# THE EFFECT OF WEB NON HOMOGENEITY AND 

 CORE EFFECTS ON WOUND ROLL STRESSBy<br>DHEEPAK RAJANNAN<br>Bachelor of Technology in Mechanical Engineering<br>Amrita Vishwa Vidyapeetham<br>Coimbatore, Tamilnadu, India<br>2007<br>Submitted to the Faculty of the<br>Graduate College of the<br>Oklahoma State University in partial fulfillment of the requirements for the Degree of<br>MASTER OF SCIENCE<br>July, 2011

# THE EFFECT OF WEB NON HOMOGENEITY AND CORE EFFECTS ON WOUND ROLL STRESS 

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## LIST OF SYMBOLS

pli - pounds per lineal inch
$\Omega-$ ohm
$\mathrm{E}_{\mathrm{c}}$ - Core Stiffness
$\mathrm{E}_{\mathrm{r}}$ - Radial Modulus
$E_{t}-$ Tangential Modulus
$\mathrm{k}_{\mathrm{r}}$ - Radial Stiffness
$\mathrm{k}_{\mathrm{t}}-$ Tangential Stiffness
$\mathrm{T}_{\mathrm{w}}-$ Winding Tension
h - Web Caliper
s - Outer Radius of Roll
$\delta \mathrm{P}$ - Incremental Radial Pressure
$\delta \mathrm{T}$ - Incremental Tangential Stress
P - Total Radial Pressure
T - Total Tangential Pressure

## CHAPTER I

## INTRODUCTION

### 1.1 Web Handling

A web is a continuous strip of material and examples of web material include paper, foil, film, non-woven and laminates. The unique nature of a web is its flexibility and the transport of through process machinery, where value is added, is called Web handling. Processing operations can include making the web, coating, drying, embossing, slitting and finally many converting operations where the web becomes a discrete component. A convenient way to store web materials is to wind them and the process is termed as winding. A schematic of winding is illustrated in Figure 1.1.


Figure 1.1: Schematic of winding process on a core

In a winding process, each layer of web that has been wound to a coil form interacts with the previously wound coil. As each layer is accreted it causes a change in deformation and stress in the layers which already have been wound onto the roll. The stress variation in wound rolls is a function of winding parameters and makes it unique from homogeneous and heterogeneous solid materials. A typical wound roll is illustrated in Figure 1.2 defining the terminologies that form the winding parameters.


Cross Machine Direction (CMD)

Figure 1.2: Typical wound roll [1]

In this research the focus will be on one type of winding called center winding. In this type of winding torque is provided to the core and causes the material to coil about the core in a spiral fashion. This torque is carefully controlled as the wound roll increases in radius. Some materials are wound at constant tension while others are wound using tapered tension, where the web tension typically is decreased as the wound roll radius increases. A constant torque applied to the core is a special case of tapered tension. The winding tension as a function of wound roll radius that results from these torque control strategies is the most influential parameter in determining the wound roll stresses and pressures that develop in the wound roll. Secondary parameters include web material parameters, geometric parameters such as the inner and outer core diameters and the final wound roll diameter, web thickness and width and core material parameters.

### 1.2 Winding Model Development

The stress state in a wound roll as a result of winding has value for predicting the defects in the web material and hence is a measure of wound roll quality. Narrow webs produce wound rolls which develop stresses that have significance in the radial and tangential directions. Surface equilibrium at the roll ends dictates the axial stresses are zero on the end boundaries. As a result no appreciable axial stresses can develop internally within a narrow roll. Winding models have evolved that predict how the radial and tangential stresses vary with radius in the roll. These models require input of the winding parameters discussed in the previous section.

This study will focus on two aspects of winding models which previously have not received attention.

The first aspect is the adaptation of current models to winding non homogeneous webs called nonwovens. Previously models have required web material properties as inputs that can be measured for homogeneous webs but are difficult to quantity for nonwoven webs. The focus of this portion of research will be to attempt to reform the winding models in terms of material parameters that can be measured easily on non homogeneous webs.

The second aspect of winding that will be studied involves the core. Webs are wound on fiber cores that must be inexpensive since they will be disposed of. The core is typically mounted on an expanding mandrel. A motor provides the winding torque to the mandrel. The core expands on the mandrel as the mandrel is pressurized. The web is then spliced to the core with adhesive or adhesive tape. Next winding begins and a controller determines how the torque will vary with radius based on operator input. The wound roll will achieve a final diameter at which point the web is cut and taped to the layer beneath. Now the mandrel is deflated and the completed roll can be extracted from the mandrel and sent to storage awaiting the next process operation. The quest in this portion of the study is the development of an extended winding model that can predict how
the wound roll stresses are impacted by the deflation and extraction of the mandrel. A thorough study of the treatment of core property measurement and the impact how the core is treated in the roll model will be attached.

## CHAPTER II

## REVIEW OF LITERATURE

Early attempts to develop winding models started about 40 years ago. These primitive models were developed as 1D models to predict stress variation in the radial direction of the wound roll. In the work by Catlow et al.[2], a wound roll was analyzed based on the principle of thick cylinder pressure vessels. They considered the wound roll as "concentric cylindrical layers" and assumed the applied web line tension to be equal to the tangential stress acting on the outer diameter of wound roll. An analytical method was then used to predict the radial stress in the roll after each layer of web material that has been wound and it resulted in the analysis of an accretive solid structure with specified boundary conditions.

The wound roll was then analyzed rigorously by Altmann[3] in his work. He utilized the elasticity equations of thick cylinders and considered the anisotropy of the web material. He assumed a constant value of radial modulus of web and developed a closed form integral solution to account for the anisotropy. His analytical solution required the use of numerical techniques to compute radial and tangential wound roll stresses. Limited computation capability at those times made this model to be less effective to produce accurate stress results.

Later Pfeiffer [4] found that pressure and strain are exponentially related in the radial direction in web materials and developed a wound roll model based on energy principles. He conducted stack compression test to develop a relation between the applied pressure $(\mathrm{P})$ and radial strain $\left(\varepsilon_{\mathrm{r}}\right)$. He would then curve fit the expression $P=K_{1} e^{\left(K_{2} \varepsilon_{r}\right)}$ by manipulating the constants $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$.

The radial modulus of elasticity can be found by taking the derivative of P with respect to strain. After simplifying this becomes,

$$
E_{r}=K_{2}\left(P+K_{1}\right)
$$

Yagoda [5] treated the core boundary condition in a precise manner and utilized the closed form expression developed by Altmann to develop an asymptotic series solution for predicting stresses. Anisotropic stress-strain and strain-displacement relations were employed in stress equilibrium equation to develop the expression for radial pressure and circumferential stress in terms of dimensionless elasticity parameters. He also established a condition on the core pressure to avoid buckling based on the relation between tangential stress, radial stress and Poisson's ratio at core vicinity.

A rigorous elasticity solution was developed by Hakiel [6] which incorporated Yagoda's core boundary condition[5] and the state dependent radial modulus of Pfeiffer [4]. The following assumptions were made in his work to develop a second order differential equation for predicting the incremental radial stresses due to the addition of the most recent lap[6].

1. The wound roll is assumed to be a geometrically perfect cylinder.
2. The length, width and thickness of web remains constant.
3. The wound roll is assumed to be made of concentric rings rather than a spiral.
4. The elastic properties of each layer of web material remain constant during the addition of a current lap but are updated after each lap is added.
5. The stresses vary as a function of radius and are independent of axial and circumferential position which is an assumption of axisymmetric plane stress where the axial stress is zero.

The differential equation for incremental pressure was then developed from the equilibrium equation for plane stress, orthotropic stress-strain and compatibility relations:

$$
\begin{equation*}
r^{2} \frac{d^{2}(\delta P)}{d r^{2}}+3 r \frac{d(\delta P)}{d r}-\left(g^{2}-1\right) \delta P=0 \tag{1}
\end{equation*}
$$

where, $g=\sqrt{\frac{E_{t}}{E_{r}}}$

Hakiel needed two boundary conditions to solve for incremental pressure from the second order differential equation including:
(a) The pressure beneath the outer most lap must be in equilibrium with the applied winding tension per the thin wall pressure vessel equation:

$$
\begin{equation*}
\left.\delta P\right|_{r=s}=\left[\frac{\left.T_{w}\right|_{r=s}}{s}\right] * h \tag{2}
\end{equation*}
$$

where,

$$
\begin{aligned}
& \left.T_{w}\right|_{r=s}-\text { winding tension (psi), } \\
& \mathrm{s}-\text { outer radius of roll (in) } \\
& \mathrm{h}-\text { web thickness (in) }
\end{aligned}
$$

(b) The radial deformation of the core normalized by the outer core radius is equal to the deformation of the first layer of web material that was wound on to the core. This condition was assumed to assure continuity between the core and first layer of web material.

$$
\begin{equation*}
\frac{u}{r_{c}}=u(1)=\frac{-\delta P}{E_{c}} \tag{3}
\end{equation*}
$$

where,

$$
\begin{aligned}
& \mathrm{u}(1) \text { - radial deformation of core/outer core radius } \\
& \delta P(1) \text { - radial pressure existing between core and first layer of web material (psi) } \\
& \mathrm{E}_{\mathrm{c}}-\text { core stiffness }(\mathrm{psi})
\end{aligned}
$$

The winding models developed to date incorporate an assumption of either a constant or a radial modulus which was state dependent on pressure and a constant tangential modulus. Non homogenous webs have undefined cross sectional area, which make material properties such as Young's modulus difficult to assess from tests. An extensive search confirmed the absence of a nonhomogeneous winding model.

It is also necessary to accurately simulate the boundary condition experienced by the core in its pre-winding and post winding processes. A coil slumping study performed by Bob and Neville et al. [7] emphasized the impact of mandrel support on wound roll quality. Coil slump is a roll defect seen in the winding of metal strip webs which may or may not be wound on cores. After winding these roll are either set in cradles or on the production floor to await transfer to the next process or to storage. With time rolls that witness coil slump will lose their circular shape and become elliptical with the minor axis aligned with the gravitational vector. The study group attempted to quantify the slumping phenomenon in terms of coil mass, inter-strip friction and
strip thickness. Edwards and Gary [8] predicted the radial and tangential bore stress as a function of equivalent radial elastic modulus for the following stages of winding.

1. Release of last wrap of web (winding tension $=0$ )
2. Release of Mandrel (Mandrel collapse) and
3. Cooling to a uniform temperature during post winding processes like storage, transportation etc.

Discussions of cores in the winding of metal strip differ from the winding of membranes such as films and paper. Metal strip undergoes inelastic bending deformation, particularly where the inner layers are wound. Often the core in winding metal strip is not even a separate structure. It is usually the first several layers of the winding roll wound at extremely high tension to ensure large scale inelastic deformation. The winding tension is reduced and winding continues until the roll is complete. Then the mandrel is deflated and can be extracted.

Fiber cores or cores of plastic or steel are a necessity when winding membranes such as film or paper. Since these materials are so thin the bending strains are negligible and do not cause inelastic deformation. If these materials were wound directly on an expanding mandrel the wound rolls would collapse immediately as the mandrel deflated. These materials require a separate core structure which is often a spiral wound composite of kraft paper and resin, for reasons of economy.

The techniques used for prediction of fiber core stiffness have been studied in order to estimate combined stiffness contributed by mandrel and core. Roisum [9] in his doctoral research classified the fiber core as an anisotropic cylinder and developed an expression to estimate an anisotropic core stiffness. He utilized stress equilibrium equations for an anisotropic cylinder and

Altman expression relating radial and tangential stresses. The expression developed for anisotropic stiffness was [9]:

$$
\begin{equation*}
E_{c}=\frac{E_{c R} E_{c T}}{E_{c R}\left(\frac{\alpha-a \beta^{*} s^{-2 \gamma}}{1+a s^{-2 \gamma}}\right)-\mu_{c R} E_{c T}} \tag{4}
\end{equation*}
$$

Gerhardt [10] developed a closed form elasticity equation to predict radial deformation in axissymmetrically loaded spirally wound paper tubes i.e. fiber cores. He validated his solution by conducting a hydraulic cavity test where external pressure is applied on the tube by a hydrostatic fluid and measured the strain on the outside and inside diameter of the core using rossette strain gages. He observed good agreement between his closed form solution and the test results. He found that the hoop stress tends to be at maximum near the outside diameter of the core and it does not decrease by increasing the thickness of tube. He concluded that this behavior is typical of fiber core tubes and is unlike the case of isotropic cores where the hoop stress remains at maximum near the inner diameter of core and can be decreased by the increasing the tube thickness. The behavior was attributed to a very high radial modulus compared to the tangential modulus of paper core tubes.

Later Gerhardt [11] et al., devised a Radial Crush Tester to measure the stiffness of fiber cores ranging from 3 " to 10 " internal diameter. He applied external pressure on the core by using a hydraulically actuated rubber bladder via ball bearings and measured the amount of radial crush. The bearings occupy the gap between the bladder and the outside diameter of core. This test estimated core stiffness based on spiral winding angle, wall thickness, tube ID, moisture content and paper strength [11]. However, certain amount of pressure applied by bladder is lost due to the tangential contact stresses experienced by the load transmitting ball bearings. Hence the actual pressure applied on the core is less than supply pressure existing in bladder. This test gave a good insight about establishing a test set up to identify core stiffness.

## RESEARCH OBJECTIVE

The first objective is to determine the importance of developing a winding model to predict wound roll stresses in nonhomogenous webs. The existing model for homogeneous webs will be modified to include web thickness parameter and the effect of web thickness on roll stress variation will be presented.

The second objective is to accurately simulate the core boundary condition in a wound roll both during and after winding. The impact of the mandrel support on the core boundary condition will be studied and the contribution of an expanding mandrel to the stiffness of the core will be predicted. Model and experimental results explaining the impact of the mandrel on the stress variation in wound rolls before and after winding will be presented.

## CHAPTER III

## THE EFFECT OF WEB NON HOMOGENEITY ON WOUND ROLL STRESS

### 3.1 Hakiel's Winding Model

The objective of a Winding model is to predict radial and tangential stress variation in a wound roll. The input parameters include roll geometry, web material properties and core properties. The development of Hakiel's winding model [6] to compute wound roll pressures and stresses in wound rolls has been discussed. The model was developed from the plane stress equilibrium equation, elastic constitutive equations and compatibility equation. The plane stress equilibrium equation is given by:

$$
\begin{equation*}
r\left(\frac{d \sigma_{r}}{d r}\right)+\sigma_{r}-\sigma_{\theta}=0 \tag{5}
\end{equation*}
$$

The subscripts r and $\theta$ refer to the radial and tangential directions respectively. The elastic constitutive equation for linear orthotropic materials is given by:

$$
\begin{gather*}
\varepsilon_{r}=\frac{\sigma_{r}}{E_{r}}-\left(\frac{v_{r \theta} \sigma_{\theta}}{E_{\theta}}\right)  \tag{6}\\
\varepsilon_{\theta}=\frac{\sigma_{\theta}}{E_{\theta}}-\left(\frac{v_{\theta r} \sigma_{r}}{E_{r}}\right) \tag{7}
\end{gather*}
$$

where,

E - elastic modulus (psi)
$v$ - Poisson ratio
$\varepsilon$ - normal strain

Maxwell's relation is given by,

$$
\begin{equation*}
\frac{v_{r \theta}}{E_{\theta}}=\frac{v_{\theta r}}{E_{r}} \tag{8}
\end{equation*}
$$

Assuming $g^{2}=\frac{E_{\theta}}{E_{r}}$ and $v=v_{r \theta}$, the expressions (6) and (7) has been rearranged as:

$$
\begin{gather*}
\varepsilon_{r}=\frac{g^{2} \sigma_{r}}{E_{\theta}}-\frac{v \sigma_{\theta}}{E_{\theta}}  \tag{9}\\
\varepsilon_{\theta}=\frac{\sigma_{\theta}}{E_{\theta}}-\frac{v \sigma_{r}}{E_{\theta}} \tag{10}
\end{gather*}
$$

The strain compatibility equation based on the linear strain definitions is given by:

$$
\begin{equation*}
r \frac{d \varepsilon_{\theta}}{d r}+\varepsilon_{\theta}-\varepsilon_{r}=0 \tag{11}
\end{equation*}
$$

By substituting the expression (9) and (10) into (11) and using the stress equilibrium condition (5), the second order governing differential equation for radial pressure was developed:

$$
\begin{equation*}
r^{2} \frac{d^{2} \sigma_{r}}{d r^{2}}+3 r \frac{d \sigma_{r}}{d r}-\left(g^{2}-1\right) \sigma_{r}=0 \tag{12}
\end{equation*}
$$

Hakiel's model utilizes this second order differential equation (12) to compute incremental radial pressures due to the addition of a new layer of web to the outside of the winding roll. The expression for incremental radial pressure ( $\delta \mathrm{P}$ ) in a wound roll is (1):

$$
\begin{equation*}
r^{2} \frac{d^{2} \delta P}{d r^{2}}+3 r \frac{d \delta P}{d r}-\left(g^{2}-1\right) \delta P=0 \tag{1}
\end{equation*}
$$

The boundary conditions that were developed to compute the incremental radial pressure has been discussed in the Chapter 2. The first boundary per the thin wall pressure vessel equation (2):

$$
\left.\delta P\right|_{r=s}=\left[\frac{\left.T_{w}\right|_{r=s}}{s}\right] * h
$$

where,
$\left.T_{w}\right|_{r=s}$ - winding tension (psi),
s - outer radius of roll (in)
h-web thickness (in)

The second boundary condition was developed by assuming displacement continuity between the core and the first layer of web material (3):

$$
\begin{gathered}
\left.u\right|_{\text {core }}=\left.u\right|_{\text {material }} \\
\frac{u}{r_{c}}=u(1)=\frac{-\delta P}{E_{c}}
\end{gathered}
$$

where,

$$
u(1) \quad-\text { radial deformation of core/outer core radius }\left(r_{c}\right)
$$

$\delta P(1)$ - radial pressure existing between core and first layer of web material (psi)

$$
\mathrm{E}_{\mathrm{c}}-\text { core stiffness (psi) }
$$

The expressions (1), (2) and (3) represent a governing second order differential equation for incremental radial pressure in a wound roll with two boundary conditions. After the incremental radial pressure was computed as a function of radius, the equilibrium expression in polar coordinates (14) was used to determine the tangential stress variation with radius.

$$
\begin{equation*}
\delta T=-\delta P-\left(\frac{d \delta P}{d r}\right) \tag{13}
\end{equation*}
$$

Hakiel's differential equation cannot be solved in closed form due to the state dependency of the radial modulus and pressure in the wound roll. This causes the $\mathrm{g}^{2}$ term in expression (1) to present a non constant coefficient in the differential equation. The finite central difference approximation method was used to approximate the derivatives in expression (1) and N number of laps wound on to the roll.

$$
\begin{equation*}
\left(1+\frac{3 h}{2 r}\right) \delta P(i+1)+\left(\frac{h^{2}}{r^{2}}\left(1-\frac{E_{\theta}}{E_{r}}\right)-2\right) \delta P(i)+\left(1-\frac{3 h}{2 r}\right) \delta P(i-1)=0 \tag{14}
\end{equation*}
$$

where,
h- thickness of each lap (in)
$\delta \mathrm{P}(\mathrm{i})$ - incremental radial pressure at the inside of the $\mathrm{i}^{\text {th }}$ layer ( psi )

The equation (14) was used to compute incremental radial pressure for (N-1) interior points with $(\mathrm{N}+1)$ unknowns. Hence two additional equations required for solving the unknowns has been derived from the boundary condition existing at the core and at the outside of the wound roll. The total pressure and total tangential pressure after winding each lap of web materials is given by:

Total radial pressure: $\quad \sum_{i=1 . . j} P(i)=P(i)+\delta P(i)$
where,
$\mathrm{P}(\mathrm{i})$ - total radial pressure in the $\mathrm{i}^{\text {th }}$ layer of web material after $\mathrm{j}^{\text {th }}$ lap has been added to the wound roll. (psi)
$\delta \mathrm{P}(\mathrm{i})$ - incremental radial pressure in the $\mathrm{i}^{\text {th }}$ layer of web due to the addition of the outermost $\mathrm{j}^{\text {th }}$ layer (psi)

Total tangential stress: $\quad \sum_{i=1 . . j} T(i)=-P(i)-\frac{d P(i)}{d r}$
where,
$\mathrm{T}(\mathrm{i})$ - total tangential stress in the $\mathrm{i}^{\text {th }}$ lap

The computational procedure is continued till the winding process is complete and the output stress distribution is then analyzed to anticipate or identify the roll defects. This process has been illustrated in the Figure 3.1[6].


Figure 3.1: Winding model [6]

### 3.2 Modification of Hakiel's Model to Include Web Thickness Parameter

The WindaRoll model developed by Good,J.K. and Roisum,D.R.,[12] based on Hakiel's winding model has been modified to include the web thickness parameter. The objective of the modified WindaRoll is to predict the impact of web thickness on wound roll pressure and tangential stress variation in wound rolls. The modifications made in the Hakiel's algorithm is illustrated:

The second order differential equation derived from the plane stress equilibrium condition, constitutive equations for orthotropic materials and compatibility equations is given in expression (1),

$$
r^{2} \frac{d^{2}\left(\delta \sigma_{r}\right)}{d r^{2}}+3 r \frac{d\left(\delta \sigma_{r}\right)}{d r}-\left(g^{2}-1\right) \delta \sigma_{r}=0
$$

where,
$g^{2}=\frac{E_{\theta}}{E_{r}}$
$\mathrm{E}_{\theta}$ - tangential modulus of web material (psi)
$E_{r}$ - radial modulus as a function of wound roll pressure $P_{i}$
$\delta \sigma_{\mathrm{r}}$ - incremental radial pressure ( psi ) (used interchangeably with $\delta \mathrm{P}$ )

The term $\mathrm{g}^{2}$ has been rearranged to include web thickness (h) and tangential stiffness $\left(\mathrm{K}_{\theta \mathrm{T}}\right)$ parameter. The tangential stiffness $\left(\mathrm{K}_{\theta \mathrm{T}}\right)$ is the slope of the tangential load versus deflection observed during the "stretch test". The stretch test is conducted to find the tangential modulus $\left(\mathrm{E}_{\theta}\right)$ of a web material.


Figure 3.2: Tangential stiffness

The tangential stiffness $\mathrm{K}_{\theta \mathrm{T}}$ is defined as,

$$
\begin{equation*}
K_{\theta T}=\frac{E_{\theta} A_{w e b}}{L_{\text {test }}} \tag{18}
\end{equation*}
$$

where,

$$
\begin{aligned}
& \mathrm{A}_{\text {web }} \text { - cross sectional area of web (in }{ }^{2} \text { ) } \\
& \mathrm{A}_{\text {web }}=\mathrm{w}^{*} \mathrm{~h} \\
& \mathrm{w} \text { - width of the web (in) } \\
& \text { h- web thickness (in) } \\
& L_{\text {test }} \text { - Length of web material used in the stretch test (in) }
\end{aligned}
$$

The radial modulus is given by the Pfeiffer's expression [4],
$\mathrm{E}_{\mathrm{r}}=\mathrm{K}_{2}\left(\mathrm{P}+\mathrm{K}_{1}\right)$

The expression (17) modified to include the web thickness:

$$
\begin{equation*}
g^{2}=\left(\frac{\frac{K_{\theta T} L_{\text {test }}}{w^{*} h}}{K_{2}\left(P+K_{1}\right)}\right) \tag{19}
\end{equation*}
$$

where,

$$
\begin{aligned}
& \mathrm{K}_{1}, \mathrm{~K}_{2}-\text { Pfeiffer constants } \\
& \mathrm{P} \text { - Total radial pressure (psi) }
\end{aligned}
$$

The outer boundary condition used by Hakiel referred in the expression (2) has been used in the modified WindaRoll model:

$$
\left.\delta \sigma_{r}\right|_{r=s}=\left[\frac{\left.T_{w}\right|_{r=s}}{s}\right] * h
$$

The core boundary condition referred in expression (3) was modified to include web thickness parameter as illustrated:

$$
\frac{u_{c}}{r_{c}}=\frac{-\delta P}{E_{c}}
$$

where,

$$
\begin{aligned}
& u_{c} \text { - deformation of exterior of the core (in) } \\
& r_{c} \text { - radius of core (in) } \\
& E_{c} \text { - core stiffness (psi) } \\
& \delta P \text { - Incremental radial pressure (psi) }
\end{aligned}
$$

$$
\begin{equation*}
u_{\text {layer } 1}=\varepsilon_{\theta} r_{c}=\frac{\delta \sigma_{\theta}-v_{r \theta} \delta P}{E_{\theta}} \tag{20}
\end{equation*}
$$

where,
$\varepsilon_{\theta}-$ tangential strain
$\delta \sigma_{\theta}-$ incremental tangential stress (psi)

The plane stress equilibrium equation in cylindrical coordinates referred in expression (5),

$$
\sigma_{\theta}=\sigma_{r}+r \frac{d \sigma_{r}}{d r}
$$

where,
$\sigma_{\theta}-$ tangential stress (psi)
$\sigma_{\mathrm{r}}-$ radial stress (psi)
r- radius (in)

The expression (3) is rearranged as,

$$
\begin{equation*}
u_{c}=\frac{-\delta P}{E_{c}} r_{c} \tag{21}
\end{equation*}
$$

The tangential strain for the web material based on orthotropic constitutive equation in the tangential direction is given by (10),

$$
\varepsilon_{\theta}=\frac{\sigma_{\theta}}{E_{\theta}}-v_{\theta r} \frac{\sigma_{r}}{E_{r}}
$$

where,

$$
\mathrm{E}_{\theta} \text { - tangential modulus of web material ( } \mathrm{psi} \text { ) }
$$

$\mathrm{E}_{\mathrm{r}}$ - radial modulus of web material (psi)

Through Maxwell relation, $\frac{v_{\theta r}}{E_{r}}=\frac{v_{r \theta}}{E_{\theta}}$

$$
\begin{equation*}
\varepsilon_{\theta}=\frac{\sigma_{\theta}}{E_{\theta}}-v_{r \theta} \frac{\sigma_{r}}{E_{\theta}} \tag{22}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\delta \varepsilon_{\theta}=\frac{\delta u}{r_{c}}=\frac{\delta \sigma_{\theta}}{E_{\theta}}-v_{r \theta} \frac{\delta \sigma_{r}}{E_{\theta}} \tag{23}
\end{equation*}
$$

Using the relation,

$$
\begin{equation*}
\delta \sigma_{\theta}=\delta \sigma_{r}+r_{c} \frac{d \sigma_{r}}{d r} \tag{24}
\end{equation*}
$$

Rearranging terms we get:

$$
\begin{equation*}
\frac{\delta u}{r_{c}}=\frac{\delta \sigma_{r}+r_{c} \frac{d \delta \sigma_{r}}{d r}}{E_{\theta}}-v_{r \theta} \frac{\delta \sigma_{r}}{E_{\theta}} \tag{25}
\end{equation*}
$$

Based on the assumption of displacement continuity used to derive the expression (4),

$$
\begin{array}{r}
\left.\frac{\delta u}{r_{c}}\right|_{w e b}=\left.\frac{\delta u}{r_{c}}\right|_{\text {core }} \\
\frac{\delta \sigma_{r}+r_{c} \frac{d \delta \sigma_{r}}{d r}-v_{r \theta} \delta \sigma_{r}}{E_{\theta}}=\frac{\delta \sigma_{r}}{r_{c}} \tag{27}
\end{array}
$$

The equation (27) rearranged to include tangential stiffness ( $\mathrm{K}_{\theta \mathrm{T}}$ ):

$$
\begin{equation*}
\frac{d \delta \sigma_{r}}{d r}=\left[\frac{\frac{K_{\theta T} L_{\text {test }}}{w^{*} h}}{E_{c}}-1+v_{r \theta}\right] \frac{\delta \sigma_{r}}{r_{c}} \tag{28}
\end{equation*}
$$

The number of laps ( $\mathrm{n}_{\text {laps }}$ ) wound onto the roll has been related to the web thickness in the following expression:

$$
\begin{equation*}
\text { nlaps }=\frac{r_{\text {roll }}-r_{c}}{h} \tag{29}
\end{equation*}
$$

where,
$r_{\text {roll }}-$ final wound roll radius (in)

The expressions (19), (2), (28) and (29) were incorporated in the WindaRoll model and the model follows the Hakiel's algorithm to compute wound roll pressures and stresses.

### 3.3 Input parameters

The inputs given to the winding model can be categorized as winding conditions, roll geometry, web material and core properties. The inputs used in the winding model have been illustrated in Table 3.1.

| Input | Components |
| :---: | :--- |
| Winding conditions | winding tension as a function of wound roll radius |
| Core and Roll geometry | Roll: Outer diameter |
| Web material | Web caliper, width, Tangential modulus, Radial modulus, Poisson <br> ratio of web |
| Core properties | Material modulus, Poisson ratio of core, core stiffness (calculated diameter <br> from Rosium's expression) |
| Number of grid points | Number of points or radial locations in the roll at which stresses are to <br> be evaluated determines a virtual web thickness |

Table 3.1 Input parameters for winding model

### 3.4 Validation

The modified version of WindaRoll model based on Hakiel's algorithm has been validated for a wound roll experiment referred by Hakiel in his work [6]. The experiment involves a 9-mil resin coated paper with a Poisson ratio of material $v_{\mathrm{r} \theta}=0$, tangential modulus of $600,000 \mathrm{psi}$ and a radial modulus given by the expression [6],

$$
\begin{equation*}
\mathrm{E}_{\mathrm{r}}=124 \mathrm{P} \tag{30}
\end{equation*}
$$

where,

$$
\mathrm{P} \text { - total radial pressure in the wound roll }(\mathrm{psi})
$$

The wound roll stresses are predicted by conducting winding experiments and the result is illustrated in the Figure 3.3. Hakiel verified the results from his winding model in comparison with the results from a winding experiment discussed earlier. The WindaRoll model and the modified WindaRoll model to include web thickness parameter were then executed for the same input conditions used in winding experiment. The modified WindaRoll model results were then compared with results from Hakiel's work, experimental results and WindaRoll as illustrated in

Figure 3.3.


Figure 3.3: In-roll radial stress distribution predicted by Hakiel model (left), WindaRoll and Modified WindaRoll model (right)

It can be inferred from the results that the modified WindaRoll model is in good agreement with Hakiel's model.

The important influential parameters of a winding model are the winding tension, tangential stiffness and web thickness. The impact of web thickness on wound roll pressures has been discussed in the following section.

### 3.5 Impact of Web Caliper on Wound roll Stress

The WindaRoll and modified WindaRoll models were executed for the same input conditions listed in Table 3.2. To predict the impact of thickness, the winding models were executed for the following web thickness values $50 \% \mathrm{~h}, 75 \% \mathrm{~h}, 100 \% \mathrm{~h}, 150 \% \mathrm{~h}$ and $200 \% \mathrm{~h}$ with respect to a reference value of thickness ' $h$ '. A thickness ' $h$ ' of 0.009 ' was chosen and the results output from the model for various web thickness are illustrated in Figures 3.4 and 3.5.

| Winding Conditions |  |  |
| ---: | ---: | :--- |
| Starting Winding Stress | 555.5556 | Psi |
| Taper | 0 | $\%$ |
| Ending Winding Stress | 556 | Psi |
| Starting Nip Load | 0 | Lb |
| Nip Taper | 0 | $\%$ |
| Ending Nip Load | 0 | Lb |
| Nip Load per unit width | 0.00 | Pli |
| Roll Geometry |  |  |
| Core ID | 1 | Inch |
| Core OD | 2 | Inch |
| Roll OD | 8 | Inch |
| Material Properties |  |  |
| Web Caliper | 0.009 | Inch |
| Web Width | 6 | Inch |
| Web-to-Web Kinetic COF | 0.16 |  |
| MD Modulus Et $=$ | $6.00 \mathrm{E}+05$ | Psi |
| Stack Modulus Er: K1= | 0.00 | Psi |
| K2 $=$ | 124.00 |  |
| Poisson's Ratio of Web | 0 |  |
| Core Properties |  |  |
| Core Material Modulus | $5.00 \mathrm{E}+05$ | Psi |
| Poisson's Ratio of Core | 0.3 |  |
| Core Stiffness | $3.66 \mathrm{E}+05$ | Psi |


| Percentage | h (in) |
| :---: | :---: |
| 50 | 0.0045 |
| 75 | 0.00675 |
| 90 | 0.0081 |
| 100 | 0.009 |
| 150 | 0.0135 |
| 200 | 0.018 |

Table 3.2 Input conditions used in comparative study

The modified WindaRoll and WindaRoll model has been executed for a web thickness range from $0.0045^{\prime \prime}$ to $0.018^{\prime \prime}$ and the results are illustrated in Figure 3.4 and Figure 3.5 respectively.



Figure 3.4: Pressure and tangential stress prediction by Modified WindaRoll



Figure 3.5: Pressure and tangential stress prediction by WindaRoll model

It can be observed from the wound roll pressure distribution output by WindaRoll and modified WindaRoll model that the thickness parameter has significant impact on wound roll pressure distribution. The pressure distribution computed by modified WindaRoll model in Figure 3.4 illustrates that high pressures exist in the wound roll as the web thickness is increased. The pressure distribution computed by WindaRoll in Figure 3.5 did not show any significant rise in pressure distribution as the web thickness is increased. The tangential stress distribution computed by modified WindaRoll illustrated a reasonable variation with thickness where as that computed by WindaRoll remained constant regardless of web thickness variation.

For a web thickness of $0.018^{\prime \prime}(200 \% \mathrm{~h})$, the pressure in the plateau region at $2.5 "$ roll radius computed by the WindaRoll model has been found to be 45 psi where as that predicted by the modified WindaRoll model is 80 psi . For a web thickness of $0.045 "(50 \% \mathrm{~h})$, the wound roll pressure computed by WindaRoll and modified WindaRoll were found to be 37 psi and 20 psi respectively. The reference value of web thickness ' $h$ ' used is 0.009 '.

The range through which the web thickness can vary in a Hakiel model to compute wound roll stresses distribution within $5 \%$ and $10 \%$ error limits has been studied. A web thickness of 0.009 " was chosen and the error limits were plotted as illustrated in Figure 3.6.The web thickness for a $5 \%$ error band in Hakiel model has been computed to range from 0.00468 " $(52 \% \mathrm{~h})$ to 0.01332 " $(148 \% \mathrm{~h})$. The web thickness for $10 \%$ error band ranges from 0.0054 " $(6 \% \mathrm{~h})$ to 0.01818 "( $202 \% \mathrm{~h}$ ). The wound roll pressure computed by Hakiel model for a web thickness of $0.0054(6 \% \mathrm{~h})$ at $2.5 "$ roll radius has been found to be 35.4 psi and that computed by the modified WindaRoll model is 2.1 psi. The wound roll pressure computed by Hakiel model for a web thickness of $0.01818(202 \% \mathrm{~h})$ at 2.5 " roll radius has been found to be 41.6 psi and that computed by the modified WindaRoll model is 78.8 psi which indicates a percentage deviation of $47.2 \%$.



Figure 3.6: Wound roll pressure distribution in a Hakiel model with $5 \%$ and $10 \%$ error bands

## CHAPTER IV

## THE IMPACT OF A MANDREL ON CORE STIFFNESS

In order to assess the impact of mandrels on core stiffness it is necessary to first study various configurations of mandrels. The following section illustrates various mandrel configurations and the mechanisms involved in it to support cores during a typical winding process. A Combined stiffness model has been developed to predict the stress variation in a center wound roll to quantify the impact of an expanding mandrel on the wound roll stress distribution. This model has been partly validated by Quall's thermoelastic model which is incorporated into the WindaRoll code and subsequently validated by experimental findings.

### 4.1 Winding Mandrels

A mandrel supporting a core shaft has two requirements. First it should engage the inner surface of the core so that the winder can provide torque to the core which is essential for the winding operation. Second, the mandrel should locate the core as concentrically as possible. A non concentric core will cause a dynamic winding tension that will cycle one per revolution. All mandrel designs must satisfy the first requirement. The degree to which the second requirement is satisfied depends largely on the design. These mandrels used in winding processes are available in different configurations and have different mechanisms built in them to support cores. A typical mandrel-core arrangement is illustrated in Figure 4.1. Several mandrel configurations used in winding applications has been discussed in this section.


1. Web (wound on to roll form)
2. Core
3. Mandrel

Figure 4.1: A typical mandrel-core arrangement during winding process [14]

Expanding mandrels also called core shafts or centering shafts based on their functionality. A typical mandrel from Tidland Corporation, Camas, Washington, used in unwind and winding processes is illustrated in Figures 4.2 and 4.3. The bladders expand upon the application of internal pressure and push the external sleeves via button screws and internal sleeves as illustrated in the Figure 4.4. The expansion of external sleeves applies a tight gripping force to hold the core during winding (Figure 4.2).


1. External Metal Sleeve (4 numbers)
2. Internal Metal Sleeve (4 numbers)
3. Bladder
4. Internal Screw (4 numbers)

Figure 4.2: Schematic of mandrel arrangement in Tidland shaft [14]


Figure 4.3: Expanding core shaft and rubber bladder (Tidland)


Figure 4.4: External cylindrical part illustrating internal and external sleeves connected via welded buttons in a Tidland shaft

Another configuration of centering shaft used to support and locate the cores made by Goldrenrod Corporation, Beacon Falls, Connecticut, is illustrated in the Figure 4.5. This shaft has three metal and three rubber ledges placed alternatively at 6 equidistant locations. These ledges extend radially upon the application of 85 to 100 psi pneumatic pressure. The shaft has an internal valve (Figure 4.5) constructed in such a way that the valves leading to metal sleeves has relatively large diameter holes compared to those leading to rubber sleeves. This arrangement causes the three
metal sleeves to expand first to concentrically locate the core and after few seconds rubber ledges expand to provide a gripping force to the core as illustrated in the Figure 4.6.


Figure 4.5: Golden rod centering shaft and internal valve construction to expand sleeves [15]


Figure 4.6: Working principle of Goldenrod shaft [15]

A third type of core shaft configuration manufactured by Tidland Corporation is illustrated in the Figure 4.7. The shaft identified as "Lug shaft" uses the principle of bladder-sleeve concept and utilizes four lugs placed at equidistant locations to support and locate the core. The lugs contact the inside of the core locally.


Figure 4.7: Lug shaft and bladder-sleeve-lug arrangement [16]

A mechanically operated expanding shaft manufactured by Goldenrod is illustrated in Figure 4.8. A screw rod enclosed by the shaft is connected to an expanding tapered sleeve and a wrench is used to rotate the screw rod. This causes the shaft to push the tapered sleeve which in turn protrudes externally to locate and support the core. These shafts have been suggested for usage in high speed winding applications.


Figure 4.8: Mechanically operated expanding core shaft [17]

### 4.2 Thermoelastic model

The thermoelastic model was developed by Good,J.K., and Qualls.W.R [13] to compute the effect of temperature on wound roll stress distribution after winding is complete. The thermoelastic model functions in the following steps:

1. A Winding model first executes based on Hakiel's algorithm and the wound roll stresses are computed.
2. Second, the thermoelastic model utilizes the temperature change experienced by the wound roll and the thermal expansion properties of web material and the core.
3. A boundary value model executes and in steps the temperature change is incorporated and the corresponding stresses are computed. These stresses are used to update the state dependent properties of wound roll and the next step in temperature change occurs. This continues till the wound roll experiences the specified temperature change in the model.

The thermoelastic model developed by Good.J.,K. et al. [13] was developed from the plane stress equilibrium condition referred in expression (10):

$$
\sigma_{\theta}=\sigma_{r}+r \frac{d \sigma_{r}}{d r}
$$

The thermal effects were then included in the elastic constitutive relations used by Hakiel [6] :

$$
\begin{gather*}
\varepsilon_{r}=\frac{\sigma_{r}}{E_{r}}-\frac{v_{r \theta} \sigma_{\theta}}{E_{\theta}}+\alpha_{r} \Delta T  \tag{31}\\
\varepsilon_{\theta}=\frac{\sigma_{\theta}}{E_{\theta}}-\frac{v_{\theta r} \sigma_{r}}{E_{r}}+\alpha_{\theta} \Delta T \tag{32}
\end{gather*}
$$

where the subscripts r refers to the radial direction and $\theta$ refers to the tangential direction.
$\varepsilon$ - normal strain
$\sigma_{\mathrm{r}}-$ radial stress (psi)
$\sigma_{\theta}-$ tangential stress (psi)
$v$ - Poisson ratio
$\alpha$ - thermal coefficient of expansion
$\Delta \mathrm{T}$ - temperature change

The plane stress equilibrium condition was then substituted in the expression (31) and (32) to give:

$$
\begin{gather*}
\varepsilon_{r}=\frac{\sigma_{r}}{E_{r}}-\frac{v_{r \theta}\left(\sigma_{r}+r \frac{d \sigma_{r}}{d r}\right)}{E_{\theta}}+\alpha_{r} \Delta T  \tag{33}\\
\varepsilon_{\theta}=\frac{\left(\sigma_{r}+r \frac{d \sigma_{r}}{d r}\right)}{E_{\theta}}-\frac{v_{\theta r} \sigma_{r}}{E_{r}}+\alpha_{\theta} \Delta T \tag{34}
\end{gather*}
$$

The expressions (33) and (34) were then substituted into the compatibility equation:

$$
\begin{equation*}
r \frac{d \varepsilon_{\theta}}{d r}+\varepsilon_{\theta}-\varepsilon_{r}=0 \tag{35}
\end{equation*}
$$

The following assumptions were made in this model,

1. The properties $\mathrm{E}_{\theta}, v_{\theta}$ and $\alpha_{\theta}$ were assumed to be constant
2. The wound roll is subjected to a homogeneous temperature change.

After substitution, the expression (35) has been rearranged to give:

$$
\begin{align*}
& \frac{r^{2}}{E_{\theta}} \frac{d^{2} \sigma_{r}}{d r^{2}}+\left(\frac{3 r}{E_{\theta}}-\frac{r v_{\theta r}}{E_{r}}+\frac{r v_{r \theta}}{E_{\theta}}\right) \frac{d \sigma_{r}}{d r}+ \\
& \left(-\frac{r}{E_{r}} \frac{d v_{\theta r}}{d r}-r v_{\theta r}\left(\frac{-1}{E_{r}^{2}} \frac{d E_{r}}{d r}\right)+\frac{1}{E_{\theta}}-\frac{v_{\theta r}}{E_{r}}-\frac{1}{E_{r}}+\frac{v_{r \theta}}{E_{\theta}}\right) \sigma_{r}=\left(\alpha_{r}-\alpha_{\theta}\right) \Delta T \tag{36}
\end{align*}
$$

Applying Maxwell relation: $\frac{v_{\theta r}}{E_{r}}=\frac{v_{r \theta}}{E_{\theta}}$, the expression (36) has been rearranged as:

$$
\begin{align*}
& \frac{r^{2}}{E_{\theta}} \frac{d^{2} \sigma_{r}}{d r^{2}}+\left(\frac{3 r}{E_{\theta}}\right) \frac{d \sigma_{r}}{d r}+ \\
& \left(-\frac{r}{E_{r}} \frac{d}{d r}\left(\frac{E_{r} v_{r \theta}}{E_{\theta}}\right) *\left(\frac{-1}{E_{r}{ }^{2}} \frac{d E_{r}}{d r}\right)+\frac{1}{E_{\theta}}-\frac{1}{E_{r}}\right) \sigma_{r}=\left(\alpha_{r}-\alpha_{\theta}\right) \Delta T \tag{37}
\end{align*}
$$

Considering the first assumption of constant modulus and Poisson ratio in the tangential direction, the expression (37) has been simplified to give the second order differential equation for radial stress including thermal effects:

$$
r^{2} \frac{d^{2} \sigma_{r}}{d r^{2}}+3 r \frac{d \sigma_{r}}{d r}+\left(1-\frac{E_{\theta}}{E_{r}}\right) \sigma_{r}=E_{\theta}\left(\alpha_{r}-\alpha_{\theta}\right) \Delta T
$$

Then the boundary conditions were modified to include thermal influences. The first boundary condition at the core assumed by Hakiel [6] for displacement continuity has been modified as:

$$
\begin{gather*}
\left.u_{c}\right|_{\text {core }}=\left.u_{c}\right|_{\text {material }} \\
\frac{\sigma_{r}}{E_{c}}+\alpha_{c} \Delta T=\varepsilon_{\theta} \tag{39}
\end{gather*}
$$

where,

$$
\mathrm{E}_{\mathrm{c}} \text { - core stiffness (psi) }
$$

Using the elastic constitutive equation for tangential strain (32) and stress equilibrium equation (10), the boundary condition at the core (39) was rearranged to give:

$$
\begin{equation*}
r \frac{d \sigma_{r}}{d r}+\sigma_{r}\left(1-v-\frac{E_{\theta}}{E_{c}}\right)=E_{\theta}\left(\alpha_{c}-\alpha_{r}\right) \Delta T \tag{40}
\end{equation*}
$$

where, $v=v_{r \theta}$

The outer boundary condition was considered to be a traction free surface:

At $\mathrm{r}=\mathrm{r}_{\text {outer }}: \quad \sigma_{r}=0$

The expressions (38), (40) and (41) represent a second order differential equation to compute radial stress with inner and outer boundary conditions. The differential equation (18) was then rearranged by applying central difference approximation by considering N number of discretized locations :
$\left(\frac{r^{2}}{h^{2}}-\frac{3 r}{2 h}\right) \sigma_{r}(i-1)+\left(\frac{2 r}{h^{2}}+1-\frac{E_{\theta}}{E_{r}}\right) \sigma_{r}(i)+\left(\frac{r^{2}}{h^{2}}+\frac{3 r}{2 h}\right) \sigma_{r}(i+1)=E_{\theta}\left(\alpha_{r}-\alpha_{\theta}\right) \Delta T$
with, $h=\frac{r_{\text {out }}-r_{\text {in }}}{N}$
where,

$$
\mathrm{r}_{\text {out }} \text { - outer radius (in) }
$$

```
rin
```

The expression (42) used to solve for all the (N-2) interior points and two boundary conditions represent a boundary value model and a tridiagonal system of equations with N unknowns. The system of equations in matrix form can be written as,

$$
\begin{equation*}
[A]\left\{\sigma_{r}\right\}=\{B\} \tag{43}
\end{equation*}
$$

This system of equations is solved by Gaussian elimination approach with ( $\mathrm{N}-1$ ) forward and ( $\mathrm{N}-1$ ) backward substitution process. The thermoelastic model can be executed only for a specified temperature change and is not an accretive solution as given by Hakiel's winding model. Since the radial modulus of a wound roll is highly non-linear, the specified temperature change has to be subdivided into steps and the model has to be executed. The temperature change in steps allows the non linear properties of wound roll to be updated and hence the model can provide accurate results for the specified temperature change.

### 4.3 Combined Stiffness model

The combined stiffness model is a variation of thermoelastic model [13] incorporated into the WindaRoll model. The objective of this model is to estimate the impact of mandrel on stress distribution in wound rolls and the combined stiffness term refers to the stiffness contribution of core and mandrel. An Excel VBA code that utilizes the wound roll stress resulting from a 1D winding model based on Hakiel code and simulates the release of mandrel from the core by assigning a decreasing core modulus to the wound roll has been developed in his work.

The combined stiffness model follows a similar algorithm and the temperature effect is replaced by step reduction in combined core stiffness till the combined core stiffness reaches the stiffness due to the core alone. The release of mandrel is modeled as a decrease in core stiffness. While winding the core stiffness is a function of both the stiffness of the core and the support provided
to the inside of the core by the mandrel. After winding is completed, it is assumed the core is extracted and the stiffness of the core is due to core stiffness alone. This model functions in the following steps:

1. A winding model executes based on Hakiel's algorithm that incorporates combined mandrel and core stiffness into the core stiffness used by the model. This is an accretive solution as described previously.
2. A boundary value model is now executed. It begins with the winding stress distribution and combined core stiffness used in the winding model. In steps the combined core stiffness is reduced and the corresponding stresses are computed. These stresses are used to update the wound roll state dependent properties and the next reduction in combined core stiffness occurs. This continues until the combined core stiffness reaches the stiffness due to the core alone.

Neglecting the temperature effects in expression (38), the governing equation for radial stress yields the expression used by Hakiel [6] in his winding model and the resulting expression is used in Combined stiffness model to develop a boundary value model:

$$
\begin{equation*}
r^{2} \frac{d^{2} \sigma_{r}}{d r^{2}}+3 r \frac{d \sigma_{r}}{d r}+\left(1-\frac{E_{\theta}}{E_{r}}\right) \sigma_{r}=0 \tag{44}
\end{equation*}
$$

Then the boundary conditions have been formulated to include the combined core stiffness into the model. The inner boundary condition existing at the core assuming the displacement between the core and first layer of web material is given in expression (3):

$$
\begin{equation*}
\frac{\sigma_{r}}{E_{c}}=\varepsilon_{\theta} \tag{45}
\end{equation*}
$$

where, $\mathrm{E}_{\mathrm{c}}$ - combined core stiffness (psi) that represents the stiffness of both the core and the mandrel. By using the elastic constitutive equation for tangential strain and plane stress equilibrium equation, the expression (42) can be rearranged to give:

$$
\begin{equation*}
\frac{d \sigma_{r}}{d r}=\frac{\left[\frac{E_{\theta}}{E_{c}}-1+v_{r \theta}\right]}{r_{c}} \sigma_{r} \tag{46}
\end{equation*}
$$

where, $r_{c}$ - outer radius of the core (in)

The second boundary condition has been developed by considering the traction free surface existing at the radius of the wound roll:

At $\mathrm{r}=\mathrm{r}_{\mathrm{o}}: \sigma_{r}=0$

By applying central difference formulation for N number of discretized locations, the expression (44) can be rewritten as,

$$
\begin{equation*}
\left(1+\frac{3 h}{2 r}\right) \sigma_{r}(i+1)+\left(\frac{h^{2}}{r^{2}}\left(1-\frac{E_{\theta}}{E_{r}}\right)-2\right) \sigma_{r}(i)+\left(1-\frac{3 h}{2 r}\right) \sigma_{r}(i-1)=0 \tag{48}
\end{equation*}
$$

The expression (48) that computes radial stress at all ( $\mathrm{N}-2$ ) interior points and the two boundary conditions (45) and (46) represent a tridiagonal system of equations with N unknowns. The equations are solved by Gaussian elimination approach with ( $\mathrm{N}-1$ ) forward and (N-1) backward substitution steps. Since the radial modulus of wound roll is non-linear the step change approach followed in thermoelastic model has been adopted in the combined stiffness model. The release of mandrel from the core after winding has been applied as a step decrease in the combined core stiffness and the state dependent properties of wound roll were updated. This process continues till the combined core stiffness reaches the stiffness due to the core alone. The algorithm following in the model is illustrated in Figure 4.9.


Figure 4.9 Combined stiffness model - Algorithm

A comparative study between wound roll stresses with a core supported by mandrel and that of an unsupported core was performed to quantify the impact of mandrel. A Dupont 37792 gage web material has been chosen for the study and results from combined stiffness model have been discussed in the following section. The input conditions used in this comparative study are illustrated in Table 4.1. The tangential modulus of chosen web material has been determined experimentally and is found to be 796,060 psi. The constants $K_{1}$ and $K_{2}$ used in the Pfeiffer's expression for radial modulus discussed in literature survey has been determined by experiments and a curve fitting technique. These values are found to be 0.7190 psi and 31.22 respectively. The
core manufactured by Sonoco Corporation, Hartsville, South Carolina, has been used. A uniaxial compression test was performed by using an Instron 8502 material testing system to compute tangential and radial modulus of the core. The tangential and radial modulus of the core has been found to be 36,314 psi and 12,973 psi respectively. The expression (4) for anisotropic core stiffness was used to compute the anisotropic core stiffness to be $38,449 \mathrm{psi}$. The experimental method followed to determine the radial and tangential modulus of core is discussed in the following Chapter.

| Layers in Roll | 1000 |  |
| :---: | :---: | :---: |
| Number of Grids | 333 |  |
| Completed Grid Calculations | 333 |  |
| Winding Conditions |  |  |
| Starting Winding Stress | 724.638 | psi |
| Taper | 0 |  |
| Ending Winding Stress | 725 | psi |
| Starting Nip Load | 0 |  |
| Nip Taper | 0 |  |
| Ending Nip Load | 0 |  |
| Nip Load per unit width | 0.00 |  |
| Roll Geometry |  |  |
| Core ID | 3.017 | in |
| Core OD | 3.551 | in |
| Roll OD | 7.551 | in |
| Material Properties |  |  |
| Web Caliper Web Width <br> Web-to-Web Kinetic COF <br> MD Modulus Et= Stack Modulus Er: K1= <br> $\mathrm{K} 2=$ <br> Poisson's Ratio of Web | 0.00092 | in |
|  | 6 | in |
|  | 0.16 |  |
|  | $7.96 \mathrm{E}+05$ | psi |
|  | 0.72 | psi |
|  | 31.22 |  |
|  | 0 |  |
| Core Properties |  |  |
| Poisson's Ratio of Core Core Stiffness | 0.3 |  |
|  | $6.12 \mathrm{E}+04$ | psi |

Table 4.1: Input conditions for comparative study of Hakiel's wound roll model and Combined stiffness model

### 4.2 Similarity between Thermoelastic model and Combined Stiffness model

WindaRoll VBA code based on Quall's thermoelastic model[13] developed by Dr.Good has been utilized to predict stresses during winding process and the similarity existing between thermoelastic model and combined stiffness code has been discussed. The thermal properties of web and core has been assigned to be zero and negative expansion coefficient respectively. This has been done to simulate shrinkage of the core on the wound roll after winding is complete. A negative coefficient of expansion assigned to the core simulates core shrinkage due to roll pressures developed during winding. The codes were executed for the same input conditions discussed in the section 4.1 and the comparison results are illustrated in Figure 4.10.



Figure 4.10: Similarity between Combined stiffness and thermoelastic winding model stress comparison

### 4.3 Comparative study

The stress variation in wound rolls during winding and post winding process has been discussed in the following section. A WindARoll FE-VBA code developed by Dr.Good has been utilized to predict stresses during winding and Combined stiffness code to predict stress variation during post winding process. The input conditions discussed earlier were used in the study and results are illustrated in Figure 4.11.


Figure 4.11: Wound roll stress variation during winding and post winding process -radial stress and tangential stress

From the model results, it can be observed that a considerable decrease in stress variation occurs near the vicinity of core. This can be primarily attributed to a decreasing radial stiffness of core when the mandrel is released. The pressures and tangential stresses away from the core are not significantly affected. This case will be studied in laboratory tests in the following Chapter.

## CHAPTER V

## EXPERIMENTAL RESULTS AND DISCUSSION

The Combined stiffness and WindaRoll winding model utilized to predict the impact of mandrel on core stiffness has been validated with experimental findings in the following section. The core stiffness, tangential and radial modulus of web material and the resulting stress distributions during winding and post winding process has been determined experimentally. The properties measured are utilized as input parameters for the winding model and results are compared with experimental findings.

### 5.1 Prediction of Core stiffness

An expression for anisotropic core stiffness developed by Rosium [9] has been utilized to estimate core stiffness. The expression referred in equation (4) is given as,

$$
E_{c}=\frac{E_{c R} E_{c T}}{E_{c R}\left(\frac{\alpha-a \beta s^{-2 \gamma}}{1+a s^{-2 \gamma}}\right)-\mu_{c R} E_{c T}}
$$

where,

$$
\mathrm{E}_{\mathrm{cR}}-\text { radial modulus of core (psi) }
$$

## $\mathrm{E}_{\mathrm{cT}}$-tangential modulus of core (psi)

The non dimensional parameters used in equation 5.1 are defined as,
$\exists_{C}=\frac{E_{c T}}{E_{c}}=0 \quad$ (In this case)
$\exists_{C R}=\frac{E_{c T}}{E_{c R}}$
$\mu=\frac{1}{2}\left(\mu_{c T}+\exists_{c R} \mu_{c R}\right)$
$\delta=\frac{1}{2}\left(\mu_{c T}-\exists_{c R} \mu_{c R}\right)$
$\gamma=\left|\sqrt{\delta^{2}+E_{c R}}\right|$
$\alpha=\gamma-\delta$
$\beta=\gamma+\delta$
$a=\frac{\gamma-\mu-\exists_{c}}{\gamma+\mu+\exists_{c}}$
$b=1-a$
where,
$\mathrm{E}_{\mathrm{c}}$ - isotropic core stiffness (psi)
$\mu_{c T}$ - Poisson ratio of core in the tangential direction
$\mu_{c R}$ - Poisson ratio of core in the radial direction
A cube specimen of a dimension equal to the thickness of the core of 0.25 " was prepared from a core manufactured by Sonoco Inc. to measure the radial and tangential modulus of fiber core as illustrated in Figure 5.1. A test setup using an Instron machine has been utilized to apply load on
the specimen as illustrated in Figure 5.2. A Uniaxial compressive load was then applied on radial and tangential direction of the specimen to establish plots of applied load versus displacement.


Figure 5.1: Cube specimen of side 0.25 in used for stiffness measurement from a Sonoco core


Figure 5.2: Compressive load applied on specimen using Instron testing machine

The radial and tangential directions were marked on the cube specimen as ' $T$ ' and ' $R$ ' as indicated in Figure 5.1. The tangential modulus was estimated from the slope of stress versus
strain data obtained by applying a uniaxial compressive load in the tangential direction. The modulus in the tangential direction ( $\mathrm{E}_{\mathrm{cT}}$ ) was found to be $36,341 \mathrm{psi}$. The radial modulus estimated by applying a compressive load in the radial direction $\left(\mathrm{E}_{\mathrm{cR}}\right)$ was found to be 12,973 psi. These modulus values were used in equation (4) to predict the anisotropic core stiffness and it has been estimated to be 38,449 psi as illustrated in Table 5.1.

| $\left.\mathrm{E}_{\mathrm{cT}} \mathrm{psi}\right)$ | 36341 |
| :---: | :---: |
| $\mathrm{E}_{\mathrm{cR}}(\mathrm{psi})$ | 12973 |
| $\mathrm{r}=\mathrm{s}(\mathrm{in})$ | 1.7755 |
| $\mu_{\mathrm{cR}}$ | 0.17 |
| $\mathrm{E}_{\mathrm{cR}}$ | 2.8012 |
| $\mathrm{E}_{\mathrm{c}}$ | 0 |
| Based on Maxwell <br> relation |  |
| $\mu_{\mathrm{cT}}$ | 0.4762 |
| $\mu$ | 0.4762 |
| $\delta$ | 0 |
| $\gamma$ | 1.6737 |
| $\alpha$ | 1.6737 |
| $\beta$ | 1.6737 |
| a | 0.5570 |
| b | 0.4430 |
|  |  |
| $\mathrm{~s}^{-2 \mathrm{~g}}$ | 0.1463 |


| $\mathrm{E}_{\mathrm{c}}(\mathrm{psi})$ | $38,449.19$ |
| :--- | :--- |

Table 5.1: Calculation of Anisotropic core stiffness

### 5.2 Web Material Properties

A Dupont 37792 gage polyester web material has been used in the experimental study that has a thickness of 0.00092 " and a width of $6 "$. The determination of tangential and radial stiffness of the chosen web material is discussed in the following section.

### 5.2.1 Stretch Test

The "stretch test" has been performed to evaluate tangential modulus of the chosen web material. This test was used because it minimizes the grip effects seen when tensile testing short coupons of web. Fifty feet of web material was used as the length in this test. One end of the specimen was constrained and the opposite end was connected to a force transducer. A gradually increasing tensile load was applied to the web using a force transducer and the corresponding change in length of the specimen was recorded. The data and the reduction of the data to stress and strain is given in Table 5.2. The experiment has been repeated for three different samples and the tangential stiffness value identified as the average of three trials has been determined to be 796,060 psi.

| Dupont 377 92 gage |  |  |  |
| :---: | :---: | :---: | :---: |
| Cross sectional area of web | 0.00552 | in^2 |  |
| Length of test specimen | 600 | In |  |
| Sample 1 |  |  |  |
| Load (lb) | Reading <br> (in) | Strain | Stress (kpsi) |
| 0 | 0 | 0 | 0 |
| 2 | 0.277 | 0.000462 | 362.3188406 |
| 4 | 0.548 | 0.000913 | 724.6376812 |
| 6 | 0.851 | 0.001418 | 1086.956522 |
| 8 | 1.13 | 0.001883 | 1449.275362 |
| 10 | 1.4 | 0.002333 | 1811.594203 |
| 12 | 1.675 | 0.002792 | 2173.913043 |
| 14 | 2.01 | 0.00335 | 2536.231884 |
| 16 | 2.236 | 0.003727 | 2898.550725 |
| 18 | 2.45 | 0.004083 | 3260.869565 |

Table 5.2 Evaluating tangential modulus from tensile stress and strain for sample 1


Figure 5.3: Slope of tensile stress vs tensile strain to predict a tangential modulus of 782023 psi for sample 1

### 5.2.2 Stack test

This test has been performed to determine the radial modulus of web material Dupont 37792 gage. A one inch stack of web sheets was prepared and an Instron 8502 material testing system was used to apply a normal compressive load on the specimen as illustrated in Figure 5.4. This machine is equipped with a data acquisition system that measures applied load and corresponding displacement in the specimen as illustrated in Figure 5.5.


Figure 5.4: Instron machine applying compressive load on the specimen


Figure 5.5: Data acquisition system that records load and corresponding displacement

The radial or normal stress and strain has been evaluated from the load versus displacement data. This data curve fit using Pfeiffer's expression: [4],

$$
\begin{equation*}
P=-K_{1}+K_{1} e^{K_{2} \varepsilon_{r}} \tag{49}
\end{equation*}
$$

A least squared error routing was used to determine the values of $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ best fit the data.

The constants $K_{1}$ and $K_{2}$ are to be determined to specify the radial modulus of the chosen material as [5],

$$
\begin{equation*}
E_{r}=\frac{d P}{d \varepsilon_{r}}=K_{1} K_{2} e^{K_{2} \varepsilon_{r}}=K_{2}\left(K_{1}+P\right) \tag{50}
\end{equation*}
$$

The stress values obtained from experimental data was compared with Pfeiffer's stress values. Using a curve fitting technique, the values of $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ were determined to be 0.7190 and 31.2166 respectively. The fit using Pfeiffer's expression (21) is illustrated in Figure 5.6.


Figure 5.6: Comparison between experimental stress data and Pfeiffer's data

### 5.3 Winding Tests

The following section discusses about the calibration of pull tabs used to measure stresses in wound rolls. Pull tabs are simple friction devices that infer pressure by measuring the force required to cause a wound-in pull tab to slip either within web layers or within an envelope of material within the web layers. The purpose of the envelope is to provide a controlled surface which has low coefficient of friction for the tab to slip within. In these tests the taps were steel shim stock $\frac{1}{2}$ " wide, greater than 6 " long such that the protruded from both sides of the wound roll when wound into the roll, and were 0.001 " thick. The envelopes were kraft paper coated with silicone.

### 5.3.1 Calibration of Pull tabs

The pull tabs are shown in Figure 5.7. The experiment involved a one inch stack of web similar to that which was described in the radial modulus test. The tabs within their envelopes were inserted into the stack of web. The Instron 8502 was used to subject the stack to known pressure levels, typically in a range of zero to 40 psi , as shown in Figure 5.8. The force required to cause the pull tab to slip was measured with a hand held force gage (Make: SHIMPO, Model: FGE-50). This data would be curve fit with a line. The expression for the line would then be used for the calibration curve for that pull tab. This calibration process was repeated for each pull tab. A set of calibration data for one pull tab and the corresponding calibration curve are shown in Figure 5.9.


Figure 5.7: Silicone pull tabs used to predict wound roll stress


Figure 5.8: Load applied using Instron machine and force applied to cause slippage in pull tab is being measured for calibration.

Wound roll pressures were then measured by inserting the pull tabs during winding. After winding was complete the force required to cause slippage was measured using force transducer. Using the measured force, the pressure existing in wound roll was estimated from a calibration curve such as that illustrated in Figure 5.9.


Figure 5.9 Calibration curve illustrating a relation existing between slippage force and pressure in wound roll for pull tab 4 used in winding experiment

### 5.4 Winding experiment

A Winding experiment was conducted to validate the wound roll stresses predicted by the WindaRoll model during winding and the Combined Stiffness winding model during post winding processes. A core manufactured by Sonoco was chosen to perform the winding tests, which had an ID of 3.017 in and an OD of 3.551 in . The stiffness of the core was determined experimentally as discussed in the section 5.1 . Three strain gages of $350 \Omega$ resistance were installed on the core at equidistant locations as illustrated in Figures 5.10 and 5.11. Thin wire leads were used along the core periphery to establish a smooth cylindrical surface and insulated wire leads were used to connect the strain gage to a strain indicator. The purpose of these gages is to measure the strain along the core periphery and estimate the deformation characteristic of the core during winding process.


Figure 5.10: Core specimen with an installed strain gage (Instrumented core)


Figure 5.11: Three strain gages placed at equidistant locations along the circumference of core

A center winding process at constant winding tension levels of $4.0,4.5$ and 5.0 lbs was performed and the experimental setup is illustrated in Figure 5.13. The setup involves an expanding mandrel,

Windroll, Unwindroll, Tension Control Unit, Instrumented Core, Calibrated Pull tabs and a Strain Indicator. An expanding cantilever type multiple bladder shaft manufactured by Goldenrod (Model no: GR49279) was utilized in the winding process as illustrated in Figure 5.12.


Figure 5.12: Golden rod expanding shaft (Model no: GR49279)

The winding has a closed loop tension controller that maintains a constant winding tension till the winding process is complete. The constant winding tension was confirmed by monitoring the tension data during the winding experiment. The tension data monitored in a 4.5 lb winding experiment is illustrated in Figure 5.14.


Figure 5.13: Experimental setup for a constant tension center winding process


Figure 5.14: Characteristic of a 4.5 lb winding tension during a tension monitor experiment

The experiment was performed by mounting the core on the mandrel and expanding the mandrel to grip the core. Strain readings observed on the core surface from the strain indicator were noted. Then the lead wires were wound around the rotating mandrel and the winder was started at a constant winding tension level. The first pull tab was inserted after winding few layers adjacent to the core and three pull tabs were inserted at an interval of 0.5 " pile height in order to measure wound roll pressures. A finished roll diameter of 7.6 " OD was wound and the winder was stopped. The strain gages were reconnected to the strain indicator and the readings were noted as illustrated in Figure 5.15. Strain measurement was performed both during inflated and deflated mandrel conditions.


Figure 5.15: Strain measurement performed after winding process

Then the calibrated pull tabs were used to measure wound roll pressure by measuring slippage force and calibration curve as illustrated in Figure 5.16. Then the mandrel was deflated and pull tab measurements were noted again along with the strain readings. The first set of measurements
done measured the wound roll pressure with the mandrel still inflated and the measurements made after the mandrel was deflated predicts final wound roll pressure after winding.


Figure 5.16: Force measurement using pull tabs

The WindaRoll model was then executed. The measured web material properties, core properties, roll geometry and winding conditions were input. The results of a test where a 4.5 lbs constant winding tension was used the test and model results are shown in Figure 5.17. To study the effect of mandrel deflation the Combined Stiffness model was then executed with the same input conditions that output is shown with the pull tab pressure test results acquired after mandrel deflation are shown in Figure 5.18. Three set of experiments were performed to ensure confidence in the data observed. The corresponding set of data points were plotted in the validation plots discussed in the following section.


Figure 5.17: Plot illustrating correlation between WindaRoll model and experimental results at a winding tension of 4.5 lbs with the mandrel in inflated condition

The new values of $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ computed for a specific pressure range of zero to 30 psi were found to be 0.6 psi and 32.0 respectively. The tangential modulus for the specified pressure range was found to be $766,234 \mathrm{psi}$. The model results were then compared with test results as illustrated in Figures 5.17 and 5.18. A good agreement can be observed among model and experimental data easured using pull tabs 2, 3 and 4 when the mandrel is in inflated condition. The pull tabs used for pressure measurements were labeled as illustrated in Figure 5.17. The sharp rise in pressure exhibited by the model in the vicinity of the core is only partially seen in the pull tab test data.


Figure 5.18: Plot illustrating correlation between Combined stiffness model and experimental results at a winding tension of 4.5 lbs with the mandrel in deflated condition With the mandrel in deflated condition, the Combined Stiffness model is in good agreement with experimental findings except for the region near the core. The strain measurements observed in the winding test and after the mandrel was deflated is illustrated in Table 5.3.

| Condition | Strain data <br> (micro strain) |
| :---: | :---: |
| After mandrel inflation | 636 |
| After winding | 511 |
| After mandrel deflation | -28 |

Table 5.3: Strain measurement data

A drop in tangential strain of 600 micro strain has been observed during winding test and after the mandrel is deflated. If an axisymmetric deformation was occurring, the 600 micro strain yielded the following drop in radius:

$$
\varepsilon_{\theta}=-600 \mu s=\frac{u}{r_{c}}=\frac{u}{1.7755}
$$

$$
u=-0.00107 \mathrm{in}
$$

The radial strain was then computed as,

$$
\begin{gathered}
\varepsilon_{r}=\frac{\ln \left(\frac{P}{K_{1}}+1\right)}{K_{2}}=\frac{\ln \left(\frac{25}{0.7190}+1\right)}{31.217} \\
\varepsilon_{r}=0.1146 \mathrm{in} / \mathrm{in}
\end{gathered}
$$

If a decrease in strain by 0.00107 in was considered in the stack test of 1 " stack, then the decreased stack strain is,

$$
\varepsilon_{r}=0.1146-0.00107=0.11352 \mathrm{in} / \mathrm{in}
$$

The pressure computed using the expression,
$P=-0.719\left(e^{31.217(0.11352)}-1\right)=24.1 p s i$

Thus the strain gage readings and pull tab measurements observed the same pressure and established a confidence in the measured data.

The pull tab 1 measured a drop in pressure of 2 psi where as the model predicted a decrease in pressure of 10 psi . This small drop in test pressure coincided with a high tangential strain decrease of $600 \mu$ s observed along the core boundary. The strain gage data measured a drop of 1 psi in pressure after the mandrel is deflated and this is in good agreement with the pull tab test data. A reasonable discrepancy has been observed between model results and test data in the vicinity of the core.

The mandrel and core used in the winding tests were then carefully examined. Strain measurements were conducted on the core periphery with the mandrel in inflated condition. The mandrel orientation was fixed and the relative position of core with respect to mandrel has been changed during strain measurement. The results of the experiment have confirmed a non axissymmetric expansion of the core on the Goldenrod mandrel and are illustrated in Figure 5.20.


Figure 5.19: Strain measurement performed on expanding mandrel

This strain behavior has been confirmed from the working principle of Goldenrod shaft illustrated in Figure 4.6.The rubber ledges have been found to protrude beyond the steel sleeves to grip the core and this causes a lobe shaped expansion of core rather than a concentric expansion. The mandrel was expanded and the offset in concentricity measured using dial indicator has been found to be 0.008 in .

The core stiffness used in winding models is based upon the assumption of asymmetric radial deformation which results from a hydrostatic pressure presented on the outer surface of an axisymmetric core.

This assumption may not be valid when considering the asymmetry the mandrel induces in the core upon inflation. The tangential strain data suggest substantial bending deformations are induced in the core by the expanding mandrel.

Based on the strain data illustrated in Table 5.3 and the working principle of golden rod mandrel, the core has been observed to be supported by the mandrel only at three equidistant locations as illustrated in Figure 5.20.


Figure 5.20: Three point support provided by the mandrel

The three point support provided by the mandrel was simulated along the interior of core using finite element method. The orthotropic properties of the core measured experimentally to compute anisotropic core stiffness discussed in section 5.1 was utilized to define the material model of the core. A pressure ( P ) of 25 psi observed in the 4.5 lbs constant tension winding process was simulated along exterior of core. The average of the tangential strains ( $\varepsilon_{\theta \theta a \mathrm{avg}}$ ) experienced by the exterior of core was evaluated from the deformation characteristic of the exterior of the core (Figure 5.21).


Figure 5.21: Deformation characteristic of mandrel

The core stiffness was then evaluated as,

$$
\begin{equation*}
E_{c}=\frac{P}{\varepsilon_{\theta \theta_{a v g}}} \tag{51}
\end{equation*}
$$

The core stiffness has been evaluated to be $3,726 \mathrm{psi}$.

To determine whether the winding models better predict the pressures in the vicinity of the core when axisymmetry does exist centerwinding experiments at a constant winding tension of 4.5 lbs were performed on a thick wall steel core. This core had a $3.00 "$ ID and a $3.4 "$ OD. The core is shown in a winding experiment in Figure 5.21. A steel core was utilized because the expanding mandrel cannot significantly deform the steel core due to its high core modulus. Hence the
pressures measured using pull tabs should compare with the model. Pressure measurement has been performed using pull tabs and the results are compared with Winding model results predicted using a steel core as illustrated in Figure 5.22.


Figure 5.22: Winding experiment performed on steel core


Figure 5.23: Comparison of radial pressure prediction between WindaRoll and experimental findings

In Figure 5.23 good agreement is shown between pull tab pressure data at all radius locations and the model. Thus as long as axisymmetry is maintained at the core during winding good agreement is seen between the models and tests.

## CHAPTER VI

## CONCLUSIONS AND FUTURE SCOPE

Based on the observation of a good agreement between experimental findings of Hakiel and results predicted by the winding model modified to include web thickness parameter, the model has been used to measure the impact of web thickness on wound roll stress distribution. It has been concluded that the wound roll pressures in a wound roll increases as the web thickness is increased. The inaccuracy of Hakiel's model to respond to web thickness variation is evident from the pressure distribution computed by the model for various web thickness. It has also been found that Hakiel's model can compute wound roll pressures within $10 \%$ error band when the web thickness varied from $6 \%$ to $202 \%$ of the reference web thickness value ' $h$ '. As long as the web thickness varies within these limits, Hakiel's model can still be applied to compute wound roll pressures and stresses in non homogeneous webs. The high sensitivity of wound roll pressure distribution to web thickness variation computed by modified WindaRoll model implies the need for the best estimate of web thickness in non wovens.

The experimental findings validated the wound roll pressure computation by Combined stiffness model at locations away from the core. A careful investigation placed to study the boundary condition existing near the core witnessed an asymmetry condition rather than an axisymmetric boundary condition. Since the winding models are one dimensional based on the assumption of
an axisymmetric boundary condition and the core stiffness has been incorporated into these models as an one dimensional radial stiffness, these models are not accurate enough to predict wound roll pressures near the core. The asymmetry condition existing at the core was confirmed by the strain and pressure measurements recorded near the core vicinity. As long as the core is subjected to a hydrostatic expansion by the mandrel to develop an axisymmetric boundary condition, the 1D winding models cannot accurately define the stress situation in the vicinity of the core.

## Future Scope

The future scope this work is to improve mandrel designs used in the web handling industry to provide an axisymmetric boundary condition in the vicinity of the core. This study also instigates the need for careful examination of the wound roll pressure distribution existing along the circumferential locations of the exterior of the core.

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## APPPENDICES

## Modified WindaRoll model

Option Base 0
Option Explicit
DefDbl A-J

Public r()$, \mathrm{p}(), \mathrm{dp}(), \mathrm{t}(), \operatorname{er}(), \mathrm{tw}(), \mathrm{ho}()$, delp(), maxpress, n , ktheta, ltest, aweb, god
Public a()$, \mathrm{b}(), \mathrm{c}(), \mathrm{d}()$, beta(), gama(), h
Public ngrids\%, Asize, $\mathrm{j} \%$, $\mathrm{i} \%$, lap\%, $\mathrm{jj} \%$, $\mathrm{k} \%$, $\mathrm{ii} \%$, NLAPS
' number of grid points '
Public cid, cod, rod, ecm, et, muweb, mucore, sten, taper, kone, ktwo, rk, nip_dia
Public velocity, vis, pli, rms_bot, rms_top, air_option, caliper, Winder_option, units_option, taper_option

Public Dt, NTemp, Acore, Arad, Atang, Ec, nipforce, nip_taper, thermal_option, wid, cof, ec_option, R1, f, viscosity

Public ten, req, hh, eq_rms, factor1, factor2, mu, mut, nit, tw_nip, r02, rc2, ERLO, ERHI
Public cc, aa1, bb1, cc1, ratio, hr, pr, Pa, erstack, rout, rinc, rin, vrt, vtr, hunit
Public CONE, CTWO
Sub hakiel()
With Application
.Calculation $=$ xlManual
End With
'Dim outarray(ngrids\%, 5)

Dim PressureChart As Object
Dim TensionChart As Object
Dim eff_tenChart As Object
Dim speedChart As Object
Dim erChart As Object
Dim torqueChart As Object
' Get model parameters from spread sheet

```
sten = Range("sten")
    taper = Range("taper")
    caliper = Range("caliper")
    nipforce = Range("nipforce")
    nip_taper = Range("nip_taper")
    wid = Range("width")
    pli \(=\) Range("pli")
    cof = Range("cof")
    vis = Range("viscosity")
    nip_dia = Range("nip_dia")
    velocity = Range("velocity")
    rms_top = Range("rms_top")
```

rms_bot = Range("rms_bot")

$$
\begin{aligned}
& \text { cid = Range("cid") } \\
& \text { cod = Range("cod") } \\
& \text { rod = Range("rod") } \\
& \text { ecm = Range("ecm") } \\
& \text { et = Range("et") } \\
& \text { kone = Range("KONE") } \\
& \text { ktwo = Range("KTWO") } \\
& \text { muweb = Range("muweb") } \\
& \text { mucore = Range("mucore") } \\
& \text { Ec = Range("ec") }
\end{aligned}
$$

ktheta = Range("C73")
ltest = Range("c74")
aweb = Range("c75")
ec_option = Range("ec_option")
Winder_option = Range("winder_option")
air_option = Range("air_option")
units_option = Range("units_option")
taper_option = Range("taper_option")
Acore = Range("acore")
Atang = Range("atang")
Arad = Range("arad")

Dt = Range("dt")
thermal_option = Range("thermal_option")
ngrids\% = Range("nn")

If (units_option = 1) Then CONE = $1 / 5$
If (units_option $=2$ ) Then CONE $=1 / 600$
If (units_option $=1$ ) Then CTWO $=0.342$
If (units_option = 2) Then CTWO $=0.01406$
'Dim arrays'
Asize $=$ ngrids $\%+1$
ReDim tw(Asize), a(Asize), b(Asize), c(Asize), d(Asize), dp(Asize), beta(Asize), gama(Asize), r(Asize), p(Asize), t(Asize), er(Asize), ho(Asize), delp(Asize)

ReDim outarray(Asize, 5)
' calculate " h " the grid spacing '
$\mathrm{h}=((\operatorname{rod}-\operatorname{cod}) / 2!) /$ ngrids $\%$
' calculate $\mathrm{r}(\mathrm{j} \%$ ) the radius array '
For $\mathrm{j} \%=0$ To ngrids $\%$
$r(j \%)=\operatorname{cod} / 2!+h * j \%$
'R1 $=(\mathrm{r}(\mathrm{j} \%)$ * nip_dia $/ 2 \#) /(\mathrm{r}(\mathrm{j} \%)+$ nip_dia $/ 2 \#)$
'If Not $\mathrm{f}=0$ Then
'f $=$ pli * $(1!-($ nip_taper / 100! $) *((r(j \%)-r(0)) / r(j \%)))$
'ho(j\%) $=4 \#$ * viscosity * velocity / 60\# * R1 / f
'Else
$\mathrm{ho}(\mathrm{j} \%)=0.65 * \mathrm{r}(\mathrm{j} \%) *(12 * 0.0000000026 * \text { velocity } *(12 / 60) /(\text { sten } * \text { caliper }))^{\wedge}(2 / 3)$
'End If
outarray $(\mathrm{j} \%, 0)=\mathrm{r}(\mathrm{j} \%)$
Next j\%
maxpress $=0$
'zero arrays'
For i\% = 0 To ngrids\%
$\operatorname{tw}(\mathrm{i} \%)=0: \mathrm{a}(\mathrm{i} \%)=0: \mathrm{b}(\mathrm{i} \%)=0: \mathrm{c}(\mathrm{i} \%)=0: \mathrm{d}(\mathrm{i} \%)=0: \mathrm{dp}(\mathrm{i} \%)=0: \operatorname{beta}(\mathrm{i} \%)=0: \operatorname{gama}(\mathrm{i} \%)=0:$ $\mathrm{p}(\mathrm{i} \%)=0: \mathrm{t}(\mathrm{i} \%)=0: \operatorname{er}(\mathrm{i} \%)=0$

Next i\%
' calculate $\mathrm{tw}(\mathrm{j} \%)$ the winding tension array ${ }^{\prime}$
For $\mathrm{j} \%=0$ To ngrids\%
If (taper_option = 1) Then

$$
\text { ten }=\operatorname{sten}+(\operatorname{sten} *(-\operatorname{taper} / 100) /(\operatorname{rod} / 2-r(0)) *(r(j \%)-r(0)))
$$

Else
ten $=\operatorname{sten} *(1!-(\operatorname{taper} / 100!) *((r(j \%)-r(0)) / r(j \%)))$
End If
pli $=$ nipforce $/$ wid
If (air_option = 1) Then
req $=r(j \%) *$ nip_dia $/ 2 /\left(r(j \%)+n i p \_d i a / 2\right)$
$\mathrm{hh}=(4 \# *$ vis $*$ velocity $*$ req $*$ CONE $) /$ pli
eq_rms $=\left(\text { rms_top } \wedge 2+\text { rms_bot }^{\wedge} 2\right)^{\wedge} 0.5$
factor1 $=$ eq_rms
factor2 $=3 \# *$ eq_rms
$\mathrm{mu}=\mathrm{cof}$
If $\mathrm{hh}<$ factor1 Then

$$
\mathrm{mut}=\mathrm{mu}
$$

ElseIf hh < factor2 Then

$$
\mathrm{mut}=\mathrm{mu} *(3 / 2-\mathrm{hh} /(2 * \text { eq_rms }))
$$

ElseIf hh $>$ factor2 Then

$$
\text { mut }=0.0001
$$

End If
End If
nit $=\operatorname{cof} *$ pli $/$ caliper
If (air_option = 1) Then
nit $=$ mut $*$ pli $/$ caliper
End If
tw_nip $=$ nit $*(1!+(-$ nip_taper $/ 100) /(\operatorname{rod} / 2-r(0)) *(r(j \%)-r(0)))$
If units_option $=2$ Then tw_nip $=$ tw_nip * 10
If Winder_option $=1$ Then $\operatorname{tw}(\mathrm{j} \%)=$ ten
If Winder_option $=2$ Then $\operatorname{tw}(j \%)=$ ten + tw_nip
If Winder_option $=3$ Then $\operatorname{tw}(\mathrm{j} \%)=$ tw_nip
$\operatorname{outarray}(\mathrm{j} \%, 3)=\operatorname{tw}(\mathrm{j} \%)$
Next j\%
' calculate ecm the core stiffness from Roisum p-25'
If ec_option $=1$ Then
$\mathrm{r} 02=(\operatorname{cod} / 2!) *(\operatorname{cod} / 2!)$
$\operatorname{rc} 2=(\operatorname{cid} / 2!) *(\operatorname{cid} / 2!)$
$\mathrm{Ec}=\mathrm{ecm}^{*}(\mathrm{r} 02-\mathrm{rc} 2) /(\mathrm{r} 02+\mathrm{rc} 2-$ mucore $*(\mathrm{r} 02-\mathrm{rc} 2))$
Range("ec") = Ec
End If
' calculate cc the core constant '
$\mathrm{cc}=((($ ktheta $*$ lest $) /(\mathrm{h} *$ wid $* \mathrm{Ec})))-1!+$ muweb
ActiveSheet.Range("c78") = cc
$\mathrm{rk}=1!+\mathrm{h} * \mathrm{cc} / \mathrm{r}(0)$
' Add lap \#1 ${ }^{\prime}$
$\mathrm{p}(0)=(\operatorname{tw}(0)+\mathrm{tw}(1)) / 2!* \mathrm{~h} / \mathrm{r}(0)$
' Add lap \#2 '
$\mathrm{p}(1)=(\operatorname{tw}(1)+\operatorname{tw}(2)) / 2!* h / r(1)$
$\mathrm{p}(0)=\mathrm{p}(0)+\mathrm{p}(1) / \mathrm{rk}$
Call calcer(p(1), er(1), r(1), tw(1))
' Add lap \#3 '
$\mathrm{p}(2)=(\operatorname{tw}(2)+\operatorname{tw}(3)) / 2!* h / r(2)$
aal $=1!-(3!* h) /(2!* r(1))$
$\mathrm{bb} 1=(\mathrm{h} * \mathrm{~h} /(\mathrm{r}(1) * \mathrm{r}(1))) *(1!-(($ ktheta $*$ lest $) /(\mathrm{h} *$ wid $)) / \operatorname{er}(1))-2!$
$\mathrm{cc} 1=1!+(3!* h) /(2!* r(1))$
$\mathrm{dp}(0)=\mathrm{cc} 1 * \mathrm{p}(2) /(-\mathrm{rk} * \mathrm{bb} 1-\mathrm{a} 1)$
$\mathrm{dp}(1)=\mathrm{rk} * \mathrm{dp}(0)$
$\mathrm{p}(0)=\mathrm{p}(0)+\mathrm{dp}(0)$
$\mathrm{p}(1)=\mathrm{p}(1)+\mathrm{dp}(1)$
Call calcer(p(1), er(1), r(1), tw(1))
Call calcer(p(2), er(2), r(2), tw(2))
' Add lap \#4 thru ngrids\% using Tri-diagonal '
For lap\% = 4 To ngrids\%
Range("n") = lap\%
Call tridiag(lap\%)
' Add dp() to p() '
For $\mathrm{jj} \%=0$ To lap\% -1
$\mathrm{p}(\mathrm{jj} \%)=\mathrm{p}(\mathrm{jj} \%)+\mathrm{dp}(\mathrm{jj} \%)$
Call calcer(p(jj\%), er(jj\%), r(jj\%), tw(jj\%))
Next jj\%
Next lap\% 'end of lap 4 thru n loop '
'Thermal Stress Analysis
If (thermal_option = 1) Then
Call ThermalStress
Else
End If

```
t(ngrids%) = sten
t(0)=-p(0) -r(0) * ((p(1) - p(0))/h)
outarray(0, 1) =p(0)
outarray(0, 2) =t(0)
outarray (0, 4) = dp(0)
For jj% = 1 To ngrids%
    t(jj%) = -p(jj%) -r(jj%) * ((p(jj% + 1) - p(jj% - 1)) / (2# * h))
    outarray(jj%,1) = p(jj%)
```

```
outarray \((\mathrm{jj} \%, 2)=\mathrm{t}(\mathrm{jj} \%)\)
\(\operatorname{outarray}(\mathrm{jj} \%, 4)=\operatorname{dp}(\mathrm{jj} \%)\)
```

Next jj\%
outarray $($ ngrids $\%, 2)=-\mathrm{p}($ ngrids $\%)-\mathrm{r}($ ngrids $\%) *((\mathrm{p}($ ngrids $\%)-\mathrm{p}($ ngrids $\%-1)) /(\mathrm{h}))$

If (ngrids $\%>100$ ) Then
ratio $=\mathrm{CDbl}($ ngrids $\%$ ) $/ 100 \#$
For $\mathrm{i} \%=0$ To 100
$\mathrm{k} \%=\operatorname{CInt}(\mathrm{i} \% *$ ratio $)$
$\operatorname{outarray}(\mathrm{i} \%, 0)=\operatorname{outarray}(\mathrm{k} \%, 0)$
$\operatorname{outarray}(\mathrm{i} \%, 1)=\operatorname{outarray}(\mathrm{k} \%, 1)$
$\operatorname{outarray}(\mathrm{i} \%, 2)=\operatorname{outarray}(\mathrm{k} \%, 2)$
$\operatorname{outarray}(\mathrm{i} \%, 3)=\operatorname{outarray}(\mathrm{k} \%, 3)$
$\operatorname{outarray}(\mathrm{i} \%, 4)=\operatorname{outarray}(\mathrm{k} \%, 4)$
Next i\%
Else
End If
$\operatorname{outarray}(100,1)=\operatorname{tw}($ ngrids $\%) * \mathrm{~h} / \mathrm{r}($ ngrids $\%)$
$\operatorname{outarray}(100,4)=\operatorname{tw}($ ngrids $\%) * h / r($ ngrids $\%)$
' Write results back to spread sheet
Range("outarray") = outarray

If ec_option $=2$ Then
Range("ec") = Ec
End If

Set PressureChart = ActiveSheet.ChartObjects("PressureChart")
PressureChart.Visible $=$ False
Set TensionChart = ActiveSheet.ChartObjects("TensionChart")
TensionChart.Visible $=$ False
Set eff_tenChart = ActiveSheet.ChartObjects("eff_tenChart")
eff_tenChart.Visible $=$ False
Set speedChart = ActiveSheet.ChartObjects("speedChart")
speedChart.Visible $=$ False
'Set erChart = ActiveSheet.ChartObjects("erChart")
'erChart. Visible $=$ False
Set torqueChart = ActiveSheet.ChartObjects("torqueChart")
torqueChart.Visible $=$ False
hunit $=\operatorname{CInt}(((\operatorname{rod}-\operatorname{cod}) / 2!) / 4)+1$

PressureChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
PressureChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
PressureChart.Chart.Axes(xlCategory).MajorUnit = hunit
If (units_option = 1) Then PressureChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option = 2) Then PressureChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then PressureChart.Chart.Axes(xlValue).AxisTitle.Caption = "Pressure (psi)"

If (units_option = 2) Then PressureChart.Chart.Axes(xlValue).AxisTitle.Caption $=$ "Pressure (kPa)"

TensionChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
TensionChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
TensionChart.Chart.Axes(xlCategory).MajorUnit = hunit
If (units_option = 1) Then TensionChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option = 2) Then TensionChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then TensionChart.Chart.Axes(xlValue).AxisTitle.Caption = "Stress (psi)"

If (units_option = 2) Then TensionChart.Chart.Axes(xlValue).AxisTitle.Caption = "Stress (kPa)"
eff_tenChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
eff_tenChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
eff_tenChart.Chart.Axes(xlCategory).MajorUnit = hunit
If (units_option = 1) Then eff_tenChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option = 2) Then eff_tenChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then eff_tenChart.Chart.Axes(xlValue).AxisTitle.Caption = "Wound-OnTension (psi)"

If (units_option = 2) Then eff_tenChart.Chart.Axes(xlValue).AxisTitle.Caption = "Wound-OnTension (kPa)"
speedChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
speedChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
speedChart.Chart.Axes(xlCategory).MajorUnit $=$ hunit
If (units_option = 1) Then speedChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option $=2$ ) Then speedChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then speedChart.Chart.Axes(xlValue).AxisTitle.Caption = "Speed (fpm)"
If (units_option = 2) Then speedChart.Chart.Axes(xIValue).AxisTitle.Caption = "Speed (mpm)"
'erChart.Chart.Axes(xlCategory).MinimumScale = 0
'If maxpress > 200 Then
'fff $=100$
'Else
'fff $=10$
'End If
'erChart.Chart.Axes(xlCategory).MaximumScale $=(\operatorname{Int}(\operatorname{maxpress} / \mathrm{fff})+1) *$ fff
torqueChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
torqueChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
torqueChart.Chart.Axes(xlCategory).MajorUnit = hunit
If (units_option $=1$ ) Then torqueChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option $=2$ ) Then torqueChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then torqueChart.Chart.Axes(xlValue).AxisTitle.Caption = "Torque (inlb)"

If (units_option = 2) Then torqueChart.Chart.Axes(xlValue).AxisTitle.Caption = "Torque (N$\mathrm{cm})^{\prime}$

With Application
.Calculation $=x$ 1Automatic
End With

PressureChart.Visible $=$ True

```
TensionChart.Visible = True
eff_tenChart.Visible = True
speedChart.Visible = True
'erChart.Visible = True
torqueChart.Visible = True
```

End Sub
$1 * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ।$

Sub tridiag(lap\%): ' START OF tridiag

```
dp(lap% - 1) = (tw(lap% - 1) + tw(lap%)) / 2! * h / r(lap% - 1)
a(0)=0!
b(0) = rk
c(0) = -1!
d(0)=0!
For iii% = 1 To lap% - 2
hr = h / r(iii%)
a(iii%) = 1!-1.5 * hr
b(iii%) = hr * hr * (1! - ((ktheta * ltest) / (h * wid)) / er(iii%)) - 2!
c(iii%) = 1! + 1.5 * hr
d(iii%)=0!
```

Next iii\%
$\mathrm{a}(\mathrm{lap} \%-1)=0$ !
$\mathrm{b}($ lap $\%-1)=1$ !

```
d(lap% % 1) = dp(lap% - 1)
c(lap%-1) = 0!
beta(0)=b(0)
gama(0)=d(0)/b(0)
```

For iii\% = 1 To lap\% - 1
beta(iii\%) $=\mathrm{b}(\mathrm{iii} \%)-\mathrm{a}(\mathrm{iii} \%) * \mathrm{c}(\mathrm{iii} \%-1) /$ beta(iii\%-1)
gama(iii\%) $=(\mathrm{d}(\mathrm{iii} \%)$ - $\mathrm{a}(\mathrm{iii} \%)$ * gama(iii\% - 1)) / beta(iii\%)
Next iii\%
$\operatorname{dp}(\operatorname{lap} \%-1)=\operatorname{gama}(\operatorname{lap} \%-1)$
For $\mathrm{iii} \%=(\mathrm{lap} \%-2)$ To 0 Step -1
$\mathrm{dp}(\mathrm{iii} \%)=\operatorname{gama}(\mathrm{iii} \%)-\mathrm{c}(\mathrm{iii} \%) * \operatorname{dp}(\mathrm{iii} \%+1) / \operatorname{beta}(\mathrm{iii} \%)$

Next iii\%
End Sub
$' * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * \prime$
Sub calcer(Press, erout, $r$, ten): ' START OF calcer
$1 * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * 1$

If Press > maxpress Then
maxpress $=$ Press
Else
End If
'If (Press > 400) Then
$' \mathrm{pr}=400$
'Else
$\mathrm{pr}=$ Press
'End If

If (air_option = 1) Then
' calculate equivalent roughness

$$
\text { eq_rms }=(\text { rms_top } \wedge 2+\text { rms_bot } \wedge 2)^{\wedge} 0.5
$$

If $($ Winder_option $=1)$ Then

```
    hh = 0.65 * r * CTWO * (12 * vis * velocity / (ten * caliper)) ^ (2 / 3)
```

Else
req $=r *$ nip_dia $/ 2 /\left(r+n i p \_d i a / 2\right)$
hh $=(4 \# *$ vis * velocity * req * CONE $) /$ pli
End If
factor $1=$ eq_rms
factor $2=3 \# *$ eq_rms
If (units_option = 1) Then

$$
\mathrm{Pa}=14.7
$$

Else

$$
\mathrm{Pa}=14.7 * 6.89
$$

End If
If $\mathrm{hh}<$ factor 1 Then

$$
\text { erout }=\text { ktwo } *(\text { kone }+ \text { pr })
$$

ElseIf hh $>$ factor 1 And $h h<$ factor2 Then

$$
\text { erstack }=\text { ktwo } *(\text { kone }+ \text { pr })
$$

'If (units_option = 1) Then
'erout $=($ caliper +hh$) /(($ caliper $/$ erstack $)+\mathrm{hh} *($ ten * caliper $/ \mathrm{r}+\mathrm{Pa}) /(\mathrm{pr}+$ ten * caliper $/$
$\mathrm{r}+\mathrm{Pa})^{\wedge} 2$ )
'Else
'erout $=($ caliper $/ 100+h h) /\left((\right.$ caliper $/ 100 /$ erstack $)+h h^{*}($ ten * caliper $/ \mathrm{r}+\mathrm{Pa}) /(\mathrm{pr}+$ ten $*$ caliper $/ \mathrm{r}+\mathrm{Pa})^{\wedge} 2$ )
'End If
erout $=$ erstack $+($ hh - eq_rms $) *\left((\mathrm{pr}+\mathrm{Pa}+\text { ten } * \text { caliper } / \mathrm{r})^{\wedge} 2 /(\right.$ ten $*$ caliper $/ \mathrm{r}+\mathrm{Pa})-$ erstack) / ( 2 * eq_rms)

## Else

$$
\text { erout }=(\mathrm{pr}+\mathrm{Pa}+\text { ten } * \text { caliper } / \mathrm{r})^{\wedge} 2 /(\text { ten } * \text { caliper } / \mathrm{r}+\mathrm{Pa})
$$

End If
Else

$$
\text { erout }=\text { ktwo } *(\text { kone }+ \text { pr })
$$

End If
End Sub

Sub ThermalStress()

```
rout = rod / 2
rinc}=\textrm{cid}/
rin}=\operatorname{cod}/
NLAPS = ngrids%
NTemp = 20
```

$$
\mathrm{Dt}=-\mathrm{Dt}
$$

$$
\begin{aligned}
& \mathrm{r}(0)=\text { rin } \\
& \text { lap }=1 \\
& \mathrm{~h}=(\text { rout }- \text { rin }) / \text { NLAPS }
\end{aligned}
$$

'XX I ran most thermal calcs w/ muweb=0
$\mathrm{vrt}=$ muweb

For $\mathrm{i}=2$ To NLAPS +1

$$
r(i-1)=r(0)+(i-2) * h
$$

Next i

$$
\mathrm{Dt}=\mathrm{Dt} / \mathrm{NTemp}
$$

For $\mathrm{k}=1$ To NTemp

$$
\begin{aligned}
& \operatorname{er}(0)=\mathrm{ktwo} *(\operatorname{kone}+\mathrm{p}(0)) \\
& \mathrm{vtr}=\mathrm{vrt} * \operatorname{er}(0) / \text { et } \\
& \mathrm{d}(0)=(1 \#-\text { et } / \mathrm{er}(0) * \mathrm{vtr}-\mathrm{et} /(\mathrm{Ec} * \operatorname{rin})-\operatorname{rin} / \mathrm{h}) \\
& \mathrm{c}(0)=\mathrm{rin} / \mathrm{h} \\
& \mathrm{~b}(0)=\mathrm{et} * \mathrm{Dt} *(\text { Acore }- \text { Atang }) \\
& \text { For i }=2 \text { To NLAPS } \\
& \operatorname{er}(\mathrm{i}-1)=\mathrm{ktwo} *(\text { kone }+\mathrm{p}(\mathrm{i}-1)) \\
& \mathrm{vtr}=\mathrm{vrt} * \operatorname{er}(\mathrm{i}-1) / \text { et }
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{a}(\mathrm{i}-2)=\left(\mathrm{r}(\mathrm{i}-1)^{\wedge} 2 / \mathrm{h} \wedge 2-\mathrm{r}(\mathrm{i}-1) /(2 \# * \mathrm{~h}) *(3 \#-\mathrm{et} / \mathrm{er}(\mathrm{i}-1) * \mathrm{vtr}+\mathrm{vrt})\right) \\
& \mathrm{d}(\mathrm{i}-1)=\left(1 \#-2 \# * \mathrm{r}(\mathrm{i}-1)^{\wedge} 2 / \mathrm{h} \wedge 2+\mathrm{vrt}-\mathrm{et} / \mathrm{er}(\mathrm{i}-1) *(1+\mathrm{vtr})\right) \\
& \mathrm{c}(\mathrm{i}-1)=\mathrm{r}(\mathrm{i}-1)^{\wedge} 2 / \mathrm{h} \wedge 2+\mathrm{r}(\mathrm{i}-1) /(2 \# * \mathrm{~h}) *(3 \#-\text { et } / \mathrm{er}(\mathrm{i}-1) * \mathrm{vtr}+\mathrm{vrt}) \\
& \mathrm{b}(\mathrm{i}-1)=\mathrm{et} * \mathrm{Dt}^{*}(\text { Arad }- \text { Atang })
\end{aligned}
$$

Next i
$\mathrm{d}($ NLAPS $)=1 \#$
$b($ NLAPS $)=0 \#$
Call SOLVETRI(NLAPS + 1)
lap $=$ NLAPS +1

For $\mathrm{i}=1$ To lap

$$
\mathrm{p}(\mathrm{i}-1)=\mathrm{p}(\mathrm{i}-1)+\mathrm{dp}(\mathrm{i}-1)
$$

Next i
Next k
End Sub
'***** Subroutine SOLVETRI
'***** -SOLVES THE TRIDIAGONAL SYSTEM OF DIMENSION IDIM
'***** FOR THE SOLUTION VECTOR X(IDIM)

## Sub SOLVETRI(IDIM)

$\mathrm{n}=\mathrm{IDIM}$
For $\mathrm{i}=2$ To n
$\mathrm{d}(\mathrm{i}-1)=\mathrm{d}(\mathrm{i}-1)-(\mathrm{a}(\mathrm{i}-2) / \mathrm{d}(\mathrm{i}-2)) * \mathrm{c}(\mathrm{i}-2)$
$\mathrm{b}(\mathrm{i}-1)=\mathrm{b}(\mathrm{i}-1)-(\mathrm{a}(\mathrm{i}-2) / \mathrm{d}(\mathrm{i}-2)) * \mathrm{~b}(\mathrm{i}-2)$
Next i
$d p(n-1)=b(n-1) / d(n-1)$
For $\mathrm{i}=(\mathrm{n}-1)$ To 1 Step -1
$d p(i-1)=(b(i-1)-c(i-1) * d p(i)) / d(i-1)$

Next i

End Sub

Sub Highlightinputs()

If Range("Winder_option") = 1 Then 'Center Winding
Range("sten").Interior.ColorIndex $=37$
Range("taper").Interior.ColorIndex $=37$
Range("nipforce").Interior.ColorIndex $=0$
Range("nip_taper").Interior.ColorIndex $=0$
End If

If Range("Winder_option") = 2 Then 'Center Winding with Nip Range("sten").Interior.ColorIndex = 37

Range("taper").Interior.ColorIndex = 37
Range("nipforce").Interior.ColorIndex = 37
Range("nip_taper").Interior.ColorIndex = 37

## End If

If Range("Winder_option") = 3 Then 'Surface Winding
Range("sten").Interior.ColorIndex $=0$
Range("taper").Interior.ColorIndex $=0$
Range("nipforce").Interior.ColorIndex $=37$
Range("nip_taper").Interior.ColorIndex = 37
End If
If Range("air_option") = 2 Then 'No air calculations
Range("viscosity").Interior.ColorIndex = 0
Range("nip_dia").Interior.ColorIndex $=0$
Range("velocity").Interior.ColorIndex $=0$
Range("rms_top").Interior.ColorIndex = 0
Range("rms_bot").Interior.ColorIndex = 0

## End If

If Range("air_option") = 1 Then
Range("viscosity").Interior.ColorIndex $=37$
Range("velocity").Interior.ColorIndex $=37$
Range("rms_top").Interior.ColorIndex = 37
Range("rms_bot").Interior.ColorIndex = 37
If Range("Winder_option") = 1 Then Range("nip_dia").Interior.ColorIndex = 0 'Air calculations w/o nip

If Range("Winder_option") > 1 Then Range("nip_dia").Interior.ColorIndex = 37 'Air calculations with nip

Else

End If
If Range("thermal_option") = 1 Then 'Thermoelastic calculations
Range("arad").Interior.ColorIndex = 37
Range("atang").Interior.ColorIndex = 37
Range("acore").Interior.ColorIndex = 37
Range("dt").Interior.ColorIndex = 37
End If

If Range("thermal_option") $=2$ Then 'No Thermoelastic calculations Range("arad").Interior.ColorIndex = 0

Range("atang").Interior.ColorIndex = 0
Range("acore").Interior.ColorIndex = 0
Range("dt").Interior.ColorIndex = 0
End If

If Range("ec_option") = 1 Then 'Calculate Core Stiffness
Range("ecm").Interior.ColorIndex = 37
Range("mucore").Interior.ColorIndex = 37
Range("ec").Interior.ColorIndex = 0
End If

If Range("ec_option") $=2$ Then 'Input Core Stiffness
Range("ecm").Interior.ColorIndex $=0$
Range("mucore").Interior.ColorIndex = 0
Range("ec").Interior.ColorIndex = 37
End If
'If Range("units_option") = 1 Then
' Range(
'End If
End Sub

## Combined Stiffness Winding Model

Option Base 0
Option Explicit
DefDbl A-J

Public r()$, \mathrm{p}(), \mathrm{dp}(), \mathrm{t}(), \operatorname{er}(), \mathrm{tw}()$, ho(), delp(), maxpress, $\mathrm{n}, \mathrm{tn}()$
Public a()$, \mathrm{b}(), \mathrm{c}(), \mathrm{d}()$, beta(), gama(), $\mathrm{h}, \mathrm{j} 1$
Public ngrids\%, Asize, $\mathrm{j} \%$, $\mathrm{i} \%$, lap\%, $\mathrm{jj} \%, \mathrm{k} \%$, iii\%, NLAPS, estep, Nmod, iter, s , inc ' number of grid points '

Public cid, cod, rod, ecm, et, muweb, mucore, sten, taper, kone, ktwo, rk, nip_dia
Public velocity, vis, pli, rms_bot, rms_top, air_option, caliper, Winder_option, units_option, taper_option

Public Dt, NTemp, Acore, Arad, Atang, Ec, nipforce, nip_taper, thermal_option, wid, cof, ec_option, R1, f, viscosity

Public ten, req, hh, eq_rms, factor1, factor2, mu, mut, nit, tw_nip, r02, rc2, ERLO, ERHI
Public cc, aa1, bb1, cc1, ratio, hr, pr, Pa, erstack, rout, rinc, rin, vrt, vtr, hunit
Public CONE, CTWO
Sub hakiel()
With Application
.Calculation $=$ xlManual
End With
'Dim outarray(ngrids\%, 5)

Dim PressureChart As Object
Dim TensionChart As Object
Dim eff_tenChart As Object
Dim speedChart As Object
Dim erChart As Object
Dim torqueChart As Object
' Get model parameters from spread sheet

```
sten = Range("sten")
taper = Range("taper")
caliper = Range("caliper")
nipforce = Range("nipforce")
    nip_taper = Range("nip_taper")
    wid = Range("width")
    pli = Range("pli")
    cof = Range("cof")
    vis = Range("viscosity")
    nip_dia = Range("nip_dia")
    velocity = Range("velocity")
    rms_top = Range("rms_top")
    rms_bot = Range("rms_bot")
```

```
cid = Range("cid")
cod = Range("cod")
rod = Range("rod")
ecm = Range("ecm")
et = Range("et")
kone = Range("KONE")
ktwo = Range("KTWO")
muweb = Range("muweb")
mucore = Range("mucore")
Ec = Range("ec")
```

ec_option = Range("ec_option")
Winder_option = Range("winder_option")
air_option = Range("air_option")
units_option = Range("units_option")
taper_option = Range("taper_option")
Acore = Range("acore")
Atang = Range("atang")
Arad = Range("arad")
Dt = Range("dt")
thermal_option = Range("thermal_option")
ngrids\% = Range("nn")

If (units_option = 1) Then CONE = $1 / 5$
If (units_option $=2$ ) Then CONE $=1 / 600$
If (units_option $=1$ ) Then CTWO $=0.342$
If (units_option = 2) Then CTWO $=0.01406$
'Dim arrays'
Asize $=$ ngrids $\%+1$
ReDim tw(Asize), a(Asize), b(Asize), c(Asize), d(Asize), dp(Asize), beta(Asize), gama(Asize), r (Asize), p (Asize), t (Asize), er(Asize), ho(Asize), delp(Asize)

ReDim outarray(Asize, 5), $\operatorname{tn}($ Asize $)$
' calculate " h " the grid spacing '
$\mathrm{h}=((\operatorname{rod}-\operatorname{cod}) / 2!) /($ ngrids $\%)$
' calculate $\mathrm{r}(\mathrm{j} \%$ ) the radius array '
For j\% = 0 To ngrids\%
$r(j \%)=\operatorname{cod} / 2!+h * j \%$
'R1 $=(\mathrm{r}(\mathrm{j} \%)$ * nip_dia / 2\#) / (r(j\%) + nip_dia / 2\# $)$
'If Not $\mathrm{f}=0$ Then
'f = pli * $(1!-($ nip_taper / 100! $) *((r(j \%)-r(0)) / r(j \%)))$
'ho(j\%) $=4 \#$ * viscosity * velocity / 60\# * R1 / f
'Else
$\mathrm{ho}(\mathrm{j} \%)=0.65 * \mathrm{r}(\mathrm{j} \%) *(12 * 0.0000000026 * \text { velocity } *(12 / 60) /(\text { sten } * \text { caliper }))^{\wedge}(2 / 3)$
'End If
outarray $(\mathrm{j} \%, 0)=\mathrm{r}(\mathrm{j} \%)$
Next j\%
maxpress $=0$
'zero arrays'
For $\mathrm{i} \%=0$ To ngrids\%
$\operatorname{tw}(\mathrm{i} \%)=0: a(\mathrm{i} \%)=0: b(\mathrm{i} \%)=0: c(\mathrm{i} \%)=0: \mathrm{d}(\mathrm{i} \%)=0: d p(\mathrm{i} \%)=0: \operatorname{beta}(\mathrm{i} \%)=0: \operatorname{gama}(\mathrm{i} \%)=0:$
$\mathrm{p}(\mathrm{i} \%)=0: \mathrm{t}(\mathrm{i} \%)=0: \operatorname{er}(\mathrm{i} \%)=0: \operatorname{tn}(\mathrm{i} \%)=0$
Next i\%
' calculate $\mathrm{tw}(\mathrm{j} \%)$ the winding tension array '
For j\% = 0 To ngrids\%
If (taper_option = 1) Then
ten $=\operatorname{sten}+($ sten $*(-\operatorname{taper} / 100) /(\operatorname{rod} / 2-r(0)) *(r(j \%)-r(0)))$
Else
ten $=\operatorname{sten} *(1!-(\operatorname{taper} / 100!) *((r(j \%)-r(0)) / r(j \%)))$
End If
pli $=$ nipforce $/$ wid
If (air_option = 1) Then
req $=r(j \%) *$ nip_dia $/ 2 /\left(r(j \%)+n i p \_d i a / 2\right)$
hh $=(4 \# *$ vis * velocity $*$ req * CONE $) /$ pli
eq_rms $=\left(\text { rms_top } \wedge 2+r m s \_b o t ~ \wedge 2\right)^{\wedge} 0.5$
factorl $=$ eq_rms
factor2 $=3 \# *$ eq_rms
$\mathrm{mu}=\mathrm{cof}$
If $\mathrm{hh}<$ factor 1 Then

$$
\mathrm{mut}=\mathrm{mu}
$$

ElseIf hh $<$ factor2 Then

$$
\text { mut }=\mathrm{mu} *(3 / 2-\mathrm{hh} /(2 * \text { eq_rms }))
$$

ElseIf hh > factor2 Then

$$
\mathrm{mut}=0.0001
$$

## End If

## End If

nit $=\operatorname{cof} *$ pli $/$ caliper
If (air_option = 1) Then
nit $=$ mut $*$ pli $/$ caliper
End If
tw_nip $=$ nit $*(1!+(-$ nip_taper $/ 100) /(\operatorname{rod} / 2-r(0)) *(r(j \%)-r(0)))$
If units_option $=2$ Then tw_nip $=$ tw_nip * 10

If Winder_option $=1$ Then $\operatorname{tw}(\mathrm{j} \%)=$ ten
If Winder_option $=2$ Then $\operatorname{tw}(j \%)=$ ten + tw_nip
If Winder_option $=3$ Then $\operatorname{tw}(\mathrm{j} \%)=$ tw_nip
$\operatorname{outarray}(\mathrm{j} \%, 3)=\operatorname{tw}(\mathrm{j} \%)$
Next j\%
' calculate ecm the core stiffness from Roisum p-25'
If ec_option = 1 Then
$\mathrm{r} 02=(\operatorname{cod} / 2!) *(\operatorname{cod} / 2!)$
$\operatorname{rc} 2=(\operatorname{cid} / 2!) *(\operatorname{cid} / 2!)$
Ec = Range("ec")
End If
' calculate cc the core constant '
$\mathrm{cc}=\mathrm{et} / \mathrm{Ec}-1!+$ muweb
$\mathrm{rk}=1!+\mathrm{h} * \mathrm{cc} / \mathrm{r}(0)$
' Add lap \#1 '
$\mathrm{p}(0)=(\operatorname{tw}(0)+\mathrm{tw}(1)) / 2!* \mathrm{~h} / \mathrm{r}(0)$
' Add lap \#2 ${ }^{\prime}$
$\mathrm{p}(1)=(\operatorname{tw}(1)+\operatorname{tw}(2)) / 2!* h / r(1)$
$\mathrm{p}(0)=\mathrm{p}(0)+\mathrm{p}(1) / \mathrm{rk}$
Call calcer(p(1), er(1), r(1), tw(1))
' Add lap \#3'
$\mathrm{p}(2)=(\operatorname{tw}(2)+\operatorname{tw}(3)) / 2!* h / r(2)$
aal $=1!-(3!* h) /(2!* r(1))$
$\mathrm{bb} 1=(\mathrm{h} * \mathrm{~h} /(\mathrm{r}(1) * r(1))) *(1!-e t / e r(1))-2!$
$\mathrm{cc} 1=1!+(3!* h) /(2!* r(1))$
$\mathrm{dp}(0)=\mathrm{cc} 1$ * $\mathrm{p}(2) /(-\mathrm{rk} * \mathrm{bb} 1-\mathrm{aa} 1)$
$\mathrm{dp}(1)=\mathrm{rk} * \mathrm{dp}(0)$
$\mathrm{p}(0)=\mathrm{p}(0)+\mathrm{dp}(0)$
$\mathrm{p}(1)=\mathrm{p}(1)+\mathrm{dp}(1)$
Call calcer(p(1), er(1), r(1), tw(1))
Call calcer(p(2), er(2), r(2), tw(2))
' Add lap \#4 thru ngrids\% using Tri-diagonal '
For lap\% = 4 To ngrids\%
Range("n") = lap\%
Call tridiag(lap\%)
' Add dp() to p() '

For $\mathrm{jj} \%=0$ To lap $\%-1$
$p(j \mathrm{j} \%)=\mathrm{p}(\mathrm{jj} \%)+\mathrm{dp}(\mathrm{jj} \%)$
Call calcer(p(jj\%), er(jj\%), r(jj\%), tw(jj\%))
Next jj\%
Next lap\% 'end of lap 4 thru n loop '
'Thermal Stress Analysis
If (thermal_option = 1) Then
Call ThermalStress
Else
End If

```
t(ngrids%) = sten
t(0) = (-p(0)-r(0) * ((p(1)-p(0)) / h))
outarray(0, 1) =p(0)
outarray(0, 2) =t(0)
outarray (0,4) = dp(0)
For jj% = 1 To ngrids%
    t(jj%)= -p(jj%) -r(jj%) * ((p(jj% + 1) - p(jj% - 1)) / (2# * h))
    outarray(jj%, 1) = p(jj%)
    outarray(jj%, 2) = t(jj%)
    outarray(jj%,4)=dp(jj%)
Next jj%
    outarray(ngrids%, 2) = -p(ngrids%) - r(ngrids%) * ((p(ngrids%) - p(ngrids% - 1)) / (h))
```

'------------Combined Stiffness analysis
'If (thermal_option = 1) Then
Call combinedstiffness
'End If

If (ngrids $\%>100$ ) Then
ratio $=$ CDbl(ngrids\%) / 100\#
For $\mathrm{i} \%=0$ To 100
$\mathrm{k} \%=\operatorname{CInt}(\mathrm{i} \%$ * ratio $)$
outarray $(\mathrm{i} \%, 0)=\operatorname{outarray}(\mathrm{k} \%, 0)$
$\operatorname{outarray}(\mathrm{i} \%, 1)=\operatorname{outarray}(\mathrm{k} \%, 1)$
outarray $(\mathrm{i} \%, 2)=\operatorname{outarray}(\mathrm{k} \%, 2)$
outarray $(\mathrm{i} \%, 3)=\operatorname{outarray}(\mathrm{k} \%, 3)$
$\operatorname{outarray}(\mathrm{i} \%, 4)=\operatorname{outarray}(\mathrm{k} \%, 4)$
Next i\%
Else
End If
outarray $(100,1)=\operatorname{tw}($ ngrids $\%) ~ * h / r($ ngrids $\%)$
$\operatorname{outarray}(100,4)=\operatorname{tw}($ ngrids $\%) * h / r($ ngrids $\%)$
' Write results back to spread sheet
Range("outarray") = outarray

If ec_option $=2$ Then
Range("ec") = Ec
End If

Set PressureChart = ActiveSheet.ChartObjects("PressureChart")
PressureChart.Visible $=$ False
Set TensionChart = ActiveSheet.ChartObjects("TensionChart")
TensionChart.Visible $=$ False
Set eff_tenChart = ActiveSheet.ChartObjects("eff_tenChart")
eff_tenChart.Visible $=$ False
Set speedChart = ActiveSheet.ChartObjects("speedChart")
speedChart.Visible $=$ False
'Set erChart = ActiveSheet.ChartObjects("erChart")
'erChart. Visible $=$ False
Set torqueChart = ActiveSheet.ChartObjects("torqueChart")
torqueChart.Visible $=$ False
hunit $=\operatorname{CInt}(((\operatorname{rod}-\operatorname{cod}) / 2!) / 4)+1$

PressureChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
PressureChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
PressureChart.Chart.Axes(xlCategory).MajorUnit = hunit
If (units_option = 1) Then PressureChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option = 2) Then PressureChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then PressureChart.Chart.Axes(xlValue).AxisTitle.Caption = "Pressure (psi)"

If (units_option = 2) Then PressureChart.Chart.Axes(xlValue).AxisTitle.Caption $=$ "Pressure (kPa)"

TensionChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
TensionChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
TensionChart.Chart.Axes(xlCategory).MajorUnit = hunit
If (units_option = 1) Then TensionChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option = 2) Then TensionChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then TensionChart.Chart.Axes(xlValue).AxisTitle.Caption = "Stress (psi)"

If (units_option = 2) Then TensionChart.Chart.Axes(xlValue).AxisTitle.Caption = "Stress (kPa)"
eff_tenChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
eff_tenChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
eff_tenChart.Chart.Axes(xlCategory).MajorUnit = hunit
If (units_option = 1) Then eff_tenChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option = 2) Then eff_tenChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then eff_tenChart.Chart.Axes(xlValue).AxisTitle.Caption = "Wound-OnTension (psi)"

If (units_option = 2) Then eff_tenChart.Chart.Axes(xlValue).AxisTitle.Caption = "Wound-OnTension (kPa)"
speedChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
speedChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
speedChart.Chart.Axes(xlCategory).MajorUnit $=$ hunit
If (units_option = 1) Then speedChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option $=2$ ) Then speedChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then speedChart.Chart.Axes(xlValue).AxisTitle.Caption = "Speed (fpm)"
If (units_option = 2) Then speedChart.Chart.Axes(xIValue).AxisTitle.Caption = "Speed (mpm)"
'erChart.Chart.Axes(xlCategory).MinimumScale = 0
'If maxpress > 200 Then
'fff $=100$
'Else
'fff $=10$
'End If
'erChart.Chart.Axes(xlCategory).MaximumScale $=(\operatorname{Int}(\operatorname{maxpress} / \mathrm{fff})+1) *$ fff
torqueChart.Chart.Axes(xlCategory).MaximumScale $=\operatorname{rod} / 2$
torqueChart.Chart.Axes(xlCategory).MinimumScale $=\operatorname{cod} / 2$
torqueChart.Chart.Axes(xlCategory).MajorUnit = hunit
If (units_option $=1$ ) Then torqueChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (in)"

If (units_option $=2$ ) Then torqueChart.Chart.Axes(xlCategory).AxisTitle.Caption = "Radius (cm)"

If (units_option = 1) Then torqueChart.Chart.Axes(xlValue).AxisTitle.Caption = "Torque (inlb)"

If (units_option = 2) Then torqueChart.Chart.Axes(xlValue).AxisTitle.Caption = "Torque (N$\mathrm{cm})^{\prime}$

With Application
.Calculation $=x$ 1Automatic
End With

PressureChart.Visible $=$ True

```
TensionChart.Visible = True
eff_tenChart.Visible = True
speedChart.Visible = True
'erChart.Visible = True
torqueChart.Visible = True
```

End Sub
$' * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * \prime$

Sub tridiag(lap\%): ' START OF tridiag
$' * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * \prime$

```
dp(lap% - 1) = (tw(lap% - 1) + tw(lap%)) / 2! * h / r(lap% - 1)
a(0)=0!
b}(0)= r
c(0) = -1!
d(0)=0!
For iii% = 1 To lap% - 2
hr = h / r(iii%)
a(iii%) = 1!-1.5 * hr
b(ii%) = hr * hr * (1! - et / er(iii%)) - 2!
c(iii%) = 1! + 1.5 * hr
d(iii%)=0!
Next iii%
a(lap%-1) = 0!
b}(lap%-1)=1
d(lap% % 1) = dp(lap% - 1)
```

$$
\mathrm{c}(\text { lap } \%-1)=0!
$$

$$
\begin{aligned}
& \operatorname{beta}(0)=b(0) \\
& \operatorname{gama}(0)=d(0) / b(0)
\end{aligned}
$$

For $\mathrm{iii} \%=1$ To lap\% - 1
beta(iii\%) $=\mathrm{b}(\mathrm{iii} \%)-\mathrm{a}(\mathrm{iii} \%) * \mathrm{c}(\mathrm{iii} \%-1) / \operatorname{beta}(\mathrm{iii} \%-1)$
gama(iii\%) $=(\mathrm{d}(\mathrm{iii} \%)-\mathrm{a}(\mathrm{iii} \%)$ * gama(iii\%-1)) / beta(iii\%)
Next iii\%
$\operatorname{dp}(\mathrm{lap} \%-1)=\operatorname{gama}(\mathrm{lap} \%-1)$
For $\mathrm{iii} \%=(\mathrm{lap} \%-2)$ To 0 Step -1
$\mathrm{dp}(\mathrm{iii} \%)=\operatorname{gama}(\mathrm{iii} \%)-\mathrm{c}(\mathrm{iii} \%) * \operatorname{dp}(\mathrm{iii} \%+1) /$ beta(iii\%)
Next iii\%
End Sub
$' * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * \prime$
Sub calcer(Press, erout, $r$, ten): ' START OF calcer
$' * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ।$
If Press > maxpress Then
maxpress $=$ Press
Else
End If
'If (Press > 400) Then
'pr $=400$
'Else
pr $=$ Press
'End If

If (air_option = 1) Then
' calculate equivalent roughness

$$
\text { eq_rms }=(\text { rms_top } \wedge 2+\text { rms_bot } \wedge 2)^{\wedge} 0.5
$$

If $($ Winder_option $=1)$ Then

```
hh = 0.65 * r * CTWO * (12 * vis * velocity / (ten * caliper)) ^ (2 / 3)
```

Else
req $=r$ * nip_dia $/ 2 /\left(r+n i p \_d i a / 2\right)$
hh $=(4 \#$ * vis * velocity * req * CONE) / pli
End If
factor1 $=$ eq_rms
factor2 $=3 \# *$ eq_rms
If (units_option = 1) Then

$$
\mathrm{Pa}=14.7
$$

Else

$$
\mathrm{Pa}=14.7 * 6.89
$$

End If
If $\mathrm{hh}<$ factorl Then

$$
\text { erout }=\text { ktwo } *(\text { kone }+\mathrm{pr})
$$

ElseIf hh $>$ factor 1 And $h h<$ factor2 Then

$$
\text { erstack }=\text { ktwo } *(\text { kone }+ \text { pr })
$$

'If (units_option = 1) Then
'erout $=($ caliper +hh$) /(($ caliper $/$ erstack $)+\mathrm{hh} *($ ten * caliper $/ \mathrm{r}+\mathrm{Pa}) /(\mathrm{pr}+$ ten * caliper $/$ $\mathrm{r}+\mathrm{Pa})^{\wedge}$ 2)
'Else
'erout $=($ caliper $/ 100+h h) /(($ caliper $/ 100 /$ erstack $)+h h *($ ten $*$ caliper $/ r+\operatorname{Pa}) /(p r+$ ten $*$ caliper $/ \mathrm{r}+\mathrm{Pa})^{\wedge} 2$ )
'End If
erout $=$ erstack $+($ hh - eq_rms $) *\left((\mathrm{pr}+\mathrm{Pa}+\text { ten } * \text { caliper } / \mathrm{r})^{\wedge} 2 /(\right.$ ten $*$ caliper $/ \mathrm{r}+\mathrm{Pa})-$ erstack) / (2 * eq_rms)

Else

$$
\text { erout }=(\mathrm{pr}+\mathrm{Pa}+\text { ten } * \text { caliper } / \mathrm{r})^{\wedge} 2 /(\text { ten } * \text { caliper } / \mathrm{r}+\mathrm{Pa})
$$

End If
Else

$$
\text { erout }=\text { ktwo } *(\text { kone }+\mathrm{pr})
$$

End If
End Sub
'XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXXXXXXXXXXX

Sub ThermalStress()

$$
\begin{aligned}
& \text { rout }=\operatorname{rod} / 2 \\
& \operatorname{rinc}=\operatorname{cid} / 2 \\
& \operatorname{rin}=\operatorname{cod} / 2 \\
& \text { NLAPS }=\text { ngrids } \% \\
& \text { NTemp }=10
\end{aligned}
$$

$$
\mathrm{Dt}=-\mathrm{Dt}
$$

$$
\begin{aligned}
& \mathrm{r}(0)=\mathrm{rin} \\
& \mathrm{lap}=1
\end{aligned}
$$

$$
\mathrm{h}=(\text { rout }- \text { rin }) / \text { NLAPS }
$$

'XX I ran most thermal calcs w/ muweb=0

$$
\mathrm{vrt}=\text { muweb }
$$

For $\mathrm{i}=2$ To NLAPS +2

$$
r(i-1)=r(0)+(i-2) * h
$$

Worksheets("combinedstiffness").Cells(i + 3, 2) = r(i-1)

## Next i

$$
\mathrm{Dt}=\mathrm{Dt} / \mathrm{NTemp}
$$

## For $\mathrm{k}=1$ To NTemp

$$
\begin{aligned}
& \operatorname{er}(0)=\text { ktwo } *(\operatorname{kone}+\mathrm{p}(0)) \\
& \mathrm{vtr}=\mathrm{vrt} * \operatorname{er}(0) / \text { et } \\
& \mathrm{d}(0)=(1 \#-\text { et } / \text { er }(0) * \operatorname{vtr}-\text { et } /(\mathrm{Ec} * \text { rin })-\text { rin } / \mathrm{h}) \\
& \mathrm{c}(0)=\text { rin } / \mathrm{h} \\
& \mathrm{~b}(0)=\text { et } * \mathrm{Dt} *(\text { Acore }- \text { Atang })
\end{aligned}
$$

For $\mathrm{i}=2$ To NLAPS

$$
\begin{aligned}
& \operatorname{er}(\mathrm{i}-1)=\mathrm{ktwo} *(\mathrm{kone}+\mathrm{p}(\mathrm{i}-1)) \\
& \mathrm{vtr}=\mathrm{vrt} * \operatorname{er}(\mathrm{i}-1) / \mathrm{et} \\
& \mathrm{a}(\mathrm{i}-2)=\left(\mathrm{r}(\mathrm{i}-1)^{\wedge} 2 / \mathrm{h} \wedge 2-\mathrm{r}(\mathrm{i}-1) /(2 \# * \mathrm{~h}) *(3 \#-\mathrm{et} / \mathrm{er}(\mathrm{i}-1) * \mathrm{vtr}+\mathrm{vrt})\right) \\
& \mathrm{d}(\mathrm{i}-1)=\left(1 \#-2 \# * \mathrm{r}(\mathrm{i}-1)^{\wedge} 2 / \mathrm{h}^{\wedge} 2+\mathrm{vrt}-\mathrm{et} / \mathrm{er}(\mathrm{i}-1) *(1+\mathrm{vtr})\right) \\
& \mathrm{c}(\mathrm{i}-1)=\mathrm{r}(\mathrm{i}-1)^{\wedge} 2 / \mathrm{h} \wedge 2+\mathrm{r}(\mathrm{i}-1) /(2 \# * \mathrm{~h}) *(3 \#-\text { et } / \mathrm{er}(\mathrm{i}-1) * \mathrm{vtr}+\mathrm{vrt}) \\
& \mathrm{b}(\mathrm{i}-1)=\mathrm{et} * \mathrm{Dt}^{*}(\text { Arad }-\operatorname{Atang})
\end{aligned}
$$

## Next i

$\mathrm{d}($ NLAPS $)=1 \#$
b(NLAPS) $=0 \#$
Call SOLVETRI(NLAPS + 1)
lap $=$ NLAPS +1

For $\mathrm{i}=1$ To lap

$$
\mathrm{p}(\mathrm{i}-1)=\mathrm{p}(\mathrm{i}-1)+\mathrm{dp}(\mathrm{i}-1)
$$

'Worksheets("Thermoelasticity").Cells(4+i, $2+\mathrm{k})=\mathrm{p}(\mathrm{i}-1)$
Next i
Next k

End Sub
' XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXXXXXXXXXXX

Sub combinedstiffness()

$$
\begin{aligned}
& \mathrm{j} 1=0 \\
& \text { rout }=\operatorname{rod} / 2 \\
& \text { rinc }=\operatorname{cid} / 2
\end{aligned}
$$

```
rin}=\operatorname{cod}/
NLAPS = ngrids%
NTemp = 10
Dt=-Dt
r(0) = rin
lap = 1
h = (rout - rin) / NLAPS
```

vrt = muweb
For $\mathrm{i}=2$ To NLAPS +2

$$
r(i-1)=r(0)+(i-2) * h
$$

Worksheets("combinedstiffness").Cells(i+3,2)=r(i-1)

## Next i

## $\mathrm{Dt}=\mathrm{Dt} / \mathrm{NTemp}$

For $\mathrm{k}=1$ To NTemp

$$
\begin{aligned}
& \operatorname{er}(0)=\mathrm{ktwo} *(\mathrm{kone}+\mathrm{p}(0)) \\
& \mathrm{vtr}=\mathrm{vrt} * \operatorname{er}(0) / \mathrm{et} \\
& \mathrm{~d}(0)=(1 \#-\text { et } / \mathrm{er}(0) * \mathrm{vtr}-\mathrm{et} /(\mathrm{Ec} * \operatorname{rin} *((10-\mathrm{k}+1) / \mathrm{NTemp}))-\mathrm{rin} / \mathrm{h}) \\
& \mathrm{c}(0)=\operatorname{rin} / \mathrm{h}
\end{aligned}
$$

$$
\mathrm{b}(0)=\mathrm{et} * \mathrm{Dt} * \text { Acore }
$$

## For $\mathrm{i}=2$ To NLAPS

$$
\begin{aligned}
& \operatorname{er}(\mathrm{i}-1)=\mathrm{ktwo} *(\mathrm{kone}+\mathrm{p}(\mathrm{i}-1)) \\
& \mathrm{vtr}=\mathrm{vrt} * \mathrm{er}(\mathrm{i}-1) / \mathrm{et} \\
& \mathrm{a}(\mathrm{i}-2)=\left(\mathrm{r}(\mathrm{i}-1)^{\wedge} 2 / \mathrm{h} \wedge 2-\mathrm{r}(\mathrm{i}-1) /(2 \# * \mathrm{~h}) *(3 \#-\text { et } / \mathrm{er}(\mathrm{i}-1) * \mathrm{vtr}+\mathrm{vrt})\right) \\
& \mathrm{d}(\mathrm{i}-1)=\left(1 \#-2 \# * r(\mathrm{i}-1)^{\wedge} 2 / \mathrm{h} \wedge 2+\mathrm{vrt}-\mathrm{et} / \mathrm{er}(\mathrm{i}-1) *(1+\mathrm{vtr})\right) \\
& \mathrm{c}(\mathrm{i}-1)=\mathrm{r}(\mathrm{i}-1)^{\wedge} \mathrm{A}^{2} / \mathrm{h} \wedge 2+\mathrm{r}(\mathrm{i}-1) /(2 \# * \mathrm{~h}) *(3 \#-\mathrm{et} / \mathrm{er}(\mathrm{i}-1) * \mathrm{vtr}+\mathrm{vrt}) \\
& \mathrm{b}(\mathrm{i}-1)=0
\end{aligned}
$$

Next i
$\mathrm{d}($ NLAPS $)=1 \#$
$b($ NLAPS $)=0 \#$

## Call SOLVETRI(NLAPS + 1)

lap $=$ NLAPS +1
inc $=0$
For $\mathrm{i}=1$ To lap

$$
\mathrm{p}(\mathrm{i}-1)=\mathrm{p}(\mathrm{i}-1)+\mathrm{dp}(\mathrm{i}-1)
$$

Worksheets("combinedstiffness").Cells( $4+\mathrm{i}, 2+\mathrm{k})=\mathrm{p}(\mathrm{i}-1)$
If $(\mathrm{i} / 10)=\operatorname{Int}(\mathrm{i} / 10) \operatorname{Or}(\mathrm{i}=1)$ Then

$$
\text { inc }=\mathrm{inc}+1
$$

$$
\text { Worksheets("sheet1").Cells( } 1+\text { inc, } 2)=\mathrm{i}
$$

$$
\text { Worksheets("sheet1").Cells( } 1+\mathrm{inc}, 1)=r(\mathrm{i})
$$

$$
\text { Worksheets("sheet1").Cells( } 1+\mathrm{inc}, 2+\mathrm{k})=\mathrm{p}(\mathrm{i}-1)
$$

## End If

## Next i

```
\(\operatorname{tn}(\) ngrids \(\%)=0\)
\(\operatorname{tn}(0)=(-p(0)-r(0) *((p(1)-p(0)) / h))\)
For \(\mathrm{jj} \%=1\) To lap -1
    \(\operatorname{tn}(\mathrm{jj} \%)=(-\mathrm{p}(\mathrm{jj} \%)-\mathrm{r}(\mathrm{jj} \%) *((\mathrm{p}(\mathrm{jj} \%+1)-\mathrm{p}(\mathrm{jj} \%-1)) /(2 \#\) * h\()))\)
```

Next jj\%
$\operatorname{tn}($ lap $)=(-\mathrm{p}($ lap $)-\mathrm{r}($ lap $) *((\mathrm{p}($ lap $)-\mathrm{p}($ lap -1$)) /(\mathrm{h})))$
inc $=0$
For $\mathrm{i}=1$ To lap -1
If $(\mathrm{i} / 10)=\operatorname{Int}(\mathrm{i} / 10) \operatorname{Or}(\mathrm{i}=1)$ Then
inc $=$ inc +1
Worksheets("sheet2").Cells( $1+$ inc, 2$)=\mathrm{i}$
Worksheets("sheet2").Cells( $1+\mathrm{inc}, 1)=\mathrm{r}(\mathrm{i})$
Worksheets("sheet2").Cells(1+inc, $2+\mathrm{k})=\operatorname{tn}(\mathrm{i}-1)$
End If
Next i

Next k

## End Sub

## $1 * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$

'***** Subroutine SOLVETRI
'***** -SOLVES THE TRIDIAGONAL SYSTEM OF DIMENSION IDIM
'***** FOR THE SOLUTION VECTOR X(IDIM)
$1 * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *$

## Sub SOLVETRI(IDIM)

$\mathrm{n}=\mathrm{IDIM}$
For $\mathrm{i}=2$ To n
$\mathrm{d}(\mathrm{i}-1)=\mathrm{d}(\mathrm{i}-1)-(\mathrm{a}(\mathrm{i}-2) / \mathrm{d}(\mathrm{i}-2)) * \mathrm{c}(\mathrm{i}-2)$
$b(i-1)=b(i-1)-(a(i-2) / d(i-2)) * b(i-2)$
Next i
$\mathrm{dp}(\mathrm{n}-1)=\mathrm{b}(\mathrm{n}-1) / \mathrm{d}(\mathrm{n}-1)$
For $\mathrm{i}=(\mathrm{n}-1)$ To 1 Step -1
$d p(i-1)=(b(i-1)-c(i-1) * d p(i)) / d(i-1)$
Next i

End Sub

Sub Highlightinputs()

If Range("Winder_option") $=1$ Then 'Center Winding
Range("sten").Interior.ColorIndex $=37$
Range("taper").Interior.ColorIndex $=37$
Range("nipforce").Interior.ColorIndex = 0
Range("nip_taper").Interior.ColorIndex $=0$
End If

If Range("Winder_option") = 2 Then 'Center Winding with Nip
Range("sten").Interior.ColorIndex = 37
Range("taper").Interior.ColorIndex = 37
Range("nipforce").Interior.ColorIndex = 37
Range("nip_taper").Interior.ColorIndex = 37
End If

If Range("Winder_option") = 3 Then 'Surface Winding
Range("sten").Interior.ColorIndex $=0$
Range("taper").Interior.ColorIndex $=0$
Range("nipforce").Interior.ColorIndex = 37
Range("nip_taper").Interior.ColorIndex = 37
End If

If Range("air_option") = 2 Then 'No air calculations
Range("viscosity").Interior.ColorIndex = 0
Range("nip_dia").Interior.ColorIndex $=0$
Range("velocity").Interior.ColorIndex = 0

Range("rms_top").Interior.ColorIndex $=0$
Range("rms_bot").Interior.ColorIndex = 0
End If

If Range("air_option") = 1 Then
Range("viscosity").Interior.ColorIndex $=37$
Range("velocity").Interior.ColorIndex $=37$
Range("rms_top").Interior.ColorIndex $=37$
Range("rms_bot").Interior.ColorIndex = 37
If Range("Winder_option") = 1 Then Range("nip_dia").Interior.ColorIndex = 0 'Air calculations w/o nip

If Range("Winder_option") > 1 Then Range("nip_dia").Interior.ColorIndex = 37 'Air calculations with nip

Else
End If

If Range("thermal_option") $=1$ Then 'Thermoelastic calculations
Range("arad").Interior.ColorIndex $=37$
Range("atang").Interior.ColorIndex $=37$
Range("acore").Interior.ColorIndex $=37$
Range("dt").Interior.ColorIndex $=37$

End If

If Range("thermal_option") = 2 Then 'No Thermoelastic calculations
Range("arad").Interior.ColorIndex $=0$
Range("atang").Interior.ColorIndex $=0$

Range("acore").Interior.ColorIndex $=0$
Range("dt").Interior.ColorIndex $=0$
End If

If Range("ec_option") = 1 Then 'Calculate Core Stiffness
Range("ecm").Interior.ColorIndex $=37$
Range("mucore").Interior.ColorIndex $=37$
Range("ec").Interior.ColorIndex $=37$
End If

If Range("ec_option") $=2$ Then 'Input Core Stiffness
Range("ecm").Interior.ColorIndex $=0$
Range("mucore").Interior.ColorIndex = 0
Range("ec").Interior.ColorIndex $=0$
End If
'If Range("units_option") = 1 Then
' Range(
'End If

End Sub

## VITA

Dheepak Rajannan
Candidate for the Degree of
Master of Science

## Thesis: THE EFFECT OF WEB NON HOMOGENEITY AND CORE EFFECTS ON WOUND ROLL STRESS

Major Field: Mechanical and Aerospace Engineering
Biographical:
Education:
Completed the requirements for the Master of Science in Mechanical and Aerospace Engineering at Oklahoma State University, Stillwater, Oklahoma in July, 2011.

Received Bachelor of Technology in Mechanical Engineering at Amrita School of Engineering, Amrita Vishwa Vidyapeetham, Coimbatore, Tamilnadu, India in April,2007.

## Experience:

Worked as Senior Design Engineer (July 2007 to July 2009) at Projects Research and Development, HED, Larsen and Toubro, India. Worked as Graduate Research Assistant (August 2010 to July 2011) at Web Handling Research Center, OSU. Also worked as Graduate Teaching Assistant for Engineering Design (August 2009 to December 2009), Mechanical Design I (January 2010 to May 2010) and Advanced Methods in Design (January 2011 to May 2011) courses.

Professional Memberships:
Member of Honor society of Phi Kappa Phi

Scope and Method of Study: Winding models developed so far have been used to compute wound roll stresses only in homogeneous webs and have not been utilized to compute roll stresses in non-homogeneous webs like laminates and non-woven webs. It is difficult to assess the thickness of non woven webs since there are many internal voids and the web is very compressible. The effect of uncertainty in web thickness will be explored in winding models. The pressure in the vicinity of the core is impacted by the core stiffness. During winding the core stiffness is often augmented by a core shaft or an expanding mandrel. The impact of core shaft deflation/extraction on wound roll stresses will be studied.

Findings and Conclusions: It was found that if web thickness was kept totally separate as an input parameter in a winding model that the wound roll stresses are very sensitive to thickness. It was found that the axisymmetric definition of core stiffness yields stiffnesses that are unrealistic when fiber cores are supported on expanding mandrels and then later extracted.

