# ALLOCATION OF FINITE BUFFER CAPACITY TO PART TYPES FOR MAXIMIZING PROFITS IN 

## SERIAL LINES

By<br>YOUSUFF ZAMAN HABIBULLAHKHAN<br>Bachelor of Engineering<br>University of Madras<br>Madras, India

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Thesis Approved:


Thesis Advisor


Wayne B. Powell
Dean of the Graduate College

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## CHAPTER I

## THE PROBLEM AND ITS SETTING

## Introduction

Significant pressure from competitors has forced manufacturers to review their present manufacturing/management techniques, such as Just-In-Time (JIT). JIT is both a philosophy and a set of techniques (Vollmann et al., 1992). The ultimate objectives of the JTT philosophy are to obtain zero inventory, zero lead-time, zero failures, zero disturbances, zero waste, and a flow process. These objectives lead to routine execution of schedule day in and day out (Vollmann et al., 1992).

JIT systems are pull systems wherein parts are produced in upstream departments whenever there is demand for those parts in the downstream departments. A pull system is employed to minimize in-process inventory and to enable all processes to know accurate timing and required quantity (Monden, 1983).

## Buffers

Buffers are included in most production systems to maintain product flow in the presence of variation. One way buffer capacities can be established is by determining the number of parts that can be accommodated in a given finite space based on the part(s)
dimensions, the maximum number of parts can then be calculated. The variability in demand for different part types can be handled by varying the capacity allocation for each part type within the buffer space. One disadvantage of this approach is that the total number of parts that can be handled may vary with varying dimensions of parts.

Another way to determine buffer capacity is to declare the maximum number of part(s) that must be accommodated. The finite space requirement is then calculated. The advantage of this approach is that each part type has a finite amount of space available within the buffer. The disadvantage of this approach is that it is difficult to trade-off allocated buffer spaces for part types with varying priority factors i.e., the capacity of part types with a high priority factor will be limited by their fixed buffer space.

One of the objectives of the JTT philosophy is to reduce the buffer capacity. This can be accomplished by reducing the floor space devoted to the in-process inventory or by reducing the number of parts. In this research, we define the buffer capacity as the maximum number of parts that can be accommodated in a finite space. The situation wherein the actual capacity is less than the desired capacity will be overcome by allocating the buffer space to the part types with higher priority factors. This aspect of buffer space allocation is discussed with an example in Chapter IV.

## Performance Measures

Performance measures can be classified into two broad categories. The first category is time-based performance measures, such as those based on job completion time, tardiness, earliness and deviation from due dates. The second category is monetary measures, such as profit per order. In this research, the primary performance measure
will be a monetary performance measure, that is, the accumulated total profit rate.
Deviation from due dates is directly linked to a penalty. If the product is delivered to the customer on time, then no lateness penalty is incurred. If the product is delivered to the customer after the expected due date, then a lateness penalty is applied. If the product is completed before the due date, the product is stored as finished goods inventory and inventory holding cost is applied until the due date. Holding costs are also applied to WIP inventories as they proceed through the manufacturing process. Since the cost performance of a system may be sensitive to the cost structure, time-based performance measures will be considered as secondary performance measures to balance the effects the cost structure will have on the system performance.

## Problem

This research models a system with finite buffer capacity where different part types compete for capacity. Therefore, the objective of this research is to determine the optimum number of part types to produce and allocation of space for a finite buffer capacity pull system to maximize profits for the manufacturer.

## Definition of Terminology

We define here some of the key terminology used in this research.
Lead Time: Lead time for a job is defined as the time difference between the completion time of job and the order arrival time of the job.

Throughput: Throughput is defined as the number of parts produced for different part types in a month.

Due date: Due date is the expected date of delivery of a part(s) to a customer(s).
Lateness Penalty: Lateness penalty is the penalty to be applied when a part(s) is delivered to a customer(s) after the expected due date.

Holding Cost: Holding cost is the cost applied when a part is completed before the due date and is held until its delivery.

WIP Holding Cost: WIP Holding cost is the cost incurred due to WIP inventories as they proceed through the manufacturing process.

## CHAPTER II

## LITERATURE REVIEW

## Introduction

In this chapter, a review of related literature is presented. This review focuses on research involving JT production systems. Although, the success of a JIT system depends on factors throughout an entire organization, this research studies its application to the shop floor. The shop floor aspect of a JIT system involves better vendor scheduling; reduction in lead-time and in-process inventory; and better quality control. The in-process inventory aspect of the JT literature is reviewed, as it is relevant to the research. Controlling the buffer capacity and the number of kanbans that are used in a system can reduce the in-process inventory. The literature review is divided into two sections. First, the problem of determining the buffer capacity is considered.

A second major issue in this type of research is consideration of the characteristics of the system to study. In many cases, results vary based on the system characteristics. Therefore, the second part of the literature review considers different experimental design factors.

## Literature Review on the Buffer Capacity

A buffer is a space where WIP or finished parts are stored. A buffer is placed between two machines and is used to store the finished goods from an upstream machine, which will be subsequently used as "raw materials" for the downstream machine. In this research, we define the buffer capacity as the maximum number of parts that can be accommodated in a finite space.

Leisten (1990) analyzes a static deterministic flowshop problem using heuristics for various buffer conditions, namely, unlimited buffers, finite buffers, and no intermediate storage. He finds that heuristics do not provide good results when job passing is allowed. Job passing is a situation wherein jobs are not processed in the same sequence at every machine center and therefore some jobs may overtake other jobs.

Koulamas et al. (1987) calculate the optimal buffer size for a two-stage machining process to maximize the profit rate under varying cutting speeds and tool replacement intervals. The processing times are deterministic in nature. The optimal buffer space is defined as the one necessary to keep the critical machine running when there is a tool change on the non-critical machine. The result show that the unit price increases as the tool variability and/or the penalty cost increases.

So and Pinault (1988) propose a method for allocating buffer storage in a single product pull system. Each machine center has two buffers, one in front of it (input material buffer) and another behind it (output material buffer). Although the input-buffer and output-buffer may correspond to the same physical buffer, they are logically treated as different buffers. So and Pinault decompose their system into individual $M / M / 1$ stations with bulk service. The authors conclude that the performance (i.e., average
percentage of demand backlogged) of their model will hold good if the performance parameter is less than 0.05 .

Berkley (1993) analyzed the change in relative performance of the first come first serve (FCFS) and shortest processing time (SPT) sequencing rules with changes in processing time variability and station input buffer capacities in a single-card kanban system. Using an example system, Berkley found that while FCFS has greater average production rates when processing times are normal and input buffer capacities are large. SPT has greater average production rates when processing times are exponential and input buffer capacities are small. The maximum input buffer capacity used in Berkley's research was ten containers.

Aligina (1996) extended the work of Berkley (1993) by incorporating other sequencing rules, such as earliest due date (EDD) and critical ratio (CR) and studied their effect under the same set of conditions. Aligina suggested the need for an algorithm, which will provide the optimum number of part types for a finite buffer capacity in a single card kanban system. This research will pursue this research question.

## Literature Review on Due Dates Assignments and Tardiness Penalty

The objective of this section is to help in designing the shop structure that will be studied in this research. In this research, due date assignment is considered endogenous in nature. Endogenous implies that the due dates are set internally by the scheduler as each job arrives on the basis of job characteristics, shop status information, and an estimate of the job flow time (Cheng and Gupta, 1989). One such due date assignment method is the Total Work Content (TWK) method. Ragatz and Mabert (1984) define due
dates using TWK as the sum of the arrival time plus the product of allowance factor (k) and total job processing time. Mathematically, due dates can be represented as

$$
D D_{j}=a_{j}+k \sum_{i=1}^{n} P_{i j}
$$

where;
$\mathrm{DD}_{\mathrm{j}}$ : is the due date of job j ;
$\mathrm{a}_{\mathrm{j}}$ : is the arrival time of job j ;
k : is the allowance factor;
n : is the number of operations for job j ; and
$P_{\mathrm{ij}}$ : is the processing time of operation i for job j .
Ragatz and Mabert (1988) used three levels for due date assignment, namely tight, medium, and loose. The three levels of due date tightness are set such that, when the FCFS dispatching rule is used, the number of tardy jobs will be $20 \%, 10 \%$, and $5 \%$ for tight, medium, and loose due dates, respectively. Abu-Suleiman (1998) studied the job shop environment and used allowance factors of 9,6 , and 3 to generate the tight, medium, and loose due dates, respectively.

Ragatz and Mabert (1988) consider average total cost per period as the primary performance measure. The total cost consists of late delivery cost (penalty tardiness cost) and holding cost. The penalty tardiness cost is estimated as total work content per time period late implying that the penalty tardiness cost is directly proportional to the work content. According to the definition of the TWK method, due date allowance is directly proportional to work content. Therefore, penalty tardiness is directly proportional to the due date allowance. Thus, penalty tardiness becomes a function of job value and absolute tardiness. Holding cost consists of both WIP and finished goods inventory cost implying
that the cost of holding the raw materials before station one is zero. The ratio between penalty tardiness cost and holding cost in these studies was $1: 20$. Ahmed and Fisher (1992) followed the same cost structure.

Kawtummachi et al. (1997) applied meta-scheduling methods in an automated flowshop. The objective of their study was to minimize the total cost. The tardiness penalty cost is represented as the cost of overtime. Work-In-Process (WIP) cost is proportional to the product of the number of jobs in the system and the average holding time. Inventory cost is calculated as being proportional to the product of average inventory of a job and inventory time.

The inventory and penalty cost that have been used in the above literature can be expressed in the following generic form:

$$
\mathrm{I}_{\mathrm{j}}=f\left(\mathrm{~V}_{\mathrm{j}}, \mathrm{t}_{\mathrm{j}}\right)
$$

where;
$\mathrm{I}_{\mathrm{j}}$ : the inventory cost for job j ;
$V_{j}$ : the value of job $j$;
$\mathrm{t}_{\mathrm{j}}$ : the time job j spent in the system; and
$\mathrm{P}_{\mathrm{j}}=f\left(\mathrm{~V}_{\mathrm{j}}, \mathrm{d}_{\mathrm{j}}-\mathrm{DD}_{\mathrm{j}}\right)$
where;
$\mathrm{P}_{\mathrm{j}}$ : is the tardiness penalty of job j ;
$\mathrm{d}_{\mathrm{j}}$ : the time job j departed the system; and
$\mathrm{DD}_{\mathrm{j}}$ : is the due date of job j .
Abu-Suleiman (1998) models the tardiness penalty as a function of job value and relative tardiness (tardiness divided by lead-time). This is a major shift from the traditional method of modeling the tardiness penalty as a function of job value and
absolute tardiness. In this research, we will follow Abu-Suleiman's approach to model the tardiness penalty cost. This issue will be discussed in detail in Chapter IV.

Koulamas et al. (1987) defines profit rate as the ratio of the difference between the selling price per job and the total cost per job to the total processing time.

Mathematically the profit rate per job is shown as:
$P A_{j}=\left(S_{j}-T C_{j}\right) / t_{j}$
where;
$P A_{j}$ : is the profit per job $j$;
$\mathrm{S}_{\mathrm{j}}$ : is the selling price for job j ;
$\mathrm{TC}_{\mathrm{j}}$ : is the total cost incurred for job j ; (The total cost incurred is the sum of inventory holding cost and penalty cost.) and
$\mathrm{t}_{\mathrm{j}}$ : is the total processing time per job j .
Different authors have used different performance measures to evaluate the finite buffer space. Most of these performance measures are time-based. This research will evaluate the problem of buffer space allocation based on monetary performance measures.

## Conclusion

The literature review provides insight into various factors affecting this research. The cost structure, which will form an integral part of this research and will determine the total profit generated was also reviewed. This research will provide operational guidelines, which will seek to maximize the profit for a manufacturer by determining the
optimum number of simultaneously processed part types and the allocation of space to those part types in a finite buffer capacity system.

## CHAPTER III

## RESEARCH GOAL AND OBJECTIVES

## Research Goal

The primary goal of this research was to determine the optimum number of simultaneously processed part types in a finite buffer capacity manufacturing system to maximize profits for the part type(s) manufacturer. The motivation for this research was the fact that the current literature does not adequately consider the case of multiple products undergoing different operations while competing for finite buffer capacity.

## Research Objectives

The primary objectives of this research are:

- to review current journals, books, and articles on finite buffer allocation methodology,
- to determine a set of experimental factors and their appropriate levels to assess the importance of the main factors on plant performance,
- to develop a simulation model using the simulation software package ARENA (Kelton et al., 1998) to execute the experimental design,
- to perform statistical analysis on the simulation results, and
- to develop a set of operational guidelines for manufacturers to determine the optimum number of part types, which will maximize profits.


## Methodology

The methodology used to accomplish the research objectives was:

1. To model a six station serial production system capable of handling ten different part types.
2. To develop the simulation model, and to execute the experimental design using the simulation software package ARENA (Kelton et al, 1998). The process of verification and validation will be conducted to determine the correctness of the model. The characteristics of the experiments namely the run length, number of replications, and warm-up period will be determined based on the pilot runs. This issue will be discussed in detail in Chapter IV.
3. To conduct the experimental design runs and perform statistical analysis of the simulation results obtained.
4. To develop conclusions and recommendations based upon the results obtained from the statistical analysis while simultaneously developing a set of operational guidelines for manufacturers to allocate finite buffer space.
5. To document the research.
6. To identify areas of future research.

## Assumptions

The following assumptions were made for this research.

1. Raw materials are available as required and space is not a problem for storing the finished goods.
2. Orders occur for a single part type with the size of orders sampled from uniform distribution with a minimum value of 1 and a maximum value of 18 . The inter-arrival time for all the orders is exponentially distributed with a mean of 144.
3. The different part types occupy the same amount of space within the buffers.
4. Material-handling time is zero.
5. All time units are in minutes.
6. Decisions regarding acceptance or rejection of an incoming order once taken cannot be revoked.
7. Machine breakdowns are not considered.
8. No scrap or rework is taken into account.
9. Set-up time for each part type is negligible.
10. Time value of money is included in the holding cost and penalty cost factors.
11. The processing cost, i.e., tools, worker's pay, and overheads are the same regardless of the buffer allocation procedure and therefore need not be modeled.

## CHAPTER IV

## RESEARCH METHODOLOGY

## System Description

Simulation is the evaluation tool used in this research. A six-station production line model is developed using the simulation software package ARENA (Kelton et al., 1998) (see Figure 1). The system is similar to and based upon the systems discovered in the literature survey. All the stations have a single buffer between them i.e., the output buffer for the upstream station becomes the input buffer for the downstream station and so on. Buffer space is one of the experimental factors and consists of two levels namely, 60 and 90 parts. Therefore, in an empty system the buffer space in front of the first machine has a capacity of 60 or 90 parts based on the experimental factor level chosen implying available capacity is fixed. In a non-empty system, the buffer space will be the difference between its capacity (i.e., 60 or 90 parts) and space occupied by the existing parts. Buffers present in front of other stations have ample space for work-in-process and finished goods. The other buffers have infinite capacity. It is assumed that there is ample space for storing the finished goods.

Orders arrive for parts in batches with the time between orders exponentially distributed with a mean of 144 minutes. An order is always for a single part type but it may be for quantity greater than one part. The order quantity for each order is generated

from a discrete uniform distribution with a minimum value of 1 and a maximum value of 18. The system can simultaneously process a maximum of " $n$ " different part types. The parameter " $n$ " is an experimental factor, which will be examined at three levels, namely, 2,6 , and 10 . This implies that when the experimental factor for maximum number of part types is set at 2 , then only the two part types with the highest priorities will be generated. The system in ARENA for part type level $=10$ is shown in Appendix 1. Processing time for each part type at each station is independent and identically distributed according to a normal distribution with a mean of sixteen. The variance of processing time, which is one of the experimental factors, will be considered at three levels namely $0.25,1.00$, and 2.00 . All times are expressed in the same time units (i.e., minutes). The average batch size, buffer size and the average inter-arrival times for the parts are fixed quantities. The probability for an order generated of a particular part type is determined using the sum of the years digits method refer to Appendix 2. The objective is to achieve higher volume of lower margin parts and lower volume of high margin parts. In other words, higher margin parts have less frequent orders (special orders) and lower margin parts have more frequent orders (commodities). The generality of the distribution is as follows:

Discrete distribution (part type 1, cumulative probability, part type 2, cumulative probability ...part type n, cumulative probability)

For a system processing two part types, the probability is
$\operatorname{Disc}(1,1 / 3,2,1)$,
For a system processing six part types, the probability is
$\operatorname{Disc}(1,1 / 21,2,3 / 21,3,6 / 21,4,10 / 21,5,15 / 21,6,1)$, and For a system processing ten part types, the probability is
$\operatorname{Disc}(1,1 / 55,2,3 / 55,3,6 / 55,4,10 / 55,5,15 / 55,6,21 / 55,7,28 / 55,8,36 / 55,9,45 / 55$, 10,1).

All orders reside in an area called "Waiting Area for Orders Received". Once each day the decision to accept or reject waiting orders (i.e., release it to the shop or choose not to accept the order) is taken. This decision is taken by considering the present shop conditions i.e., the available space in the finite buffer (buffer preceding machine 1) and by taking into account the previously accepted orders that have not yet completed machine 1 processing.

## Order Acceptance Logic

Whenever an order is to be accepted, preference is given to orders with high priority with respect to accumulated total profit rate potential and for those, which will not violate the available space with full orders (order splitting is not allowed). For example, if the available space is 2 , and an order for 3 parts with the highest priority is evaluated, it will be rejected. Once an order is accepted, a due date for that order is set. The due date applies to all parts in the order.

## Order Sequencing Logic

A hybrid logic is used for shuffling the queue. Initially, first in first out (FIFO) within the priority factor will be used to schedule the flow of parts. For example, in a system comprising of two part types, part type 1 will be scheduled ahead of part type 2 . Orders from any prior day's acceptance decisions may remain in the buffer at machine 1 at the time the current day's acceptance decisions are being evaluated. If an order for a
part type has crossed its due date then the priority is given to that order to minimize the penalty cost. The order sequencing logic rules are applicable only for scheduling of orders for machine-1. Downstream machines always use the FIFO rule.

Material-handling time is assumed to be zero implying that there occurs an instantaneous transfer of orders from an input buffer to the processing station and, after completion of work on all parts within an order, from the processing station to the output buffer. The decision regarding acceptance or rejection of an order from a customer is taken once every 1440 minutes ( 24 hours). This is done to avoid disturbances in the production system caused by frequent modification of available orders. Orders arriving between decision points are held until a decision is made. Once a decision is taken, it cannot be reversed. A station can produce parts of various part type mixes. Set up time is assumed to be zero.

## Experimental Factors

The effect of the following factors on cost performance will be considered:

1. Buffer size,
2. Due date allowance factor (k),
3. Penalty tightness factor (ptf),
4. Variation of processing time, and
5. Maximum number of part types.

Buffer size consists of two levels namely 60 and 90 . With respect to Figure 1 this implies that the first buffer in front of the first machine (machine 1 buffer) has a capacity of 60 or 90 parts based on the experimental factor level chosen. Due date allowance
factor levels are 3,6, and 9. Two levels of penalty tightness factors are considered, namely, $\mathrm{ptf}=1$ and $\mathrm{ptf}=2$ (a factor related to lateness, defined later in this section). The mean processing time per part per station is fixed at 16.0 . The variance of the processing time is analyzed at 3 levels namely, $0.25,1.00$, and 2.00 . The three levels of number of part types are 2,6 , and 10 . For each of these combinations, the average total profit is determined experimentally. A total of 108 experimental combinations are examined. The focus of this research is a preliminary investigation of the importance of the main factors.

Due dates are set using the Total Work Content method (TWK). The value of the constant k is chosen to be 3,6 , and 9 for low, medium, and high due dates, respectively. As discussed in the literature review, tardiness penalty cost and inventory holding cost have been modeled in the following generic forms:

$$
\begin{aligned}
& P_{j}=f\left(V_{j}, d_{j}-D D_{j}\right) \text {, and } \\
& I_{j}=f\left(V_{j}, t_{j}\right) .
\end{aligned}
$$

In this research, the inventory carrying cost will follow the same generic form mentioned above. The tardiness penalty is considered a function of job value and relative tardiness. Relative tardiness will be modeled with respect to lead-time. Therefore, the tardiness penalty cost will be modeled as:
$\mathrm{P}_{\mathrm{j}}=f\left(\mathrm{~V}_{\mathrm{j}},\left(\left(\mathrm{d}_{\mathrm{j}}-\mathrm{DD}_{\mathrm{j}}\right) /\left(\mathrm{DD}_{\mathrm{j}}-\mathrm{a}_{\mathrm{j}}\right)\right)\right)$, and $\mathrm{I}_{\mathrm{j}}=f\left(\mathrm{Vj}, \mathrm{t}_{\mathrm{j}}\right)$
where;
$P_{j}$ : is the tardiness penalty of job $j$;
$V_{j}$ : the value of job $j ;$
$\mathrm{d}_{\mathrm{j}}$ : the time job j departed the system;
$\mathrm{DD}_{\mathrm{j}}$ : due date of job j ;
$a_{j}$ : the order arrival time of job $j$;
$\mathrm{I}_{\mathrm{j}}$ : the inventory cost for job j ; and
$\mathrm{t}_{\mathrm{j}}$ : the time job j spent in the system.
Figure 2 illustrates how the tardiness penalty cost is modeled. Two levels of penalty tightness factor (ptf) are used; 1 and 2 . If the penalty tightness is set to $1, a$ job will incur a tardiness penalty cost equal to its selling price if it is late for the period of its lead-time. Similarly, a job will incur a tardiness penalty cost equal to its selling price if its lateness is twice its lead-time when ptf is set to 2 .

Abu-Suleiman's (1998) approach is used to determine the job value, overhead expenses, and profits from each job. The raw material cost of a job $j\left(\mathbf{R}_{\mathrm{j}}\right)$ is initially assumed and follows the generic form

Raw material cost $=1000-100^{*}($ part type -1$)$
where part type $=1,2, \ldots 10$
The value added after each processing step is $5 \%$ of its raw material cost. The profits for different part types are set individually. Abu-Suleiman had arbitrarily chosen the various percentages described above but assumed them to be the representative of realistic scenarios. This research also assumes the same.

## Cost Structure

The performance measure in this research is total profit and is defined as:
$\mathrm{ATPR}=\sum_{n} \sum_{i=1}^{n} P A_{j}$ and

$$
P A_{j}=\left(S_{j}-T C_{j}\right) / t_{j}
$$



Figure 2: Modeling Tardiness Penalty Cost
where;
ATPR: is the accumulated total profit rate based on experimental processing time;
$P A_{j}$ : is the profit rate per job $j$;
n : number of part types;
$\mathrm{S}_{\mathrm{j}}$ : is the selling price for job j (selling price is $1.5^{*}$ the total value of the job);
$\mathrm{TC}_{\mathrm{j}}$ : is the total cost incurred for job j (the total cost incurred is the sum of inventory holding cost and penalty cost); and $\mathrm{t}_{\mathrm{j}}$ : is the expected total processing time for job j .

Two types of costs are considered in this research, namely, the inventory holding cost and the penalty cost. The inventory holding cost per job is defined as follows:
$\mathrm{I}_{\mathrm{j}}=\int_{r j}^{d j} H V_{\mathrm{j}} \mathrm{dt}$
where;
$\mathrm{I}_{\mathrm{j}}$ : is the inventory holding cost for job j .
H : is the holding cost factor,
$V_{j}$ : is the value of job $j$,
$\mathrm{r}_{\mathrm{j}}$ : is the release time for job j (the time at which job j is released to the shop floor), and
$\mathrm{d}_{\mathrm{j}}$ : is the time job j departed the system.
Since the system under study is discrete in nature, the above integration can be expressed as follows (Abu-Suleiman, 1998):
$\mathrm{I}_{\mathrm{j}}=\sum_{\mathrm{i}=1}^{n+1} H V_{\mathrm{i}, \mathrm{j}}\left(\mathrm{t}_{\mathrm{i}, \mathrm{j}}-\mathrm{t}_{\mathrm{i}-1, \mathrm{j}}\right)$
where;
$\mathrm{I}_{\mathrm{j}}$ : is the inventory holding cost for job j ,
H : is the holding cost factor,
$V_{i, j}$ is the value of job $j$ before being processed on machine $i$, and
$t_{i, j}$ : is the time at which job $j$ leaves machine $i, t_{0 j}=r_{j}$.
Here $V_{l, j}$ is the cost of raw material for job $j\left(R_{j}\right)$. The storage area where jobs wait until their due date is modeled as machine number $(\mathrm{n}+1)$. The holding cost factor is set arbitrarily as $0.01 \%$ of the selling price of the order. The value of the holding cost factor does not affect the generality of the study that is conducted.

Penalty cost is the second type of cost and is defined as follows:

$$
\mathrm{P}_{\mathrm{j}}=\mathrm{p}_{\mathrm{j}}\left[\mathrm{dj}-\mathrm{DD}_{\mathrm{j}}\right]^{+}=\mathrm{p}_{\mathrm{j}}\left(\max \left[0, \mathrm{~d}_{\mathrm{j}}-\mathrm{DD}_{\mathrm{j}}\right]\right)
$$

where;
$P_{j}$ : is the penalty cost for job $j$,
$\mathrm{p}_{\mathrm{j}}$ : is the penalty cost factor for $\mathbf{j o b} \mathbf{j}$,
$\mathrm{d}_{\mathrm{j}}$ : the time job j departed the system, and
$\mathrm{DD}_{\mathrm{j}}$ : is the due date of job j .
Since the tardiness penalty is proportional to the job's lead-time, penalty cost factor $p_{j}$ is calculated as follows:
$\mathrm{p}_{\mathrm{j}}=\mathrm{S}_{\mathrm{j}} /\left(\mathrm{ptf}\left(\mathrm{DD}_{\mathrm{j}}-\mathrm{a}_{\mathrm{j}}\right)\right)$
where;
$p_{j}$ : is the penalty cost factor for job j ,
$\mathrm{S}_{\mathrm{j} \text { : }}$ is the selling price of the job j ,
ptf : is the level of penalty, when $\mathrm{ptf}=1$, the penalty cost is equal to the selling price if a job tardiness is equal to its lead-time,
$\mathrm{DD}_{\mathrm{j}}$ : is the due date of job j , and
$\mathrm{a}_{\mathrm{j}}$ : is the arrival time of job j .

## Simulation Verification and Validation

Verification is the process of ensuring that the ARENA model behaves in the way it is intended according to the modeling assumptions made (Kelton et al., 1998). Animation is an effective tool to perform the verification process. The path of the entities is traced as they progress through the system. This ensures that the entities go through the proper sequence of events and proper assignment of attribute values. The implementation of priority factors into the model can be observed using the animation technique.

Validation is the process of ensuring that the model behaves consistent with the real system. Since there is no existing real system that can be used to compare the
simulation results, the validation process cannot be conducted in this manner. Instead, the process of parts accounting, wherein all the parts that entered the system were accounted for, as accepted, work-in-process, and processed parts, is used to validate the model.

## Simulation Model

The ARENA simulation software package is used to simulate the six-stage production line. All the job's attributes are assigned as soon as the job enters the system. This is done to ensure that the jobs in different simulation scenarios have the same attributes based on consistent use of the random numbers. The flow chart of the flow of parts is shown in Appendix 1. A disk copy of the ARENA model is available from the author or from the School of Industrial Engineering and Management at Oklahoma State University.

## Simulation Characteristics

The characteristics essential to ensure good simulation results are run length, number of replication, and warm-up period. Pilot runs were conducted to determine the above characteristics. The system is started in an "empty and idle" condition. The period until the system reaches steady state is known as the warm-up period. To determine the warm-up period Welch's procedure (Law and Kelton, 1991) is used. This method consists of several steps that are described below. The number of replications is set at 7 (arbitrarily chosen within the recommended range of 5 to 10). It is determined that the
system reaches its steady state at around 65,000 time units (refer Appendix-3 for additional information).

## Determination of Run Length

The cycle time for a part type 1 order is collected after the warm up period. A confidence interval (CI) with a half-width less than or equal to $5 \%$ of the mean is desired. Three different combinations of run-length and warm-up period were considered. The number of replications is set at 7 . The results are summarized in the Table 1 below.

TABLE I
Effects of Different Run Lengths and Warm Up Period on the Ratio of Half-Length over the Mean

| Warm Up Period | Run length | Mean | SD | O.5 WIDTH | Ratio (\%) |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 65000 | 1000000 | 1800 | 481 | 5.05 | 0.280556 |
| 65000 | 50000 | 1810 | 478 | 7.09 | 0.391713 |
| 65000 | 60000 | 1820 | 477 | 6.45 | 0.354396 |

Note: SD-Standard Deviation

Since all values tested satisfied the confidence interval, the value (i.e., 65000 ), which led to warm being approximately $10 \%$ of the run length (rule of thumb) is chosen. The warm up period graph is shown in Appendix 3.

## CHAPTER V

## RESULTS

As discussed previously, 108 different system configurations were studied. The primary performance measure of this research is accumulated total profit rate. For each experiment, ATPR is determined by knowing the raw material cost, processing cost, and the selling price. The results can be summarized by the following table:

TABLE II
Table of Means with Varying Number of Parts

| No. of Parts | Profit Mean |
| :---: | :---: |
| 2 | 41148 |
| 6 | 28581 |
| 10 | 17108 |

ANOVA tests were performed using the SAS software to determine the statistical significance of the main factors. For each experiment, the ATPR for individual replication were found and used. The General Linear Model (GLM) procedure in SAS produced an F value and a probability value ( Pr ) for each of the main factors. The F value is the ratio produced by dividing the Mean Square for the model by the Mean Square for Error. For a confidence interval of $95 \%$, i.e., with an alpha ( $\propto$ ) value of 0.05 ,

General Linear'Wodets Procedure

Dependent Variable: PROFITS

| Source | DF | - Sua of Squaras | - Mean Square | F value | $\mathrm{Pr}>\mathrm{F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Model | $B$ | .135584358853,0030000 | 18945544856.6254000 | 126.20 | 0.0001 |
| Error | 747 | - 100300273375.6880000 | 134270730.8580800 |  |  |
| Corrected Total | 755 | 225864632228.6890000. |  |  |  |
| , | A. Square | c.v. | Aoot use | 'PROF I | rs mean |
| . . . | 0.574753 | 40.03218 | 11587.52695800 | 28945.3 | 32017480 |
| * |  |  | . . |  |  |
| Soure: | - OF | - Type I 35 | Mean Squara | F value | Pr $\boldsymbol{P r}$ |
| PARTTYPE - | 2 | 728736-40003.21450000 | - 3643s820001.60720000 | 271.37 | 0.0001 |
| D0 | 2 | 44452430390.77010000 | 24226215195.38500000 | 180.43 | 0.0001 |
| VAR | 2 | 280598862.50103700 | 130299431.25051800 | 0.97 | 0.3794 |
| тIGMT | 1 | 12781549214.38680000 | 12781548214.26880000 | 95.18 | 0.0001 |
| BuFSIEE | 1 | 1198140382.15113000 | 1198140382.15113000 | 8.91 | 0.0029 |

Figure 3: Anova Test Results


Figure 4: Duncan Test Result for the Experimental Factor Part Type

General Linear Models Procedure
Duncan's Hultiple Range Test for variable: PAOFITS
NOTE: This test controls the type I comparisonmise error rate, nat the experimentinise error rate

Alphas 0.05 dfa 747 USE* $1.3427 E B$

$$
\text { Nutber of Heans } \quad 2 \quad 3
$$ Critical Range 20272134

Weans with the sae letter are not significantly differant.

| Duncan Grouping | Mean | N | do |
| ---: | ---: | ---: | ---: |
| A | 38487 | 252 | 0 |
| B | 32487 | 252 | 6 |
| C |  | 17882 | 252 |

Figure 5: Duncan Test Result for the Experimental Factor Due Date

The sas syatem $\quad 18: 49$ Sarurday, May 1, 199930
General Linear Models Procedure
Duncan's Multiple Range Tast for variable: PROFITS
WOTE: This test controls the type I comparisomise orror rate, nat the experisentwise error rate

Alpha= $0.05 \mathrm{df}=747$ uSE $=1.3427 E 8$
Number of Heans 2 . 3 Critical Range 2027 2:34

Heans with the same letter are not sagnificantly diffarant.

| Duncan Grouping | Masn | N | VAA |
| ---: | ---: | ---: | ---: |
| A | 29775 | 252 | 2 |
| A |  |  |  |
| A | 28570 | 252 | 1 |
| A |  |  |  |
| A | 28492 | 252 | 0.25 |

Figure 6: Duncan Test Result for the Experimental Factor Variance
the probability of acceptance of the main factors is 0.05 . A difference between the Pr value and the F value greater than 0.05 for a factor implies that the experimental factor is insignificant.

Figure 3 displays the result of the ANOVA test. It is observed that the $\operatorname{Pr}$ value for the experimental factor "variance" yielded a value of 0.3794 , which is greater than 0.05 implying that variance is insignificant for this system. The other experimental factors namely, part type, due date allowance factor, penalty tightness factor, and buffer size yielded a Pr value lesser than 0.05 implying that these were statistically significant factors. High order interactions (i.e., two-way and greater) were not considered in this research. Preliminary analysis showed all the 2-way and 3-way interaction were significant. By not including the subsequent higher level interactions as part of the model in SAS, the degrees of freedom for these terms are pooled into the Error term. There is risk in this approach since the significance of high order interactions may be lost; however, the focus of this research is a preliminary investigation of the importance of the main factors.

In addition to the above test, Duncan's multiple-range tests are also conducted with the same alpha $(\propto)$ value of 0.05 to group the factor levels within each experimental factor. Since each mean falls into a different group for each factor except variance (refer to Figure 4,5, and 6), there are significant differences between the means. For example, in Figure 4, the different levels of the number of part types fall in different grouping levels namely, A, B, and C implying that these three levels of number of part types are significantly different from one another. This interpretation holds good for Figure 5. The means with the different letter implies that levels of the above said experimental factors are significantly different. It is found that there is no significant difference between the
means for different levels of variance (refer Figure 6). The means with the same letter implies that different levels of the experimental factor, variance are not significantly different. From the Anova test, the Ptf and the buffer size results confirm the obvious, i.e., there is a significant difference between the two means.

The results can be consolidated using graphs, which provide a summary of the performance of the system with respect to ATPR under varying due date levels and numbers of part types. The data points for these graphs are shown in Appendix 4. Figure 7 displays the ATPR generated under different due dates and number of part types but under the same variance, penalty tightness factor, and buffer level. It is observed that for a given number of part types, having tighter due date level (i.e., 9) generated more ATPR. This can be seen by observing the third bar in each group. Moreover, maximum ATPR is generated when fewer part types are used. The first group of bars is greater than their respective bar in other groups. It is shown by the Duncan's test of means that the different levels of the experimental factors were statistically significantly different. Therefore the condition of maximum ATPR is used to determine the part type level which generated maximum profits (refer to Figure 7).

The only difference between Figure 7 and Figure 8 is the change in the buffer size level. By comparing Figure 7 and Figure 8, it is observed that having a bigger buffer size generated greater ATPR since each of the respective bars is taller except in cases when the ptf. The only difference between Figure 7 and Figure 9 is the change in the penalty tightness factor level. By comparing Figure 7 and Figure 9, it is observed by looking at the first bar in each group that greater ATPR was generated for a due date tightness level of 3 and by having a higher penalty tightness factor (i.e., 2 ). It is observed that maximum


Figure 7: Average Total Proflt for Var=0.25, Ptt=1, Buffer=60


Figure 8 : Average Total Proflt for Var=0.25, Ptf=1, Buffer=90


Figure 9 : Average Total Profit for Var=0.25, Ptf=2, Buffer=60


Figure 10: Average Total Profit for Var=0.25, Ptf=2, Buffer=90


Figure 11: Average Total Profit for Var=1.00, Ptt=1, Buffer=60


Figure 12: Average Total Profit for Var=1.00, $\mathrm{Pt}=1$, Buffer=90


Figure 13: Average Total Proflif for Var=1.00, Ptf=2, Buffer=60


Figure 14: Average Total Profit for Var=1.00, $\mathrm{Ptf}=2$, Buffer=90


Figure 15: Average Total Proflt for Var=2.00, Ptf=1, Buffer=60


Figure 16: Average Total Profit for Var=2.00, Ptf=1, Buffer=90


Figure 17: Average Total Proflt for Var=2.00, Ptf=2, Buffer=60


Figure 18: Average Total Profit for Var=2.00, $\mathrm{Ptf}=2$, Buffer=90

ATPR is generated when fewer part types are used. The break up of the ATPR with respect to different part types indicated that higher numbered (but lower priority) part types generated more ATPR than the lower part types. For example, in an experimental configuration involving 6 part types the higher part types (i.e., $4,5,6$ ) generated greater ATPR than the lower part types (i.e., 1,2,3) although lower part types generated more profit than the higher part types with respect to profit per part (i.e., profit margin). This can be attributed to the experimental assumption of increasing the probability of generation of orders with respect to the part types. In other words higher margin parts have less frequent orders (special orders) and lower margin parts have more frequent orders (commodities). These trends hold good for all the experimental combinations.

It is observed that processing fewer part types with tighter due date levels, higher penalty tightness factors, and larger buffer sizes generated maximum ATPR. This result was expected due to the fact that the high margin part types were being processed. An increased buffer size provides additional opportunity for higher margin part types to be accepted and it also reduces the number of part types to be rejected.

The only surprise is the fact that although higher numbered part types (i.e., 4, 5, and 6) generated more profits than the lower part types (i.e., 1, 2, and 3) maximum ATPR is not generated by the higher part type experimental factor levels (6 and 10). This can be attributed to the loose raw material cost structure wherein the percent difference between the highest numbered part type and the lowest numbered part type is $90 \%$.

It can be concluded that processing fewer numbers of part types with tighter due date levels, higher penalty tightness factors, and larger buffer size generates maximum ATPR.

## CHAPTER VI

## CONCLUSIONS AND FUTURE RESEARCH

This chapter concludes this research report by presenting the conclusions and future directions of this research. A simulation model is developed using the simulation software, ARENA. It is found that four of the experimental factors namely, due date allowance factor, buffer size, penalty tightness factor, and number of part types were significant with respect to ATPR.

This research has also identified the experimental settings that will maximize profit. It is observed that processing fewer numbers of part types with tighter due date levels, higher penalty tightness factors, and larger buffer size generated maximum ATPR. This finding is important for part type manufacturers as it provides operational guidelines to improve the profits generated by a manufacturing system. For an upstart manufacturing firm whose initial aim is to generate maximum ATPR, the operational guidelines provided are that they should use lesser number of part types, looser due dates level, higher penalty tightness level, and a higher buffer size to maximize profits. For an established manufacturing firm, which wants to use a smaller buffer size (say 60 ) due to implementation of a pull system like JIT, the operational guidelines provided are that they should use looser due date level, higher tightness level, and low number of part types to maximize ATPR. From the research point of view, to further generalize the results of this research these experimental factors should be further analyzed to determine
their effectiveness on the system under additional operational parameters such as machine breakdowns, and arrival of rush orders.

Another important finding of this research is that variation in processing time (denoted by the factor, variance) was not a significant factor in this study. Variance is relatively small compared to due dates level to have any effect on the system. It is possible the higher levels of variation relative to the mean (highest level in this study was $2 / 16=12.5 \%$ ) with tighter due date factors might indicate significance. Higher levels of variation could be the focus of additional studies in this area.

## Future Research

Some possible directions for future research are given below:

- In this research, no penalty cost was attached to the orders that are rejected. In a more general situation, penalty cost for the order rejected can be considered during the decision making process. If some part types 1 were rejected due to unavailability of buffer space, it might have reduced the ATPR generated, thereby providing an opportunity for different levels of part types to generate maximum profits individually.
- A more complicated flow shop system may be considered; for example, a system with machine failures and rework. This will increase the inventory holding cost thereby cutting down on the ATPR generated.
- The decision to accept or reject an order is based on the profit rate generated per part, which is assumed to be known apriori. Instead a more complicated profitability factor can be incorporated by considering the actual profit rate per part by considering
the penalty cost and inventory holding cost generated at the completion of manufacturing of each part.
- Rather than processing orders in batches at each station, they could be split into parts and subsequently processed at individual stations. This will smooth the flow since the parts are transferred individually.
- Increasing the numerical value of the variance of the processing time. This could bring about the significance of the variance as an experimental factor.
- Since virtually all the main factors were significant, additional studies including high order interactions is recommended.
- In this research, the percent difference between the raw material cost of the higher profit rate per part generating part type (i.e., 1) and that of the lower profit rate per part generating part type (i.e., 10) was $90 \%$. This percent difference is due to the assumptions made for the initial raw material cost. Changing this percent difference might bring about a change in the final results.
- Further research can be conducted by considering the possibility of investigating two parts where one has the highest priority (part which generates comparatively maximum profit rate per part) and another has the lowest priority (part which generates comparatively minimum profit rate per part).


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## Appendix-1: The Block Diagram of the System



## Appendix-2: Sum Of The Years Digit Method Calculation

The generality of the distribution is as follows:
Discrete distribution (part type 1, cumulative probability part type 2, cumulative probability ...part type n, cumulative probability)
(i) For a system processing two part types, the sum of the two numbers is 3 (i.e., 1 and 2), therefore the probability is

Disc (part type 1, (part type 1/sum of the two numbers), part type 2, (part type 2 /sum of the two numbers))
i.e., $\operatorname{Disc}(1,1 / 3,2,1)$
(ii) For a system processing six part types, the sum of the six numbers is 21 (i.e., $1,2,3,4,5$, and 6 ), therefore the probability is

Disc (part type 1, (part type 1/sum of the six numbers), part type 2, (part type 2/sum of the six numbers)... part type 6, (part type 6/sum of the six numbers))
i.e., $\operatorname{Disc}(1,1 / 21,2,3 / 21,3,6 / 21,4,10 / 21,5,15 / 21,6,1)$, and
(iii) For a system processing ten part types, the sum of the ten numbers is 55 (i.e., $1,2,3,4,5,6,7,8,9$, and 10 ), therefore the probability is Disc (part type 1, (part type 1/sum of the six numbers), part type 2, (part type 2/sum of the six numbers)... part type 10 , (part type $10 /$ sum of the six numbers)) i.e., $\operatorname{Disc}(1,1 / 55,2,3 / 55,3,6 / 55,4,10 / 55,5,15 / 55,6,21 / 55,7,28 / 55,8,36 / 55,9,45,55$, 10,1 ).

## Appendix-3: Warm Up Period Determination Procedure

Step 1: Simulation runs were made for the worst conditions for seven replications each of $1,000,000\left(x_{n}\right)$ number of observations (arbitrary chosen). The time-in-system of part type 1 order is recorded as single observation data $\left(\mathrm{Y}_{\mathrm{ij}}, \mathrm{j}=1,2, \ldots, 7 ; \mathrm{i}=1,2, \ldots, \mathrm{x}_{\mathrm{n}}\right)$.

Step 2: For the 7 replications, the average time-in-system ( $\overline{\mathrm{Y}}_{\mathrm{ij}}$ ) of part type 1 order is determined using the formula:
$\bar{Y}_{i j}=\sum_{j=1}^{7} Y_{i j} / n, \mathrm{n}=7$ and $\mathrm{i}=1,2, \ldots, \mathrm{x}_{\mathrm{n}}$.
$\mathrm{x}_{\mathrm{n}}=$ number of observations.
Step 3: To smooth out the high frequency oscillation in the time in system measure, the moving average method is used. The window (w) of the moving average is a positive integer such that $w \leq\left(x_{n} / 2\right)$. The bigger the window values the smoother the curve. Based on the selected window, the moving average values $\left(\mathrm{Y}_{\mathrm{i}}(\mathrm{w})\right)$ are calculated as follows:


Step 4: Plot $Y_{i}(w)$ where $i=1,2, \ldots\left(x_{n}-w\right)$ for different window sizes, $w$. Window size of 200 is used to determine the truncation point. Microsoft Excel spreadsheet software is
used to plot the graph. In the graph (refer Appendix-3), the $x$-axis represents the total time and the $y$-axis represents the time-in-system for part type 1 order. It is determined that the system reaches its steady state at around 65,000 time units.



## Appendix-4: Data Points for the Graphs

| Part types | D.D. level | Var. level | Ptf level | Buffer Size | ATPR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 0.25 | 1 | 60 | 13468.29 |
| 2 | 6 | 0.25 | 1 | 60 | 39222.43 |
| 2 | 9 | 0.25 | 1 | 60 | 45763.43 |
| 6 | 3 | 0.25 | 1 | 60 | 10751.17 |
| 6 | 6 | 0.25 | 1 | 60 | 31416.14 |
| 6 | 9 | 0.25 | 1 | 60 | 36669.43 |
| 10 | 3 | 0.25 | , | 60 | 6246.771 |
| 10 | 6 | 0.25 | 1 | 60 | 16921.57 |
| 10 | 9 | 0.25 | 1 | 60 | 19616.43 |
|  |  |  |  |  |  |
| 2 | 3 | 0.25 | 1 | 90 | 11328.1 |
| 2 | 6 | 0.25 | 1 | 90 | 44747.29 |
| 2 | 9 | 0.25 | 1 | 90 | 53236.57 |
| 6 | 3 | 0.25 | 1 | 90 | 9142.314 |
| 6 | 6 | 0.25 | 1 | 90 | 35970 |
| 6 | 9 | 0.25 | 1 | 90 | 42789.57 |
| 10 | 3 | 0.25 | 1 | 90 | 4838.771 |
| 10 | 6 | 0.25 | 1 | 90 | 19137.14 |
| 10 | 9 | 0.25 | 1 | 90 | 22750 |
|  |  |  |  |  |  |
| 2 | 3 | 0.25 | 2 | 60 | 32237.57 |
| 2 | 6 | 0.25 | 2 | 60 | 45114.43 |
| 2 | 9 | 0.25 | 2 | 60 | 48385 |
| 6 | 3 | 0.25 | 2 | 60 | 25816.29 |
| 6 | 6 | 0.25 | 2 | 60 | 36148.86 |
| 6 | 9 | 0.25 | 2 | 60 | 38775.43 |
| 10 | 3 | 0.25 | 2 | 60 | 13985.86 |
| 10 | 6 | 0.25 | 2 | 60 | 19323.43 |
| 10 | 9 | 0.25 | 2 | 60 | 20671 |
|  |  |  |  |  |  |
| 2 | 3 | 0.25 | 2 | 90 | 35813.43 |
| 2 | 6 | 0.25 | 2 | 90 | 52523.14 |
| 2 | 9 | 0.25 | 2 | 90 | 56767.71 |
| 6 | 3 | 0.25 | 2 | 90 | 28801.86 |
| 6 | 6 | 0.25 | 2 | 90 | 42216 |
| 6 | 9 | 0.25 | 2 | 90 | 45625.57 |
| 10 | 3 | 0.25 | 2 | 90 | 15315 |
| 10 | 6 | 0.25 | 2 | 90 | 22464.43 |
| 10 | 9 | 0.25 | 2 | 90 | 24270.86 |


| Part types | D.D. level | Var. leve! | Ptf.level | Buffer Size | ATPR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 1 | 1 | 60 | 14407.86 |
| 2 |  | 1 | 1 | 60 | 39801.29 |
| 2 | 9 | 1 | 1 | 60 | 46282.71 |
| 6 | 3 | 1 | 1 | 60 | 11662.54 |
| 6 | 6 | 1 | 1 | 60 | 32006.71 |
| 6 | 9 | 1 | 1 | 60 | 37191.57 |
| 10 | 3 | 1 | 1 | 60 | 6384.529 |
| 10 | 6 | 1 | 1 | 60 | 16942.57 |
| 10 | 9 | 1 | 1 | 60 | 19610.14 |
|  |  |  |  |  |  |
| 2 | 3 | 1 | 1 | 90 | 10450.26 |
| 2 | 6 | 1 | 1 | 90 | 44338 |
| 2 | 9 | 1 | 1 | 90 | 52949.43 |
| 6 | 3 | 1 | 1 | 90 | 8504.614 |
| 6 | 6 | 1 | 1 | 90 | 35661.86 |
| 6 | 9 | 1 | 1 | 90 | 42564.43 |
| 10 | 3 | 1 | 1 | 90 | 4943.614 |
| 10 | 6 | 1 | 1 | 90 | 19220 |
| 10 | 9 | 1 | 1 | 90 | 22797.86 |
|  |  |  |  |  |  |
| 2 | 3 | 1 | 2 | 60 | 32887.71 |
| 2 | 6 | 1 | 2 | 60 | 45584.57 |
| 2 | 9 | 1 | 2 | 60 | 48825.14 |
| 6 | 3 | 1 | 2 | 60 | 26458.57 |
| 6 | 6 | 1 | 2 | 60 | 36630.57 |
| 6 | 9 | 1 | 2 | 60 | 39223.29 |
| 10 | 3 | 1 | 2 | 60 | 14037.71 |
| 10 | 6 | 1 | 2 | 60 | 19317.14 |
| 10 | 9 | 1 | 2 | 60 | 20651 |
|  |  |  |  |  |  |
| 2 | 3 | 1 | 2 | 90 | 35289.57 |
| 2 | 6 | 1 | 2 | 90 | 52233.57 |
| 2 | 9 | 1 | 2 | 90 | 56539.86 |
| 6 | 3 | 1 | 2 | 90 | 28412.57 |
| 6 | 6 | 1 | 2 | 90 | 41991.57 |
| 6 | 9 | 1 | 2 | 90 | 45442.86 |
| 10 | 3 | 1 | 2 | 90 | 15372.57 |
| 10 | 6 | 1 | 2 | 90 | 22510.71 |
| 10 | 9 | 1 | 2 | 90 | 24299.86 |


| Part types | D.D. level | Var. level | Ptf level | Buffer Size | ATPR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 2 | 1 | 60 | 14625.14 |
| 2 | 6 | 2 | 1 | 60 | 39662.29 |
| 2 | 9 | 2 | 1 | 60 | 46036.71 |
| 6 | 3 | 2 | 1 | 60 | 11734.67 |
| 6 | 6 | 2 | 1 | 60 | 31834 |
| 6 | 9 | 2 | 1 | 60 | 36949.57 |
| 10 | 3 | 2 | 1 | 60 | 5988.557 |
| 10 | 6 | 2 | 1 | 60 | 16824.86 |
| 10 | 9 | 2 | 1 | 60 | 19580.57 |
|  |  |  |  |  |  |
| 2 | 3 | 2 | 1 | 90 | 11431.07 |
| 2 | 6 | 2 | 1 | 90 | 44843.86 |
| 2 | 9 | 2 | 1 | 90 | 53253.86 |
| 6 | 3 | 2 | 1 | 90 | 9357.771 |
| 6 | 6 | 2 | 1 | 90 | 36133.57 |
| 6 | 9 | 2 | 1 | 90 | 42862.71 |
| 10 | 3 | 2 | 1 | 90 | 4235.229 |
| 10 | 6 | 2 | 1 | 90 | 19049.71 |
| 10 | 9 | 2 | 1 | 90 | 22791.86 |
|  |  |  |  |  |  |
| 2 | 3 | 2 | 2 | 60 | 35837.71 |
| 2 | 6 | 2 | 2 | 60 | 45353 |
| 2 | 9 | 2 | 2 | 60 | 48540.57 |
| 6 | 3 | 2 | 2 | 60 | 26348 |
| 6 | 6 | 2 | 2 | 60 | 36397.86 |
| 6 | 9 | 2 | 2 | 60 | 38955.29 |
| 10 | 3 | 2 | 2 | 60 | 13880.29 |
| 10 | 6 | 2 | 2 | 60 | 19298.86 |
| 10 | 9 | 2 | 2 | 60 | 20676.57 |
|  |  |  |  |  |  |
| 2 | 3 | 2 | 2 | 90 | 35837.71 |
| 2 | 6 | 2 | 2 | 90 | 52544.43 |
| 2 | 9 | 2 | 2 | 90 | 56749.29 |
| 6 | 3 | 2 | 2 | 90 | 28902.86 |
| 6 | 6 | 2 | 2 | 90 | 42290.86 |
| 6 | 9 | 2 | 2 | 90 | 45655.43 |
| 10 | 3 | 2 | 2 | 90 | 15076.86 |
| 10 | 6 | 2 | 2 | 90 | 22484.29 |
| 10 | 9 | 2 | 2 | 90 | 24355.29 |

## Appendix-5: Example Calculation for the Cost Structure

Consider a part that goes through 3 operations. The initial raw material cost is $\$ 1000$ and the value added after each operation is $5 \%$. The job arrives at time 0 and an allowance factor of 9 is used. The average processing time per operation is 1.5 hrs .

$$
\begin{aligned}
& \text { Due Date }=D D_{j}=a_{j}+k \sum_{i=1}^{n} P_{i j} \\
& =0+9^{*}\left(3^{*} 1.5\right) \\
& =40.5 \mathrm{hrs}
\end{aligned}
$$

Selling Price $=1.5^{*}$ the total value of the job

$$
\begin{aligned}
& =1.5^{*}\left(\text { Value } 12+1.05^{*}(\text { value } 12)+1.05^{*}(\text { value } 13)+1.05^{*}(\text { value } 14)\right) \\
& =\$ 6465.19
\end{aligned}
$$

Selling Price (excluding cost of raw material cost) $=\$ 5465.19$
Holding Cost Factor $=0.01 / 100 * 1736.44$

$$
=0.546519
$$

$$
\text { Inventory Cost }=\mathrm{I}_{\mathrm{j}}=\sum_{i=1}^{n+1} H V_{\mathrm{i}, \mathrm{j}}\left(\mathrm{t}_{\mathrm{i}, \mathrm{j}}-\mathrm{t}_{\mathrm{i}-\mathrm{l}, \mathrm{j}}\right)
$$

$$
\begin{aligned}
& =0.1736^{*}\left(\left(1000^{*} 1.5\right)+\left(1.05^{*} 1000^{*} 1.5\right)+\left(1.05^{2} * 1000^{*} 1.5\right)\right) \\
& =\$ 2584.36
\end{aligned}
$$

Penalty time $=\mathrm{p}_{\mathrm{j}}\left(\max \left[0, \mathrm{~d}_{\mathrm{j}}-\mathrm{DD}_{\mathrm{j}}\right]\right)$

$$
=0
$$

Penalty time factor $=6465.19 / 2 * 40.5$

$$
=79.82
$$

Penalty cost = Penalty time factor * Penalty time

$$
=0
$$

Total cost $=$ Inventory cost + Penalty cost

$$
=\$ 2584.36
$$

Profit $=($ Selling Price - Total cost $) /$ total Processing time
$=(6465.19-2584.36) / 4.5$
$=\$ 862.41$

## VITA

## Yousuff Zaman Habibullahkhan

Candidate for the degree of
Master of Science
Thesis: ALLOCATION OF FINITE BUFFER CAPACITY TO PART TYPES FOR MAXIMIZING PROFITS IN SERIAL LINES

Major field: Industrial Engineering and Management
Biographical:
Personal Data: Born in Madras, India, January 27, 1976, the son of Mr. Habibullah Khan and Mrs. Iqbal Begum

Education: Graduated from Gill Adarsh Matriculation Higher Secondary School, Madras, India, in May 1993; received Bachelor of Engineering degree in Mechanical Engineering from University of Madras, Madras, India in May 1997; completed the requirement for the Master of Science degree in Industrial Engineering and Management at Oklahoma State University in December 1999.

Experience: Graduate Teaching Assistant, Oklahoma State University, from August 1998 to May 1999. Production Engineer, Ingersoll Products Company, Chicago, Illinois, from June 1999 to Present.

Affiliations: Institute For Operations Research and Management Science (INFORMS).

