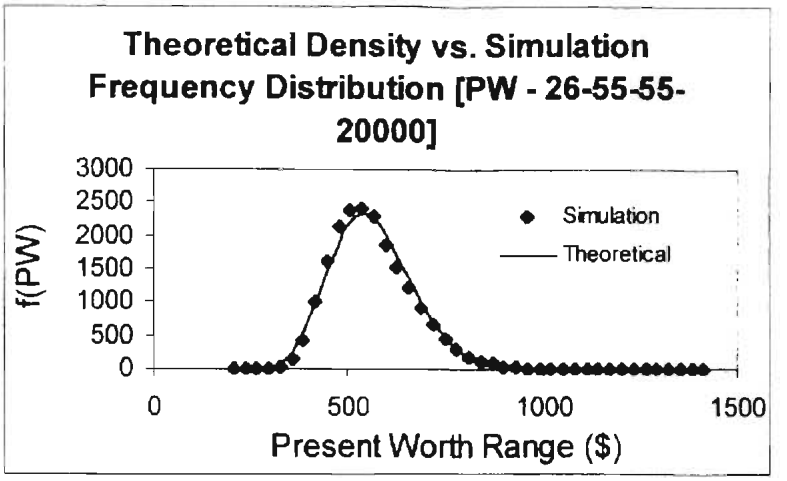

Figure 114. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

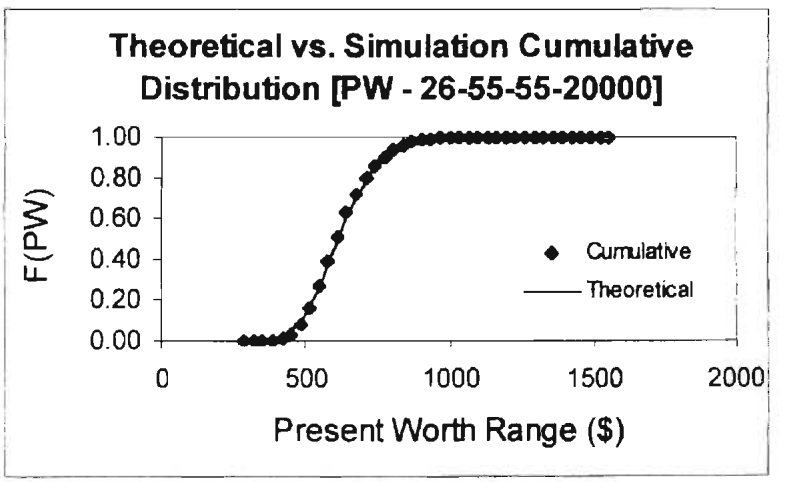

Figure 115. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Table 95

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 26-55-55-20000]				Test Statistic	Critical Value
Predicted Alpha			7.79		
Predicted Beta			19.72		
Number Of Data Points			20000	370.67	32.67
alpha = 0.05					
Degrees of freedom			21		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
31	4.61E-03	92.27	-61.27	3,754.41	40.69
147	1.22E-02	243.60	-96.60	9,331.56	38.31
435	2.74E-02	548.80	-113.80	12,950.44	23.60
1,009	4.88E-02	976.00	33.00	1,089.00	1.12
1,635	7.28E-02	1,456.40	178.60	31,897.96	21.90
2,158	9.47E-02	1,894.20	263.80	69,590.44	36.74
2,401	1.10E-01	2,201.60	199.40	39,760.36	18.06
2,433	1.16E-01	2,327.00	106.00	11,236.00	4.83
2,292	1.13E-01	2,263.80	28.20	795.24	0.35
1,863	1.02E-01	2,044.80	-181.80	33,051.24	16.16
1,517	8.63E-02	1,725.40	-208.40	43,430.56	25.17
1,232	6.83E-02	1,366.00	-134.00	17,956.00	13.14
911	5.09E-02	1,017.40	-106.40	11,320.96	11.13
685	3.57E-02	714.20	-29.20	852.64	1.19
467	2.36E-02	472.80	-5.80	33.64	0.07
296	1.48E-02	295.00	1.00	1.00	0.00
195	8.67E-03	173.42	21.58	465.64	2.68
121	4.79E-03	95.79	25.21	635.47	6.63
82	2.48E-03	49.58	32.42	1,050.82	21.19
43	1.20E-03	23.96	19.04	362.61	15.14
24	5.38E-04	10.75	13.25	175.52	16.33
10	2.23E-04	4.45	5.55	30.77	6.91
7	8.44E-05	1.69	5.31	28.21	16.70
6	4.11E-05	0.82	5.18	26.81	32.62

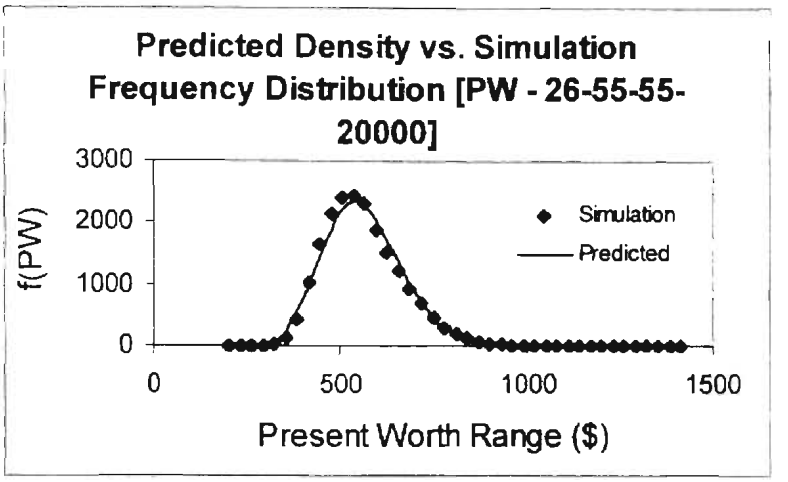

Figure 116. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

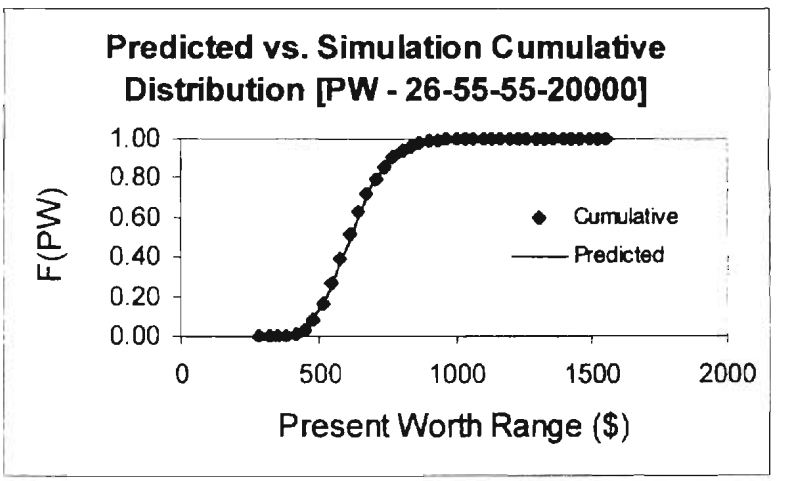

Figure 117. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(5,5)

Table 96

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	1	1
324.22	4	12	12
354.46	14	54	53
384.69	72	161	159
414.92	282	368	365
445.16	585	685	681
475.39	1,099	1,085	1,082
505.63	1,640	1,510	1,509
535.86	2,173	1,885	1,887
566.09	2,406	2,142	2,145
596.33	2,331	2,240	2,244
626.56	2,110	2,171	2,176
656.80	1,853	1,963	1,966
687.03	1,471	1,661	1,663
717.26	1,195	1,319	1,320
747.50	884	985	984
777.73	655	691	690
807.97	444	456	454
838.20	325	282	281
868.43	175	164	162
898.67	131	88	88
928.90	69	44	44
959.14	44	21	20
989.37	23	9	9
1,019.60	7	3	3
1,049.84	5	1	1
1,080.07	3	0	0
1,110.31	0	0	0
1,140.54	0	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 97

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 26-55-26-20000]					
Theoretical Alpha			9.16	Test Statistic	Critical Value
Theoretical Beta			18.80		
Number Of Data Points			20000	384.09	32.67
alpha = 0.05					
Degrees of freedom			21		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
18	3.35E-03	66.97	-48.97	2,397.70	35.80
72	8.06E-03	161.23	-89.23	7,961.67	49.38
282	1.84E-02	368.00	-86.00	7,396.00	20.10
585	3.42E-02	684.60	-99.60	9,920.16	14.49
1,099	5.42E-02	1,084.80	14.20	201.64	0.19
1,640	7.55E-02	1,510.00	130.00	16,900.00	11.19
2,173	9.43E-02	1,885.00	288.00	82,944.00	44.00
2,406	1.07E-01	2,142.00	264.00	69,696.00	32.54
2,331	1.12E-01	2,239.60	91.40	8,353.96	3.73
2,110	1.09E-01	2,171.00	-61.00	3,721.00	1.71
1,853	9.81E-02	1,962.60	-109.60	12,012.16	6.12
1,471	8.30E-02	1,660.80	-189.80	36,024.04	21.69
1,195	6.60E-02	1,319.00	-124.00	15,376.00	11.66
884	4.92E-02	984.60	-100.60	10,120.36	10.28
655	3.46E-02	691.00	-36.00	1,296.00	1.88
444	2.28E-02	455.80	-11.80	139.24	0.31
325	1.41E-02	282.20	42.80	1,831.84	6.49
175	8.18E-03	163.51	11.49	131.96	0.81
131	4.42E-03	88.43	42.57	1,812.41	20.50
69	2.22E-03	44.43	24.57	603.90	13.59
44	1.03E-03	20.62	23.38	546.85	26.53
23	4.39E-04	8.77	14.23	202.45	23.08
7	1.70E-04	3.39	3.61	13.03	3.84
8	8.31E-05	1.66	6.34	40.18	24.19

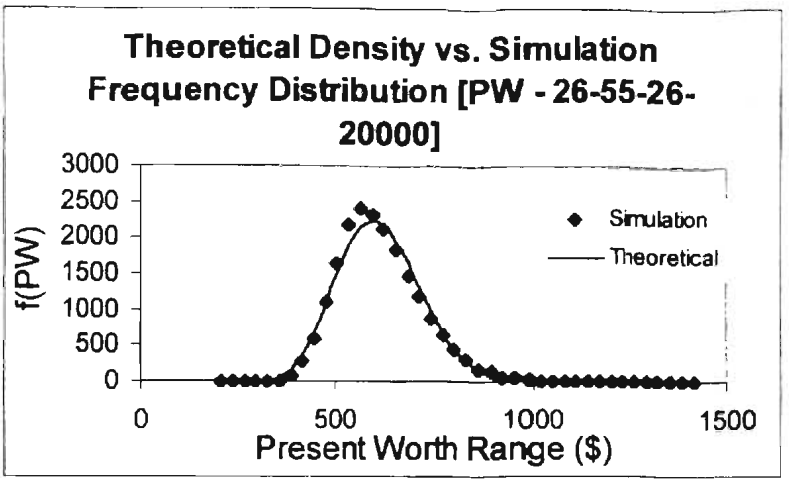

Figure 118. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

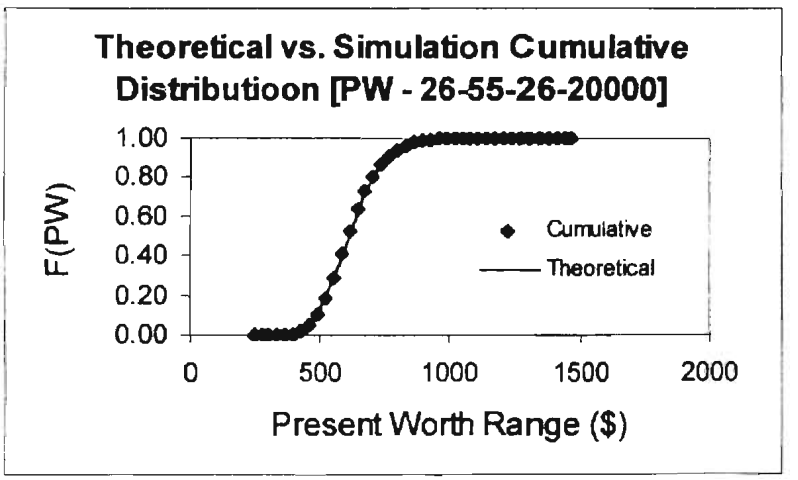

Figure 119. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Table 98

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 26-55-26-20000]					
Predicted Alpha			9.20	Test Statistic	Critical Value
Predicted Beta			18.88		
Number Of Data Points			20000	385.26	32.67
alpha = 0.05					
Degrees of freedom			21		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
18	3.29E-03	65.76	-47.76	2,280.60	34.68
72	7.97E-03	159.39	-87.39	7,637.12	47.91
282	1.83E-02	365.40	-83.40	6,955.56	19.04
585	3.41E-02	681.40	-96.40	9,292.96	13.64
1,099	5.41E-02	1,082.40	16.60	275.56	0.25
1,640	7.55E-02	1,509.40	130.60	17,056.36	11.30
2,173	9.43E-02	1,886.60	286.40	82,024.96	43.48
2,406	1.07E-01	2,145.40	260.60	67,912.36	31.65
2,331	1.12E-01	2,244.00	87.00	7,569.00	3.37
2,110	1.09E-01	2,175.60	-65.60	4,303.36	1.98
1,853	9.83E-02	1,966.20	-113.20	12,814.24	6.52
1,471	8.32E-02	1,663.00	-192.00	36,864.00	22.17
1,195	6.60E-02	1,319.60	-124.60	15,525.16	11.77
884	4.92E-02	984.00	-100.00	10,000.00	10.16
655	3.45E-02	689.60	-34.60	1,197.16	1.74
444	2.27E-02	454.20	-10.20	104.04	0.23
325	1.40E-02	280.60	44.40	1,971.36	7.03
175	8.11E-03	162.24	12.76	162.79	1.00
131	4.38E-03	87.52	43.48	1,890.63	21.60
69	2.19E-03	43.84	25.16	632.84	14.43
44	1.01E-03	20.28	23.72	562.63	27.74
23	4.30E-04	8.60	14.40	207.41	24.12
7	1.66E-04	3.31	3.69	13.61	4.11
8	8.06E-05	1.61	6.39	40.81	25.33

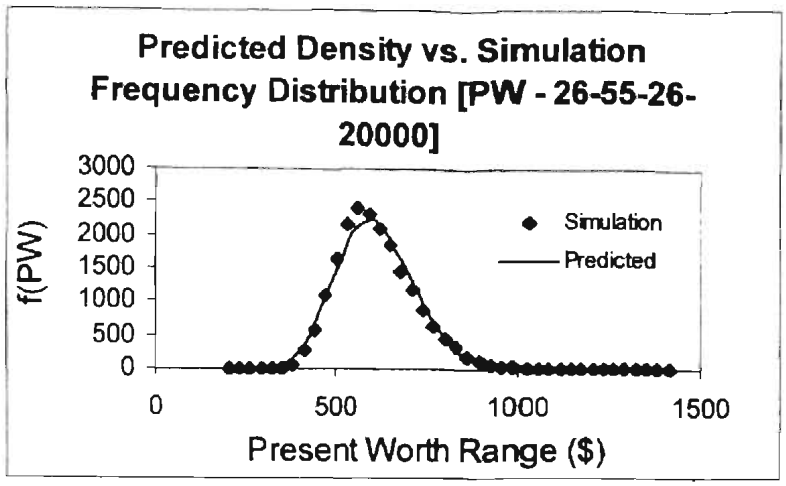

Figure 120. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

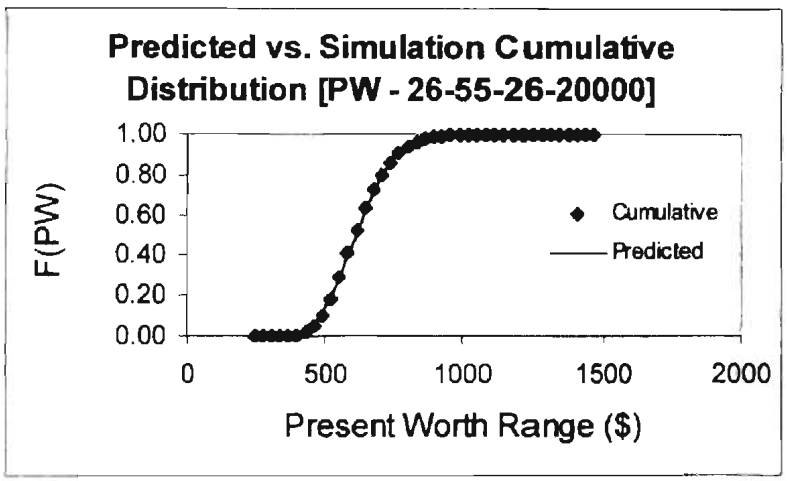

Figure 121. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(5,5) and the Interest Rate ~B(2,6)

Table 99

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	2	1
293.99	2	22	20
324.22	50	111	107
354.46	203	335	327
384.69	639	720	710
414.92	1,198	1,222	1,215
445.16	1,980	1,741	1,740
475.39	2,378	2,159	2,166
505.63	2,549	2,392	2,404
535.86	2,592	2,407	2,422
566.09	2,121	2,227	2,240
596.33	1,746	1,910	1,919
626.56	1,388	1,529	1,532
656.80	987	1,146	1,145
687.03	766	807	803
717.26	517	534	529
747.50	344	333	328
777.73	227	195	191
807.97	137	107	104
838.20	80	55	53
868.43	51	27	25
898.67	21	12	11
928.90	17	5	5
959.14	3	2	2
989.37	3	1	1
1,019.60	0	0	0
1,049.84	1	0	0
1,080.07	0	0	0
1,110.31	0	0	0
1,140.54	0	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 100

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 26-26-62-20000]					
Theoretical Alpha			7.77	Test Statistic	Critical Value
Theoretical Beta			21.10		
Number Of Data Points			20000	338.22	30.14
alpha = 0.05					
Degrees of freedom			19		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
52	6.71E-03	134.23	-82.23	6,761.26	50.37
203	1.68E-02	335.00	-132.00	17,424.00	52.01
639	3.60E-02	720.20	-81.20	6,593.44	9.16
1,198	6.11E-02	1,222.40	-24.40	595.36	0.49
1,980	8.70E-02	1,740.80	239.20	57,216.64	32.87
2,378	1.08E-01	2,159.20	218.80	47,873.44	22.17
2,549	1.20E-01	2,391.80	157.20	24,711.84	10.33
2,592	1.20E-01	2,407.00	185.00	34,225.00	14.22
2,121	1.11E-01	2,227.00	-106.00	11,236.00	5.05
1,746	9.55E-02	1,910.40	-164.40	27,027.36	14.15
1,388	7.64E-02	1,528.80	-140.80	19,824.64	12.97
987	5.73E-02	1,146.00	-159.00	25,281.00	22.06
766	4.03E-02	806.60	-40.60	1,648.36	2.04
517	2.67E-02	534.00	-17.00	289.00	0.54
344	1.66E-02	332.80	11.20	125.44	0.38
227	9.75E-03	194.95	32.05	1,027.39	5.27
137	5.36E-03	107.27	29.73	883.88	8.24
80	2.77E-03	55.31	24.69	609.74	11.02
51	1.33E-03	26.63	24.37	593.76	22.29
21	5.96E-04	11.93	9.07	82.32	6.90
17	2.47E-04	4.94	12.06	145.43	29.44
7	1.40E-04	2.81	4.19	17.57	6.26

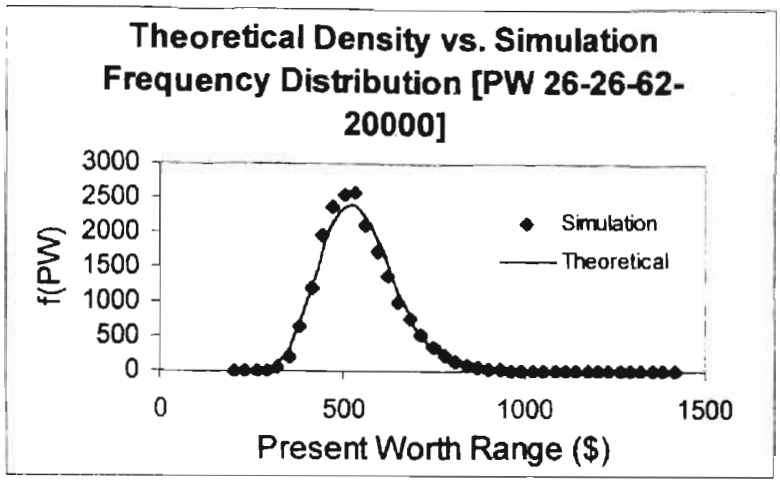

Figure 122. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

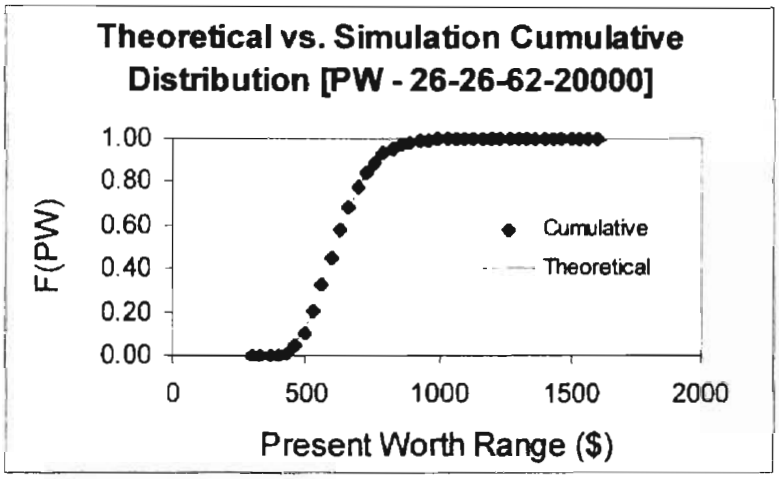

Figure 123. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Table 101

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Chi-squared Test - [PW - 26-26-62-20000]				Test Statistic	Critical Value
Predicted Alpha			7.75		
Predicted Beta			21.32		
Number Of Data Points			20000	339.53	30.14
alpha = 0.05					
Degrees of freedom			19		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
52	6.42E-03	128.36	-76.36	5,831.00	45.43
203	1.63E-02	326.80	-123.80	15,326.44	46.90
639	3.55E-02	710.00	-71.00	5,041.00	7.10
1,198	6.08E-02	1,215.00	-17.00	289.00	0.24
1,980	8.70E-02	1,739.80	240.20	57,696.04	33.16
2,378	1.08E-01	2,165.80	212.20	45,028.84	20.79
2,549	1.20E-01	2,404.40	144.60	20,909.16	8.70
2,592	1.21E-01	2,421.60	170.40	29,036.16	11.99
2,121	1.12E-01	2,239.80	-118.80	14,113.44	6.30
1,746	9.60E-02	1,919.00	-173.00	29,929.00	15.60
1,388	7.66E-02	1,532.20	-144.20	20,793.64	13.57
987	5.73E-02	1,145.00	-158.00	24,964.00	21.80
766	4.01E-02	802.80	-36.80	1,354.24	1.69
517	2.65E-02	529.00	-12.00	144.00	0.27
344	1.64E-02	327.80	16.20	262.44	0.80
227	9.54E-03	190.82	36.18	1,308.82	6.86
137	5.21E-03	104.25	32.75	1,072.54	10.29
80	2.67E-03	53.32	26.68	711.75	13.35
51	1.27E-03	25.45	25.55	652.83	25.65
21	5.64E-04	11.29	9.71	94.37	8.36
17	2.31E-04	4.62	12.38	153.16	33.12
7	1.29E-04	2.58	4.42	19.52	7.56

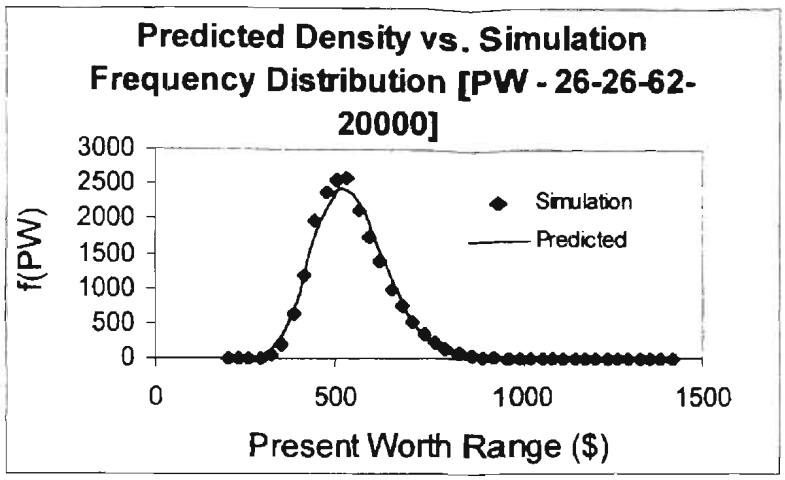

Figure 124. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

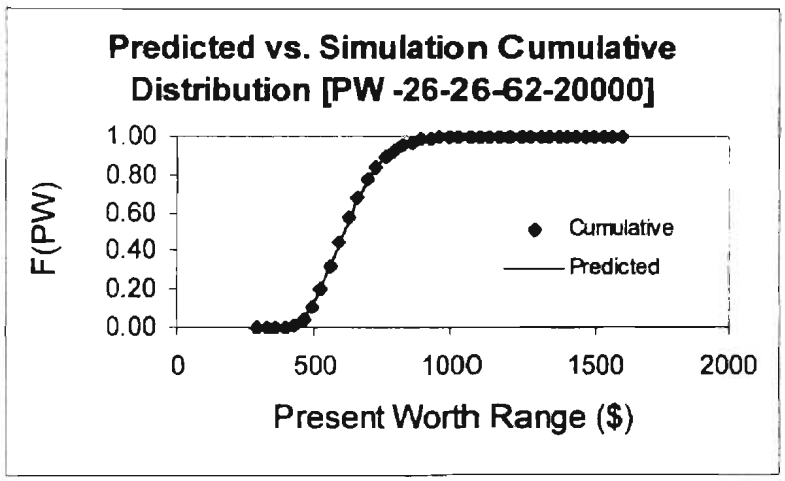

Figure 125. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(6,2)

Table 102

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	1	1
324.22	1	12	12
354.46	20	54	53
384.69	74	161	159
414.92	249	368	365
445.16	583	685	681
475.39	1,151	1,085	1,082
505.63	1,768	1,510	1,509
535.86	2,165	1,885	1,887
566.09	2,307	2,142	2,145
596.33	2,261	2,240	2,244
626.56	2,071	2,171	2,176
656.80	1,860	1,963	1,966
687.03	1,479	1,661	1,663
717.26	1,223	1,319	1,320
747.50	854	985	984
777.73	692	691	690
807.97	445	456	454
838.20	308	282	281
868.43	200	164	162
898.67	125	88	88
928.90	84	44	44
959.14	37	21	20
989.37	28	9	9
1,019.60	6	3	3
1,049.84	5	1	1
1,080.07	2	0	0
1,110.31	1	0	0
1,140.54	1	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 103

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 26-26-55-20000]					
Theoretical Alpha			9.16	Test Statistic	Critical Value
Theoretical Beta			18.8		
Number Of Data Points			20000	439.85	32.67
alpha = 0.05					
Degrees of freedom			21		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
21	3.35E-03	66.97	-45.97	2,112.91	31.55
74	8.06E-03	161.23	-87.23	7,608.76	47.19
249	1.84E-02	368.00	-119.00	14,161.00	38.48
583	3.42E-02	684.60	-101.60	10,322.56	15.08
1,151	5.42E-02	1,084.80	66.20	4,382.44	4.04
1,768	7.55E-02	1,510.00	258.00	66,564.00	44.08
2,165	9.43E-02	1,885.00	280.00	78,400.00	41.59
2,307	1.07E-01	2,142.00	165.00	27,225.00	12.71
2,261	1.12E-01	2,239.60	21.40	457.96	0.20
2,071	1.09E-01	2,171.00	-100.00	10,000.00	4.61
1,860	9.81E-02	1,962.60	-102.60	10,526.76	5.36
1,479	8.30E-02	1,660.80	-181.80	33,051.24	19.90
1,223	6.60E-02	1,319.00	-96.00	9,216.00	6.99
854	4.92E-02	984.60	-130.60	17,056.36	17.32
692	3.46E-02	691.00	1.00	1.00	0.00
445	2.28E-02	455.80	-10.80	116.64	0.26
308	1.41E-02	282.20	25.80	665.64	2.36
200	8.18E-03	163.51	36.49	1,331.33	8.14
125	4.42E-03	88.43	36.57	1,337.54	15.13
84	2.22E-03	44.43	39.57	1,566.13	35.25
37	1.03E-03	20.62	16.38	268.46	13.02
28	4.39E-04	8.77	19.23	369.74	42.15
6	1.70E-04	3.39	2.61	6.81	2.01
9	8.31E-05	1.66	7.34	53.86	32.42

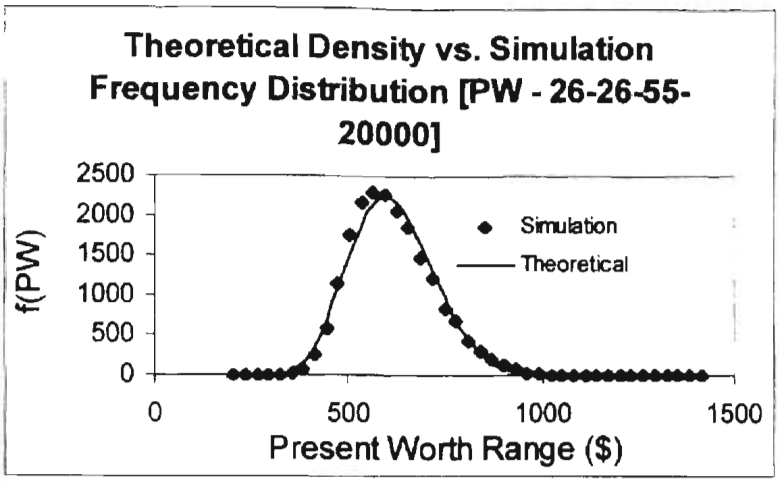

Figure 126. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

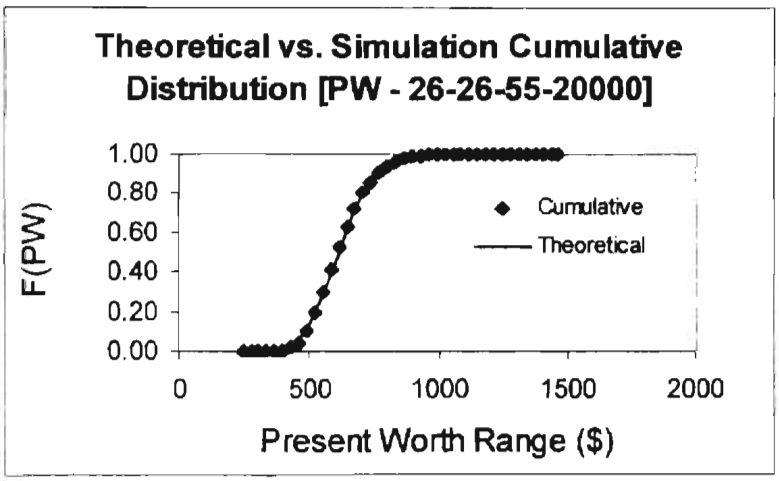

Figure 127. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Table 104

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Chi-squared Test - [PW - 26-26-55-20000]					
Predicted Alpha			9.11	Test Statistic	Critical Value
Predicted Beta			18.92		
Number Of Data Points			20000	443.02	32.67
alpha = 0.05					
Degrees of freedom			21		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
21	3.29E-03	65.76	-44.76	2,003.07	30.46
74	7.97E-03	159.39	-85.39	7,291.55	45.75
249	1.83E-02	365.40	-116.40	13,548.96	37.08
583	3.41E-02	681.40	-98.40	9,682.56	14.21
1,151	5.41E-02	1,082.40	68.60	4,705.96	4.35
1,768	7.55E-02	1,509.40	258.60	66,873.96	44.30
2,165	9.43E-02	1,886.60	278.40	77,506.56	41.08
2,307	1.07E-01	2,145.40	161.60	26,114.56	12.17
2,261	1.12E-01	2,244.00	17.00	289.00	0.13
2,071	1.09E-01	2,175.60	-104.60	10,941.16	5.03
1,860	9.83E-02	1,966.20	-106.20	11,278.44	5.74
1,479	8.32E-02	1,663.00	-184.00	33,856.00	20.36
1,223	6.60E-02	1,319.60	-96.60	9,331.56	7.07
854	4.92E-02	984.00	-130.00	16,900.00	17.17
692	3.45E-02	689.60	2.40	5.76	0.01
445	2.27E-02	454.20	-9.20	84.64	0.19
308	1.40E-02	280.60	27.40	750.76	2.68
200	8.11E-03	162.24	37.76	1,425.73	8.79
125	4.38E-03	87.52	37.48	1,404.86	16.05
84	2.19E-03	43.84	40.16	1,612.54	36.78
37	1.01E-03	20.28	16.72	279.55	13.78
28	4.30E-04	8.60	19.40	376.43	43.78
6	1.66E-04	3.31	2.69	7.23	2.19
9	8.06E-05	1.61	7.39	54.59	33.88

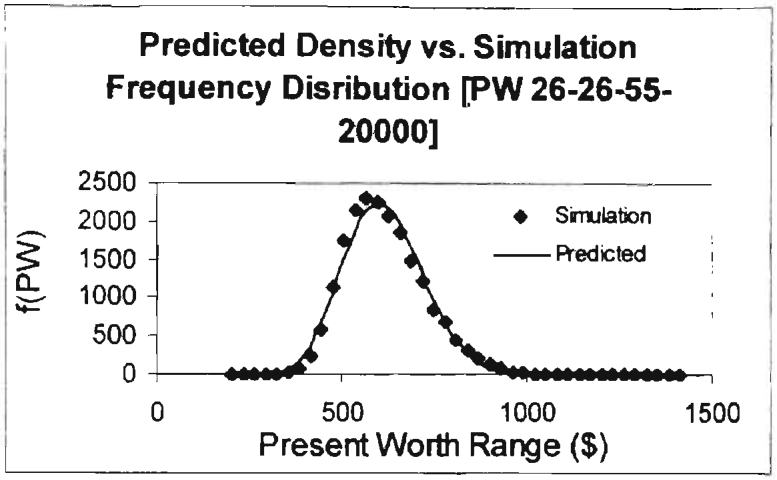

Figure 128. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

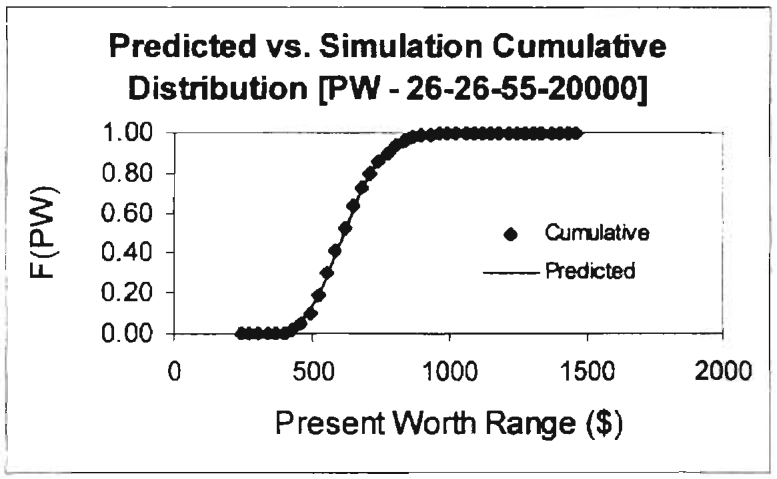

Figure 129. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(5,5)

Table 105

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	0	0
324.22	0	1	1
354.46	1	7	7
384.69	6	28	28
414.92	26	82	83
445.16	101	192	194
475.39	255	377	379
505.63	550	643	645
535.86	965	973	975
566.09	1,494	1,332	1,332
596.33	2,019	1,668	1,666
626.56	2,223	1,929	1,925
656.80	2,288	2,074	2,070
687.03	2,108	2,085	2,081
717.26	1,818	1,966	1,963
747.50	1,527	1,743	1,741
777.73	1,304	1,454	1,454
807.97	978	1,143	1,143
838.20	736	845	846
868.43	542	587	589
898.67	398	382	384
928.90	269	233	234
959.14	178	132	133
989.37	92	69	70
1,019.60	53	33	34
1,049.84	32	14	15
1,080.07	19	6	6
1,110.31	10	2	2
1,140.54	6	1	1
1,170.77	1	0	0
1,201.01	1	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 106

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 26-26-26-20000]					
Theoretical Alpha			10.29	Test Statistic	Critical Value
Theoretical Beta			16.36		
Number Of Data Points			20000	613.36	35.17
alpha = 0.05					
Degrees of freedom			23		
Interval Frequency	Theoretical	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	1.80E-03	36.02	-29.02	842.07	23.38
26	4.10E-03	81.96	-55.96	3,131.95	38.21
101	9.60E-03	192.08	-91.08	8,295.71	43.19
255	1.89E-02	377.40	-122.40	14,981.76	39.70
550	3.22E-02	643.00	-93.00	8,649.00	13.45
965	4.87E-02	973.40	-8.40	70.56	0.07
1,494	6.66E-02	1,331.80	162.20	26,308.84	19.75
2,019	8.34E-02	1,667.60	351.40	123,481.96	74.05
2,223	9.64E-02	1,928.60	294.40	86,671.36	44.94
2,288	1.04E-01	2,074.40	213.60	45,624.96	21.99
2,108	1.04E-01	2,085.40	22.60	510.76	0.24
1,818	9.83E-02	1,966.20	-148.20	21,963.24	11.17
1,527	8.72E-02	1,743.00	-216.00	46,656.00	26.77
1,304	7.27E-02	1,454.40	-150.40	22,620.16	15.55
978	5.71E-02	1,142.60	-164.60	27,093.16	23.71
736	4.22E-02	844.80	-108.80	11,837.44	14.01
542	2.94E-02	587.00	-45.00	2,025.00	3.45
398	1.91E-02	382.20	15.80	249.64	0.65
269	1.16E-02	232.60	36.40	1,324.96	5.70
178	6.58E-03	131.57	46.43	2,156.06	16.39
92	3.44E-03	68.79	23.21	538.57	7.83
53	1.65E-03	32.99	20.01	400.38	12.14
32	7.18E-04	14.37	17.63	310.89	21.64
19	2.81E-04	5.61	13.39	179.26	31.94
10	9.67E-05	1.93	8.07	65.07	33.66
8	3.76E-05	0.75	7.25	52.52	69.78

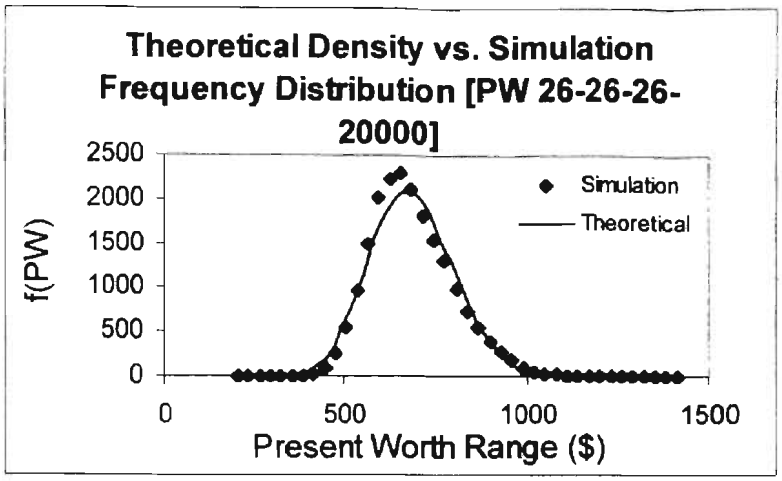

Figure 130. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

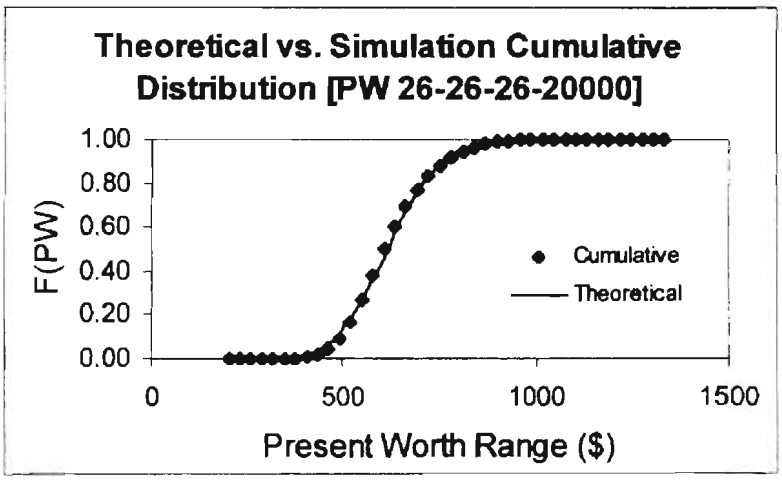

Figure 131. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Table 107

Chi-Squared Goodness-of-Fit Test Comparing the Simulation Frequency Values to the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

Chi-squared Test - [PW - 26-26-26-20000]				Test Statistic	Critical Value
Predicted Alpha			10.18		
Predicted Beta			16.36		
Number Of Data Points			20000	610.29	35.17
alpha = 0.05					
Degrees of freedom			23		
Interval Frequency	Predicted	Ej	fj - Ej	(fj - Ej)^2	(fj - Ej)^2/Ej
7	1.83E-03	36.70	-29.70	882.01	24.03
26	4.15E-03	82.98	-56.98	3,246.63	39.13
101	9.69E-03	193.73	-92.73	8,599.15	44.39
255	1.90E-02	379.40	-124.40	15,475.36	40.79
550	3.23E-02	645.00	-95.00	9,025.00	13.99
965	4.87E-02	974.60	-9.60	92.16	0.09
1,494	6.66E-02	1,331.60	162.40	26,373.76	19.81
2,019	8.33E-02	1,665.80	353.20	124,750.24	74.89
2,223	9.63E-02	1,925.20	297.80	88,684.84	46.07
2,288	1.04E-01	2,070.00	218.00	47,524.00	22.96
2,108	1.04E-01	2,081.00	27.00	729.00	0.35
1,818	9.81E-02	1,962.60	-144.60	20,909.16	10.65
1,527	8.70E-02	1,740.80	-213.80	45,710.44	26.26
1,304	7.27E-02	1,453.60	-149.60	22,380.16	15.40
978	5.72E-02	1,143.20	-165.20	27,291.04	23.87
736	4.23E-02	846.40	-110.40	12,188.16	14.40
542	2.95E-02	589.00	-47.00	2,209.00	3.75
398	1.92E-02	384.40	13.60	184.96	0.48
269	1.17E-02	234.40	34.60	1,197.16	5.11
178	6.65E-03	132.90	45.10	2,033.76	15.30
92	3.48E-03	69.69	22.31	497.69	7.14
53	1.68E-03	33.53	19.47	379.09	11.31
32	7.33E-04	14.66	17.34	300.80	20.52
19	2.87E-04	5.75	13.25	175.62	30.55
10	9.95E-05	1.99	8.01	64.16	32.25
8	3.90E-05	0.78	7.22	52.12	66.81

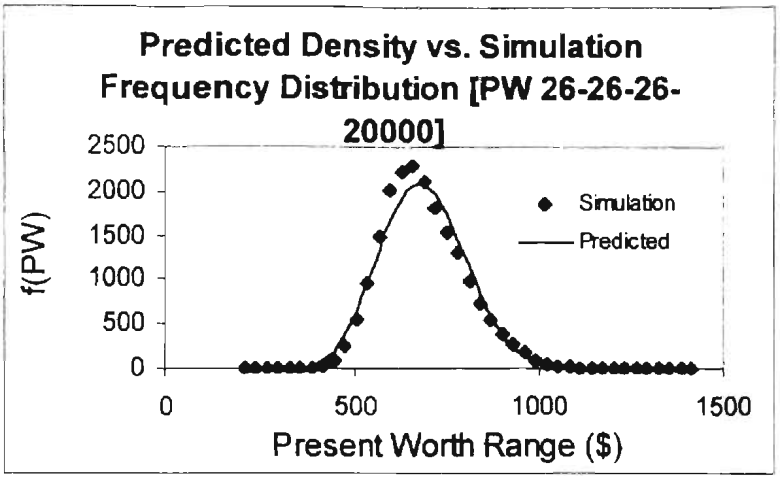

Figure 132. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

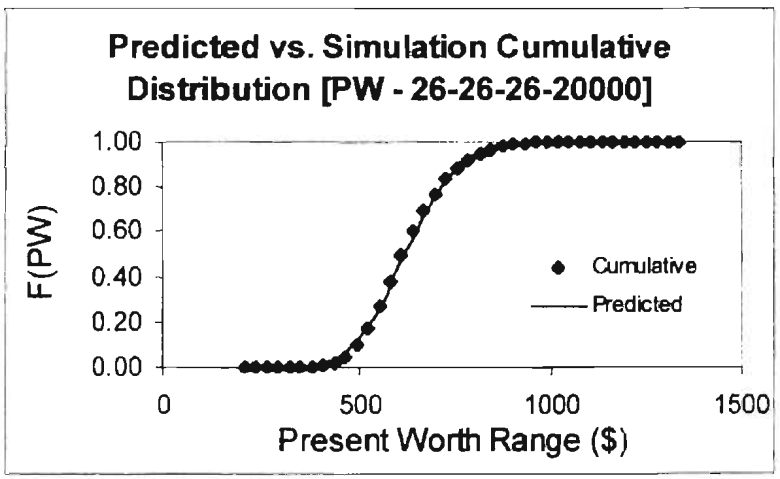

Figure 133. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(2,6), the Lump-Sum Cash Flow Timing ~B(2,6) and the Interest Rate ~B(2,6)

APPENDIX D – ADDITIONAL SIMULATIONS

Table 108

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(1,10) and the Interest Rate ~B(1,10)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	0	0	0
263.75	0	0	0
293.99	0	0	0
324.22	0	0	0
354.46	0	0	0
384.69	0	0	0
414.92	0	0	0
445.16	1	0	0
475.39	0	0	0
505.63	1	1	1
535.86	17	4	5
566.09	36	24	27
596.33	102	97	108
626.56	277	308	329
656.80	616	764	791
687.03	1,308	1,514	1,531
717.26	2,278	2,434	2,422
747.50	3,641	3,206	3,162
777.73	4,243	3,487	3,429
807.97	3,143	3,144	3,102
838.20	1,939	2,353	2,345
868.43	1,104	1,460	1,478
898.67	632	748	774
928.90	360	314	335
959.14	169	107	118
989.37	80	29	34
1,019.60	28	6	8
1,049.84	14	1	1
1,080.07	8	0	0
1,110.31	2	0	0
1,140.54	1	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 109

Data Sheet for Simulation for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(1,10) and the Interest Rate ~B(1,10)

Number of Runs	20,000
Alpha - Cash Flow	1
Beta - Cash Flow	10
Assumed Cash Flow Mean	1,000
Assumed Cash Flow Range	750
Calculated Cash Flow Minimum	931.82
Calculated Cash Flow Maximum	1,681.82
Simulation Cash Flow Mean	999.63
Simulation Cash Flow Variance	3,684.56
Alpha - Timing	1
Beta - Timimg	10
Assumed Timing Mean	4.50
Assumed Timing Range	2.00
Calculated Timing Minimum	4.32
Calculated Timing Maximum	6.32
Simulation Timing Mean	4.50
Simulation Timing Variance	0.028
Alpha - Interest Rate	1
Beta - Interest Rate	10
Assumed Interest Rate Mean	0.20
Assumed Interest Rate Range	0.15
Calculated Interest Rate Minimum	0.19
Calculated Interest Rate Maximum	0.34
Simulation Interest Rate Mean	0.20
Simulation Interest Rate Variance	0.00015
Simulation Present Worth Mean	407.37
Simulation Present Worth Variance	1,287.62
Predicted Present Worth Mean	407.43
Predicted Present Worth Variance	1,341.20
Theoretical Present Worth Mean	407.39
Theoretical Present Worth Variance	1,297.15

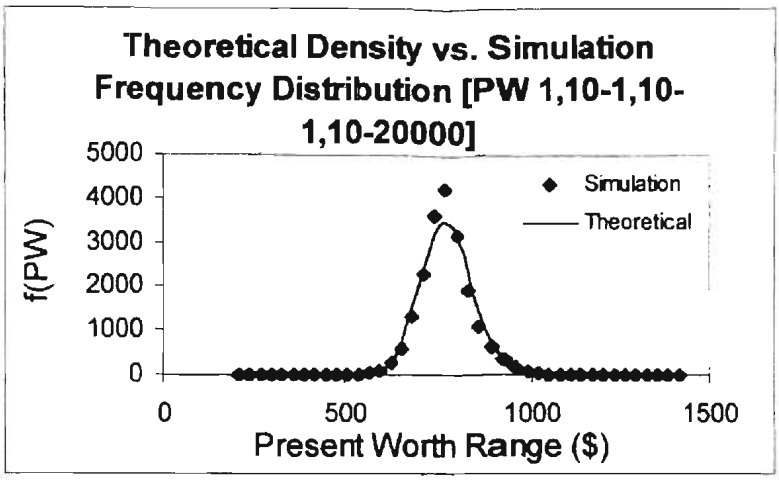

Figure 134. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(1,10) and the Interest Rate ~B(1,10)

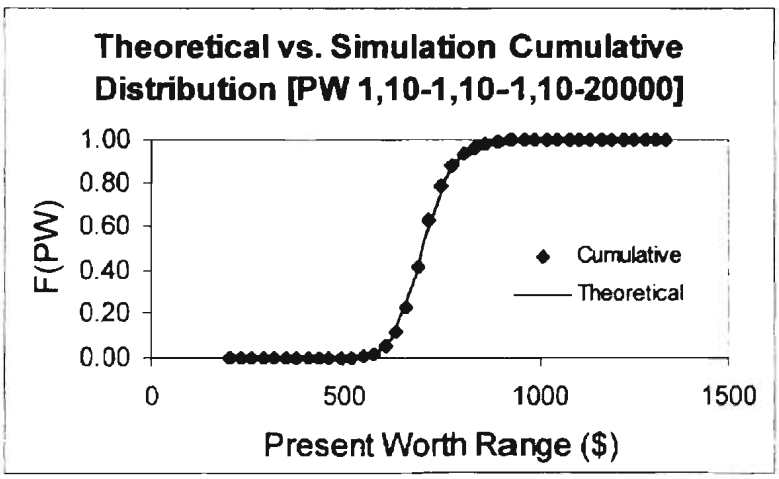

Figure 135. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(1,10) and the Interest Rate ~B(1,10)

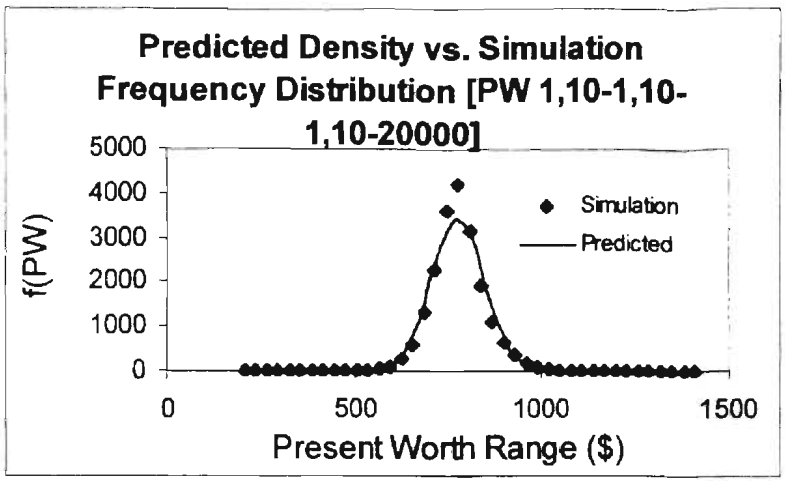

Figure 136. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(1,10) and the Interest Rate ~B(1,10)

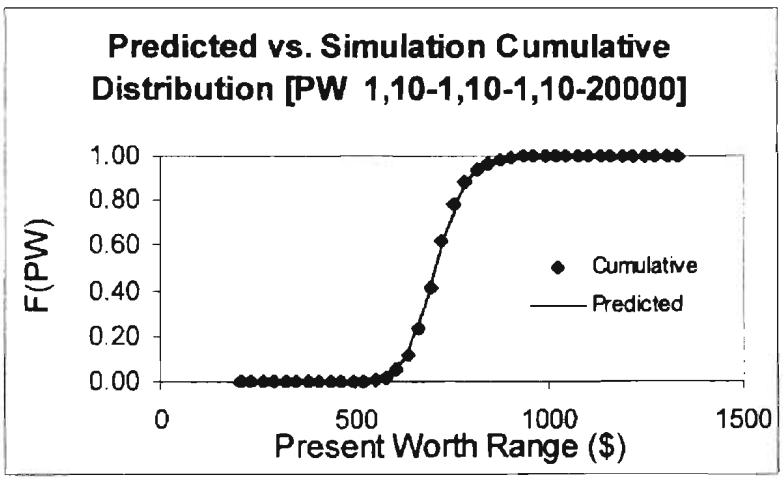

Figure 137. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(1,10) and the Interest Rate ~B(1,10)

Table 110

Simulation Frequency, Theoretical Density and Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(10,1) and the Interest Rate ~B(13,2)

PW Range Cell Boundary	Simulation Frequency	Theoretical Density	Predicted Density
203.28	0	0	0
233.52	2,192	2,605	2,536
263.75	6,462	5,899	5,921
293.99	5,266	5,130	5,187
324.22	3,071	3,239	3,265
354.46	1,583	1,724	1,722
384.69	759	819	808
414.92	398	356	346
445.16	150	144	138
475.39	66	55	51
505.63	36	19	18
535.86	12	7	6
566.09	1	2	2
596.33	2	1	1
626.56	0	0	0
656.80	1	0	0
687.03	1	0	0
717.26	0	0	0
747.50	0	0	0
777.73	0	0	0
807.97	0	0	0
838.20	0	0	0
868.43	0	0	0
898.67	0	0	0
928.90	0	0	0
959.14	0	0	0
989.37	0	0	0
1,019.60	0	0	0
1,049.84	0	0	0
1,080.07	0	0	0
1,110.31	0	0	0
1,140.54	0	0	0
1,170.77	0	0	0
1,201.01	0	0	0
1,231.24	0	0	0
1,261.48	0	0	0
1,291.71	0	0	0
1,321.94	0	0	0
1,352.18	0	0	0
1,382.41	0	0	0
1,412.65	0	0	0

Table 111

Data Sheet for Simulation for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(10,1) and the Interest Rate ~B(13,2)

Number of Runs	20,000
Alpha - Cash Flow	1
Beta - Cash Flow	10
Assumed Cash Flow Mean	1,000
Assumed Cash Flow Range	1,250
Calculated Cash Flow Minimum	886.36
Calculated Cash Flow Maximum	2,136.36
Simulation Cash Flow Mean	1,001.53
Simulation Cash Flow Variance	10,312.59
Alpha - Timing	10
Beta - Timing	1
Assumed Timing Mean	5.00
Assumed Timing Range	4.00
Calculated Timing Minimum	1.36
Calculated Timing Maximum	5.36
Simulation Timing Mean	5.00
Simulation Timing Variance	0.104
Alpha - Interest Rate	13
Beta - Interest Rate	2
Assumed Interest Rate Mean	0.15
Assumed Interest Rate Range	0.07
Calculated Interest Rate Minimum	0.09
Calculated Interest Rate Maximum	0.16
Simulation Interest Rate Mean	0.15
Simulation Interest Rate Variance	0.00004
Simulation Present Worth Mean	473.32
Simulation Present Worth Variance	3,232.27
Predicted Present Worth Mean	473.16
Predicted Present Worth Variance	3,151.73
Theoretical Present Worth Mean	473.18
Theoretical Present Worth Variance	3,219.73

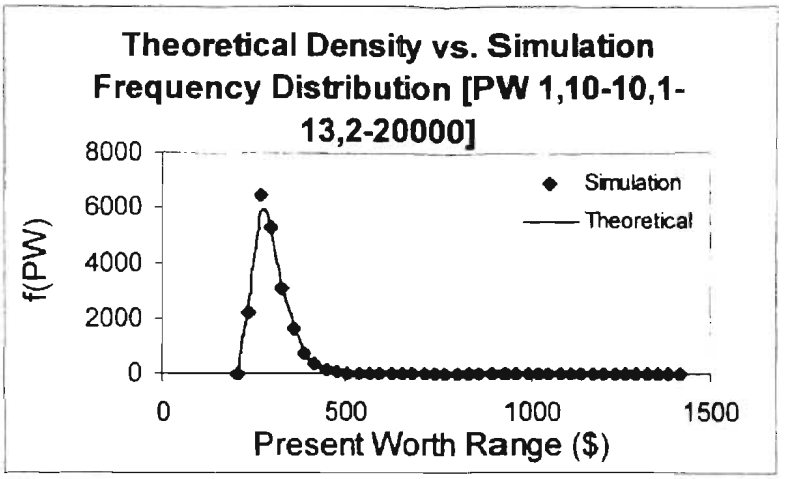

Figure 138. Graph Showing the Simulation Frequency versus the Theoretical Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(10,1) and the Interest Rate ~B(13,2)

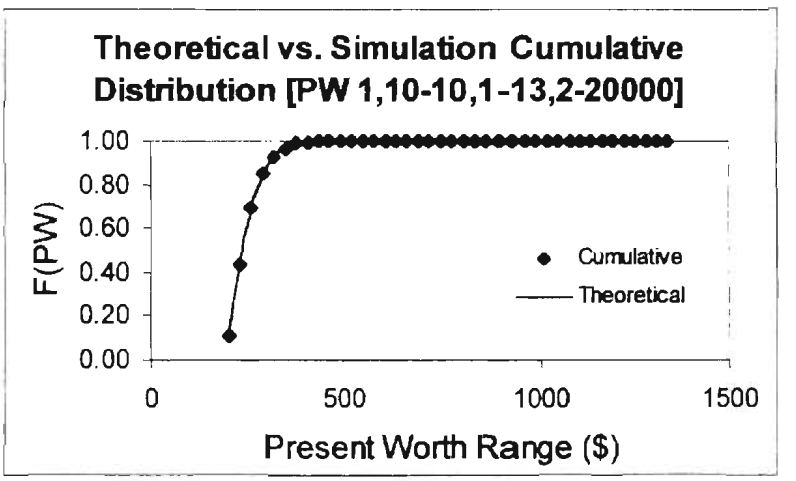

Figure 139. Graph Showing the Simulation versus the Theoretical Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(10,1) and the Interest Rate ~B(13,2)

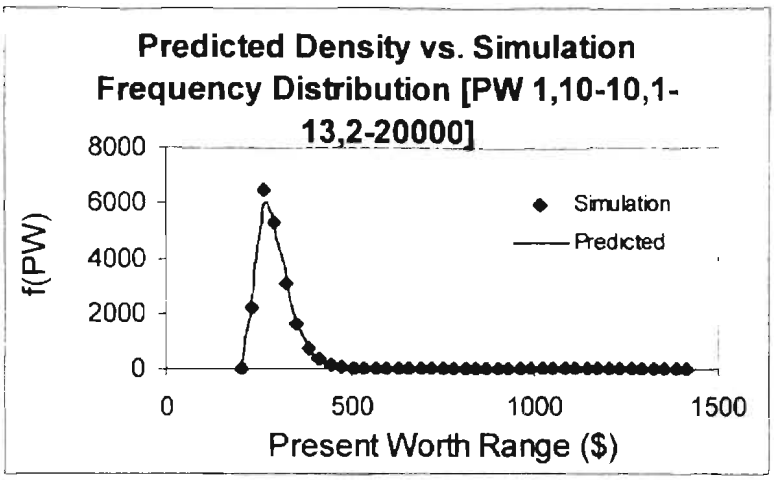

Figure 140. Graph Showing the Simulation Frequency Values versus the Predicted Density Values for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(10,1) and the Interest Rate ~B(13,2)

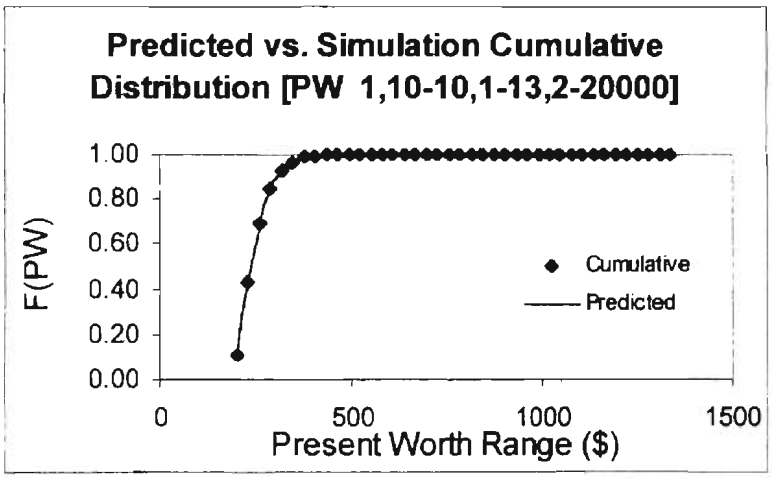

Figure 141. Graph Showing the Simulation versus the Predicted Cumulative Distribution for the Present Worth when the Lump-Sum Cash Flow ~B(1,10), the Lump-Sum Cash Flow Timing ~B(10,1) and the Interest Rate ~B(13,2)

VITA

Edward Leon McCombs

Candidate for the Degree of

Master of Science

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