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FLUID DYNAMICS OF WELL-BORE BOTTOM-HOLE CLEANING

The University of Oklahoma

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THE UNIVERSITY OF OKLAHOMA
GRADUATE COLLEGE

FLUID DYNAMICS OF WELL-BORE BOTTOM-HOLE CLEANING

A DISSERTATION
SUBMITTED TO THE GRADUATE FACULTY
in partial fulfillment of the requirements for the
degree of
DOCTOR OF PHILOSOPHY

By
MOHAMED SADIK BIZANTI

Norman, Oklahoma

1983

FLUID DYNAMICS OF WELL-BORE BOTTOM-HOLE CLEANING

A DISSERTATION

APPROVED FOR THE DEPARTMENT OF
PETROLEUM AND GEOLOGICAL ENGINEERING

By

By Edward J. Blech
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Plus. Wicken

ABSTRACT

A comprehensive series of tests were run on the University of Oklahoma drilling research laboratory rig investigating the variables which influence the rate of cutting removal. No rock cutting or penetration was done in these tests as the bit was held at a preselected height, above a packed column of cuttings. The parameters which were allowed to vary in the test included nozzle diameter, chip size, cutting density, mud density, mud plastic viscosity, yield point, mud flow rate, drill pipe RPM, bottom-hole pressure, pressure drop through the nozzles, nozzle distance from bottom of hole, hydraulic diameter of annulus and bore hole inclination angle. A regression analysis of the data indicated how the rate of cutting removal was a function of the above variable expressed in terms of dimensionless parameters such as Reynolds number, Froude number, rotational Reynolds number, etc. A study was also conducted on the effect of twelve different nozzle configurations on the rate of cutting removal.

The use of the curves and regression equations developed in this study should allow an engineer the opportunity to more closely optimize bottom-hole cleaning and, therefore, improve the rate of penetration and minimize drilling cost per foot.

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Finally, I would like to dedicate this dissertation to my dear mother, Salha E. Krait.

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CHAPTER I

INTRODUCTION

Fluid Dynamics of Bottom-Hole Cleaning

Bottom-hole cleaning is a major factor affecting the rate of penetration. Usually a good bottom-hole cleaning yields an increase in the rate of penetration. Most of the energy available at the rig surface is expended at the drill bit and, in order to minimize the cost of drilling operations, both energy and time must be conserved.

Bottom-hole cleaning is a function of the drilling fluid in removing cuttings from underneath the bit face in order to prevent bit-balling and floundering. At the bit face, fluid flows through the nozzles, across the bottom, and back to the surface through the annulus. The resultant friction and turbulence of mixing causes a great loss of energy. Efficient bottom-hole cleaning must accomplish at least three basic functions:

1. Washing at the mud-rock interface
2. Controlling "crossflow" and avoiding the regrinding of cuttings
3. Fast-lifting of cuttings around the bit face

The objective of this laboratory study is to investigate the main parameters that control or effect bottom-hole cleaning, which is closely related to the rate of penetration and to point out how each parameter is related to bottom-hole cleaning.

A better understanding of bottom-hole cleaning should allow an engineer the opportunity to more closely optimize bottom-hole cleaning and therefore improve the rate of penetration and minimize drilling costs per foot.

Background Literature

It is widely accepted that bit hydraulics has a major effect on the rate of penetration. The actual rate of penetration is always equal to or less than the rate which can be achieved if chips are removed as soon as they are generated.^{1,2} Today's jet bits consume the hydraulic horsepower available at the bit by the fluid velocity that exists from the nozzles that are located at certain distances from the bottom of the hole and which are comparable to the cone diameter. It has been established³ that extension of bit nozzles improved bottom-hole cleaning. Sutko⁴ studied the effect of extended nozzles on bottom-hole pressures. Five nozzle extensions were used (0 to 3.5 inches) and several velocities (90 to 321 ft/sec) were used. It was observed that the pressure at large values of distance between the nozzle and hole bottom per nozzle diameter is extremely low and increases at an increasing rate. It was concluded that as the number of nozzles is decreased, the rate of penetration should increase. Jet hydraulics has been studied extensively by several authors. Kendall and Goins,⁵ have presented hydraulics programs for maximum jet velocity, maximum jet impact, and maximum hydraulic horsepower. They did not point out or indicate which criteria had the greatest effect on the rate of penetration.

Feenstra and VanLeeuwen⁶ have shown, in a full scale experiment on jets in impermeable rock drilling, that increasing jet velocity influences the rate of penetration more than increasing the flow rate.

Randall⁷ presented the following guidelines for optimum hydraulics:

Flow rate - 30 to 50 gallons per minute per inch of bit diameter. Jet horsepower - 2-1/2 to 5 horsepower per square inch of bottom area. Maximum bit hydraulics 50% to 65% of available pump pressure across the bit jet nozzles. Impact is maximized when about 50% of the pump pressure is across the jets (i.e., 50% lost in the rest of the drill string). Two jets for smaller bits at lower penetration rates; otherwise, use three jets.⁷

McLean^{8,9} has written on the improvement of jet bit hydraulics, suggesting maximizing cross flow velocity for maximizing hydraulic horsepower in current jet bit. He described the jet impingement action on the hole bottom and classified it as two mechanisms. One is an impact-pressure wave in the immediate area of the jet impingement. The second is crossflow which spreads across the bottom away from the pressure wave. Equation (1.1) shows the expression developed.^{8,9} for the cross-flow velocity at the edge of the immediate area of jet impingement:

$$V_c = \frac{K(Q_n V_n)^{0.5}}{D} \text{ --- (1.1)}$$

where:

V_c = across flow velocity

Q_n = flow rate through the nozzle

V_n = fluid velocity at the nozzle

D = hole diameter

K = constant.

Equation (1.1) was developed for three-nozzle jet bits. When one of the nozzles is blank, the flow rate of the other two nozzles will be increased by 50%. The corresponding increase in cross-flow velocity is 22.5%, based on the square root of $Q_n V_n$, assuming the nozzle velocity is constant for either a two- or three-jet system. This increase in the cross-flow velocity will yield a higher chip force and, therefore, will result in improvement in bottom-hole cleaning.

Sutko⁴ suggested that the crossflow velocity of a constant bit impact force could be increased by blanking one or two nozzles. This corresponds to maximizing impact force per nozzle. The logic behind the above statement is as follows:

$$\begin{aligned} JF &= (\rho Q V_n / 60 g_c) \text{ ----- (1.2.A)} \\ &= 0.000516 \quad Q V_n \\ &= K Q V_n \end{aligned}$$

$$JF_n = K \frac{Q}{n} V_n \text{ ----- (1.2.B)}$$

Where:

JF = hydraulic impact force

JF_n = hydraulic impact force per nozzle

n = number of jet nozzles

Q = volume rate of flow, gpm

ρ = mud density, lb/gal

g_c = 32.17 (lb_m - ft/lb_f - sec²)

K = a proportionately constant including mud weight and the conversion constant in equation (1.2.A)

V_n = nozzle velocity, ft/sec

By decreasing the number of nozzles (n), JF_n increases and this, on the other hand, will increase the cross-flow velocity which is equal to the square root of $Q_n V_n$.

Eckel,¹⁰ through his microbit drilling study, has shown that the rate of penetration - other conditions being constant - can be described as an exponential function of a Reynolds number function involving flow rate, mud

density, nozzle diameter, and bit or bottom-hole viscosity:

$$R \propto \left(\frac{Q \rho}{d_n \mu} \right)^a \text{----- (1.3)}$$

Where: R = rate of penetration

Q = flow rate

ρ = fluid density

d_n = nozzle diameter

μ = bit viscosity

Figure (1-1) shows the results of Eckel's microbit study. Figure (1-2) shows how apparent viscosity changes by shear rate. Figure (1-3) shows the sensitivity of Reynolds number to viscosity under the fixed conditions of a field test. William and Bruce,¹¹ reported the following conclusions from their laboratory experiment on the carrying capacity of drilling muds:

1. Turbulent flow in the well annulus is most desirable from the standpoint of removing cuttings.
2. Low viscosity and low gel strength are advantageous in removing cuttings.
3. Increase in mud weight is effective in increasing carrying capacity.
4. Rotation of the drill pipe increases carrying capacity.
5. If turbulent flow can be maintained, an annular velocity slightly greater than the slip velocity of the largest cuttings to be transported should keep the bore hole clean. According to William and Bruce,¹¹ this implies velocities of 100 to 125 feet per minute rather than the presently used 175 to 225 feet per minute.

On the subject of jet bit hydraulic controversy Doiron and Deane¹² have

Microbit studies gave these results

Figure (1-1) Drilling Rate vs Reynolds number function (after Eckel¹⁰).

These shear rates apply to mud systems

Figure (1-2) Viscosity vs Shear rate (after Eckel¹⁰).

Figure (1-3) Reynolds number vs Viscosity
(after Eckel¹⁰).

stated the following:

Most of the maximizing theories produce conditions near the maximum jet impact force and maximum hydraulic horsepower operating conditions. From the standpoint of designing hydraulic programs, quibbling over maximum hydraulic horsepower or maximum jet impact does not appear to be justified.

From the derivation of bit hydraulics presented by Kendall and Goins,⁵ it appears that either parameter (maximum bit hydraulic horsepower and maximum jet impact force) will be within about 92 percent of its maximum value when the other is maximized. It has been shown^{13, 14} that the rate of penetration is proportional to the circulating rate and to the nozzle velocity and is, therefore, proportional to the product of these two quantities. It can be shown that the fluid impact force is directly proportional to this product. Therefore, it appears that the jet impact force should be maximized rather than hydraulic horsepower when designing bit hydraulic criteria. Field results¹⁵ have confirmed the advantages of using two nozzle bits and indicated an average of 15 to 20 percent improvement in the rate of penetration by using two nozzle bits. Warren and Winters¹⁶ conducted a laboratory experiment with an 8-1/2 inch IADC series 6-1-7 bit to determine the area and magnitude of the jet impact under realistic flow rates and jet velocities. Their tests have indicated that for the same flow rate through each nozzle, a smaller fraction of the jet energy reaches the hole bottom for larger diameter jets than for smaller diameter jets. Their tests have also indicated that with two jets, a larger fraction of energy reaches the hole bottom than with three jets. On the issue of blanking one nozzle, Warren and Winters¹⁶ have stated the following:

Examination of a bit hydraulics reveals that the available area for fluid to return from under the bit is mostly to either side of the nozzles because of the volume occupied by the cones and bit legs. The area open to fluid return is approximately 15 percent of the total hole cross-sectional area. This means that the jet is not issuing into a

stagnant mass of fluid, but is flowing into a nearly counterflowing fluid. This causes a tendency for the jet to be deflected toward the wall of the hole as well as to significantly increase the entrainment of fluid into the jet.

On the issue of the entrainment effect, Platten and Keffer¹⁷ have found the entrainment to be increased two to five times that of a single jet flowing into a stagnant fluid.

Warren and Winters¹⁶ have also conducted bottom-hole erosion tests using 8-1/2 inch IADC series 6-1-7 bit. Seven thousand pounds weight-on-bit was applied and rotation was stopped when the bit had drilled approximately 8 inches. Their tests indicated that, for the same flow through each nozzle for the same impact force, smaller diameter jets give more erosion. It was also stated that, at the same energy level per nozzle, the two jet case gave more bottom-hole erosion than the three jet case. Their results indicated that there is a detrimental interaction between the jets when the larger jets are used.

On the effect of drill pipe rotation speed on the rate of penetration, Black, et al,¹⁸ have conducted a laboratory study of shale drilling. They concluded that an increase in rotational speed showed a significant effect on rate of penetration for the 60 to 120 RPM range used in their study. Additionally, Pratt¹⁹ has tested a new extended nozzle bit effect on the rate of penetration and made the following statement:

A large percentage of the hydraulic force is required to transfer the energy from the nozzle to the bottom of the hole with the normal jet configuration. As the distance between the nozzle and the bottom of the hole is reduced, the fluid jet passes through less fluid. The energy required to move the fluid is less, dispersion of the jet stream and mixing are reduced. Extended nozzles, by reducing the distance between the nozzle and the bottom of the hole, increases the hydraulic effect for better penetration.

Cholet, et al^{20, 21} presented a unique bit design which improves hydraulics and performance. Their drill bit shows a radical bit design in which

two nozzles of a three-cone bit were extended and the third nozzle was diverted upward. The new bit provided better cleaning at the bit's face resulting in increased rate of penetration of up to 20 percent and a longer bit life. Figure (1-4) shows the principle of hydraulic concept for this new bit. Figure (1-5) shows the advantages in bottom-hole cleaning obtained by the new bit over the standard jet bit. Figure (1-6) shows a schematic diagram of the reverse jet bit. Townsend,²² in his Master's Thesis, studied two-nozzle and three-nozzle drilling under simulated field conditions. It was concluded that less regrinding of chip occurs when drilling with two jets under all the conditions of differential pressures and weight on bits investigated. Cunningham,²³ in a paper published on rock bit selection, has stated the following:

Once the formation is broken loose from the undisturbed material, it may still be held at the bottom by differential pressures. Two methods are available to clean the bottom: (1) use of high hydraulic horsepower jetting to blast the material from the bottom so that the teeth can penetrate undisturbed material and, (2) mud weight is reduced as much as possible but still consistent with safe blowout procedures. This will reduce the differential pressures to a minimum and make the cleaning job of the jets much easier.

Tibbitts et al²⁴ studied the effects of bit hydraulics while drilling shale with a standard three-cone bit. Their investigations indicated that the rate of penetration continually increases with increasing hydraulic horsepower and weight on bit, and there are certain hydraulic horsepower levels, corresponding to the total nozzle flow area, at which effective bottom-hole cleaning occurs. Figures (1-7) and (1-8) show the rate of penetration versus total flow area for various hydraulic horsepower.

It appears from the literature cited herein that the objective of bit hydraulic optimizing methods is the increase of energy needed at the bit face

Figure (1-4) - PRINCIPLE OF A NEW HYDRAULIC CONCEPT.
(after Cholet²¹).

(from laboratory research on models)

Figure (1-5) - Capacity for scavenging out cuttings. Results of laboratory tests. (after Cholet²¹).

Vertical cross-section
of reverse nozzle of downward nozzle

Figure (1-6) - Diagram of a reverse jet bit. (after Cholet²¹).

Figure (1-7) - Rate of penetration vs. total nozzle flow area for various hydraulic horsepowers and 30,000-lbm weight on bit.
(after Tibbitts et al²⁴)

Figure (1-8) - Rate of penetration vs. total nozzle flow area for various hydraulic horsepowers and 40,000-lbm weight on bit.
(after Tibbitts et al²⁴)

to remove the cuttings. Jet impact, bit hydraulic horsepower, and Reynold's number are all related to the fluid velocity at the nozzle. At the nozzle the fluid flow is accelerated and a pressure drop occurs. This hydraulic energy conversion provides the hydraulic influence that cleans the bit and the bottom of the hole. Therefore, we are indeed looking for establishing hydraulic cleaning criteria in which we will be able (1) to remove the cuttings held on the borehole by formation fluids, (2) transport the cuttings as soon as they are generated in order to avoid regrinding (bit floundering), and (3) clean the bit face to prevent bit balling.

CHAPTER II

DIMENSIONAL ANALYSIS

Dimensional analysis is simply a mathematical tool. To use it in any situation the engineer will need to know the following:²⁵

1. The variables believed to be related to a particular situation,
2. Dimensions of the variables,
3. Fundamental algebra.

The benefits of the dimensional analysis are to save time, trouble and expense in the experimental investigation or a correlation of a problem.

Dimensional Analysis generates a relationship which can be written as an equation which describes how the dependent variable varies with the independent variables.

The solution to the problem at hand will be performed by the Buckingham π method. The Buckingham π theorem states that the number of dimensionless groups required to specify completely the relationship is the number of variables (n) minus the number of dimensions (m) involved in the variables. These dimensionless groups are often called π groups.

In order to apply the Buckingham π theorem to the problem of predicting the mass rate of cuttings removed (M_c), it is postulated that the mass rate of cuttings removed would be expected to depend on the nozzle diameter (d_n), the chip size (S), the cutting density (ρ_c), the plastic viscosity (μ_p), the mud density

(ρ_f), the bit diameter (d_b), the mud flow rate (Q), the yield point of the drilling fluid (T_y), the drillpipe rotational speed (RPM), the bottom-hole pressure (P_{BH}), the pressure drop through the bit nozzles (ΔP_n), the nozzle distance from the bottom of hole (L), the hydraulic diameter of annulus (d_h), and the bore hole inclination angle (g).

Thus:

$$M_c = f(d_n, S, \rho_c, \mu_p, \rho_f, d_b, Q, T_y, \text{RPM}, P_{BH}, \Delta P_n, L, d_h, g) \text{ --- (2.1)}$$

The dimensions of the variables, d_n , S , ρ_c , μ_p , ρ_f , d_b , Q , T_y , RPM, P_{BH} , ΔP_n , L , d_h , and g , include length (L), force (F), and time (T). Thus, we have fourteen variables and three primary quantities. Therefore, there will be eleven dimensionless groups.

$$\text{Thus: } n = 14, \quad m = 3, \quad \pi\text{'s} = n - m = 11$$

Substituting the dimensions of the variables in equation (2.1) yields the following:

$$\begin{aligned} FL^{-1}T &= (L)^{c_1} (L)^{c_2} (FL^{-4}T^2)^{c_3} (FL^{-2}T)^{c_4} (FL^{-4}T^2)^{c_5} \\ &(L)^{c_6} (L^3T^{-1})^{c_7} (FL^{-2})^{c_8} (T^{-1})^{c_9} (FL^{-2})^{c_{10}} (FL^{-2})^{c_{11}} \\ &(L)^{c_{12}} (L)^{c_{13}} (LT^{-2})^{c_{14}} \text{ --- (2.2)} \end{aligned}$$

Equating the exponents of F, L, and T on both sides of equation (2.2)

$$1 = c_3 + c_4 + c_5 + c_8 + c_{10} + c_{11} \text{ --- (2.3)}$$

$$\begin{aligned}
 -1 = c_1 + c_2 - 4c_3 - 2c_4 - 4c_5 + c_6 + 3c_7 - 2c_8 \\
 - 2c_{10} - 2c_{11} + c_{12} + c_{13} + c_{14} \text{ ----- (2.4)}
 \end{aligned}$$

$$1 = 2c_3 + c_4 + 2c_5 - c_7 - c_9 - 2c_{14} \text{ ----- (2.5)}$$

From equation (2.3)

$$c_5 = 1 - c_3 - c_4 - c_8 - c_{10} - c_{11} \text{ ----- (2.6)}$$

From equation (2.5)

$$c_7 = 2c_3 + c_4 + 2c_5 - c_9 - 2c_{14} - 1 \text{ ----- (2.7)}$$

Substitute equation (2.5) in (2.7)

$$\begin{aligned}
 c_7 &= 2c_3 + c_4 + 2 [1 - c_3 - c_4 - c_8 - c_{10} - c_{11}] - c_9 - 2c_{14} - 1 \\
 &= 2c_3 + c_4 + 2 - 2c_3 - 2c_4 - 2c_8 - 2c_{10} - 2c_{11} - c_9 - 2c_{14} - 1 \\
 &= -c_4 - 2c_8 - c_9 - 2c_{10} - 2c_{11} - 2c_{14} + 1 \text{ ----- (2.8)}
 \end{aligned}$$

From equation (2.4)

$$\begin{aligned}
 c_1 &= -1 - c_2 + 4c_3 + 2c_4 + 4c_5 - c_6 - 3c_7 + 2c_8 \\
 &+ 2c_{10} + 2c_{11} - c_{12} - c_{13} - c_{14} \text{ ----- (2.9)}
 \end{aligned}$$

Substitute the values of c_5 and c_7 in equation (2.9)

$$c_1 = -1 - c_2 + 4c_3 + 2c_4 + 4 [1 - c_3 - c_4 - c_8 - c_{10} - c_{11}]$$

$$- c_6 - 3 [-c_4 - 2c_8 - c_9 - 2c_{10} - 2c_{11} - 2c_{14} + 1]$$

$$+ 2c_8 + 2c_{10} + 2c_{11} - c_{12} - c_{13} - c_{14}$$

$$c_1 = -1 - c_2 + 4c_3 + 2c_4 + 4 - 4c_3 - 4c_4 - 4c_8 - 4c_{10}$$

$$- 4c_{11} - c_6 + 3c_4 + 6c_8 + 3c_9 + 6c_{10} + 6c_{11} + 6c_{14} - 3$$

$$+ 2c_8 + 2c_{10} + 2c_{11} - c_{12} - c_{13} - c_{14}$$

$$c_1 = -c_2 + c_4 - c_6 + 4c_8 + 3c_9 + 4c_{10} + 4c_{11}$$

$$- c_{12} - c_{13} + 5c_{14} \text{ ----- (2.10)}$$

Substitute equations (2.6), (2.8) and (2.10) in equation (2.1)

$$Mc = f(d_n)^{-c_2 + c_4 - c_6 + 4c_8 + 3c_9 + 4c_{10} + 4c_{11} - c_{12} - c_{13} + 5c_{14}} (S)^{c_2}$$

$$(\rho_c)^{c_3} (\mu_p)^{c_4} (\rho_f)^{1 - c_3 - c_4 - c_8 - c_{10} - c_{11}} (d_b)^{c_6}$$

$$(Q)^{-c_4-2c_8-c_9-2c_{10}-2c_{11}-2c_{14}+1} (T_y)^{c_8} (RPM)^{c_9}$$

$$(P_{BH})^{c_{10}} (P_n)^{c_{11}} (L)^{c_{12}} (d_h)^{c_{13}} (g)^{c_{14}}$$

$$\frac{M_c}{\rho_f Q} = f \left[\frac{S^{c_2}}{d_n} \frac{\rho_c^{c_3}}{\rho_f} \frac{d_n \mu_p^{c_4}}{\rho_f Q} \frac{d_b^{c_6}}{d_n} \frac{d_n^4 T_y^{c_8}}{\rho_f Q^2} \frac{d_n^3 RPM^{c_9}}{Q} \right. \\ \left. \frac{d_n^4 P_{BH}^{c_{10}}}{\rho_f Q^2} \frac{d_n^4 P_n^{c_{11}}}{\rho_f Q^2} \frac{L^{c_{12}}}{d_n} \frac{d_h^{c_{13}}}{d_n} \frac{d_n^5 g^{c_{14}}}{Q^2} \right] \dots (2.11)$$

$$\frac{M_c}{\rho_f Q} = f \left[\frac{S}{d_n} \frac{\rho_c}{\rho_f} \frac{d_n \mu_p}{\rho_f Q} \frac{d_b}{d_n} \frac{d_n^4 T_y}{\rho_f Q^2} \frac{d_n^3 RPM}{Q} \right. \\ \left. \frac{d_n^4 P_{BH}}{\rho_f Q^2} \frac{d_n^4 P_n}{\rho_f Q^2} \frac{L}{d_n} \frac{d_h}{d_n} \frac{d_n^5 g}{Q^2} \right] \dots (2.11^1)$$

The left hand side term is the mass flow rate of cutting per mass flow rate of drilling fluid, the LHS is a dimensionless term.

Simplify equation (2-11¹) by manipulating RHS terms as follows:

$$\frac{d_n^4 P_n}{\rho_f Q^2} / \frac{d_n^4 P_{BH}}{\rho_f Q^2} = \frac{P_n}{P_{BH}} \dots \dots \dots (2.12)$$

$$\frac{d_n^3 RPM}{Q} / \frac{d_n^4 T_y}{\rho_f Q^2} = \frac{RPM Q \rho_f}{d_n T_y} \dots \dots \dots (2.13)$$

$$\left(\frac{L}{d_n} \right) / \left(\frac{d_b}{d_n} \right) = \left(\frac{L}{d_b} \right) \text{-----} \quad (2.14)$$

Substituting equations (2.12), (2.13) and (2.14) in equation (2.11)

$$\frac{M_c}{M_f} = f \left\{ \left(\frac{S}{d_n} \right), \left(\frac{\rho_c}{\rho_f} \right), \left(\frac{d_n \mu_p}{\rho_f Q} \right), \left(\frac{L}{d_b} \right), \left(\frac{RPM Q \rho_f}{d_n T_y} \right), \left(\frac{\Delta P_n}{P_{BH}} \right), \left(\frac{d_h}{d_n} \right), \left(\frac{d_n^5 g}{Q^2} \right) \right\} \text{--} \quad (2.15)$$

$$\left(\frac{RPM Q \rho_f}{d_n T_y} \right) / \left(\frac{d_h}{d_n} \right) = \left(\frac{RPM Q \rho_f}{d_h T_y} \right) \text{-----} \quad (2.16)$$

Substitute equation (2.16) in equation (2.15)

$$\frac{M_c}{M_f} = f \left\{ \left(\frac{S}{d_n} \right), \left(\frac{\rho_c}{\rho_f} \right), \left(\frac{L}{d_b} \right), \left(\frac{d_n \mu_p}{\rho_f Q} \right), \left(\frac{RPM Q \rho_f}{d_h T_y} \right), \left(\frac{\Delta P_n}{P_{BH}} \right), \left(\frac{d_n^5 g}{Q^2} \right) \right\} \text{--} \quad (2.17)$$

or:

$$\frac{M_c}{M_f} = f \left\{ \left(\frac{S}{d_n} \right), \left(\frac{\rho_c}{\rho_f} \right), \left(\frac{L}{d_b} \right), \left(\frac{\rho_f Q}{d_n \mu_p} \right), \left(\frac{RPM Q \rho_f}{d_h T_y} \right), \left(\frac{\Delta P_n}{P_{BH}} \right), \left(\frac{d_n^5 g}{Q^2} \right) \right\} \text{--} \quad (2.18)$$

SOLUTION APPROACH

From the final form of dimensional analysis:

$$\frac{M_c}{M_f} = f \left\{ \left(\frac{S}{d_n} \right), \left(\frac{\rho_c}{\rho_f} \right), \left(\frac{L}{d_b} \right), \left(\frac{\rho_f Q}{d_n \mu_p} \right), \left(\frac{RPM Q \rho_f}{d_h T_y} \right), \left(\frac{\Delta P_n}{P_{BH}} \right), \left(\frac{d_n^5 g}{Q^2} \right) \right\} \text{--} \quad (2.18)$$

A simplified solution approach can be written in the following forms:

$$\frac{M_c}{M_f} = f \left(\frac{S}{d_n} \right) \text{-----} \quad (2.19)$$

By varying the chip size (S) only, a relationship between mass rate ratios and $\frac{S}{d_n}$ can be investigated without affecting the rest of the variables that appear in equation (2.18). Similarly, the following relations can be written as:

$$\frac{M_c}{M_f} = f \left(\frac{\rho_c}{\rho_f} \right) \text{-----} (2.20)$$

Where ρ_c is the variable parameter:

$$\frac{M_c}{M_f} = f \left(\frac{L}{d_b} \right) \text{-----} (2.21)$$

Where L is the variable parameter:

$$\frac{M_c}{M_f} = f \left(\frac{\text{RPM } Q \rho_f}{d_h T_y} \right) \text{-----} (2.22)$$

Where RPM is the variable parameter:

$$\frac{M_c}{M_f} = f \left(\frac{d_n^5 g}{Q^2} \right) \text{-----} (2.23)$$

Where g is the variable parameter:

$$\frac{M_c}{M_f} = f \left[\left(\frac{\rho_f Q}{d_n \mu_p} \right), \left(\frac{\Delta P_n}{P_{BH}} \right) \right] \text{-----} (2.24)$$

Where Q and d_n are the variable parameters.

Geometric similarity in this research was not satisfied due to the usage of flat bottom bits. It is felt that geometric similarity is not important from the point of view of the relative amount of cutting material removed by the fluid jet action. The amount of cuttings removed from underneath the simulated bit equipped with one, two, or three nozzles appears to be relatively similar to the amount that is removed under a tri-cone shaped bit equipped with one, two, or three nozzles.

The kinematic similarity which signifies the similarity of motion⁵⁴ is also not satisfied, since the velocity vectors and the acceleration vectors of the fluid motion underneath and around the simulated bit are going to be different from the velocity and acceleration vectors underneath and around a tri-cone shaped bit.

A model and a prototype will be similar when the prediction equations are similar. Therefore, all that is necessary is to make the corresponding terms equal. The scaling similarity with Reynolds number can be illustrated by observing that

$$(NR)_m = \frac{\rho_m Q_m}{d_{n_m} \mu_m} \quad (NR)_P = \frac{\rho_P Q_P}{d_{n_P} \mu_P}$$

Assuming the density is constant for these to be equal

$$Q_m d_{n_P} \mu_P = Q_P d_{n_m} \mu_m$$

where P indicates the prototype and m the model

$$\frac{d_{n_P}}{d_{n_m}} \approx 4 \quad (\text{scale ratio used}) = \frac{\mu_P}{\mu_m} \quad 10 \quad (\text{scale ratio used})^=$$

Therefore, $40 Q_m = Q_P$.

From the eleven dimensionless groups obtained by the Buckingham π method, the nozzle diameter variable appeared in ten dimensionless groups. Only a few of the dimensionless groups that appeared among the eleven groups were familiar. Therefore, it is difficult to relate the other groups to the actual field conditions. In Equation (2.12) the division of the two dimensionless groups gave a new dimensionless group which contains only $(\frac{P_n}{P_{BH}})$, which is a more realistic term to relate to the mass rate cleaning function. This new dimensionless group $(\frac{P_n}{P_{BH}})$ plus one of the two groups used in the division should appear in the final set of dimensionless groups.^{54,55,56,57} For the purpose of simplicity, only the new product of the division is shown and investigated in this research. This procedure was repeated four times for the same purpose.

In summary, reducing the dimensionless groups was done on the basis of eliminating the interdependency between the dimensionless groups, to investigate only the important dimensionless groups that are felt to be the most influential. Three of the four divisions performed on the dimensionless groups were done to eliminate the nozzle diameter which can be studied in one dimensionless group commonly known as the Reynolds number. The division of these dimensionless groups yielded familiar products. From the beginning, it was felt that these familiar products would have an important influence on the well-bore bottom-hole cleaning.

The set of dimensionless groups used in this study are not unique, but they were judged by the author as a useful set that encompasses all of the important parameters. It should be noted that in this study geometric and kinematic similarity were not met.

CHAPTER III

EXPERIMENTAL APPARATUS AND EXPERIMENTAL TECHNIQUE

The experimental research was conducted on the University of Oklahoma Drilling Research Laboratory Rig. The drilling simulation model consists of an Acker Drilling Unit powered hydraulically, capable of 22,000 pounds of force on the bit with a variable bit rotation rate of up to 500 revolutions per minute. The force on the bit as well as the rotation speed are governed by electronic controls. A schematic diagram of the experimental equipment used is shown in figure (3-1).

Data acquisition equipment includes a Geolograph Drilling Recorder which records footage, bit rotation speed, torque, pump pressure, and weight on the bit, all as a function of time. Mud pump pressure and bit rotational speed readings are double checked by means of pressure gauge and electronic digital reader.

A circulating mud system consists of a fifty-five gallon mud tank, mud pump, flow meter, and mud mixing unit.

Mud Pump Unit

The mud pump used for this research is a Halliburton Model No. 9000 BC6 worm pump equipped with Western Manufacturing gearbox. The gearbox has four gear ratios: 1.2:1, 1:1, 1.9:1, and 1.5:1. A maximum discharge rate

Figure (3-1) SCHEMATIC DIAGRAM OF THE
CIRCULATING MUD SYSTEM

provided by the pump is 60 gallons per minute. The mud pump is powered by a Halliburton motor with the following characteristics:

G. E. Induction Motor
Model No. 99E50G1
22V
47A
875-1750 rpm
3 phase, 60 cycle
20 Hp continuous

The mud pump was tested under various pressures and the corresponding flow rates were recorded. The procedure was repeated for all four gears in order to establish the desired operating conditions. Figure (3-2) shows the mud pump characteristics.

Flow Meter

A Halliburton Model No. 458.80001 Turbine Flow Meter which is a rugged, versatile instrument capable of handling a wide variety of liquids, including many types of slurries and suspensions, is installed in the mud flow line. Flow rates were recorded using Halliburton Flow Totalizer instrument.

Well Bore Configuration

The bore hole was simulated by using clear plexiglas tubes of 1-1/2 and 1-6/8 inches inside diameter, 2 inches outside diameter, and 61 inches long. Cast iron pipe of 3/4 inches inside diameter, one inch outside diameter, and 76 inches long was used to simulate a drill pipe. A 1-1/4 inch flat-bottom surface, no water courses were simulated; bits made of cast iron were used to simulate drill bits. The bits were designed to fit the lower end of the drill pipe similar to screw type configuration. Figure (3-3) shows a side view of the drilling simulation model used in this research.

Figure (3-2) MUD PUMP CHARACTERISTICS

- Gear Number 4
- Gear Number 3
- Gear Number 2
- Gear Number 1

Figure (3-3) DRILLING SIMULATION MODEL.

Experimental Technique

An experimental run time was determined by trial and error procedure. After the cutting material was deposited at the bottom of the annulus, plexiglas tube, the bottom surface of the simulated drill bit was set at one inch above the cutting material surface except in the experiment where the nozzle distance above the cutting material was variable. The mud pump was started and run for three seconds. Hence, all run times were three seconds. The amount of cutting removal by the nozzle's jet action was observed by recording the decrease in the cuttings column level immediately at the end of the run time. A measuring tape around the clear plexiglas tube was used for this purpose. Cutting heights were measured to the nearest $1/8$ of an inch. Several observations were made for each data point collected and a mean value of the observations taken to improve the accuracy of data acquisition.

The decrease in the sand cuttings column level is then converted to grams of cuttings removed by a conversion factor. The conversion factor was determined in a separate experiment where, with the use of an identical plexiglas tube, the weight of one inch of wet column cuttings was determined.

The three seconds run time was determined by a trial and error procedure. Longer run time (greater than three seconds) showed that the amount of sand particles removed was less affected by the jet stream action due to the decrease in the sand column in the annulus underneath the bit face. The jet intensity is lower as the distance between the nozzles and bottom hole increases.

The one inch spacing was selected since it represents an average value of the amount of cutting removal. This average value was taken from the low and high extreme values of amount of sand removed at the corresponding large and small spacing values tried.

CHAPTER IV

EXPERIMENTAL RESULTS

Six different experiments were conducted to study the relationships among the dimensionless groups that are related to the mass flow rate of cuttings per the mud mass flow rate.

Experiment Number 1

$$\frac{M_c}{M_f} = f\left(\frac{S}{d_n}\right) \quad (4.1)$$

To study the effect of chip size per nozzle diameter on the mass flow rate of cuttings per mass flow rate of drilling fluid, the chip sizes used were 6, 10, 12, and 20 U.S. Series mesh. The corresponding chip sizes in inches are listed in Table (4.1).

Experimental Data

Run Time = 3 seconds

Flow Rate = 6 gallons per minute

BHP = 6.3 psig

Annulus Size = 1-3/4 inches (ID)

Pump Pressure = 360 psig

Bit Size = 1-1/4 inches

Drill Pipe = 3/4 x 1 inch diameter

TABLE (4.1) STANDARD SIEVE OPENINGS

STANDARD SIEVE OPENINGS	
U.S. Series Mesh	Sieve Opening Inches
4	0.187
6	0.132
8	0.0937
10	0.0787
12	0.0661
14	0.0555
16	0.0469
18	0.0394
20	0.0331
25	0.0280
30	0.0232
35	0.0197
40	0.0165
60	0.0098
70	0.0083
100	0.0059
170	0.0035

Spacing Between Nozzles and Top of Cuttings = 1 inch

Nozzle Sizes = 3 x 3/32 inches

Fluid = Water

Sand Conversion Factor = $\frac{60 \text{ grams of dry material}}{\text{inch of wet sand column}}$

Gear Used = number 4

Table (4.2) shows the experimental data. The terms that appear in Table (4.2) are explained in Appendix (B).

Experiment Number 2

$$\frac{M_c}{M_f} = f \left(\frac{d_n^5 g}{Q^2} \right) \quad (4.2)$$

To investigate the relationship between the mass rate ratios and reciprocal of the Froude number:

$$\frac{d_n^5 g}{Q^2}$$

the bore hole angle of inclination was allowed to vary in three positions, perpendicular, horizontal, and 45 degree angle, while other variables were kept constant throughout the experiment. Figures (4.1) and (4.2) show a picture of zero and 45 degree angles set up.

Experimental Data:

Run Time = 3 seconds

Flow Rate = 4.75 gallons per minute

BHP = 2.5 psig in the horizontal position

BHP = 4 psig in the vertical position

BHP = 3.2 psig at 45° angle

TABLE (4.2) CHIP SIZE RESULTS

OBS	MESH	S	D ₅₀	INCHES	Q	BHP
1	6	0.1320	0.16237	2.5	6	6.3
2	6	0.1320	0.16237	2.5	6	6.3
3	6	0.1320	0.16237	2.5	6	6.3
4	6	0.1320	0.16237	2.5	6	6.3
5	6	0.1320	0.16237	2.5	6	6.3
6	10	0.0787	0.16237	2.5	6	6.3
7	10	0.0787	0.16237	2.5	6	6.3
8	10	0.0787	0.16237	2.5	6	6.3
9	10	0.0787	0.16237	2.5	6	6.3
10	10	0.0787	0.16237	2.5	6	6.3
11	12	0.0661	0.16237	2.5	6	6.3
12	12	0.0661	0.16237	2.5	6	6.3
13	12	0.0661	0.16237	2.5	6	6.3
14	12	0.0661	0.16237	2.5	6	6.3
15	12	0.0661	0.16237	2.5	6	6.3
16	20	0.0331	0.16237	2.5	6	6.3
17	20	0.0331	0.16237	2.5	6	6.3
18	20	0.0331	0.16237	2.5	6	6.3
19	20	0.0331	0.16237	2.5	6	6.3
20	20	0.0331	0.16237	2.5	6	6.3

OBS	SSPD	GRAMS	MDCT	MMDCT	RATION
1	0.812958	150	6.6139	50.04	0.132172
2	0.812958	150	6.6139	50.04	0.132172
3	0.812958	150	6.6139	50.04	0.132172
4	0.812958	150	6.6139	50.04	0.132172
5	0.812958	150	6.6139	50.04	0.132172
6	0.484695	150	6.6139	50.04	0.132172
7	0.484695	150	6.6139	50.04	0.132172
8	0.484695	150	6.6139	50.04	0.132172
9	0.484695	150	6.6139	50.04	0.132172
10	0.484695	150	6.6139	50.04	0.132172
11	0.407095	150	6.6139	50.04	0.132172
12	0.407095	150	6.6139	50.04	0.132172
13	0.407095	150	6.6139	50.04	0.132172
14	0.407095	150	6.6139	50.04	0.132172
15	0.407095	150	6.6139	50.04	0.132172
16	0.203855	150	6.6139	50.04	0.132172
17	0.203855	150	6.6139	50.04	0.132172
18	0.203855	150	6.6139	50.04	0.132172
19	0.203855	150	6.6139	50.04	0.132172
20	0.203855	150	6.6139	50.04	0.132172

Figure (4-1) Top: Horizontal Set Up
Figure (4-2) Bottom: 45 degrees Angle Set Up

Annulus Size = 1-1/2 inches (ID)

Pump Pressure = 430 psig

Bit Size = 1-1/4 inches

Drill Pipe = 3/4 x 1 inch diameter

Spacing = 1 inch

Nozzle Sizes = 3 x 5/32 inches

Fluid = water

Sand Conversion Factor = $\frac{43 \text{ grams of dry sand}}{\text{inch of wet sand column}}$

Sand Size = 12 mesh

Gear Used = Number 3

Table (4.3) shows the experimental data. Appendix (B) contains information on the terms appearing in Table (4.3).

Experiment Number 3

$$\frac{M_c}{M_f} = f \left(\frac{L}{d_b} \right)$$

(4.3)

To study the effect of nozzle spacing, distance between bit nozzles and top layer of cuttings, on the mass flow rate of cuttings per mass flow rate of drilling fluid, the experiment was conducted by varying the nozzle spacing (0.125 to 3.0 inches) while other parameters were kept constant throughout the experiment.

Experimental Data:

Run Time = 3 seconds

Flow Rate = 3 gallons per minute

BHP = 5.3 psig

Annulus Size = 1-3/4 inches (ID)

TABLE (4.3) RECIPROCAL FROUDE'S NUMBER RESULTS

OBS	ANGLE	G	INCHES	DE	Q	BHP
1	0	0.0000	3.25	0.27063	4.75	2.5
2	0	0.0000	3.25	0.27063	4.75	2.5
3	0	0.0000	3.25	0.27063	4.75	2.5
4	0	0.0000	3.25	0.27063	4.75	2.5
5	0	0.0000	3.25	0.27063	4.75	2.5
6	45	22.7688	3.25	0.27063	4.75	3.2
7	45	22.7688	3.25	0.27063	4.75	3.2
8	45	22.7688	3.25	0.27063	4.75	3.2
9	45	22.7688	3.25	0.27063	4.75	3.2
10	45	22.7688	3.25	0.27063	4.75	3.2
11	90	32.2000	3.25	0.27063	4.75	4.0
12	90	32.2000	3.25	0.27063	4.75	4.0
13	90	32.2000	3.25	0.27063	4.75	4.0
14	90	32.2000	3.25	0.27063	4.75	4.0
15	90	32.2000	3.25	0.27063	4.75	4.0

OBS	GRAMS	MDOT	MMDOT	RATUM	RFROUDE
1	139.75	6.16195	39.615	0.155546	C.00000000
2	139.75	6.16195	39.615	0.155546	0.00000000
3	139.75	6.16195	39.615	0.155546	C.00000000
4	139.75	6.16195	39.615	0.155546	0.00000000
5	139.75	6.16195	39.615	0.155546	C.00000000
6	139.75	6.16195	39.615	0.155546	0.00146498
7	139.75	6.16195	39.615	0.155546	C.00146498
8	139.75	6.16195	39.615	0.155546	0.00146498
9	139.75	6.16195	39.615	0.155546	C.00146498
10	139.75	6.16195	39.615	0.155546	0.00146498
11	139.75	6.16195	39.615	0.155546	C.00207180
12	139.75	6.16195	39.615	0.155546	0.00207180
13	139.75	6.16195	39.615	0.155546	C.00207180
14	139.75	6.16195	39.615	0.155546	0.00207180
15	139.75	6.16195	39.615	0.155546	C.00207180

Sand Size = 6 mesh (0.132 inch)

Bit Size = 1-1/4 inch

Drill Pipe = 3/4 x 1 inch diameter

Nozzle Size = 3 x 3/32 inches

Fluid = Water

Pump Pressure = 380 psig

Sand Conversion Factor = $\frac{60 \text{ grams of dry sand}}{\text{inch of wet sand column}}$

Gear Used = number 4

The experimental data are shown in Table (4.4). Appendix (B) shows an explanation of the terms appearing in Table (4.4).

Experiment Number 4

$$\frac{M_c}{M_f} = f \left(\frac{\text{RPM } Q \rho_f}{d_h T_y} \right) \quad (4.4)$$

To investigate the relationship between the mass flow rate ratios and the rotational Reynolds number $\frac{\text{RPM } Q \rho_f}{d_h T_y}$ the drill pipe rotary speed was allowed to vary (0 to 40 RPM), while the other parameters (Q , ρ_f , d_h , T_y) were kept constant throughout the experiment.

Experimental Data:

Run Time = 3 seconds

Flow Rate = 8.5 gallons per minute

Annulus Size = 1-1/2 inches (ID)

Sand Size = 12 mesh

Bit Size = 1-1/4 inches

Hydraulic Diameter (d_h) = 0.5 inch

TABLE (4.4) NOZZLE SPACING RESULTS

OBS	L	DB	INCHES	Q	BHP	SPBD	GRAMS	NDOT	MNDCT	RATION
1	3.000	1.25	0.250	3	5.3	2.4	15.0	0.66139	25.02	0.026434
2	3.000	1.25	0.250	3	5.3	2.4	15.0	0.66139	25.02	0.026434
3	3.000	1.25	0.250	3	5.3	2.4	15.0	0.66139	25.02	0.026434
4	3.000	1.25	0.250	3	5.3	2.4	15.0	0.66139	25.02	0.026434
5	2.500	1.25	0.375	3	5.3	2.0	22.5	0.99209	25.02	0.039652
6	2.500	1.25	0.375	3	5.3	2.0	22.5	0.99209	25.02	0.039652
7	2.500	1.25	0.375	3	5.3	2.0	22.5	0.99209	25.02	0.039652
8	2.500	1.25	0.375	3	5.3	2.0	22.5	0.99209	25.02	0.039652
9	2.000	1.25	0.500	3	5.3	1.6	30.0	1.32278	25.02	0.052869
10	2.000	1.25	0.500	3	5.3	1.6	30.0	1.32278	25.02	0.052869
11	2.000	1.25	0.500	3	5.3	1.6	30.0	1.32278	25.02	0.052869
12	2.000	1.25	0.500	3	5.3	1.6	30.0	1.32278	25.02	0.052869
13	1.500	1.25	0.625	3	5.3	1.2	37.5	1.65348	25.02	0.066086
14	1.500	1.25	0.625	3	5.3	1.2	37.5	1.65348	25.02	0.066086
15	1.500	1.25	0.625	3	5.3	1.2	37.5	1.65348	25.02	0.066086
16	1.500	1.25	0.625	3	5.3	1.2	37.5	1.65348	25.02	0.066086
17	1.000	1.25	0.750	3	5.3	0.8	45.0	1.98417	25.02	0.079303
18	1.000	1.25	0.750	3	5.3	0.8	45.0	1.98417	25.02	0.079303
19	1.000	1.25	0.750	3	5.3	0.8	45.0	1.98417	25.02	0.079303
20	1.000	1.25	0.750	3	5.3	0.8	45.0	1.98417	25.02	0.079303
21	0.750	1.25	0.875	3	5.3	0.6	52.5	2.31487	25.02	0.092521
22	0.750	1.25	0.875	3	5.3	0.6	52.5	2.31487	25.02	0.092521
23	0.750	1.25	0.875	3	5.3	0.6	52.5	2.31487	25.02	0.092521
24	0.750	1.25	0.875	3	5.3	0.6	52.5	2.31487	25.02	0.092521
25	0.625	1.25	1.000	3	5.3	0.5	60.0	2.64556	25.02	0.105738
26	0.625	1.25	1.000	3	5.3	0.5	60.0	2.64556	25.02	0.105738
27	0.625	1.25	1.000	3	5.3	0.5	60.0	2.64556	25.02	0.105738
28	0.625	1.25	1.000	3	5.3	0.5	60.0	2.64556	25.02	0.105738
29	0.500	1.25	1.125	3	5.3	0.4	67.5	2.97626	25.02	0.118955
30	0.500	1.25	1.125	3	5.3	0.4	67.5	2.97626	25.02	0.118955
31	0.500	1.25	1.125	3	5.3	0.4	67.5	2.97626	25.02	0.118955
32	0.500	1.25	1.125	3	5.3	0.4	67.5	2.97626	25.02	0.118955
33	0.375	1.25	1.250	3	5.3	0.3	75.0	3.30695	25.02	0.132172
34	0.375	1.25	1.250	3	5.3	0.3	75.0	3.30695	25.02	0.132172
35	0.375	1.25	1.250	3	5.3	0.3	75.0	3.30695	25.02	0.132172
36	0.375	1.25	1.250	3	5.3	0.3	75.0	3.30695	25.02	0.132172
37	0.250	1.25	1.500	3	5.3	0.2	90.0	3.96834	25.02	0.158607
38	0.250	1.25	1.500	3	5.3	0.2	90.0	3.96834	25.02	0.158607
39	0.250	1.25	1.500	3	5.3	0.2	90.0	3.96834	25.02	0.158607
40	0.250	1.25	1.500	3	5.3	0.2	90.0	3.96834	25.02	0.158607
41	0.125	1.25	2.000	3	5.3	0.1	120.0	5.29112	25.02	0.211476
42	0.125	1.25	2.000	3	5.3	0.1	120.0	5.29112	25.02	0.211476
43	0.125	1.25	2.000	3	5.3	0.1	120.0	5.29112	25.02	0.211476
44	0.125	1.25	2.000	3	5.3	0.1	120.0	5.29112	25.02	0.211476

Pump Pressure = 400 psig

Drill Pipe = 3/4 x 1 inch diameter

Spacing = 1 inch

Sand Conversion Factor = $\frac{43 \text{ grams of dry sand}}{\text{inch of wet sand column}}$

Nozzle Size = 3 x 3/32 inches

Gear Used = Number 3

Fluid = water-base bentonite

Fluid Weight = 8.6 pounds per gallon

Apparent Viscosity = 5.5 cp

Plastic Viscosity = 5 cp

Yield Point = 1 lb/100 ft²

API Filtrate Rate = 21 cc/30 min

A chemical product, "Dykem," is used for sand coloring in order to improve the visibility of the sand removal motion. The data are shown in Table (4.5). Appendix (B) shows an explanation of the terms appearing in Table (4.5).

Experiment Number 5

$$\frac{M_c}{M_f} = f \left(\frac{\rho_c}{\rho_f} \right) \quad (4.5)$$

To study the effect of cutting density per fluid density on the mass flow rate ratios, the cutting density was allowed to vary while other parameters were kept constant throughout the experiment. The cutting materials used for the experiment were:

Sand:

Specific gravity = 2.5982 gm/cc

Size = 6 mesh (0.132 inch)

Steel Shots:

Specific gravity = 7.763 gm/cc

Size = 0.130 inch diameter

Lead Shots:

Specific gravity = 10.539 gm/cc

Size = 0.130 inch diameter

Experimental Data:

Run Time = 3 seconds

Flow Rate = 6.5 gallons per minute

Annulus Size = 1-3/4 inch (ID)

Bit Size = 1-1/4 inch

Pump Pressure = 360 psig

Drill Pipe = 3/4 x 1 inch diameter

Spacing = 1 inch

Nozzle Size = 3 x 3/32 inch

Gear Used = number 4

Fluid = Water

Sand Conversion Factor = $\frac{60 \text{ grams of dry sand}}{\text{inch of wet sand column}}$

Steel Conversion Factor = $\frac{174.125 \text{ grams of dry steel}}{\text{inch of wet steel column}}$

Lead Conversion Factor = $\frac{275.4 \text{ grams of dry lead}}{\text{inch of wet lead column}}$

The data are shown in Table (4.6). Appendix (B) shows an explanation of the terms appearing in Table (4.6).

TABLE (4.5) ROTATIONAL REYNOLDS NUMBER RESULTS

OBS	RPM	INCHES	Q	GRAMS	MDOT	MMDOT	RATIO	RNR
1	0	1.750	8.5	75.250	3.31797	73.1	0.0453895	0
2	0	1.875	8.5	80.625	3.55497	73.1	0.0486316	0
3	0	1.750	8.5	75.250	3.31797	73.1	0.0453895	0
4	0	1.875	8.5	80.625	3.55497	73.1	0.0486316	0
5	10	2.250	8.5	96.750	4.26597	73.1	0.0583580	1462
6	10	2.250	8.5	96.750	4.26597	73.1	0.0583580	1462
7	10	2.250	8.5	96.750	4.26597	73.1	0.0583580	1462
8	10	2.250	8.5	96.750	4.26597	73.1	0.0583580	1462
9	20	2.375	8.5	102.125	4.50297	73.1	0.0616001	2924
10	20	2.375	8.5	102.125	4.50297	73.1	0.0616001	2924
11	20	2.375	8.5	102.125	4.50297	73.1	0.0616001	2924
12	20	2.375	8.5	102.125	4.50297	73.1	0.0616001	2924
13	30	2.500	8.5	107.500	4.73996	73.1	0.0648422	4386
14	30	2.500	8.5	107.500	4.73996	73.1	0.0648422	4386
15	30	2.500	8.5	107.500	4.73996	73.1	0.0648422	4386
16	30	2.500	8.5	107.500	4.73996	73.1	0.0648422	4386
17	40	2.375	8.5	102.125	4.50297	73.1	0.0616001	5848
18	40	2.375	8.5	102.125	4.50297	73.1	0.0616001	5848
19	40	2.375	8.5	102.125	4.50297	73.1	0.0616001	5848
20	40	2.500	8.5	107.500	4.73996	73.1	0.0648422	5848

TABLE (4.6) CUTTING DENSITY RESULTS

OBS	INCHES	CUTDENSITY	Q	GRAMS	VOLRATIO	RATIO
1	5.750	2.5982	6.5	345.00	0.184221	0.31153
2	5.875	2.5982	6.5	352.50	0.188226	0.31153
3	5.750	2.5982	6.5	345.00	0.184221	0.31153
4	5.875	2.5982	6.5	352.50	0.188226	0.31153
5	5.750	2.5982	6.5	345.00	0.184221	0.31153
6	5.875	2.5982	6.5	352.50	0.188226	0.31153
7	5.750	2.5982	6.5	345.00	0.184221	0.31153
8	5.875	2.5982	6.5	352.50	0.188226	0.31153
9	4.500	7.7630	6.5	783.56	0.144173	0.93082
10	4.625	7.7630	6.5	805.33	0.148178	0.93082
11	4.500	7.7630	6.5	783.56	0.144173	0.93082
12	4.625	7.7630	6.5	805.33	0.148178	0.93082
13	4.500	7.7630	6.5	783.56	0.144173	0.93082
14	4.625	7.7630	6.5	805.33	0.148178	0.93082
15	4.500	7.7630	6.5	783.56	0.144173	0.93082
16	4.625	7.7630	6.5	805.33	0.148178	0.93082
17	3.750	10.5390	6.5	1032.75	0.120144	1.26367
18	3.875	10.5390	6.5	1067.17	0.124149	1.26367
19	3.750	10.5390	6.5	1032.75	0.120144	1.26367
20	3.875	10.5390	6.5	1067.17	0.124149	1.26367
21	3.750	10.5390	6.5	1032.75	0.120144	1.26367

OBS	MDOT	MDCT	RATIO	CUTTVOL	FLUIDVOL
1	15.2120	54.21	0.280612	13.8304	75.075
2	15.5427	54.21	0.286712	14.1311	75.075
3	15.2120	54.21	0.280612	13.8304	75.075
4	15.5427	54.21	0.286712	14.1311	75.075
5	15.2120	54.21	0.280612	13.8304	75.075
6	15.5427	54.21	0.286712	14.1311	75.075
7	15.2120	54.21	0.280612	13.8304	75.075
8	15.5427	54.21	0.286712	14.1311	75.075
9	34.5494	54.21	0.637325	10.8238	75.075
10	35.5091	54.21	0.655028	11.1245	75.075
11	34.5494	54.21	0.637325	10.8238	75.075
12	35.5091	54.21	0.655028	11.1245	75.075
13	34.5494	54.21	0.637325	10.8238	75.075
14	35.5091	54.21	0.655028	11.1245	75.075
15	34.5494	54.21	0.637325	10.8238	75.075
16	35.5091	54.21	0.655028	11.1245	75.075
17	45.5367	54.21	0.840006	9.0198	75.075
18	47.0546	54.21	0.868006	9.3205	75.075
19	45.5367	54.21	0.840006	9.0198	75.075
20	47.0546	54.21	0.868006	9.3205	75.075
21	45.5367	54.21	0.840006	9.0198	75.075

TABLE (4.6) Cont'd

OBS	INCHES	CUTDNSTY	Q	GRAMS	VOLRATIC	RATIO D
22	3.875	10.539	6.5	1067.17	0.124149	1.26367
23	3.750	10.539	6.5	1032.75	0.120144	1.26367
24	3.875	10.539	6.5	1067.17	0.124149	1.26367
OBS	MDO1	MMDOT	RATIO H	CUTTVOL	FLUIDVOL	
22	47.0546	54.21	0.868006	9.32049	75.075	
23	45.5367	54.21	0.840006	9.01983	75.075	
24	47.0546	54.21	0.868006	9.32049	75.075	

Experiment Number 6

$$\frac{M_c}{M_f} = f \left\{ \left(\frac{\rho_f Q}{d_n \mu_p} \right) \left(\frac{\Delta P_n}{BHP} \right) \right\} \quad (4.6)$$

The experiment was conducted to investigate the effects of Reynolds number and the ratio of the pressure drop through the nozzles and bottom hole pressure on the mass flow rate ratios. Four different flow rates were used, 4.75, 8.5, 12, and 16 gpm, and five nozzle sizes were used, 4/32, 5/32, 6/32, 7/32, and 8/32 inches. The rest of the parameters were kept constant throughout the test.

Experimental Data:

Run Time = 3 seconds

Annulus Size = 1-3/4 inches (ID)

Bit Size = 1-1/4 inches

Drill Pipe = 3/4 x 1 inch diameter

Spacing = 1 inch

Gear Used = number 3

Sand Size = 6 mesh (0.132 inch)

Fluid = water (8.34 ppg)

Room Temperature = 78°F

$\mu_p = 0.8904$ cp

It is assumed that the water viscosity is equal to the plastic viscosity.

The experimental data are shown in Table (4.7). Appendix (B) shows an explanation of the terms appearing in Table (4.7).

TABLE (4.7) RESULTS OF EXPERIMENT NUMBER 6

OBS	DN	Q	INCHES	BHP	DE	GRAMS
1	0.12500	4.75	2.250	4.1	0.21650	135.0
2	0.12500	4.75	2.250	4.1	0.21650	135.0
3	0.12500	4.75	2.250	4.1	0.21650	135.0
4	0.12500	4.75	2.250	4.1	0.21650	135.0
5	0.12500	4.75	2.250	4.1	0.21650	135.0
6	0.12500	8.50	2.750	4.8	0.21650	165.0
7	0.12500	8.50	2.750	4.8	0.21650	165.0
8	0.12500	8.50	2.875	4.8	0.21650	172.5
9	0.12500	8.50	2.875	4.8	0.21650	172.5
10	0.12500	8.50	2.750	4.8	0.21650	165.0
11	0.12500	12.00	3.750	6.5	0.21650	225.0
12	0.12500	12.00	3.750	6.5	0.21650	225.0
13	0.12500	12.00	3.875	6.5	0.21650	232.5

OBS	MDOT	RMDOT	RAT ION	PDROP	RATIO P
1	5.9525	39.615	0.150259	12.773	3.1153
2	5.9525	39.615	0.150259	12.773	3.1153
3	5.9525	39.615	0.150259	12.773	3.1153
4	5.9525	39.615	0.150259	12.773	3.1153
5	5.9525	39.615	0.150259	12.773	3.1153
6	7.2753	70.890	0.102628	40.901	8.5211
7	7.2753	70.890	0.102628	40.901	8.5211
8	7.6060	70.890	0.107293	40.901	8.5211
9	7.6060	70.890	0.107293	40.901	8.5211
10	7.2753	70.890	0.102628	40.901	8.5211
11	9.9209	100.080	0.099129	81.519	12.5414
12	9.9209	100.080	0.099129	81.519	12.5414
13	10.2515	100.080	0.102434	81.519	12.5414

OBS	HHP	TA	JP	NR	PRODC T
1	0.03540	0.0368156	2.6476	44976	140114
2	0.03540	0.0368156	2.6476	44976	140114
3	0.03540	0.0368156	2.6476	44976	140114
4	0.03540	0.0368156	2.6476	44976	140114
5	0.03540	0.0368156	2.6476	44976	140114
6	0.20284	0.0368156	8.4782	80483	685804
7	0.20284	0.0368156	8.4782	80483	685804
8	0.20284	0.0368156	8.4782	80483	685804
9	0.20284	0.0368156	8.4782	80483	685804
10	0.20284	0.0368156	8.4782	80483	685804
11	0.57073	0.0368156	16.8976	113623	1425000
12	0.57073	0.0368156	16.8976	113623	1425000
13	0.57073	0.0368156	16.8976	113623	1425000

TABLE (4.7) Cont'd

OBS	DN	Q	INCHES	BHP	DE	GRAMS
14	0.12500	12.00	3.875	6.5	0.21650	232.5
15	0.12500	12.00	3.750	6.5	0.21650	225.0
16	0.12500	16.00	4.625	8.0	0.21650	277.5
17	0.12500	16.00	4.750	8.0	0.21650	285.0
18	0.12500	16.00	4.500	8.0	0.21650	270.0
19	0.12500	16.00	4.750	8.0	0.21650	285.0
20	0.12500	16.00	4.750	8.0	0.21650	285.0
21	0.15625	4.75	3.250	4.0	0.27063	195.0
22	0.15625	4.75	3.375	4.0	0.27063	202.5
23	0.15625	4.75	3.125	4.0	0.27063	187.5
24	0.15625	4.75	3.250	4.0	0.27063	195.0
25	0.15625	4.75	3.250	4.0	0.27063	195.0
26	0.15625	8.50	3.500	4.4	0.27063	210.0

OBS	MDOT	HMMDOT	RATIOH	PDRDP	RATIOP
14	10.2515	100.080	0.102434	81.519	12.5414
15	9.9209	100.080	0.099129	81.519	12.5414
16	12.2357	133.440	0.091695	144.923	18.1154
17	12.5664	133.440	0.094173	144.923	18.1154
18	11.9050	133.440	0.089216	144.923	18.1154
19	12.5664	133.440	0.094173	144.923	18.1154
20	12.5664	133.440	0.094173	144.923	18.1154
21	8.5981	39.615	0.217041	5.231	1.3078
22	8.9288	39.615	0.225389	5.231	1.3078
23	8.2674	39.615	0.208693	5.231	1.3078
24	8.5981	39.615	0.217041	5.231	1.3078
25	8.5981	39.615	0.217041	5.231	1.3078
26	9.2595	70.890	0.130617	16.752	3.8073

OBS	HHP	TA	JF	NR	PRODC
14	0.57073	0.0368156	16.8976	113623	1425000
15	0.57073	0.0368156	16.8976	113623	1425000
16	1.35284	0.0368156	30.0403	151498	2744445
17	1.35284	0.0368156	30.0403	151498	2744445
18	1.35284	0.0368156	30.0403	151498	2744445
19	1.35284	0.0368156	30.0403	151498	2744445
20	1.35284	0.0368156	30.0403	151498	2744445
21	0.01450	0.0575244	1.6945	35981	47057
22	0.01450	0.0575244	1.6945	35981	47057
23	0.01450	0.0575244	1.6945	35981	47057
24	0.01450	0.0575244	1.6945	35981	47057
25	0.01450	0.0575244	1.6945	35981	47057
26	0.08308	0.0575244	5.4260	64387	245136

TABLE (4.7) Cont'd

OBS	DN	Q	INCHES	BHP	DE	GRAMS
27	0.15625	8.50	3.375	4.4	0.27063	202.5
28	0.15625	8.50	3.375	4.4	0.27063	202.5
29	0.15625	8.50	3.750	4.4	0.27063	225.0
30	0.15625	8.50	3.625	4.4	0.27063	217.5
31	0.15625	12.00	4.250	5.2	0.27063	255.0
32	0.15625	12.00	4.000	5.2	0.27063	240.0
33	0.15625	12.00	4.125	5.2	0.27063	247.5
34	0.15625	12.00	4.250	5.2	0.27063	255.0
35	0.15625	12.00	4.250	5.2	0.27063	255.0
36	0.15625	16.00	5.250	6.2	0.27063	315.0
37	0.15625	16.00	5.250	6.2	0.27063	315.0
38	0.15625	16.00	5.250	6.2	0.27063	315.0
39	0.15625	16.00	5.500	6.2	0.27063	330.0

OBS	MDOT	EMDOT	RAT IOM	PDROP	RATIO P
27	8.9288	70.890	0.125952	16.7519	3.80725
28	8.9288	70.890	0.125952	16.7519	3.80725
29	9.9209	70.890	0.139947	16.7519	3.80725
30	9.5902	70.890	0.135282	16.7519	3.80725
31	11.2436	100.080	0.112346	33.3879	6.42075
32	10.5822	100.080	0.105738	33.3879	6.42075
33	10.9129	100.080	0.109042	33.3879	6.42075
34	11.2436	100.080	0.112346	33.3879	6.42075
35	11.2436	100.080	0.112346	33.3879	6.42075
36	13.8892	133.440	0.104086	59.3562	9.57358
37	13.8892	133.440	0.104086	59.3562	9.57358
38	13.8892	133.440	0.104086	59.3562	9.57358
39	14.5506	133.440	0.109042	59.3562	9.57358

OBS	BHP	TA	JP	NR	PRODCT
27	0.083075	0.057524	5.4260	64387	245136
28	0.083075	0.057524	5.4260	64387	245136
29	0.083075	0.057524	5.4260	64387	245136
30	0.083075	0.057524	5.4260	64387	245136
31	0.233754	0.057524	10.8145	90899	583637
32	0.233754	0.057524	10.8145	90899	583637
33	0.233754	0.057524	10.8145	90899	583637
34	0.233754	0.057524	10.8145	90899	583637
35	0.233754	0.057524	10.8145	90899	583637
36	0.554084	0.057524	19.2258	121198	1160301
37	0.554084	0.057524	19.2258	121198	1160301
38	0.554084	0.057524	19.2258	121198	1160301
39	0.554084	0.057524	19.2258	121198	1160301

TABLE (4.7) Cont'd

OBS	DN	Q	INCHES	BHP	DE	GRAMS
40	0.15625	16.00	5.125	6.2	0.27063	307.5
41	0.18750	4.75	4.000	3.9	0.32476	240.0
42	0.18750	4.75	4.000	3.9	0.32476	240.0
43	0.18750	4.75	4.000	3.9	0.32476	240.0
44	0.18750	4.75	4.000	3.9	0.32476	240.0
45	0.18750	4.75	4.000	3.9	0.32476	240.0
46	0.18750	8.50	5.125	4.0	0.32476	307.5
47	0.18750	8.50	5.125	4.0	0.32476	307.5
48	0.18750	8.50	5.125	4.0	0.32476	307.5
49	0.18750	8.50	5.125	4.0	0.32476	307.5
50	0.18750	8.50	5.125	4.0	0.32476	307.5
51	0.18750	12.00	5.750	4.9	0.32476	345.0
52	0.18750	12.00	5.750	4.9	0.32476	345.0
OBS	MDOT	MMDOT	RATIOA	PDROP	RATIOB	
40	13.5585	133.440	0.101607	59.3562	9.57358	
41	10.5822	39.615	0.267127	2.5227	0.64685	
42	10.5822	39.615	0.267127	2.5227	0.64685	
43	10.5822	39.615	0.267127	2.5227	0.64685	
44	10.5822	39.615	0.267127	2.5227	0.64685	
45	10.5822	39.615	0.267127	2.5227	0.64685	
46	13.5585	70.890	0.191261	8.0783	2.01956	
47	13.5585	70.890	0.191261	8.0783	2.01956	
48	13.5585	70.890	0.191261	8.0783	2.01956	
49	13.5585	70.890	0.191261	8.0783	2.01956	
50	13.5585	70.890	0.191261	8.0783	2.01956	
51	15.2120	100.080	0.151998	16.1006	3.28584	
52	15.2120	100.080	0.151998	16.1006	3.28584	
OBS	HHP	TA	JP	NR	PRODC	
40	0.554084	0.057524	19.2258	121198	1160301	
41	0.006991	0.082835	1.1767	29984	19395	
42	0.006991	0.082835	1.1767	29984	19395	
43	0.006991	0.082835	1.1767	29984	19395	
44	0.006991	0.082835	1.1767	29984	19395	
45	0.006991	0.082835	1.1767	29984	19395	
46	0.040061	0.082835	3.7681	53655	108361	
47	0.040061	0.082835	3.7681	53655	108361	
48	0.040061	0.082835	3.7681	53655	108361	
49	0.040061	0.082835	3.7681	53655	108361	
50	0.040061	0.082835	3.7681	53655	108361	
51	0.112723	0.082835	7.5101	75749	248899	
52	0.112723	0.082835	7.5101	75749	248899	

TABLE (4.7) Cont'd

OBS	DN	Q	INCHES	BHP	DZ	GRAMS
53	0.18750	12.00	5.750	4.9	0.32476	345.0
54	0.18750	12.00	5.750	4.9	0.32476	345.0
55	0.18750	12.00	5.750	4.9	0.32476	345.0
56	0.18750	16.00	6.000	5.4	0.32476	360.0
57	0.18750	16.00	6.000	5.4	0.32476	360.0
58	0.18750	16.00	6.000	5.4	0.32476	360.0
59	0.18750	16.00	6.000	5.4	0.32476	360.0
60	0.18750	16.00	6.000	5.4	0.32476	360.0
61	0.21875	4.75	3.875	3.9	0.37888	232.5
62	0.21875	4.75	3.750	3.9	0.37888	225.0
63	0.21875	4.75	3.875	3.9	0.37888	232.5
64	0.21875	4.75	3.750	3.9	0.37888	225.0
65	0.21875	4.75	3.625	3.9	0.37888	217.5
OBS	MDOT	MMDOT	RATIOH	PDROP	RATIOP	
53	15.2120	100.080	0.151998	16.1006	3.28584	
54	15.2120	100.080	0.151998	16.1006	3.28584	
55	15.2120	100.080	0.151998	16.1006	3.28584	
56	15.8734	133.440	0.118955	28.6233	5.30061	
57	15.8734	133.440	0.118955	28.6233	5.30061	
58	15.8734	133.440	0.118955	28.6233	5.30061	
59	15.8734	133.440	0.118955	28.6233	5.30061	
60	15.8734	133.440	0.118955	28.6233	5.30061	
61	10.2515	39.615	0.258779	1.3618	0.34918	
62	9.9209	39.615	0.250432	1.3618	0.34918	
63	10.2515	39.615	0.258779	1.3618	0.34918	
64	9.9209	39.615	0.250432	1.3618	0.34918	
65	9.5902	39.615	0.242084	1.3618	0.34918	
OBS	BHP	TA	JP	NR	PRODCT	
53	0.112723	0.082835	7.5101	75749	248399	
54	0.112723	0.082835	7.5101	75749	248899	
55	0.112723	0.082835	7.5101	75749	248899	
56	0.267195	0.082835	13.3512	100998	535354	
57	0.267195	0.082835	13.3512	100998	535354	
58	0.267195	0.082835	13.3512	100998	535354	
59	0.267195	0.082835	13.3512	100998	535354	
60	0.267195	0.082835	13.3512	100998	535354	
61	0.003774	0.112746	0.8645	25701	8974	
62	0.003774	0.112748	0.8645	25701	8974	
63	0.003774	0.112748	0.8645	25701	8974	
64	0.003774	0.112748	0.8645	25701	8974	
65	0.003774	0.112748	0.8645	25701	8974	

TABLE (4.7) Cont'd

OBS	DN	Q	INCHES	BHP	DE	GRAMS
66	0.21875	8.50	4.250	4.0	0.37888	255.0
67	0.21875	8.50	4.125	4.0	0.37888	247.5
68	0.21875	8.50	4.250	4.0	0.37888	255.0
69	0.21875	8.50	4.125	4.0	0.37888	247.5
70	0.21875	8.50	4.125	4.0	0.37888	247.5
71	0.21875	12.00	4.375	4.3	0.37888	262.5
72	0.21875	12.00	4.500	4.3	0.37888	270.0
73	0.21875	12.00	4.625	4.3	0.37888	277.5
74	0.21875	12.00	4.500	4.3	0.37888	270.0
75	0.21875	12.00	4.625	4.3	0.37888	277.5
76	0.21875	16.00	4.875	4.8	0.37888	292.5
77	0.21875	16.00	5.000	4.8	0.37888	300.0
78	0.21875	16.00	4.875	4.8	0.37888	292.5
OBS	MDOT	MMDOT	RATIOH	PDROP	RATIOF	
66	11.2436	70.890	0.158607	4.3607	1.09019	
67	10.9129	70.890	0.153942	4.3607	1.09019	
68	11.2436	70.890	0.158607	4.3607	1.09019	
69	10.9129	70.890	0.153942	4.3607	1.09019	
70	10.9129	70.890	0.153942	4.3607	1.09019	
71	11.5743	100.080	0.115651	8.6913	2.02124	
72	11.9050	100.080	0.118955	8.6913	2.02124	
73	12.2357	100.080	0.122259	8.6913	2.02124	
74	11.9050	100.080	0.118955	8.6913	2.02124	
75	12.2357	100.080	0.122259	8.6913	2.02124	
76	12.8971	133.440	0.096651	15.4512	3.21901	
77	13.2278	133.440	0.099129	15.4512	3.21901	
78	12.8971	133.440	0.096651	15.4512	3.21901	
OBS	HHP	TA	JF	NR	PRODT	
66	0.021626	0.112748	2.76838	45990.4	50138	
67	0.021626	0.112748	2.76838	45990.4	50138	
68	0.021626	0.112748	2.76838	45990.4	50138	
69	0.021626	0.112748	2.76838	45990.4	50138	
70	0.021626	0.112748	2.76838	45990.4	50138	
71	0.060849	0.112748	5.51760	64927.6	131234	
72	0.060849	0.112748	5.51760	64927.6	131234	
73	0.060849	0.112748	5.51760	64927.6	131234	
74	0.060849	0.112748	5.51760	64927.6	131234	
75	0.060849	0.112748	5.51760	64927.6	131234	
76	0.144236	0.112748	9.80906	86570.1	278670	
77	0.144236	0.112748	9.80906	86570.1	278670	
78	0.144236	0.112748	9.80906	86570.1	278670	

TABLE (4.7) Cont'd

OBS	DL	Q	INCHES	BHP	DE	GRAMS
79	0.21875	16.00	5.125	4.8	0.3788E	307.5
80	0.21875	16.00	5.000	4.8	0.3788E	300.0
81	0.25000	4.75	3.250	3.7	0.43301	195.0
82	0.25000	4.75	3.000	3.7	0.43301	180.0
83	0.25000	4.75	3.125	3.7	0.43301	187.5
84	0.25000	4.75	3.250	3.7	0.43301	195.0
85	0.25000	4.75	3.250	3.7	0.43301	195.0
86	0.25000	8.50	3.750	3.9	0.43301	225.0
87	0.25000	8.50	3.500	3.9	0.43301	210.0
88	0.25000	8.50	3.625	3.9	0.43301	217.5
89	0.25000	8.50	3.750	3.9	0.43301	225.0
90	0.25000	8.50	3.750	3.9	0.43301	225.0
91	0.25000	12.00	4.000	4.0	0.43301	240.0
OBS	MDOT	MMDOT	RATIOH	PDROP	RATIOF	
79	13.5585	133.440	0.101607	15.4512	3.21901	
80	13.2278	133.440	0.099129	15.4512	3.21901	
81	8.5981	39.615	0.217041	0.7982	0.21574	
82	7.9367	39.615	0.200345	0.7982	0.21574	
83	8.2674	39.615	0.208693	0.7982	0.21574	
84	8.5981	39.615	0.217041	0.7982	0.21574	
85	8.5981	39.615	0.217041	0.7982	0.21574	
86	9.9209	70.890	0.139947	2.5561	0.65541	
87	9.2595	70.890	0.130617	2.5561	0.65541	
88	9.5902	70.890	0.135282	2.5561	0.65541	
89	9.9209	70.890	0.139947	2.5561	0.65541	
90	9.9209	70.890	0.139947	2.5561	0.65541	
91	10.5822	100.080	0.105738	5.0945	1.27362	
OBS	BHP	TA	JF	NR	PRODCI	
79	0.144236	0.112748	9.80906	86570.1	278670	
80	0.144236	0.112748	9.80906	86570.1	278670	
81	0.002212	0.147262	0.66190	22487.9	4851	
82	0.002212	0.147262	0.66190	22487.9	4851	
83	0.002212	0.147262	0.66190	22487.9	4851	
84	0.002212	0.147262	0.66190	22487.9	4851	
85	0.002212	0.147262	0.66190	22487.9	4851	
86	0.012676	0.147262	2.11954	40241.6	26375	
87	0.012676	0.147262	2.11954	40241.6	26375	
88	0.012676	0.147262	2.11954	40241.6	26375	
89	0.012676	0.147262	2.11954	40241.6	26375	
90	0.012676	0.147262	2.11954	40241.6	26375	
91	0.035667	0.147262	4.22441	56811.6	72357	

TABLE (4.7) Cont'd

OBS	DN	Q	INCHES	BHP	DE	GRAMS	MDOT	MMDOT	RATION
92	0.25	12	4.000	4.0	0.43301	240.0	10.5822	100.08	0.105738
93	0.25	12	4.125	4.0	0.43301	247.5	10.9129	100.08	0.109042
94	0.25	12	4.125	4.0	0.43301	247.5	10.9129	100.08	0.109042
95	0.25	12	4.000	4.0	0.43301	240.0	10.5822	100.08	0.105738
96	0.25	16	4.500	4.5	0.43301	270.0	11.9050	133.44	0.089216
97	0.25	16	4.500	4.5	0.43301	270.0	11.9050	133.44	0.089216
98	0.25	16	4.500	4.5	0.43301	270.0	11.9050	133.44	0.089216
99	0.25	16	4.500	4.5	0.43301	270.0	11.9050	133.44	0.089216
100	0.25	16	4.500	4.5	0.43301	270.0	11.9050	133.44	0.089216

OBS	PDROP	RATIOF	BHP	TA	JP	NR	PRODUCT
92	5.09449	1.27362	0.0356674	0.147262	4.22441	56811.6	72357
93	5.09449	1.27362	0.0356674	0.147262	4.22441	56811.6	72357
94	5.09449	1.27362	0.0356674	0.147262	4.22441	56811.6	72357
95	5.09449	1.27362	0.0356674	0.147262	4.22441	56811.6	72357
96	9.05687	2.01264	0.0845449	0.147262	7.51006	75748.9	152455
97	9.05687	2.01264	0.0845449	0.147262	7.51006	75748.9	152455
98	9.05687	2.01264	0.0845449	0.147262	7.51006	75748.9	152455
99	9.05687	2.01264	0.0845449	0.147262	7.51006	75748.9	152455
100	9.05687	2.01264	0.0845449	0.147262	7.51006	75748.9	152455

Nozzle Configuration Experiment

The purpose of this experiment was to determine the effect of nozzle configuration, number of nozzles, nozzle orientation, and nozzle size on the rate of cutting removal. Twelve bits with different nozzle configuration were investigated. No water courses were simulated. Description of the twelve bit configurations are shown in Table (4.8). Figure (4-3) shows the bits used.

A plexiglas tube, 61 inches long and 1-1/2 x 2 inches in diameter, was used to simulate the borehole. Sand size of 12 mesh was used to simulate the cuttings. A 3/4 x 1 inch diameter drill pipe was used to conduct the drilling fluid to the drill bit. The spacing between the bits and cutting surface was consistently kept at one inch throughout the entire experiment. Mud pump gear Number 3 was selected to pump the drilling fluid (water) at four different flow rates (4.75, 8.5, 12, and 16 gallons per minute). The procedure for determining the amount of cutting removal is similar to the procedure used in the previous experiments. The results of the mass flow rates of cutting removal for each bit nozzle configuration for the various flow rates used are shown in Tables (4.9), (4.10), (4.11), and (4.12). The terms appearing in these tables are shown in Appendix (B).

TABLE 4.8

NOZZLE CONFIGURATION

Bit Number	Description
1	3 x 5/32 nozzles directed downward
2	Two nozzles downward with area equal to area of 3 x 5/32 nozzles
3	One nozzle downward with an area equal to area of 3 x 5/32 nozzles
4	Two nozzles downward, each is 5/32
5	One corner nozzle directed downward with diameter equal 5/32
6	One center nozzle directed downward with diameter equal 5/32
7	Two small and one large nozzle, all directed downward
8	Two large and one small nozzle all directed downward
9	3 x 5/32 nozzles pointing 45 degrees outward
10	3 x 5/32 nozzles pointing 45 degrees outward and 60 degrees sideward
11	3 x 5/32 nozzles with one nozzle pointing upward
12	3 x 5/32 nozzles with two nozzles pointing upward

Figure (4-3) BIT CONFIGURATION USED

TABLE (4.9) CLEANING ACTION BY VARIOUS NOZZLE COMBINATIONS
 MASS RATE OF CUTTING REMOVED (LB/MIN) VS BIT NO
 Q=4.75 GPM

OBS	BITNUMBR	Q	INCHES	GRAMS	MDOT
1	1	4.75	3.250	139.750	6.1620
2	2	4.75	4.500	193.500	8.5319
3	3	4.75	3.500	150.500	6.6359
4	4	4.75	4.500	193.500	8.5319
5	5	4.75	9.875	424.625	18.7229
6	6	4.75	6.000	258.000	11.3759
7	7	4.75	5.125	220.375	9.7169
8	8	4.75	3.750	161.250	7.1099
9	9	4.75	3.000	129.000	5.6880
10	10	4.75	3.125	134.375	5.9250
11	11	4.75	4.000	172.000	7.5839
12	12	4.75	3.125	134.375	5.9250

TABLE (4.10) CLEANING ACTION BY VARIOUS NOZZLE COMBINATIONS
 MASS RATE OF CUTTING REMOVED (LB/MIN) VS BIT NO
 Q=8.50 GPM

OBS	BITNUMBR	Q	INCHES	GRAMS	MDOT
1	1	3.5	3.750	161.250	7.1099
2	2	8.5	5.250	225.750	9.9539
3	3	8.5	3.625	155.875	6.8729
4	4	8.5	5.375	231.125	10.1909
5	5	8.5	10.000	430.000	18.9599
6	6	8.5	6.750	290.250	12.7979
7	7	8.5	5.500	236.500	10.4279
8	8	8.5	4.000	172.000	7.5839
9	9	8.5	3.125	134.375	5.9250
10	10	8.5	3.250	139.750	6.1620
11	11	8.5	4.750	204.250	9.0059
12	12	8.5	3.750	161.250	7.1099

TABLE (4.11) CLEANING ACTION BY VARIOUS NOZZLE COMBINATIONS
 MASS RATE OF CUTTING REMOVE (LB/MIN) VS BIT NO
 Q=12.0 GPM

OBS	BITNUMBER	Q	INCHES	GRAMS	MDOT
1	1	12	4.00	172.00	7.5839
2	2	12	5.75	247.25	10.9019
3	3	12	4.25	182.75	8.0579
4	4	12	6.50	279.50	12.3239
5	7	12	6.00	258.00	11.3759
6	8	12	4.50	193.50	8.5319
7	9	12	3.25	139.75	6.1620
8	10	12	3.50	150.50	6.6359
9	11	12	5.25	225.75	9.9539
10	12	12	4.75	204.25	9.0059

TABLE (4.12) CLEANING ACTION BY VARIOUS NOZZLE COMBINATIONS
 MASS RATE OF CUTTING REMOVED (LB/MIN) VS BIT NO
 $Q=16.0$ GPM

OBS	BITNUMBER	Q	INCHES	GRAMS	ADDT
1	1	16	4.750	204.250	9.0059
2	2	16	6.000	258.000	11.3759
3	3	16	5.000	215.000	9.4799
4	4	16	7.000	301.000	13.2719
5	7	16	6.500	279.500	12.3239
6	8	16	4.750	204.250	9.0059
7	9	16	3.500	150.500	6.6359
8	10	16	4.000	172.000	7.5839
9	11	16	6.000	258.000	11.3759
10	12	16	5.625	241.875	10.6649

CHAPTER V

STATISTICAL ANALYSIS

Discussion of Method Used:

Experimental data is usually plotted to see if there is any type of relation that exists among the variables. This type of relation is commonly referred to as the correlation of the data.²⁶ The word "correlation" refers to the degree of association between the variables involved in the study performed.

The solution to the final equation generated by the dimensional analysis method discussed in Chapter II can be written in the following manner:

$$\pi = A (\pi_1)^{B_1} (\pi_2)^{B_2} \dots (\pi_k)^{B_k} \dots \dots \dots (5-1)$$

in which $\pi, \pi_1, \pi_2, \dots, \pi_k$ are dimensionless groups and $A, B_1, B_2, \dots, B_k$ are constants to be estimated. For simplification, equation (5-1) can be transposed into the standard logarithms:²⁷

$$\text{Log } \pi = \text{Log } A + B_1 \text{Log } \pi_1 + B_2 \text{Log } \pi_2 + \dots + B_k \text{Log } \pi_k \dots \dots \dots (5-2)$$

If $Y = \text{Log } \pi, X_1 = \text{Log } \pi_1, X_2 = \text{Log } \pi_2 \dots X_k = \text{Log } \pi_k$

and $B_0 = \text{Log } A$, then equation (5-2) can be written as:

$$Y = B_0 + B_1 X_1 + B_2 X_2 + \dots + B_k X_k \dots \dots \dots (5-3)$$

The values of $B_0, B_1, B_2, \dots, b_k$ remain to be determined by the method of least squares. The least square method assists in the prediction of a response variable from a function of regressor variables and parameters and adjusting these parameters until equation (5-3) is optimized. Adjusting the parameters means minimizing the sum of the squares of the difference between the actual response value and the value predicted by the equation.²⁸

The optimizing procedures is performed as follows:

$$SS = \sum_{i=1}^n E_i^2 = \sum_{i=1}^n (y_i - b_0 - b_1 X_i - \dots - b_k X_{ki})^2 \quad \text{--- (5-4)}$$

Where E is the error term, and $b_0, b_1, \dots, b_k$ are the values for $B_0, B_1, \dots$, and B_k that minimize the sum of squares.

The result of the regression program is then converted to the form of equation (5-1) by taking the antilog of B_0 . A computer program called REG, which is a part of the Statistical Analysis Systems Package available in the University of Oklahoma, was utilized to perform the regression analysis for this study. Details of the program and its output format are listed in Appendix (A). The main statistical parameters²⁹ which are considered important in terms of analyzing the regression results are: 1) R-Square, and 2) F-Value. Discussion of these parameters will be included in Chapter VIII.

CHAPTER VI

THEORETICAL SCOURING MODEL

The removal of cuttings from the bottom of a well hole is similar to the removal of sediment from streambeds. The movement of sediment³⁰ in alluvial streams is such a complex problem that it may never be completely subject to rational solution. It represents, in fact, the most extreme degree of unsteady, nonuniform flow, since the streambed as well as the water's surface may be continuously changing in form. With the current information, an approximate understanding of the general transport mechanism can be obtained only by isolating particular details or by so simplifying the boundary conditions that only the most significant variables need to be considering.

The scouring criteria can be analyzed³¹ as follows: Consider a plain, stationary bed of cuttings consisting of loose particles of uniform size with liquid flowing over it. As soon as the liquid starts flowing, hydrodynamic forces are exerted upon the solid particles. A further increase in the flow intensity causes an increase in the magnitude of these forces. Hence, for a particular stationary bed, a condition is eventually reached at which particles in the movable bed are unable to resist the hydrodynamic forces and, thus, begin to be dislodged and eventually to start to move. This movement is not an instantaneous one for all particles of a given size resting in the top layer. In fact, at any given hydrodynamic condition, some move and some do not. This is

due to the statistical nature of the problem which implicitly brings out the fact that the flow is usually turbulent. The condition of the "initial movement of the bed" is determined by observation; therefore, its definition is a very subjective one.

The initial movement of the bed, frequently described by civil engineers as the critical condition of the initial scour, will be explained by the critical velocity equation, considering the impact of the liquid on the particles.

Condition of Incipient Motion:

The condition of incipient movement for an assembly of loose solid particles (spheres) is described in terms of the forces acting on the particles by:

$$\tan \phi = \frac{F_t}{F_n} \quad (6.1)$$

where F_t = parallel forces to the angle of repose

F_n = normal forces to the angle of repose

ϕ = angle of repose

$\tan \phi$ = coefficient of friction

F_t and F_n are results of the hydrodynamic drag F_D , the lift force F_L , the buoyant force F_B , and the submerged weight W . The condition of incipient movement under the action of these forces becomes, according to Equation (6.1):

$$\tan \phi = \frac{F_d}{W - F_B - F_L} \quad (6.2)$$

In the usual way, drag, buoyant force, and lift force are expressed by

$$F_D = C_D K_1 d^2 \frac{\rho_f V_b^2}{2} \quad (6.3)$$

$$F_B = \rho_f \frac{\pi}{6} d^3 g \quad (6.4)$$

$$F_L = C_L K_2 d^2 \frac{\rho_f V_b^2}{2} \quad (6.5)$$

Where the weight of the particle is expressed as

$$W = K_3 \rho_s \frac{\pi}{6} g d^3 \quad (6.6)$$

Where:

- V_b = fluid velocity at the bottom of fluid layer
- C_D, C_L = drag and lift coefficient, respectively
- d = particle diameter
- K_1, K_2, K_3 = particle shape factors (equal 1 for sphere)
- ρ_f = liquid density
- ρ_s = particle density
- g = acceleration of gravity

Introducing Equations (6.3), (6.4), (6.5), and (6.6) into Equation (6.2) yields:

$$\tan \phi = \frac{C_D \frac{1}{2} \rho_f V^2 d^2}{\rho_s \frac{\pi}{6} g d^3 - \rho_f \frac{\pi}{6} d^3 g - C_L d^2 \frac{\rho_f V^2}{2}}$$

$$\tan \phi = \frac{C_D \frac{1}{2} \rho_f V^2 d^2}{(\rho_s - \rho_f) \frac{\pi}{6} g d^3 - C_L d^2 \frac{\rho_f V^2}{2}}$$

Solving for critical V at the bottom, $(V_b)_{cr}$ yields:

$$\frac{\rho_f (V_b)_{cr}^2}{(\rho_s - \rho_f) \frac{\pi}{6} g d \tan \phi} = \frac{2}{C_D + C_L \tan \phi}$$

$$(V_b)_{cr}^2 = \frac{2 (\rho_s - \rho_f) \frac{\pi}{6} g d \tan \phi}{\rho_f (C_D + C_L \tan \phi)} \quad (6.7)$$

with $(V_b)_{cr}$ as the critical bottom velocity at which, according to Equation (6.1) incipient particle motion takes place.

From Equation (6.7), high drag forces or high lift forces seem to lower the critical velocity at which incipient particle motion takes place.

Equation of Motion:

Once the cutting is swept off the bottom, its motion with respect to the fluid velocity is described by Newton's second law:

Mass x Acceleration = ΣF (assuming no lift or weight forces in the x-direction).

$$\left(\rho_s \frac{\pi}{6} d^3 \right) \frac{dv_s}{dt} = C_D \frac{1}{2} \rho_f (V_f - V_s)^2 \frac{\pi}{4} d^2$$

Let $K_1 = \rho_s \left(\frac{\pi}{6} d^3 \right)$

$$K_2 = \frac{1}{2} \rho_f \frac{\pi}{4} d^2$$

Thus, $K_1 \frac{dv_s}{dt} = C_D K_2 (V_f - V_s)^2$

$$\frac{dv_s}{(V_f - V_s)^2} = C_D \frac{K_2}{K_1} dt$$

Integrating the above equation:

$$\int_0^{V_s} \frac{dV_s}{(V_f - V_s)^2} = C_D \frac{K_2}{K_1} \int_0^t dt$$

Solving for V_s :

$$V_s = V_f \left[1 - \frac{1}{C_D \frac{K_2}{K_1} V_f t + 1} \right] \quad (6.9)$$

It appears from Equation (6.9) that high drag coefficients cause the particle to approach the fluid velocity faster. Therefore, it can be stated that once the cutting is swept off the bottom, it would seem that high velocity is helpful in that it will accelerate the cutting and improve bottomhole cleaning.

It has been established^{32, 34, 35, 36.} that drag coefficient is almost inversely proportional to Reynolds number for Reynolds number below 1.0.

From the concepts outlined in this chapter, it appears that if the fluid velocity near the bottom of the particle is near zero, the drag force in this case is a function of Reynolds number which is defined as $\frac{\rho_f V_f d}{\mu}$. Once the cutting is swept off the bottom, the drag force in this case is a function of Reynolds number which is defined as $\rho_f \left(\frac{V_f - V_s}{\mu} \right) d$. To clarify further, lift force and drag force are discussed below.

Lift Force: The particles on the bed are lifted when the upward hydraulic force, the lift force, exceeds the submerged weight of the particle. When the particles rest on the bed, the velocity at the bottom is zero; while the velocity at the top of the particle is greater than zero. As

a result, there is a high pressure under the particle and low pressure at the top (Bernouille Principle). This pressure difference creates an upward force. As the particles travel upward, the velocity difference between the top and bottom of the particles will reduce, thereby reducing the lift. After the particles reach a certain height, the lift force becomes smaller than the submerged weight and the particles begin to fall.

Drag Force: The drag force is the force component created by a moving fluid on any object submerged in that fluid. The direction of the force is the same as that of the free stream of fluid.

CHAPTER VII

DISCUSSION OF THE EXPERIMENTAL RESULTS

In Chapter IV the experimental data was presented in a tabular form. In order to simplify the evaluation, the experimental results are presented in graphic form in this Chapter. An attempt will be made to describe some of the finer points observed in the experimental runs.

The experimental data relating to chip size are plotted versus mass flow rate of cuttings per mass flow rate of drilling fluid as shown in Figure (7-1). Figure (7-1) shows that the mass flow rate of cuttings per mass flow rate of drilling fluid is independent of the cutting size for the sizes tested. The cuttings tested in the experiments ranged in size from 20 to 6 U.S. sears mesh (0.0331 to 0.132 inches). This independency between the small sizes of cuttings and the mass flow rate of cuttings per mass flow rate of drilling fluid, did occur under the same turbulent jet flow condition, same annular velocity, same pump flow rate, and same borehole configuration.

Sutko³⁶ has shown that for chip sizes of 0 to 0.02 inches, the chip removal force remains very small and no significant change in the chip removal force was observed. It appears that the flow pattern in the vicinity of the chip is consistent for the range of the chip size tested.

The experimental data relating to Froude number is plotted versus mass flow rate of cuttings removed per mass flow rate of drilling as shown in

Figure (7-1) MASS RATE RATIOS VS CHIP SIZE PER NOZZLE DIAMETER

Figure (7-2). Figure (7-2) shows that reciprocal Froude's number, including the gravity term, has no effect on the mass flow rate of cuttings removed per mass flow rate of drilling fluid. The reciprocal Froude's number ($\frac{gL}{V^2}$) when multiplied and divided by ρA , is a ratio of weight to dynamic (or inertial) force. Therefore, the results would indicate that the dynamic force is more dominant than the weight. One could infer from this that the angle of borehole inclination in deviated hole does not affect the mass rate of cutting removed.

Cheatham and Yarbrough³⁷ conducted an experimental investigation of the jetting action of a single central jet in removing loose particles from the bottom of a cylindrical hole. They have found that the ratio of inertial forces to gravitational forces given by a modified Froude number, ($\frac{\rho Q^2}{g \Delta \rho D^5}$), is more significant than the ratio of inertial to frictional forces given by Reynolds number, ($\frac{\rho Q}{\mu D}$). The reason for the contradiction in Froude number effect is that Cheatham and Yarbrough have included the density ratio term in their modified Froude number whereas, in this study, the modified Froude number does not include the density term.

In other words, Cheatham and Yarbrough have investigated the effect of the following term:

$$\frac{g D^5}{Q^2} \frac{\Delta \rho}{\rho}$$

Which is equal to

$$\frac{g D^5}{Q^2} \frac{\rho_s - \rho_f}{\rho_f} = \frac{g D^5}{Q^2} \left(\frac{\rho_s}{\rho_f} - 1 \right) \quad (7.1)$$

Figure (7-2) MASS RATE RATIOS VS RECIPROCAL FROUDE NUMBER ($d_n^5/g/Q^2$)

whereas, in this research, $\frac{g D^5}{Q^2}$ and $\frac{\rho_s}{\rho_f}$ have been investigated separately; Cheatham and Yarbrough have found that the product of the terms appearing in Equation (7.1) have an effect in the removal of loose particles from the bottom of a cylindrical hole. In this research, it has been shown that $\frac{g D^5}{Q^2}$ had no effect on the mass rate of cutting removed. It will be shown shortly herein that the term $(\frac{\rho_s}{\rho_f})$ does have a strong effect on the mass rate of cuttings removed.

Figure (7-3) shows the dimensionless mass flow rate (mass flow rate of cuttings removed per mass flow rate of drilling fluid) as a function of the dimensionless length (off bottom spacing per bit diameter). The experimental data shows that for a large value of spacing per bit diameter, the mass rate ratios (mass flow rate of cutting removed per mass flow rate of drilling fluid) are extremely low and increase at an increasing rate. As the nozzles move toward the bottom of the hole, $(\frac{L}{d_b})$ decreasing, the jet works its way through less fluid to reach the bottom of the hole. Thus, the amount of mixing and energy loss is minimized. This finding agrees with that of Van Lingen³ whose laboratory drilling in impermeable Belgian limestone, showed a 20 percent increase in penetration rate at reduced hydraulic horsepower and a 30 percent increase for unchanged power resulted with nozzles close to bottom. The finding also agrees with Feenstra's findings⁶ which resulted from his full scale experiments on jets in impermeable rock drilling. Feenstra made the following statement regarding those experiments:

At high bit load, bottom-balling limits the penetration of insert-type bits. In this case bottom scavenging is required, which is most effectively performed with velocity jets from nozzles extended close to bottom. As a result of this measure, penetration may improve 70 percent in non-friable rock, and less in friable rock. This will make the insert-type bit more versatile.

Figure (7-3) MASS RATE RATIOS VS SPACING PER BIT DIAMETER

Feenstra also indicated, that further intensification of bottom hole scavenging, obtained by extension of the nozzles to within an inch of the bottom did not improve tooth scavenging but did provide, even at moderate horsepower, higher fluid velocities in that bottom area which tends to ball-up. For the above reasons, it was recommended that extended nozzles with insert-type bits be utilized due to the fact that such bits provide more space, available between bit matrix and formation bottom, than any other three-cone bits.

These findings were also found to coincide with those of McLean⁸ who conducted a "cross-flow and impact under jet bits" study in which he concluded that the extended nozzles will diminish the impact area. From the edge of this smaller impact area to the edge of the non-extended nozzles impact area the velocity of the cross-flow will be higher. Therefore, an increase in the sweeping capacity of the cuttings along the hole bottom will result. Experiments by Pekarek et al.³⁸ also confirmed and agreed with our findings. They observed that the power of a submerged jet stream decreases with respect to distance from the nozzle primarily because of the uncontrollable divergence of the stream. The power of the jet stream decreased with distance to a value at which the jet stream ceases to remove the cuttings. This value of power was referred to as "threshold power of specific formation material."

Figure (7-4) shows an increase in rotational Reynolds number, $\left(\frac{\text{RPM } Q \rho_f}{D_H T_y}\right)$, yields an increase in mass flow rate ratios (mass flow rate of cuttings per mass flow rate of drilling fluid), up to a rotary speed of 30 RPM where the mass flow rate ratios tend to level off. This agrees in part with Maurer² whose laboratory observations indicated that the drilling rate is directly

Figure (7-4) MASS RATE RATIOS VS ROTATIONAL REYNOLDS NUMBER

proportional to the rotary speed. The experimental observations seem to correlate with the study performed by Outmans³⁹ in which he observed that the drilling rate to be increasing with the rotary speed, but the higher the rotary speed, the smaller the incremental response. Gray et al.⁴⁰ indicated that the rotation of the drill pipe improves the cutting transport ratio because it imparts a helical motion to the cuttings in the vicinity of the drill pipe. VanLingen,³ Gatlin,⁴¹ Moore,⁴² Cunningham,⁴³ and Brantley⁴⁴ have all found that an increase in the rotary speed results in an increase of rate of penetration.

The experimental data relating to cutting density is plotted versus mass rate ratios (mass flow rate of cuttings per mass flow rate of drilling fluid) as shown in figure (7-5). Figure (7-5) shows that increasing the cutting density increases the mass flow rate ratios. As the specific gravity of cuttings increases from 2.59 to 7.76 to 10.53 (sand to steel to lead) per the same unit of mass of drilling fluid, the mass flow rate of cuttings per mass flow rate of drilling fluid is expected to increase sharply due to the increase in the cutting density.

The relationship between the volume of cuttings removed per volume of fluid circulated and the density ratio (cutting density per fluid density) appearing in Figure (7-6) indicate that doubling the cutting density results in a 2 percent reduction in volume of cutting removal. For all practical purposes, the cutting volume is considered constant regardless of the cutting density since the cutting density experienced in drilling operations hardly changes by this magnitude.

The experimental data relating to Reynolds number and the ratio of pressure drop through the nozzles and the bottom hole pressure are plotted

Figure (7-5) MASS RATE RATIOS VS DENSITY RATIOS

Figure (7-6) VOLUME RATIOS VS DENSITY RATIOS.

versus mass flow rate of cutting removed as shown in Figure (7-7). Figure (7-7) shows that as the product of Reynolds number and the ratio of pressure drop through the nozzles and the bottom-hole pressure increases, the mass flow rate of cutting removed increases for all the different bits used. The increase in the mass flow rate of cutting removed is attributed to the higher values of Reynolds number (higher jet velocities) and to the correspondingly higher pressure drop through the nozzles. The jet action was for some time believed to merely improve the transport of cuttings from bottom to annulus, resulting in a clean hole bottom. Memorial Institute⁴⁵ showed, however, that jet velocities of the order 40 ft/sec sufficed to clear even large (0 - 0.4 inch) chips from bottom in the time between two successive tooth actions.

The above finding seems to correlate with the work of Feenstra and Van Leeuwen⁶ who have shown, through their full scale experiments on jets in impermeable rock drilling, that the penetration rate tends to increase with nozzle velocity. They have stated that this favorable action of the jets would intensify bottom scavenging as the result of increased fluid velocity along the bottom. Measurements of the vertical distribution of the cross-flow velocity at several positions beneath a 4-3/4 inch tricone jet bit in a laboratory model⁸ showed that the maximum velocity in the cross-flow is directly proportional to the square root of the product of the rate of flow and the velocity of the jets through the nozzles. The data⁸ also showed that maximizing the product of the rate of flow and the velocity of the jets through the nozzles maximizes velocities in the cross-flow beneath a conventional jet bit. This action maximizes the drag on the cuttings which protrude from the bottom.

Figure (7-7) MASS RATE OF CUTTING REMOVED VS $NR \times (NPD/BHP)$

Since the drag forces are proportional to $\rho V^2 A$, therefore, maximizing the product of Reynolds number and the pressure drop through the nozzles which is proportional to $\rho^2 V^3 A^2$, will tend to maximize the drag on the cutting. An increase in the drag forces would seem to enhance the sweeping action of the particles.

Cunningham and Eenink⁴⁶ have concluded, through their laboratory study, that the drilling rates are improved with increased jet velocities and decreased pressure differential (hydrostatic minus formation pressure). This increase in jet velocities which agrees in part with data related to the product of Reynolds number and the pressure drop through the nozzles, helps remove the layers of cuttings as soon as they are generated and, thus, minimize the recutting by the bit. This will allow the rock bit teeth to penetrate relatively undamaged formations.

The dimensionless mass flow rate ratios (mass flow rate of cuttings per mass flow rate of drilling fluid) are plotted versus the product of Reynolds number and the ratio of pressure drop through the nozzles and bottom-hole pressure as shown in Figure (7-8).

Figure (7-8) shows that at higher values of $(NR \times \frac{\Delta P_n}{BHP})$, the mass rate ratios tend to stabilize or become constant. The mass rate of cuttings per mass rate of drilling fluid is proportional to particle velocity per fluid velocity. Equation (6.9) shows that at high values of fluid velocities the particle velocity to the fluid velocity approaches a constant value. A possible explanation for the lower end of the curve can be explained by Equation (6.7) which shows that critical velocity will be less at small fluid velocity. Since small fluid velocity is associated with small Reynolds number and higher drag coefficient, more particles are picked up for that particular Reynolds number.

Figure (7-8) MASS RATE RATIOS VS NR*(NPD/BHP)

Figures (7-9) and (7-10) show the effect of Reynolds number on the mass flow rate ratios and the mass flow rate of cutting removal. An increase in Reynolds number tends to increase the sweeping particle action. As stated in Chapter VI, Equation (6.9), once the cutting is swept off the bottom, it would seem that high fluid velocity associated with high Reynolds number is helpful in that it will accelerate the cutting and improve bottom-hole cleaning.

The increase in the mass rate of cutting removal with Reynolds number agrees in part with the microbit study conducted by Eckel.¹⁰ Eckel has shown that the rate of penetration, other conditions being constant, can be described as an exponential function involving flow rate and density, nozzle diameter, and bit or bottom-hole viscosity.

Figures (7-11) and (7-12) show the effect of ratios of pressure drop through the nozzles and bottom-hole pressure on the mass flow ratios and the mass flow rate of cutting removal. The trend of Figures (7-11) and (7-12) are similar to the trends observed in Figures (7-7), (7-8), (7-9), and (7-10) due to the common factor that exists among these figures which is the fluid velocity. The higher the fluid velocity through the bit the higher the pressure drop through the bit nozzles.

Numerous authors^{6, 19, 36, 47, 48} have shown that the drilling rate is dependent upon the hydraulic effect of the bit. The rotary bit hydraulic jet system performs two functions at the bit:¹⁹ (1) preventing bit-balling -- keeping the bit clean so that it does not lock up when the area between the teeth becomes clogged with cuttings and, (2) bottom-hole cleaning -- removing the formation chip from the bottom before the next cone gets to it so that the teeth do not have to redrill the chip.

Figure (7-9) MASS RATE RATIOS VS REYNOLDS NUMBER

Figure (7-10) MASS RATE OF CUTTING REMOVED VS REYNOLDS NUMBER

Figure (7-11) MASS RATE RATIOS VS PRESSURE RATIOS

Figure (7-12) MASS RATE OF CUTTING REMOVED VS PRESSURE RATIOS

Figures (7-13) and (7-14) show an increase in the mass rate of cutting removal with the hydraulic horsepower and the hydraulic impact force. Bit hydraulic horsepower and jet impact are related to the fluid velocity at the nozzle. At the nozzle, the fluid flow is accelerated and a pressure drop occurs. This energy conversion provides the hydraulic influence that cleans the bit and the bottom of the hole. Generally, an increase in either bit horsepower or impact will increase the drilling rate. The primary purpose of either is to clean the bottom of the hole and to prevent regrinding of cuttings.

The jet impact force is proportional to ρV^2 , therefore maximizing the jet impact force will tend to maximize the drag forces exerted on the cutting since drag forces are also proportional to ρV^2 .

The hydraulic horsepower available at the bit is proportional to ρV^3 ; hence, it appears that also increasing the hydraulic horsepower will tend to improve the drag forces required to dislodge and carry the cutting particles off the bottom.

Sufficient flow rate is needed to clean the bit and that jet velocity is needed to free the drilled chip held by differential pressure⁴⁹ to the bottom of the hole. This jet velocity has a greater effect on penetration rate than the flow rate alone; however, for a given hole size, there is a limited range of flow rates that can be used for hole cleaning. Therefore, jet velocity for a given hole size is somewhat dependent upon the hydraulic horsepower at the bottom of the hole. Early investigation⁵⁰ has shown that penetration rate is dependent upon the circulation rate and also upon the jet velocity. This requirement gave rise to the maximum (QV_n) program commonly called "jet impact" program, justified by its proponents who believe that cutting removal

Figure (7-13) MASS RATE OF CUTTING REMOVED VS HYDRAULIC HORSEPOWER

Figure (7-14) MASS RATE OF CUTTING REMOVED VS HYDRAULIC IMPACT FORCE

is dependent upon the blasting effect of the jet striking the bottom of the hole. Later investigators, who were concerned primarily with down hole mud operated devices, chose to consider hydraulic horsepower which has a greater physical significance to oilfield engineers than jet impact. The term "bit hydraulic horsepower" came into usage and was justified by its proponents who believed that cutting removal is dependent upon the amount of fluid energy expended at the bit.

Figures (7-15) and (7-16) show how the mass flow rate ratio (mass flow rate of cuttings removed per mass flow rate of drilling fluid) related to hydraulic horsepower and hydraulic impact force. The trends were similar to the one observed with Reynolds number, but the magnitude effects are different. The magnitude effects in terms of the correlation of these parameters with the mass flow rate ratios set forth in Chapter VIII.

Figure (7-15) MASS RATE RATIOS VS HYDRAULIC HORSEPOWER

Figure (7-16) MASS RATE RATIOS VS HYDRAULIC IMPACT FORCE

CHAPTER VIII

REGRESSION ANALYSIS

Regression procedures were conducted on the experimental data to obtain the nature of the relation between variables. With two variables that can be correlated linearly, the regression coefficient is the slope of the line used to correlate the variables. If a higher-order correlation is used, there are regression coefficients for each of the ordered terms relating to the variables. If more than two variables are involved in the correlation, there are regression coefficients for each order of each of the variables used to predict the variation in some other variables.

The degree of association between one variable and another, or between one variable and several others, is referred to as the correlation. A correlation coefficient is defined²⁷ as the measure of the amount of relation between variables. In regression analysis, a whole function is estimated (the regression equation), but correlation analysis yields only one number — an index designed to give an immediate picture of how closely two variables move together. Although correlation is a less powerful technique than regression, the two are so closely related that correlation often becomes a useful tool in interpreting regression.

Private consultations⁵¹ with professional statisticians indicated that R^2 - and F-values are good measures of the correlation significance.

R^2 - Square is a measure that indicates the portion of the (corrected) total that is attributed to the fit rather than left to residual error. It is called the coefficient of multiple determination. It is the square of the multiple correlation: In other words, it is the square of the correlation between the dependent variable and the predicted values. It is also referred to as a measure of the proportion of the total variation of the mean, $\bar{y}$, explained by the regression. A coefficient of multiple determination of unity implies a perfect association between the variables, or the regression fit, and will explain 100 percent of the variation in the dependent variable. A zero coefficient refers to no association among the variables.

The resulting⁵² R^2 indicates how well the model fits the data. The data will always dictate which is the best model, as the model cannot dictate the data. R^2 , though, is not artificially inflated by the increase in sample size. Increasing sample size merely produces a more stable and closer estimate of the population R^2 value. This is one of the reasons that researchers have been reporting R^2 values along with their indices of statistical significance.

The coefficient of multiple determination can be calculated as follows:²⁹

$$R^2 = \frac{\text{sum of squares due to regression}}{\text{total sum of squares corrected for the mean}} = \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

Where:

$\hat{Y}_i$ = is an estimated value of y for an observed value of y_i

$\bar{Y}$ = overall mean

Y_i = observed value

When there is a perfect correlation between the variables, there is no residual between the variables: there is no residual deviation of Y from $\hat{Y}$ and $R^2 = 1.0$. When there is no correlation and more of the sum of squares of deviation is removed by a linear relationship and $R^2 = 0.0$, sometimes R^2 is referred to as the fraction of the sum of squares of deviation that is removed by the correlation.

F-Value: F-test is conducted for significance of the model. F-value can be calculated as follows:

$$F = \frac{MS_{reg}}{S^2} = \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n - K}}$$

Where: MS_{reg} = mean square due to regression
 S^2 = Mean square due to residual
 n = number of observations

From F-tables, with probability levels of $(1 - \alpha)$ and $(K, N-K-1)$ degree of freedom, a critical F-value is obtained. The K and N imply the number of variables and the number of observations. From the comparison between the calculated and the critical F-value, the following can be deduced:

1. If, F (calculated) $>$ F (critical), then R^2 is significant at α level running a risk of less than α percent being wrong.
2. If, F (calculated) $>$ F (critical), then we reject the null hypothesis that the parameter (slope) is equal to zero running a risk of less than α percent of being wrong.

Null Hypothesis:

$$H_0 : B_1 = 0$$

$$H_1 : B_1 \neq 0$$

This hypothesis relates to the significance of regression. Failing to reject $H_0 : b_1 = 0$, implies that there is no linear relationship between the dependent and the independent variable. It may also imply that the independent variable is of little value in explaining the variation in the dependent variable. Alternatively, if $H_0 : B_1 = 0$ is rejected, this implies that X is of value in explaining the variability in Y .

Remember, deciding that (slope) $B_1 = 0$ is a very important conclusion that is only aided by the F-test. The inability to show that the slope is not statistically different from zero may not necessarily mean that Y and X are unrelated. It may simply mean that our ability to detect this relationship has been obscured by the variance of the measurement process or, that the range of values of X is inappropriate. A great deal of non-statistical evidence and knowledge of the subject matter in the field is required in order to make the conclusion that $B_1 = 0$.

Results of the regression analysis conducted on the experimental data are summarized as shown in Tables (8.1) and (8.2). Tables (8.1) and (8.2) show that the chip size and Froude number have a slope of zero (for all practical purposes) with respect to the mass flow rate of cuttings removed per the mass flow rate of drilling fluid. Perfect R^2 -values are obtained with chip size and reciprocal Froude number experiments. Repeated observations are the reasons for such high R^2 -values.

In the chip size experiment, there are twenty observations which occur in four groups each of five repeats. There are, effectively, four pieces of

information, represented by four mean values, and sixteen error degrees of freedom for pure error, four at each repeat point. If the slope, $B_1 = 0$, then $R^2 = 1.0$ and this is the special case where R^2 value can be 1.0. Therefore, the statistics of chip size and Froude number experiments indicate that the chip size and Froude number (regression coefficients equal zero) are not related to the mass flow rate ratios.

Private consultations⁵¹ indicated that R^2 -value greater than 0.9 is reasonable for physical data, where R^2 -value lower than 0.3 is considered poor correlation. The mass flow rate ratio was found to be strongly dependent upon Reynolds number (regression coefficient equals - 0.9583), nozzle height above bottom of hole (regression coefficient equals 0.60067). The mass flow rate ratio was also found from the regression analysis, to be weakly dependent upon (nozzle pressure drop/bottom-hole pressure) ratio (regression coefficient equals 0.1708) and on the rotational number (regression coefficient equals 0.03438).

Results of the correlation conducted on the derived dimensionless groups effecting the dimensionless mass flow rate ratio (mass flow rate of cutting removed per mass flow rate of drilling fluid) is defined in tables (8.1) and (8.2).

The relationship developed between the chip size and the mass flow rate ratio is supported by Sutko's³⁶ work, although Sutko did not show the magnitude effect of chip size on the forces acting on a chip removal.

The relationship developed between the reciprocal Froude number and the mass flow rate ratios is in disagreement with the work done by Cheatham and Yarbrough.³⁷ The reason for the difference is that they included the density term in the Froude number whereas, in this study the density effect was treated separately. The degree of association between the off-bottom spacing per bit diameter and the mass flow rate ratios is supported by the

TABLE 8.1
MASS FLOW RATE RATIO CORRELATIONS

	a_1	a_2	a_3	R^2	F-Value	Std Error Residual
$\frac{M_c}{M_f} = a_1 \left(\frac{S}{d_n} \right)^{a_2}$	0.13217	4.8×10^{-16}		1.0	999999.99	0.0
$M_c = a_1 \left(\frac{S}{d_n} \right)^{a_2}$	6.6139	-9.614×10^{-16}		1.0	0	0.0
$\frac{M_c}{M_f} = a_1 \left(\frac{d_n^5 g}{Q^2} \right)^{a_2}$	0.15554	2.794×10^{-16}		1.0	999999.99	0.0
$M_c = a_1 \left(\frac{d_n^5 g}{Q^2} \right)^{a_2}$	6.1619	-2.79×10^{-16}		1.0	999999.99	0.0
$\frac{M_c}{M_f} = a_1 \left(\frac{L}{d_b} \right)^{a_2}$	0.063215	-0.60067		0.9424	686.569	0.0628

TABLE 8.1 (continued)
MASS FLOW RATE RATIO CORRELATIONS

	a_1	a_2	a_3	R^2	F-Value	Std Error Residual
$M_c = a_1 \left(\frac{L}{d_b} \right)^{a_2}$	1.58165	-0.60067		0.9424	686.569	0.0628
$\frac{M_c}{M_f} = a_1 \left(\frac{RPM Q \rho_f}{d_H T_y} \right)^{a_2}$	0.046815	0.03438		0.9386	275.038	0.012796
$M_c = a_1 \left(\frac{RPM Q \rho_f}{d_H T_y} \right)^{a_2}$	3.42218	0.03438		0.9386	275.038	0.012796
$\frac{M_c}{M_f} = a_1 \left(\frac{\rho_c}{\rho_f} \right)^{a_2}$	0.699089	0.7777		0.9978	9932.967	0.00973
$M_c = a_1 \left(\frac{\rho_c}{\rho_f} \right)^{a_2}$	37.8977	0.7777		0.9978	9932.967	0.00973

TABLE 8.2
MASS FLOW RATE RATIO CORRELATIONS

	a_1	a_2	a_3	R^2	F-Value	Std Error Residual
$\frac{M_c}{M_f} = a_1 (NR)^{a_2} \left(\frac{\Delta P_n}{BHP}\right)^{a_3}$	5820.576	-0.9583	0.1708	0.7818	173.76	0.068207
$M_c = a_1 (NR)^{a_2} \left(\frac{\Delta P_n}{BHP}\right)^{a_3}$	0.000109	1.02555	-0.3408	0.5899	69.749	0.066588
$\frac{M_c}{M_f} = a_1 (NR)^{a_2}$	61.55268	-0.55088		0.7352	272.04	0.0767
$M_c = a_1 (NR)^{a_2}$	0.95988	0.21924		0.2258	28.581	0.090995
$\frac{M_c}{M_f} = a_1 \left(\frac{\Delta P_n}{BHP}\right)^{a_2}$	0.123376	-0.19104		0.5476	118.622	0.09759

TABLE 8.2 (continued)
MASS FLOW RATE RATIO CORRELATIONS

	a_1	a_2	a_3	R^2	F-Value	Std Error Residual
$M_c = a_1 \left(\frac{\Delta P_n}{BHP} \right)^{a_2}$	11.0323	0.0465		0.0636	6.659	0.10024
$\frac{M_c}{M_f} = a_1 (JF)^{a_2}$	0.212929	-0.27704		0.7352	272.041	0.07467
$M_c = a_1 (JF)^{a_2}$	9.03618	0.10962		0.2258	28.581	0.091148
$\frac{M_c}{M_f} = a_1 (HHP)^{a_2}$	0.0869	-0.163158		0.6749	203.408	0.082741
$M_c = a_1 (HHP)^{a_1}$	12.5747	0.055979		0.1558	18.093	0.095166

work conducted by Van Lingen,³ Feenstra,⁶ McLean,⁸ Pekarek et al,³⁸ and Sutko et al.⁴ However, none of the above researchers have indicated the degree of association between the nozzle extension and bottom-hole cleaning on rate of penetration. The degree of association between the rotational Reynolds number which includes the RPM term and mass flow rate ratios is lower than the degree of association between the rotational speed and the rate of penetration indicated by Wardroup and Cannon.⁵³ Wardroup and Cannon showed that the rate of penetration obtained in Carthage marble is related to the weight on bit and the rotary speed by the following relationship:

$$R = aWN^{0.4}$$

where: R = rate of penetration

W = weight on bit

N = rotary speed

a = constant

The above relationship was developed under a wide range of bit weight (0 to 1500 lbs) and rotary speed (0 to 90 RPM) whereas, in this research bit weight was not simulated and the range of RPM was zero to forty. The degree of association between Reynolds number and the mass flow rate ratio seems to correlate well with the work conducted by Eckel¹⁰. Eckel has shown in a microbit study that rate of penetration is related to Reynolds number raised to half power.

Two other significant findings can be shown from the correlation study. First, the mass flow rate ratio correlates more closely with Reynolds number than with jet impact or hydraulic horsepower. Second, the significant use of dimensionless parameters in performing the correlation analysis. The statistical criteria, R^2 -value, was favorable when the correlation was conducted on

dimensionless function $(\frac{M_c}{M_f})$, while it was less favorable when the correlations were conducted on non-dimensionless function (M_c) .

Figure (8-1) shows the experimentally determined ratio of cutting-mass flow rate per mud-mass flow rate as a function of the semi-empirical equation $A_1 (NR)^{a_2} (\frac{\Delta^p n}{BHP})^{a_3}$, which represents the calculated mass flow rate ratio. A best line was drawn through the experimental and calculated values. The slope of the line was observed to be nearly equal 1.0 and the standard error between the experimental and calculated points is 0.068207 which is associated with R^2 -value of 0.7818.

Figure (8-2) shows the experimentally determined mass flow rate ratio as a function of the semi-empirical equation $A_1 (NR)^{a_2}$, which represents the calculated mass flow rate ratio. The standard error between the experimental and calculated values is 0.0767 which is associated with R^2 -value of 0.7352.

Figure (8-3) shows the experimentally determined mass flow rate ratio as a function of the semi-empirical equation $A_1 (\frac{\Delta^p n}{BHP})^{a_2}$, which represents the calculated mass flow rate ratio. The Standard error between the experimental and calculated values is 0.09759 which is associated with R^2 -value of 0.5476. Figure (8-4) shows the experimentally determined mass flow rate ratio as a function of the semi-empirical equation $A_1 (JF)^{a_2}$, which represents the calculated mass flow rate ratio. The standard error between the experimental and the calculated values is 0.07467 which is associated with R^2 -value of 0.7352.

Figure (8-1) EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

Figure (8-2) EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

Figure (8-3) EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

Figure (8-4) EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

Finally, Figure (8-5) shows the experimentally determined mass flow rate ratio as a function of the semi-empirical equation $A_1(\text{HHP})^{a_2}$, which represents the calculated mass flow rate ratio. The standard error between the experimental and the calculated values is 0.082741 which is associated with R^2 -value of 0.6749.

Figures (8-1) through (8-5) show the significance of R^2 -value in terms of the degree of association between the variables. The closer R^2 -value to unity the better (closer) the association.

Figure (8-5) EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

CHAPTER IX

COMPARISON OF BIT NOZZLE CONFIGURATION

From the literature surveyed it is evident that nozzle geometry has an effect on the penetration rates of tricone bits. The hydraulic conditions under a typical tricone bit are quite complicated. Several bit configurations were tested in the laboratory under similar conditions to understand the effect of bit nozzle configurations on the mass flow rate of cutting removal.

Warren and Winters¹⁶ have stated that the jets on the bit are aimed toward the corner of the bottom of the hole with a slight inclination of approximately ten to fifteen degrees. The flow field of the jet may be influenced by the wellbore wall, bottom of the hole, cones on the bit, and returning fluid from under the bit, as well as stagnation zones under the bit due to the effect of adjacent jets.

Twelve different nozzle configurations (discussed in Chapter IV) were tested at four different flow rates (4.75, 8.5, 12, and 16 gallons per minute).

Laboratory results were plotted as mass flow rate of cutting removal versus bit number. Figures (9-1), (9-2), (9-3), and (9-4) show these results.

Bits number five (one corner nozzle directed downward with diameter equal $5/32$) and six (one center nozzle directed downward with diameter equal $5/32$) were tested under the lower two flow rates only because of current equipment limitations. These two bits have shown better bottom-hole cleaning

Figure (9-1) MASS RATE OF CUTTING REMOVED (LB/MIN) VS BIT NO

Figure (9-2) MASS RATE OF CUTTING REMOVED (LB/MIN) VS BIT NO

Figure (9-3) MASS RATE OF CUTTING REMOVED (LB/MIN) VS BIT NO

Figure (9-4) MASS RATE OF CUTTING REMOVED (LB/MIN) VS BIT NO

than the rest of the bit nozzle configurations tested. This improvement in bottom-hole cleaning was attributed to the higher hydraulic jet impact and hydraulic horsepower associated with these two bits. The higher hydraulic jet impact and hydraulic horsepower were due to higher nozzle velocities. According to Equation (6.9), the higher the fluid velocity, the particle approaches the fluid velocity faster. Figures (9-5) and (9-6) show the improvement of mass flow rate of cutting removal (lb/min) as a function of the hydraulic jet impact force and hydraulic horsepower for bits one through six.

Bit number two (two nozzles downward with an area equal area of $3 \times 5/32$ nozzles) and bit number four (two nozzles downward each is $5/32$) have also shown an improvement in bottom-hole cleaning over the standard bit ($3 \times 5/32$ nozzles directed downward). However, this improvement was less than the improvement obtained by bit number five and bit number six.

Bits number one and number two have the same hydraulic jet impact force and the same hydraulic horsepower. However, use of bit number two resulted in improved bottom-hole cleaning, in agreement with Sutko's³⁶ work. The reason for the improvement in bottom-hole cleaning with one or two jets is parallel to the reason given by Sutko in which he stated that with one or two nozzles operating the fluid passes over the bottom-hole and moves into the annulus through the space where the blanked-off nozzle or nozzles are. This is a relatively "easy" path as compared to three-nozzle case where the fluid hits the hole bottom and must return to the annulus past the flow coming from the nozzles.

Bit number three (one nozzle with area equal area of $3 \times 5/32$ nozzles) generally has an improvement over the standard bit. The improvement is also

Figure (9-5) MASS RATE OF CUTTING REMOVED VS HYDRAULIC IEPACT FORCE

Figure (9-6) MASS RATE OF CUTTING REMOVED VS HYDRAULIC HORSEPOWER

attributed to better flow pattern underneath the bit face.

The results of the second set of bit configurations (bit numbers 7 through 12) are plotted [Figures (9-7) and (9-8)] as a function of mass flow rate of cutting removed versus the hydraulic jet impact and hydraulic horsepower.

Bit number nine (3 x 5/32 inch nozzles pointing 45° outward) and bit number ten (3 x 5/32 inch nozzles pointing 45° outward and 60° sideward) have not showed an improvement in bottom-hole cleaning over the standard bit. On the contrary, they have generally shown a reduction in bottom-hole cleaning. A possible explanation for the reduction in bottom-hole cleaning associated with bits number nine and number ten is that part of the jet impacts near the wall of the hole before it reaches the bottom. Therefore, the intensity of the impact force is unable to dislodge as many cuttings from the bottom of the hole.

Bit number seven (two small and one large nozzle) and bit number eight (two large and one small nozzle) showed an improvement in the amount of cutting removal over the standard bit. Bits number seven and eight had higher values of hydraulic jet impact force than the standard bit. The higher values of jet impact force seems to be the reason for the improvement in bottom-hole cleaning.

Another interesting observation is the bottom-hole cleaning ability of bit number eleven (one nozzle pointing upward and two nozzles directed downward) has shown an improvement of about twenty-five percent in the amount of cutting mass flow rate removed over bit number one (standard bit).

The hydraulic jet impact force is lower in bit number eleven. The improvement in bottom-hole cleaning might possibly be attributed to the

Figure (9-7) MASS RATE OF CURRING REMOVED VS HYDRAULIC IMPACT FORCE

Figure (9-8) MASS RATE OF CUTTING REMOVED VS HYDRAULIC HORSEPOWER

better washing at the bottom of the bit face and to the reduction in bottom-hole pressure caused by the upward jet action. The upward jet action creates a vacuum region above the bit which tends to transport the cutting particles away from the bit faster. This finding, associated with bit number eleven, correlates well with the field results obtained by a recent bit design by Cholet et al²⁰. At least a fifteen percent increase in rate of penetration was obtained by this bit design and it is shown in figure (9-9).

Finally, bit number twelve (two upward nozzles and one directed downward) has shown a slight improvement in bottom-hole cleaning over the standard bit. The improvement that appeared in spite of the lowered jet forces associated with bit number twelve could be explained by the favorable flow pattern and to the reduced fluid mixing that existed underneath the bit face.

These test results show that large impact forces or bit hydraulic horsepower are not the primary factors required for good bottom-hole cleaning, but good tangential sweeping velocities across the face of the bottom of the hole and possible bottom-hole pressure reductions due to upward jets can play a large part in cleaning the hole.

Figure (9-9) Reverse Nozzle Three-Cone Bit
(after Cholet²⁰).

CHAPTER X

CONCLUSIONS

This is the first time a parametric analysis was conducted of all the fluid dynamic parameters which affect bottom-hole cleaning. Several definite conclusions may be made from the experimental study of the important parameters that influence bottom-hole cleaning.

1. The ratio of (cutting-mass flow rate/mud-mass flow rate) was found to be strongly dependent upon Reynolds number, nozzle height above the bottom of the hole and cutting density, while it was weakly dependent on (nozzle-pressure drop/bottom-hole pressure) ratio and rotational Reynolds number.
2. No dependency was found with chip size per nozzle diameter and Froude number.
3. The best nozzle configuration for bottom-hole cleaning was a single nozzle and two blank nozzles.
4. 25 percent improvement in bottom-hole cleaning was obtained with a bit equipped with one nozzle pointing upward and two nozzles directed downward compared to a bit with three nozzles pointing downward.

5. From the regression analysis conducted on the experimental results, it is shown that the hydraulic impact force correlates better than the hydraulic horsepower with the ratio of (cutting-mass flow rate/mud-mass flow rate).
6. The regression analysis showed that the ratio of cutting-mass flow rate/mud-mass flow rate, correlated better with Reynolds number than jet impact or hydraulic horsepower.
7. This study showed that many variables contribute to bottom-hole cleaning and that merely varying a single parameter such as hydraulic horsepower or jet impact force will not optimize bottom-hole cleaning.

CHAPTER XI

RECOMMENDATIONS FOR FUTURE RESEARCH

In order to gain future insight into the parameters that affect the bottom hole cleaning, additional investigations are needed.

1. Simulate formation pore pressure and investigate its variation effect on bottom-hole cleaning.
2. Use a microbit to study the effect of bit face water courses.
3. Study the effect of temperature, other than room temperature, on bottom-hole cleaning.
4. Investigation of jet angle effect on bottom-hole cleaning.
5. Use polymers to study the effect of drilling fluid viscosity on bottom-hole cleaning.
6. More investigations are needed for bit number 5 (one corner nozzle directed downward) and bit number 11 (one nozzle pointing upward and two pointing downward) since these bits show a promising improvement in bottom-hole cleaning over the standard 3-jet bits.
7. Drill actual samples and investigate the effect of weight-on-bit on the generation of cuttings and cutting removal.

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APPENDIX A

Regression Programs

Listings

NOTE: SAS OPTIONS SPECIFIED ARE:
 SORT=4

```

1      OPTION PAGESIZE=50 LINESIZE=84 NODATE NOAUTOREM;
2      DATA MS1;
3      INPUT DN Q INCHES OHP DE;
4      BHF=BHF+14.7;
5      GRAMS=INCHES*60;
6      MDCT=GRAMS*20/453.59;
7      MDDOT=E.34*Q;
8      RATIOF=MDCT/MDDOT;
9      PDECP=((Q**2*8.34)/(7450*.95**2*DE**4));
10     RATIOF=EDB0E/RHP;
11     RHP=(EDBCE*Q)/1714;
12     TA=((3.1416*3)/4)*(DN**2);
13     JF=(0.000518*8.34*Q**2)/1A;
14     NR=(379.0849*8.34*(Q/3)/(0.8904*DN));
15     PRODC1=NR*RATIOF;
16     CARDS;

```

NOTE: DATA SET WORK.MS1 HAS 100 OBSERVATIONS AND 16 VARIABLES. 1

44 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.16 SECONDS AND 92K.

```

117     PROC PRINT DATA=MS1;

```

NOTE: THE PROCEDURE PRINT USED 0.50 SECONDS AND 144K
 AND PRINTED PAGES 1 TO 8.

```

118     DATA MS7;
119     SET MS1;

```

NOTE: DATA SET WORK.MS7 HAS 100 OBSERVATIONS AND 16 VARIABLES. 1

44 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.08 SECONDS AND 64K.

```

120     PROC SORT DATA=MS1;
121     BY DN Q;

```

NOTE: 4 CYLINDERS DYNAMICALLY ALLOCATED ON SYSDA FOR EACH OF 3 SORT WORK DATA SETS

NOTE: DATA SET WORK.MS1 HAS 100 OBSERVATIONS AND 16 VARIABLES. 1

44 OBS/TRK

S A S L O G OS SAS 82.23 VS2/1VS JOB SADIK

NOTE: THE PROCEDURE SORT USED 0.28 SECONDS AND 152K.

```

122      PROC MEANS;
123      BY DN C;
124      VAR RATION RATIOF NM PRODCI MDCT JFE JF;
125      OUTPUT OUT=STATS MEAN=RATIONM BALLOPM NEM PRODCIA MDCT
TM HHFM JFM;
126      SYMBOL1 I=JOIN V=STAR C=3;
127      SYMBOL2 I=JOIN V=SQUARE C=3;
128      SYMBOL3 I=JOIN V=TRIANGLE C=3;
129      SYMBOL4 I=JOIN V=DIAMOND C=3;
130      SYMBOL5 I=JOIN V=PLUS C=3;

```

NOTE: DATA SET WORK.STATS HAS 20 OBSERVATIONS AND 9 VARIABLES. 2
50 OBS/TAK

NOTE: THE PROCEDURE MEANS USED 0.27 SECONDS AND 150K
AND PRINTED PAGES 9 TO 13.

```

131      PROC GPLOT DATA=STATS;
132      PLOT RATIONM*HHFM=DN/HHFM=1.4 VREF=0.30 VPOS=22 HPOS=
70;
133      TITLE .H=1 .C=2 MASS RATE RATIOS VS HYDRAULIC HORSEPO
WER;
134      LABEL HHFM=HYDRAULIC HORSEPOWER;
135      LABEL RATIONM=MASS RATE RATIOS;

```

NOTE: THE PROCEDURE GPLOT USED 0.50 SECONDS AND 416K.

```

136      PROC GPLOT DATA=STATS;
137      PLOT RATIONM*JFM=LN/HHFM=35 VREF=0.30 VPOS=22 HPOS=70
;
138      TITLE .H=1 .C=2 MASS RATE RATIOS VS HYDRAULIC IMPACT
FORCE;
139      LABEL JFM=HYDRAULIC IMPACT FORCE;
140      LABEL RATIONM=MASS RATE RATIOS;

```

NOTE: THE PROCEDURE GPLOT USED 0.40 SECONDS AND 624K.

```

141      PROC GPLOT DATA=STATS;
142      PLOT MDCT*HHFM=DN/HHFM=1.4 VREF=17.5 VPOS=22 HPOS=70
;
143      TITLE .H=1 .C=2 MASS RATE OF CUTTING REMOVED VS HYDRA
ULIC HORSEPOWER;
144      LABEL HHFM=HYDRAULIC HORSE POWER;
145      LABEL MDCT=MASS RATE OF CUTTING REMOVED;

```

NOTE: THE PROCEDURE GPLOT USED 0.40 SECONDS AND 624K.

S A S L O G OS SAS 82.25 VS2/MVS JCB SADIK

```
146      PROC GFLCT DATA=STATS;
147      PLOT MDCTM*JFM=DN/HREF=35 VREF=17.5 VPCS=22 HPOS=70;
148      TITLE .H=1 .C=2 MASS RATE OF CUTTING REMOVED VS HYDRA
ULIC IMPACT FORCE;
149      LABEL JFM=HYDRAULIC IMPACT FORCE;
150      LABEL MDCTM=MASS RATE OF CUTTING REMOVED;
```

NOTE: THE PROCEDURE GFLCT USED 0.45 SECONDS AND 624K.

```
151      PROC GFLCT DATA=STATS;
152      PLOT RATIOCM*PHODCTM=DN/HREF=2600000 VREF=0.30 VPOS=2
2 HPOS=70;
153      TITLE .H=1 .C=2 MASS RATE RATIOS VS NR*(NED/3h2);
154      LABEL PHODCTM=NR*(NPD/BHP);
155      LABEL RATIOCM=MASS RATE RATIOS;
```

NOTE: THE PROCEDURE GFLCT USED 0.36 SECONDS AND 624K.

```
156      PROC GFLCT DATA=STATS;
157      PLOT RATIOCM*NRN=DN/HREF=160000 VREF=0.30 VPCS=22 HPO
S=70;
158      TITLE .H=1 .C=2 MASS RATE RATIOS VS REYNOLDS NUMBER;
159      LABEL NRN=REYNOLDS NUMBER;
160      LABEL RATIOCM=MASS RATE RATIOS;
```

NOTE: THE PROCEDURE GFLCT USED 0.39 SECONDS AND 624K.

```
161      PROC GFLCT DATA=STATS;
162      PLOT MDCTM*NRN=DN/HREF=160000 VREF=17.5 VPCS=22 HPCS=
70;
163      TITLE .H=1 .C=2 MASS RATE OF CUTTING REMOVED VS REYNO
LDS NUMBER;
164      LABEL NRN=REYNOLDS NUMBER;
165      LABEL MDCTM=MASS RATE OF CUTTING REMOVED;
```

NOTE: THE PROCEDURE GFLCT USED 0.43 SECONDS AND 624K.

```
166      PROC GFLCT DATA=STATS;
167      PLOT RATIOCM*RATIOPM=DN/HREF=19 VREF=0.30 VPCS=22 HPO
S=70;
168      TITLE .H=1 .C=2 MASS RATE RATIOS VS PRESSURE RATIOS;
169      LABEL RATIOCM=MASS RATE RATIOS;
170      LABEL RATIOPM=NPD/BHP;
```

NOTE: THE PROCEDURE GFLCT USED 0.38 SECONDS AND 624K.

```
171      PROC GFLCT DATA=STATS;
172      PLOT MDCTM*RATIOCM=DN/HREF=19 VREF=17.5 VPOS=22 HPOS=
70;
```

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173 TITLE .H=1 .C=2 MASS RATE OF CUTTING REMOVED VS PRESS
 174 LABEL MLCIN=MASS RATE OF CUTTING REMOVED;
 175 LABEL RATIOPM=NPD/BHP;

NOTE: THE PROCEDURE GPLCI USED 0.43 SECONDS AND 624K.

176 PROC GPLCI DATA=STATS;
 177 PLOT MLCIN*PRODCTD=CM/HREF=2600000 VREF=17.5 VPOS=22
 HPOS=70;
 178 TITLE .H=1 .C=2 MASS RATE OF CUTTING REMOVED VS MR*(N
 PD/BHP);
 179 LABEL MDCIN=MASS RATE OF CUTTING REMOVED;
 180 LABEL PRODCIN=MR*(NPD/BHP);

NOTE: THE PROCEDURE GPLCI USED 0.40 SECONDS AND 624K.

181 PROC PRINT DATA=MS7;

NOTE: THE PROCEDURE PRINT USED 0.49 SECONDS AND 424K
 AND PRINTED PAGES 14 TO 21.

182 DATA MS7;
 183 SET MS1;

NOTE: DATA SET WORK.MS7 HAS 100 OBSERVATIONS AND 16 VARIABLES. 1
 44 OBS/TRK
 NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 424K.

184 PROC SORT DATA=MS1;
 185 BY Q DE;

NOTE: DATA SET WORK.MS1 HAS 100 OBSERVATIONS AND 16 VARIABLES. 1
 44 OBS/TRK
 NOTE: THE PROCEDURE SORT USED 0.14 SECONDS AND 424K.

186 PROC MEANS;
 187 BY Q DE;
 188 VAR RATIOC MLCIN;
 189 OUTPUT OUT=FLC MEAN=RATIOC MLCIN DNM;
 190 SYMBOL1 I=JOIN V=STAR C=3;
 191 SYMBOL2 I=JOIN V=SQUARE C=3;
 192 SYMBOL3 I=JOIN V=TRIANGLE C=3;
 193 SYMBOL4 I=JOIN V=DIAMOND C=3;

NOTE: DATA SET WORK.FLOW HAS 20 OBSERVATIONS AND 5 VARIABLES. 43
 3 OBS/TRK
 NOTE: THE PROCEDURE MEANS USED 0.21 SECONDS AND 442K

S A S L O G O S SAS 82.25 VS2/SVS JCL SAEIK
AND PRINTED PAGES 22 TO 24.

```
194      PROC GPLOT DATA=FLG4;
195      PLOT RATIOHM*DNM=Q/HREF=0.250 VREF=0.30 VEGS=22 HEGS=
70;
196      TITLE .H=1 .C=2 MASS RATE RATIOVS VS NOZZLE DIAMETER ;
197      LABEL RATIOHM=MASS RATE RATIOVS;
198      LABEL DNM=DIAMETER OF THREE EQUAL NOZZLES;
```

NOTE: THE PROCEDURE GPLOT USED 0.35 SECONDS AND 624K.

```
199      PROC GELCI DATA=FLG4;
200      PLOT PDCIM*DNM=Q/HREF=0.25 VREF=17.5 VPOS=22 HPOS=70;
201      TITLE .H=1 .C=2 MASS RATE OF CUTTING REMOVED VS NOZZLE
DIAMETER ;
202      LABEL MDOIM=MASS RATE OF CUTTING REMOVED;
203      LABEL DNM=DIAMETER OF THREE EQUAL NOZZLES;
```

NOTE: THE PROCEDURE GELCI USED 0.40 SECONDS AND 624K.

```
204      PROC PRINT DATA=MS7;
```

NOTE: THE PROCEDURE PRINT USED 0.49 SECONDS AND 424K
AND PRINTED PAGES 25 TO 32.

```
205      DATA MS2;
206      SET MS1;
207      PRODCI=PRCDCT;
208      RATIOH=LOG10(RATIOH);
209      RATIOF=LOG10(RATIOF);
210      NR=LOG10(NR);
```

NOTE: DATA SET WORK.MS2 HAS 100 OBSERVATIONS AND 16 VARIABLES. 1
44 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 424K.

```
211      PROC PRINT DATA=MS2;
```

NOTE: THE PROCEDURE PRINT USED 0.42 SECONDS AND 424K
AND PRINTED PAGES 33 TO 38.

```
212      PROC REG DATA=MS2;
213      MODEL RATIOH=NR RATIOF/ALL COLLIN PARTIAL;
214      OUTPUT OUT=C P=PRED;
```

NOTE: DATA SET WORK.C HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/TRK

NOTE: THE PROCEDURE REG USED 0.74 SECONDS AND 424K

S A S L O G O S SAS 82.2b VS2/4VS CCB SADIK
AND PRINTED PAGES 39 TO 51.

```
215      PROC GPLOT DATA=C;
216      PLOT RATIOH*PRED/VPOS=22 HPOS=70;
217      SYMBOL1 I=RL V=STAR C=3;
218      LABEL RATIOH=LOG MASS RATE RATIOS;
219      LABEL FREQ=LCG (A1(NR)**A2(RATIOH)**A3);
220      TITLE .h=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
      ULATED;
```

NOTE: THE PROCEDURE GPLOT USED 0.43 SECONDS AND 626K.

```
221      DATA MS3;
222      MERGE MS2 C;
223      MODEL=10**PRED;
224      EXPRMT=10**RATIOH;
225      PRODC1=PRCDCT;
226      LABEL EXPRMT=MASS RATE RATIOS;
```

NOTE: DATA SET WORK.MS3 HAS 100 OBSERVATIONS AND 19 VARIABLES. 1
22 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.11 SECONDS AND 432K.

```
227      PROC PRINT DATA=MS3;
```

NOTE: THE PROCEDURE PRINT USED 0.54 SECONDS AND 424K
AND PRINTED PAGES 52 TO 59.

```
228      PROC GPLOT DATA=MS3;
229      PLOT EXPRMT*MODEL/VPOS=22 HPOS=70 HREF=0.23 VREF=0.3;
230      SYMBOL1 I=RL C=3;
231      LABEL EXPRMT=MASS RATE RATIOS;
232      LABEL MODEL=A1(NR)**A2*(RATIOH)**A3;
233      TITLE .h=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
      ULATED;
```

NOTE: THE PROCEDURE GPLOT USED 0.43 SECONDS AND 626K.

```
234      DATA MS5;
235      SET MS1;
236      PRODC1 =PRCDCT;
237      RATIOH=LOG10(RATIOH);
238      RATIOF=LCG10(RATIOH);
239      NR=LCG10(NR);
```

NOTE: DATA SET WORK.MS5 HAS 100 OBSERVATIONS AND 16 VARIABLES. 1
44 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 424K.

S A S I C G OS SAS 62.2E VS2/MVS JOB SADIK

240 PROC PRINT DATA=MS5;

NOTE: THE PROCEDURE PRINT USED 0.43 SECONDS AND 424K
AND PRINTED PAGES 60 TO 65.

241 PROC REG DATA=MS5;
242 MODEL RATION=NR/ALL COLLIN PARTIAL;
243 OUTPUT OUT=D P=PBED;

NOTE: DATA SET WORK.D HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/TAK
NOTE: THE PROCEDURE REG USED 0.61 SECONDS AND 424K
AND PRINTED PAGES 66 TO 77.

244 PROC GPLOT DATA=D;
245 PLOT RATION*PBED/VPOS=22 H=CS=70;
246 SYMBOL1 I=BL V=STAR C=3;
247 LABEL RATION=LOG MASS RATE RATIOS;
248 LABEL PBED= LCG(A1(NR)**A2);
249 TITLE .H=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
ULATED;

NOTE: THE PROCEDURE GPLOT USED 0.42 SECONDS AND 626K.

250 DATA MS6;
251 MERGE MS5 D;
252 MODEL=10**PBED;
253 EXPRT=10**RATION;
254 PBODCT=PBODCT;
255 LABEL EXPRT=MASS RATE RATIOS;

NOTE: DATA SET WORK.MS6 HAS 100 OBSERVATIONS AND 19 VARIABLES. 1
22 OBS/TAK
NOTE: THE DATA STATEMENT USED 0.12 SECONDS AND 432K.

256 PROC PRINT DATA=MS6;

NOTE: THE PROCEDURE PRINT USED 0.55 SECONDS AND 424K
AND PRINTED PAGES 78 TO 85.

257 PROC GPLOT DATA=MS6;
258 PLOT EXPRT*MODEL/VPOS=22 H=CS=70 HREF=0.24 VDEF=0.3;
259 SYMBOL1 I=BL C=J;
260 LABEL EXPRT=MASS RATE RATIOS,
261 LABEL MODEL=A1(NR)**A2;
262 TITLE .H=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
ULATED;

S A S L O G O S SAS 82.25 VS2/MVS CCS SADIK

NOTE: THE PROCEDURE GPLOT USED 0.43 SECONDS AND 626K.

```
263 DATA MS7;
264 SET MS1;
265 RATIOH=LOG10(RATIOH);
266 RATIOF=LOG10(RATIOF);
```

NOTE: DATA SET WORK.MS7 HAS 100 OBSERVATIONS AND 16 VARIABLES. 144 OBS/TK

NOTE: THE DATA STATEMENT USED 0.08 SECONDS AND 424K.

```
267 PROC PRINT DATA=MS7;
```

NOTE: THE PROCEDURE PRINT USED 0.43 SECONDS AND 424K AND PRINTED PAGES 86 TO 91.

```
268 PROC REG DATA=MS7;
269 MODEL RATIOH=RATIOF/ALL COLLIN PARTIAL;
270 OUTPUT OUT=E P=PEED;
```

NOTE: DATA SET WORK.E HAS 100 OBSERVATIONS AND 17 VARIABLES. 136 OBS/TK

NOTE: THE PROCEDURE REG USED 0.63 SECONDS AND 424K AND PRINTED PAGES 92 TO 103.

```
271 PROC GPLOT DATA=E;
272 PLOT RATIOH*PEED/VPOS=22 HPOS=70;
273 SYMBOL1 I=RL V=STAR C=3;
274 LABEL RATIOH=LOG MASS RATE RATIOS;
275 LABEL SPEED=LOG(A1(RATIOF)**A2);
276 TITLE .H=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALCULATED;
```

NOTE: THE PROCEDURE GPLOT USED 0.43 SECONDS AND 626K.

```
277 DATA MS8;
278 DELETE MS7 E;
279 MODEL=10**PEED;
280 EXPRNT=10**RATIOH;
```

NOTE: DATA SET WORK.MS8 HAS 100 OBSERVATIONS AND 19 VARIABLES. 122 OBS/TK

NOTE: THE DATA STATEMENT USED 0.11 SECONDS AND 432K.

```
281 PROC PRINT DATA=MS8;
```

NOTE: THE PROCEDURE PRINT USED 0.55 SECONDS AND 424K

S A S L C G JS SAS 82.26 VS2/MVS CCB SALLK
AND PRINTED PAGES 104 TO 111.

```
282      PROC GPLOT DATA=MS8;
283      PLOT EXPMT*MODEL/VPOS=22 HPOS=70 HREF=0.23 VREF=0.3;
284      SYMBOL1 I=RL V=STAR C=3;
285      LABEL EXPMT=MASS RATE RATIOS;
286      LABEL MODEL=A1(RATIO)*A2;
287      TITLE .H=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
      ULATED;
```

NOTE: THE PROCEDURE GPLOT USED 0.43 SECONDS AND 626K.

```
288      DATA MS9;
289      SET MS1;
290      RATIO=LGG10(RATIO);
291      JF=LOG10(JF);
```

NOTE: DATA SET WORK.MS9 HAS 100 OBSERVATIONS AND 16 VARIABLES. 1
44 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.08 SECONDS AND 424K.

```
292      PROC PRINT DATA=MS9;
```

NOTE: THE PROCEDURE PRINT USED 0.42 SECONDS AND 424K
AND PRINTED PAGES 112 TO 117.

```
293      PROC REG DATA=MS9;
294      MODEL RATIO=JF/ALL COLLIN PARTIAL;
295      OUTPUT OUT=F F=PRED;
```

NOTE: DATA SET WORK.F HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/TRK

NOTE: THE PROCEDURE REG USED 0.57 SECONDS AND 424K
AND PRINTED PAGES 118 TO 129.

```
296      PROC GPLOT DATA=F;
297      PLOT RATIO*PRED/VPOS=22 HPOS=70.
298      SYMBOL1 I=RL V=STAR C=3;
299      LABEL RATIO=LOG MASS RATE RATIOS;
300      LABEL PRED=LGG(A1(JF)*A2);
301      TITLE .H=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
      ULATED;
```

NOTE: THE PROCEDURE GPLOT USED 0.42 SECONDS AND 626K.

```
302      DATA MS10;
303      MERGE MS9 F;
304      MODEL =10**PRED;
```

S A S L O G OS SAS 82.28 VS2/AVS JOB SASIK

305 EXPRT=10**RAT10;

NOTE: DATA SET WORK.MS10 HAS 100 OBSERVATIONS AND 19 VARIABLES.
122 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.11 SECONDS AND 432K.

306 PROC PRINT DATA=MS10;

NOTE: THE PROCEDURE PRINT USED 0.54 SECONDS AND 424K
AND PRINTED PAGES 130 TO 137.

307 PROC Gplot DATA=MS10;

308 PLOT EXPRT*MODEL/VPOS=22 HPOS=70 HREF=0.24 VREF=0.3;

309 SYMBOL1 I=RL V=STAR C=J;

310 LABEL EXPRT=MASS RATE RATIOS;

311 LABEL MODEL=A1(J1)**A2;

312 TITLE .H=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
ULATED;

NOTE: THE PROCEDURE Gplot USED 0.43 SECONDS AND 626K.

313 DATA MS11;

314 SET MS1;

315 RAT10=LCG10(RAT10);

316 HHP=LCG10(HHP);

NOTE: DATA SET WORK.MS11 HAS 100 OBSERVATIONS AND 16 VARIABLES.
144 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 424K.

317 PROC PRINT DATA=MS11;

NOTE: THE PROCEDURE PRINT USED 0.42 SECONDS AND 424K
AND PRINTED PAGES 138 TO 143.

318 PROC REG DATA=MS11;

319 MODEL RAT10=HHP/ALL COLLIN PARTIAL;

320 OUTPUT OUT=G F=PBEC;

NOTE: DATA SET WORK.G HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/TRK

NOTE: THE PROCEDURE REG USED 0.61 SECONDS AND 424K
AND PRINTED PAGES 144 TO 155.

321 PROC Gplot DATA=G;

322 PLOT RAT10*FRED/VPOS=22 HPOS=70;

323 SYMBOL1 I=RL V=STAR C=J;

324 LABEL RAT10=LOG MASS RATE RATIOS;

SAS LCG OS SAS 82.25 VS2/AVS JOB SADIK

```
325 LABEL PRED=LCG10(A1(HHP)**A2);
326 TITLE .H=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
      ULATED;
```

NOTE: THE PROCEDURE GPLCT USED 0.41 SECONDS AND 626K.

```
327 DATA MS12;
328 MERGE MS11 G;
329 MODEL=10**PRED;
330 EXPMT=10**EATLON;
```

NOTE: DATA SET WORK.MS12 HAS 100 OBSERVATIONS AND 19 VARIABLES.
122 OBS/IRK

NOTE: THE DATA STATEMENT USED 0.11 SECONDS AND 432K.

```
331 PROC PRINT DATA=MS12;
```

NOTE: THE PROCEDURE PRINT USED 0.55 SECONDS AND 424K
AND PRINTED PAGES 150 TO 163.

```
332 PROC GELCT DATA=MS12;
333 PLCT EXPMT*MODEL/VPOS=22 HPOS=70 HREF=0.24 VREF=0.3;
334 SYMBOL1 I=RL V=STAR C=J;
335 LABEL EXPMT=MASS RATE RATIOS;
336 LABEL MODEL=A1(HHP)**A2;
337 TITLE .H=1 .C=2 EXPERIMENTAL MASS RATE RATIOS VS CALC
      ULATED;
```

NOTE: THE PROCEDURE GPLCT USED 0.43 SECONDS AND 626K.

```
338 DATA MS15;
339 SET MS1;
340 MDOT=LOG10(MDGT);
341 RATIOF=LCG10(EATLONF);
342 NR=LCG10(NR);
```

NOTE: DATA SET WORK.MS15 HAS 100 OBSERVATIONS AND 16 VARIABLES.
144 OBS/IRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 424K.

```
343 PROC REG DATA=MS15;
344 MODEL MDOT=NR/ALL COLLIN PARTIAL;
345 OUTPUT CUT=T F=PBED;
```

NOTE: DATA SET WORK.T HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/IRK

NOTE: THE PROCEDURE REG USED 0.61 SECONDS AND 424K
AND PRINTED PAGES 164 TO 175.

S A S L O G OS SAS 82.2B VS2/EVS JCU SADIK

```
346      DATA MS16;
347      SET MS1;
348      MDOT=LOG10(MDOT);
349      RATIOF=LOG10(RATIOF);
```

NOTE: DATA SET WORK.MS16 HAS 100 OBSERVATIONS AND 16 VARIABLES.
144 OBS/TRK
NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 424K.

```
350      PROC REG DATA=MS16;
351      MODEL MDOT=RATIOF/ALL COLLIN PARTIAL;
352      OUTPUT OUT=X F=PRED;
```

NOTE: DATA SET WORK.X HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/TRK
NOTE: THE PROCEDURE REG USED 0.61 SECONDS AND 424K
AND PRINTED PAGES 176 TO 187.

```
353      DATA MS17;
354      SET MS1;
355      MDOT=LOG10(MDOT);
356      JF=LOG10(JF);
```

NOTE: DATA SET WORK.MS17 HAS 100 OBSERVATIONS AND 16 VARIABLES.
144 OBS/TRK
NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 424K.

```
357      PROC REG DATA=MS17;
358      MODEL MDOT=JF/ALL COLLIN PARTIAL;
359      OUTPUT OUT=Y F=PRED;
```

NOTE: DATA SET WORK.Y HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/TRK
NOTE: THE PROCEDURE REG USED 0.61 SECONDS AND 424K
AND PRINTED PAGES 188 TO 199.

```
360      DATA MS18;
361      SET MS1;
362      MDOT=LOG10(MDOT);
363      HHE=LOG10(HHE);
```

NOTE: DATA SET WORK.MS18 HAS 100 OBSERVATIONS AND 16 VARIABLES.
144 OBS/TRK
NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 424K.

```
364      PROC REG DATA=MS18;
365      MODEL MDOT=HHE/ALL COLLIN PARTIAL;
```

S A S L O G OS SAS 82.2B VS2/EVS JCB JALIK

366 OUTPUT OUT=Z P=PBED;

NOTE: DATA SET WORK.Z HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/TRK

NOTE: THE PROCEDURE REG USED 0.61 SECONDS AND 424K
AND PRINTED PAGES 200 TO 211.

367 DATA MS13;
368 SET MS1;
369 MDOT=LOG10(MDOT1);
370 RATIOF=LOG10(RATIOF);
371 NR=LOG10(NR);

NOTE: DATA SET WORK.MS13 HAS 100 OBSERVATIONS AND 16 VARIABLES.
144 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 424K.

372 PROC REG DATA=MS13;
373 MODEL MDCT=NR RATIOF/ALL COLLIN PARTIAL;
374 OUTPUT OUT=S P=PBED;

NOTE: DATA SET WORK.S HAS 100 OBSERVATIONS AND 17 VARIABLES. 136
OBS/TRK

NOTE: THE PROCEDURE REG USED 0.76 SECONDS AND 424K
AND PRINTED PAGES 212 TO 224.

NOTE: SAS USED 626K MEMCHY.

NOTE: SAS INSTITUTE INC.
SAS CIRCLE
PO BOX 8000
CARY, N.C. 27511-8000

MASS RATE OF CUTTING REMOVED VS NOZZLE DIAMETER

MODEL CROSSPRODUCTS $X'X$ $X'Y$ $Y'Y$

$X'X$	INTERCEP	NR	RATIOF	RATIOH
INTERCEP	100	478.5226	-24.3822	-86.2168
NR	478.5226	2294.829	-104.865	-415.342
RATIOF	-24.3822	-104.865	37.21636	15.04763
RATIOH	-86.2168	-415.342	15.04763	76.42112

SEQUENTIAL PARAMETER ESTIMATES

INTERCEP -0.862168
 NR 1.78925 -0.554088
 RATIOF 3.76497 -0.958263 0.170816

 $X'X$ INVERSE, B, SSE

INVERSE	INTERCEP	NR	RATIOF	RATIOH
INTERCEP	44.76218	-9.17522	3.472505	3.764966
NR	-9.17522	1.88121	-0.710375	-0.958263
RATIOF	3.472505	-0.710375	0.3002248	0.1708162
RATIOH	3.764966	-0.958263	0.1708162	0.4548053

MASS RATE OF CUTTING REMOVED VS NOZZLE DIAMETER

DEP VARIABLE: RATIO

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR>F
MODEL	2	1.629480	0.814740	173.766	0.0001
ERROR	97	0.454805	0.004683714		
C TOTAL	99	2.084285			
ROOT MSE		0.068474	R-SQUARE	0.7818	
DEP MEAN		-0.862188	ADJ R-SQ	0.7773	
C.V.		-7.94151			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PR> T
INTERCEP	1	3.764966	0.458123	8.218	0.0001
NR	1	-0.952263	0.093917	-10.203	0.0001
RATIOF	1	0.170816	0.037515	4.553	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	74.336831	0.316673	0.000000
NR	1	1.532292	0.468126	-1.482654
RATIOF	1	0.097168	0.097188	0.001605

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
NR	1	0.106507
RATIOF	1	0.106507

COVARIANCE OF ESTIMATES

COVB	INTERCEP	NR	RATIOF
INTERCEP	0.2098771	-0.04302	0.01628159
NR	-0.04302	0.008820458	-0.00333074
RATIOF	0.01628159	-0.00333074	0.001407668

MASS RATE OF CUTTING REMOVAL VS NOZZLE DIAMETER

CORRELATION OF ESTIMATES

CORRE	INTERCEP	NR	BATICP
INTERCEP	1.0000	-0.9999	0.9472
NR	-0.9999	1.0000	-0.9452
BATICP	0.9472	-0.9452	1.0000

COLLINEARITY DIAGNOSTICS

VARIANCE PROPORTIONS

NUMBER	EIGENVALUE	CONDITION INDEX	PC1: INTERCEP	PC2: NR	PC3: BATICP
1	2.232	1.000	0.0000	0.0000	0.0000
2	0.767496	1.705	0.0000	0.0000	0.0000
3	0.00113602	140.182	0.9999	0.9999	0.9999

OBS	ACTUAL	PREDICT VALUE	STL ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	LOWER95% PREDICT
1	-.823159	-.722488	0.016619	-.755472	-.689505	-.862336
2	-.823159	-.722488	0.016619	-.755472	-.689505	-.862336
3	-.823159	-.722488	0.016619	-.755472	-.689505	-.862336
4	-.823159	-.722488	0.016619	-.755472	-.689505	-.862336
5	-.823159	-.722488	0.016619	-.755472	-.689505	-.862336
6	-.663459	-.695448	0.013135	-.721516	-.669379	-.833828
7	-.647068	-.695448	0.013135	-.721516	-.669379	-.833828
8	-.680492	-.695448	0.013135	-.721516	-.669379	-.833828
9	-.663459	-.695448	0.013135	-.721516	-.669379	-.833828
10	-.663459	-.695448	0.013135	-.721516	-.669379	-.833828
11	-.573282	-.673279	0.012326	-.697744	-.648814	-.811367
12	-.573282	-.673279	0.012326	-.697744	-.648814	-.811367
13	-.573282	-.673279	0.012326	-.697744	-.648814	-.811367
14	-.573282	-.673279	0.012326	-.697744	-.648814	-.811367
15	-.573282	-.673279	0.012326	-.697744	-.648814	-.811367
16	-.587070	-.654864	0.013387	-.681433	-.628294	-.793339
17	-.601311	-.654864	0.013387	-.681433	-.628294	-.793339
18	-.587070	-.654864	0.013387	-.681433	-.628294	-.793339
19	-.601311	-.654864	0.013387	-.681433	-.628294	-.793339
20	-.616034	-.654864	0.013387	-.681433	-.628294	-.793339
21	-.663459	-.638117	0.015349	-.668581	-.607653	-.777392
22	-.698221	-.638117	0.015349	-.668581	-.607653	-.777392
23	-.680492	-.638117	0.015349	-.668581	-.607653	-.777392
24	-.663459	-.638117	0.015349	-.668581	-.607653	-.777392

MASS RATE OF CUTTING REMOVED VS NOZZLE DIAMETER

OBS	ACTUAL	PREDICT VALUE	STD ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	LOWER95% PREDICT
25	-.663459	-.638117	0.015349	-.668581	-.607653	-.777392
26	-.988735	-.881038	0.013086	-.907010	-.855066	-1.315
27	-.988735	-.881038	0.013086	-.907010	-.855066	-1.019
28	-.969429	-.881038	0.013086	-.907010	-.855066	-1.019
29	-.969429	-.881038	0.013086	-.907010	-.855066	-1.019
30	-.988735	-.881038	0.013086	-.907010	-.855066	-1.019
31	-.883999	-.852856	0.008462	-.869650	-.836062	-.989793
32	-.899793	-.852856	0.008462	-.869650	-.836062	-.989793
33	-.899793	-.852856	0.008462	-.869650	-.836062	-.989793
34	-.854036	-.852856	0.008462	-.869650	-.836062	-.989793
35	-.868759	-.852856	0.008462	-.869650	-.836062	-.989793
36	-.718373	-.829515	0.007068	-.843543	-.815487	-.966140
37	-.718373	-.829515	0.007068	-.843543	-.815487	-.966140
38	-.718373	-.829515	0.007068	-.843543	-.815487	-.966140
39	-.718373	-.829515	0.007068	-.843543	-.815487	-.966140
40	-.718373	-.829515	0.007068	-.843543	-.815487	-.966140
41	-.799678	-.811100	0.008640	-.828247	-.793952	-.948080
42	-.812643	-.811100	0.008640	-.828247	-.793952	-.948080
43	-.799678	-.811100	0.008640	-.828247	-.793952	-.948080
44	-.812643	-.811100	0.008640	-.828247	-.793952	-.948080
45	-.812643	-.811100	0.008640	-.828247	-.793952	-.948080
46	-.854036	-.794757	0.011350	-.817283	-.772231	-.932514
47	-.883999	-.794757	0.011350	-.817283	-.772231	-.932514
48	-.868759	-.794757	0.011350	-.817283	-.772231	-.932514
49	-.854036	-.794757	0.011350	-.817283	-.772231	-.932514
50	-.854036	-.794757	0.011350	-.817283	-.772231	-.932514
51	-1.004	-.979587	0.012882	-1.005	-.954020	-1.118
52	-1.004	-.979587	0.012882	-1.005	-.954020	-1.118
53	-.989558	-.979587	0.012882	-1.005	-.954020	-1.118
54	-.989558	-.979587	0.012882	-1.005	-.954020	-1.118
55	-1.004	-.979587	0.012882	-1.005	-.954020	-1.118
56	-.949441	-.948248	0.008940	-.965991	-.930505	-1.085
57	-.975770	-.948248	0.008940	-.965991	-.930505	-1.085
58	-.962406	-.948248	0.008940	-.965991	-.930505	-1.085
59	-.949441	-.948248	0.008940	-.965991	-.930505	-1.085
60	-.949441	-.948248	0.008940	-.965991	-.930505	-1.085
61	-.818162	-.925350	0.007813	-.940857	-.909843	-1.062
62	-.818162	-.925350	0.007813	-.940857	-.909843	-1.062
63	-.818162	-.925350	0.007813	-.940857	-.909843	-1.062
64	-.818162	-.925350	0.007813	-.940857	-.909843	-1.062
65	-.818162	-.925350	0.007813	-.940857	-.909843	-1.062
66	-.930851	-.904628	0.009150	-.922788	-.886469	-1.042
67	-.924617	-.904628	0.009150	-.922788	-.886469	-1.042
68	-.912718	-.904628	0.009150	-.922788	-.886469	-1.042

MASS RATE OF CUTTING REMOVAL VS NOZZLE DIAMETER

OBS	ACTUAL	PREDICT VALUE	STD ERR PREDICT	LOWER95% MEAN	UPPER95% MEAN	LOWER95% PREDICT
69	-.924617	-.904628	0.009150	-.922788	-.886469	-1.042
70	-.912718	-.904628	0.009150	-.922788	-.886469	-1.042
71	-.975770	-.887503	0.011593	-.910512	-.864493	-1.025
72	-.975770	-.887503	0.011593	-.910512	-.864493	-1.025
73	-.962406	-.887503	0.011593	-.910512	-.864493	-1.025
74	-.962406	-.887503	0.011593	-.910512	-.864493	-1.025
75	-.975770	-.887503	0.011593	-.910512	-.864493	-1.025
76	-1.038	-1.062	0.014555	-1.091	-1.033	-1.201
77	-1.026	-1.062	0.014555	-1.091	-1.033	-1.201
78	-1.050	-1.062	0.014555	-1.091	-1.033	-1.201
79	-1.026	-1.062	0.014555	-1.091	-1.033	-1.201
80	-1.026	-1.062	0.014555	-1.091	-1.033	-1.201
81	-.982609	-1.029	0.011427	-1.052	-1.006	-1.167
82	-.982609	-1.029	0.011427	-1.052	-1.006	-1.167
83	-.982609	-1.029	0.011427	-1.052	-1.006	-1.167
84	-.962406	-1.029	0.011427	-1.052	-1.006	-1.167
85	-.993074	-1.029	0.011427	-1.052	-1.006	-1.167
86	-.924617	-1.004	0.010612	-1.025	-.983199	-1.142
87	-.924617	-1.004	0.010612	-1.025	-.983199	-1.142
88	-.924617	-1.004	0.010612	-1.025	-.983199	-1.142
89	-.924617	-1.004	0.010612	-1.025	-.983199	-1.142
90	-.924617	-1.004	0.010612	-1.025	-.983199	-1.142
91	-1.015	-.983596	0.011622	-1.007	-.960530	-1.121
92	-1.004	-.983596	0.011622	-1.007	-.960530	-1.121
93	-1.015	-.983596	0.011622	-1.007	-.960530	-1.121
94	-.993074	-.983596	0.011622	-1.007	-.960530	-1.121
95	-1.004	-.983596	0.011622	-1.007	-.960530	-1.121
96	-1.050	-.966501	0.013653	-.993598	-.939404	-1.105
97	-1.050	-.966501	0.013653	-.993598	-.939404	-1.105
98	-1.050	-.966501	0.013653	-.993598	-.939404	-1.105
99	-1.050	-.966501	0.013653	-.993598	-.939404	-1.105
100	-1.050	-.966501	0.013653	-.993598	-.939404	-1.105

OBS	UPPER95% PREDICT	RESIDUAL	STD ERR RESIDUAL	STUDENT RESIDUAL	-2-1-0 1 2
1	-.582640	-.100671	0.066427	-1.516	***
2	-.582640	-.100671	0.066427	-1.516	***
3	-.582640	-.100671	0.066427	-1.516	***
4	-.582640	-.100671	0.066427	-1.516	***
5	-.582640	-.100671	0.066427	-1.516	***
6	-.557067	0.031989	0.067203	0.476	
7	-.557067	0.048380	0.067203	0.720	*

MASS RATE OF CUTTING REMOVAL VS NOZZLE DIAMETER

UPPER95%		STD 2AA		STUDENT			
OBS	PREDICT	RESIDUAL	RESIDUAL	RESIDUAL	RESIDUAL	-2-1-0	1 2
8	-.557007	0.014956	0.067203	0.223	1		1
9	-.557007	0.031989	0.067203	0.476	1		1
10	-.557007	0.031989	0.067203	0.476	1		1
11	-.535192	0.099997	0.067356	1.485	1	**	1
12	-.535192	0.099997	0.067356	1.485	1	**	1
13	-.535192	0.099997	0.067356	1.485	1	**	1
14	-.535192	0.099997	0.067356	1.485	1	**	1
15	-.535192	0.099997	0.067356	1.485	1	**	1
16	-.516388	0.067793	0.067153	1.010	1	**	1
17	-.516388	0.053553	0.067153	0.797	1	*	1
18	-.516388	0.067793	0.067153	1.010	1	**	1
19	-.516388	0.053553	0.067153	0.797	1	*	1
20	-.516388	0.038630	0.067153	0.578	1	*	1
21	-.498841	-.025342	0.066732	-0.380	1		1
22	-.498841	-.060104	0.066732	-0.901	1	*	1
23	-.498841	-.042375	0.066732	-0.635	1	*	1
24	-.498841	-.025342	0.066732	-0.380	1		1
25	-.498841	-.025342	0.066732	-0.380	1		1
26	-.742676	-.107696	0.067212	-1.602	1	***	1
27	-.742676	-.107696	0.067212	-1.602	1	***	1
28	-.742676	-.088391	0.067212	-1.315	1	**	1
29	-.742676	-.088391	0.067212	-1.315	1	**	1
30	-.742676	-.107696	0.067212	-1.602	1	***	1
31	-.715919	-.031143	0.067949	-0.458	1		1
32	-.715919	-.046537	0.067949	-0.691	1	*	1
33	-.715919	-.046937	0.067949	-0.691	1	*	1
34	-.715919	-.001180	0.067949	-0.017	1		1
35	-.715919	-.015903	0.067949	-0.234	1		1
36	-.692890	0.111142	0.068108	1.632	1	***	1
37	-.692890	0.111142	0.068108	1.632	1	***	1
38	-.692890	0.111142	0.068108	1.632	1	***	1
39	-.692890	0.111142	0.068108	1.632	1	***	1
40	-.692890	0.111142	0.068108	1.632	1	***	1
41	-.674119	0.011421	0.067927	0.168	1		1
42	-.674119	-.001544	0.067927	-0.023	1		1
43	-.674119	0.011421	0.067927	0.168	1		1
44	-.674119	-.001544	0.067927	-0.023	1		1
45	-.674119	-.001544	0.067927	-0.023	1		1
46	-.657000	-.059279	0.067527	-0.878	1	*	1
47	-.657000	-.089242	0.067527	-1.322	1	**	1
48	-.657000	-.074002	0.067527	-1.096	1	**	1
49	-.657000	-.059279	0.067527	-0.878	1	*	1
50	-.657000	-.059279	0.067527	-0.878	1	*	1
51	-.841300	-.024211	0.067252	-0.360	1		1

MASS RATE OF CUTTING REMOVED VS NOZZLE DIAMETER

OBS	UPPER95%		STD ERR		STUDENT				
	PREDICT	RESIDUAL	RESIDUAL	RESIDUAL	RESIDUAL	RESIDUAL	-2-1-0	1	2
52	-.841300	-.024211	0.067252	-0.360	1				
53	-.841300	-.009971	0.067252	-0.148	1				
54	-.841300	-.009971	0.067252	-0.148	1				
55	-.841300	-.024211	0.067252	-0.360	1				
56	-.811192	-.001192	0.067888	-0.018	1				
57	-.811192	-.027521	0.067888	-0.405	1				
58	-.811192	-.014157	0.067888	-0.209	1				
59	-.811192	-.001192	0.067888	-0.018	1				
60	-.811192	-.001192	0.067888	-0.018	1				
61	-.788565	0.107189	0.068027	1.576	1	***			
62	-.788565	0.107189	0.068027	1.576	1	***			
63	-.788565	0.107189	0.068027	1.576	1	***			
64	-.788565	0.107189	0.068027	1.576	1	***			
65	-.788565	0.107189	0.068027	1.576	1	***			
66	-.767517	-.032223	0.067860	-0.475	1				
67	-.767517	-.019989	0.067860	-0.295	1				
68	-.767517	-.008090	0.067860	-0.119	1				
69	-.767517	-.019989	0.067860	-0.295	1				
70	-.767517	-.008090	0.067860	-0.119	1				
71	-.749666	-.088267	0.067486	-1.308	1	**			
72	-.749666	-.088267	0.067486	-1.308	1	**			
73	-.749666	-.074903	0.067486	-1.110	1	**			
74	-.749666	-.074903	0.067486	-1.110	1	**			
75	-.749666	-.088267	0.067486	-1.308	1	**			
76	-.922760	0.024043	0.066909	0.559	1				
77	-.922760	0.035625	0.066909	0.532	1	*			
78	-.922760	0.012144	0.066909	0.182	1				
79	-.922760	0.035625	0.066909	0.532	1	*			
80	-.922760	0.035625	0.066909	0.532	1	*			
81	-.891144	0.046317	0.067514	0.686	1	*			
82	-.891144	0.046317	0.067514	0.686	1	*			
83	-.891144	0.046317	0.067514	0.686	1	*			
84	-.891144	0.066521	0.067514	0.985	1	*			
85	-.891144	0.035652	0.067514	0.531	1	*			
86	-.866735	0.079643	0.067647	1.177	1	**			
87	-.866735	0.079643	0.067647	1.177	1	**			
88	-.866735	0.079643	0.067647	1.177	1	**			
89	-.866735	0.079643	0.067647	1.177	1	**			
90	-.866735	0.079643	0.067647	1.177	1	**			
91	-.845750	-.031197	0.067481	-0.462	1				
92	-.845750	-.020202	0.067481	-0.299	1				
93	-.845750	-.031197	0.067481	-0.462	1				
94	-.845750	-.009478	0.067481	-0.140	1				
95	-.845750	-.020202	0.067481	-0.299	1				

MASS RATE OF CUTTING REMOVED VS NOZZLE DIAMETER

	UPPER95%		STL ERR	STUDENT			
OBS	PREDICT	RESIDUAL	RESIDUAL	RESIDUAL	-2-1-C	1	2
96	-.827923	-.063054	0.067099	-1.238		**	
97	-.827923	-.063054	0.067099	-1.238		**	
98	-.827923	-.063054	0.067099	-1.238		**	
99	-.827923	-.063054	0.067099	-1.238		**	
100	-.827923	-.063054	0.067099	-1.238		**	

OBS	COOK'S D
1	0.048
2	0.048
3	0.048
4	0.048
5	0.048
6	0.003
7	0.007
8	0.001
9	0.003
10	0.003
11	0.025
12	0.025
13	0.025
14	0.025
15	0.025
16	0.014
17	0.008
18	0.014
19	0.008
20	0.004
21	0.003
22	0.014
23	0.007
24	0.003
25	0.003
26	0.032
27	0.032
28	0.022
29	0.022
30	0.032
31	0.001
32	0.002
33	0.002
34	0.000

MASS FATE OF CUTTING REMOVED VS NOBLE ELAETERA

OBS	COOK'S J
35	0.000
36	0.010
37	0.010
38	0.010
39	0.010
40	0.010
41	0.000
42	0.000
43	0.000
44	0.000
45	0.000
46	0.007
47	0.016
48	0.011
49	0.007
50	0.007
51	0.002
52	0.002
53	0.000
54	0.000
55	0.002
56	0.000
57	0.001
58	0.000
59	0.000
60	0.000
61	0.011
62	0.011
63	0.011
64	0.011
65	0.011
66	0.001
67	0.001
68	0.000
69	0.001
70	0.000
71	0.017
72	0.017
73	0.012
74	0.012
75	0.017
76	0.002
77	0.004
78	0.001

MASS RATE OF CUTTING REMOVED VS NOZZLE DIAMETERS

OBS	LOGK'S D
79	0.004
80	0.004
81	0.004
82	0.004
83	0.004
84	0.009
85	0.003
86	0.011
87	0.011
88	0.011
89	0.011
90	0.011
91	0.002
92	0.001
93	0.002
94	0.000
95	0.001
96	0.021
97	0.021
98	0.021
99	0.021
100	0.021

SUM OF RESIDUALS	1.13656E-12
SUM OF SQUARED RESIDUALS	0.4548053

EXPERIMENTAL MASS DATA RATIOS VS CALCULATED

DEP VARIABLE: RATIO

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	1.532292	1.532292	272.041	0.0001
ERROR	98	0.551993	0.005632562		
C TOTAL	99	2.084285			
RCCT MSE		0.075051	R-SQUARE	0.7352	
DEP MEAN		-0.862186	ADJ R-SQ	0.7325	
C.V.		-8.70466			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR HO: PARAMETER=0	PROB > T
INTERCEP	1	1.789247	0.160930	11.118	0.0001
NR	1	-0.554088	0.033594	-16.454	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	74.336831	0.690265	0.000000
NR	1	1.532292	1.532292	-0.857417

VARIABLE	DF	TOLERANCE
INTERCEP	1	-
NR	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	NR
INTERCEP	0.02589842	-0.00540039
NR	-0.00540039	0.00112855

EXPERIMENTAL MASS RATE RATIOS VS CALCULATION

DEP VARIABLE: RATIO

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PRCE>F
MODEL	1	1.141354	1.141354	118.622	0.0001
ERROR	98	0.942932	0.009621751		
C TOTAL	99	2.084285			
ROOT MSE		0.098091	R-SQUARE	0.5476	
DEP MEAN		-0.862188	ADJ R-SQ	0.5430	
C.V.		-11.3769			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR HO: PARAMETER=0	PROB > T
INTERCEP	1	-0.908768	0.010701	-84.925	0.0001
RATIOF	1	-0.191039	0.017540	-10.891	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	74.336831	69.394325	0.000000
RATIOF	1	1.141354	1.141354	-0.740000

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
RATIOF	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	RATIOF
INTERCEP	0.000114508	.00007501575
RATIOF	.00007501575	0.0003076655

EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

DEP VARIABLE: RATIO

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	1.532292	1.532292	272.041	0.0001
ERROR	98	0.551993	0.005632582		
C TOTAL	99	2.084285			
REGT MSE		0.075051	R-SQUARE	0.7352	
DEP MEAN		-0.862188	ADJ R-S	0.7325	
C.V.		-8.70466			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR HO: PARAMETER=0	PROB > T
INTERCEP	1	-0.671764	0.013770	-48.784	0.0001
JF	1	-0.277044	0.016797	-16.494	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	74.336831	13.404725	0.000000
JF	1	1.532292	1.532292	-0.857417

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
JF	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	JF
INTERCEP	0.0001896195	-0.000193926
JF	-0.000193926	0.0002821358

EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

DEP VARIABLE: LATICM

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	1.406600	1.406600	203.408	0.0001
ERROR	98	0.677685	0.006915158		
C TOTAL	99	2.084285			
ROOT MSE		0.083157	R-SQUARE	0.6749	
DEP MEAN		-0.662168	ADJ R-SQ	0.6715	
C.V.		-9.64493			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR HO: PARAMETER=0	PROB > T
INTERCEP	1	-1.060872	0.016224	-65.389	0.0001
HHP	1	-0.163158	0.011440	-14.262	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	74.336831	29.507005	0.000000
HHP	1	1.406600	1.406600	-0.621498

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
HHP	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	HHP
INTERCEP	0.0002632205	0.0001593685
HHP	0.0001593685	0.0001308726

EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

DEP VARIABLE: MDCT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.239896	0.239896	28.581	0.0001
ERROR	98	0.822554	0.008393409		
C TOTAL	99	1.062450			
RCOT MSE		0.091616	R-SQUARE	0.2258	
DEP MEAN		1.031331	ADJ R-S	0.2179	
C.V.		8.883231			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PAGE > T
INTERCEP	1	-0.017760	0.196450	-0.091	0.9281
NR	1	0.219240	0.041009	5.346	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	106.364	0.0000675164	0.000000
NR	1	0.239896	0.239896	0.475179

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
NR	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	NR
INTERCEP	0.03859261	-0.00804741
NR	-0.00804741	0.00168172

EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

DEP VARIABLE: MDCI

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.067603	0.067603	6.659	0.0113
ERROR	98	0.994846	0.010151		
C TOTAL	99	1.062450			
RCOT MSE		0.100755	R-SQUARE	0.0636	
DEP MEAN		1.031331	ADJ R-SQ	0.0541	
C.V.		9.769374			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR HO: PARAMETER=0	PROB > T
INTERCEP	1	1.042668	0.010991	94.862	0.0001
RATIOF	1	0.046494	0.018017	2.561	0.0113

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	106.364	91.350300	0.000000
RATIOF	1	0.067603	0.067603	0.252250

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
RATIOF	1	1.000000

COVARIANCE OF ESTIMATES

COV	INTERCEP	RATIOF
INTERCEP	0.0001208125	.00007914588
RATIOF	.00007914588	0.0003246046

EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

DEP VARIABLE: MDCT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PR > F
MODEL	2	0.626686	0.313343	69.749	0.0001
ERROR	97	0.435764	0.00449241		
C TOTAL	99	1.062450			
ROOT MSE		0.667025	R-SQUARE	0.5899	
DEP MEAN		1.031331	ADJ R-SQ	0.5814	
C.V.		6.458924			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PR > T
INTERCEP	1	-3.959242	0.448431	-8.829	0.0001
NR	1	1.025550	0.091930	11.156	0.0001
RATIOF	1	-0.340770	0.036725	-9.279	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	106.364	0.350197	0.000000
NR	1	0.239896	0.559083	2.222771
RATIOF	1	0.386790	0.386790	-1.848320

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
NR	1	0.106507
RATIOF	1	0.106507

COVARIANCE OF ESTIMATES

COVB	INTERCEP	NR	RATIOF
INTERCEP	0.2010901	-0.0412189	0.01559992
NR	-0.0412189	0.008451168	-0.0031913
RATIOF	0.01559992	-0.0031913	0.001348733

EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

DEP VARIABLE: MDCT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.239896	0.239896	28.581	0.0001
ERROR	98	0.822554	0.008393409		
C TOTAL	99	1.062450			
ROOT MSE		0.091616	R-SQUARE	0.2258	
DEP MEAN		1.031331	ADJ R-SQ	0.2179	
C.V.		8.883231			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	0.955985	0.016810	56.871	0.0001
JF	1	0.109620	0.020504	5.346	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	106.364	27.147298	0.000000
JF	1	0.239896	0.239896	0.475179

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
JF	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	JF
INTERCEP	0.0002825621	-0.00028898
JF	-0.00028898	0.0004204301

EXPERIMENTAL MASS RATE RATIOS VS CALCULATED

DEP VARIABLE: MDOT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.165578	0.165578	18.093	0.0001
ERROR	98	0.896872	0.009151751		
C TOTAL	99	1.062450			
ROOT MSE		0.095665	R-SQUARE	0.1558	
DEP MEAN		1.031331	ADJ R-SQ	0.1472	
C.V.		9.275853			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	1.099499	0.018664	58.909	0.0001
HHP	1	0.055979	0.013161	4.254	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	106.364	31.759392	0.000000
HHP	1	0.165578	0.165578	0.394773

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
HHP	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	HHP
INTERCEP	0.0003483548	0.0002109135
HHP	0.0002109135	0.0001732012

NOTE: SAS OPTIONS SPECIFIED ARE:
SORT=4

```
1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA MS1;
3      INPUT MESH S DNE INCHES Q BHP;
4      SSPND=S/DNE;
5      LABEL SSPND=CHIPPING SIZE PER NOZZLE DIAMETER;
6      GRAMS=INCHES*60;
7      MMDOT=GRAMS*20/453.59;
8      MMDOT=8.34*Q;
9      RATION=MMDOT/MMDOT;
10     CARDS;
```

NOTE: DATA SET WORK.MS1 HAS 20 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 92K.

```
31     PROC PRINT DATA=MS1;
```

NOTE: THE PROCEDURE PRINT USED 0.15 SECONDS AND 136K
AND PRINTED PAGE 1.

```
32     DATA MS2;
33     SET MS1;
34     RATION=LOG10(RATION);
35     SSPND=LGG10(SSPND);
```

NOTE: DATA SET WORK.MS2 HAS 20 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 84K.

```
36     PROC PRINT DATA =MS2;
```

NOTE: THE PROCEDURE PRINT USED 0.15 SECONDS AND 136K
AND PRINTED PAGE 2.

```
37     PROC REG DATA=MS2;
38     MODEL RATION=SSPND/ALL COLLIN PARTIAL;
39     OUTPUT OUT=C P=PRED;
```

NOTE: DATA SET WORK.C HAS 20 OBSERVATIONS AND 12 VARIABLES. 190
OBS/TRK

S A S L O G OS SAS 82.28 VS2/MVS JCB SADIK

NOTE: THE PROCEDURE REG USED 0.32 SECONDS AND 176K
AND PRINTED PAGES 3 TO 8.

```
40      DATA MS3;
41      MERGE MS2 C;
42      MODEL=10**PRED;
43      EXPRMT=10**RATIOH;
44      SSPND=10**SSPND;
45      LABEL EXPRMT=MASS RATE RATIOS;
```

NOTE: DATA SET WORK.MS3 HAS 20 OBSERVATIONS AND 14 VARIABLES. 16
4 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 120K.

```
46      PROC PRINT DATA=MS3;
```

NOTE: THE PROCEDURE PRINT USED 0.16 SECONDS AND 144K
AND PRINTED PAGE 9.

```
47      PROC FLCT DATA=MS3;
48      PLOT EXPRMT*SSPND='A' MODEL*SSPND='P'/OVERLAY;
```

NOTE: THE PROCEDURE PLOT USED 0.15 SECONDS AND 136K
AND PRINTED PAGE 10.

```
49      PROC GFLCT;
50      PLOT EXPRMT*SSPND MODEL*SSPND/OVERLAY HAXIS=0 TO 0.9
BY 0.05 VAXIS=0 TO 0.28 BY
51      0.02 VREF=0.28 HREF=0.9 VPOS=22 HPOS=70;
52      SYMBOL1 V=STAR C=3;
53      SYMBOL2 1=SPLINE V=NONE C=3 L=2;
54      TITLE .H=1 .C=2 MASS RATE RATIOS VS CHIP SIZE PER NOZ
ZLE DIAMETER;
```

NOTE: THE PROCEDURE GPLCT USED 0.54 SECONDS AND 512K.

```
55      PROC PLOT DATA=C;
56      PLOT RATIOH*SSPND='A' PRED*SSPND='P'/OVERLAY;
```

NOTE: THE PROCEDURE PLOT USED 0.16 SECONDS AND 340K
AND PRINTED PAGE 11.

```
57      DATA MS4;
58      SET MS1;
59      MDCT=LOG10(MDCT);
60      SSPND=LOG10(SSPND);
```

NOTE: DATA SET WORK.MS4 HAS 20 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK

S A S L C G CS SAS 82.28 VS2/MVS JOB SADIK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 292K.

61 PROC REG DATA=MS4;
62 MODEL MDOT=SSPND/ALL COLLIN PARTIAL;

NOTE: THE PROCEDURE REG USED 0.29 SECONDS AND 512K
AND PRINTED PAGES 12 TO 16.

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DEP VARIABLE: RATIOH

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.0001	0.0001	999999.999	0.0001
ERROR	18	3.33067E-15	1.85037E-16		
C TOTAL	19	0.0001			
ROOT MSE		1.36028E-08	R-SQUARE	1.0000	
DEP MEAN		-0.878860	ADJ R-SQ	1.0000	
C.V.		-0.000015478			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR HO: PARAMETER=0	PROB > T
INTERCEP	1	-0.878860	6.07256E-09	-99999.999	0.0001
SSPND	1	4.80736E-16	1.41530E-08	0.000	1.0000

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	15.447881	3.875739	0.000000
SSPND	1	2.46519E-31	2.13489E-31	0.000000

VARIABLE	DF	TOLEBRANCE	VARIABLE LABEL
INTERCEP	1	.	INTERCEP1
SSPND	1	1.000000	CUTTING SIZE PER NOZZLE DIAMETER

COVARIANCE OF ESTIMATES

COVB	INTERCEP	SSPND
INTERCEP	3.68760E-17	7.43861E-17
SSPND	7.43861E-17	2.00306E-16

MASS RATE RATIOS VS CHIP SIZE PER NOZZLE DIAMETER

DEP VARIABLE: MDO1

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.0001	0.0001	0.000	0.0000
ERROR	18	0	0		
C TOTAL	19	0.0001			
ROOT MSE		.	R-SQUARE	1.0000	
DEP MEAN		0.820458	ADJ R-SQ	1.0000	
C.V.		0			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	0.820458	0	.	1.0000
SSPND	1	-9.61471E-16	0	.	1.0000

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	13.463019	3.377755	0.000000
SSPND	1	8.38165E-31	8.53958E-31	-0.000000

VARIABLE	DF	TOLERANCE	VARIABLE LABEL
INTERCEP	1	.	INTERCEPT
SSPND	1	1.000000	CUTTING SIZE PER NOZZLE DIAMETER

COVARIANCE OF ESTIMATES

COVB	INTERCEP	SSPND
INTERCEP	0	0
SSPND	0	0

```

1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA MS1;
3      INPUT ANGLE G INCHES DE Q BHP;
4      GRAMS=INCHES*43;
5      MDCT=GRAMS*20/453.59;
6      MMDOT=6.34*Q;
7      RATION=MDCT/MMDOT;
8      RFROUDE=DE**5*G/Q**2;
9      CARDS;

```

NOTE: DATA SET WORK.MS1 HAS 15 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK
NOTE: THE DATA STATEMENT USED 0.11 SECONDS AND 92K.

```

25      PROC PRINT DATA=MS1;

```

NOTE: THE PROCEDURE PRINT USED 0.14 SECONDS AND 136K
AND PRINTED PAGE 1.

```

26      DATA MS2;
27      SET MS1;
28      RATION=LOG10(RATION);
29      IF RFROUDE NE 0 THEN RFROUDE=LOG10(RFROUDE);

```

NOTE: DATA SET WORK.MS2 HAS 15 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK
NOTE: THE DATA STATEMENT USED 0.08 SECONDS AND 84K.

```

30      PROC PRINT DATA =MS2;

```

NOTE: THE PROCEDURE PRINT USED 0.16 SECONDS AND 136K
AND PRINTED PAGE 2.

```

31      PROC REG DATA=MS2;
32      MODEL RATION=RFROUDE/ALL COLLIN PARTIAL;
33      OUTPUT OUT=C P=PHET;

```

NOTE: DATA SET WORK.C HAS 15 OBSERVATIONS AND 12 VARIABLES. 190
OBS/TRK
NOTE: THE PROCEDURE REG USED 0.29 SECONDS AND 176K

S A S L O G O S S A S 82.23 VS2/HVS JOB SADIK
AND PRINTED PAGES 3 TO 8.

```
34      DATA MS3;
35      MERGE MS2 C;
36      MODEL=10**PRED;
37      EXPRMT=10**RATICH;
38      IF RFROUDE NE 0 THEN RFRCUDE=10**RFROUDE;
39      LABEL EXPRMT=MASS RATE RATIOS;
```

NOTE: DATA SET WORK.MS3 HAS 15 OBSERVATIONS AND 14 VARIABLES. 16
4 OBS/TRK
NOTE: THE DATA STATEMENT USED 0.10 SECONDS AND 120K.

```
40      PROC PRINT DATA=MS3;
```

NOTE: THE PROCEDURE PRINT USED 0.17 SECONDS AND 144K
AND PRINTED PAGE 9.

```
41      PROC PLOT DATA=MS3;
42      PLOT EXPRMT*RFROUDE='A' MODEL*RFROUDE='P'/OVERLAY;
```

NOTE: THE PROCEDURE PLOT USED 0.16 SECONDS AND 136K
AND PRINTED PAGE 10.

```
43      PROC GPLOT;
44      PLOT EXPRMT*RFROUDE MODEL*RFROUDE/OVERLAY HAXIS=0 TO
0.0025 BY 0.0005 VAXIS=
45      0.0 TO 0.3 BY 0.05 VREF=0.3 HREF=0.0025 VECS=22 HPCS=
70;
46      SYMBOL1 V=STAR C=3;
47      SYMBOL2 I=SPLINE V=NONE C=3 L=2;
48      TITLE _H=1 _C=2 MASS RATE RATIOS VS RECIPROCAL RFROUDE
NUMBER;
```

NOTE: THE PROCEDURE GPLOT USED 0.55 SECONDS AND 512K.

```
49      PROC PLOT DATA=C;
50      PLOT RATICH*RFROUDE='A' PRED*RFROUDE='P'/OVERLAY;
```

NOTE: THE PROCEDURE PLOT USED 0.16 SECONDS AND 340K
AND PRINTED PAGE 11.

```
51      DATA MS4;
52      SET MS1;
53      MDOT=LOG10(MDOT);
54      IF RFRCUDE NE 0 THEN RFROUDE=LOG10(RFROUDE);
```

NOTE: DATA SET WORK.MS4 HAS 15 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK

S A S L O G OS SAS 82.28 VS2/MVS JCB SADIK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 292K.

```
55        PROC REG DATA=MS4;  
56        MODEL MDOT=RFROUDE/ALL CCLLIN PARTIAL;  
57        OUTPUT OUT=D P=PEED;
```

NOTE: DATA SET WORK.C HAS 15 OBSERVATIONS AND 12 VARIABLES. 190
OES/TRK

NOTE: THE PROCEDURE REG USED 0.31 SECONDS AND 380K
AND PRINTED PAGES 12 TO 17.

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DEP VARIABLE: RATICM

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.0001	0.0001	999999.999	0.0001
ERROR	13	2.88658E-15	2.22045E-16		
C TOTAL	14	0.0001			
ROOT MSE		1.49012E-08	R-SQUARE	1.0000	
DEP MEAN		-0.808141	ADJ R-SQ	1.0000	
C.V.		-0.000018439			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	-0.808141	6.85905E-09	-99999.999	0.0001
RFROUDE	1	2.79427E-16	2.95501E-09	0.000	1.0000

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	9.796386	3.270318	0.000000
RFROUDE	1	2.02146E-30	1.98545E-30	0.000000

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
RFROUDE	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	RFROUDE
INTERCEP	4.43430E-17	1.60607E-17
RFROUDE	1.60607E-17	8.73209E-18

MASS RATE RATIOS VS RECIPROCAL FROUDE NUMBER

DEP VARIABLE: MDCI

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.0001	0.0001	999999.999	0.0001
ERROR	13	1.11022E-15	8.54018E-17		
C TOTAL	14	0.0001			
ROOT MSE	9.24131E-09	R-SQUARE	1.0000		
DEP MEAN	0.789718	ADJ R-SQ	1.0000		
C.V.	0.0000011702				

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	0.789718	4.12977E-09	99999.999	0.0001
RFROUDE	1	-2.79427E-16	1.83262E-09	-0.000	1.0000

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	9.354826	3.122912	0.000000
RFROUDE	1	2.02146E-30	1.98545E-30	-0.000000

VARIABLE	DF	TOLERANCE
INTERCEP	1	.
RFROUDE	1	1.000000

COVARIANCE OF ESTIMATES

COVB	INTERCEP	RFROUDE
INTERCEP	1.70550E-17	6.17719E-18
RFROUDE	6.17719E-18	3.35850E-18

NOTE: SAS OPTIONS SPECIFIED ARE:
SORT=4

```
1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA MS1;
3      INPUT RPM INCHES Q;
4      GRAMS=INCHES*43;
5      MDCT=GRAMS*20/453.59;
6      MMDOT=8.6*Q;
7      RATION=MDCT/MMDOT;
8      RNE=((RPM*Q*8.6)/(0.5*1.0));
9      LABEL RNR=ROTATIONAL REYNOLDS NUMBER;
10     CARDS;
```

NOTE: DATA SET WORK.MS1 HAS 20 OBSERVATIONS AND 8 VARIABLES. 280
OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 92K.

```
31     PROC PRINT DATA=MS1;
```

NOTE: THE PROCEDURE PRINT USED 0.16 SECONDS AND 132K
AND PRINTED PAGE 1.

```
32     DATA MS2;
33     SET MS1;
34     RATION=LOG10(RATION);
35     IF RNE NE 0 THEN RNR=LOG10(RNE);
```

NOTE: DATA SET WORK.MS2 HAS 20 OBSERVATIONS AND 8 VARIABLES. 280
OBS/TRK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 84K.

```
36     PROC PRINT DATA =MS2;
```

NOTE: THE PROCEDURE PRINT USED 0.15 SECONDS AND 132K
AND PRINTED PAGE 2.

```
37     PROC REG DATA=MS2;
38     MODEL RATION=RNE/ALL COLLIN PARTIAL;
39     OUTPUT OUT=C P=PRED;
```

NOTE: DATA SET WORK.C HAS 20 OBSERVATIONS AND 9 VARIABLES. 250 O
BS/TRK

S A S L O G OS SAS 82.2B VS2/MVS JCB SADIK

NOTE: THE PROCEDURE REG USED 0.35 SECONDS AND 176K
AND PRINTED PAGES 3 TO 8.

```
40      DATA MS3;
41      MERGE MS2 C;
42      MODEL=10**PRED;
43      EXPMT=10**RATIOH;
44      IF RNR NE 0 THEN RNR=10**RNR;
45      LABEL EXPMT=MASS RATE RATIOS;
```

NOTE: DATA SET WORK.MS3 HAS 20 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 120K.

```
46      PROC PRINT DATA=MS3;
```

NOTE: THE PROCEDURE PRINT USED 0.17 SECONDS AND 136K
AND PRINTED PAGE 9.

```
47      PROC PLOT DATA=MS3;
48      PLOT EXPMT*RNR='A' MODEL*RNR='P'/OVERLAY;
49      TITLE .H=1 .C=2 MASS RATE RATIOS VS ROTATIONAL REYNOL
DS NUMBER;
50      FOOTNOTE .H=1 .F=DUPLEX .C=2 -----REGRESSION;
```

NOTE: THE PROCEDURE PLOT USED 0.18 SECONDS AND 136K
AND PRINTED PAGE 10.

```
51      PROC GPLOT;
52      PLOT EXPMT*RNR MODEL*RNR/OVERLAY HAXIS=C TO 6000 BY
1500 VAXIS=.04 TO
53      0.085 BY 0.01 VREF=0.08 HREF=6000 VPOS=22 HPOS=80 VPO
S=22 HPOS=70;
54      SYMBOL1 V=STAR C=3;
55      SYMBOL2 I=SPLINE V=NONE C=3 L=2;
```

NOTE: THE PROCEDURE GPLOT USED 0.65 SECONDS AND 512K.

```
56      PROC PLOT DATA=C;
57      PLOT RATIOH*RNR='A' PRED*RNR='P'/OVERLAY;
```

NOTE: THE PROCEDURE PLOT USED 0.17 SECONDS AND 344K
AND PRINTED PAGE 11.

```
58      DATA MS4;
59      SET MS1;
60      MDOT=LOG10(MDOT);
61      IF RNR NE 0 THEN RNR=LOG10(RNR);
```

S A S L O G US SAS 82.2E VS2/MVS JOB SADIK

NOTE: DATA SET WORK.MS4 HAS 20 OBSERVATIONS AND 8 VARIABLES. 230
OBS/TRK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 296K.

62 PROC REG DATA=MS4;
63 MODEL MDOT=BNR/ALL COLLIN PARTIAL;
64 OUTPUT OUT=D P=PBED;

NOTE: DATA SET WORK.D HAS 20 OBSERVATIONS AND 9 VARIABLES. 250 O
BS/TRK

NOTE: THE PROCEDURE REG USED 0.35 SECONDS AND 384K
AND PRINTED PAGES 12 TO 17.

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DEP VARIABLE: RATION

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.047566	0.047566	275.038	0.0001
ERROR	18	0.003112979	0.0001729433		
C TOTAL	19	0.050679			
ROOT MSE		0.013151	R-SQUARE	0.9386	
DEP MEAN		-1.233075	ADJ R-SQ	0.9352	
C.V.		-1.0665			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	-1.329614	0.006521714	-203.875	0.0001
RNR	1	0.034360	0.002073053	16.584	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	30.409464	7.188384	0.000000
RNR	1	0.047566	0.047566	0.968801

VARIABLE	DF	TOLERANCE	VARIABLE LABEL
INTERCEP	1	.	INTERCEPT
RNR	1	1.000000	ROTATIONAL REYNOLDS NUMBER

COVARIANCE OF ESTIMATES

COVB	INTERCEP	RNR
INTERCEP	.00004253275	-.0000120675
RNR	-.0000120675	.00000429755

MASS RATE RATIOS VS ROTATIONAL REYNOLDS NUMBER

DEP VARIABLE: MDCT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.047566	0.047566	275.038	0.0001
ERROR	18	0.003112979	0.0001729433		
C TOTAL	19	0.050679			
ROOT MSE		0.013151	R-SQUARE	0.9386	
DEP MEAN		0.630843	ADJ R-SQ	0.9352	
C.V.		2.084636			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	0.534303	0.006521714	81.927	0.0001
RNR	1	0.034380	0.002073053	16.534	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	7.959250	1.160796	0.000000
RNR	1	0.047566	0.047566	0.968801

VARIABLE	DF	TOLERANCE	VARIABLE LABEL
INTERCEP	1	.	INTERCEP1
RNR	1	1.000000	ROTATIONAL REYNOLDS NUMBER

COVARIANCE OF ESTIMATES

COVB	INTERCEP	RNR
INTERCEP	.00004253275	-.0000120675
RNR	-.0000120675	.00000429755

```

1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA MS1;
3      INPUT L DB INCHES Q BHP;
4      SPBD=L/DB;
5      LABEL SPBD=SPACING PER BIT DIAMETER;
6      GRAMS=INCHES*60;
7      MDCT=GRAMS*20/453.59;
8      MMDOT=8.34*Q;
9      RATION=MDCT/MMDOT;
10     CARDS;

```

NOTE: DATA SET WORK.MS1 HAS 44 OBSERVATIONS AND 10 VARIABLES. 22
6 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 92K.

```

55     PROC PRINT DATA=MS1;

```

NOTE: THE PROCEDURE PRINT USED 0.18 SECONDS AND 136K
AND PRINTED PAGE 1.

```

56     DATA MS2;
57     SET MS1;
58     RATION=LOG10(RATION);
59     SPBD=LOG10(SPBD);

```

NOTE: DATA SET WORK.MS2 HAS 44 OBSERVATIONS AND 10 VARIABLES. 22
6 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.08 SECONDS AND 84K.

```

60     PROC PRINT DATA =MS2;

```

NOTE: THE PROCEDURE PRINT USED 0.18 SECONDS AND 136K
AND PRINTED PAGE 2.

```

61     PROC REG DATA=MS2;
62     MODEL RATION=SPBD/ALL COLLIN PARTIAL;
63     OUTPUT OUT=C P=PRED;

```

NOTE: DATA SET WORK.C HAS 44 OBSERVATIONS AND 11 VARIABLES. 207
OBS/TRK

S A S L C G OS SAS 82.2E VS2/NVS JOB SADIK

NOTE: THE PROCEDURE REG USED 0.41 SECONDS AND 176K
AND PRINTED PAGES 3 TO 10.

```
64      DATA MS3;
65      MERGE MS2 C;
66      MODEL=10**PRED;
67      EXPRNT=10**RATIOH;
68      SPBD=10**SPBD;
69      LABEL EXPRNT=MASS RATE RATIOS;
```

NOTE: DATA SET WORK.MS3 HAS 44 OBSERVATIONS AND 13 VARIABLES. 17
6 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.10 SECONDS AND 120K.

```
70      PROC PRINT DATA=MS3;
```

NOTE: THE PROCEDURE PRINT USED 0.23 SECONDS AND 144K
AND PRINTED PAGES 11 TO 13.

```
71      PROC PLOT DATA=MS3;
72      PLOT EXPRNT*SPBD='A' MODEL*SPBD='P'/OVERLAY;
```

NOTE: THE PROCEDURE PLOT USED 0.18 SECONDS AND 136K
AND PRINTED PAGE 14.

```
73      PROC GPLOT;
74      PLOT EXPRNT*SPBD MODEL*SPBD/OVERLAY VREF=0.26 HREF=2.
75      VPOS=22 HPOS=70;
76      SYMBOL1 V=STAR C=3;
77      SYMBOL2 I=SPLINE V=NONE C=3 L=2;
78      TITLE .H=1 .C=2 MASS RATE RATIOS VS SPACING PER BIT D
79      IANETER;
```

NOTE: THE PROCEDURE GPLOT USED 0.57 SECONDS AND 512K.

```
78      PROC PLOT DATA=C;
79      PLOT RATIOH*SPBD='A' PRED*SPBD='P'/OVERLAY;
```

NOTE: THE PROCEDURE PLOT USED 0.16 SECONDS AND 344K
AND PRINTED PAGE 15.

```
80      DATA MS4;
81      SET MS1;
82      MDCT=LCG10(MDOT);
83      SPBD=LOG10(SPBD);
```

NOTE: DATA SET WORK.MS4 HAS 44 OBSERVATIONS AND 10 VARIABLES. 22
6 OBS/TRK

S A S L C G OS SAS 82.2E VS2/1VS JOB SADIK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 296 K.

84 PROC REG DATA=MS4;
85 MODEL MDCT=SFBD/ALL COLLIN PARTIAL;
86 OUTPUT OUT=D P=PRED;

NOTE: DATA SET WORK.D HAS 44 OBSERVATIONS AND 11 VARIABLES. 207
OBS/TRK

NOTE: THE PROCEDURE REG USED 0.44 SECONDS AND 384 K
AND PRINTED PAGES 16 TO 23.

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DEP VARIABLE: RATION

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	2.716332	2.716332	686.569	0.0001
ERROR	42	0.166168	0.003956386		
C TOTAL	43	2.882500			
ROOT MSE		0.062900	R-SQUARE	0.9424	
DEP MEAN		-1.074945	ADJ R-SQ	0.9410	
C.V.		-5.85145			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	-1.199176	0.010602	-113.111	0.0001
SPBD	1	-0.600670	0.022924	-26.202	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	50.842258	50.618579	0.000000
SPBD	1	2.716332	2.716332	-0.970749

VARIABLE	DF	TOLERANCE	VARIABLE LABEL
INTERCEP	1	.	INTERCEPT
SPBD	1	1.000000	SPACING PER BIT DIAMETER

COVARIANCE OF ESTIMATES

COVB	INTERCEP	SPBD
INTERCEP	0.000112397	0.0001086885
SPBD	0.0001086885	0.0005255188

MASS RATE RATIOS VS SPACING PER BIT DIAMETER

DEP VARIABLE: MDCT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	2.716332	2.716332	686.569	0.0001
ERROR	42	0.166168	0.003956386		
C TOTAL	43	2.882500			
ROOT MSE		0.062900	R-SQUARE	0.9424	
DEP MEAN		0.323343	ADJ R-SQ	0.9410	
C.V.		19.45298			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	0.199111	0.010602	18.731	0.0001
SPBD	1	-0.600670	0.022924	-26.202	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	4.600223	1.395519	0.000000
SPBD	1	2.716332	2.716332	-0.970749

VARIABLE	DF	TOLERANCE	VARIABLE LABEL
INTERCEP	1	.	INTERCEPT
SPBD	1	1.000000	SPACING PER BIT DIAMETER

COVARIANCE OF ESTIMATES

COVB	INTERCEP	SPBD
INTERCEP	0.000112397	0.0001086885
SPBD	0.0001086885	0.0005255188

```

1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA MS1;
3      INPUT INCHES CUTDNSTY Q;
4      IF CUTDNSTY=2.59818 THEN GRAMS=INCHES*60;
5      IF CUTDNSTY=7.763 THEN GRAMS=INCHES*174.125;
6      IF CUTDNSTY=10.539 THEN GRAMS=INCHES*275.4;
7      LABEL VOLRATIO=VOLUME RATIOS;
8      LABEL RATIOD=CUTTING DENSITY PER FLUID DENSITY;
9      MDCT=GRAMS*20/453.59;
10     MMDOT=8.34*Q;
11     RATION=MDCT/MMDOT;
12     RATIOD=CUTDNSTY/8.34;
13     LABEL RATIOC=CUTTING DENSITY PER FLUID DENSITY;
14     LABEL RATION=MASS RATE RATIOS;
15     CUTTVCL=(INCHES*3.1416/4)*1.75**2;
16     FLUIDVOL=0.325*231;
17     VOLRATIO=CUTTVOL/FLUIDVOL;
18     CARDS;

```

NOTE: DATA SET WORK.MS1 HAS 24 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.11 SECONDS AND 92K.

```

43      PROC PRINT DATA=MS1;

```

NOTE: THE PROCEDURE PRINT USED 0.17 SECONDS AND 136K
AND PRINTED PAGES 1 TO 2.

```

44      PLOT VOLRATIO*RATIOD/HAXIS=0.25 TO 1.35 BY 0.55 VAXIS
45      =0.11 TO 0.19 BY 0.01
46      VREF=0.19 HREF=1.35 VPOS=22 HPOS=70;
47      SYMBOL3 I=RQ V=STAR C=3;
48      TITLE .H=1 .C=2 VOLUME RATIOS VS DENSITY RATIOS;

```

NOTE: THE PROCEDURE GLOT USED 0.43 SECONDS AND 418K.

```

49      DATA MS2;
50      SET MS1;
51      RATION=LOG10(RATION);

```

S A S L O G O S SAS 82.2b VS2/MVS JCB 321X

52 RATIOC=LOG10(RATIOC);

NOTE: DATA SET WORK.MS2 HAS 24 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 296K.

53 PROC PRINT DATA =MS2;

NOTE: THE PROCEDURE PRINT USED 0.16 SECONDS AND 348K
AND PRINTED PAGES 3 TO 4.

54 PROC REG DATA=MS2;

55 MODEL RATION=RATIOC/ALL COLLIN PARTIAL;

56 OUTPUT OUT=C P=PRED;

NOTE: DATA SET WORK.C HAS 24 OBSERVATIONS AND 12 VARIABLES. 190
OBS/TRK

NOTE: THE PROCEDURE REG USED 0.35 SECONDS AND 384K
AND PRINTED PAGES 5 TO 11.

57 PROC PRINT DATA=MS2;

NOTE: THE PROCEDURE PRINT USED 0.17 SECONDS AND 348K
AND PRINTED PAGES 12 TO 13.

58 DATA MS3;

59 MERGE MS2 C;

60 MODEL=10**PRED;

61 EXPRT=10**RATION;

62 RATIOD=10**RATIOC;

63 LABEL EXPRT=MASS RATE RATIOS;

NOTE: DATA SET WORK.MS3 HAS 24 OBSERVATIONS AND 14 VARIABLES. 16
4 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 324K.

64 PROC PRINT DATA=MS3;

NOTE: THE PROCEDURE PRINT USED 0.17 SECONDS AND 352K
AND PRINTED PAGES 14 TO 15.

65 PROC PLOT DATA=MS3;

66 PLOT EXPRT*RATIOD='A' MODEL*RATIOD='P'/CVERLAY;

NOTE: THE PROCEDURE PLOT USED 0.16 SECONDS AND 344K
AND PRINTED PAGE 16.

67 PROC GPLOT;

S A S I C G CS SAS 82.2E VS2/MVS JOB SADIK

```

68      PLOT EXPRMT*RATIOD MODEL*RATIOD/OVERLAY HAXIS=0.25 TO
1.35 BY 0.55
69      VAXIS=0.25 TO 0.85 BY 0.10 VREF=0.85 HREF=1.35 VPOS=2
2 HPOS=70;
70      SYMBOL1 V=STAR C=J;
71      SYMBOL2 I=SPLINE V=NONE C=3 L=2;
72      TITLE .H=1 .C=2 MASS RATE RATIOS VS DENSITY RATIOS;

```

NOTE: THE PROCEDURE GELCT USED 0.37 SECONDS AND 512K.

```

73      PROC PLOT DATA=C;
74      PLOT RATIONH*RATIOD='A' PRED*RATIOD='P'/OVERLAY;

```

NOTE: THE PROCEDURE PLOT USED 0.13 SECONDS AND 344K
AND PRINTED PAGE 17.

```

75      DATA MS4;
76      SET MS1;
77      MDOT=LCG10 (MDOT) ;
78      RATIOC=LOG10 (RATIOD) ;

```

NOTE: DATA SET WORK.MS4 HAS 24 OBSERVATIONS AND 11 VARIABLES. 20
7 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 296K.

```

79      PROC REG DATA=MS4;
80      MODEL MDOT=RATIOC/ALL COLLIN PARTIAL;
81      OUTPUT OUT=D P=PRED;

```

NOTE: DATA SET WORK.D HAS 24 OBSERVATIONS AND 12 VARIABLES. 190
OBS/TRK

NOTE: THE PROCEDURE REG USED 0.29 SECONDS AND 384K
AND PRINTED PAGES 18 TO 24.

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VOLUME RATIOS VS DENSITY RATIOS

DEP VARIABLE: RATIOH MASS RATE RATIOS

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.989544	0.989544	9932.967	0.0001
ERROR	22	0.002191688	.00009962218		
C TOTAL	23	C.991736			
ROOT MSE		0.009981091	R-SQUARE	0.9978	
DEP MEAN		-0.268504	ADJ R-SQ	0.9977	
C.V.		-3.7173			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	-0.155467	0.002331798	-66.672	0.0001
RATIOH	1	0.777777	0.007803969	99.664	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	1.730261	0.442842	0.000000
RATIOH	1	0.989544	0.989544	0.998894

VARIABLE	DF	TOLERANCE	VARIABLE LABEL
INTERCEP	1	.	INTERCEPT
RATIOH	1	1.000000	CUTTING DENSITY PER FLUID DENSITY

COVARIANCE OF ESTIMATES

COVB	INTERCEP	RATIOH
INTERCEP	.00000543728	.00000885108
RATIOH	.00000885108	.00006090154

MASS RATE RATIOS VS DENSITY RATIOS

DEP VARIABLE: MDCT

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	1	0.989544	0.989544	9932.967	0.0001
ERROR	22	0.002191688	0.00009962218		
C TOTAL	23	0.991736			
ROOT MSE		0.009981091	R-SQUARE	0.9978	
DEP MEAN		1.465576	ADJ R-SQ	0.9977	
C.V.		0.6810355			

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > T
INTERCEP	1	1.578613	0.002331798	676.994	0.0001
RATIOD	1	0.777777	0.007803969	99.664	0.0001

VARIABLE	DF	TYPE I SS	TYPE II SS	STANDARDIZED ESTIMATE
INTERCEP	1	51.549897	45.658901	0.000000
RATIOD	1	0.989544	0.989544	0.998894

VARIABLE	DF	TOLERANCE	VARIABLE LABEL
INTERCEP	1	.	INTERCEPT
RATIOD	1	1.000000	CUTTING DENSITY PER FLUID DENSITY

COVARIANCE OF ESTIMATES

COVB	INTERCEP	RATIOD
INTERCEP	.00000543728	.00000885108
RATIOD	.00000885108	.00006090194

NOTE: SAS OPTIONS SPECIFIED ARE:
 SORT=4

```

1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA FLOW1;
3      INPUT BITNUMBER Q INCHES;
4      GRAMS=INCHES*43;
5      MDCT=GRAMS*20/453.59;
6      TITLE1 .H=1 CLEANING ACTION BY VARIOUS NOZZLE COMBINA
TIONS;
7      TITLE2 .H=1 .C=2 MASS RATE OF CUTTING REMOVED (LB/MIN
) VS BIT NO;
8      TITLE3 .H=1 .C=2 Q=4.75 GPM;
9      CARDS;

```

NOTE: DATA SET WORK.FLOW1 HAS 12 OBSERVATIONS AND 5 VARIABLES. 4
 33 OBS/TEK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 84K.

```

22      PROC PRINT DATA=FLOW1;

```

NOTE: THE PROCEDURE PRINT USED 0.14 SECONDS AND 132K
 AND PRINTED PAGE 1.

```

23      PROC GCHART DATA=FLOW1;
24      VBAR BITNUMBER/TYPE= MEAN SUMYAS=MDOT GROUP=BITNUMBER;
25      LABEL MDOT=MASS RATE OF CUTTING REMOVED;

```

NOTE: THE PROCEDURE GCHART USED 0.66 SECONDS AND 408K.
 NOTE: SAS USED 408K MEMORY.

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```

1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA FLOW2;
3      INPUT BITNUMBER Q INCHES;
4      GRAMS=INCHES*43;
5      MDOT=GRAMS*20/453.59;
6      TITLE1 .H=1 CLEANING ACTION BY VARIOUS NOZZLE COMBINA
TIONS;
7      TITLE2 .H=1 .C=2 MASS RATE OF CUTTING REMOVED (LB/MIN
) VS BIT NO;
8      TITLE3 .H=1 .C=2 Q=8.50 GPM;
9      CARDS;

```

NOTE: DATA SET WORK.FLOW2 HAS 12 OBSERVATIONS AND 5 VARIABLES. 4
33 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.08 SECONDS AND 84K.

```

22      PROC PRINT DATA=FLOW2;

```

NOTE: THE PROCEDURE PRINT USED 0.14 SECONDS AND 132K
AND PRINTED PAGE 1.

```

23      PROC GCHART DATA=FLOW2;
24      VBAR BITNUMBER/TYPE= MEAN SUMVAR=MDOT GROUP=BITNUMBER;
25      LABEL MDOT=MASS RATE OF CUTTING REMOVED;

```

NOTE: THE PROCEDURE GCHART USED 0.67 SECONDS AND 408K.

NOTE: SAS USED 408K MEMORY.

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NOTE: SAS OPTIONS SPECIFIED ARE:
SORT=4

```

1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA FLOW3;
3      INPUT BITNUMBER Q INCHES;
4      GRAMS=INCHES*43;
5      MDCT=GRAMS*20/453.59;
6      TITLE1 .H=1 CLEANING ACTION BY VARIOUS NOZZLE COMBINA
TIONS;
7      TITLE2 .H=1 .C=2 MASS RATE OF CUTTING REMOVED (LB/MIN
) VS BIT NO;
8      TITLE3 .H=1 .C=2 Q=12.0 GPM;
9      CARDS;

```

NOTE: DATA SET WORK.FLOW3 HAS 10 OBSERVATIONS AND 5 VARIABLES. 4
33 OBS/TK

NOTE: THE DATA STATEMENT USED 0.09 SECONDS AND 84K.

```

20     PROC PRINT DATA=FLOW3;

```

NOTE: THE PROCEDURE PRINT USED 0.13 SECONDS AND 132K
AND PRINTED PAGE 1.

```

21     PROC GCHART DATA=FLOW3;
22     VBAR BITNUMBER/TYPE= MEAN SUMVAR=MDCT GRGUP=BITNUMBER;
23     LABEL MDCT=MASS RATE OF CUTTING REMOVED;

```

NOTE: THE PROCEDURE GCHART USED 0.68 SECONDS AND 406K.

NOTE: SAS USED 406K MEMORY.

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NOTE: SAS OPTIONS SPECIFIED ARE:
SORT=4

```

1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA FLOW4;
3      INPUT BITNUMBER Q INCHES;
4      GRAMS=INCHES*43;
5      MDOT=GRAMS*20/453.59;
6      TITLE1 .H=1 CLEANING ACTION BY VARIOUS NOZZLE COMBINA
TIONS;
7      TITLE2 .H=1 .C=2 MASS RATE OF CUTTING REMOVED (LB/MIN
) VS BIT NO;
8      TITLE3 .H=1 .C=2 Q=16.0 GPM;
9      CARDS;

```

NOTE: DATA SET WORK.FLOW4 HAS 10 OBSERVATIONS AND 5 VARIABLES. 4
33 OBS/TRK
NOTE: THE DATA STATEMENT USED 0.08 SECONDS AND 84K.

```

20     PROC PRINT DATA=FLOW4;

```

NOTE: THE PROCEDURE PRINT USED 0.14 SECONDS AND 132K
AND PRINTED PAGE 1.

```

21     PROC GCHART DATA=FLOW4;
22     VBAR BITNUMBER/TYPE= MEAN SUMVAR=MDOT GROUP=BITNUMBER;
23     LABEL MDOT=MASS RATE OF CUTTING REMOVED;

```

NOTE: THE PROCEDURE GCHART USED 0.67 SECONDS AND 408K.
NOTE: SAS USED 408K MEMCHY.

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NOTE: SAS OPTIONS SPECIFIED ARE:
SORT=4

```

1      OPTION PAGESIZE=50 LINESIZE=64 NODATE NONUMBER;
2      DATA MS1;
3      INPUT Q BITNG ED&OP HHP JF INCHES;
4      GRAMS=INCHES*43;
5      MDOT=GRAMS*20/453.59;
6      MMDOT=8.34*Q;
7      RATION=MDCI/MMDOT;
8      CARDS;

```

NOTE: DATA SET WORK.MS1 HAS 24 OBSERVATIONS AND 10 VARIABLES. 22
6 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.10 SECONDS AND 92K.

```

33     PRGC PRINT DATA=MS1;

```

NOTE: THE PROCEDURE PRINT USED 0.18 SECONDS AND 136K
AND PRINTED PAGES 1 TO 2.

```

34     DATA MS7;
35     SET MS1;

```

NOTE: DATA SET WORK.MS7 HAS 24 OBSERVATIONS AND 10 VARIABLES. 22
6 OBS/TRK

NOTE: THE DATA STATEMENT USED 0.07 SECONDS AND 84K.

```

36     PRGC SORT DATA=MS1;
37     BY BITNG C;
38     SYMBOL1 I=JOIN V=STAR C=3;
39     SYMBOL2 I=JOIN V=SQUARE C=3;
40     SYMBOL3 I=JOIN V=TRIANGLE C=3;
41     SYMBOL4 I=JOIN V=DIAMOND C=3;
42     SYMBOL5 I=JOIN V=PLUS C=3;
43     SYMBOL6 I=JOIN V=HASH C=3;
44     LABEL RATION=MASS RATE RATIOS;
45     LABEL HHP=HYDRAULIC HORSEPOWER;
46     LABEL JF=HYDRAULIC JET IMPACT FORCE;
47     LABEL BITNG=BIT NUMBER;
48     LABEL MDOT=MASS RATE OF CUTTING REMOVED;

```

NOTE: 4 CYLINDERS DYNAMICALLY ALLOCATED ON SYSDA FOR EACH OF 3 S
ORT WORK DATA SETS

S A S I C G OS SAS 82.2E VS2/MVS JOB SADIK

NOTE: DATA SET WORK.MS1 HAS 24 OBSERVATIONS AND 10 VARIABLES. 22
6 OBS/TRK

NOTE: THE PROCEDURE SORT USED 0.33 SECONDS AND 152K.

```
49      PROC GELOT DATA=MS7;
50      PLOT MDOT*HHP=EITNO/HREF=1.3 VREF=16 VPOS=22 HPOS=70;
51      TITLE1 .H=1 .C=2 MASS RATE OF CUTTING REMOVED VS HYDR
AULIC HORSEPOWER;
```

NOTE: THE PROCEDURE GELOT USED 0.56 SECONDS AND 416K.

```
52      PROC GPLOT DATA=MS7;
53      PLOT MDOT*JF=BITNO/HREF=70 VREF=16 VPOS=22 HPOS=70;
54      TITLE1 .H=1 .C=2 MASS RATE OF CUTTING REMOVED VS HYDR
AULIC IMPACT FORCE;
```

NOTE: THE PROCEDURE GPLOT USED 0.47 SECONDS AND 624K.

```
55      PROC GELOT DATA=MS7;
56      PLOT RATION*HHP=EITNO/HREF=0.6 VREF=0.25 VPOS=22 HPOS
=70;
57      TITLE1 .H=1 .C=2 MASS RATE RATIOS VS HYDRAULIC HORSEP
OWER;
```

NOTE: THE PROCEDURE GELOT USED 0.43 SECONDS AND 624K.

```
58      PROC GELOT DATA=MS7;
59      PLOT RATION*JF=BITNO/HREF=70 VREF=0.25 VPOS=22 HPOS=7
0;
60      TITLE1 .H=1 .C=2 MASS RATE RATIOS VS HYDRAULIC IMPACT
FORCE;
```

NOTE: THE PROCEDURE GPLOT USED 0.43 SECONDS AND 624K.

NOTE: SAS USED 624K MEMORY.

NOTE: SAS INSTITUTE INC.
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APPENDIX B

Definition of Terms and Calculation Techniques

Definition of Terms

200

S	=	Chip size in inches
N	=	Number of nozzles
DN	=	Nozzle diameter in inches
DNE	=	Effective nozzle diameter
Inches	=	Inches of sand removed
Q	=	Flow rate, gallons per minute
BHP	=	Bottom-hole pressure, psig
SSPND	=	Chip size per nozzle diameter
MDOT	=	Mass flow rate of cuttings, lb/min (M_c)
MMDOT	=	Mass flow rate of drilling fluid, lb/min (M_f)
G	=	Gravity of acceleration, ft/sec ²
Angle	=	Bore hole angle of inclination (zero angle means horizontal position.)
L	=	Nozzle spacing, inches
DB	=	Bit diameter, inches
RPM	=	Rotational speed, revolution per minute
RNR	=	Rotational Reynolds number, dimensionless
OBS	=	Observation
CUTDNSTY	=	Cutting density, gm/cc
DE	=	DNE
HHP	=	Hydraulic horsepower

PDROP = Pressure drop through the bit
 TA = Total nozzle area, in²
 ρ = Fluid density, lb/gal

CALCULATION TECHNIQUE

$$DNE = \sqrt{N \left(\frac{DN}{32} \right)^2}$$

GRAMS = Inches of sand removed x conversion factor ($\frac{\text{grams}}{\text{inch}}$)

$$MDOT = \frac{\text{Grams}}{3 \text{ sec}} \times \frac{60 \text{ sec}}{\text{min}} \times \frac{\text{lb}}{453.59 \text{ grams}} = \text{lb/min}$$

$$MMDOT = Q \left(\frac{\text{gal}}{\text{min}} \right) \times \rho \left(\frac{\text{lb}}{\text{gal}} \right) = \text{lb/min}$$

$$RATIOM = MDOT/MMDOT$$

$$SPBD = L/DB$$

$$SSPND = S/DN$$

$$VOLRATIO = \text{Cutting volume/fluid volume}$$

$$\text{Cutting Volume} = \text{Inches} \times \frac{\pi}{4} \times \text{inside diameter of annulus squared}$$

$$\text{Fluid Volume} = Q \left(\frac{\text{gal}}{\text{min}} \right) \times \text{run-time} (3 \text{ sec}) \times \left(\frac{\text{min}}{60 \text{ sec}} \right) \times \left(\frac{231 \text{ in}^3}{\text{gal}} \right)$$

$$RATIOD = \text{Cutting density/fluid density}$$

$$PDROP = \frac{\rho Q^2}{7430 \times 0.95^2 \times DE^4}$$

$$RATIOP = PDROP/BHP$$

$$PRODCT = NR \times RATIOP$$

$$HHP = \frac{PDROP \times Q}{1714}$$

$$TA = \left(\frac{\pi}{4} \right) \times 3 \times DN^2$$

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$$JF = \frac{0.000518 \times \rho \times Q^2}{TA}$$

$$NR = 379.0849 \times \rho \times \left(\frac{Q}{3} \right) / .8904 \times DN$$

Hydraulic jet impact force in bit configuration study

$$JF = 0.000518 \times \rho \times q^2 \text{ (through a single nozzle)/nozzle area}$$

$$\text{Nozzle Area} = \frac{\pi}{4} DN^2$$

$$\text{Total Jet Impact} = JF \times \text{number of nozzles pointing downward}$$