COMPARISON OF FRICTIONAL PRESSURE DROP CORRELATIONS FOR ISOTHERMAL TWO-PHASE HORIZONTAL FLOW

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COMPARISON OF FRICTIONAL PRESSURE DROP

CORRELATIONS FOR ISOTHERMAL

TWO-PHASE HORIZONTAL FLOW

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ABSTRACT: An extensive literature search was made for the available two-phase frictional pressure drop correlations and isothermal two-phase horizontal flow experimental data. The experimental data was systematically refined and 2,429 horizontal two-phase flow pressure drop data points from 11 authors were selected for this study. Computer codes of 42 pressure drop correlations were written in Engineering Equation Solver (EES). The performance of the pressure drop correlations was evaluated against the diverse experimental data using statistical tools. Review of previously done comparisons by other authors is also presented. Appropriate recommendations are forwarded both for wide and narrow sets of applications.

Comparisons between the correlations are presented using relative error bands and graphical probability density functions. The best performing correlations in the ±30% error band for each experimental data base are presented. The analysis showed that the performance of most of the correlations available in literature is restricted only for narrow range of applications. Performance of the correlations was also found to vary with void fraction ranges and flow pattern. For a reader who is interested in more specific sets of conditions, results of the comparison are summarized and presented for more specific sets of flow conditions based on void fraction ranges.

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NOMENCLATURE

Symbols	**Description, [unit]**
A	Area, [m^2]
B	Chisholm (1973) parameter, [-]
C	Chisholm (1967) constant, [-]
C_0	Two-phase distribution parameter, [-]
D	Diameter, [m]
f	Friction factor, [-]
Fr	Froude number, $Fr = \frac{G^2}{gD\rho^2}$ [-]
g	Acceleration due to gravity, [m/s^2]
G	Mass flux, [kg/m^2s]
$H_l(\theta)$	Liquid hold up as a function of inclination angle, [-]
k	Absolute roughness, [mm]
$\dot{m}$	Mass flow rate, [kg/sec]
n	Blasius constant, [-]
P	Pressure, [Pa]
p	Perimeter, [m]
Q	Volumetric flow rate, [m^3/s]
Re	Reynolds number, $\mathrm{Re} = \frac{GD}{\mu}$ [-]
S	Slip ratio, [-]
T	Temperature, [ºC]
U	Velocity, [m/s]

w_i	Interfacial surface width, [m]
We	Weber number, $We = \dfrac{\rho_l U_l^2 D}{\sigma_l}$ [-]
x	Flow quality or Dryness fraction, $\left[\dfrac{\dot{m}_g}{\dot{m}_g + \dot{m}_l}\right]$[-]
X	Lockhart and Martinelli (1949) parameter, [-]

Greek symbols

α	Void fraction, [-]
β	Volumetric flow quality, $\left[\dfrac{Q_g}{Q_l + Q_g}\right]$[-]
δ	Film thickness, [m]
$\left[\dfrac{\Delta P}{\Delta L}\right]$	Pressure drop per length, [Pa/m]
λ	Input liquid content, $\left[\dfrac{Q_l}{Q_l + Q_g}\right]$[-]
μ	Dynamic viscosity, [kg/m-s]
$\overline{\mu}$	Mean, [-]
υ	Kinematic viscosity, [m^2/s]
ρ	Density, [kg/m^3]
ϕ	Two-phase multiplier, [-]
σ	Surface tension, [kg/s^2]
σ_{SD}	Standard deviation, [-]

τ	Shear stress, [N/m^2]
Γ	Parameter in Chisholm (1972) correlation, [-]
θ	Angle of inclination, [radians]

Subscripts

a	Acceleration component
atm	Atmospheric condition
g	Gas phase
go	Gas phase only
i	Interfacial
l	Liquid phase
lo	Liquid phase only
ns	No-slip
sg	Based on superficial gas velocity
sl	Based on superficial liquid velocity
sys	System
tp	Two-phase mixture
wg	Between wall and gas
wl	Between wall and liquid

CHAPTER 1

INTRODUCTION

Two-phase flow is a term used to define an area of fluid mechanics that deals with the flow of two different phases flowing simultaneously. The term phase refers to a state of the matter. It can either be gas, liquid or solid in most practical applications. Therefore, two-phase flow is a particular example of multiphase flow where any two of the three phases exist in a flow system.

Combinations of the phases can be mentioned as gas-liquid, gas-solid, liquid-solid. The combination of the phases can be formed from a single component or a mixture of two different components. Steam-water flow is an example of a single component fluid whereas air-water mixture can be mentioned as two component two-phase flow. Gas-liquid flow is the most popular among the other phase combinations, for most engineering applications of two-phase flow. From here on, the term two-phase in this study refers to single or two component gas-liquid flows.

Two-phase flow has a very wide application in the modern industry. From the vast industrial application, we can mention refrigeration, air conditioning, petroleum, and food process industries just to name a few of them where two-phase flow is extensively employed. Most of the applications require the ability to predict the two-phase frictional pressure drop accurately for the design and optimization of components such as pumps and pipe lines.

Due to this crucial nature of the application quite a few literature can be found dealing with two-phase flow pressure drop analysis. The fact that two-phase pressure drop is dependent on numerous parameters makes the analysis more complicated. In an effort to address this issue a lot of two-phase flow pressure drop correlations have been developed by different authors for more than half a century. Although there are quite a number of improvements in understanding of the two-phase flow phenomena, a gap still exists between experimental investigation and the predicted pressure drops available in the literature.

Before going forward, it would be a worthwhile to highlight some of the most common terminologies and definitions of parameters that would be encountered throughout this work to facilitate understanding of the discussions in the coming chapters.

1.1 Basic Definitions and Terminologies

Volumetric flow rate (Q) has SI unit of [m^3/s] and it is defined as the ratio of mass flow rate $(\dot{m})$ to density (ρ).

$$Q = \frac{\dot{m}}{\rho} \tag{1.1}$$

The total mass flow rate $(\dot{m}_{tp})$ has SI unit of [kg/s] and it is defined as the sum of the mass flow rate of the liquid phase $(\dot{m}_l)$ and the gas phase $(\dot{m}_g)$.

$$\dot{m}_{tp} = \dot{m}_l + \dot{m}_g \tag{1.2}$$

The total mass flux (G_{tp}) has SI unit of [kg/m^2-s] and it is defined as the sum of the mass flux of the liquid phase (G_l) and the gas phase (G_g).

$$G_{tp} = G_l + G_g \tag{1.3}$$

Flow quality (*x*) is defined as the ratio of gas phase mass flow rate to the total mass flow rate.

$$x = \left[\frac{\dot{m}_g}{\dot{m}_g + \dot{m}_l}\right] = \left[\frac{\dot{m}_g}{\dot{m}_{tp}}\right] \tag{1.4}$$

Slip ratio (*S*) is defined as the ratio of the average velocity of the gas phase (U_g) to the average velocity of the liquid phase (U_l)

$$S = \frac{U_g}{U_l} \tag{1.5}$$

1.2 <u>Definition of Void Fraction</u>

Void fraction (α) is defined as the ratio of the cross-sectional area occupied by the gas (A_g) to the total cross-sectional area of the pipe (A_t). This ratio also gives the volume of space the gas phase occupies in two-phase flow in a pipe.

$$\alpha = \frac{A_g}{A_t} = \frac{A_g}{A_l + A_g} \tag{1.6}$$

There are many correlations in the open literature to predict void fraction. Ghajar and Tang (2012) compared 54 void fraction correlations against a diverse experimental data in vertical and horizontal two-phase flows. They recommended the Woldesemayat and

Ghajar (2007) void fraction correlation for horizontal two-phase flows. Therefore, the Woldesemayat and Ghajar (2007) correlation was used to calculate void fraction values in this study unless otherwise mentioned. The correlation is shown below.

$$\alpha = \frac{U_{sg}}{C_0 \left(U_{sg} + U_{sl} \right) + U_{gu}} \tag{1.7}$$

$$C_0 = \frac{U_{sg}}{U_{sg} + U_{sl}} \left[1 + \left(\frac{U_{sl}}{U_{sg}} \right)^{\left(\frac{\rho_g}{\rho_l} \right)^{0.1}} \right] \tag{1.8}$$

$$U_{gu} = 2.9 \left(1.22 + 1.22 \sin\theta \right)^{P_{atm}/P_{sys}} \left[\frac{gD\sigma \left(1 + \cos\theta \right) \left(\rho_l - \rho_g \right)}{\rho_l^2} \right]^{0.25} \tag{1.9}$$

The leading constant of 2.9 in equation (1.9) carries a unit of $m^{-0.25}$

1.3 <u>Flow Patterns in Horizontal Two-Phase Flow</u>

Forces of gravity, buoyancy, interfacial tension, friction and pressure play a major role in shaping the form of a flow. The various geometrical shapes the flow takes in two or three dimensions are often referred to as flow pattern. There are many types of flow patterns that could exist in two-phase flow depending on a specific set of flow parameters including diameter or pipe inclination.

Interpretation of flow patterns is usually subjective. According to Tang (2011) there is no uniform procedure to describe and classify flow patterns. Usually flow pattern maps are used to identify the flow regime if visual observation is not possible. There are different flow pattern maps suggested by several researchers. Figure 1 shows a flow pattern map suggested by Taitel and Dukler (1976) as well as Kim and Ghajar (2002).

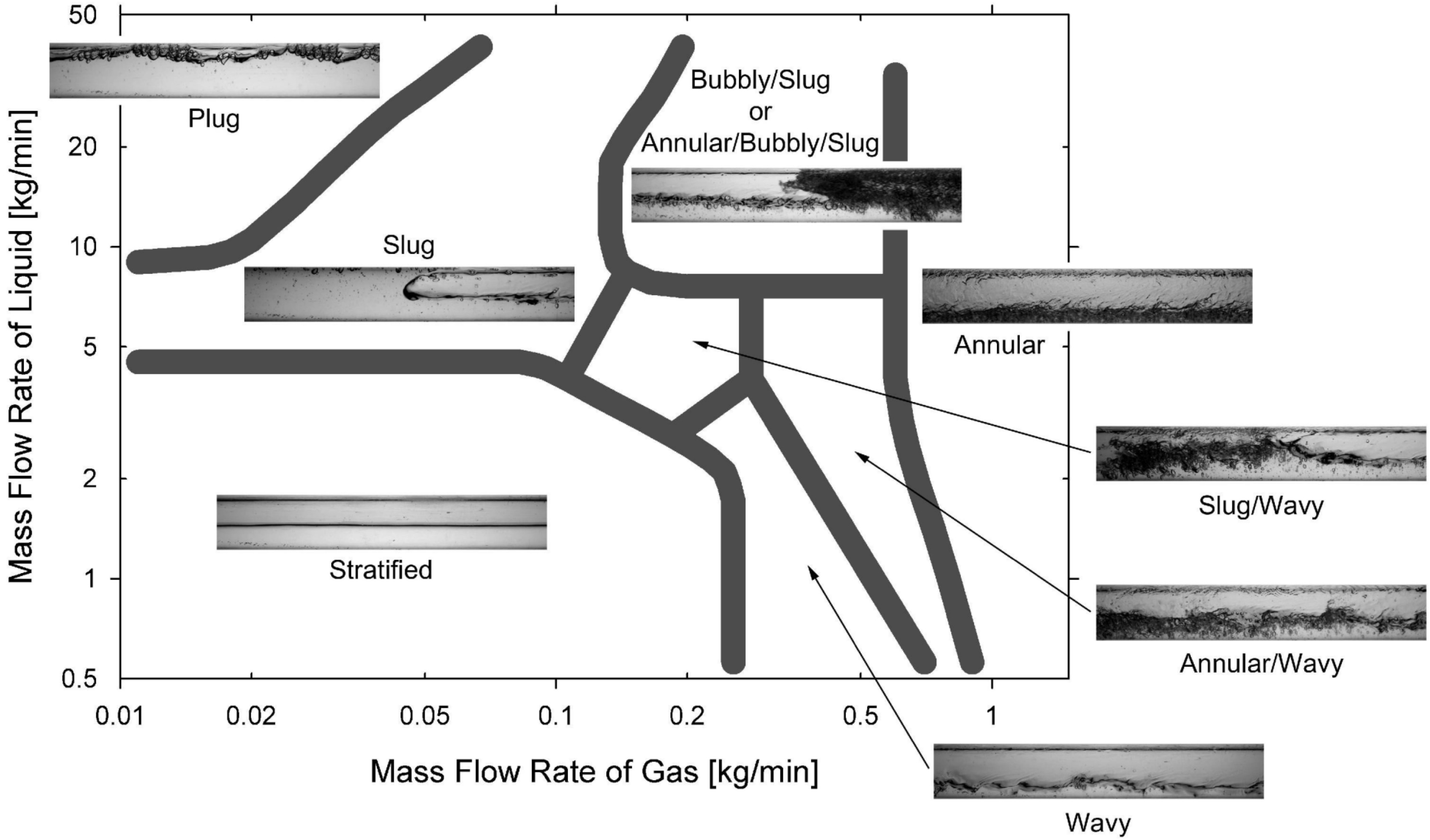

Figure 1: Flow map of horizontal pipe with photographs of representative flow patterns (Taken from Tang, 2011)

The pictures in Figure 1 represent each flow pattern. For brevity and focus of the study, only the following major flow patterns that occur in horizontal flow are considered in this study. Variations of flow patterns that are reported by authors of experimental data bases are grouped and treated in the major flow patterns indicated in this section. Descriptions of the major flow patterns that occur in horizontal flow follow from here.

Stratified Flow: The gas and liquid phases flow separately one on top of the other at low gas and liquid velocity. The liquid flows along the bottom of the pipe while the gas flows in the top section of the pipe.

Wavy Flow: Increased gas velocity in stratified flow creates wave on the interface in the flow direction. The amplitude of the wave depends on the relative velocity but it normally does not touch the upper side of the pipe wall.

Plug Flow: Elongated gas bubbles and liquid plugs appear alternatively on top of the pipe during this flow type in horizontal two phase flow. The diameters of the elongated bubbles are smaller than the pipe which allows for a continuous liquid phase to appear on the bottom of the pipe.

Slug Flow: Large amplitude wave or splashes of liquid occasionally pass through the upper side of the pipe with a higher velocity than the average liquid velocity. Pressure fluctuations are very typical in such type of flows.

Bubble Flow: Gas bubbles are dispersed in the liquid phase. Usually high concentration of the gas bubbles appear in the upper half of the pipe due to buoyancy effect. However, when shear forces are dominant, uniform distribution of bubbles occur in the pipe. Bubble flows usually appear when both phases have high mass flow rates.

Annular Flow: The liquid phase forms a continuous film around the inside wall of the pipe and the gas flows as a central core with higher velocity. Due to effect of gravity, usually the liquid film is thicker at the bottom side of the pipe in horizontal flows.

Mist Flow: The annular liquid film is thinned and destroyed at higher gas flow rates due to shear force at interface. Liquid droplets are entrained in a continuous gas phase during a mist flow. Some authors refer this flow pattern as spray flow or dispersed flow.

1.4 Pressure Drop in Two-Phase Flow

Similar to single phase flow, the total two-phase flow pressure drop may be written as sum of three major components.

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \left[\frac{\Delta P}{\Delta L}\right]_{a} + \left[\frac{\Delta P}{\Delta L}\right]_{f} + \left[\frac{\Delta P}{\Delta L}\right]_{h} \tag{1.10}$$

Where:

$\left[\frac{\Delta P}{\Delta L}\right]_{tp}$ = Total two-phase pressure drop

$\left[\frac{\Delta P}{\Delta L}\right]_{a}$ = Two-phase flow pressure drop due to acceleration

$\left[\frac{\Delta P}{\Delta L}\right]_{f}$ = Two-phase flow pressure drop due to frictional losses

$\left[\frac{\Delta P}{\Delta L}\right]_{h}$ = Hydrostatic pressure drop

Hydrostatic pressure drop is the two-phase flow pressure drop due to change in elevation. Since the focus of this study deals with horizontal flow only, the hydrostatic pressure drop is neglected. Also the pressure drop due to acceleration in isothermal flows in relatively uniform diameter and short pipes is very small and it is often negligible.

Therefore, from now onwards, the term two-phase flow pressure drop refers to the frictional pressure drop component only.

Frictional pressure drop is the result of an irreversible work done due to shear at the pipe wall and at the gas-liquid interface. The frictional pressure drop in two phase flow is much more complex to predict than single phase flow. This is due to the fact that it is dependent on many flow parameters such as pipe diameter, mass flux, pipe orientation, pipe surface roughness, fluid properties and interfacial contact area between the phases.

Generally, due to simultaneous presence of both phases in the pipe, smaller cross sectional area is available for these phases to flow. Therefore, higher pressure drop in two-phase flow is expected as a result of the friction between the phases.

There are many two-phase flow pressure drop correlations in the open literature. Sometimes it becomes difficult to know which correlation would be more accurate or suitable for the task at hand. Moreover, the lack of good understanding of the two-phase flow behavior had led many researchers to develop correlations that are limited to a certain range of flow parameters. Therefore, the user of the correlation must understand those restrictions and must make sure the task at hand is within the restrictions. For instance, one of the most common restrictions to the correlations is flow pattern specification. However, flow pattern is usually subjective and even in most applications observation of flow pattern may not be practical.

Therefore, the purpose of this study is to find a single or group of two-phase flow pressure drop correlations that could acceptably predict most of the experimental data collected for wide range of flow conditions with no or very few restrictions for its use.

A search is made in the open literature to collect pressure drop correlations. During the collection process, preference was given to correlations where all the required information is clearly stated by the authors. Graphical correlations without plotting data information are also excluded for the sake of accuracy. Also correlations without too many restrictions were given preference in order to achieve the goal of the study. Moreover, correlations that require a lot of assumptions or correlations with complicated iterative procedures are excluded from this study. And also some correlations had to be excluded because they require inputs that are not reported in most experimental data bases or in some cases that are difficult to measure experimentally.

Predictive performance of the collected two phase pressure drop correlations is then compared against experimental data collected from literature. Based on the results of the comparison, the best performing correlations for wide range of applications were selected and also recommendations were also given for narrower range of applications where higher accuracy is required.

Contents of this study have been organized in such a way that the pressure drop correlations collected from literature are briefly presented in Chapter 2. Review of previously done comparison work carried out by different previous researchers is also presented in this chapter. In Chapter 3, characteristics of the experimental data base are presented along with the associated fluid physical properties. Chapter 4 focuses on presenting detailed predictive performance comparison of the correlations reported in Chapter 2. And finally, concise conclusions and recommendations are forwarded in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

Literature review on two-phase flow pressure drop correlations is presented in this chapter. This chapter contains three main sections. In the first section, a number of pressure drop correlations collected from the open literature are presented. And in the second section, some of previously done comparison works are discussed. And by the end of the chapter, a brief summary of the chapter is presented.

2.1 Two-Phase Flow Pressure Drop Correlations

Literature on two-phase flow pressure drop correlations can be classified in different ways. The correlations can be classified based on several criterions such as inclination angle, flow pattern or method of development. Even though there are several types of criterions to classify the correlations, only selected criterions are considered here to classify the correlations in order to facilitate a systematic approach in the study.

Classifying the correlations based on their applicability for certain angle of inclination will yield correlations grouped based on horizontal flow, inclined flow, vertical flow or correlations that can be used for any type of inclination.

In this study only correlations which are proposed for horizontal flow and correlations that can be used for any inclination are collected from the open literature to investigate their performance in horizontal two-phase flows.

In order to stay within the scope of this study, classification of correlations based on flow pattern is discussed briefly for major flow patterns observed in horizontal two-phase flows. Usually, flow patterns tend to be vague due to the fact that identification of specific flow pattern is often subjected to personal opinion based on visual analysis of the flow. Moreover, formulation of the transition zones from one flow pattern to another is still in development stage and in most cases it is not generally applicable for wide range of two-phase flow conditions. From the literature gathered at hand, it has been noted that some correlations are developed for specific flow patterns and restrictions are set based on flow patterns. Moreover, investigation of the performance of frictional pressure drop correlations has shown that most correlations exhibit dependence on flow pattern. Therefore grouping correlations based on flow patterns was found to be necessary in this study.

Other two-phase flow parameters such as pipe diameter, test section length, flow rates of the fluids, some relatively minor restrictions used by authors are not considered to be as a classification criterion for the correlations. Instead these parameters are used as an investigation tool in this study.

Rather than using the physical parameters mentioned in the preceding paragraph to classify the correlations, we have decided to classify the correlations into five categories based on the method the authors used to develop their correlations. They are separated flow models, homogeneous flow models, empirical models, phenomenological models and numerical models. A brief description of these categories and the two-phase flow pressure drop correlations that fall into them is presented in the following sections.

2.1.1 __Separated Flow Models__

In separated flow models each phase/fluid is assumed to flow separately from one another. Most separated flow models assume different velocities for each phase unlike homogeneous flow models where both of the fluids are assumed to have the same velocity.

A method of using a two-phase frictional pressure drop multiplier, ϕ, is a very popular method of developing a separated flow model pressure drop correlation. This type of analysis was found to be appealing for most researchers because single-phase flow techniques and results are analogically related to two-phase flows by this method. This instance has a benefit of avoiding ambiguity over which physical property of the phases to use, such as which viscosity of either of the phases to use during calculation of two-phase pressure drop.

There are two ways of modeling the two-phase friction multiplier. The first one is assuming all the flow to be as one of the single phases such as all flow as liquid or all flow as gas. The implication here is to use the total mass flux (the sum of the mass fluxes for each phase) instead of the mass flow for each phase. Subscripts ‘lo’ and ‘go’ are used to indicate liquid only and gas only, respectively.

$$\phi_{lo}^2 = \frac{\left[\frac{\Delta P}{\Delta L}\right]_{tp}}{\left[\frac{\Delta P}{\Delta L}\right]_{lo}} \tag{2.1}$$

$$\phi_{go}^2 = \frac{\left[\frac{\Delta P}{\Delta L}\right]_{tp}}{\left[\frac{\Delta P}{\Delta L}\right]_{go}} \tag{2.2}$$

The second method is to assume as if only one of the phases exist and use the respective mass flux only while calculating the Reynolds number. Therefore in this case, only the respective mass flux is used to calculate the Reynolds number.

$$\phi_{l}^2 = \frac{\left[\frac{\Delta P}{\Delta L}\right]_{tp}}{\left[\frac{\Delta P}{\Delta L}\right]_{l}} \tag{2.3}$$

$$\phi_{g}^2 = \frac{\left[\frac{\Delta P}{\Delta L}\right]_{tp}}{\left[\frac{\Delta P}{\Delta L}\right]_{g}} \tag{2.4}$$

Usually using the liquid two-phase friction multiplier is preferred because the liquid density generally does not vary too much in most of the applications as compared to the gas density. The concept of two-phase friction multiplier was introduced by Martinelli et al. (1944). And later Martinelli and Nelson (1948) developed the concept of using the parameter ϕ_{lo} claiming it is more convenient for boiling and condensation flows.

Martinelli and Nelson (1948) proposed a graphical pressure drop correlation for forced circulation of boiling water. It is an empirical correlation based on experimental data for the flow combination of air and various liquids. The authors assumed turbulent-turbulent flow insisting that will be the most dominant case for all practical purposes involving forced circulation. The authors assumed the static pressure drop of the liquid phase and the vapor phase to be the same. This assumption makes the correlation to be well suited for annular flows. The authors provided a graph where the two-phase pressure drop can be determined when values of exit mass flow quality, system pressure and the single phase pressure drop of the liquid are known. However, since the correlation is developed empirically based on a very limited data, the authors stated that the correlation is merely an extrapolation of the experimental data.

Lockhart and Martinelli (1949) proposed a two-phase flow pressure drop correlation based on experimental data collected from several two-phase flow researchers. Their correlation development is based on two-phase pressure drop data with simultaneous flow of air with several types of liquids including water, benzene, diesel, kerosene and various oils flowing in a diameter ranging from 1.5mm to 25mm. The absolute pressure ranges from 110.3 kPa to 358.5 kPa.

They developed their correlation based on two basic assumptions. The first assumption states that the static pressure drop of the liquid phase and the gas phase must be equal for all the flow patterns when there is no appreciable radial static pressure difference. The second assumption states that the sum of the volume occupied by each phase must be equal to the total volume of the pipe. According to the authors these two assumptions imply that the flow pattern does not change along the pipe length. Therefore, the authors indicated that alternate slugs of liquid and gas moving down the pipe termed as the slug flow is excluded from their investigation.

Four types of flow mechanisms were assumed during the development of their correlation. They categorized the flow mechanisms as:

Viscous-Viscous (vv): when the flow of both the liquid and the gas is laminar

Viscous-Turbulent (vt): when the flow of the liquid is laminar and the gas is turbulent

Turbulent-Viscous (tv): when the flow of the liquid is turbulent and the gas is laminar

Turbulent-Turbulent (tt): when the flow of both the liquid and the gas is turbulent

The authors introduced a new parameter called *X*. This parameter *X* is a function of the ratios of the mass fluxes, densities, and viscosities of the liquid and the gas phase in addition to the diameter of the pipe. The parameter *X* relates the single phase pressure drops for liquid and gas as if each fluid is flowing alone in the pipe.

$$\left[\frac{\Delta P}{\Delta L}\right]_l = X^2 \left[\frac{\Delta P}{\Delta L}\right]_g \tag{2.5}$$

Lockhart and Martinelli (1949) proposed a correlation to calculate the value of *X* for each type of flow mechanism listed above. They showed that for the four types of flow mechanisms, the value of *X* can be calculated as (the subscripts of *X* are as given in the previous paragraph along with explanation of the flow mechanisms):

$$X_{vv}^2 = \left(\frac{\dot{m}_l}{\dot{m}_g}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right) \tag{2.6}$$

$$X_{vt}^2 = \mathrm{Re}_g^{-0.8}\left(\frac{C_l}{C_g}\right)\left(\frac{\dot{m}_l}{\dot{m}_g}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right) \tag{2.7}$$

$$X_{tv}^{2} = \mathrm{Re}_l^{0.8}\left(\frac{C_l}{C_g}\right)\left(\frac{\dot{m}_l}{\dot{m}_g}\right)\left(\frac{\rho_g}{\rho_l}\right)\left(\frac{\mu_l}{\mu_g}\right) \tag{2.8}$$

$$X_{tt}^{1.11} = \left(\frac{\dot{m}_l}{\dot{m}_g}\right)\left(\frac{\rho_g}{\rho_l}\right)^{0.555}\left(\frac{\mu_l}{\mu_g}\right)^{0.111} \tag{2.9}$$

Re_l and Re_g are the Reynolds number of the liquid and gas respectively, as if each fluid is flowing alone in the pipe. C_l and C_g are constants in the general form of Blasius equation for friction factor of the liquid and gas, respectively. The general form of the Blasius equation is expressed in the form of:

$$f_l = \left(\frac{C_l}{\mathrm{Re}_l^{n}}\right) \tag{2.10}$$

$$f_g = \left(\frac{C_g}{\mathrm{Re}_g^{m}}\right) \tag{2.11}$$

The authors determined the values of the constants C_l and C_g from experimental data and they specified the value of C_l and C_g for a smooth pipe. The values of n, m, C_l and C_g to be used for calculating X from the above equations is as given in Table 1.

Table 1: Values of exponents and constants in Lockhart and Martinelli (1949) correlation

Variables	Viscous-Viscous (v-v)	Viscous-Turbulent (v-t)	Turbulent-Viscous (t-v)	Turbulent-Turbulent (t-t)
n	1	1	0.2	0.2
m	1	0.2	1	0.2
C_l	0.046	16	0.046	16
C_g	0.046	0.046	16	16

The two-phase and the single phase pressure drops are correlated with the two-phase friction multiplier ϕ. The parameter ϕ is a function of the dimensionless variable X. Single phase pressure drops are calculated assuming either only the liquid or the gas phase exist in the pipe.

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \phi_l^2 \left[\frac{\Delta P}{\Delta L}\right]_l \tag{2.12}$$

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \phi_g^2 \left[\frac{\Delta P}{\Delta L}\right]_g \tag{2.13}$$

Lockhart and Martinelli (1949) correlation was presented graphically as plot of the two-phase friction multiplier ϕ versus the dimensionless parameter X. As has been noted above, the parameter X has unique value for each of the four flow mechanisms. The authors suggested tentative criteria for determining the flow mechanism based on Reynolds number of each single phase.

They suggested Re=1000 to be the end of laminar two-phase flow and Re=2000 to be start of turbulent flow mechanism. The authors claim Re=2000 is a conservative criterion because of the following reasons. If we consider the gas phase flowing alone in a pipe at Re=2000 the flow mechanism of the gas is turbulent. An introduction of a liquid phase in the pipe will increase the Reynolds number and will eventually lower the transition point of the flow mechanism. This way, the transition point ensures that at Re=2000 the flow is turbulent.

Chenoweth and Martin (1955) argued that the accuracy of Lockhart and Martinelli (1949) correlation deteriorates for larger diameter pipes and higher system pressures. Chenoweth and Martin (1955) used pressure drop data collected from several authors with diameters ranging from 15.2mm up to 77.9mm and a maximum system pressure of 689.4 kPa. They reported that Lockhart and Martinelli (1949) correlation exhibits the maximum error when the maximum system pressure was applied in the maximum diameter. They also reported Lockhart and Martinelli (1949) correlation's performance weakens when pipe diameter or gas density are increased beyond the range of the data Lockhart and Martinelli (1949) used to develop their correlation.

The prediction of Lockhart and Martinelli (1949) correlation resulted in over prediction up to 250% relative error from the measure experimental data. Chenoweth and Martin (1955) then proposed an empirical correlation graphically to predict the two-phase pressure drop when both of the fluids are turbulent. They reported their correlation predicted 92% of the experimental within an error band of $\pm 50\%$.

Baroczy (1966) suggested a complex graphical correlation based on steam, air-water, and mercury-nitrogen data. The author introduced a term called property index which is a function of viscosity and density of each phases a shown in equation (2.14).

$$Z = \left[\frac{\left(\frac{\mu_l}{\mu_g} \right)^{0.2}}{\left(\frac{\rho_l}{\rho_g} \right)} \right] \tag{2.14}$$

The author stated that when each phase flowing alone at the total mass flow rate is turbulent, the reciprocal of the property index is equal to the ratio of the pressure drop gradient for all gas flow to that for all liquid flow as shown in equation (2.15).

$$\left[\frac{\left[\frac{\Delta P}{\Delta L}\right]_{go}}{\left[\frac{\Delta P}{\Delta L}\right]_{lo}}\right] = \left[\frac{\left(\frac{\rho_l}{\rho_g}\right)}{\left(\frac{\mu_l}{\mu_g}\right)^{0.2}}\right] \tag{2.15}$$

The two-phase pressure drop correlation is given graphically as a plot of two-phase multiplier versus the property index. The two-phase friction multiplier if the total flow is assumed liquid in the pipe $\left(\phi_{lo}^2\right)$ is shown to be a function of property index, mixture quality and mass flux. The correlation is plotted for mass flux of $1\text{x}10^6$ lb/hr-ft^2 only. For other mass flow rates, correction factors are proposed in separate complex plots. The correction factors were proposed at four different specific values of mass fluxes and interpolation to other values may lead to errors because of the complex graphical nature of the correlation.

Chisholm (1967) presented a theoretical analysis of Lockhart and Martinelli (1949) correlation by including the effect of interfacial shear forces. Considering the interfacial shear force between the phases while developing the correlation enabled to predict the hydraulic diameters of the phases more accurately than Lockhart and Martinelli (1949).

Although Lockhart and Martinelli developed and plotted the relationship between ϕ and X, using graphs to calculate values is inconvenient and raises concerns in degree of accuracy while reading from the plots. This made the Chisholm (1967) correlation more useful and convenient for two-phase flow pressure drop calculation in many practical applications. Simplified equations were proposed by Chisholm (1967) in terms of Lockhart and Martinelli (1949) parameters.

$$\phi_l^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \tag{2.16}$$

The values of C are given for the four different flow mechanisms, shown in Table 2.

Table 2: Values of the constant *C* in Chisholm (1967) correlation

Flow mechanism (Liquid-Gas)	Value of *C*
Viscous-viscous	5
Turbulent-viscous	10
Viscous-turbulent	12
Turbulent-turbulent	20

In a later investigation, **Chisholm (1973)** transformed the graphical Baroczy (1966) correlation into sets of equations to predict the pressure drop of turbulent flow in evaporating two-phase mixtures in smooth tubes. Two parameters designated by letters *B* and Γ were introduced in the equations.

$$\Gamma^2 = \left[\frac{\left[\frac{\Delta P}{\Delta L}\right]_{go}}{\left[\frac{\Delta P}{\Delta L}\right]_{lo}} \right] \tag{2.17}$$

$$\phi_{lo}^2 = 1 + (\Gamma^2 - 1)\left\{Bx(1-x) + x^2\right\} \tag{2.18}$$

Values of the coefficient B to transform Baroczy (1966) graphical correlation into equations were given as follows:

$$B = \frac{55}{G^{1/2}} \qquad 0 < \Gamma < 9.5 \tag{2.19}$$

$$B = \frac{520}{\Gamma G^{1/2}} \qquad 9.5 < \Gamma < 28 \tag{2.20}$$

$$B = \frac{15000}{\Gamma^2 G^{1/2}} \qquad \Gamma > 28 \tag{2.21}$$

However, Chisholm (1973) insisted that the Baroczy (1966) correlation underestimates the magnitude of friction in certain situations. Therefore, the author recommended values of B for smooth tubes as shown in Table 3.

Table 3: Values of Coefficient B from Chisholm (1973) correlation

Γ	G_{tp} [kg/m^{2}s]	B
$0 < \Gamma < 9.5$	$G_{tp} \leq 500$	4.8
	$500 < G_{tp} < 1900$	$2400/G$
	$G_{tp} \geq 1900$	$55/G^{1/2}$
$9.5 < \Gamma < 28$	$G_{tp} \leq 600$	$520/\Gamma G^{1/2}$
	$G_{tp} > 600$	$21/\Gamma$
$\Gamma > 28$	$G_{tp} > 0$	$15000/\Gamma^2 G^{1/2}$

Chisholm (1978) studied the influence of pipe surface roughness and proposed an equation to extrapolate the frictional pressure drop for rough surface pipes from his correlations to smooth pipes.

$$\frac{B_R}{B_S} = \left[0.5 \left\{ 1 + \left(\frac{\mu_g}{\mu_l} \right)^2 + 10^{-600\left(k/D\right)} \right\} \right]^{\frac{0.25-n}{0.25}} \tag{2.22}$$

B_R and B_S are the B coefficients in Chisholm (1973) for rough and smooth pipes, respectively. It can be seen that B_R approaches the smooth pipe value B_S at n=0.25. The Blasius exponent, *n*, *is* evaluated from:

$$\frac{f_{lo}}{f_{go}} = \left(\frac{\mathrm{Re}_{go}}{\mathrm{Re}_{lo}} \right)^n \tag{2.23}$$

The author mentioned that for rough pipe flows the Blasius exponent, n=0. The above expression was introduced to determine the value of n in the transitional region where the rough pipe value (n=0) may approach the smooth pipe value (n=0.25).

Chawla (1968) proposed a correlation to predict two-phase flow pressure drop of gas-liquid flows based the momentum exchange between the two phases. The author used an assumption of gas velocity to be greater than liquid velocity ($U_g > U_l$). This assumption is usually valid for annular and wavy stratified flows. The correlation is shown below.

$$\left[\frac{\Delta P}{\Delta L} \right]_{tp} = \frac{0.3164}{\mathrm{Re}_{go}^{0.25}} \frac{G_{tp}^2 x^{7/8}}{2D\rho_g} \left[1 + \frac{1-x}{x} \xi \frac{\rho_g}{\rho_l} \right]^{19/8} \tag{2.24}$$

Where:
$$\frac{1}{\xi^3}=\frac{1}{\xi_1^3}+\frac{1}{\xi_2^3} \tag{2.25}$$

$$\xi_1^3=\exp\left[0.9592+\ln(B)\right] \tag{2.26}$$

$$\xi_2^3=\exp\left[\left(0.1675-0.055\ln\left(\frac{k}{D}\right)\right)\ln(B)-0.67\right] \tag{2.27}$$

$$B=\left(\frac{1-x}{x}\right)\left(\frac{1}{\mathrm{Re}_l\,Fr_l}\right)^{1/6}\left(\frac{\rho_g}{\rho_l}\right)^{0.9}\left(\frac{\mu_g}{\mu_l}\right)^{0.5} \tag{2.28}$$

$$Fr_l=\frac{G_{tp}^2(1-x)^2}{gD\rho_l^2} \tag{2.29}$$

Wallis (1969) indicated that the value of liquid only friction factor multiplier decreases for increasing system pressure at a given value of the flow quality *x*. He proposed a correlation and reported good prediction results for bubbly flow steam water data using his correlation which is expressed in terms of the liquid only friction factor multiplier. However, the correlation suffers under prediction for the annular flow pattern. The correlation was proposed for turbulent flows (Re_{sl}>2,000) in smooth pipes and it is shown in equation (2.30).

$$\phi_{lo}^2=\left(1+x\frac{\rho_l-\rho_g}{\rho_g}\right)\left(1+x\frac{\mu_l-\mu_g}{\mu_g}\right)^{-1/4} \tag{2.30}$$

Friedel (1979) proposed a correlation for horizontal and vertical upward flows in small pipe diameters as small as 4mm. He used Froude number *(Fr)* to include the effect of gravity and also Weber number *(We)* to account for the effect of surface tension in small pipes.

$$\phi_{lo}^2 = \mathrm{E} + \frac{3.24\,\mathrm{FH}}{Fr^{0.045} We^{0.035}} \tag{2.31}$$

$$\mathrm{E} = (1-x)^2 + x^2\left(\frac{\rho_l f_{go}}{\rho_g f_{lo}}\right) \tag{2.32}$$

$$\mathrm{F} = x^{0.78}(1-x)^{0.224} \tag{2.33}$$

$$\mathrm{H} = \left(\frac{\rho_l}{\rho_g}\right)^{0.91}\left(\frac{\mu_g}{\mu_l}\right)^{0.19}\left(1-\frac{\mu_g}{\mu_l}\right)^{0.7} \tag{2.34}$$

Grønnerud (1979) proposed a separated flow model correlation specifically for refrigerants. The author defined his own two-phase frictional multiplier (ϕ_{gd}). Calculation of the two-phase friction multiplier is a function of fluid properties and a unique friction factor term that is mainly dependent on values of Froude number. The correlation is presented below.

$$\phi_{gd}^2 = 1 + \left(\frac{dp}{dl}\right)_{gd}\left[\frac{\left(\frac{\rho_l}{\rho_g}\right)}{\left(\frac{\mu_g}{\mu_l}\right)^{0.25}} - 1\right] \tag{2.35}$$

$$\left(\frac{dp}{dl}\right)_{gd} = f_{Fr}\left[x + 4\left(x^{1.8} - x^{10} f_{Fr}^{0.5}\right)\right] \tag{2.36}$$

The value of the friction factor term f_{Fr} in the above equation depends on liquid Froude number. A value of $f_{Fr} = 1$ is used for liquid Froude number is greater than 1, Otherwise the equation (2.37) has to be used.

$$f_{Fr} = Fr_l^{0.3} + 0.0055\left(\ln\frac{1}{Fr_l}\right)^2 \tag{2.37}$$

Theissing (1980) proposed an improvement for Lockhart and Martinelli (1949) correlation theory. The author studied the interaction between the phases and suggested a unique friction multiplier technique.

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \left[\left[\frac{\Delta P}{\Delta L}\right]_{lo}^{\frac{1}{N\varphi}}(1-x)^{1/\varphi} + \left[\frac{\Delta P}{\Delta L}\right]_{go}^{\frac{1}{N\varphi}} x^{1/\varphi}\right]^{N\varphi} \tag{2.38}$$

$$\varphi = 3 - 2\left(\frac{2\sqrt{\rho_l/\rho_g}}{1+\rho_l/\rho_g}\right)^{0.7/n} \tag{2.39}$$

Where:

$$N = \frac{N_1 + N_2\left(\left[\frac{\Delta P}{\Delta L}\right]_g \Big/ \left[\frac{\Delta P}{\Delta L}\right]_l\right)^{0.1}}{1+\left(\left[\frac{\Delta P}{\Delta L}\right]_g \Big/ \left[\frac{\Delta P}{\Delta L}\right]_l\right)^{0.1}} \tag{2.40}$$

$$N_1 = \frac{\ln\left(\left[\frac{\Delta P}{\Delta L}\right]_l \Big/ \left[\frac{\Delta P}{\Delta L}\right]_{lo}\right)}{\ln(1-x)} \tag{2.41}$$

$$N_2 = \frac{\ln\left(\left[\frac{\Delta P}{\Delta L}\right]_g \Big/ \left[\frac{\Delta P}{\Delta L}\right]_{go}\right)}{\ln(x)} \tag{2.42}$$

Chen and Spedding (1981) conducted experiments on air-water mixture flowing in a horizontal pipe of 45.5mm Perspex pipe. The authors made analytical study of the Lockhart and Martinelli (1949) correlation and developed a semi-empirical correlation to predict the two-phase gas friction factor. Chen and Spedding (1981) proposed a simple correlation for steam-water systems given as a function of superficial Reynolds number of gas and liquid.

$$\phi_g^2 = 4050\,\mathrm{Re}_{sg}^{-0.91}\,\mathrm{Re}_{sl}^{0.44} \tag{2.43}$$

Hasan and Rhodes (1984) studied the effect of mass flux and pressure on the two phase friction multiplier in case of horizontal boiling water flow for pressures up to 825kPa. They proposed a modification to the Chisholm parameter (C) to be calculated as shown below.

$$C = 3.218\left(\frac{2000}{G_{tp}}\right)^{0.3602}\left(\frac{\upsilon_g}{\upsilon_l}\right)^{0.262} \tag{2.44}$$

Awad and Muzychka (2004a) proposed a correlation for liquid and gas frictional multipliers using an asymptotic model. The asymptotic model relates the two-phase frictional pressure gradient to the single-phase frictional pressure gradients of the liquid and gas flowing alone. The authors recommended Churchill's (1977) correlation to calculate the fanning friction factor for the single phase pressure drop calculations.

The values for the parameter X are to be evaluated using Lockhart and Martinelli (1949) correlation. The value of the contant q was determined to be 0.25 from experimental data.

Where:

$$\phi_l = \left[1 + \left(\frac{1}{X^2} \right)^q \right]^{\frac{1}{q}} \tag{2.45}$$

$$\phi_g = \left[1 + \left(X^2 \right)^q \right]^{\frac{1}{q}} \tag{2.46}$$

Awad (2007) adjusted the value of q in the correlations above. Depending on the pipe diameter the value of q=0.307 was proposed for regular size pipes while q=0.5 was found to predict mini and micro-channel pipes.

Sun and Mishima (2009) compared eleven correlations using 2,092 data from literature on mini and micro channels. The working fluids include R123, R134a, R22, R236ea, R245fa, R404a, R407C, R410a, R507, CO_2, water and air. The regular pipe size correlations of Lockhart and Martinelli (1949), Chisholm (1973), Friedel (1979) and Muller-Steinhagen & Heck (1986) were compared against other six mini and micro channel correlations. They reported that Lockhart and Martinelli (1949) correlation and the mini-channel correlations gave similar results in the laminar-laminar flow region whereas the turbulent-turbulent region was predicted well by Muller-Steinhagen & Heck (1986) correlation.

Sun and Mishima (2009) proposed a new correlation based on the Chisholm parameter (C) as a function of the flow quality and superficial Reynolds number of the liquid and the gas phases. They reported that Muller-Steinhagen & Heck (1986) correlation and the new correlation achieved superior results for refrigerant fluids. The Sun and Mishima (2009) correlation is shown below.

$$\phi_l^2 = 1 + \frac{C_{sm}}{X^{1.19}} + \frac{1}{X^2} \tag{2.47}$$

Where

$$C_{sm} = 1.79\left(\frac{\mathrm{Re}_g}{\mathrm{Re}_l}\right)^{0.4}\left(\frac{1-x}{x}\right)^{0.5} \tag{2.48}$$

Awad and Muzychka (2010) studied pressure drop in mini and micro-channels where they proposed a similar approach to Sun and Mishima (2009). They stated that the total frictional pressure drop gradient is the sum of the frictional pressure drop of each single phase and the interfacial pressure drop gradient as shown below.

$$\left(\frac{\Delta P}{\Delta L}\right)_{tp} = \left(\frac{\Delta P}{\Delta L}\right)_l + \left(\frac{\Delta P}{\Delta L}\right)_i + \left(\frac{\Delta P}{\Delta L}\right)_g \tag{2.49}$$

The two phase liquid multiplier in Lockhart and Martinelli (1949) correlation is then expressed in terms of interfacial friction multiplier $\phi_{l,i}$ as shown below.

$$\phi_l^2 = 1 + \phi_{l,i}^2 + \frac{1}{X^2} \tag{2.50}$$

$$\phi_{l,i}^2 = \frac{C_A}{X^m} \tag{2.51}$$

The authors claim that better control of prediction results can be achieved when two parameters are used instead of the single parameter as in the case of the C parameter in Chisholm (1967) correlation. Awad and Muzychka (2010) empirically evaluated values for the constants C_A and m. The values of C_A and m were given in a table for different mass flow rates, flow patterns and flow mechanisms separately. This indicates that the method could be appropriate for specific application but using this type of empirical correlation with many different values for a constant usually poses a danger of large errors when the correlation is applied for a two-phase problem other than the data set in which the constants are evaluated.

2.1.2 <u>Homogeneous Flow Models</u>

Homogeneous flow model is the simplest approximation of a two-phase mixture flow where the two-phases are assumed to have the same flow velocity. Based on this assumption, a friction factor term similar to a single phase flow may be applied to solve for the frictional pressure drop. A term called two-phase friction factor, f_{tp}, is used in the two-phase pressure drop correlations and a pressure drop calculation technique and formulation structure that is similar to the single phase friction factor implemented shown in the equation (2.52).

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \frac{2 f_{tp} G_{tp}^2}{D\rho_{tp}} \tag{2.52}$$

The homogenous density $\left(\rho_{tp}\right)$ in the equation above is usually calculated as follows:

$$\frac{1}{\rho_{tp}} = \left(\frac{x}{\rho_g} + \frac{1-x}{\rho_l}\right) \tag{2.53}$$

In equation (2.52) f_{tp} has to be known in order to calculate the frictional two-phase flow pressure drop. The two alternative approaches that had been proposed by previous investigators are summarized by Chen (1979) as follows:

1. Defining a relationship between f_{tp} and Re_{tp}
2. Defining the two-phase viscosity μ_{tp} such that the Moody chart used commonly for single phase flow may also be applied to the two-phase flow problem

Several authors in early studies of gas-oil mixture have tried the first alternative approach to the problem. Poettman and Carpenter (1952) proposed a correlation for vertical two-phase flows by attempting a correlation between f_{tp} and Re_{tp}. However their correlation did not include μ_{TP} in the relationship.

Bertuzzi et al. (1956) proposed a correlation for f_{tp} as a function of the gas and liquid superficial Reynolds numbers for specific range of gas-liquid mass flow rate ratios.

$$f_{tp} = \mathrm{f}\left(\mathrm{Re}_{sg}^{x}\ \mathrm{Re}_{sl}^{a}\right) \tag{2.54}$$

$$\text{Where: } a = \exp\left(-0.1\left(\frac{x}{1-x}\right)\dot{m}_{tp}\right) \tag{2.55}$$

The authors tabulated a list of limitations specifying the applicable range of variables in their correlation. The correlation is presented in a complex graphical form.

Shannak (2008) studied the effect of relative surface roughness on the two-phase frictional pressure drop and suggested a correlation for the two-phase Reynolds number and two phase friction factor. He indicated that the frictional pressure drop increases with increasing mass flux, increasing vapor quality and increasing surface roughness. He stated that the influence of surface roughness to be more significant at higher vapor quality and higher mass flux. The correlations given by Shannak (2008) are shown below.

$$Re_{tp} = \frac{G_{tp} D\left[x^2 + \left(1-x^2\right)\left(\rho_g / \rho_l\right)\right]}{\mu_g x + \mu_l (1-x)\left(\rho_g / \rho_l\right)} \tag{2.56}$$

$$\frac{1}{\sqrt{f_{TP}}} = -2log\left[\frac{1}{3.7065}\frac{k}{D} - \frac{5.0452}{Re_{tp}} log\left(\frac{1}{2.2857}\left(\frac{k}{D}\right)^{1.1098} + \frac{5.8506}{Re_{tp}^{\ 0.8981}}\right)\right] \tag{2.57}$$

The second alternative approach used in the development of a homogeneous correlation is done by defining a two-phase viscosity (μ_{tp}). After defining the two-phase viscosity, the two-phase pressure drop is calculated using one of the single phase friction factor correlations or a specific one suggested by the author of the two-phase correlation. Homogeneous correlations developed by using this alternative method are presented below.

McAdams et al. (1942) defined the two-phase viscosity as a function of the dryness fraction and the viscosity of each single phase.

$$\frac{1}{\mu_{tp}} = \left(\frac{x}{\mu_g} + \frac{1-x}{\mu_l} \right) \tag{2.58}$$

$$\mathrm{Re}_{tp} = \frac{G_{tp} D}{\mu_{tp}} \tag{2.59}$$

The Reynolds number of the two-phase mixture is calculated using the total mass flux of the mixture. The authors suggested using fanning friction factor given in Table 4.

Table 4: Suggested friction factor methods in McAdams et al. (1942) correlation

Re_{tp}	f_{tp}
$\mathrm{Re}_{tp} < 2000$	$f_{tp} = \frac{16}{\mathrm{Re}_{tp}}$
$\mathrm{Re}_{tp} \geq 2000$	$f_{tp} = \frac{0.046}{\mathrm{Re}_{tp}^{0.2}}$

Davidson et al. (1943) conducted experiments on high-pressure (3.6MPa up-to 23.9 MPa) steam-water two-phase two phase flow and proposed a two-phase mixture viscosity. They compared all liquid flow Reynolds number against two-phase steam-water flow Reynolds number and found a better agreement with the Blasius equation for single phase flow when they plotted the two-phase friction factor against the Reynolds number of all liquid flow multiplied by the ratio of the densities of the phases at inlet and outlet of the pipe. However, the correlation does not approach the gas viscosity when the flow quality approaches x=1. Davidson et al. (1943) correlation is then expressed as shown below.

$$\mu_{TP} = \mu_l \left(1 + x \left(\frac{\rho_l}{\rho_g} - 1 \right) \right) \tag{2.60}$$

James and Silberman (1958) studied bubbly flow pattern and indicated the friction factor is approximately equal or slightly greater than the friction factor for liquid flowing alone. They recommended two phase viscosity expression given by Weinig (1953) which is shown in equation (2.61) as stated in Chen (1979) thesis.

$$\mu_{tp} = \frac{\mu_l}{\left[1 - x \left(\frac{U_{sg}}{U_{sl}} \right) \right]^{1/3}} \tag{2.61}$$

Cicchitti et al. (1960) developed a homogeneous two-phase correlation for upward vertical tube for adiabatic and non adiabatic flows. Their study focused on spray/ dispersed flow where the liquid phase is fully dispersed in the gas phase. The two-phase viscosity is defined as:

$$\mu_{tp} = \mu_g x + \mu_l (1 - x) \tag{2.62}$$

And the frictional two-phase pressure drop is then calculated from the following equation.

$$\left[\frac{\Delta P}{\Delta L}\right]_f = \frac{0.092 G_{tp}^{1.8} \mu_{tp}^{0.2}}{D^{1.2} \rho_{tp}} \tag{2.63}$$

Owens (1961) proposed the liquid viscosity to be the two-phase viscosity claiming that in most two-phase flows the liquid is the dominant phase.

$$\mu_{tp} = \mu_l \tag{2.64}$$

Dukler et al. (1964) proposed two methods to calculate the two phase pressure frictional drop based on similarity analysis. Equations to calculate Reynolds number and friction factor were suggested by using analogy between single phase and two phase flows. The authors proposed two types of correlations for two cases. In the first case, the slip velocity was assumed to be zero and hence equations for a homogeneous flow are given as below. This correlation will be referred as **Dukler et al. (1964) – (Case I)** in this study.

$$\left(\frac{\Delta p}{\Delta L}\right)_{tp} = \frac{2 f U_{tp}^2 D \rho_{ns}}{\mu_{ns}} \tag{2.65}$$

$$\mu_{ns} = \mu_l \lambda + \mu_g (1 - \lambda) \tag{2.66}$$

$$\rho_{ns} = \rho_l \lambda + \rho_g (1 - \lambda) \tag{2.67}$$

Where: $$\lambda = \left[\frac{Q_l}{Q_l + Q_g}\right] = \frac{1}{1 + \left(\frac{x}{1-x}\right)\frac{\rho_l}{\rho_g}} \tag{2.68}$$

$$\mathrm{Re}_{ns} = \frac{U_{tp}\rho_{ns}D}{\mu_{ns}} \tag{2.69}$$

$$f = 0.0014 + \frac{0.125}{\mathrm{Re}_{ns}^{0.32}} \tag{2.70}$$

In the second case, the authors indicated a slip may occur during the flow and an equation based on the homogeneous (non-slip) model was proposed. This correlation will be referred as **Dukler et al. (1964) – (Case II)** in this study. Further details of the equations and calculation procedures can be found in Dukler (1969).

$$\left(\frac{\Delta p}{\Delta L}\right)_{tp} = \frac{2 f_{tp} D \rho_{tp} U_{tp}^2}{\mu_{tp}} \tag{2.71}$$

Where: $$\frac{f_{tp}}{f_0} = 1.0 + \frac{-\ln\lambda}{S} \tag{2.72}$$

$$f_0 = 0.0014 + \frac{0.125}{\mathrm{Re}_{tp}^{0.32}} \tag{2.73}$$

$$S = 1.281 - 0.478(-\ln\lambda) + 0.444(-\ln\lambda)^2 - 0.094(-\ln\lambda)^3 + 0.00843(-\ln\lambda)^4 \tag{2.74}$$

$$\mathrm{Re}_{tp} = \frac{U_{tp}\rho_{tp}D}{\mu_{tp}} \tag{2.75}$$

Where:
$$\mu_{tp} = \mu_l \lambda + \mu_g (1-\lambda) \tag{2.76}$$

$$\rho_{tp} = \rho_l \frac{\lambda^2}{(1-\alpha)} + \rho_g \frac{(1-\lambda)}{\alpha} \tag{2.77}$$

Beggs and Brill (1973) developed a correlation based on experimental measurement they made in 25.4mm and 38.1mm (1 inch and 1.5 inch) pipes in different inclinations. They stated that the no-slip two-phase friction factor (f_{ns}) can be obtained from a Moody (1944) chart or for smooth pipe as shown below in equation (2.78) as a function of the no-slip Reynolds number (Re_{ns}).

$$f_{ns} = \left[4\log\left(\frac{\mathrm{Re}_{ns}}{4.5223\log(\mathrm{Re}_{ns}) - 3.8215} \right) \right]^{-2} \tag{2.78}$$

$$\mathrm{Re}_{ns} = \frac{G_{tp} D}{\mu_l \lambda + \mu_g (1-\lambda)} \tag{2.79}$$

The no-slip Reynolds number will approach Reynolds number for gas or liquid as input liquid content (λ) approaches zero or one, respectively.

The authors correlated the liquid hold up $(H_l(\theta))$ as a function of pipe inclination (θ) for three groups of flow patterns namely segregated, intermittent and distributed flows. Stratified, wavy and annular flows are grouped as 'segregated flow'. Plug and slug are grouped as 'intermittent flow'. Bubble and mist flows are grouped as 'distributed flow'.

Using regression analysis, Beggs and Brill (1973) showed that the input liquid content and liquid hold up is a natural logarithmic function of ratio of the two-phase friction factor to the no-slip friction factor.

$$\frac{f_{tp}}{f_{ns}} = e^{s} \tag{2.80}$$

$$y = \frac{\lambda}{\left(H_l(\theta)\right)^2} \tag{2.81}$$

For y values in the interval $1<y<1.2$,

$$s = \ln(2.2y - 1.2) \tag{2.82}$$

And for all other y values outside the interval $1<y<1.2$, the parameter s was given by the following equation.

$$s = \left[\frac{\ln(y)}{-0.0523 + 3.182\ln(y) - 0.8725(\ln(y))^2 + 0.01853(\ln(y))^4}\right] \tag{2.83}$$

Beattie and Whalley (1982) proposed a correlation in a simple form to predict the two-phase mixture viscosity (μ_{tp}) as a function of the single phase viscosities and the volumetric flow quality (β). The authors reported that the correlation predicts the pressure drop in all flow patterns with reasonable accuracy. The authors claim that the correlation is capable of predicting most diabatic flow conditions except condensation through any complex geometry pipes.

$$\mu_{tp} = \mu_l(1-\beta)(1+2.5\beta) + \mu_g\beta \tag{2.84}$$

$$\beta = \frac{\rho_l x}{\rho_l x + \rho_g(1-x)} \tag{2.85}$$

Garcia et al. (2003) proposed the use of liquid viscosity to define the Reynolds number of the two-phase mixture. They claim that the main frictional resistance is generated from the liquid phase. The equation they proposed is shown below as it is stated in Awad and Muzychka (2008).

$$\mu_{tp} = \mu_l \left(\frac{\rho_m}{\rho_l} \right) = \frac{\mu_l \rho_g}{x\rho_l + (1-x)\rho_g} \tag{2.86}$$

Awad and Muzychka (2004b) developed a correlation to predict the two-phase multiplier for liquid only flow $\left(\phi_{lo}^2\right)$. They started with the basic definition of ϕ_{lo}^2 from the separated flow model and combined it with the frictional pressure drop methods for homogeneous models.

$$\phi_{lo}^2 = \frac{\left[\frac{\Delta P}{\Delta L}\right]_{tp}}{\left[\frac{\Delta P}{\Delta L}\right]_{lo}} \tag{2.87}$$

Where:

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \frac{2 f_{tp} G_{tp}^2}{d\rho_{tp}} \tag{2.88}$$

$$\left[\frac{\Delta P}{\Delta L}\right]_{lo} = \frac{2 f_{lo} G_{tp}^2}{d\rho_l} \tag{2.89}$$

Then homogeneous mixture definitions for density $\left(\rho_{tp}\right)$ and viscosity $\left(\mu_{tp}\right)$ from equations (2.53) and (2.58) are substituted in the above equations. The authors then rearranged the variables to get an expression for the two-phase friction multiplier $\left(\phi_{lo}^2\right)$ as function of friction factors and physical properties as shown below. Churchill's (1977) was recommended to calculate the two-phase friction factor (f_{tp}) and the single phase friction factor (f_{lo}).

$$\phi_{lo}^2 = \left(\frac{f_{tp}}{f_{lo}}\right)\left(1 + x\frac{\rho_l - \rho_g}{\rho_g}\right) \tag{2.90}$$

Awad and Muzychka (2008) used analogy between thermal conductivity in porous media and viscosity in two-phase flow to develop four new two phase viscosity definitions. The new definitions are made to satisfy liquid viscosity for *x*=0 and gas viscosity for *x*=1 flow qualities.

The authors claim that these definitions are applicable for wide range of diameter including mini and micro channels. They reported good agreement with experimental data was achieved for refrigerant flows. The two-phase viscosity definitions are listed in Table 5.

Table 5: Two-phase mixture viscosity definitions given by Awad and Muzychka (2008)

Definition Number	Two-phase mixture viscosity definition
1	$$\mu_{tp} = \mu_l \frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} \tag{2.91}$$
2	$$\mu_{tp} = \mu_g \frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)} \tag{2.92}$$
3	$$\mu_{tp} = \frac{1}{4}\left[\omega + \sqrt{\omega^2 + 8\mu_l\mu_g}\right] \qquad \omega = (3x-1)\mu_g + (3(1-x)-1)\mu_l \tag{2.93}$$
4	$$\mu_{tp} = \frac{1}{2}\left[\mu_l \frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} + \mu_g \frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)}\right] \tag{2.94}$$

2.1.3 Empirical Models

Almost all correlations listed in the previous sections have some constant or parameter that had to be evaluated empirically from experimental data. Even if the degree of empiricism could vary from one correlation to another, in this study we believed it is important to dedicate a specific section for correlations that have been developed solely by relating the two pressure drop to some selected parameter empirically.

During the early stage of two-phase flow study, a number of correlations have been developed by curve fitting of data based on experimental pressure drop measurements. Correlating the experimental data by using some carefully selected variables is a convenient way of developing a correlation with a minimum analytical knowledge of the problem.

Since the two-phase flow problem involves several independent physical quantities, analyzing their relationship and developing dimensionless parameters is not an easy task as it may seem. Different studies have shown some dimensionless groups to play a dominant role in determining liquid holdup and pressure drop in variety of applications.

The main drawback of this method is that the prediction capability heavily relies on the quality of the data and vastness of the experimental data employed in the study. Several previous authors, including Dukler et al. (1964), indicated that most of the empirical correlations give poor prediction when they are used beyond the range of data that they were developed.

Lombardi and Pedrocchi (1972) proposed a simple empirical correlation after studying the influence of frictional pressure drop on the total pressure drop of vertical two phase flow.

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \frac{c_1 G_{tp}^{C_2} \sigma^{0.4}}{D^{1.2} \rho_{tp}^{0.866}} \tag{2.95}$$

Where: $c_1 = 0.83$ and $c_2 = 1.4$

Muller-Steinhagen and Heck (1986) proposed a more advanced empirical correlation that relates the two-phase pressure drop to the liquid only single phase pressure drop based on 9,300 data points. The authors claim their correlation is relatively simple and yet it gives a competitive accuracy as compared with most of previously suggested correlations with complicated calculation procedures. The authors plotted the two phase frictional pressure drop against increasing flow quality and they found out that the frictional pressure drop increases with increasing flow quality up to a maximum value of x=0.85 and then falls to the frictional pressure drop for single phase gas phase flow. The authors developed a relationship through curve fitting and proposed the followings sets of equations.

$$\mathrm{G}_{\mathrm{MSH}} = \left[\frac{\Delta P}{\Delta L}\right]_{lo} + 2\left[\left(\frac{\Delta P}{\Delta L}\right)_{go} - \left(\frac{\Delta P}{\Delta L}\right)_{lo}\right]x \tag{2.96}$$

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \mathrm{G}_{\mathrm{MSH}}(1-x)^{1/3} + \left[\frac{\Delta P}{\Delta L}\right]_{go} x^3 \tag{2.97}$$

The pressure drop correlation proposed by Muller-Steinhagen and Heck (1986) has two restrictions. The authors indicated that the liquid only Reynolds number must be greater than one hundred and also the single phase pressure drop of the gas must be greater than the single phase pressure drop for the liquid.

2.1.4 Phenomenological Models

Phenomenological models are developed based on information of certain geometrical configuration of the liquid and the gas phase that is observed for a range of flow parameters, which is as termed as flow pattern. Flow patterns not only impose unstable and complex geometry to the system but also critically affect the relative magnitudes of several force systems active to varying extents. Information such as interfacial shear stress and slug frequency are usually used to formulate phenomenological models.

Concentrating modeling efforts on certain selected flow patterns helps to investigate the mechanisms involved in momentum and energy transfer with more detail. However, the primary challenge in using phenomenological models is the model by itself is dependent on prediction of flow pattern maps. The prediction of flow patterns is still on developmental stage and flow pattern by itself is mostly inclined to visual perception of the investigator. According to Ferguson and Spedding (1995), there are as many as 16 flow patterns mentioned in literature. Endeavour to define accurate transition boundaries between flow patterns is still an ongoing task. Two-phase flow pressure drop correlations developed for specific flow patterns are presented in the next sections.

2.1.4.1 Stratified flow pattern

Johannessen (1972) made a theoretical analysis of the stratified and wavy flows based on Lockhart and Martinelli (1949) correlation. He compared his prediction results against experimental data of two-phase flow pressure drop measurement in 52.5 mm, 140 mm and 197 mm diameter pipes. Air-water, air-oil and natural gas-oil fluid combinations were included in the experimental data. The author reported the proposed method was found to give better predictions than the generalized Lockhart and Martinelli (1949) method. However, Chen and Spedding (1981) indicated interfacial friction factor was not included in Johannessen (1972) correlation. Also, Awad (2007) reported that the good performance of Johannessen (1972) correlation is limited to $0.3 < X < 2$ range of the X parameter in Lockhart and Martinelli (1949) correlation. Awad (2007) indicated that the effect of higher gas velocities in wavy flow and the resulting energy transfer from the gas to the liquid was not included in Johannessen (1972) correlation. Moreover, Awad (2007) raised some concerns on the accuracy of the flow pattern identification in the wavy flow saying that part of the experimental data was most likely measured in breaking wave flow pattern. In this flow pattern, liquid droplets are accelerated by the gas phase which leads to energy loss of the gas phase.

Agrawal et al. (1973) developed a model using a mechanistic approach for stratified flow regime. The model employs the definition of equivalent diameter for the gas and the liquid phases and it also includes the effect of interfacial stress between the phases. Assuming a flow between two parallel plates, the authors provided velocity profile integration equations for laminar liquid-turbulent gas flow and turbulent liquid and turbulent gas flows. However, the authors indicated that they were unable to find the transition point from laminar to turbulent flow of the liquid. The model involves an iterative procedure which starts by assuming the liquid hold up and then iterating until the two phase pressure drop found using the gas phase pressure drop (equation 2.98) matches the liquid phase pressure drop (equation 2.99).

$$\left(\frac{\Delta P}{\Delta L}\right)_{tp} = \frac{\tau_{wg} p_g + \tau_i w_i}{A_g} \tag{2.98}$$

$$\left(\frac{\Delta P}{\Delta L}\right)_{tp} = \frac{\tau_{wl} p_l - \tau_i w_i}{A_l} \tag{2.99}$$

The variables p_l and p_g are the perimeter of the pipe cross section occupied by the liquid phase and the gas phase respectively. Width of the gas-liquid interface is designated by w_i. The shear stress between the pipe and the fluids is represented by τ . Details for each term in equations can be found in Agrawal et al. (1973).

2.1.4.2 Bubble flow pattern

Bankoff (1960) proposed a pressure drop correlation in bubble flow assuming a variable density model. The authors observed concentration of bubbles on the central axis of the pipe during a steam-water flow in vertical pipes. Radial gradient of bubbles concentration where the maximum concentration is located at the center of the pipe is assumed to develop the two-phase model. The author stated that the relative velocity of the bubbles with respect to the surrounding liquid is negligible as compared to the stream velocity. Therefore in a similar manner to homogeneous models, the gas and the liquid were assumed to have same velocity. Bankoff (1960) correlation for two-phase frictional pressure gradient is expressed in terms of the two-phase friction multiplier as shown below.

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \phi^{7/4}\left[\frac{\Delta P}{\Delta L}\right]_{l} \tag{2.100}$$

$$\phi = \frac{1}{1-x}\left[1-\gamma\left(1-\frac{\rho_g}{\rho_l}\right)\right]^{\frac{3}{7}}\left[1+x\left(\frac{\rho_l}{\rho_g}-1\right)\right] \tag{2.101}$$

$$\gamma = \frac{0.71+2.35\left(\frac{\rho_g}{\rho_l}\right)}{1+\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)} \tag{2.102}$$

2.1.4.3 Composite flow pattern models

Methods that propose separate correlation for different flow patterns and present the correlations in a combined form are grouped as composite models in this study. In composite methods, the authors suggest separate correlations for each or group of flow patterns using one or more of correlation development methods mentioned in the previous sections.

Hoogendoorn (1959) made two-phase flow experiments in horizontal smooth and rough pipes with diameters 50 mm, 91 mm and 140 mm. The flows of air-water and air-oil mixtures in adiabatic conditions were investigated. The author compared the prediction of Lockhart and Martinelli (1949) correlation against their experimental data and proposed three sets of new correlations based on flow patterns where large deviations from their experimental data were observed. The first set contains plug, slug and froth flow patterns in which it was observed that when the gas density of the fluid is different than density of air at atmospheric pressure error in prediction occur. In the second set, stratified and wavy flow patterns are studied and the author proposed a new correlation for wavy flow. And in the third set, a new correlation was proposed for mist-annular flow. The Hoogendoorn (1959) correlation for the three groups of flow patterns is shown below.

For plug, slug and froth flows:

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \left[\frac{\Delta P}{\Delta L}\right]_{lo}\left[1+230\left(\frac{G_g}{G_l}\right)^{0.84}\right]\left[0.00138\left(\frac{\rho_l}{\rho_g}\right)\right]^{\eta} \tag{2.103}$$

$$\eta = 9.5\left(\frac{G_g}{G_l}\right)^{0.5} - 62.6\left(\frac{G_g}{G_l}\right)^{1.3} \quad \text{For } \left(\frac{G_g}{G_l}\right) < 0.03 \tag{2.104}$$

$$\eta = 1 \quad \text{For } \left(\frac{G_g}{G_l}\right) \geq 0.03 \tag{2.105}$$

For wavy flow:

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = C_H\left(\frac{G_g}{G_l}\right)^{1.45}\left(\frac{G_l^2}{2d\rho_g}\right) \quad \text{for} \quad \left(\frac{G_g}{G_l}\right) < 0.8 \tag{2.106}$$

For annular mist flow:

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = 0.12G_g^{-0.25}\left(\frac{G_g^2}{2d\rho_g}\right) \quad \text{for} \quad 30 < G_l < 200 \tag{2.107}$$

The effect of pipe diameter, pipe roughness and fluid combination was included for wavy flow through the empirical constant C_H in separate tables. Hoogendoorn (1959) proposed different values of C_H for different fluid combinations and different surface roughness at the three different diameters he used in his experiments.

Mandhane et al. (1977) proposed improvement to Lockhart and Martinelli (1979) equation for slug and bubble flow. The authors recommended Colebrook (1939) equation to calculate the single phase friction parameter.

For slug flow:

$$X = \left(\frac{U_{sl}}{U_{sg}}\right)^{0.5}\left(\frac{\rho_l}{\rho_g}\right)^{0.375}\left(\frac{\mu_l}{\mu_g}\right)^{0.1} \tag{2.108}$$

For bubbly flow:

$$X = \left(\frac{U_{sl}}{U_{sg}}\right)^{0.875}\left(\frac{\rho_l}{\rho_g}\right)^{0.375}\left(\frac{\mu_l}{\mu_g}\right)^{0.125} \tag{2.109}$$

$$\phi_g = 2.20X^{0.862} \tag{2.110}$$

Olujic (1985) insisted that there are two extremely different flow regimes based on the relative velocity difference of the two-phases in a horizontal two-phase. He named the two regions as Alpha region and Beta region. In the Alpha region, the velocity of the gas phase is higher than that of the liquid. Flow patterns such as wavy, slug and annular-dispersed fall usually appear in this region. In the second region which was referred as the Beta region, the velocities of the two-phases are nearly equal. He stated that flow patterns such as bubble and plug fall in this group. Therefore, the author claims that his correlation is valid for all flow patterns except the dispersed flow pattern.

The author attempted to divide the two flow regimes based on a modified Froude number *(N_{Fr})* and phase volume flow ratio (Q_g/Q_l). If the value of the phase volume ratio (Q_g/Q_l) is less than or equal to the right hand side term in the equation (2.111) the flow will be identified as Beta region flow. Otherwise, the flow is regarded as Alpha region flow.

$$\left[\frac{Q_g}{Q_l}\right] \le \left[\frac{12\left(N_{Fr}^{\frac{1}{2}}\right)}{1+\left(\frac{N_{Fr}^{\frac{1}{2}}}{7}\right)}\right] \tag{2.111}$$

Where:

$$N_{Fr} = \left(\frac{G_{tp}x}{\rho_l^2}\right)^2 \left(\frac{\rho_l / \rho_g}{gD}\right) \tag{2.112}$$

The frictional pressure drop in the Beta region is given by:

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = \left(\frac{f_{tp}G_{tp}^2}{2D\rho_l}\right)\left[1+x\left(\left(\frac{\rho_l}{\rho_g}\right)-1\right)\right]\left[1-x\left(\left(\frac{\rho_l}{\rho_g}\right)-1\right)(J-1)\right] \tag{2.113}$$

$$f_{tp} = \left\{-0.5\log\left[\frac{k/D}{3.7} - \frac{5.02}{\mathrm{Re}_{tp}}\log\left(\frac{k/d}{3.7} + \frac{14.5}{\mathrm{Re}_{tp}}\right)\right]\right\} \tag{2.114}$$

$$\mathrm{Re}_{tp} = \frac{G_{tp}D}{\mu_l\left[1-x\left(1-\frac{\mu_l}{\mu_g}\right)\right]^{-1}} \tag{2.115}$$

$$J = 1.2\left[\frac{(7+8n)(7+15n)}{(7+9n)(7+16n)}\right] \tag{2.116}$$

Where:

$$n = \left(\frac{0.671}{Q_l / Q_g} \right) \left[1 + \left(1 + 0.907 \frac{Q_l}{Q_g} \right)^{1/2} \right] \quad (2.117)$$

On the other hand the frictional pressure drop in the Alpha region is given by:

$$\left[\frac{\Delta P}{\Delta L} \right]_{tp} = \left(\frac{f_g G_g^2}{2D\rho_g} \right) \left[1 + \left(\frac{\rho_g (1-x)}{\rho_l x \varepsilon_0} \right) \right]^{19/8} \quad (2.118)$$

$$f_g = \frac{0.3164}{\mathrm{Re}_{sg}^{0.25}} \quad (2.119)$$

$$\mathrm{Re}_{sg} = \frac{G_g D}{\mu_g} \quad (2.120)$$

Where:

$$\varepsilon_0 = \left(\varepsilon_1^{-3} + \varepsilon_2^{-3} \right)^{-1/3} \quad (2.121)$$

$$\varepsilon_1 = 0.77 \left(\frac{\rho_l}{\rho_g} \right)^{-0.55} \Gamma_0^{n_1} \quad (2.122)$$

$$\varepsilon_2 = 2.19 \left(\frac{\rho_l}{\rho_g} \right)^{-0.61} \Gamma_0^{n_2} \quad (2.123)$$

$$\Gamma_0 = \left(\frac{1-x}{x} \right) \left(\frac{G_{tp}^2}{\rho_l^2 g D} \right)^{-1/4} \left(\frac{\rho_l}{\rho_g} \right)^{-1/2} \left(\frac{\mu_l}{\mu_g} \right)^{-1/8} \quad (2.124)$$

$$n_1 = 0.266 \left(\frac{\rho_l}{\rho_g} \right)^{0.057} \quad (2.125)$$

$$n_2 = 1.78 \left(\frac{\rho_l}{\rho_g} \right)^{-0.078} \quad (2.126)$$

For relative surface roughness of 0.006 or more, Olujic (1985) suggested using a modification of the two phase parameter (ε_R) as shown in the equation below.

$$\varepsilon_R = \varepsilon_0 \left\{ \exp\left[\left(\frac{0.006}{k/D} - 1 \right) x^{0.2} \right] \right\} \tag{2.127}$$

Quiben and Thome (2007) proposed analytical models for annular, stratified, wavy, annular-mist and slug flow. A model to predict the frictional pressure drop for transition between flow patterns of slug and stratified is also suggested. However, except for annular and annular-mist flow, all the suggested models require information for mass flux and void fraction at transition boundaries. Because this information is not provided in the experimental data sets in this study, only annular and mist flow pattern correlations are validated in this study. Equations to predict the two-phase frictional pressure drop in annular flow pattern were developed assuming uniform thickness film thickness δ and neglecting entrainment.

$$\left[\frac{\Delta P}{\Delta L} \right]_{tp} = 2 f_i \frac{\rho_g U_g^2}{D} \tag{2.128}$$

$$f_i = 0.67 \left(\frac{\delta}{D} \right)^{1.2} \left(\frac{\delta^2 g (\rho_l - \rho_g)}{\sigma} \right)^{-0.4} \left(\frac{\mu_g}{\mu_l} \right)^{0.08} We_l^{-0.034} \tag{2.129}$$

$$U_g = \frac{G_{tp} x}{\alpha \rho_g} \tag{2.130}$$

$$\delta = \frac{D(1-\alpha)}{4} \tag{2.131}$$

$$We_l = \frac{\rho_l U_l^{\,2} D}{\sigma} \tag{2.132}$$

For mist flow, the authors suggested homogeneous flow using Blasius (1913) equation for friction factor and Cicchitti et al. (1960) model for two-phase viscosity as shown in equation (2.134) and (2.135), respectively.

$$\left[\frac{\Delta P}{\Delta L}\right]_{tp} = 2 f_{tp} \frac{G_{tp}^2}{D\rho_{tp}} \tag{2.133}$$

$$f_{tp} = \frac{0.079}{\mathrm{Re}_{tp}^{0.25}} \tag{2.134}$$

$$\mu_{tp} = \mu_g x + \mu_l (1 - x) \tag{2.135}$$

$$\rho_{tp} = \rho_g \beta + (1 - \beta)\rho_l \tag{2.136}$$

2.1.5 Numerical Models

Numerical models usually involve the solution of the equations of continuity and momentum for two-phase flow on a three dimensional grid. Direct solution to the Navier-Stokes equations is becoming available for relatively low Reynolds numbers in case of single phase turbulent pipe flows. However, in case of two-phase flows the problem becomes more complicated. The presence of a second phase with a different transport properties and the fact that the second phase is usually not distributed across the interior of the pipe makes solution very challenging. Even if attempts to modify the single-phase energy correlations to suit gas-liquid flow has been made by some authors, rigorous numerical models for two-phase flow do not contribute in most practical applications today.

Awad (2007) mentioned efforts made to solve the two-phase flow problem using methods such as integral analysis, differential analysis, computational fluid dynamics (CFD) and artificial neural network (ANN). In one-dimensional integral analysis, some forms of certain functions are assumed to describe flow parameters such as velocity or concentration distribution in a pipe first. Then the functions are made to satisfy fluid mechanics equations and appropriate boundary conditions in integral form.

Differential analysis is a method by which velocity and concentration fields are deduced from suitable differential equations. Time-averaged quantities are usually employed to write the equations like single phase turbulence theories.

Computational fluid dynamics (CFD) solutions are usually challenged by stability, convergence and accuracy. The importance of well-placed mesh is also crucial in implementing computational fluid dynamics methods.

Artificial neural network (ANN) employs the prior acquired knowledge to respond to a new information rapidly and automatically. Large data sets are usually required to develop artificial neural network methods.

2.2 Previous Comparison Work

Comparison of pressure drop correlations done by other authors earlier than this study is presented in this section. Some of the authors are interested in specific application while others are interested in a wider application of pressure drop correlations.

Almost all of the authors used different type of experimental data set and different correlations in their comparison work. Not all of the information such as experimental data is available in the open literature. Summarizing these works will definitely give some insight about the pressure drop correlations and therefore it is presented in this section.

2.2.1 Dukler et al. (1964) Comparison

Dukler et al. (1964) used 2,620 data points selected from AGA/API Data Bank that was compiled by University of Houston to compare five pressure drop correlations. The data points contain horizontal pressure drop experimental investigations in pipe diameters ranging from 1 inch to $5\frac{1}{2}$ inch and liquid densities from 1 to 20 centipoises.

The correlations compared were Baker (1954), Bankoff (1960), Chenoweth and Martin (1955), Lockhart and Martinelli (1949) and Yagi (1954). The authors indicated all the five correlations contain constants evaluated from experimental data. The authors also pointed out in some cases the constants are evaluated from a data representing a narrow range of conditions. As an example they mentioned an empirical constant in Bankoff's correlation is derived from a steam-water data for two-phase flow.

Dukler et al. (1964) employed statistical tools such as arithmetic mean deviation, standard deviation and they also developed a new statistical variable to account for the fractional deviation which includes 68% of the population to measure the spread of data. As the result of the comparison using the statistical parameters, Dukler et al. (1964) concluded that Lockhart and Martinelli (1949) correlation is the best correlation as compared to the rest four correlations.

The authors indicated that Bankoff (1960) and Yagi (1954) correlations were not sufficient for wide range of applications. Bankoff (1960) was found to perform well only for single component two-phase flow with higher pressures. Furthermore, Dukler et al. (1964) investigation based on diameter indicated the prediction performance of Chenoweth and Martin (1955) correlation and Lockhart and Martinelli (1949) correlation decreases as the pipe diameter increases. They also indicated that Baker (1954) exhibited better performance for large diameter pipe sizes with more viscous fluids because the development of the correlation was based on data of crude oil flowing in large diameter pipes.

A comparison done based on flow pattern indicated that Lockhart and Martinelli (1949) correlation is better than all the other four correlations except for the plug flow where Chenoweth and Martin (1955) was found to be better.

2.2.2 Idsinga et al. (1976) Comparison

Idsinga et al. (1976) used 2,220 steam-water pressure drop data points to compare eighteen pressure drop correlations. The diameter of the pipes ranged from 2.3 mm to 33 mm.

The authors employed statistical tools such as average error, root mean square error and standard deviation to evaluate performance of the correlations. The authors also stated that they have considered uncertainties of the experimental data. They indicated the measured mass velocity and pressure drop were the major components for adiabatic data while inlet sub cooling and change of the flow quality through the test section influence the error range in diabatic data.

Several groups of the experimental data were established based on source of data, fluid properties and flow conditions. Two groups of pressure ranges and three groups of mass velocity were formed. The flow quality was also grouped in seven groups. The correlations were compared in each group and on overall experimental database. Owens (1961) and Cicchitti et al. (1960) correlations followed by Thom (1964) and Baroczy (1966) were found to predict the experimental data set better than the rest of the correlations compared.

2.2.3 Mandhane et al. (1977) Comparison

Mandhane et al. (1977) used the University of Calgary Multiphase Pipe Flow Data Bank which contains 10,583 data points. The authors used a flow pattern map developed by themselves in 1974 to group the data into six flow patterns namely bubbly, stratified, wave, slug, annular and dispersed bubble.

Properties of the experimental data base were given in graphical form. The flow pattern map developed by the authors predicted most of the data to be in slug and annular flow regimes with 4,057 in slug flow and 3,058 data points in annular region. There were 1,651 data points in bubbly and 827 points in stratified flow regimes. However, the authors indicated that the number of points given by other authors flow pattern maps differ from this prediction.

Sixteen correlations were compared against the experimental data bank. Namely; Lockhart and Martinelli (1949), Chisholm (1967), Baker (1961), Dukler et al. (1964), Chawla (1968), Hoogendoorn (1959), Bertuzzi et al. (1956), Chenoweth and Martin (1955), Baroczy (1966), Beggs and Brill (1973), Govier and Aziz (1972), Agrawal et al. (1973), Hughmark (1965) and Levy (1952). In addition to the correlations listed above the authors included two new correlations proposed by them. The authors, Mandhane et al. (1977), proposed two new correlations for slug and dispersed bubble flow regimes.

The authors recommended Chenoweth and Martin (1955) for Bubble and annular flows. Agrawal et al. (1973) correlation was recommended for stratified flow regime. Dukler et al. (1964) with no-slip correlation was found to perform well for stratified wave flows. Slug and dispersed bubble flows were reported to be best predicted by the new correlations that were proposed by Mandhane et al. (1977).

2.2.4 Behnia (1991) Comparison

Behnia (1991) used 197 data points from a data bank sponsored by the American Gas Association compiled by Gregory (1980). The data bank contains data collected from several company records representing either normal production or special test conditions. Most of the data points came from oil and natural gas flowing in large pipes with internal diameter of 484 mm. Behnia (1991) compared seven correlations; Fancher and Brown (1963), Hagedorn and Brown (1965), Mukherjee and Brill (1985), Duns and Ros (1963), Dukler et al. (1964), Aziz et al. (1972), and Beggs and Brill (1973).

The author used average deviation, standard deviation and root mean square (RMS) statistical methods to evaluate the correlations. The author indicated that the data base was best predicted by the Beggs and Brill (1973) correlation. The author also reported the Dukler et al. (1964) correlation over predicted the data base while the Fancher and Brown (1963), Hagedorn and Brown (1965) correlations suffer under prediction.

2.2.5 Ferguson and Spedding (1995) Comparison

Ferguson and Spedding (1995) used their own pressure drop measurement data measured in 0.0935m diameter pipe for air-water two-phase flow and another air-water data set from Nguyen and Spedding (1976) in 0.045m diameter Perspex pipe.

Relative error analysis is used as a statistical tool to compare the performance of the correlations. The authors claim that more sophisticated statistical techniques mask the excessive deviation from the experimental data. The authors argued that most statistical methods used by previous authors are appropriate for a wide natural variation such as weight or age of a population that exist naturally. However, those statistical techniques can mask the valuable insight while reporting the comparison of correlations performance.

Ferguson and Spedding (1995) focused their comparison on the performance of correlations for specific type of flow patterns. The flow patterns observed in their study were mostly in transition zones such as stratified roll waves, stratified inertial waves, annular-droplets, annular-roll waves, etc. The authors reported that Olujic (1985) was in best agreement with the data set for annular, droplet, plug and most transitional stratified flow patterns. Hashizume et al. (1985) correlation gave the second best prediction for annular flows. Dukler et al. (1964) and Hanratty (1987) were also reported to be the second best correlations for droplet and transitional stratified flows, respectively.

Ferguson and Spedding (1995) indicated two-phase flow pressure prediction in smooth stratified flow is difficult because the pressure loss is usually small due to the presence of interfacial level gradient. They recommended the Hanratty (1987) correlation for smooth stratified correlation.

No correlation was suggested for slug, bubbly and blow-through slug flow patterns. The authors stated that low prediction performance of the correlations arises from the intermittent nature of these flow patterns.

2.2.6 Momoki et al. (2000) Comparison

Momoki et al. (2000) used the flow pattern map developed by Taitel and Dukler (1976) to group the experimental data they used for comparison. The experimental data contains 460 points from air-water, steam-water and three types of refrigerants (R134a, R22 and R114). Based on the Taitel and Dukler (1976) flow pattern map, 346 points were in the annular region, 59 were in the intermittent flow and the remaining 18 points were in the bubbly flow regime. The diameter of the pipes ranged from 7.9 to 24.3 mm.

The authors compared five correlations; Lockhart and Martinelli (1949), Chisholm (1973), Thom (1964), Martinelli and Nelson (1948), Owens (1961) and Cicchitti et al. (1960) correlations. Average error, root mean square error and standard deviation statistical tools were used to compare the performance of the correlations.

The authors reported the Homogeneous Model using liquid viscosity as two-phase viscosity was able to predict the annular flows better than the other correlations. Chisholm (1973) was found to perform better for the intermittent flow regimes. Thom (1964) gave the best prediction for steam-water data while large deviation from experimental data was observed on the air-water data. Momoki et al. (2000) argued that the large errors could be from the fact that the correlation was developed from steam-water data.

2.2.7 Tribbe and Muller-Steinhagen (2000) Comparison

Tribbe and Muller-Steinhagen (2000) made a comparison focused on flow pattern specific correlations. The authors used an experimental data base containing 7,000 data points from Dukler Data Bank from Dukler et al. (1964). They presented the results of six selected empirical correlations and twenty one flow pattern specific phenomenological methods. Based on the Taitel and Dukler (1976) flow map five flow pattern regimes; stratified, wavy, annular, intermittent and dispersed bubble flow patterns, were considered.

Agrawal et al. (1973) correlation was found to give accurate prediction in the stratified region and it was also reported that this correlation is weakly influenced by variation of fluid system. The authors also indicated the performance of other phenomenological correlations is poor in this flow regime and empirical methods such as Bandel (1973) correlations gave more accurate prediction than the phenomenological methods. Olujic (1985) and Bandel (1973) correlations were found to give more accurate prediction in the stratified wavy flows. The authors reported the prediction of Friedel (1979) correlation in smooth and wavy stratified flow regimes resulted in large deviation from the experimental data. Lockhart and Martinelli (1949) correlation gave accurate prediction for air-water and air-oil systems.

The authors reported that the predictive performance of most of the correlations was poor in the intermittent flow regime. However, empirical models such as Beattie and Whalley

(1982) and Bandel (1973) correlations were found to be in good agreement with the experimental data in this flow regime. The phenomenological model proposed by Nicholson et al. (1978) gave similar accuracy with the Bandel (1973) correlation in this intermittent flow regime. The refrigerant flow data was reported to be poorly predicted by all the correlations in this region.

In the annular flow regime, Hashizume et al. (1985) was found to be in good agreement with the experimental data. It was also reported that Hashizume et al. (1985) correlation exhibited low sensitivity to changes in fluid combinations. Most empirical correlations showed system sensitivity to fluid combination where air-oil data is particularly poorly predicted.

In the summary of the study, Tribbe and Muller-Steinhagen (2000) stated that low prediction is observed in flow pattern transition zones. As a general note, the authors indicated that even if the phenomenological models have similar accuracy to the empirical models, there is a remarkable improvement on reduction of sensitivity to changes in fluid combinations when phenomenological models are employed.

2.3 Chapter Summary

In this chapter review of frictional pressure drop correlations and review of previously done comparison work is done. Five groups of methods used in developing frictional two-phase pressure drop are presented. Review of the correlations indicates that almost all correlations require validation of empirical constant(s) obtained based on experimental data at some point during the development of the correlation. This indicates that there is a concern of accuracy when the correlations are used for a two-phase flow problem other than the experimental data the author of correlations used.

Review of previously done comparison works to investigate the performance of the pressure drop correlations revealed that different correlations are suggested for use by different authors. This could be due to the different data sets the authors used to validate the correlations.

In addition to the experimental data, there is also a difference on correlations suggested for a specific flow pattern. This could be due to the fact that identifying flow patterns visually or by using flow pattern maps is still resulting in ambiguous definitions of flow patterns.

In general, review of the two-phase pressure drop correlations and study of the previously done comparison works indicated that validation of two-phase frictional pressure drop correlations is a task that has to be done on a continuous basis whenever there is a new experimental data or a new correlation is available in literature.

CHAPTER 3

EXPERIMENTAL DATA BASE AND FLUID PROPERTIES

The main focus of this chapter will be presenting the characteristics of the experimental data base that was collected from the literature and the various correlations used to determine the physical properties of the fluids used in the experiments. A brief discussion of the two phase pressure drop measurement techniques used by the authors of the experimental data will also be discussed towards the end of this chapter. In the collection process of the experimental data, effort has been made to include a wide range of two phase flow conditions and various types of working fluids.

3.1 Experimental Database

A total of 2,429 data points of experimental pressure drop measurements from eleven different authors are used in this study. The data is collected from the open literature and checked for completeness of the required inputs for pressure drop calculation. After initial screening to check completeness of the data set, the data of Reid et al. (1957), Wicks (1958), Gregory and Scott (1969), Beggs (1972), Nguyen (1975), Chen (1979), Mukherjee (1979), Hashizume (1983), Bhattacharyya (1985), Adritsos (1986), Gokcal (2005) were selected. The range of pipe diameter, fluid combination and pipe surface roughness are summarized in Table 6. More information on the experimental data base can also be found in appendix A.

Table 6: Characteristics of Data Base Sources

Source	Mixture considered	Diameter [mm]	L/D	No. of data points	Surface roughness [mm]
Reid et al. (1957) [1]	Air-Water (AW)	101.6mm (4 inch) 152.4mm (6 inch)	168 112	43	$4.57x10^{-2}$
Wicks (1958) [1]	Air-Water (AW)	25.4mm (1 inch)	243	225	$1.50x10^{-3}$
Gregory and Scott (1969) [2]	CO_2-Water (CW)	19.05 mm (3/4 inch)	216	109	$1.50x10^{-3}$
Beggs (1972) [2]	Air-Water (AW)	25.4mm (1 inch) 38.1mm (1.5 inch)	540 360	58	$1.50x10^{-3}$
Nguyen (1975) [1]	Air-Water (AW)	45.5mm (1.79 inch)	44	250	$1.50x10^{-3}$

Measurement techniques: [1] Manometer, [2] Pressure Transducers

Table 6: (Contd.) Characteristics of Data Base Sources

Source	**Mixture considered**	**Diameter [mm]**	**L/D**	**No. of data points**	**Surface roughness [mm]**
Chen (1979) [1]	Air-Water (AW)	45.5mm (1.79 inch)	44	293	$1.50\text{x}10^{-3}$
Mukherjee (1979) [2]	Air-Kerosene (AK) Air-Lube Oil (AO)	38.1mm (1.5 inch)	242	90	$4.57\text{x}10^{-2}$
Hashizume (1983) [2]	R-12 R-22	10 mm (0.39 inch)	200	170	$1.50\text{x}10^{-3}$
Bhattacharyya (1985) [1]	Air-Water (AW)	25.4 mm (1 inch)	121	463	$1.50\text{x}10^{-3}$ $8.89\text{x}10^{-3}$
Andritsos (1986) [2]	Air-Water (AW) Air-Glycerol Soln. (AG)	25.2 mm (1 inch) 95.3 mm (3.75 inch)	615 258	545	$1.50\text{x}10^{-3}$
Gokcal (2005) [2]	Air-Lube Oil (AO)	50.8 mm (2 inch)	192	183	$1.50\text{x}10^{-3}$

Measurement techniques: [1] Manometer, [2] Pressure Transducers

All the required information from the experimental data base is first typed in an Excel sheet and then unit conversion is made in the EES (Engineering Equation Solver) program. Most of the authors reported their measurements in British Units but most of the pressure drop correlations in this study have to be done in SI units. Therefore, in the first stage all the experimental data has to be converted to SI units. On the other hand, some of the pressure drop correlations require calculations to be done in British Units therefore all the data has again to be converted to standard British Units in order to evaluate those correlations.

Each data set was tested against some essential requirements to determine if the pressure drop measurements are within the realistic range of values. The value of the two phase measurement was plotted against the mass flow rates of each phase to check if measurement neighboring points have consistent trends. Large scatters were observed for measurements where very small pressure drops were reported.

Following this lead, data points with a pressure drop measurement of less than five times the expected accuracy of the measurement technique were discarded. Those discarded points were also checked using all of the better performing correlations and consistent large deviation from the predictions were observed. These yielded 4 points from Andritsos (1986), 3 points from Beggs (1972), 5 points from Chen (1979), 15 points from Nguyen (1976) and 2 points from Mukherjee (1979). These points were suspected to be unreliable and therefore they were excluded from this study.

3.2 <u>Fluid Physical Properties</u>

Ten types of fluids are considered in the experimental data bases collected for this study. The types of fluids studied in each experimental data base are shown in Table 6 in the previous section.

Properties of air, water, carbon dioxide (CO_2), R12 and R22 were calculated directly from EES built in fluid property functions. The properties of glycer solutions are given by the authors along with the experimental data points and it is therefore typed in an Excel sheet along with experimental data. However, the properties of kerosene, natural gas and the different types of lube oils are calculated either from correlations given by the authors of the specific data set or by other correlations taken from the open literature. The correlations used for these fluid property calculations are discussed in this section.

3.2.1 Properties of Kerosene

The flow of kerosene with air has been studied by Mukherjee (1979). They determined physical properties from laboratory measurement and plotted the data as a function of temperature. The author gave equations for the physical properties of liquid kerosene as flows:

$$\rho_{\mathrm{ker}o} = 52.8858 - 0.0289T \qquad [\mathrm{lb/ft^3}] \qquad (3.1)$$

$$\mu_{\mathrm{ker}o} = Exp(1.4344 - 0.0115T) \qquad [\mathrm{dynes/cm}] \qquad (3.2)$$

$$\sigma_{\mathrm{ker}o} = 29.198 - 0.05T \qquad [\mathrm{cP}] \qquad (3.3)$$

The units of temperature *T* is in degree Fahrenheit (ºF)

3.2.2 Properties of Lube Oil

Mukherjee (1979) and Gockal (2005) used different types of lube oils in their studies. The respective equations suggested by each author have been used to determine the physical properties of lube oil for each data set.

Mukherjee (1979) gave equations for the physical properties of lube oil used in their study as follows:

$$\rho_{\mathrm{lub}e} = 54.6061 - 0.0263T \qquad [\mathrm{lb/ft^3}] \tag{3.4}$$

$$\mu_{\mathrm{lub}e} = Exp(4.7220 - 0.0229T) \qquad [\mathrm{dynes/cm}] \tag{3.5}$$

$$\sigma_{\mathrm{lub}e} = 38.6894 - 0.0650T \qquad [\mathrm{cP}] \tag{3.6}$$

The units of temperature *T* is in degree Fahrenheit (ºF)

Gockal (2005) used Citgo Sentry 220 oil in their study. The density and the surface tension are provided by the author and it is directly typed in the experimental data base. However, the author gave an equation to calculate the viscosity of the lube oil as follows:

$$\mu_{\mathrm{lub}e} = \frac{7\times 10^{8}(1.8T + 32)^{-3.2932}}{1000} \qquad \text{in [Pa.s]} \tag{3.7}$$

The units of temperature *T* is in degree Centigrade (ºC).

3.3 <u>Measurement Techniques</u>

A brief discussion of the measurement techniques used by the authors of the experimental data bases used in this study is presented in this section. Pressure measurement in two phase flow systems is usually done using either a manometer or pressure transducers. The type of pressure measurement used by each author is given in Table 6.

Manometers are often used to refer specifically to liquid column hydrostatic instrument. The liquid column gauges consist of a vertical column of liquid in a tube that has ends which are exposed to different pressures. The column will rise or fall until its weight is in equilibrium with the pressure differential between the two ends of the tube. For more accurate readings, inclined column may be used to further amplify the liquid movement. Depending on the pressure range to be covered different types of fluid such as mercury, oil or water may be used as a working fluid in the manometer. Mercury is preferred for most applications because of its high density (13,534 kg/m^3) and low vapor pressure. Manometers are usually convenient for pressure measurements near atmospheric pressure measurements and also not reliable for highly fluctuating pressure measurements.

Pressure transducers are used more widely in recent experiments to measure pressure drops in two phase flow systems. Pressure transducers have the advantage of being used with automated pressure recording systems and better handling of fluctuating pressures as compared to manometers. For two phase pressure drop measurements, differential pressure transducers are usually used. Differential pressure transducers involve the use of two pressure tapping lines attached to two chambers separated by a diaphragm whose movement then generates electrical signals to indicate the differential pressure. This also makes pressure transducers more favorable than manometers because only a small movement of the diaphragm occurs so that the movement of fluid in and out of the measurement lines is minimized. Different types of diaphragms based on the pressure range to be measured are used to increase the accuracy and response time of transducers. Pressure transducers must be calibrated periodically to maintain accuracy.

Generally, pressure transducers have an advantage over manometers because of their quick response and the fact that they can be integrated in electronically automated measurement systems.

To wrap up this chapter, the different data sets used in the database were briefly presented. The equations used to determine the physical properties of the fluids used in this study were also presented. The types of measurement techniques used by authors of the experimental data have also been briefly discussed. In the next chapter, methodology and results of the comparison work done in this study will be presented.

CHAPTER 4

EVALUATION OF TWO-PHASE PRESSURE DROP CORRELATIONS

Performance of the two-phase flow pressure drop correlations that were reported in Chapter 2 are compared against the experimental data base discussed in Chapter 3. The results of the comparisons are presented in this chapter.

The results of the comparisons are presented for each experimental data base. For each experimental data base, the performance of the pressure drop correlations is reported for the entire data base and again for data points grouped based on fluid type, pipe diameter, flow pattern and void fraction.

The main objective of this study is to identify a single or a group of correlations that can handle a wide range of two-phase flow conditions. The restriction of the correlations indicated by the author(s) of the correlations is removed on all correlations in an effort to see if any of the correlations could handle more than the range its author(s) recommended. Therefore, it has to be noted that some correlations are stretched beyond their limit. Hence, the results of this study which may dictate unfavorable conclusions on some or all correlations should be understood from this perspective only. As discussed in the literature review, different parameters have been used by previous authors to show the relative accuracy of one correlation over the others.

In this study, relative percentage error bands, mean and standard deviation along with probability density function plots are used to facilitate identification of the best performing correlation for each data set.

The relative percentage error (e_i) is calculated as:

$$e_i = \left[\frac{Predicted - Measured}{Measured}\right] \times 100 \tag{4.1}$$

The mean ($\bar{\mu}$) is calculated as:

$$\bar{\mu} = \frac{1}{N}\sum_{i=1}^{N} e_i \times 100 \tag{4.2}$$

The standard deviation (σ_{SD}) is calculated as:

$$\sigma_{SD} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(e_i^2 - \bar{\mu}^2\right)} \times 100 \tag{4.3}$$

The probability density function (PDF) is expressed as:

$$\mathrm{PDF} = \frac{1}{\sigma_{SD}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{e_i - \bar{\mu}}{\sigma_{SD}}\right)^2\right] \tag{4.5}$$

4.1 <u>Comparison of Correlations against the Data Sets</u>

The results of performance of the pressured drop correlations for each data set are presented in the following sections in a chronological order of the publication year of the experimental data. The results of the comparison work are presented using tables and plots. Only those correlations that are in best agreement with the experimental data are shown in the tables sorted down with decreasing performance in the ±30% error band.

Probability density function plots are used to show the spread of the relative percentage error. Vertical dotted lines will be used to indicate zero relative percentage error. Curves with maximum point close to zero mark vertical line (zero relative error) indicate minimum mean. Also curves that are tall and narrow indicate minimum standard deviation. Therefore, curves with minimum offset of maximum point from the zero indicator vertical line, taller and narrower indicate better prediction of the data in concern.

Performance comparison based on void fraction is done for each data set. The void fraction is calculated using Woldesemayat and Ghajar (2007) correlation. Four groups of void fraction are created based on 0.25 increments (i.e. 0-0.25, 0.25-0.5, 0.5-0.75 and 0.75-1). The best performing correlations for each void fraction group are then presented in the discussion. Moreover, analysis based on flow pattern is done where flow pattern information is provided by the authors of the data.

4.1.1 Comparison with the data of Reid et al. (1957)

Reid et al. (1957) studied the flow of air-water mixture in 101.6mm (4 inch) and 152.4mm (6 inch). Flow pattern is not reported by the authors. There are 5 points with void fraction less than 0.5 and all the rest of the data lies between 0.5 and 0.75 void fraction. From the results shown in Table 7, Cicchitti et al. (1960) and McAdams et al. (1942) correlations predicted 100% of the pressure drop data within the ±30% error band. However, Sun and Mishima (2009) correlation gave the highest accuracy within the ±15% error band. From Figure 2, it can be seen that Sun and Mishima (2009) correlation slightly over predicted the data while McAdams et al. (1942) under predicted the data. Grønnerud (1979) and Olujic (1985) correlations have also predicted more than 80% of the data in the ±30% error band. The rest of the correlations resulted in less accurate prediction. Comparison by diameter showed that Sun and Mishima (2009) and Dukler et al. (1964) - (Case II) correlations predicted 100%

of the 101.6mm (4 inch) and 152.4mm (6 inch)diameter data within the ±30% error band, respectively.

Table 7: Performance of correlations that are in best agreement with Reid et al. (1957) data

Selected Correlations	**Reid et al. (1957) Data (Air-Water) Total Points 43**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Cicchitti et al (1960)	62.8%	100.0%	-1.4	14.8
McAdams et al. (1942)	62.8%	100.0%	-8.6	11.1
Awad (2007) (For regular size pipes)	55.8%	97.7%	-0.4	16.4
Sun and Mishima (2009)	79.1%	97.7%	7.1	9.6
Dukler et al. (1964) - (Case II)	67.4%	95.3%	-1.1	14.7

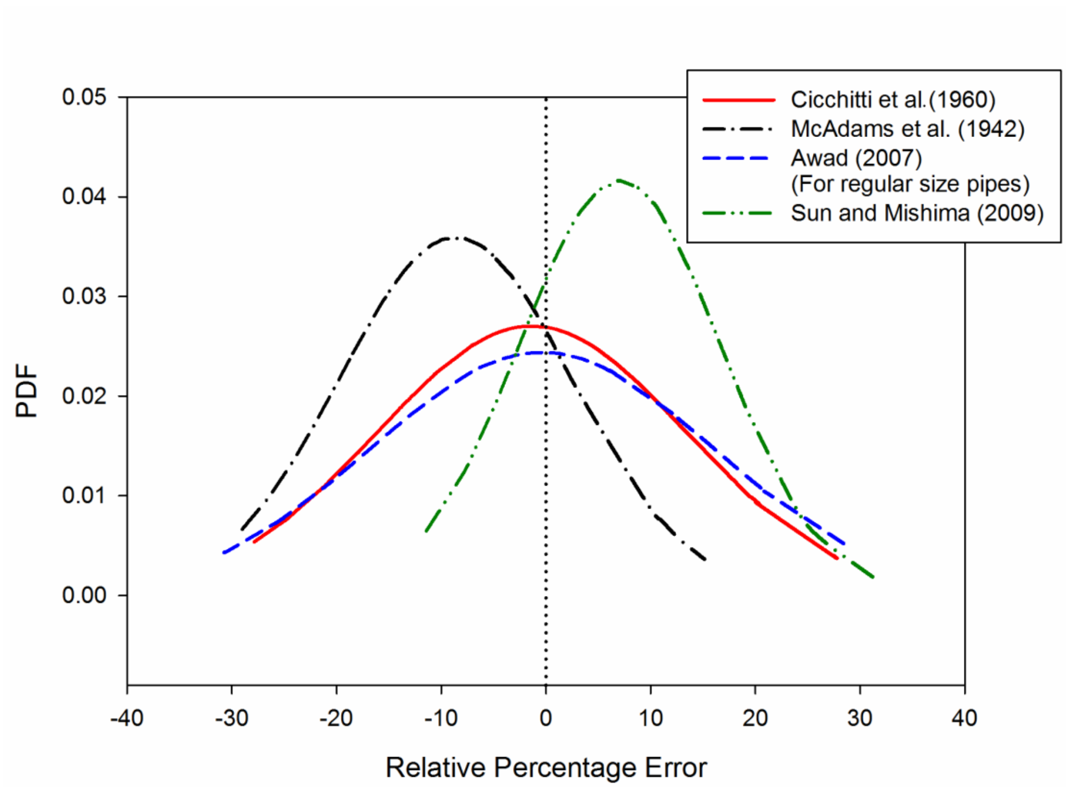

Figure 2: Probability density function for Reid et al. (1957) data comparison

4.1.2 Comparison with the data of Wicks (1958)

Wicks (1958) reported two-phase pressure drop measurement of air-water flow in 25.4mm (1 inch) diameter pipe. Flow pattern is not reported and all the points have a void fraction between 0.75 and 1. Eleven correlations predicted over 75% of the pressure drop data within the ±30% error band.

The correlations in best agreement with the data are presented in Table 8. Sun and Mishima (2009) gave the best prediction in the ±30% error band while Awad and Muzychka (2004a) predicted maximum number of points in the ±15% error band. Looking at Figure 3, Sun and Mishima (2009) correlation and Dukler et al. (1964) - (Case I) correlation under predicted the data. The data is slightly over predicted by Awad and Muzychka (2004a) correlation while Chawla (1968) gave minimum mean. Theissing (1980) and Dukler et al. also predicted 86.7% and 78.7% of the data within the ±30% error band, respectively.

Table 8: Performance of correlations that are in best agreement with Wicks (1958) data

Selected Correlations	**Wicks (1958) Data (Air-Water) Total Points 225**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Sun and Mishima (2009)	61.8%	99.6%	-10.6	9.7
Chawla (1968)	66.7%	97.3%	2.1	14.4
Dukler et al. (1964) - (Case I)	69.8%	97.3%	-4.0	13.7
Awad and Muzychka (2004a) (General case)	78.2%	93.3%	7.8	12.9

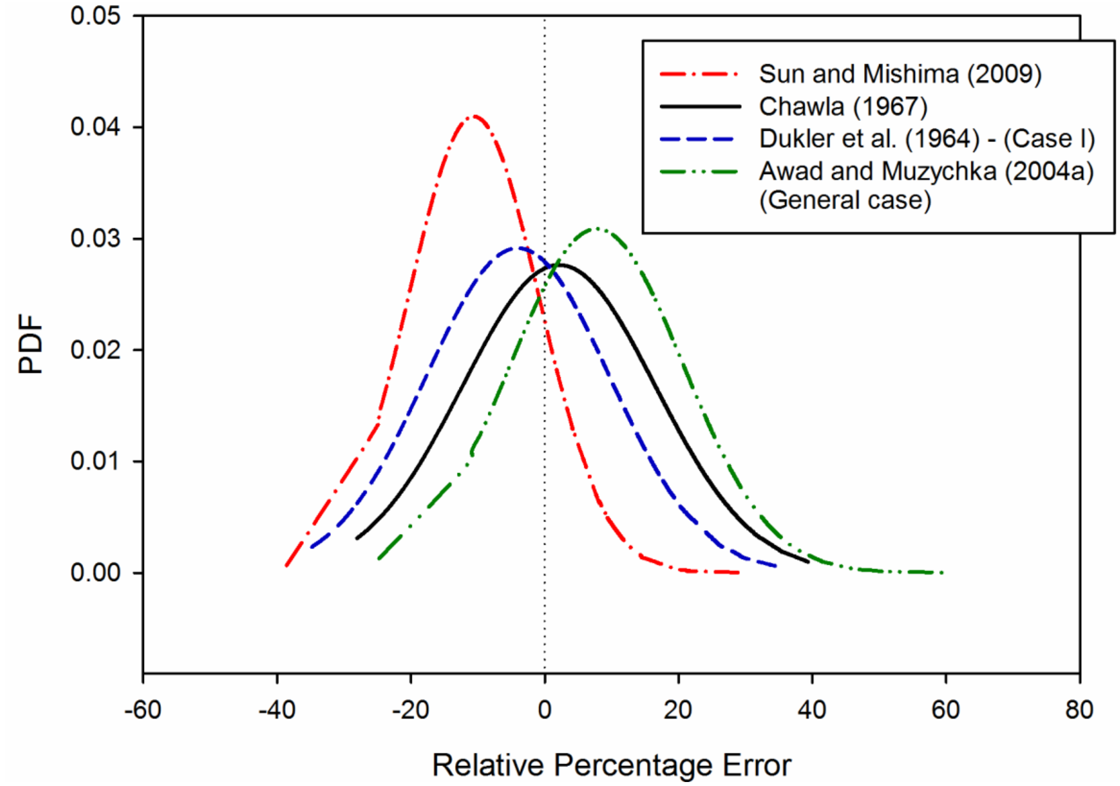

Figure 3: Probability density function for Wicks (1958) data comparison

4.1.3 Comparison with the data of Gregory and Scott (1969)

Gregory and Scott (1969) reported pressure drop data for carbon dioxide and water two-phase flow in 19.05 mm (3/4 inch) pipe. Table 9 shows that Theissing (1980) correlation predicted the maximum number of data points in the ±30% error band. In the ±15% error band, the maximum number of points is predicted by Beattie and Whalley (1982) correlation. Awad and Muzychka (2004a) and Dukler et al. (1964) - (Case II) equally predicted 91.7% of the data points in the ±30% error band. McAdams et al. (1942) and Awad and Muzychka (2004b) also predicted 84.4% and 82.6% of the data in the ±30% error band, respectively.

From Figure 4, it can be seen that all the three best performing correlations tend to under predict most of the data. Since flow pattern is not reported by the authors of the data, further analysis has been made based on void fraction only. All the data points have void fraction greater than 0.5.

Table 9: Performance of correlations that are in best agreement with Gregory and Scott (1969) data

Selected Correlations	**Gregory & Scott (1969) Data (CO_2-Water) Total Points 109**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Theissing (1980)	47.7%	93.6%	-6.4	21.9
Beattie and Whalley (1982)	53.2%	91.7%	-2.9	24.5
Olujic (1985)	50.5%	91.7%	-7.8	19.1

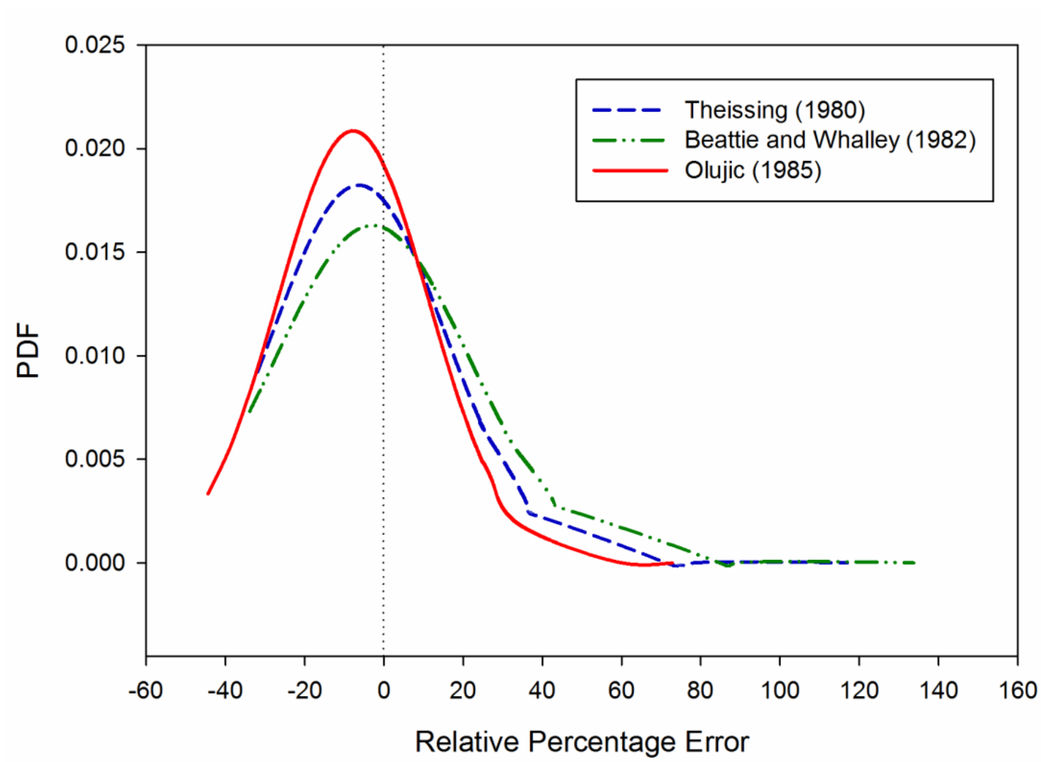

Figure 4: Probability density function for Gregory and Scott (1969) data comparison

The best performing correlations for each void fraction group are listed in Table 10. The shaded entries indicate the maximum percentage achieved for the specific void fraction range. Theissing (1980) correlation predicted both group of void fractions well in the ±30% error band. However, in the ±15% error band, Dukler et al. (1964) - (Case II) and Beattie and Whalley (1982) correlations predicted the highest number of points in 0.5-0.75 and 0.75-1 void fraction groups, respectively. It can also be seen that data points with void fraction between 0.75-1 are predicted better than those points with 0.5-0.75 void fraction.

Table 10: Comparison based on void fraction for Gregory and Scott (1969) data

Selected Correlations	**Gregory & Scott (1969) Data (CO_2-Water)**			
Void Fraction Range	**0.50 - 0.75 (31 pts.)**		**0.75 - 1.00 (78 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**
Dukler et al. (1964) (Case II)	59.0%	92.3%	16.1%	87.1%
Olujic (1985)	57.7%	92.3%	32.3%	90.3%
Theissing (1980)	41.0%	91.0%	64.5%	100.0%
Awad and Muzychka (2004a) (General case)	33.3%	87.2%	61.3%	100.0%
Beattie and Whalley (1982)	44.9%	89.7%	74.2%	96.8%

4.1.4 <u>Comparison with the data of Beggs (1972)</u>

Beggs (1972) reported two-phase pressure drop of air-water flow in 25.4mm (1 inch) and 38.1mm (1.5 inch) pipe diameters. The summary of the results in Table 11 indicate that Theissing (1980) correlation is in best agreement both in the ±15% and ±30% error bands. However, it can be seen from Figure 5 that Theissing (1980) correlation slightly under predicted the data. On the other hand, Dukler et al. (1964) - (Case II) predicted the data base with the minimum mean.

Table 11: Performance of correlations that are in best agreement with Beggs (1972) data

Selected Correlations	**Beggs (1972) Data (Air-Water) Total Points 58**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Theissing (1980)	82.8%	100.0%	-5.8	9.2
Awad and Muzychka (2008) (Viscosity Expression 4)	46.6%	96.6%	-5.7	17.9
Sun and Mishima (2009)	37.9%	94.8%	-9.1	17.5
Dukler et al. (1964) (Case II)	74.1%	93.1%	-0.9	15.3

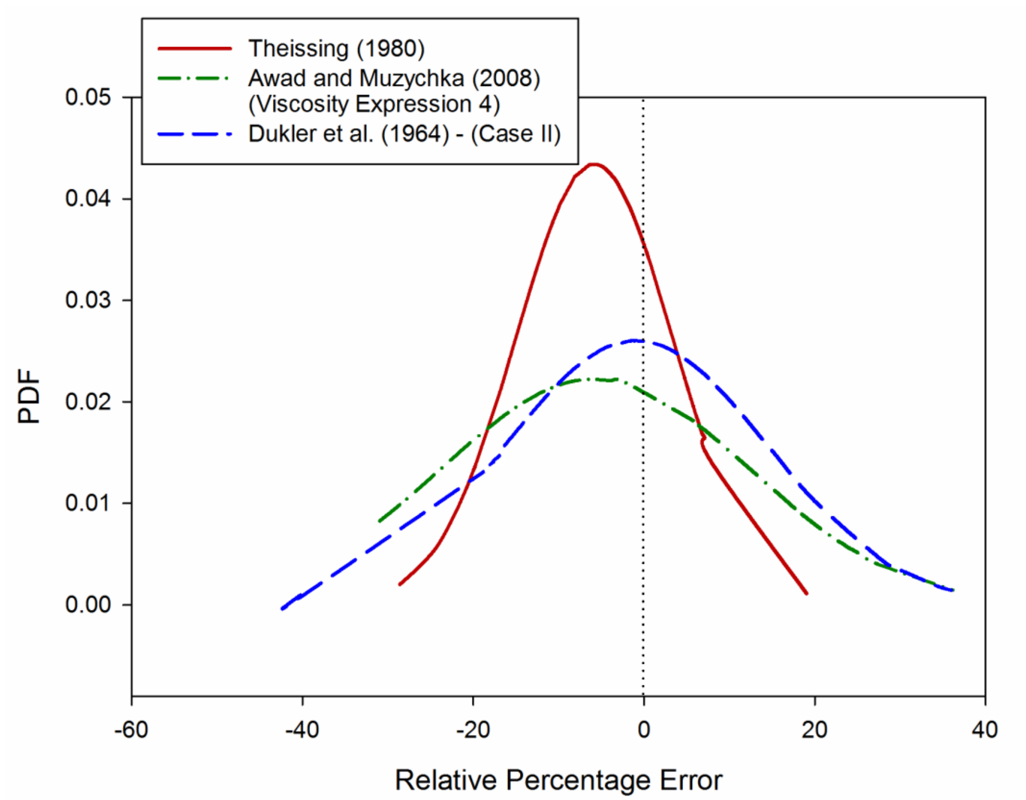

Figure 5: Probability density function for Beggs (1972) data comparison

4.1.5 ***Comparison with the data of Nguyen (1975)***

Nguyen (1975) reported pressure drop of air-water two-phase flow in 45.5mm (1.79 inch) pipe. Results presented in Table 12 show that Sun and Mishima (2009) correlation predicted maximum number of points in both ±15% and ±30% error bands. Figure 6 shows that Theissing (1980) correlation over predicted most of the data while Sun and Mishima (2009) yielded the minimum mean. The success rate of all correlations in accurately predicting the pressure drop for this data set is relatively low as compared to the other data sets.

During presentation of the experimental data bases in Chapter 3, it has been indicated in Table 6 that Nguyen (1976) experimental set up has a length to diameter ratio (L/D) of 44, which is the least among the other data sets. This may affect the quality of the experimental data, which would in turn affect the conclusion to be drawn regarding the performance of the correlations. Further investigation of the performance of the correlations has been made to see if there is any pattern that can be learned from this data base.

Table 12: Performance of correlations that are in best agreement with Nguyen (1975) data

Selected Correlations	**Nguyen (1975) Data (Air-Water) Total Points 250**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Sun and Mishima (2009)	38.0%	60.4%	0.3	52.2
Dukler et al. (1964) (Case II)	28.8%	58.8%	7.9	53.5
Theissing (1980)	32.4%	58.4%	26.8	67.9
Awad (2007) (For regular size pipes)	28.4%	57.6%	-13.9	39.6
Chisholm (1967)	33.6%	56.0%	26.3	84.0

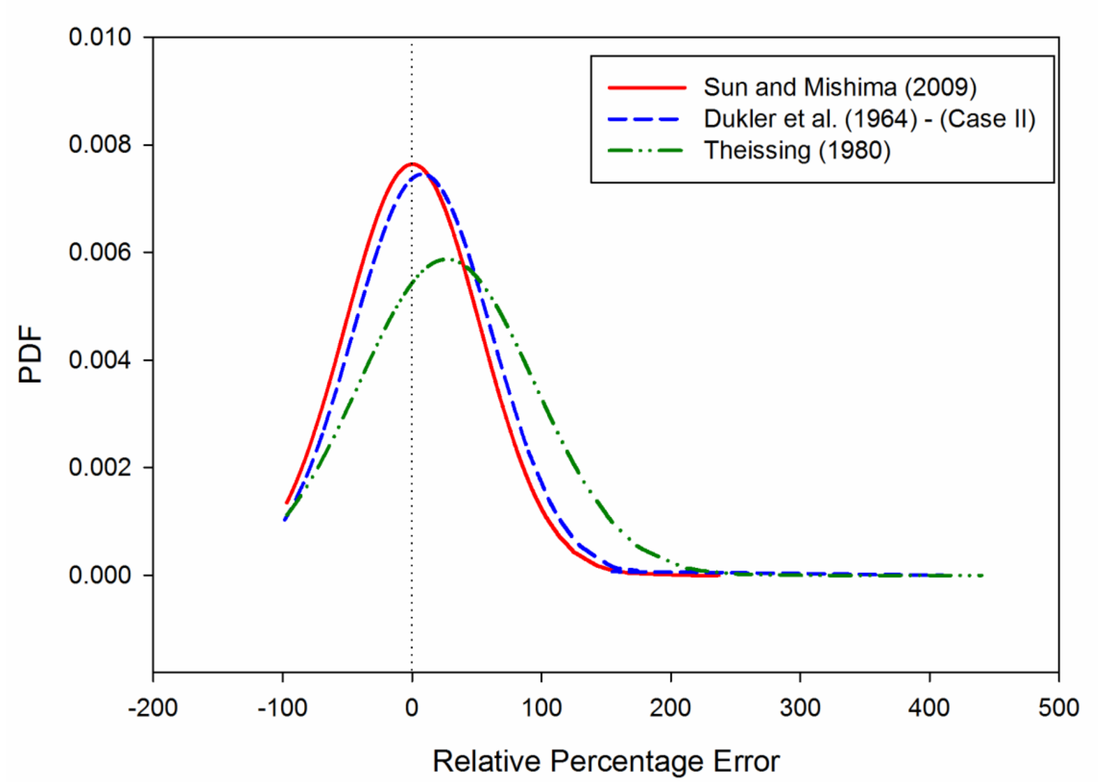

Figure 6: Probability density function for Nguyen (1975) data comparison

Performance of the pressure drop correlations has been analyzed based on flow pattern after the data points were grouped based on flow patterns as reported by authors of the data base. Table 13 summarizes prediction performance of the correlations that predicted the maximum number of points in each group flow patterns. Two of the best correlations for each of the flow patterns are shown in Table 13. The shaded entries indicate the maximum percentage achieved for the specific flow pattern. It can be seen that no correlation predicted more than two flow patterns correctly.

Most of the data points fall in the annular and stratified flow patterns. Theissing (1980) predicted maximum number of data points in annular regime followed by Awad and Muzychka (2004a). Bubbly and misty flow patterns were predicted well by Sun and

Mishima (2009), Baroczy (1966) and Chisholm (1973) correlations. Wallis (1969) and Garcia et al. (2003) correlations predicted the maximum number of data points in bubbly and slug/ plug regimes within the ±30% error band. Stratified flow pattern was found to be the least accurately predicted region. This is partly because the pressure drop in stratified flow pattern is very small such that a small error in prediction results in larger relative percentage error. Awad (2007) - (For regular size pipes) predicted the maximum percentage of data points in the stratified region in ±30% error band.

Grouping the data points based on flow pattern yielded results summarized in Table 14. Most of the data points fall in the 0.75-1 void fraction range where the maximum number of data points are predicted by Theissing (1980) and Dukler et al. (1964) - (Case II) correlations. The void fraction rage of 0.5-0.75 is the least predicted group whereas 0.25-0.5 range is the relatively well predicted zone. Figure 7 gives a graphical visualization of how the correlations performed in the ±30% error band. It can be seen that the performance of the correlations varies for the different void fraction ranges. Dukler et al. (1964) - (Case II) gives a steady prediction performance over the full void fraction range.

It has to be noted that some of the correlations such as Bankoff (1960) has been stretched beyond the limits set by the author. In this instance it can be seen that the performance of Bankoff (1960) correlation declines as void fraction generally increases. This is due to the fact that Bankoff (1960) was proposed for bubbly flows and usually bubbly flows exist in low void fraction ranges.

Generally, separated flow models predicted Nguyen (1975) data better than homogeneous models. But this could be from the fact that many of the data points are in the stratified and annular flow regimes where the velocity of the gas and the liquid are usually different.

Table 13: Comparison based on flow pattern for Nguyen (1975) data

Selected Correlations	**Nguyen (1975) Data (Air-Water)**									
Flow Pattern	**Annular (91 pts.)**		**Bubbly (8 pts.)**		**Misty (19 pts.)**		**Plug & Slug (46 pts.)**		**Stratified (86 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Theissing (1980)	46.2%	79.1%	12.5%	50.0%	84.2%	100.0%	19.6%	37.0%	15.1%	39.5%
Awad and Muzychka (2004a) (General case)	36.3%	70.3%	12.5%	25.0%	42.1%	84.2%	23.9%	63.0%	16.3%	29.1%
Sun and Mishima (2009)	33.0%	59.3%	75.0%	100.0%	73.7%	100.0%	34.8%	58.7%	33.7%	50.0%
Baroczy (1966)	44.0%	56.0%	75.0%	100.0%	100.0%	100.0%	30.4%	52.2%	15.1%	20.9%
Chisholm (1973)	44.0%	56.0%	75.0%	100.0%	100.0%	100.0%	30.4%	54.3%	16.3%	22.1%
Wallis (1969)	6.6%	17.6%	75.0%	100.0%	0.0%	0.0%	43.5%	69.6%	17.4%	24.4%
Garcia et al. (2003)	14.3%	22.0%	62.5%	100.0%	0.0%	15.8%	37.0%	67.4%	10.5%	25.6%
Awad (2007) (For regular size pipes)	18.7%	46.2%	50.0%	87.5%	68.4%	100.0%	23.9%	60.9%	30.2%	55.8%
Olujic (1985)	33.0%	44.0%	12.5%	62.5%	0.0%	0.0%	39.1%	45.7%	27.9%	53.5%

Note: The shaded entries indicate the maximum percentage achieved in the specific flow pattern.

Table 14: Comparison based on void fraction for Nguyen (1975) data

Selected Correlations	**Nguyen (1975) Data (Air-Water)**							
Void Fraction Range	**0.0 - 0.25 1.0 (14 pts.)**		**0.25 - 0.50 (21 pts.)**		**0.50 - 0.75 (35 pts.)**		**0.75 - 1.00 (180 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Bankoff (1960)	42.9%	57.1%	19.0%	52.4%	5.7%	8.6%	0.0%	0.0%
Chisholm (1967)	35.7%	50.0%	47.6%	81.0%	17.1%	34.3%	35.0%	57.8%
Muller-Steinhagen and Heck (1986)	35.7%	50.0%	47.6%	81.0%	14.3%	20.0%	20.0%	35.6%
Wallis (1969)	35.7%	50.0%	52.4%	76.2%	31.4%	51.4%	11.1%	20.0%
Garcia et al. (2003)	35.7%	50.0%	52.4%	76.2%	28.6%	48.6%	10.0%	24.4%
Theissing (1980)	7.1%	28.6%	14.3%	38.1%	17.1%	25.7%	39.4%	69.4%
Dukler et al. (1964) - (Case II)	21.4%	42.9%	14.3%	47.6%	20.0%	42.9%	32.8%	64.4%

Note: The shaded entries indicate the maximum percentage achieved in the specific void fraction range.

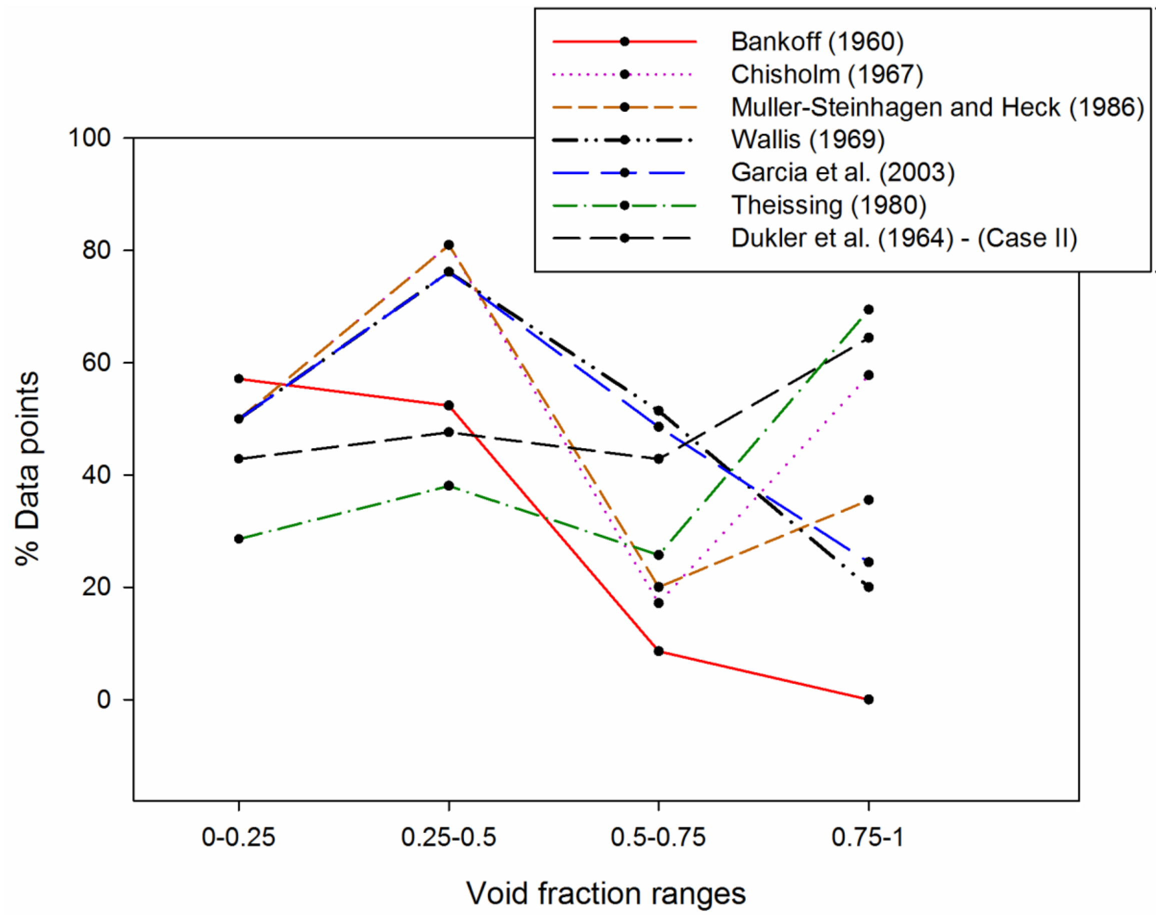

Figure 7: Performance of the top correlations within the ±30% error band in the full void fraction range for Nguyen (1975) data

4.1.6 Comparison with the data of Chen (1979)

Chen (1979) reported the pressure drop of air-water two-phase flow in 45.5mm (1.79 inch) pipe. Chen (1979) used the same experimental set up used by Nguyen (1975) data set and both have an L/D of 44. The maximum number of data points predicted in the ±30% error band is similar to Nguyen (1975) data.

From Table 15, it can be seen that Theissing (1980) and Dukler et al. (1964) - (Case II) are the correlations that predicted the maximum number of data points from the data set in the ±30% error band.

Table 15: Performance of correlations that are in best agreement with Chen (1979) data

Selected Correlations	**Chen (1979) Data (Air-Water) Total Points 293**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Theissing (1980)	41.6%	66.6%	19.3	55.5
Dukler et al. (1964) (Case II)	30.0%	60.1%	4.0	43.1
Awad and Muzychka (2004a) (General case)	30.0%	53.6%	15.1	50.4

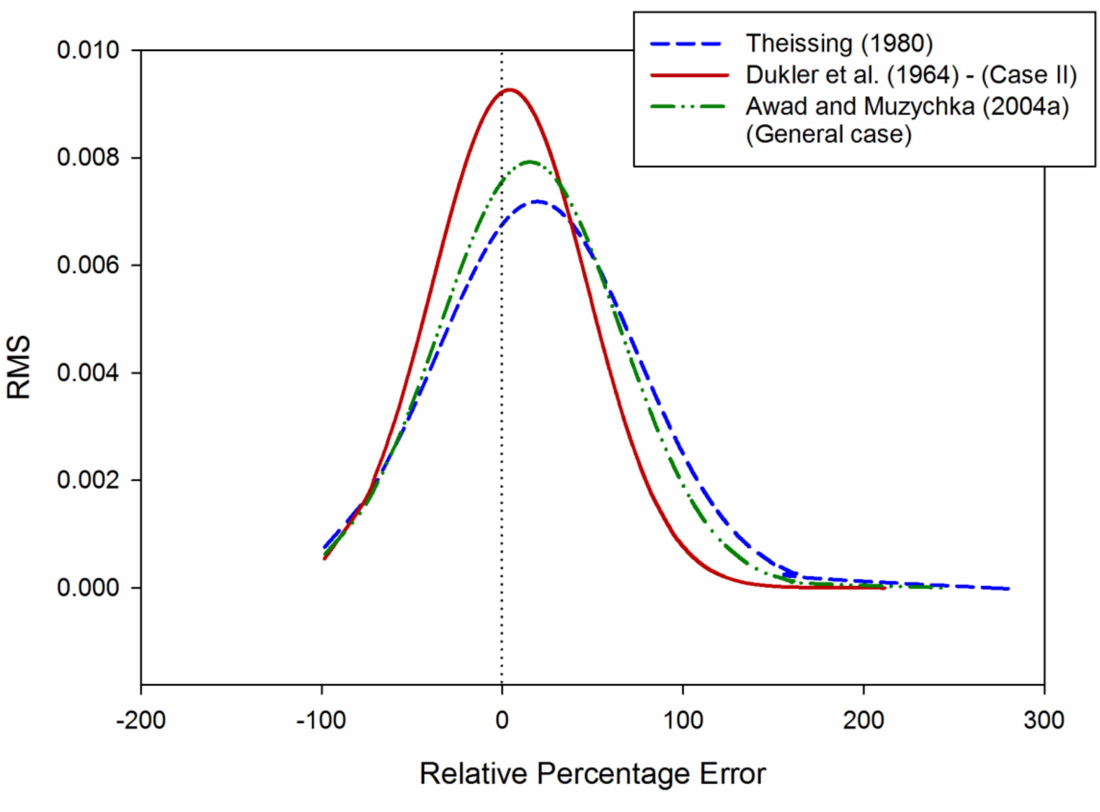

Figure 8: Probability density function for Chen (1979) data comparison

Figure 8 shows that all the top correlations tend to slightly over predict the data. Dukler et al. (1964) - (Case II) has also the minimum mean and standard deviation. Further analysis based on flow pattern shows that the plug and slug flow region is the least predicted flow pattern. Table 16 shows that Dukler et al. (1964) predicted 51.9% of the plug & slug region followed by Theissing (1980) correlation with only 38.9% of the data points. This low performance of the correlations in these regions could be due to two reasons. First, there is high fluctuation in pressure drop in slug and plug region that makes it more difficult to either get a good measurement or make good prediction. Second, since Chen (1979) used manometer measurement device, there may have been challenges to make accurate measurements in these regions.

Table 16: Comparison based on flow pattern for Chen (1979) data

Selected Correlations	**Chen (1979) Data (Air-Water)**					
Flow Pattern	**Annular (167 pts.)**		**Plug & Slug (54 pts.)**		**Stratified (72 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Theissing (1980)	58.7%	88.6%	27.8%	38.9%	16.7%	48.1%
Friedel (1979)	49.7%	76.6%	11.1%	24.1%	0.0%	7.4%
Dukler et al. (1964) - (Case II)	33.5%	67.1%	24.1%	51.9%	35.2%	66.7%
Olujic (1985)	35.9%	50.3%	18.5%	31.5%	58.3%	72.2%
Awad (2007) (For regular size pipes)	28.1%	46.7%	14.8%	35.2%	40.3%	68.1%

Grouping the data based on void fraction showed that the data points lie in the void fraction range of 0.2 to 1. Most of the data points fall in the 0.75-1 void fraction range. Summary of the results in Table 17 shows that void fraction range of 0.5-0.75 is the least predicted zone. The shaded entries in the table indicate the maximum percentage achieved for the specific void fraction range.

Table 17: Comparison based on void fraction for Chen (1979) data

Selected Correlations	**Chen (1979) Data (Air-Water)**					
Void Fraction Range	**0.25 - 0.50 (17 pts.)**		**0.50 - 0.75 (27 pts.)**		**0.75 - 1.00 (249 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Bankoff (1960)	47.1%	76.5%	11.1%	18.5%	0.0%	0.0%
Dukler et al. (1964) (Case II)	35.3%	76.5%	7.4%	25.9%	32.1%	62.7%
Awad (2007) (For regular size pipes)	5.9%	17.6%	11.1%	33.3%	32.1%	53.8%
Theissing (1980)	47.1%	64.7%	18.5%	25.9%	43.8%	71.1%
Dukler et al. (1964) - (Case II)	35.3%	76.5%	7.4%	25.9%	32.1%	62.7%

Similar to Nguyen (1975) data analysis, Chen (1979) data was predicted relatively well by separated flow models. This may be due to the fact that most of the data points are in the annular flow region where the velocity of the liquid and gas is significantly different.

4.1.7 <u>Comparison with the data of Mukherjee (1979)</u>

Mukherjee (1979) conducted experiments with air-kerosene and air-oil fluids in 38.1mm (1.5 inch) diameter pipe. Table 18 summarizes the performance of the correlations that are in best agreement with air-kerosene data. Figure 9 shows that all the best correlations over predict the data.

Table 19 summarizes the performance of the best correlations for Mukherjee (1979) air-oil data. The lube oil has a dynamic viscosity of 51.8 cP at 40ºC. Garcia et al. (2003) predicted the data relatively well both in the ±15% and ±30% error bands with minimum mean. Figure 10 shows that Awad (2007) - (For mini-channels) and Muller-Steinhagen and Heck (1986) under predicted the data.

Table 18: Performance of best correlations for Mukherjee (1979) air-kerosene data

Selected Correlations	**Mukherjee (1979) Data (Air-Kerosene) Total Points 58**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
McAdams et al. (1942)	37.9%	72.4%	35.5	145.9
Dukler et al. (1964) - (Case I)	44.8%	70.7%	21.4	116.3
Muller-Steinhagen and Heck (1986)	29.3%	56.9%	60.4	188.0

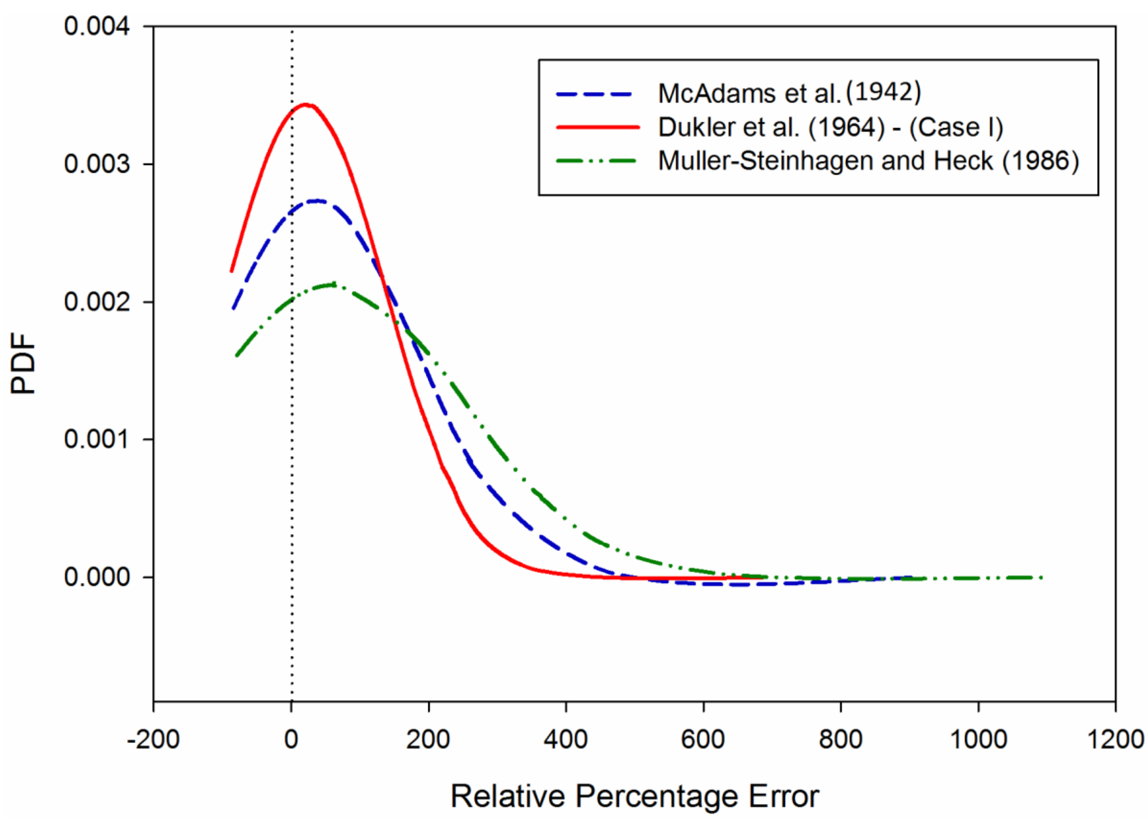

Figure 9: Probability density function for Mukherjee (1979) air-kerosene data comparison

Table 19: Performance of best correlations for Mukherjee (1979) air-oil data

Selected Correlations	Mukherjee (1979) Data (Air-Oil) Total Points 32		Mean	Standard Deviation
Percentage within	**±15%**	**±30%**		
Garcia et al. (2003)	59.4%	78.1%	-1.4	23.1
Awad (2007) (For mini-channels)	56.3%	71.9%	-17.9	20.5
Muller-Steinhagen and Heck (1986)	37.5%	62.5%	-25.1	32.9

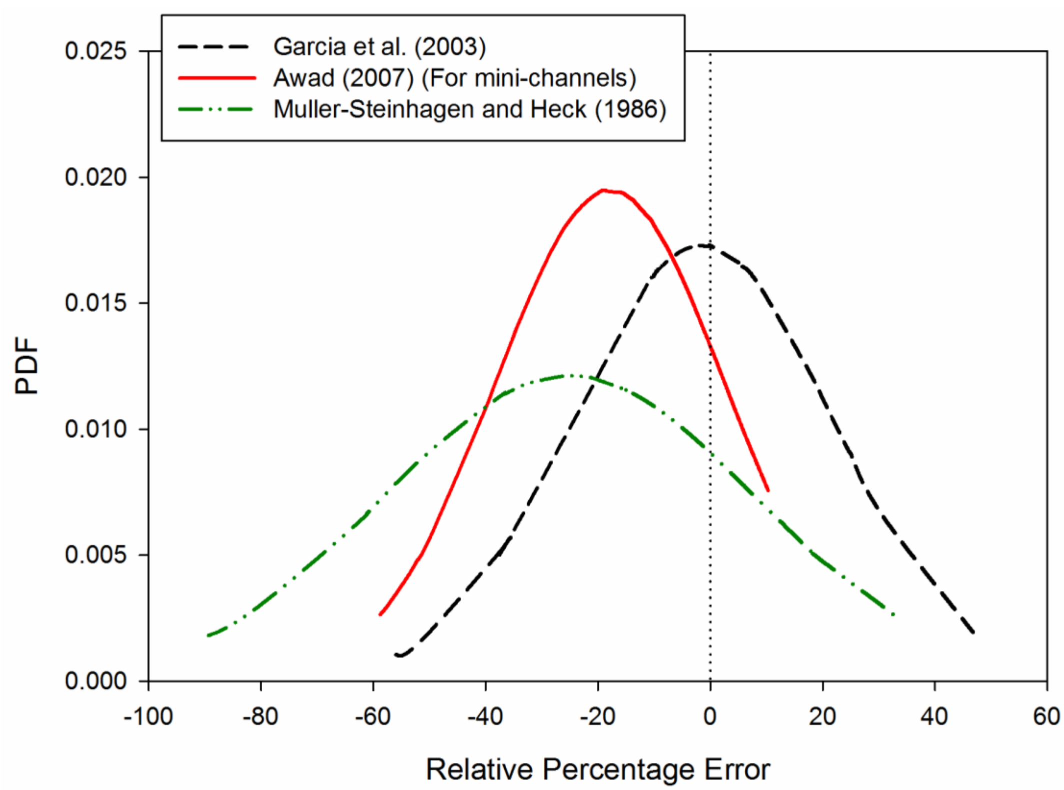

Figure 10: Probability density function for Mukherjee (1979) air-oil data comparison

The numbers of data points in Mukherjee (1979) data set are relatively small as compared to the other data sets with only 58 and 32 for air-kerosene and air-oil data, respectively. Drawing conclusions with very few numbers of data points may lead to erroneous results. Therefore, lower predictive performance of pressure drop correlations may not necessarily mean the correlations have low predictive capability for air-kerosene and air-oil mixtures. Further investigation of the performance of the correlations with larger data set would be required to get a better understanding of how the correlations behaved for these fluid combinations.

4.1.8 <u>Comparison with the data of Hashizume (1983)</u>

Hashizume (1983) conducted experiments with boiling flow of refrigerants R-12 and R-22 in 10 mm (0.39 inch) diameter pipe at high pressures ranging from 5.7 to 19.6 bars. Table 20 shows that Muller-Steinhagen and Heck (1986) predicted the maximum number of data points of the R-12 data both in the ±15% and ±30% error bands. Beggs and Brill (1973) and Theissing (1980) correlations also gave predictions that are comparable to Muller-Steinhagen and Heck (1986). The plots in Figure 11 show that Muller-Steinhagen and Heck (1986) and Beggs and Brill (1973) slightly over predict the data whereas Theissing (1980) is in a better agreement with the data over the ±40% error band.

The R-22 data is not predicted well by any of the correlations. The summary of the results in Table 21 shows that the top correlations predicted similar maximum number of points in the ±30% error band. However, Theissing (1980) correlation predicted more data points in the ±15% error band. Figure 12 shows that Beattie and Whalley (1982) and Awad and Muzychka (2008) - (Viscosity Expression 2) under predict the data whereas

Sun and Mishima (2009) over predict the data. Theissing (1980) correlation predicted the data with minimum mean.

Table 20: Performance of best correlations for Hashizume (1983) R-12 data

Selected Correlations	**Hashizume (1983) Data (R-12) Total Points 85**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Muller-Steinhagen and Heck (1986)	67.1%	89.4%	5.3	21.0
Beggs and Brill (1973)	60.0%	89.4%	5.5	19.9
Theissing (1980)	55.3%	88.2%	-1.9	21.4

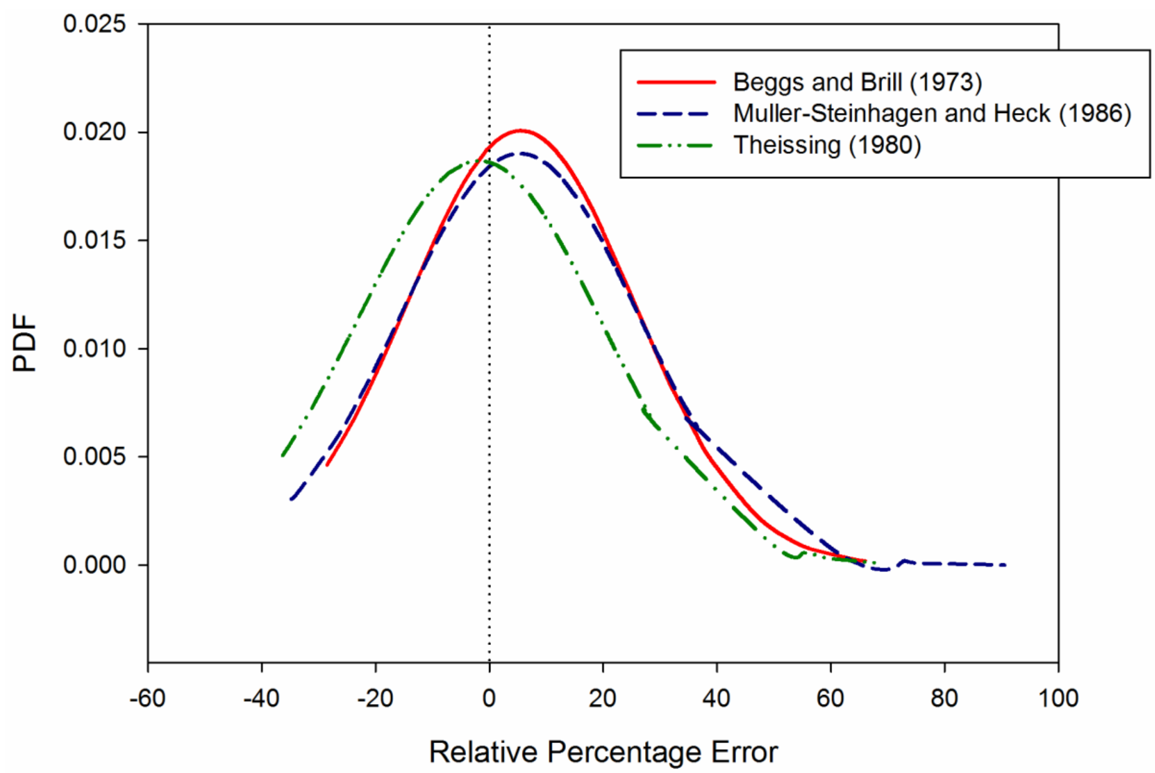

Figure 11: Probability density function for Hashizume (1983) R-12 data comparison

Table 21: Performance of best correlations for Hashizume (1983) R-22 data

Selected Correlations	**Hashizume (1983) Data (R-22) Total Points 85**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Beattie and Whalley (1982)	25.3%	59.0%	-9.9	44.7
Awad and Muzychka (2008) (Viscosity Expression 2)	27.7%	57.8%	-10.5	46.1
Sun and Mishima (2009)	27.7%	57.8%	11.8	57.0
Theissing (1980)	41.0%	57.8%	5.0	54.5

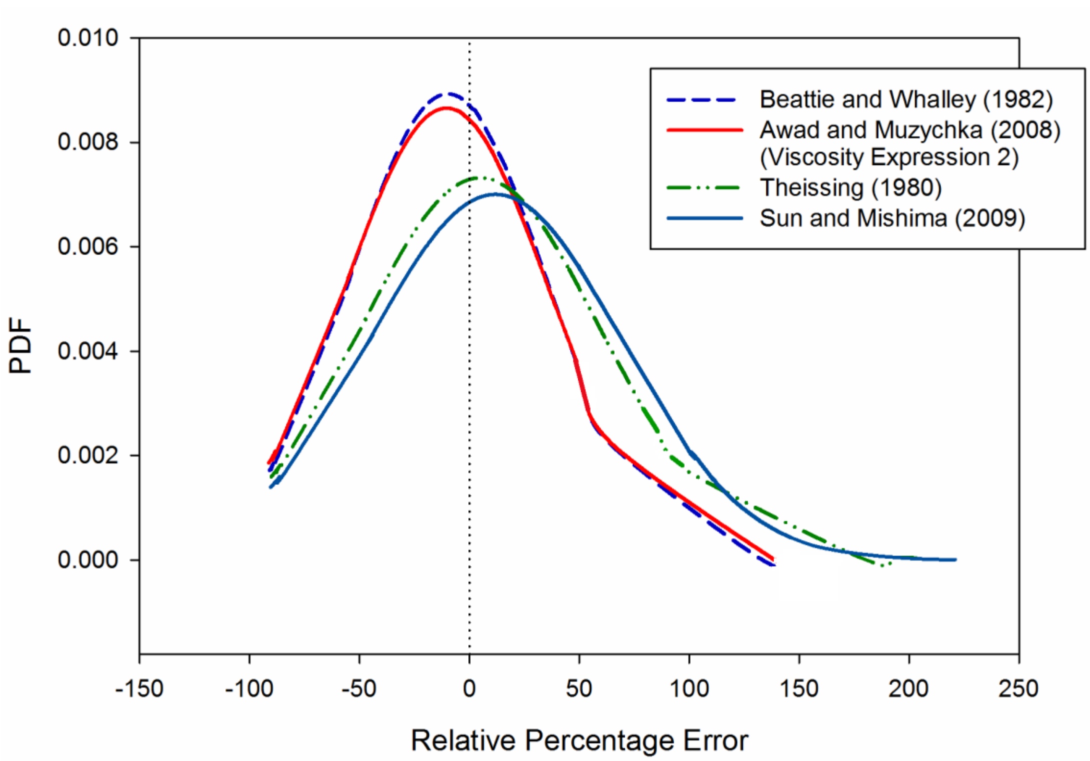

Figure 12: Probability density function for Hashizume (1983) R-22 data comparison

4.1.9 ***Comparison with the data of Bhattacharyya (1985)***

Bhattacharyya (1985) conducted air-water two-phase flow experiments in 25.4 mm (1 inch) diameter smooth and rough brass pipes. The smooth and the rough pipes have an absolute surface roughness of 1.50×10^{-3} mm and 8.89×10^{-3} mm, respectively.

The top correlation in best agreement with the smooth pipe data are homogeneous flow models. Table 22 shows that Beattie and Whalley (1982) predicted the maximum number of data points in the ±30% error band whereas Dukler et al. (1964) - (Case I) yielded the maximum accuracy in the ±15% error band. Figure 13 shows that Beattie and Whalley (1982) over predicted the data and Dukler et al. (1964) - (Case I) and Garcia et al. (2003) predicted most of the data with better deviation.

Table 22: Performance of best correlations for Bhattacharyya (1985) smooth pipe data

Selected Correlations	**Bhattacharyya (1985) Data (Air-Water) Total Points 178**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Beattie and Whalley (1982)	46.1%	78.1%	24.1	56.7
Dukler et al. (1964) (Case I)	51.1%	74.7%	3.0	45.0
Garcia et al. (2003)	49.4%	73.6%	0.4	40.7

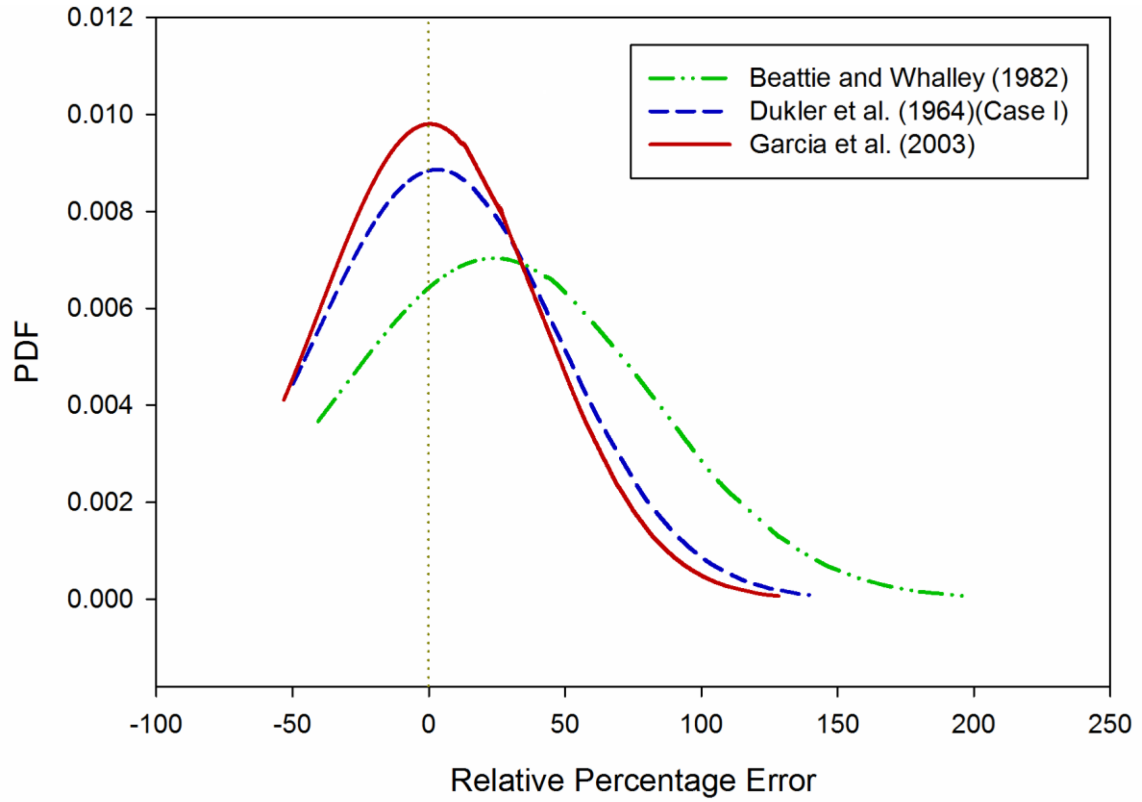

Figure 13: Probability density function for Bhattacharyya (1985) smooth pipe data

Grouping the smooth pipe data based on flow pattern yielded three groups. The authors reported the flow patterns for annular, stratified and a third group containing slug, plug, bubbly and mist flow all in one. Table 23 shows that slug, plug, bubble and mist flow patterns are well predicted by Muller-Steinhagen and Heck (1986) correlation with Beattie and Whalley (1982) being the second best. According to the results shown in Table 23, the stratified flow pattern is the least predicted. However, it has to be noted that the number of data points in the stratified regime is also very small. The maximum number of data points in the stratified flow region is predicted by Grønnerud (1979) correlation. The shaded entries in the table indicate the maximum percentage achieved for the specific flow pattern.

Table 23: Comparison based on flow pattern for Bhattacharyya (1985) smooth pipe data

Selected Correlations	**Bhattacharyya (1985) Smooth Pipe Data (Air-Water)**					
Flow Pattern	**Annular (46 pts.)**		**Bubbly, Mist, Slug & Plug, (110 pts.)**		**Stratified (22 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Dukler et al. (1964) (Case II)	34.8%	87.0%	13.6%	42.7%	9.1%	9.1%
Theissing (1980)	45.7%	84.8%	25.5%	49.1%	0.0%	9.1%
Muller-Steinhagen and Heck (1986)	26.1%	34.8%	69.1%	95.5%	0.0%	0.0%
Beattie and Whalley (1982)	52.2%	78.3%	52.7%	93.6%	0.0%	0.0%
Grønnerud (1979)	39.1%	52.2%	24.5%	60.9%	54.5%	77.3%

Table 24 shows the summary of performance comparison by grouping the smooth pipe data based on void fraction. It can be seen that no correlation performed well throughout the full range of void fraction. Most of the points in the void fraction range of 0.25-0.5 are predicted well by homogeneous models. Also it can be seen that the maximum number of points predicted in the other two void fraction ranges is relatively lower. The shaded entries in the table indicate the maximum percentage achieved for the specific void fraction range.

The rough pipe data is poorly predicted by all the correlations as compared to the smooth data. Table 25 shows that the maximum number of data points is predicted by Dukler et al. (1964) - (Case I) correlation. Figure 14 shows that most of the data points are over predicted by all the best performing correlations.

Table 24: Comparison based on void fraction for Bhattacharyya (1985) smooth pipe data

Selected Correlations	Bhattacharyya (1985) Smooth Pipe Data (Air-Water)					
Void Fraction Range	**0.25 - 0.50 (68 pts.)**		**0.50 - 0.75 (70 pts.)**		**0.75 - 1.00 (40 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Awad and Muzychka (2004b) (Homogeneous equations)	79.4%	92.6%	44.3%	64.3%	22.5%	50.0%
Awad and Muzychka (2008) (Viscosity Expression 1)	73.5%	92.6%	37.1%	67.1%	7.5%	12.5%
Chisholm (1967)	38.2%	88.2%	27.1%	75.7%	10.0%	30.0%
Theissing (1980)	13.2%	23.5%	32.9%	74.3%	42.5%	67.5%
Grønnerud (1979)	42.6%	85.3%	8.6%	27.1%	55.0%	77.5%
Awad and Muzychka (2004a) (General case)	16.2%	33.8%	35.7%	72.9%	37.5%	70.0%

Table 25: Performance of best correlations for Bhattacharyya (1985) rough pipe data

Selected Correlations	Bhattacharyya (1985) Data (Air-Water) Total Points 285		Mean	Standard Deviation
Percentage within	**±15%**	**±30%**		
Dukler et al. (1964) (Case I)	34.0%	63.2%	13.7	52.3
Awad (2007) (For regular size pipes)	37.2%	62.8%	-6.9	37.8
Garcia et al. (2003)	45.6%	62.5%	20.4	51.4

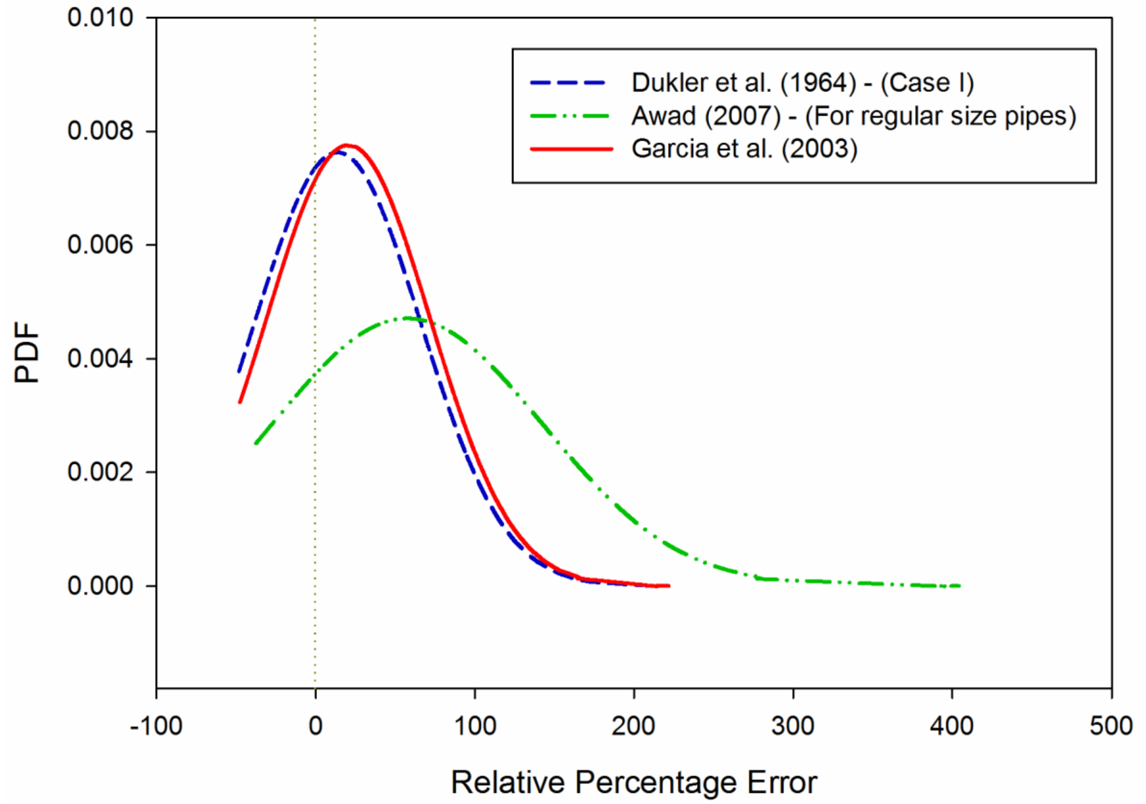

Figure 14: Probability density function for Bhattacharyya (1985) rough pipe data

Performance of the best correlations for each flow pattern is shown in Table 26. Similar to the smooth pipe data, the flow pattern group containing bubbly, mist, slug and plug flow patterns is predicted well. The maximum number of points predicted for annular flow regime is achieved by separated flow models. Stratified flow pattern is the least predicted flow pattern. Grønnerud (1979) correlation predicted 75.5% of the data points in the ±30% error band. Other than that, the rest of the correlations predicted less than 40% of the data for stratified flow. The shaded entries in the table indicate the maximum percentage achieved for the specific flow pattern.

Table 26: Comparison based on flow pattern for Bhattacharyya (1985) rough pipe data

Selected Correlations	**Bhattacharyya (1985) Rough Pipe Data (Air-Water)**					
Flow Pattern	**Annular (84 pts.)**		**Bubbly,Mist, Slug & Plug (148 pts.)**		**Stratified (53 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Dukler et al. (1964) (Case II)	47.6%	82.1%	14.2%	55.4%	3.8%	3.8%
Awad (2007) (For regular size pipes)	47.6%	72.6%	40.5%	67.6%	11.3%	34.0%
Beattie and Whalley (1982)	20.2%	36.9%	89.2%	97.3%	0.00%	0.00%
Awad and Muzychka (2004b) (Homogeneous equations)	15.5%	22.6%	92.6%	96.6%	0.00%	0.00%
Grønnerud (1979)	9.5%	21.4%	9.5%	45.3%	54.7%	75.5%

Table 27 summarizes the results of comparison based on void fraction for the rough pipe data. The void fraction range of 0.25-0.5 is predicted relatively better than the rest of the groups. It has to be noted that Awad and Muzychka (2004b) predicted bubbly, mist, slug and plug flow pattern group (in Table 26) and now as shown in Table 27, it is in best agreement with the void fraction of 0.25-0.5. Correlations that predicted the annular flow regime have also predicted the 0.75-1 void fraction range. These results indicate that there is a relationship between void fraction and flow pattern. This indication may help towards a new line of research to further investigate limitation of correlations based on void fraction or liquid hold up instead of flow pattern because flow patterns are usually subjective. The shaded entries in the table indicate the maximum percentage achieved for the specific void fraction range.

Table 27: Comparison based on void fraction for Bhattacharyya (1985) rough pipe data

Selected Correlations	**Bhattacharyya (1985) Rough Pipe Data (Air-Water)**					
Void Fraction Range	**0.25 - 0.50 (92 pts.)**		**0.50 - 0.75 (98 pts.)**		**0.75 - 1.00 (95 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Awad and Muzychka (2004b) (Homogeneous equations)	79.3%	80.4%	65.3%	70.4%	13.7%	20.0%
Awad (2007) (For regular size pipes)	64.1%	80.4%	6.1%	42.9%	43.2%	66.3%
Chisholm (1967)	70.7%	80.4%	62.2%	71.4%	10.5%	16.8%
Chisholm (1973)	77.2%	79.3%	50.0%	71.4%	14.7%	16.8%
Dukler et al. (1964) (Case II)	2.2%	37.0%	21.4%	50.0%	42.1%	73.7%
Awad (2007) (For regular size pipes)	64.1%	80.4%	6.1%	42.9%	43.2%	66.3%

Generally, the results of the comparison between the smooth and the rough pipe data indicate that the performance of the correlations is relatively lower for the rough pipe data. This indicates that the effect of pipe surface roughness has not been well taken care of by most of the correlations.

4.1.10 Comparison with the data of Andritsos (1986)

Andritsos (1986) conducted experiments with air-water and air-glycerol solutions in 25.2 mm (1 inch) and 95.3 mm (3.75 inch) pipes. To obtain detailed results, analysis is done separately based on fluid combination and pipe diameter in addition to flow pattern and void fraction groupings. Table 28 summarizes the results of the comparison for air-water data. The maximum number of points is predicted by Olujic (1985) correlation and the rest of the correlations predicted much lower number of data points. Figure 15 shows that most of the data points are predicted relatively better by Olujic (1985) correlation.

Table 28: Performance of correlations in best agreement with Andritsos (1986) air-water data

Selected correlations	Andritsos (1986) Data (Air-Water) Total Points 359		Mean	Standard Deviation
Percentage within	**±15%**	**±30%**		
Olujic (1985)	48.4%	72.7%	-11.2	67.3
Dukler et al. (1964) (Case II)	24.2%	45.4%	31.4	93.8
Awad and Muzychka (2004a) (General case)	19.2%	41.2%	82.0	179.6

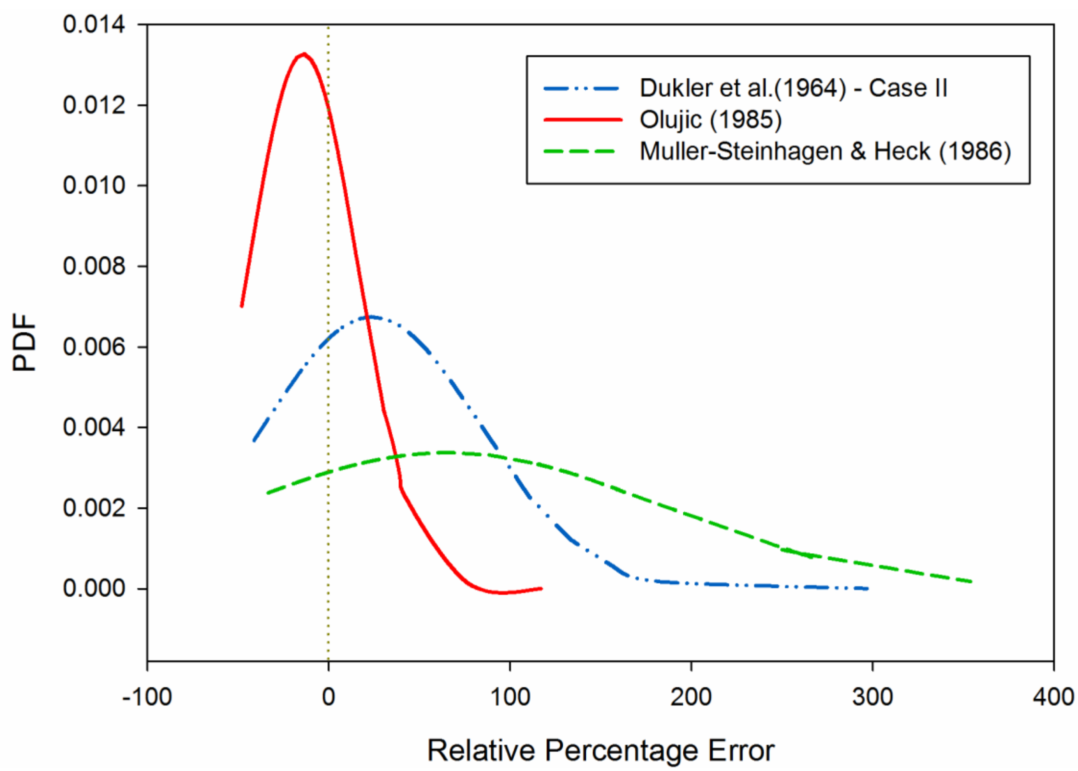

Figure 15: Probability density function for Andritsos (1986) air-water data

Grouping the air-water data based on flow pattern information yielded the results shown in Table 29. No single correlation is in best agreement with the data for all the flow patterns. Annular flow regime is predicted better than slug and stratified flow regimes. Muller-Steinhagen and Heck (1986) correlation predicted the annular data well both in the ±15% and ±30% error bands whereas the performance of Theissing (1980) correlation dropped in the ±15% error band. Similarly, Olujic (1985) predicted the slug flow pattern much better than the second best correlations in the ±15% error band. The shaded entries in the table indicate the maximum percentage achieved for the specific flow pattern.

Table 29: Comparison based on flow pattern for Andritsos (1986) air-water data

Selected Correlations	**Andritsos (1986) Data (Air-Water)**					
Flow pattern	**Annular (37 pts.)**		**Slug (36 pts.)**		**Stratified (215 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Muller-Steinhagen and Heck (1986)	64.9%	91.9%	5.6%	16.7%	4.7%	12.1%
Theissing (1980)	21.6%	91.9%	19.4%	50.0%	18.6%	27.0%
Olujic (1985)	27.0%	35.1%	72.2%	86.1%	40.9%	64.2%
Sun and Mishima (2009)	21.6%	43.2%	36.1%	77.8%	18.6%	33.5%

Sorting the air-water data based on void fraction values yielded the results shown in Table 30. The void fraction range of 0.5-0.75 is poorly predicted by all the correlations. A better performance is achieved in the 0.75-1 void fraction range. The maximum number of points predicted in 0.5-0.75 void fraction range is only 33% by Awad (2007) correlation. However, since the number of data points in 0.5-0.75 range is very small the conclusion for that range could not be reliable.

Table 30: Comparison based on void fraction for Andritsos (1986) air-water data

Selected Correlations	**Andritsos (1986) Data (Air-Water)**			
Void Fraction Range	**0.5 - 0.75 (30 pts.)**		**0.75 - 1 (329 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**
Olujic (1985)	10.0%	16.7%	39.5%	59.3%
Dukler et al. (1964) (Case II)	16.7%	20.0%	24.9%	47.7%

The performance of the correlations in best agreement with the air-glycerol solution is summarized in Table 31. The performance of Olujic (1985) correlation dropped down for air-glycerol data set as compared to its performance for the air-water data set. But for the other correlations the prediction remained more or less the same. Figure 16 shows that Olujic (1985) under predicted most of the data whereas Muller-Steinhagen and Heck (1986) over predicted.

Table 31: Performance of correlations in best agreement with Andritsos (1986) air-glycerol data

Selected Correlations	**Andritsos (1986) Data (Air-Water) Total Points 186**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Muller-Steinhagen and Heck (1986)	21.5%	45.2%	33.9	105.9
Dukler et al. (1964) - (Case II)	22.6%	44.1%	-8.5	61.8
Olujic (1985)	17.7%	43.0%	-33.5	39.4

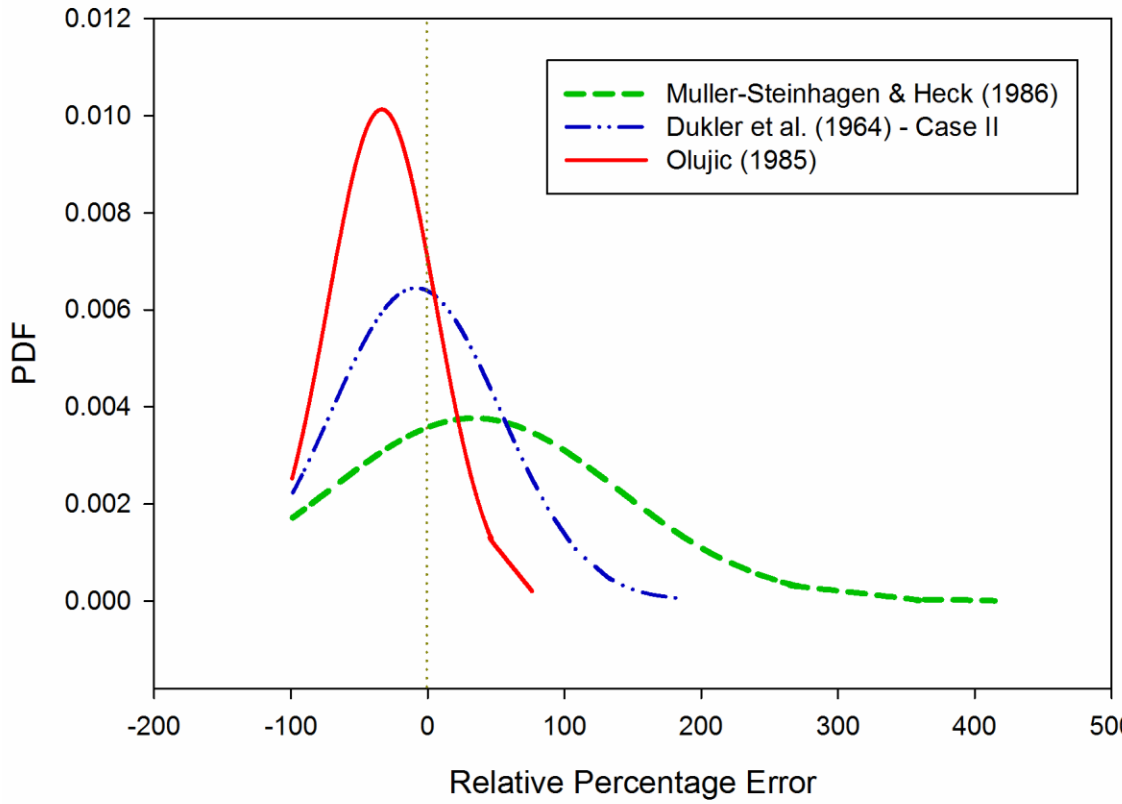

Figure 16: Probability density function for Andritsos (1986) air-glycerol data

Since the overall air-glycerol data is poorly predicted, further analysis based on flow pattern and void fraction is done in an effort to find if there is a meaningful reason for the low performance of the correlations. However, checking the flow pattern information indicated that all the points for air-glycerol fall in the stratified flow pattern. This could be the main reason for the low performance of the correlations because it has been observed from the other data sets that the performance of the correlations is relatively low in the stratified flow pattern.

Grouping the data points based on void fraction showed that all the data points have a void fraction greater than 0.5. Void fraction range of 0.75-1 is predicted slightly better than the 0.5-0.75 void fraction range even if the maximum number of points are still low (only 51% in the ±30% error band.). The maximum number of data points in 0.75-1 void fraction range is predicted by Muller-Steinhagen and Heck (1986) correlation followed by Dukler et al. (1964) - (Case II).

4.1.11 __Comparison with the data of Gokcal (2005)__

Gockal (2005) conducted experiments with high viscosity lube oil flowing with air in 50.8 mm (2 inch) diameter pipe. The oil has a dynamic viscosity of 159.4cP at 40ºC. Table 32 shows that Beggs and Brill (1973) correlation is in best agreement with the data. Figure 17 shows that Chisholm (1978) under predicted most of the data points and Awad (2007) – (Regular size pipes) over predicted the data.

Table 32: Performance of correlations in best agreement with Gockal (2005) data

Selected Correlations	**Gockal (2005) Data (Air-Oil) Total Points 183**		**Mean**	**Standard Deviation**
Percentage within	**±15%**	**±30%**		
Beggs and Brill (1973)	53.0%	79.2%	1.1	45.3
Awad (2007) (For regular size pipes)	55.2%	78.1%	21.6	85.6
Chisholm (1978)	28.4%	66.7%	-27.9	22.8

Summary of the results of comparison based on flow pattern is shown in Table 33. Plug and slug flow region is predicted better than the annular flow regime. However it has to be noted that the number of data points that fall in annular flow regime are very few.

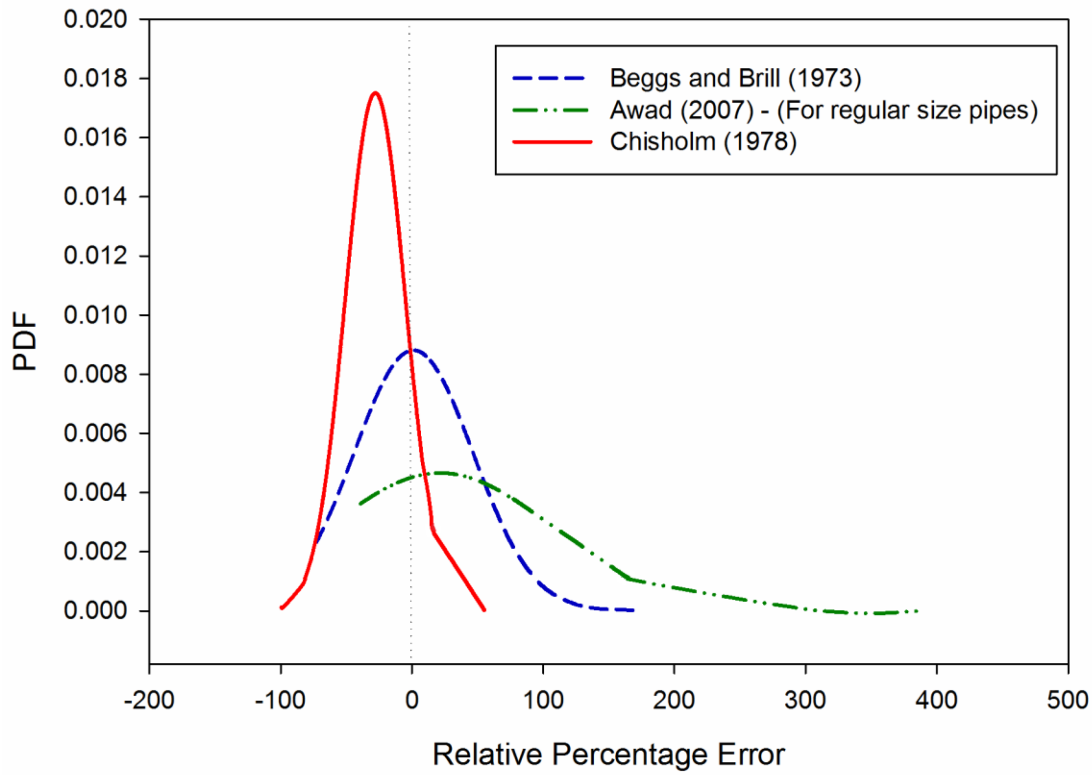

Figure 17: Probability density function for Gockal (2005) data

Table 33: Comparison based on flow pattern for Gockal (2005) data

Selected Correlations	**Gockal (2005) Data (Air-Oil)**			
Flow Pattern	**Annular (33 pts.)**		**Plug & Slug (147 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**
Cicchitti et al (1960)	36.4%	66.7%	2.0%	6.8%
Awad (2007) (For regular size pipes)	24.2%	51.5%	62.6%	85.0%
Beggs and Brill (1973)	30.3%	45.5%	59.2%	88.4%
Chisholm (1967)	33.3%	45.5%	51.0%	83.0%

Note: The shaded entries in the table indicate the maximum percentage of data points predicted for the specific flow pattern.

Summary of the results of comparison based on void fraction is shown in Table 34. Unlike air-water data presented in previous sections, most correlations predicted fair amount of data for more than two void fraction ranges. All data points within the void fraction range of 0.25-0.5 are predicted within the ±30% error band by the top four correlations. Void fraction range of 0.75-1 is the least predicted zone. The shaded entries in the table indicate the maximum percentage achieved in the specific void fraction range.

Table 34: Comparison based on void fraction for Gockal (2005) data

Selected Correlations	**Gockal (2005) Data (Air-Oil)**							
Void Fraction Range	**0 - 0.25 (20 pts.)**		**0.25 - 0.50 (36 pts.)**		**0.50 - 0.75 (66 pts.)**		**0.75 - 1.00 (61 pts.)**	
Percentage within	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**	**±15%**	**±30%**
Awad (2007) (For regular size pipes)	85.0%	95.0%	94.4%	100.0%	50.0%	83.3%	27.9%	54.1%
Chisholm (1967)	65.0%	95.0%	55.6%	100.0%	53.0%	80.3%	29.5%	47.5%
McAdams et al. (1942)	65.0%	95.0%	55.6%	100.0%	53.0%	80.3%	29.5%	47.5%
Beggs and Brill (1973)	45.0%	85.0%	72.2%	100.0%	71.2%	87.9%	24.6%	55.7%

4.2 Summary of Comparison Results

Several correlations have been mentioned to be in best agreement with certain data set or flow parameter such as void fraction range or flow pattern. However, it has to be noted that the main objective of this study is to suggest a correlation that is capable of accurately predicting two-phase flow pressure drop for a wide range of flow conditions.

For most of the data, the pressure drop correlations showed poor performance within the narrow error index of ±15%. This may not be due to the weaknesses of the correlations alone but the accuracy of the data sets which for most of the cases was not explicitly reported and should also be questioned. It was observed that most correlations were in best agreement with the data on which they were developed. This may be due to the fact that some parameters has to be determined experimentally making the correlations more or less empirical. More research and understanding would eventually lead to less empirically determined constants.

The performance of all correlations generally improved as the percentage error index was relaxed. But no correlation predicted all the data bases accurately within the ±15% or ±30% error bands. Instead results show that for the 11 data sets over 18 correlations have been mentioned to be in best agreement with specific flow phenomena. This indicates that most of the correlations are suitable for specific range of application. This would also mean a result of any correlation performance comparison depends on the type of data at hand. Therefore further investigation of the selected correlations against a greater number of data sets is recommended.

Although all data sets could not be predicted well by a single correlation, some correlations predicted more data sets better than the rest. Table 35 and Table 36 show the three correlations that predicted the maximum number of data sets within the ±30% error band. Dukler et al. (1964) - (Case II) predicted the highest number of the data sets followed by Theissing (1980) correlation. These two correlations could be a good starting point for a researcher who is interested to develop a two-phase pressure drop correlation that is capable of handling a wide range of flow conditions.

Table 35 and Table 36 show only the data bases where the top correlations were found to be the best among the other correlations. On the other hand, looking at the nature of the experimental data bases that are not mentioned in the tables gives clues where the performance of the top correlations declined. Information on the characteristics of the experimental data bases is presented in Appendix A.

Dukler et al. (1964) - (Case II) correlation performed well for all the data where the liquid component is water. The correlation also gave a consistent result for all the diameters of the air-water flow. However the performance declined for fluid combinations other than air-water data.

Theissing (1980) correlation over predicted the 101.6mm (4 inch) and 152.4mm (6 inch) pipe diameter air-water data of Reid et al. (1957). Therefore more data of large pipe diameter is required to validate the performance of Theissing (1980) correlation for over 2 inch diameter pipes.

Theissing (1980) and Dukler et al. (1964) - (Case II) correlations predicted Wicks (1958) data with an accuracy of 86.7% and 78.7% within the ±30% error band, respectively. However, since other correlations predicted that data base better than these two correlations, this data base is not is not shown in Table 35 and 36. Checking the nature of the data base indicated that Wicks (1958) data has very high gas superficial velocity. Dukler et al. (1964) - (Case II) is based on homogeneous flow model; therefore, lower performance is expected where large velocity difference between the phases exist. Modification of the correlation to incorporate larger velocity differences may help fix this issue.

High pressure refrigerant boiling data is not predicted well by most of the correlations in this study. This may be due to the fact that most of the correlations were developed based on experimental data close to atmospheric conditions and larger pipe diameters. On the other hand also, mini and micro channel correlations failed to predict the regular size pipe data. This clearly indicates there should be a clear diameter specification indicating the allowable diameter ranges for a certain correlation.

Most of the correlations could not predict rough surface pipe data. This indicates that the pipe surface roughness is not well taken care of for wider ranges. Usually two phase pressure drop correlations account for the pipe surface through the single phase pressure drop calculations. This would also mean the accuracy of single phase friction factor correlation that is used along with the two phase pressure drop has to be investigated further. Variation of prediction performances as much as 5% has been observed by changing the single phase friction factor correlation.

Very complex correlations and correlations that are developed for a very specific flow condition did not show greater superiority in prediction performance. Rather, there is a high risk of staying within the limiting range of the correlations. This could result in very large errors. Therefore, for practical purposes where a significant accuracy is not required, using the correlations suggested for the wide range of flow conditions is advisable instead of taking the risk.

For a reader who is interested only in a narrow range of applications, Tables 37, Table 38 and Table 39 may provide guiding information to select the most suitable correlation for a more specific set of flow conditions. To avoid ambiguity of flow patterns, recommendation is presented based on void fraction range. The correlations listed in the tables are the ones that predicted the maximum number of data points in the ±30% error band. Unless maximum accuracy is required, using the best correlations that are recommended for the wide range of flow conditions is recommended because they tend to give more consistent results over larger ranges.

Overall summary of the study and conclusions are presented in the next chapter. Recommendations based on the findings of this research are also summarized and forwarded.

Table 35: Data sets predicted by Dukler et al. (1964) - (Case II) correlation

Correlation	Experimental data	Fluid Combination	Pipe Diameter	System Pressure [kPa]	Void Fraction Range
Dukler et al. (1964) (Case II)	Bhattacharyya (1985)	Air-Water	25.4 mm (1 inch)	108.2-218.5	0.19-0.95
	Andritsos (1986)		25.2 mm (1 inch) 95.3 mm (4 inch)	98.6-196	0.56-0.99
	Beggs (1972)		25.4mm (1 inch) 38.1mm (1.5 inch)	357.9-679.7	0.20-0.97
	Nguyen (1975)		45.5mm (1.79 inch)	99.7-114.7	0.13-0.99
	Chen (1979)		45.5mm (1.79 inch)	101.3-120.6	0.32-0.98
	Reid et al. (1957)		101.6mm (4 inch) 152.4mm (6 inch)	146.8-210.9	0.40-0.80
	Gregory and Scott (1969)	CO_2-Water	19.05 mm (3/4 inch)	98.1-118.4	0.47-0.84
	Andritsos (1986)	Air-Glycerol soln.	25.2 mm (1 inch) 95.3 mm (3.75 inch)	98.6-123	0.47-0.98

Table 36: Data sets predicted by Theissing (1980) and Sun and Mishima (2009) correlations

Correlation	Experimental data	Fluid Combination	Pipe Diameter	System Pressure [kPa]	Void Fraction Range
Theissing (1980)	Bhattacharyya (1985)	Air-Water	25.4 mm (1 inch)	108.2-218.5	0.19-0.95
	Beggs (1972)		25.4mm (1 inch) 38.1mm (1.5 inch)	357.9-679.7	0.20-0.97
	Nguyen (1975)		45.5mm (1.79 inch)	99.7-114.7	0.13-0.99
	Chen (1979)		45.5mm (1.79 inch)	101.3-120.6	0.32-0.98
	Gregory and Scott (1969)	CO_2-Water	19.05 mm (3/4 inch)	98.1-118.4	0.47-0.84
	Hashizume (1983)	R-12	10 mm (0.39 inch)	570-1,220	0.41-0.95
		R-22	10 mm (0.39 inch)	920-1,960	0.38-0.95
Sun and Mishima (2009)	Wicks (1958)	Air-WAter	25.4mm (1 inch)	101.5-118.2	0.86-0.96
	Beggs (1972)		25.4mm (1 inch) 38.1mm (1.5 inch)	357.9-679.7	0.20-0.97
	Nguyen (1975)		45.5mm (1.79 inch)	99.7-114.7	0.13-0.99
	Reid et al. (1957)		101.6mm (4 inch) 152.4mm (6 inch)	146.8-210.9	0.40-0.80
	Hashizume (1983)	R-12 data only	10 mm (0.39 inch)	570-1,220	0.41-0.95

Table 37: Best performing correlations for air-water flow within the indicated flow parameters

Fluid Property	Pipe Diameter	System Pressure [kPa]	Void Fraction Range	Correlation
Air-Water	25.4 mm (1 inch)	108.2-218.5	0.25-0.5	Awad and Muzychka (2004b) (Homogeneous equations)
			0.5-0.75	Theissing (1980)
			0.75-1	Sun and Mishima (2009)
	38.1mm (1.5 inch)	357.9-679.7	0.5-1	Theissing (1980)
	45.5mm (1.79 inch)	99.7-114.7	0-0.25	Bankoff (1960)
			0.25-0.5	Chisholm (1967)
			0.5-0.75	Wallis (1969)
			0.75-1	Theissing (1980)
	95.3mm (3.75 inch)	98.6-164	0.75-1	Olujic (1985)
	101.6mm (4 inch)	146.8-210.9	0.5-0.75	Sun and Mishima (2009)
	152.4mm (6 inch)	146.8-210.9	0.5-0.75	Dukler et al. (1964) - (Case II)

Table 38: Best performing correlations for different fluid combinations and within the indicated flow parameters

Fluid Combination	**Pipe Diameter**	**System Pressure [kPa]**	**Void Fraction Range**	**Correlation**
CO_2-Water	19.05 mm (3/4 inch)	98.1-118.4	0.5-0.75	Dukler et al. (1964) - (Case II)
			0.75-1	Theissing (1980)
Air-Glycerol	25.2 mm (1 inch)	98.6-123	0.5-0.75	Dukler et al. (1964) - (Case I)
			0.75-1	Muller-Steinhagen and Heck (1986)
	95.3 mm (4 inch)	98.6-123	0.75-1	Dukler et al. (1964) - (Case II)
Air-Kerosene	38.1mm (1.5 inch)	194.4-633.6	0.25-0.5	Chisholm (1978)
			0.5-0.75	McAdams et al. (1942)
			0.75-1	McAdams et al. (1942)
Air-Oil (51.8 cP @ 40°C)	38.1mm (1.5 inch)	275.1-616.4	0-0.25	Garcia et al. (2003)
			0.5-0.75	Awad and Muzychka (2008) (Viscosity Expression 4)
			0.75-1	Muller-Steinhagen and Heck (1986)

Table 39: Best performing correlations for different fluid combinations and within the indicated flow parameters

Fluid Combination	Pipe Diameter	System Pressure [kPa]	Void Fraction Range	Correlation
Air-Oil (159.4 cP @ 40 °C)	50.8 mm (2 inch)	99-102	0-0.25	Awad (2007) (For regular size pipes)
			0.25-0.5	Beggs and Brill (1973)
			0.5-0.75	Beggs and Brill (1973)
			0.75-1	Cicchitti et al. (1960)
R-12 (Boiling flow)	10 mm (0.39 inch)	570-1,220	0.5-0.75	Beattie and Whalley (1982)
			0.75-1	Muller-Steinhagen and Heck (1986)
R-22 (Boiling flow)	10 mm (0.39 inch)	920-1,960	0.25-0.5	Chisholm (1973)
			0.5-0.75	Beattie and Whalley (1982)
			0.75-1	Beattie and Whalley (1982)

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary and Conclusions

An extensive comparison of 42 two-phase frictional pressure drop correlations are made against experimental pressure drop data collected from different sources. A total of 2,429 data points of experimental pressure drop measurements from eleven different sources are considered. The best performing correlations for each data set are indicated using error bands and probability density function plots.

The main objective of this study was to identify a correlation or group of correlations that could predict experimentally measured pressure drop accurately for a wide range of flow conditions. However, instead of rushing to validate correlations for many flow parameters at once, it was believed better results could be achieved by focusing on specific set of flow conditions. Then building the investigation block by block would yield a much better result in achieving the goal of identifying or developing a correlation that is capable of handling a wide range of flow conditions.

A good deal of time and effort has been spent in searching, collecting and organizing literature for much wider application beyond isothermal and horizontal two phase flow.

The soft copies of the documentation for this study contain well organized literature and computer codes that will considerably simplify and speed up the next level of investigation that could gear towards evaluation of more correlations over a wider flow conditions or to improve/ develop correlations.

Experimental data base that features a wide and diverse flow conditions for isothermal horizontal two-phase flow has been compiled and presented in Chapter 2. The experimental data base covers wide range of pipe diameters, system pressures as well as several types of fluid combinations.

The pressure drop correlations collected from the open literature were briefly presented in Chapter 3 along with review of previously done comparison works to investigate the performance of the pressure drop correlations. Review of previously done comparisons revealed that different correlations are suggested for use by different authors. This could be due to the different data set the authors used to validate the correlations. In addition to the experimental data, there is also a difference on correlations suggested for a specific flow pattern. This could be due to the fact that identifying flow patterns visually or by using flow pattern maps is still resulting in ambiguous definitions of flow patterns. This also indicates there is a need to establish a unified frame of reference to evaluate the performance of correlations.

Computer codes were written using Engineering Equation Solver Program (EES). The program has a capability of solving sophisticated set of equations over a large number data with many parameters. Built in mathematical and thermo-physical fluid property functions make it very suitable for this kind of study. Parametric Table features of the program facilitate a quick learning and understanding of the two-phase flow problem by doing parametric analysis of the problem. Besides, the program is capable of handling and organizing great deal of data at once. A sample code written in EES is shown in Appendix B.

Detailed results are generated and presented in Chapter 4 with the help of statistical tables and plots. The overall best performing correlations that predicted most of the experimental data bases within the ±30% error band as well as their relative consistency in performance were highlighted. Appropriate recommendations were given accordingly.

Pressure drop correlations suitable for general flow condition of isothermal horizontal two-phase flow are recommended. Dukler et al. (1964) - (Case II) correlation was found to be in best agreement with 6 of the 7 air-water data bases. Separated flow models by Theissing (1980) and Sun and Mishima (2009) turned out to be the second and third best correlation for the experimental data bases considered in this study. Moreover, for a reader who is interested in more specific flow conditions, a number of correlations have been suggested based on several flow parameters including void fraction ranges.

5.2 Recommendations

This study was made with a bigger plan in mind to expand it beyond isothermal two-phase horizontal flow. Provisions have been made to make it easily adaptable for expansion of the research in the future. More experimental data with more flow parameters such as inclined and vertical two-phase flows can be easily accommodated in the study. More correlations can be quickly added and evaluated against the existing experimental data base.

Due to limited availability of literature, the experimental data is more biased to air-water flow. More data with more diverse fluid combinations would give more insight to the dependency of the two-phase pressure drop on physical properties like viscosity and surface tension.

Understanding of void fraction and flow pattern is very essential in two-phase flow pressure drop. Most correlations require input of void fraction or flow pattern information at some point during the calculation. Errors in void fraction accuracy or identifying the correct flow pattern would often result in an erroneous pressure drop prediction. This implies that more improved void fraction or flow pattern identification correlations would result in better two-phase pressure drop correlations.

The performance of each correlation was found to vary throughout the different void fraction ranges. And also, it has been noted that the performance of the correlations are usually dependent on flow patterns. A brief investigation during the study showed that there is a relationship between flow pattern and void fraction range. Therefore to avoid the usual ambiguity in flow pattern definition, defining the performance of correlations based on void fraction ranges is suggested. Void fraction has advantage over flow pattern because flow patterns have discontinuity and also the transition zones between flow patterns are not defined well with the current knowledge of two-phase flow. Moreover, it is believed that more research and development of a relationship between void fraction and flow pattern would contribute a great deal in improving performance of the two-phase pressure drop correlations.

This study could be used as a starting point to develop or improve two-phase flow correlations for a specific or wide range of applications. Further investigation can be done to find out the strength and weakness of the correlations by adding more experimental data sets or more correlations. Moreover, this study can also be easily extended to inclined and vertical flows using the documents and computer codes prepared for this research.

In general, review of the two-phase pressure drop correlations and study of the previously done comparison works indicated that validation of two-phase frictional pressure drop correlations is a task that has to be done on a continuous basis in order to identify or develop correlations with improved performance.

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APPENDIX A

CHARACTERSTICS OF THE EXPERIMENTAL DATA

Summary of the characteristics of each of the experimental data base is presented in Table A1. The table includes data set source, pipe diameter, total data points, fluid combinations and void fraction ranges. Moreover, the minimum and maximum values for parameters like pressure and temperature along with the minimum and maximum ranges of the superficial velocity of each phase is also given.

Figure A1 and Figure A2 give a visual clue of the fluid combination and pipe diameter proportions in the experimental data base. Air-water fluid combination takes the major portion of the data. This is due to the fact that most of the two-phase flow experiments available in the open literature are for air-water flow.

Table A-1: General characteristics of the experimental data bases

No.	Experimental data	Fluid Combination	Pipe Diameter	Pressure Range [kPa]	U_{sl} [m/s]	U_{sg} [m/s]	Void Fraction Range	No. of Data Points
1	Reid et al. (1957)	Air-Water	101.6mm (4") 152.4mm (6")	146.8-210.9	0.67-1.74	1.26-15.23	0.40-0.80	43
2	Wicks (1958)	Air-Water	25.4mm (1")	101.5-118.2	0.13-0.78	23.6-112.3	0.86-0.96	225
3	Gregory and Scott (1969)	CO_2-Water	19.05 mm (3/4")	98.1-118.4	0.22-0.78	0.97-6.98	0.47-0.84	109
4	Beggs (1972)	Air-Water	25.4mm (1") 38.1mm (1.5")	357.9-679.7	0.23-2.62	0.31-24.96	0.20-0.97	58
5	Nguyen (1975)	Air-Water	45.5mm (1.79")	99.7-114.7	0.001-1.04	0.096-65.47	0.13-0.99	250
6	Chen (1979)	Air-Water	45.5mm (1.79")	101.3-120.6	0.004-1.232	0.51-53.5	0.32-0.98	293

Table A-1 (Contd.): General characteristics of the experimental data bases

No.	Experimental data	Fluid Combination	Pipe Diameter	Pressure Range [kPa]	U_{sl} [m/s]	U_{sg} [m/s]	Void Fraction Ranges	No. of Data Points
7	Mukherjee (1979)	Air-Kerosene	38.1mm (1.5")	194.4-633.6	0.016-4.13	0.23-24.06	0.21-0.96	58
		Air-Oil	38.1mm (1.5")	275.1-616.4	0.027-3.03	0.09-21.14	0.14-0.95	32
8	Hashizume (1983)	R-12	10 mm (0.39")	570-1,220	0.014-0.263	0.113-4.41	0.41-0.95	85
		R-22	10 mm (0.39")	920-1,960	0.015-0.3	0.1-5.19	0.38-0.95	85
9	Bhattacharyya (1985)	Air-Water	25.4 mm (1")	108.2-218.5	0.095-3.3	0.29-40.2	0.19-0.95	463
10	Andritsos (1986)	Air-Water	25.2 mm (1") 95.3 mm (3.75")	98.6-196	0.0014-0.33	0.81-163.2	0.56-0.99	359
		Air-Glycerol	25.2 mm (1") 95.3 mm (3.75")	98.6-123	0.002-0.07	2.02-32.7	0.47-0.98	186
11	Gockal (2005)	Air-Oil	50.8 mm (2")	99-102	0.01-1.76	0.09-20.3	0.11-0.97	183

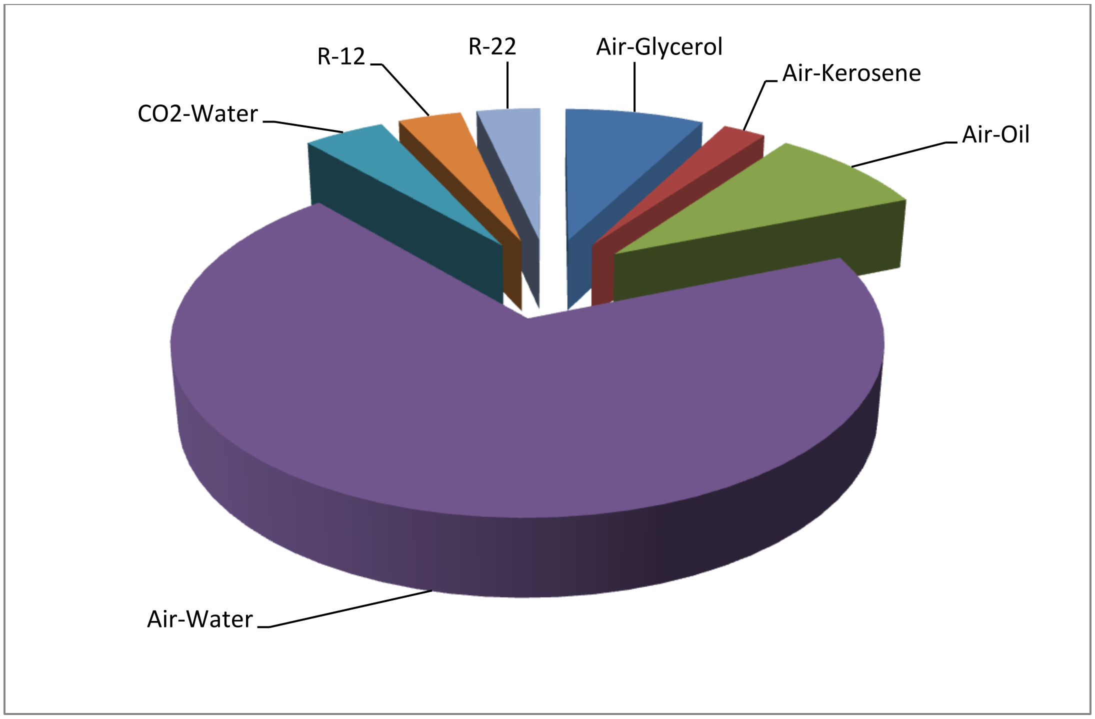

Figure A-1: Fluid combinations proportion of the experimental data base

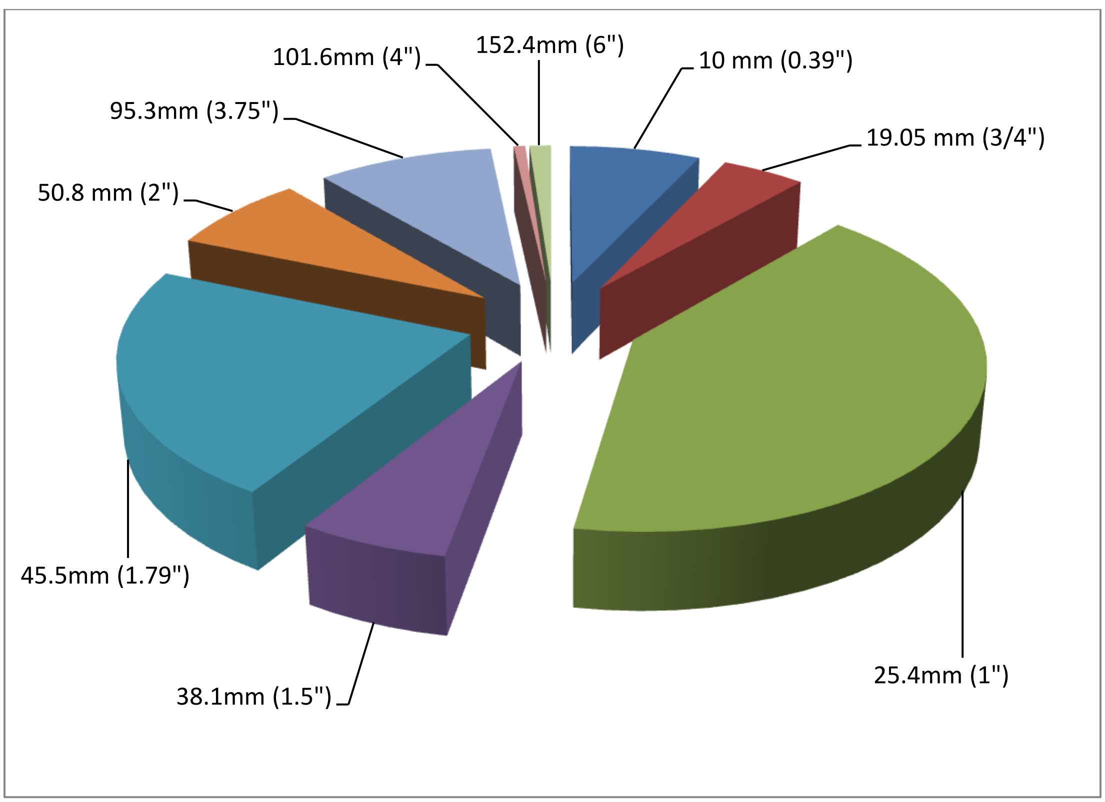

Figure A-2: Grouping of the experimental data base based on pipe diameter

APPENDIX B

SAMPLE OF EES PROGRAM CODE

Engineering Equation Solver computer program (EES) was chosen for this study. A sample code from the study is shown here.

```
"Two-phase pressure drop calculation using Thessing (1980) correlation"

"Input-Known Flow Parameters - From Parametric Table"

"D=0.0455"                                   "Internal pipe diameter”
"e=0"                                        "Pipe surface absolute roughness"

"T_avg=20"                                   "Average Temperature"
"P_sys=101325"                               "System Pressure"

"m_dot_l=0.001736111"                        "Mass flow rate of the liquid"
"m_dot_g=0.000255"                           "Mass flow rate of the gas"

"mu_g=Viscosity(Air,T=T_g)"                  "Dynamic viscosity of the gas"
"mu_l=Viscosity(Water,P=P_sys,T=T_l)"        "Dynamic viscosity of the Liquid"

"rho_g=Density(Air,T=T_g,P=P_sys)"           "Density of the gas"
"rho_l=Density(Water,T=T_l,P=P_sys)"         "Density of the liquid"

"Sigma_l=SurfaceTension(Water,T=T_avg)"      "Surface tension of the liquid"

"Calculation of Basic Parameters"
A=(pi#*(D^2))/4                              "Area of the pipe cross-section"
```

```
"Calculate Mass Fluxes"
G_l=m_dot_l/A                         "Mass flux of the liquid"
G_g=m_dot_g/A                         "Mass flux of the liquid"
G_tp=G_l+G_g                          "Two-phase mixture mass flux"

"Calculate Reynolds number, Re"
Re_l=(G_l*D)/mu_l                     "Liquid Reynolds number"
Re_g=(G_g*D)/mu_g                     "Gas Reynolds number"

Re_lo=(G_tp*D)/mu_l                   "Liquid only Reynolds number"
Re_go=(G_tp*D)/mu_g                   "Gas only Reynolds number"

"Calculate superficial velocities"
U_sl=m_dot_l/(rho_L*A)                "Superficial liquid velocity"
U_sg=m_dot_g/(rho_g*A)                "Superficial liquid velocity"
U_tp=U_sg+U_sl                        "Average mixture velocity"

x=m_dot_g/(m_dot_l+m_dot_g)           "Flow quality, x"


"Start Pressure Drop Calculation Procedures"

"Churchill equation-for single phase friction factor"

"Single phase friction factor for liquid phase"
B1= (-2.457*(ln((((7/Re_l)^0.9)+ (0.27*(e/d)) ))))^16
B2= (37530/Re_l)^16
f_l=8*((((8/Re_l)^(12))+(1/((B1+B2)^1.5)))^(1/12))

"Single phase friction factor for gas phase"
B3 = (-2.457*(ln((((7/Re_g)^0.9)+ (0.27*(e/d)) ))))^16
B4= (37530/Re_g)^16
f_g=8*((((8/Re_g)^(12))+(1/((B3+B4)^1.5)))^(1/12))

"Single phase friction factor for liquid only flow"
B5= (-2.457*(ln((((7/Re_lo)^0.9)+ (0.27*(e/d)) ))))^16
B6= (37530/Re_lo)^16
f_lo=8*((((8/Re_lo)^(12))+(1/((B5+B6)^1.5)))^(1/12))
```

```
"Single phase friction factor for liquid only flow"
B7= (-2.457*(ln((((7/Re_go)^0.9)+ (0.27*(e/d)) ))))^16
B8= (37530/Re_go)^16
f_go=8*((((8/Re_go)^(12))+(1/((B7+B8)^1.5)))^(1/12))

"Calculate single phase pressure drops"
DELTAP_l=(f_l*G_l^2)/(2*D*rho_l)              "Liquid Only pressure drop"
DELTAP_g=(f_g*G_g^2)/(2*D*rho_g)              "Gas Only pressure drop"

DELTAP_lo=(f_lo*G_tp^2)/(2*D*rho_l)           "Liquid Only pressure drop"
DELTAP_go=(f_go*G_tp^2)/(2*D*rho_g)           "Gas Only pressure drop"

"Two phase multiplier exponent"
n=(n_1+n_2*(DELTAP_g/DELTAP_l)^0.1)/(1+(DELTAP_g/DELTAP_l)^0.1)
n_1=ln(DELTAP_l/DELTAP_lo)/ln(1-x)            "Ratio of Liquid Pressure Drops"
 n_2=ln(DELTAP_g/DELTAP_go)/ln(x)             "Ratio of Gas Pressure Drops"

"Two phase friction multiplier"
psi=3-2*((2*sqrt(rho_l/rho_g))/(1+(rho_l/rho_g)))^(0.7/n)

"Two phase flow pressure drop gradient"
DELTAP_perLength=(DELTAP_lo^(1/(n*psi))*(1-x)^(1/psi) +
                      DELTAP_go^(1/(n*psi))*x^(1/psi))^(n*psi)

"End of pressure drop calculation"
```

APPENDIX C

EXPLANATION OF DATA ANALYSIS

Probability density function plots of the error distribution were done by assuming the error distribution to be Gaussian. Statistical analysis was done for the first data set (Reid et al., 1958) in the study. Detailed data analysis has been done using the Descriptive Statistics tool in Excel application.

Gaussian distribution will have a Skewness equal to zero and also a Kurtosis equal to zero. Table C-1 shows that the Descriptive Statistics for the data are very close to a good Gaussian distribution. The Skewness values are all small, so it is safe to assume the errors to be symmetric. The Kurtosis for the Awad and Cicchitti data are just a little bit high. Kurtosis (also called Flatness Factor) is a measure of the relative width of the distribution - narrow or wide.

Figures from C-1 up to C-4 present histograms for each of the correlations in best agreement with the data. Looking at the histograms, it can be seen that a Gaussian distribution is an acceptable means of representing the error distribution.

Table C-1: Histogram for McAdams et al. (1942) correlation with Reid et al. (1958) data

Statistical Parameter	**Awad & Muzychka (2007)**	**Cicchitti et al. (1960)**	**McAdams et al. (1942)**	**Sun & Mishima (2009)**
Mean	-0.37	-1.35	-8.63	7.06
Standard Error	2.49	2.25	1.69	1.46
Median	1.48	-1.94	-7.70	6.90
Standard Deviation	16.35	14.76	11.11	9.59
Sample Variance	267.41	217.75	123.33	92.04
Kurtosis	-1.03	-0.85	-0.56	-0.22
Skewness	-0.23	0.12	0.09	0.36
Range	59.38	55.36	44.18	42.94
Minimum	-30.72	-27.88	-29.02	-11.45
Maximum	28.66	27.48	15.16	31.50
Sum	-15.84	-58.21	-371.14	303.52

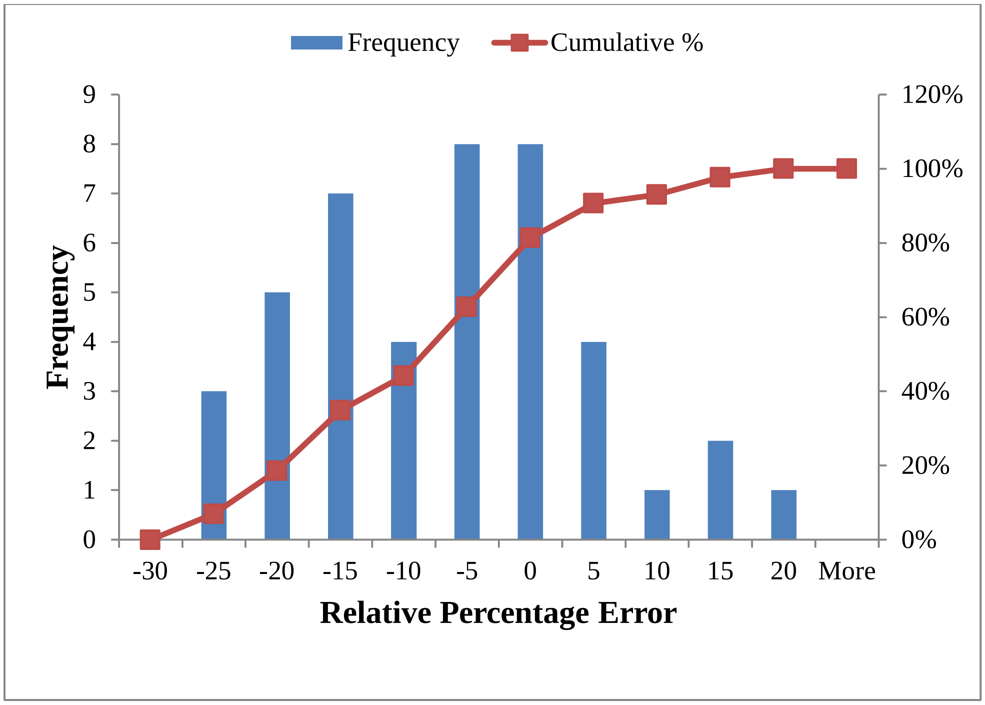

Figure C-1: Histogram for McAdams et al. (1942) correlation

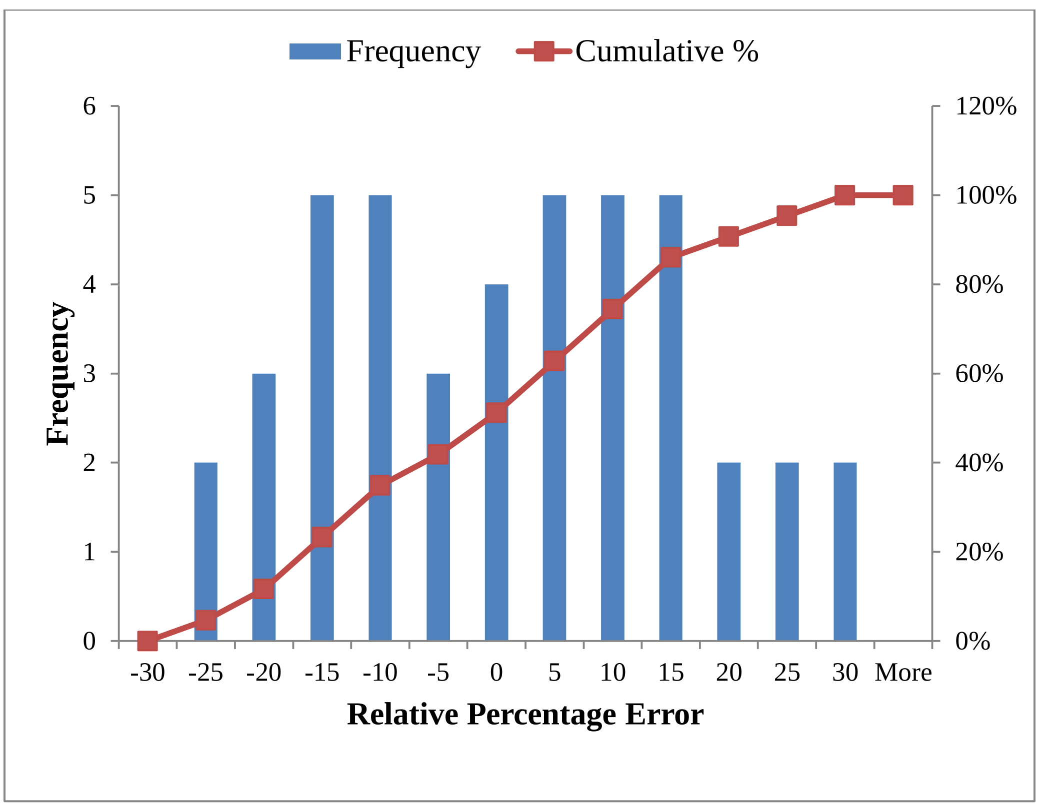

Figure C-2: Histogram for Cicchitti et al. (1960) correlation

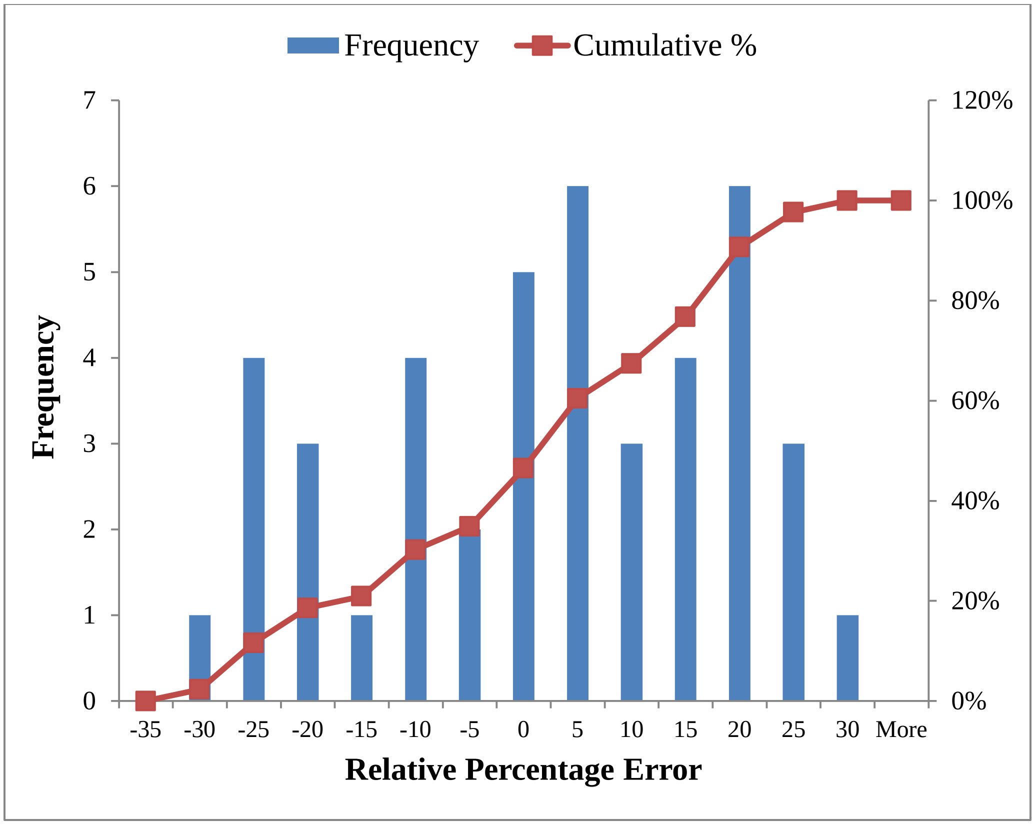

Figure C-3: Histogram for Awad & Muzychka (2007) correlation

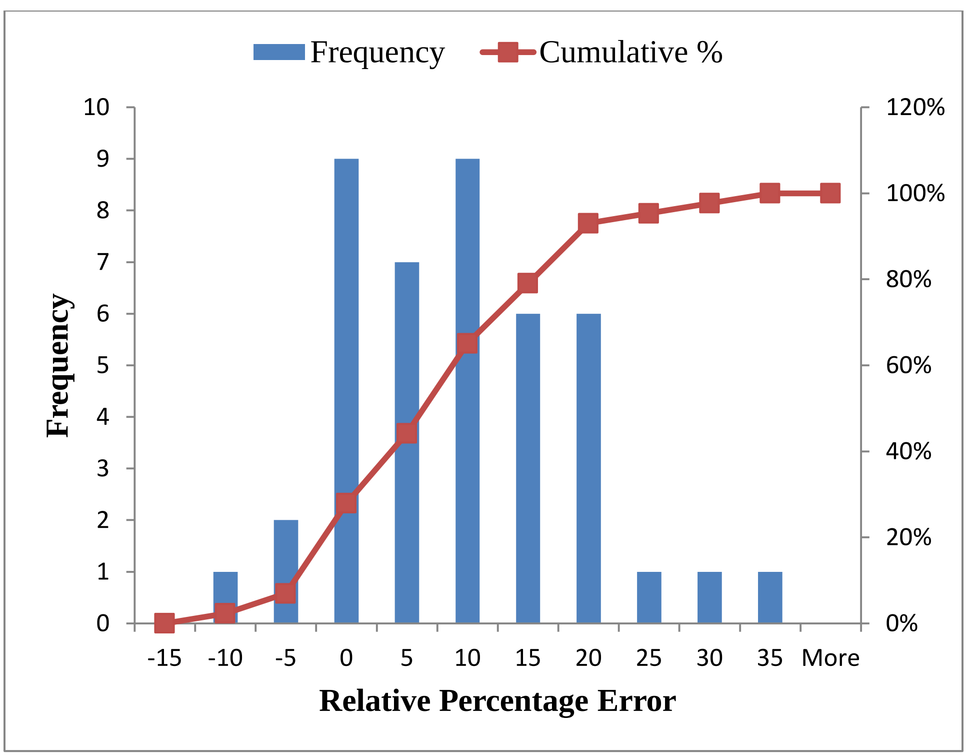

Figure C-4: Histogram for Sun and Mishima (2009) correlation

APPENDIX D

MERGED RESULTS BASED ON VOID FRACTION RESULTS

Detailed results of the correlation comparison for each data set based on void fraction groups have been presented in Chapter 4. However, for a reader who is interested only in the bigger picture, all the results comparison based on void fraction have been merged in one table.

Table D-1, presents the correlations that performed well for the specified void fraction ranges. Correlations and the type of fluid combination the result was observed is presented in the tables. Since other parameters such as diameter and pressure are excluded in this table, correlations that are reported in this table are only the ones that predicted over 70% of the data in the ±30% error band for specific group of void fraction. Correlations with less than 70% prediction performance are excluded for safer and more accurate report when important parameters such as diameter and system pressure are ignored.

Homogeneous correlations are indicated in bold fonts. It can be seen that both homogeneous and separated flow model correlations appear equally in the list for all void fraction ranges. The fluid combinations are abbreviated as follows: [AG]: air-glycerol, [AK]: air-kerosene, [AO]: air-oil, [AW]: air-water, [CO_2-W]: Carbon dioxide-water, and [R12]: Refrigerant-12.

Table D-1: Merged results of comparison based on void fraction

Void Fraction Range	**Correlation - [Fluid Combination]**
0-0.25	Awad (2007)- (For regular size pipes) - [AO], **Bankoff (1960) - [AW],** Chisholm (1967) - [AO], **Garcia et al. (2003) - [AO]**
0.25-0.5	**Awad and Muzychka (2004b) - (Homogeneous equations) -[AW],** Awad (2007)- (For regular size pipes) - [AW, AO] **Awad and Muzychka (2008) - (Viscosity Expression 4)-[AW, AO],** Bankoff (1960) - [AW], **Beggs and Brill (1973) - [AO],** Chisholm (1967) - [AO], Chisholm (1978) - [AK], **McAdams et al. (1942) - [AO]**
0.5-0.75	Awad (2007)- (For regular size pipes) - [AW, AO] **Awad and Muzychka (2008) - (Viscosity Expression 4) - [AO],** **Beggs and Brill (1973) - [AO, R12],** Chisholm (1967) - [AW, AO], Chisholm (1973) - [AW], **Cicchitti et al (1960) - [AW, R12],** **Dukler et al. (1964) - (Case II) - [AW, CO_2-W],** **McAdams et al. (1942) - [AW, AK, AO]** Muller-Steinhagen and Heck (1986) - [R12] Olujic (1985) - [CO_2-W], Theissing (1980) - [AW, CO_2-W, R12]
0.75-1	Awad and Muzychka (2004a) - (General Case) - [AW, CO_2-W], **Beattie and Whalley (1982) - [CO_2-W],** **Beggs and Brill (1973) - [R12],** **Cicchitti et al (1960) - [AW, R12],** **Dukler et al. (1964) - (Case II) - [AW],** Grønnerud (1979) - [AW], **McAdams et al. (1942) [AW],** Muller-Steinhagen and Heck (1986) - [R12, AG], Sun and Mishima (2009) - [AW], Theissing (1980) - [AW, CO_2-W, R12]

VITA

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