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Scope of Study: Multiplication by electronic analog becomes a problem when four quadrant operations on fast time scales are necessary. This problem has been realized and a study of the present totally-electronic multiplying schemes has been made. The presently known schemes do not furnish sufficient accuracy over the entire range of operation and they are limited in general to input voltage polarities which make them two quadrant devices. Also, the presently developed multiplying schemes were mostly developed by persons primarily interested in servomechanisms and, therefore, involve mechanical, optical, accoustical or other physical systems besides the necessary electronic apparatus. Since these systems are very slow to respond the multiplying devices have limited speed of response. A search in the present literature on this subject has been made to ascertain the possibility of constructing a device which does not include the above undesirable characteristics of most presently developed electronic analogue multipliers. Laboratory measurements were made to substantiate any conclusions obtained by a search through published material.

Summarized Conclusions: A four quadrant all electronic multiplier which will operate on fairly fast time scales can be developed. A development of this multiplier may begin with a presently known two quadrant multiplier. The two quadrant device can be modified to yield operation which is sufficiently accurate regardless of the polarity of the input variables. It is necessary to use great care in eliminating noise, distortion and power supply variations when a multiplier for four quadrant operation is developed from a two quadrant multiplier. A second type of four quadrant, fast responding multiplier can be built without using presently developed devices. Development of a four quadrant multiplier which is totally electronic will include the development of a very effective feedback system if the product is to be obtained with accuracies better than 1 per cent of full scale. However, there are some instances when it is not necessary to obtain the product with this degree of accuracy; in which case, a multiplier which is simple and easily developed can be built.

ADVISER'S APPROVAL Herbert L. Jones

MULTIPLICATION BY ELECTRONIC ANALOGUE

By

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Bachelor of Science

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Stillwater, Oklahoma

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PREFACE

Certain types of calculations which must be made by continuously operated electronic calculators have brought forth a need for development of an electronic analogue multiplier. It is hoped that the information presented here will stimulate interest in the search for an all electronic analogue multiplying device by making it obvious that the need for such a device exists. This paper is to show the general direction that others have taken in this quest so that their work will be used by any newcomers with minimized duplication of efforts. It should also show that a new and different type of multiplying device is needed with the application of the many new electronic techniques.

Geophysical Research Corporation deserves a note of thanks for their generous loan of laboratory facilities and needed materials. Grateful credit is given to John E. Owen for making himself so readily available for consultation.

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CHAPTER I

INTRODUCTION

It is a well known fact that the process of multiplication can be carried out by electrical or mechanical means. Desk type computers have been doing this for us for many years. The desk type computer or calculating machine is a digital type multiplier in that its accuracy is limited by the number of significant figures provided for in the construction of the machine. However, the analogue computer is limited by the percentage errors of the devices used multiplied by the full ranges of the variables that they represent. The accuracy limitations almost define the two types of systems. Further, it can be seen that the digital computer is a stepping device whose response is limited by the number of possible steps in a unit of time while the analogue computer is limited only by the frequency limits of the analogue signal representing one or both of the elements of the product which then gives a continuously operating computer which operates by identifying variables in one physical system with those in another physical system obeying equations of the same form. The analogue multiplier will take the form of figure 1. This block diagram shows the two inputs corresponding to the variables to be multiplied.

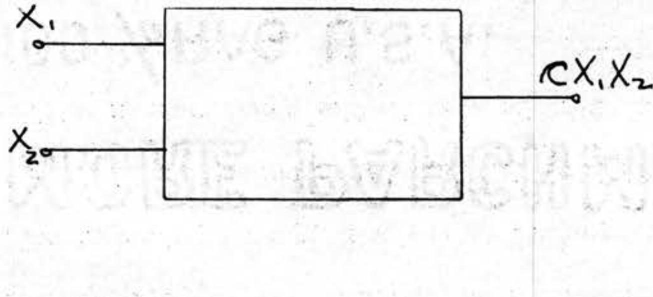

Figure 1.

General Form of a Multiplier

The output $X_o = cX_1X_2$ will be proportional to the product of X_1 and X_2. C is a constant and can be either positive or negative.

At first it appears that the only requirement is a linear circuit to perform the process of multiplication, but further study will show that it is necessary to have a device whose gain can be varied in a linear manner with a voltage or current.

All of the literature referred to has made an effort to show that the development of an analogue multiplier with improved performance is much needed. Persons primarily interested in servomechanisms have developed multiplying devices which are simple and well known; but these devices have many limitations such as high scale time, low speed of response and restrictions on input voltage polarity.

Korn and Korn says, "It would be desirable to have purely electronic multiplying devices of fair accuracy (0.1 to 0.5 per cent of full scale or .001 to .005 machine units) and as simple in design as a good d.c. intergrater (six or less tubes)".[1]

It has been further found that other disadvantages exist in most of the multiplying schemes investigated such as severe limitations on the range of input voltages, polarity of the input voltages, frequency of the input voltages and maximum ratio of one input voltage to the other input voltage.

Since electronic devices are characteristically non-linear, most previous investigators have used other types of systems such as optical, accoustical, mechanical or more often one of the above combined with an electrical system. As one would suspect, the above systems are heavy, space consuming and slow to

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[1]Korn, Granind A. and Theresa M. Korn, Electronic Analog Computers, McGraw-Hill, New York, 1952, p. 211.

respond due to their high mass.

As in most computer design undertakings, the designer of a multiplying system will find it necessary to know more than a little about the fundamentals of computers and a basic knowledge of servomechanisms. This paper does not contain these fundamentals. It only intends to point out the need for a device such as an electronic analogue multiplier and present some of the products of others in the development of multipliers employing all types of systems.

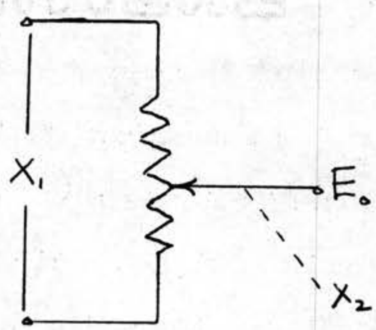
CHAPTER II

SIMPLE MULTIPLYING DEVICES

Potentiometer as a Multiplier

Figure 2 shows a potentiometer with a fixed center tap connected for use as a multiplier. The arm of the potentiometer must have motion proportional to X_2.

Figure 2.



Potentiometer as a Multiplier

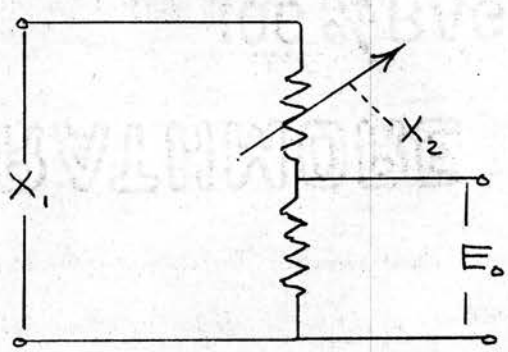
That is, when $X_2 = 0$ the arm is at ground potential and at any other position its displacement must be so that the resistance to ground is linearly proportional to X_2. The arm of the potentiometer will be on one side of the ground point for positive values of X_2 and on the opposite side for negative values of X_2. The arm of the potentiometer would be driven by a servo motor with X_2 for its input and its output to meet the above requirements of linearity.

This type of system is simple but it is heavy and slow to respond. Its accuracy is limited by the ability of the motor to return to the zero position as well as its linearity.

Multiplication Based on Ohm's Law

Figure 3 is a simple multiplier which is based on Ohm's law. The value of R_1 is controlled by X_2. The function of R_1 is to control the current through R_2 so the ratio of R_1/R_2 should be as high as practical in order to have the operating range great. The value of R_1 should be inversely proportional to X_2 so that when $X_2 = 0$, R_1 approaches an infinitely large number, the current through R_2 nears zero and the output voltage is correspondingly near zero.

Figure 3.



Multiplication Based on Ohm's Law

The Ohm's law type calculator is a very apt type for portable equipment such as an airborne navigation computer where the value of R_1 is manually adjusted. However, the possibility of using electronic resistances should not be overlooked.

<u>Vacuum Tubes as Multipliers</u>

Since vacuum tubes have much non-linearity and questionable reliability, one would not quickly decide on the use of a tube as a multiplying device. However, schemes employing tubes have been used with enough success to warrant mention. For applications where great accuracy is not needed, the tube should certainly be considered because of its extreme simplicity. When a multigrid tube is connected in a manner similar to Figure 4 it can be made to act as a multiplier. The two input variables, X_1 and X_2 are signals on two grids which have control of the plate current. The plate current then is equal to $cX_1X_2 = I_p$. Of course, the accuracy of operation is greatly dependent upon the choice of the tube and the operating point. The operating point should be selected so that the grid voltages swing the plate current over the most linear portion of their transfer characteristics. Tubes which can be used to approximately satisfy the equation $I_p = cX_1X_2$, have been found to be the 6AS6, 6SA7 and 6BE6 when the first and third grids are used for X_1 and X_2.

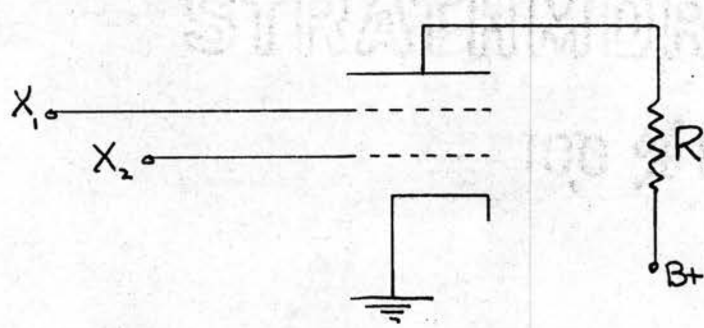


Figure 4.

Product Obtained by Multigrid Tube

Another relation more nearly satisfied for the 6AS6 is $I_p = cX_1 (X_1 - X_2)$ if this type of relation can be used.[1] The above discussion has been confined to tubes with more than one grid, however, triodes are also used as multipliers. In the case of the triode plate current will still be represented by

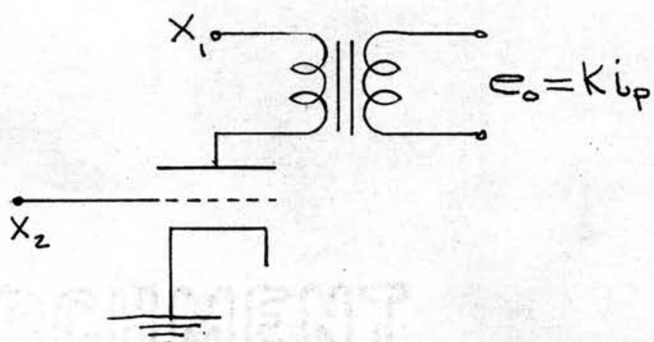


Figure 5.

A Triode Operation

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[1]Chance, B., F.C. Williams, V. Hughes, D. Sayre, and E. F. Macnichol, Waveforms, Vol. 19, MIT Radiation Laboratory Series, McGraw-Hill, New York, 1949, p. 670.

the equation $I_p = cX_1X_2$ but the elements to become the product are applied in the circuit as shown in figure 5.

The most often mentioned triode for multiplier service is the 6K6 so connected. The conditions for operation are certainly not the same as those for the same tube used in amplifier service. Operating conditions can be found in the literature.[2]

There are other types of schemes which use more than one tube and show no particular advantage over the use of a single tube. For this reason they are omitted here.

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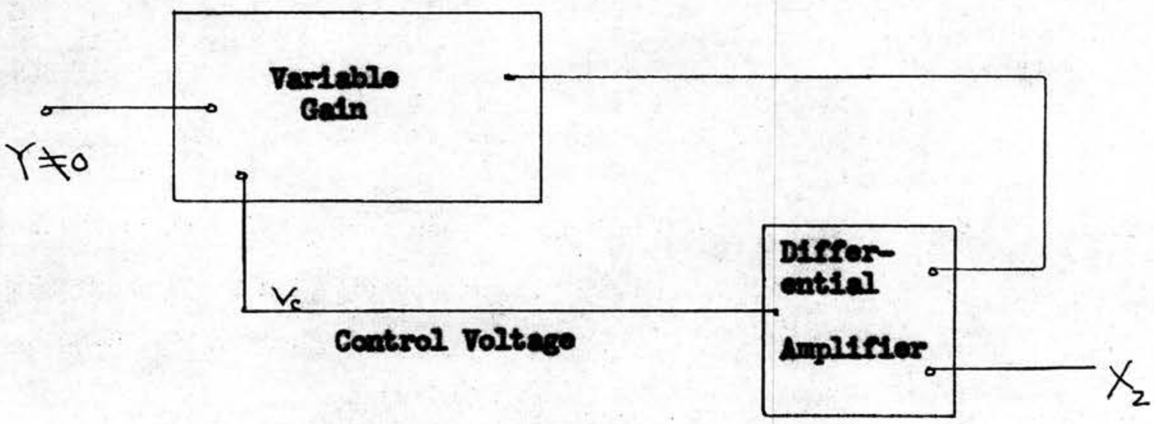
[2] Ibid, p. 669.

CHAPTER III

LINEARIZED GAIN CHARACTERISTICS BY FEEDBACK

It has been often mentioned that the electronic variable gain devices are naturally non-linear, particularly vacuum tubes. The gain of a variable gain element can be made more nearly proportional to a control voltage if an extra stabilizing voltage is introduced and if feedback is employed. If Y is fed

Figure 6.



Linearized Gain

constantly into the variable gain device, operated on by the device, fedback negatively to control the gain of the device then the system is one as shown in figure 6. When $X_2 = 0$ the gain is stable and constant.

Let G = Gain of the variable gain device

Vo = Voltage from variable gain device

A = Gain of differential amplifier

Vc = Voltage controlling gain of device

X_2 = Input voltage

To show that the gain of the variable gain element is accurately proportioned to X_2, let it first be said that the gain varies only as a function of Vc.

Then $$G = cV_c = cA\ (X_2 - YG)$$

where c is a function relating the gain of the variable gain device to the control voltage. It is not necessarily a constant.

Solving $$G = cA\ (X_2 - YG)$$

$$G = cAX_2 - cAYG$$

$$G(1 - cAY) = cAX_2$$

$$G = \frac{cAX_2}{1 - cAY}$$

By making A very large

$$G \approx X_{2/Y}$$

and the requirement that the gain be proportional to X_2 has been met. Notice that the above calculations hold only if the gain of the variable gain device is made independent of its input voltage. Feedback can be used to linearize all types of multiplying schemes. A few of which employ feedback will be shown on the following pages.

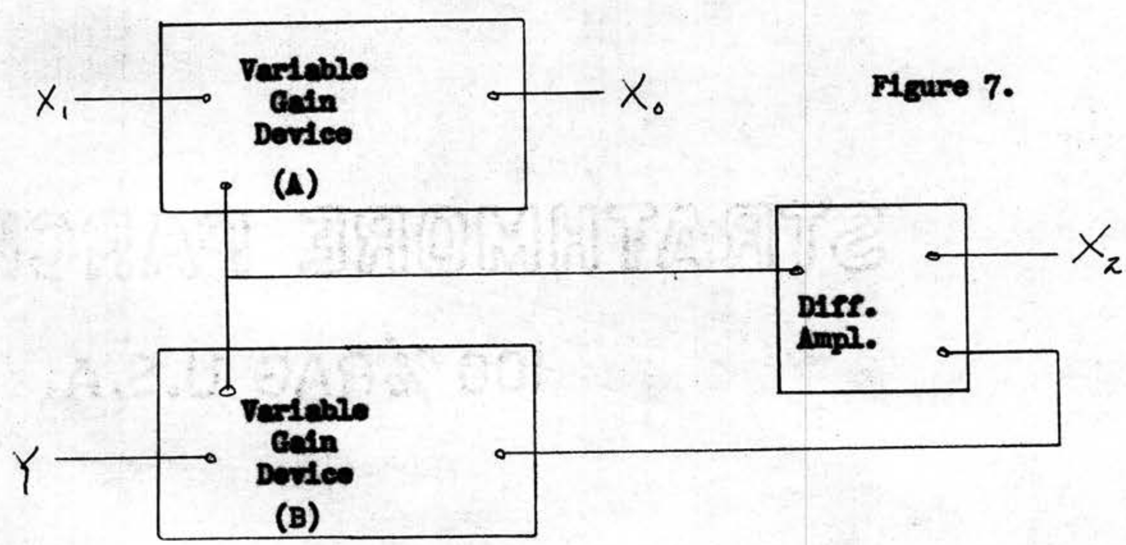


Figure 7.

Matched Variable Gain Devices

If an additional variable gain device is introduced in the manner shown in figure 7, the gain of both variable gain devices will be accurately proportional to X_2 if both devices are identical. This type of scheme, using matched devices, has found most use in servomechanisms where motor driven potentiometers are used for matched variable gain devices. It can be seen that tubes would not be readily usable as the variable gain elements since tubes are very difficult to accurately match. Even if they are matched they both will change in use and cannot be expected to remain accurately matched without sufficient aging.

The feedback arrangement can be used with modification so that it becomes what is known as the frequency sharing scheme.[1] This type system is shown in figure 8. X_1 and Y are simply added before they are applied to the

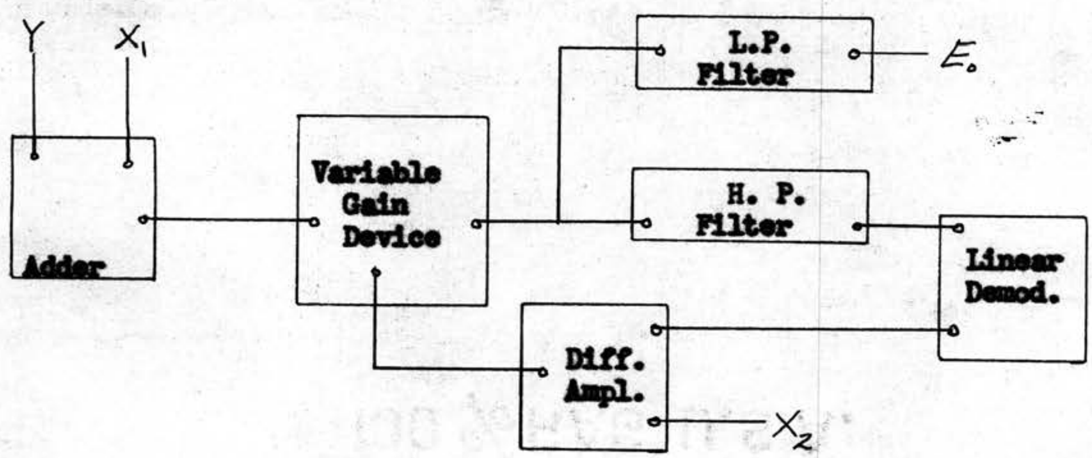


Figure 8.

Frequency Sharing Feedback System

variable gain device. A simple resistor adder may be used. Y is the ampli-

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[1]Korn, Granind A. and Theresa M. Korn, Electronic Analog Computers, McGraw-Hill, New York, 1952, p. 229.

tude of a carrier signal whose frequency is well above the highest expected frequency of X_1 or X_2. Thus, by making the feedback loop responsive only to Y signal the same variable gain device can be used for both the linearizing circuit and the multiplier circuit. It can be shown that by the circuit of figure 7 the gain of the variable gain device is accurately proportional to X_2/Y if the gain of the differential amplifier is sufficiently high. Systems of this type have been constructed with accuracies of .2% of full scale.[2,3,4]

[2] Ibid, p. 230.

[3] Chance, B., F. C. Williams, V. Hughes, D. Sayre, and E. F. MacNichol, Waveforms, Vol. 19, MIT Radiation Laboratory Series, McGraw-Hill, New York, 1949, p. 674.

[4] A multiplier which performs according to the frequency-sharing scheme was constructed by the author for his employer, Geophysical Research Corporation, but cannot be detailed to the public for reasons of company security. Accuracy measurements on this device are somewhat less than 1% of full scale when in two quadrant operation.

CHAPTER IV

PULSED ATTENUATOR MULTIPLIER

Multiplying devices based on the pulsed attenuator have received enough attention to be considered important. Many variations of the simple scheme presented here have been developed and used. Figure 9 shows the basic pulsed

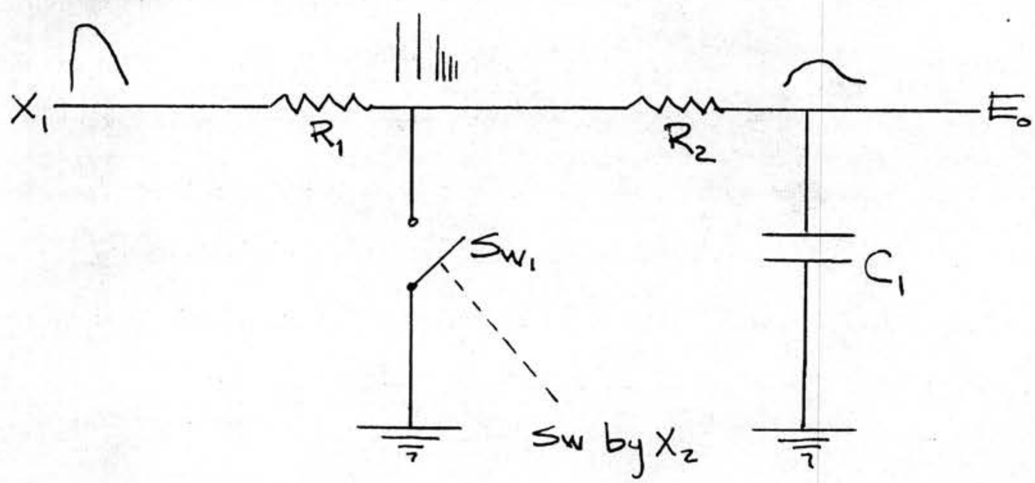

Figure 9.

Simplified Pulsed Attenuator Multiplier

attenuator multiplier. X_1 is applied to the switch through the series resistor (R_1). R_1 is large compared to the contact resistance of the switch when it is closed. X_2 varies the off-time of the switch. The off-time is considered the time when the switch is closed. The on-time is a constant and must be short compared to the period of the highest frequency input. When $X_2 = 0$, $E_o = 0$ and t_{off} is infinite. The gain G is determined by the ratio of the off-time to the on-time or

$$G = c\ t_{off}/t_{on}$$

In actual use the pulsed attenuators have the frequency of the chopping voltage equal to a constant and vary the off-time. This makes the gain equal to

$$G = c \frac{t_{on}}{t_{off} + t_{on}}$$

which is

$$c \frac{t_{off} + t_{on} + t_{off}}{t_{on} + t_{off}}$$

but $1/t_{off} + t_{on} = f$, frequency of chopping voltage

then $G = c \left(1 - ft_{off}\right)$.

The resistance-capacitance section after the attenuator serves to smooth the output voltage after it has been chopped at a constant rate with off-time proportional to X_2. At the output of this filter E_o equals cX_1X_2 which is the required operation when multiplication is performed.

Pulse attenuators have appeared in many different forms.[1] At least one such multiplier has been built and operated with good results.[2] Accuracies of better than .2% of full scale have been reported with pulsed attenuator multipliers. They have the benefit of most all electronic devices in that they can be operated at frequencies above 2000 cps. This type multiplier has its weak points as do others. The ratio of the magnitudes of the two input voltages is limited to somewhat less than 10 to 1 due to noise and grid swing limits. Approximately eight tubes are required and it may be necessary to select and match some of these tubes.

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[1] Korn, Granind A. and Theresa M. Korn, <u>Electronic Analog Computers</u>, McGraw-Hill, New York, 1952, p. 223.

[2] Greenwood, I. A., J. V. Holdam, and D. MacRae, <u>Electronic Instruments</u>, MIT Radiation Laboratory Series, Vol. 21, McGraw-Hill, New York, 1948, pp. 50-53.

CHAPTER V

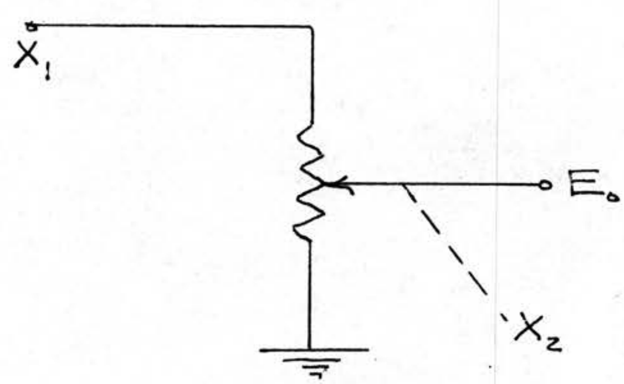
INPUT POLARITY CONSIDERATIONS

The simple laws of algebra obviously should hold during the multiplication process $(-X_1)$ (X_2) should yield a negative product, etc. When both inputs are either positive or negative with respect to a chosen reference the output should be positive. If the inputs are opposite in sign then the output should be negative with respect to the reference. If the above is true, all laws of sign are obeyed, the device is said to be a four quadrant multiplier.

Two quadrant multipliers have the polarity of either X_1 or X_2 restricted to a single sign while the other may have either positive or negative sign. Obviously, the output voltage should change sign with the input voltage that bears no restrictions.

One quadrant multipliers are product taking circuits which perform correctly only when both X_1 and X_2 have a particular polarity. Both are restricted and cannot change their sign. Then, the output always has the same sign.

Figure 10.

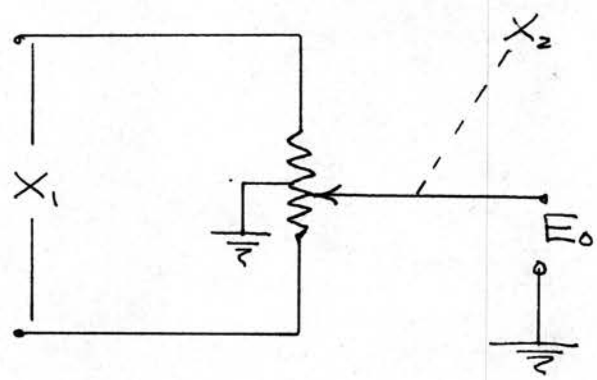


A Two Quadrant Multiplier

Figure 10 shows a device which is a multiplier. X_1 can be either positive or negative with respect to ground. X_2 can control the value of the resistance in only one direction from the zero resistance position. Therefore, X_1 can change polarity, X_2 can never change polarity and the output must always have the same sign as X_1. This is a two quadrant multiplier.

Figure 11 is a variation of figure 10. Here again, X_1 has no limitation on its sign. But, X_2 may control the arm of the potentiometer in either

Figure 11.



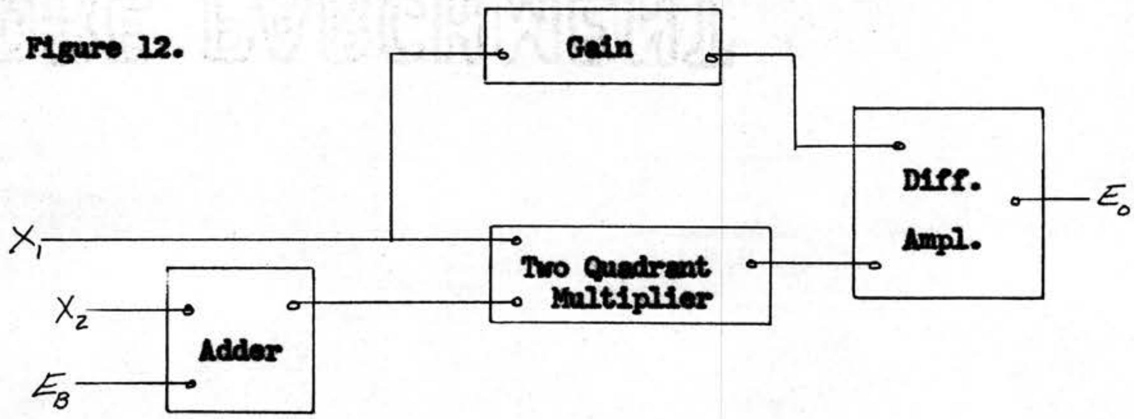
A Four Quadrant Device

direction from ground thus choosing portions of X_1 which are either positive or negative with respect to ground. The device has no polarity restrictions and is a four-quadrant multiplier.

One quadrant multipliers appear only in the form of function generators such as logarithmic devices where the logarithm of a negative number is not permitted. Multipliers in the form of function generators are briefly discussed later in this paper.

There are a number of schemes which can be used to enable two quadrant multipliers to operate as four quadrant devices. One such scheme is presented in figure 12. This diagram shows a bias voltage being added to X_2 so that the output of the two quadrant multipliers before going into the differential amplifier is equal to $X(X_2 + E_B) = X_1X_2 + X_1X_B$. The bias voltage is a constant so that the X_1E_B term depends only on X_1; this makes it possible to subject X_1 to a constant gain device with its gain adjusted so that its output is equal to X_1E_B. The output of the two quadrant multiplier is then fed into

Figure 12.



A Two Quadrant Device in Four Quadrant Operation

the differential amplifier with the X_1E_B signal so that the differential amplifier has for its output X_1X_2.

It is fairly obvious that the two X_1E_B terms which are to be cancelled out in the differential amplifier must be exactly 180° out of phase and must be free from all distortion. Any distortion (including phase distortion) will result in a portion of this voltage in the output voltage due to insufficient cancellation. The remaining voltage must be large enough to enable the X_2 signal to have its maximum value without $X_2 + E_B$ changing sign, i.e., E_B must be greater than the maximum peak X_2. Figure 13 diagramatically shows two multipliers of the two quadrant type may be combined for four quadrand operation.[1] The inputs to the top multiplier are $X_2 + E_B$ and X_1 while the inputs to the lower multiplier are $-X_2 + E_B$ and X_1. The two outputs are then

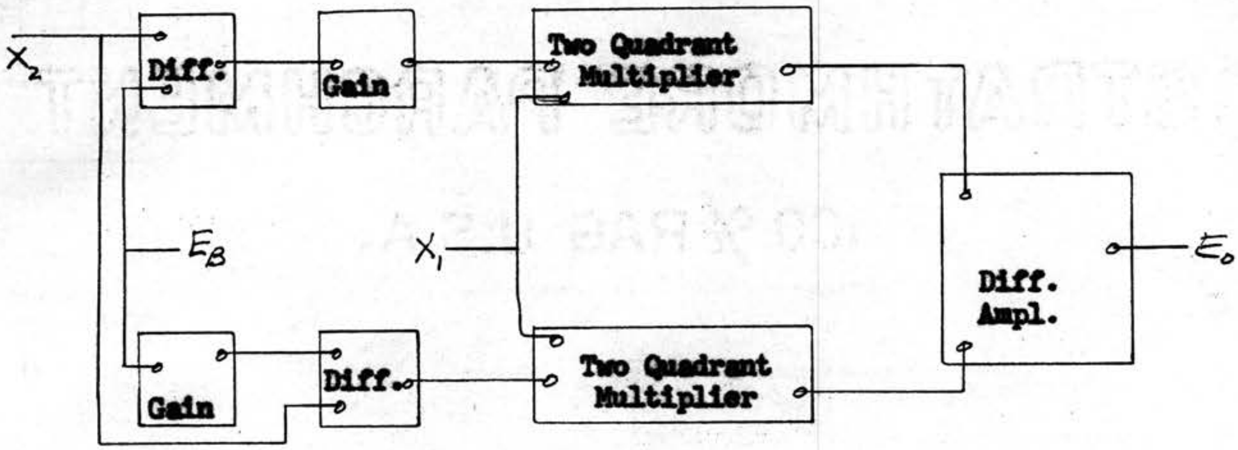


Figure 13.

Combining Two Quadrant Multipliers

$X_1X_2 + X_1X_B$ and $-X_1X_2 + X_1E_B$ respectively. After the difference of these outputs is taken by means of the differential amplifier the X_1E_B terms are no longer present and the output contains only the desired X_1X_2 terms.

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[1] Korn, Granind A. and Theresa M. Korn, Electronic Analog Computers, McGraw-Hill, New York, 1952, p. 214.

CHAPTER VI

MODULATORS AS ANALOGUE MULTIPLIERS

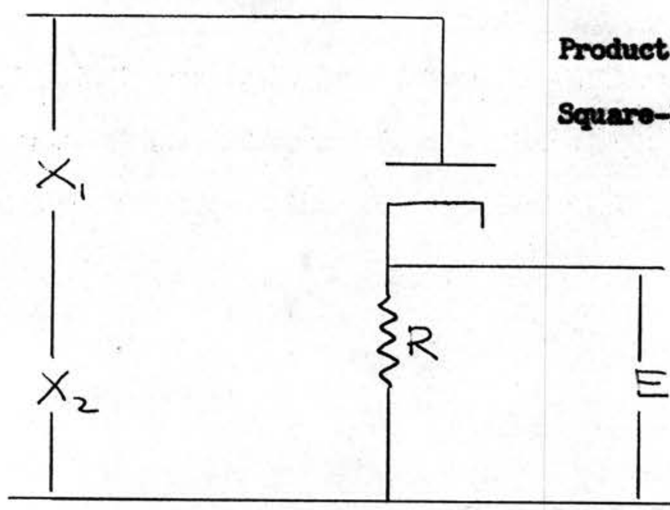
Linear Amplitude Modulator

A linear amplitude modulator may be used to perform the process of multiplication without any modification of the basic modulator circuit. This is readily seen by an inspection of the equations relating the inputs and outputs in the linear modulator circuit. In these equations the modulation voltage can be called the X_2 voltage and the carrier then becomes X_1. If four quadrant operation is desired, a phase sensitive linear modulator should be selected.[1,2]

Square-law Modulator

The common square law modulator has found most use when one of the input variables is d-c. This can be understood readily by taking a look at the

Figure 14.



Product Obtained From Square-Law Modulator

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[1] Electrical Engineering Staff of Massachusetts Institute of Technology, Applied Electronics, John Wiley & Sons, New York, 1943, pp. 638-640.

[2] Chance, B., F. C. Williams, V. Hughes, D. Sayre, and E. F. MacNichol, Waveforms, Vol. 19, MIT Radiation Laboratory Series, McGraw-Hill, New York, 1949, pp. 49-53.

output of a diode square law modulator.

$$X_1 = E_1 \text{Cos } w_1 t$$

$$X_2 = E_2 \text{Cos } w_2 t$$

The voltage from the plate of the diode is then $(X_1 + X_2)$ which equals $E_1 \text{Cos } w_1 t + E_2 \text{Cos } w_2 t = e_p$ and the voltage-current characteristic of the diode and its resistive load is described by Taylor's series.

$$i_p = a_1 e_p + a_2 e_p^2$$

which becomes $i_p = a_1 E_1 \text{Cos } w_1 t + a_1 E_2 \text{Cos } w_2 t + a_2 E_1^2 \text{Cos}^2 \, w_1 t + 2 a_2 E_1 E_2$

$$\text{Cos } w_1 t \text{Cos } w_2 t + a_2 E_2^2 \text{Cos}^2 \, w_2 t$$

when $E_1 \text{Cos } w_1 t + E_2 \text{Cos } w_2 t$ is substituted for e_p. From the complete expression of the plate current the $2a_2 E_1 E_2 \text{Cos } w_1 t \text{ Cos } w_2 t$ term is proportional to the product of the two input voltages. The remaining terms must be eliminated if they are not needed in the final solution.

If one of the inputs is to be d-c then the only terms remaining besides the product are a d-c component and a second harmonic component. The d-c can be eliminated from the output by a blocking condenser and the second harmonic can be balanced out. If one of the inputs is not d-c then both of the second harmonic terms must be eliminated by balancing and a balanced modulator becomes useful.

A balanced modulator has an output voltage which has the form of

$$A \left[a_1 X_1 + a_2 (X_2 \pm X_1) \right]$$

$$\text{or } A \left[a_1 X_2 + a_2 (X_1 \pm X_2) \right]$$

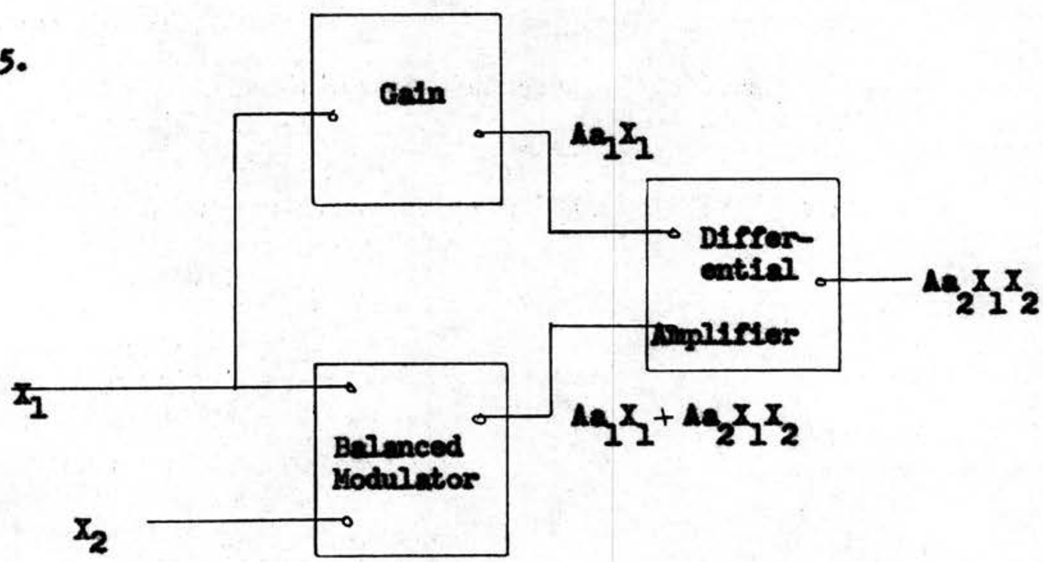
where the sum and difference frequencies represent the cross product of the input voltages. This means that the output voltage of the balanced modulator has the forms of

$$A\left(a_1X_1 + a_2X_1X_2\right)$$

$$\text{or } A\left(a_1X_2 + a_1X_1X_2\right)^*$$

If inputs are arranged so that the output is $A(a_1X_1 + a_2X_1X_2) = Aa_1X_1 + Aa_2X_1X_2$ the Aa_1X_1 term must be cancelled from the final output. Figure 15 shows a method for cancelling a term which is proportional to one of the input voltage from the output voltage. In this figure the X_1 input is subjected to gain so a voltage equal to Aa_1X_1 is available to the input of the differential amplifier along with the $Aa_1X_1 + Aa_2X_1X_2$ voltage. Of course their difference is

Figure 15.



Product Taken With Balanced Modulator

the desired product.

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*A general discussion of balanced modulator operation may be found in Electrical Engineering Staff of Massachusetts Institute of Technology, Applied Electronics, John Wiley & Sons, New York, 1943.

CHAPTER VII

FUNCTION GENERATORS IN MULTIPLIER SCHEMES

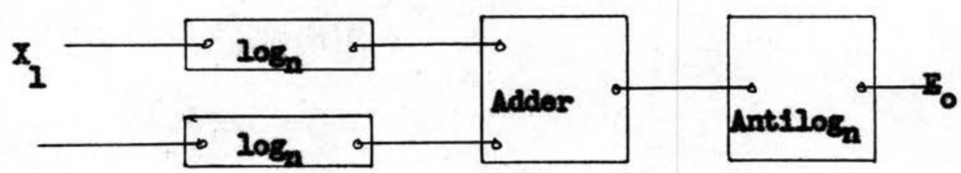
There are a great number of possible multiplier schemes which require non-linear devices. These non-linear outputs must be specific functions and, certainly there should be known methods of generating these functions. The product taking systems using these generators are discussed with very little explanation as to how the function generators operate.

Addition of Logarithms

It is certainly a fact that one may perform the process of multiplication by taking the logarithm of the two numbers whose product is desired, add these two logarithms, and then take the antilogarithm to obtain the product. The following equation should recall the above statement of fact.

$$\log_n X_1 + \log_n X_2 = \log_n X_1 X_2$$

Figure 16 illustrates this operation. The logarithm circuits may be found



Addition of Logarithms Figure 16.

in the reference material.[1] The added may be either a differential amplifier or a simple resistor adder.

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[1] Chance, B., F. C. Williams, V. Hughes, D. Sayre, and E. F. MacNichol, Waveforms, MIT Radiation Laboratory Series, McGraw-Hill, New York, 1949, p. 676.

Since logarithms are not defined for negative values of X_1 or X_2 the system of figure 16 is restricted to one quadrant operation.

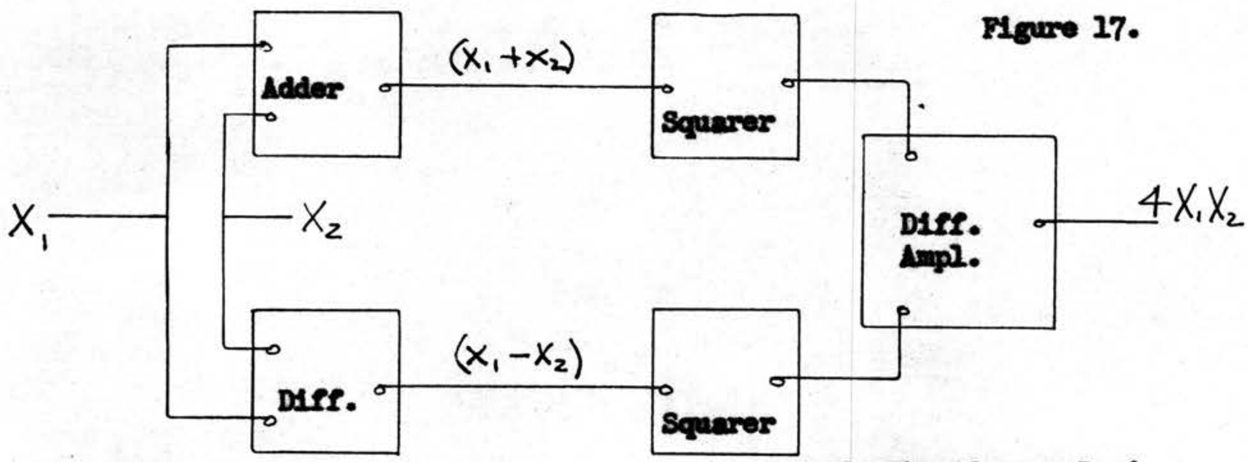
Quarter-Square Multiplier

Another equality that furnishes a method of obtaining a product is

$$4X_1X_2 = (X_1 + X_2)^2 - (X_1 - X_2)^2$$

where X_1 and X_2 are again to be the input voltages to the multiplier. Figure

Figure 17.



Quarter-Square Device

17 is a block presentation of a quarter-square multiplier. The gain through the adder and the first differential amplifier must be equal.

This device is also degraded because it cannot maintain high accuracy over its full ranges of input voltages and, therefore, must be restricted to a range of operation for a specified accuracy.

Intergration

It has been reported that the process of multiplication can be carried out directly with great precision by using a standard differential analyzer.[2,3]

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[2]Bush, V., and S. Caldwell, "A New Type of Differential Analyzer", Journal of Franklin Institute, pp. 240, 255-326, October, 1945.

[3]Greenwood, I. A., J. V. Holdam, and D. MacRae, Electronic Instruments, MIT Radiation Laboratory Series, McGraw-Hill, New York, 1948, p. 56.

In this method the product of the two input variables is represented by the following relation.

$$X_1X_2 = \int X_1dX_2 + \int X_2dX_1$$

No known circuit has been developed specifically for multiplication which operates on this basis. Certainly a differential analyzer could not be used for multiplication in small and portable equipment. However, a small compact unit utilizing the above equation might be developed.

CHAPTER VIII

SUMMARY

All of the multiplication schemes discussed in the body of this paper have assumed that the inputs to the device was on electrical input. Actually, a change of representation may be necessary since the variations to be multiplied could be mechanical, hydraulic or any other physical changes. For instance, the inputs may be hydraulic pressure in which case these pressure changes would be represerted by voltage changes. The voltage changes would then be operated on by a device similar to one of those in the preceding pages.

When direct conversion from one type of representation to electrical representation is not possible or if this conversion is very inaccurate, a third representation can be introduced to link the two systems. A simple example of this type of conversion follows. Assume that the variables to be multiplied are mechanical translational motion. Difficulties arise when these types of changes are converted to voltage variations. However, it is not difficult to convert this translational movement to rotational displacement. As shaft rotation a potentiometer may be used to obtain voltage variations proportional to the original translational changes.

It may also be necessary to tax the art of servomechanisms when these conversion methods are necessary. When direct conversion methods do not apply, then the conversion can be "looped" so that the system is shown in figure 18.

Before further summary is given it may be well to illustrate the type of mathematical operation which has caused much of the investigation of multiplying devices. When making computations involving auto-correlation

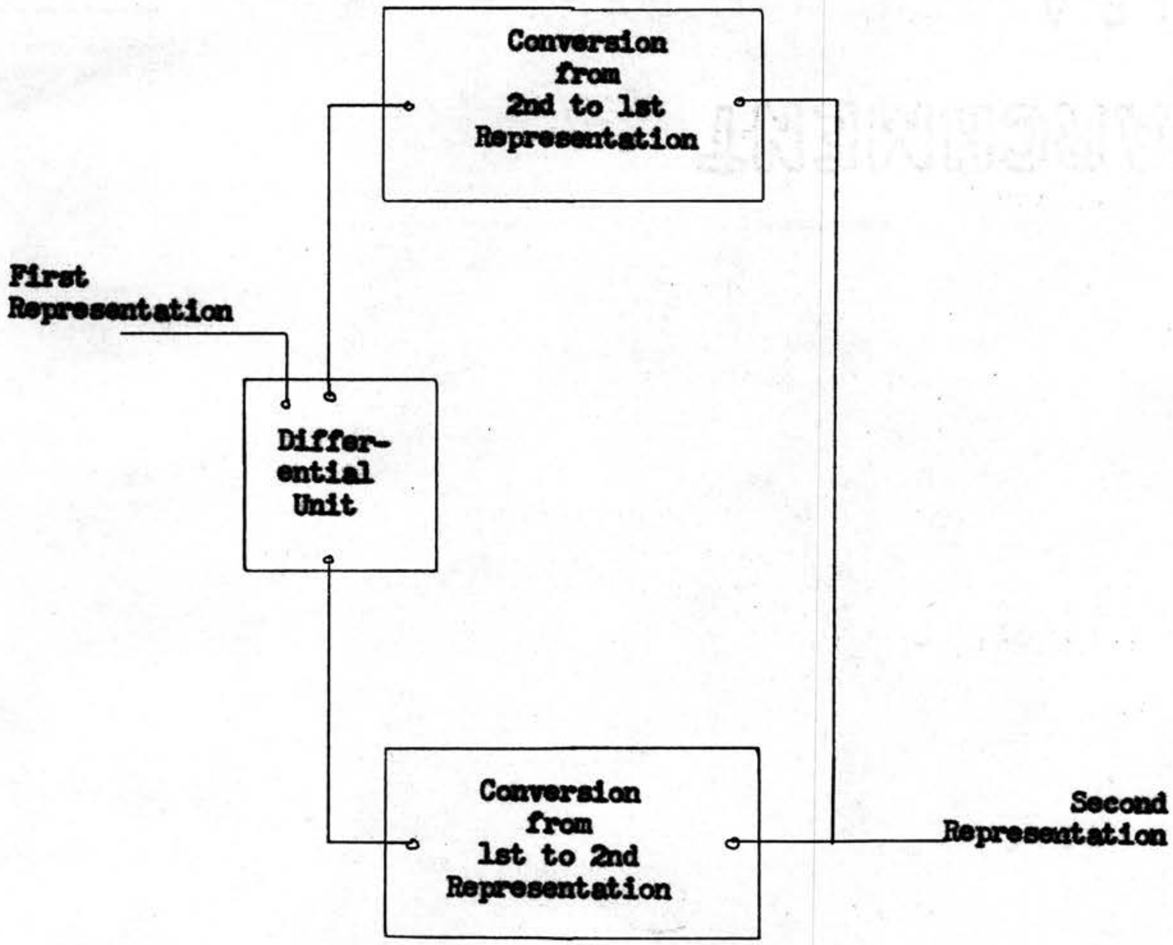

Figure 18.

Change of Representation

a particular product is to be taken. This product is

$$\left[f(t)\right] \left[f(t+d)\right]$$

and is a voltage function of time multiplied by a voltage function of time and a constant delay. These signals both may have positive and negative values. They are both continuously varying signals. They will have ratios of their amplitude which are quite large. From the devices shown in the body of this paper it can be noted that the very necessities in taking this product are the difficult achievements for most multipliers presently developed. There are some instances when the product can be very inaccurate, an example of an instance where a multiplier is needed but the accuracy of multiplication need not be great can be found in a technique used by some designers of radar equipment. In this technique the transmitted signal consists of two pulses spaced in time by an interval (t). The received signal reflected from a target then will also consist of two pulses spaced in time by the same interval (t). The received signal is delayed by means of a delay line and then multiplied by the received signal without delay. With this technique an improved signal to noise ratio is realized since theoretically the output of the multiplier is zero except when the first pulse in the returned echo signal which has been delayed coincides in time with the second echoed pulse which has witnessed no delay. It can be seen that it is necessary to have the output of the multiplier equal to zero when either of the input signals is equal to zero. This requirement is of primary importance when a multiplier is designed for this type of operation. It will also be noted that the linearity which determines the accuracy of the multiplier is not too important and may be as great as 3 per cent of full scale.

Multipliers which are all electronic having accuracies better than 1/2 of 1 per cent have been developed and are referred to in the body of this paper. These schemes are the frequency sharing multiplier and the pulsed attenuator multiplier. A careful study of the literature referred to will show the reader that these schemes are not designed for four quadrant operation. The frequency sharing scheme so boldly presented by Korn and Korn is a two quadrant multiplier in that the X_1 signal must be d.c. This is not pointed out in this literature. The scheme presented can be made to operate in four quadrant operation by selecting a variable gain device which is capable of four quadrant operation and cannot be expected to operate with the same accuracy as that quoted (0.2 per cent) in four quadrant operation since the greatest error can be expected by a device whose operation passes through zero. The pulsed attenuator system referred to has a limitation which may be severe in general computer service in that the ratio of the input voltages can never exceed about 10 to 1 if the boasted accuracy is maintained. Also, a push-pull system must be devised for the most accurate four quadrant operation.

If a two quadrant device is available and it is desired to have four quadrant operation, a system similar to that in figure 12 can be used. It must be remembered that the phase shift of the X_1 signal through the two quadrant multiplier must be equal to the phase shift of the device which passes X_1 and subjects it to a gain of E_B. If this is not true the out of phase voltage will appear in the output as error. All components used in these two channels must have phase shift which is independent of the level at which the device is operated. If a component has phase shift which is different at one level than at a previous level the X_1E_B terms cannot be totally eliminated from the output of the differential amplifier. It also is necessary to have these components as linear as possible, e.g., the gain of these two channels must be independent of the level at which they are operated. Both channels must be free from all other types of distortion and noise. If either of these channels has noise or distortion which does not exist in the other with the exact phase and amplitude, appreciable error can be expected. Experimental work has shown that extreme precautions must be taken if these errors are to be minimized.

Great care should be taken to select a variable gain device which has its output exactly equal to zero when either or both of the input variables is equal to zero. Experience has shown that many times it is not possible to realize this condition if the ratio of the two input variables is made large. Multiplier schemes sometimes function only when the inputs have a large ratio. Schemes of this nature should be avoided since one signal will nearly always be fed to the other channel.

Ordinarily a system which employs feedback will be the best all around multiplier. The general theory of feedback amplifiers readily explains why this is true since the gain of a system employing feedback can be made linear with respect to some control voltage. The important factor in a feedback multiplier is the selection of the variable gain device. This device should be one which is characteristically a four quadrant device such as a balanced ring modulator. With the balanced ring modulator one input can be balanced from the output and the other can be eliminated from the output by proper addition of a third signal whose frequency is high compared to either of the input signals. Ring modulators, if properly balanced, have output only when a modulating signal exists. This means that the ring modulator can easily meet the requirement of having zero output when one or both of the input variables is equal to zero. The balanced ring modulator must always consist of four elements which are sufficiently matched to exhibit zero output for one zero input. The difficulty in this system is now readily obvious, that is, these elements must be very accurately matched. Experimentation shows that it is sensible to match elements that have been aged to a point where no further changes in their characteristics are exhibited by further aging. After this aging process it sometimes becomes laborious to obtain four elements sufficiently alike.

If it becomes necessary to match devices then a system comparable to that in figure 7 may be a simpler approach. If the variable gain devices of figure 7 have the desired control characteristics when phase is considered, then it may result that this scheme can be developed with fewer tubes. It is known that this is true if two quadrant operation is desired. Further investigation would be necessary to ascertain the complexity of this type system when four quadrant operation is desired.

Sometimes it is necessary to have a solution for each input which has no relation to previous inputs. A computer of this type would be the airborne computer mentioned under the section entitled Multiplication Based on Ohm's Law. This type of computer has only one set of data put into the device and a meter reading is the product. It has been said that the accuracy of this type device is severely hampered by the ability of the human to read the meter dial. However, this is not true since the portion of the solution which appears on the dial does not necessarily have to represent the entire solution. In example, a device which approaches the solution in successive approximations can be made as accurate as the designer desires. This should eliminate the false idea that a digital computer is desired in this type operation compared to the analogue computer. To show the possible accuracy of this computer let us say that a meter reading of the output from the multiplier must be obtained accurately to five places. Next, assume that the human can be expected to read only two significant figures with the desired accuracy. In this case a voltage which represents the first three significant figures must be subtracted from the multiplier output before it is applied to the meter terminal. Then the voltage read by the meter is only the last two significant figures. By successively approximating the output voltage and subtracting this approximation from the output voltage extreme accuracies are possible.

The important factors governing the operation of a multiplying device are not remote from the factors governing the design of other types of electronic computers, servomechanisms and communications equipment. The success of the design will be determined primarily by the inventive ability applied if a knowledge of feedback systems and general communications circuitry is possessed.

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