# DELAY/LOSS ANALYSIS FOR VIRTUAL NETWORKS 

WITH BURSTY BANDWIDTH

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## I. INTRODUCTION

This decade is seeing a computing trend towards more and more distributed processing. Corporations are experiencing changing communication needs, requiring more bandwidth, more cost effective services, and more flexible subscription policies. Internetworking between distributed computing devices of business corporations is increasing as their Local Area Networks (LANs) now need connectivity across a larger geographic area. To meet these client requirements, new networking products and services are emerging. Geographically dispersed LANs now can be connected through different switching technologies. Some of these emerging data communication services are: Frame Relay, Switched Multimegabit Data Services (SMDS), and Asynchronous Transfer Mode (ATM).

The subject of this research relates to the network design problems in the environment mentioned above. The objective is a delay/loss analysis for virtual networks with bursty traffic and bursty bandwidth. Virtual networks can be briefly described as networks that can be shared by multiple users, but to each user it seems like a dedicated private network. A more detailed definition is offered in chapter II. In these types of networks, service providers
allow users to temporarily use extra bandwidth, greater than their guaranteed bandwidth. We call this extra bandwidth "bursty bandwidth". Bursty bandwidth is offered by network services such as Frame Relay, SMDS, and ATM. This research is focussed on Frame Relay, which is a data packet switching technology that relies on fast digital transmission links such as nearly error free fiber optics, but should also be applicable--with some modifications--to SMDS and ATM.

To help clarify the discussion to follow, some of the important parameters involved in this research are defined.

## A. Delay

When a network begins to slow down because of buffering, retransmissions, and/or any other time affecting phenomenon, the network begins to experience delay. Delay will cause response time to degrade, application time-outs and retransmissions, and may even cause users to loose data. The primary contributors to delay include: line speed (bandwidth), hardware and software interfaces, memory and buffers, processing frames (packets), and the load on every component across the transmission path. Decreasing the load or increasing the line bandwidth will decrease the delay. The delay is a very important performance measure for realtime communication services such as fast request-reply communication: e.g. data base queries, information retrieval requests, remote procedure calls, and urgent electronic-mail
messages.

## B. Blocking probability

Every network will encounter some kind of network congestion. Congestion happens when the available bandwidth is exceeded and additional information cannot be passed. Congestion can have three major effects on user traffic. Some network service providers delay the user traffic flow, by storing it in buffers until sufficient bandwidth is available to continue the transmission. Other services notify the user and simply block the transmission flow until the congestion clears or the user slows down transmission of data. Others just discard (drop) the information during congestion conditions, assuming that the user has employed some form of transmission protocols at the customer premise equipment (CPE) to stop the flow of information until congestion clears.

Because the completeness of information delivery is essential in some communication services such as file transfer applications and transmission of urgent messages in real time distributed systems, the blocking probability is very important to the user.

## C. Bursty Bandwidth

Burst bandwidth is short in duration and serves two main purposes. It relieves momentary congestion at the access point and it provides additional throughput to
interactive data applications such as those applications with high degree of delay sensitivity. However if a user sends out a large file and creates congestion, the transmission will be delayed or discarded depending on the policy used by the carrier.

## D. Bursty Traffic

Bursty traffic is generated by sources that can transmit bursts that contain more than one packet. A bursty source may generate packets at a high rate for a very short period of time. Immediately afterwards, such a source may become inactive, generating no packets. Severe network congestion may occur because of this dynamic behavior [3]. A Frame Relay service with bursty bandwidth can reduce the effect of input traffic burstiness, and is becoming a major player in the interconnection of LANs, which are known for their bursty traffic.

## I.1. Contribution of this Research

In this research two of the most important performance requirements are considered, delay and blocking probability. Decreasing the delay and blocking probability is a major responsibility for the design engineer. A good network design should maximize utilization (load) of resources while keeping the delay and/or blocking probability in line with the user expectation.

A queuing model is developed to calculate the delay and blocking probability in order to allow assessment of different strategies for different Frame Relay services. The model will allow a network designer to estimate in advance whether a particular Frame Relay service is likely to meet the communication requirements.

Many models have been developed for a bursty input traffic, but there is no model for bursty bandwidth. Dr. G. Scheets was confronted with this problem while he was developing a network design tool for Frame Relay called "Frelay" at Williams Telecommunications [2]. Because of the lack of a model for bursty bandwidth, he used the classical Kleinrock model [8], which is designed for fixed bandwidth, to calculate the delay. However, he noted that this model will lose some of its accuracy in case of intense bandwidth burstiness, and proposed that the development of a model to calculate the delay for networks with bursty bandwidth would be beneficial. The model developed in this research considers both bursty traffic input and bursty bandwidth.

## I. 2 Overview of this Paper

This work is organized as follows. Chapter II defines the virtual data network and how it compares to private line networks and switched networks. Chapter III discusses Frame Relay and provides a historical background of packet
switching technology and its evolution to Frame Relay and other recent fast packet switching technologies such as ATM and SMDS. Chapter IV presents the concept and subscription policies of Frame Relay services. Bursty bandwidth capabilities allow the user to use a greater bandwidth than the guaranteed one. But if many users try to use that extra bandwidth at the same time, they create congestion. Examples of congestion management and subscription policies are also provided in chapter IV. These examples are the basis for the development of the queuing model.

Chapter $V$ explains some of the basic models of queuing theory, then the specific queuing model used in this research is developed. A Frame Relay node includes an access point called a "port connection", and transmission lines emanating from this port connection called Permanent Virtual Circuits (PVCs). In this research, these nodes are modeled by two queues in tandem. One input queue modeling different information sources attached to the same port connection, and one output queue modeling the bursty bandwidth available for a PVC. The average waiting time is calculated for the subscription policy "delay not discard". The blocking probability is calculated for the policy "discard not delay". Examples are provided for both cases. The results are verified by a simulation model of a PVC with variable bandwidth. In chapter VI a real Frame Relay network
simulation model is developed, a trunk of N PVCs is simulated based on a given subscription policy. Chapter VII presents concluding remarks and the future work proposed to complete this research. And finally the Glossary defines common terms of telecommunication technologies, encountered throughout this paper.

## II. VIRTUAL DATA NETWORK

Many data network providers are building or have built large public data networks, and offer portions of these to customers as virtual private data networks. This section will describe the virtual network. To understand the difference between a virtual network and other types of networks, private line networks and switched networks are defined first.

## II. 1 Private (Leased) Line Networks

Private lines are the simplest form of dedicated point-to-point communications links. Private line bandwidths vary, but typically follow standard speed conventions of 9600 bps , 19.2 Kbps, 56 Kbps , and $1,544 \mathrm{Mbps}$. Users generally lease a private line from a public carrier when they want to guarantee the link between any two points of choice at any time. They do not want to share this link with anyone else. Private lines are leased based upon 24 hours a day utilization. Thus, a user who leases a private line must justify the cost by a high traffic volume.

## II. 2 Switched Networks

Switched networks can range from simple circuit
switching to advanced packet and cell switching and includes new technologies such as ATM. The main characteristics of switched networks include: addressing capability, multiple protocol and interface support, and one to many connectivity. Two examples of new switched service offerings are Frame Relay and SMDS. The switched networks allow users to select from a pool of multiple public service lines with fixed bandwidths, to flexible switched access networks where bandwidth is only allocated and used when needed (bandwidth on demand).

## II. 3 The Virtual Network

The virtual network is composed of virtual paths (in this paper the virtual paths will be also called fiber optic pipes, or trunks). A virtual path is a logical connection between the virtual path terminators, and is composed of a bundle of virtual circuits (called also virtual channels). A virtual circuit is a virtual connection established through the network from origination to destination, where packets, frames, or cells are routed over the same path for the duration of the call. These connections seem like dedicated paths to the users, but are actually network resources shared by all users. The virtual circuit is not a continuous physical transmission link, but it behaves like one, hence the term "virtual". This creates virtual leased lines.

The virtual path terminators can be switching systems, LAN gateways, or private network gateways [11] as shown in Figure II.1. Each virtual path is assigned a certain bandwidth that determines the number of virtual channels (circuits) it can support. The capacity of a virtual path can be deterministic or statistical. In the later case, statistical multiplexing gain leads to an optimum allocation of the bandwidth based on the traffic characteristics of each call.

The virtual network is a subset of the carriers larger network, but provides the image of a complete data network to the user. Virtual network services have the benefits of a private network and provide the economies and cost savings of a shared public data network.

With the bandwidth sharing capability of new technologies such as Frame Relay and cell switching, virtual networks are becoming more attractive as compared to private lines. The virtual network is often run and managed by software, making reconfiguration simple. Users may be provided the capability of altering their virtual private network to accommodate individual changing patterns, this level of control is not available through common switched services.


Figure II.1. A virtual data network and the interconnection devices [26].

## III. FRAME RELAY AND tHE EVOLUTION OF PACKET SWITCHED NETWORKS

This section provides an overview of packet switching technology. To understand the evolution of packet switching, first the historical background is summarized [26], then the packet switching technologies such as X.25, Frame Relay, fast packet, and ATM are described.

Packet switching was invented by Paul Baran and his research team in the early 1960s and published in 1964 as a secure method of transmitting military voice communications [26]. This was actually a project to enable the United States military communications system to survive after a nuclear attack. By segmenting an entire message into many pieces of data and wrapping routing and protocol information around these pieces of data, "packets" were created. These packets had a fixed maximum size assigned, 128 or 256 bytes long. Through the use of multiple packets, the entire message could be transmitted over multiple paths and diverse facilities and reassembled in its original order at the destination. In this manner of transmission, voice wire tapping was virtually impossible because only a portion of the entire transmission could be tapped, and even that portion would be incomplete and garbled.

The next step in packet-switching history was taken when the Advanced Research Project Agency of the United State Department of Defence (ARPA DOD) implemented packet switching to handle computer communications. The network was called ARPANET (ARPA-Network). ARPA was funding computer system projects at a number of university and industrial research centers, and in 1969 the first embryonic one-node network came to life, when two computers were connected at UCLA [8].

The first packet switching protocol was developed in the late 70's. It is called X .25 , and is still in use today. Packet switching continues to play an important role in the data communications of the 90's, and packet switching concepts continue to be the basis of the newer transport technologies needed to provide higher throughput, higher bandwidth, and the ability to take advantage of new fiber optic transmission facilities. The next step in packet technology transport was based on fiber optic facilities, and is called Frame Relay.

After discussing the historical background of packet switching technology, a discussion of different standards will follow.

## III.1. X. 25

X. 25 was the first protocol issued defining packet
switching. Access speed ranges up to 56 Kbps . Trunks between network nodes are limited to $56 \mathrm{Kbps} / 64 \mathrm{Kbps}$ (with the capability for fractional T1 speeds under proprietary implementations). One of the causes of the low transmission speed of X .25 is the error detection, correction, and flow control algorithms needed for the older analog transport networks of the late 1970 s.

Multiple users were allowed to share the data network facilities and bandwidth, rather than providing a dedicated bandwidth to each user as in circuit switching. The traffic on packet switched networks is bursty in nature and statistical multiplexing is used to maximize the on-demand bandwidth resources. The packet-switching is connectionless. The intelligence of the network nodes will reroute packets in the case of a link failure. In circuit switching the entire circuit would need to be switched leading to service interruption. While there is much more overhead associated with packet switching compared to circuit switching, this overhead guarantees error free delivery by the use of addressed packets.

## III.2. Frame Relay

Frame Relay is used primarily as an interface to a service, but defines also a protocol and a service. Frame Relay fills the technology gap between $X .25$ packet services
and SMDS and ATM broadband services. It is also in competition with emerging broadband standards. It is currently designed to handle speeds up to 2 Mbps , and a 45 Mbps access speed is under development. Frame Relay provides cost-effective bandwidth for bursty types of traffic (see Figure III.1). Frame Relay is suited for the interconnection of LANs and WANs, and with its statistical multiplexing capability lowers the cost of both CPE hardware and network service costs. It also provides dynamic bandwidth allocation (bandwidth-on-demand) which cannot be achieved through private-line or circuit-switched networks. The bandwidth is allocated only when it is needed. This bandwidth allocation method works best for the connection of networks such as LANs. LANs have bursty traffic and could use the entire channel during a given time, with little or no bandwidth requirement the rest of the time. In Frame Relay networks multiple logical circuits are combined within a single physical circuit, thus utilizing network bandwidth effectively, better than Time Division Multiplexing (TDM). In a Time Division Multiplexer, channels are allocated to a specific user and bandwidth is wasted during idle times, Figure III. 2 provides a comparison of Frame Relay to conventional T1 multiplexing.

Frame Relay does not provide the error control as does X.25. It relies on higher-level user protocols such as


Figure III. 1 Frame Relay compared to competing services for bursty traffic [26].

Transmission Control Protocol/Internet Protocol (TCP/IP), to provide the retransmission and guarantee of delivery, rather than duplicating it at the network layer. It relies on virtually error-free fiber optic transmission media. The result is decreased overhead and increased performance, since more information can be transmitted over the physical channel. Frames can vary in size. The larger the frame size the greater the throughput. Throughput being the number of data bits successfully transferred between two nodes in a fixed time interval.

Some of the benefits of Frame Relay are:

* true international network interface standard
* multiple users per physical access line
* reduces network access hardware cost
* wide industry support
* cost effective for transport of bursty data traffic
* high speed of access due to low packet overhead
* fills gaps between X. 25 and broadband services
* meets throughput requirements of enhanced computing power applications
* reduces delays (better than X.25)
* improves bandwidth utilization.

Frame Relay is offered on both private and public networks. The architecture specifies that Frame Relay utilizes Permanent Virtual Circuits (PVCs) or Switched

TDM MULTIPLEXER vs. FRAME RELAY DEVICE


Figure III.2. Frame Relay compared to Time Division Multiplexing (TDM) [26].

Virtual Calls (SVCs), which are defined below.

## III.2.1 Permanent Virtual Circuits

PVCs are virtual circuits permanently established between a source and a destination node. To the end user this is similar to having a private or leased line dedicated all the time between two specific nodes or users. The PVCs guarantee a connection between two points when demanded by the users. The user always sees the virtual circuit as a dedicated circuit for his or her use only, whereas the network provides the same circuit as a shared resource to multiple users upon demand (see Figure III.3). Multiple sessions, up to 1000 PVCs, can take place over a single physical circuit called a virtual path. PVCs require a dedicated access line, a pre-established virtual path between sites, and a pre-established reserved bandwidth which is called the Committed Information Rate (CIR).

## III.2.2 Switched Virtual Calls (svc)

So far, none of Frame Relay providers offers SVC based services. SVCs are connected and disconnected after data has been sent between the source and the destination node. Therefore, one source can connect to many destinations at different times, as opposed to always being connected to one destination. This is similar to the method of making a phone

## what the user qees - dedicated circuits



WHAT THE PACKET SWITCH NETWORK PFOVIDES


Figure III.3. User's perception of the packet switch network with virtual circuits[26].
call. With SVCs, Frame Relay users would be able to build more flexible meshed networks with connection possible among all nodes. An SVC-based network could be dynamically configured, and bandwidth would be allocated on an as-needed basis, eliminating leased-line cosțs. It also allows dial-up connectivity between any sites without pre-established virtual paths, uses switched access, and lets the CIR and burst size be determined at the start of each call. But while PVC-based Frame Relay was easy to implement as an overlay to existing leased-line networks, SVCs represent a much more significant development effort.

## III.3. Fast Packet

Fast packet is a term which stands for a data transmission technique. It is not a defined standard, protocol or service. Fast packet is a backbone technology as opposed to Frame Relay access technology. Fast packet combines attributes of both circuit switching and packet switching. It resembles a circuit switch for constantbandwidth traffic such as voice and video, and a packet switch for bursty data traffic such as LAN and WAN traffic, dynamically increasing the bandwidth for high bandwidth requirements and decreasing it for low bandwidth requirements. Fast packet uses fixed cell size (SMDS and ATM), or variable packet sizes (Frame Relay) to transmit
data. Fast packet does not do error detection and correction in the intermediate nodes, this reduces the overhead and speeds up packet transfer. Fast packet technologies use advanced fiber optic transport media, such as T3 and SONET.
III.4. Asynchronous Transfer Mode (ATM)

Asynchronous transfer mode is a form of fast packet switching. It uses a fixed cell size and fast packet multiplexing (FPM). Fast packet multiplexing is a general term for providing the capabilities of fast packet switching through multiplexing various types of traffic onto the transmission medium. ATM packetizes voice, data, and video and then statistically multiplexes the packets onto the same high-speed data channel. ATM provides two types of connections: virtual channels which provide logical packet connections between two users, and virtual paths which define source to destination routes for users.

## IV. CONCEPT AND SUBSCRIPTION POLICIES FOR FRAME RELAY

Customers can access the network through the point of presence (POP) of the carrier. The POP includes a port connection and the associated Permanent Virtual Circuits (PVC) that provide the connectivity between nodes.

## IV.1. Port connection

Data frames may be transmitted into the Frame Relay service from the port connection (see Figure V.1). The access device to a port connection is typically a router. The maximum amount of data that can be transmitted through the port connection on the PVCs assigned to it is defined by the port connection speed (or access rate). The port connection speed can be assigned from $56 / 64$ Kbps up to 2 Mbps speed, including T1 fractions. A port connection can have multiple PVCs assigned to it.

## IV.2. Permanent Virtual Circuit (PVC)

PVCs are logical dedicated circuits between user port connections. PVCs are assigned in simplex fashion (transmitonly direction) and they are always associated with a single port connection. Since data applications require duplex transmissions, two PVCs are required between any two port


Figure IV.1. Typical Frame Relay configuration showing PVCs associated with router 1.
connections, one in each direction. A PVC is assigned to the originating port connection, and a destination port connection must be specified. Data transmitted on a particular PVC is statistically multiplexed onto the network trunk (or fiber pipe) along with data from other users. Data frames from multiple PVCs of different users are queued into a common buffer for transmission over a particular trunk of the carrier network. Trunk bandwidth is allocated to data in the trunk buffer on a first-come, first-served basis (FCFS). There is no active management of the data flow on individual PVCs, and the actual information rate across a particular PVC will vary considerably, from the access rate in the absence of contention on the trunk to a very low rate in the case of heavy contention. Because of the lack of an active management of the data flow on individual PVCs, the service providers have introduced the concept of a Committed Information Rate (CIR). To define a PVC a CIR must also be specified.

## IV.3. Committed Information Rate (CIR)

The committed information rate can sometimes have different meanings, but most of the time it is defined to be the bandwidth guaranteed to the customer at any given time. The user can transmit data up to the CIR within a given PVC between predefined end-points at one time without any chance
of blockage or loss of data. Some providers allow the user to exceed the CIR, this will be discussed shortly.

The CIR is assigned to each PVC by the provider based on the user's (the subscriber's) expected average traffic volume to the destination port connection. The CIR cannot exceed the port connection speed of either port it is assigned between. A PVC can exceed its CIR, this process is called the burstiness capability and is discussed below.

## IV.4. Burstiness Capability

As it was mentioned before some carriers allow the user to exceed the CIR when capacity is available on the network and on the port connection. A PVC can momentarily increase transmission speed to its port connection speed.

This research is focussed on this burstiness capability of the virtual network. The burstiness capability is based on statistical approach in that not all the customers will need bandwidth at all times. There is statistically a good chance that a burst exceeding the CIR will pass without data loss. However, if an application sends out a large file and excess capacity is not available at that instant, the data will be transmitted at the committed information rate, and excess data will be delayed or discarded depending on the policy adopted by the carrier. This excess data can also cause congestion.

## IV.5. Congestion Management

There are two types of congestion on a public network. The first is momentary congestion. It occurs when several packets with the same destination arrive at a switch node at the same time. This congestion will be cleared faster if a momentary burst is available, else some packets will be delayed and will wait in a FCFS queue until the packets that arrived first are transmitted.

The second type of congestion is caused by two or more applications trying to transmit large amounts of data such that they exceed the reserved bandwidth, i.e. the input data rate becomes greater than the exit data rate.

Congestion is managed differently by different providers. Which policy the network adopts is very important for the user. Some providers use the "discard policy", in which all the packets in excess are discarded. Some use the "delay not discard policy", in which case the packets in excess are delayed and sent later. Some notify the end user of congestion by the use of FECN and BECN bits (Forward/Backward Explicit Congestion Indicator).

How the following questions are solved is important for both users and providers. When does congestion occur? Does it occur when the queues on the provider trunk are $50 \%$ full? 75\% full? What is the threshold for discarding or delaying packets? If the threshold is too low, then the throughput
could be negatively impacted when no explicit congestion is occurring on the provider's network trunk. If providers wait until explicit congestion begins occurring, many packets could be dropped before the applications begin to slow their output. If FECN/BECN alarms are used, when will they be actually sent? Will the user obey them? Each user will most likely attempt to pay for the lowest CIR possible and use as much as possible the burst bandwidth.

## IV.6. Subscription Policies

Some carriers allow the user to exceed the CIR but do not guarantee delivery of data. Others simply discard data that exceed the CIR. Some combine both policies, they allow the user to use extra bandwidth while being fair to the users who respect the reserved bandwidth; they mark the data in excess as "DE" (discard eligible), and in case of congestion, the packets marked as "DE" will be discarded until the level of congestion is brought down. So the users who intend to use extra bandwidth at very low cost (or bandwidth for nothing) should prioritize their data. Some providers use the strategy "delay not discard", the packets in excess are allowed on the network, but in case of congestion they will be momentarily delayed. On some Frame Relay networks all packets are marked as discard eligible, and all data is sent at burst rate; it is not possible for a
user to request that some critical data be sent at the committed information rate, so that no packets would be marked as discard eligible.

An example of a mechanism to implement bandwidth sharing within Frame Relay service is a credit manager, a software process which allocates credits to each PVC to transmit a certain amount of data on the network trunk [28]. Data bytes enter the network at the access rate and are stored in a buffer dedicated to the PVC addressed in the data frame, then it is taken out of that buffer and put into the appropriate network trunk buffer at a rate controlled by the availability of credits. Credits are given to the PVC at a rate proportional to the CIR of that PVC, and according to the availability of bandwidth on the network trunks which depends on the intensity of the traffic of other users. A credit manager controls the rate of a PVC, so that shared trunk buffers will not become congested. It is a congestion avoidance, not just a congestion control mechanism. The major advantages of the credit manager approach are lower delay across the network and fairness among subscribers. With a credit manger implementation, a burst of data for a particular PVC may still enter the network at the access rate, and this data will be buffered for transmission onto the network trunk, just as in the case without a credit manager. What is different is that the data is stored in an
individual buffer for that PVC and the flow of data into the shared network trunk buffer is strictly regulated. If congestion occurs it will affect only the PVC with a sustained rate higher than its CIR. Other subscribers, and even users of others PVCs from the same subscriber will be unaffected.

It is very important to understand that the committed information rate may not always be an accurate measure for the actual transmission rate. While some Frame Relay service implementations may offer predictability close to that available from a leased line (which has a fixed transmission rate), others, especially those which allow data to momentarily burst and transmit a rates greater than the CIR, may have difficulty predicting the transmission rate, reliability, and delay through the network.

The major contribution of this research is to develop a tool for the evaluation process for bursty Frame Relay services which are quite different from the evaluation process for fixed bandwidth leased lines. Specifically a queuing model is developed for evaluating the average delay of a packet transmitted through a Frame Relay network, and the blocking probability for the "discard not delay" policy. This problem of bursty bandwidth has not been pursued before. The model will allow a network planner to predict the behavior of a Frame Relay service based on the following
evaluations of the network.

* Is data in excess of the CIR marked by the network as discard eligible? While most LAN protocols will recover from lost frames, the recovery and retransmission process greatly reduces efficiency, and performance characteristics such as delay and blocking probability can be greatly affected. In case of a "discard not delay policy", the blocking probability should be evaluated, and care should be taken to assure that applications transmit only in limited burst sizes, burst sizes that keep the blocking probability within the boundaries tolerated by a particular application.
* Does a credit manager function exist to provide fair allocation of trunk bandwidth and to prevent congestion in shared trunk buffers? Without some fairness mechanisms, a few users with low CIRs but high port connection speed can fill the network trunk buffers. This will cause significant network delay for other users and reduces their transmission information rates even if their own CIRs are high.
* And finally and more important for our model, does a mechanism exist to detect spare capacity on the network trunk? What are the probabilities of having a specific burst bandwidth? This bandwidth could be proportional to the CIR and measured in percentage of the CIR $(200 \%, 300 \%, \ldots$.$) .$ What are the chances of having a transmission information rate less that the CIR?

Based on the subscription policies mentioned above, and using our model, examples of numerical results are provided for the following cases:
A. Delay evaluation for cases where
a) The Committed Information Rate (CIR) is guaranteed, extra bandwidth is allowed, the packets in excess are marked delay eligible, and will be delayed in case of congestion.
b) The CIR is not guaranteed, extra bandwidth is available.
B. Blocking probability for the case where the CIR is guaranteed, extra bandwidth is allowed,the packets in excess are marked discard eligible and will be discarded in case of congestion.

## V. QUEUING MODELS

This chapter presents some basics of queuing theory and describes the queuing models developed in this research for modeling Frame Relay services with bursty bandwidth. Queuing is the method of allowing data to be stored in buffers until there is sufficient bandwidth to transmit it to its destination. Queuing in Frame Relay is a function of the network device buffers which process the frames.

The use of the queuing theory in modeling and analysis of computer communications has been pursued by a great number of people. Applications of queuing theory in computer communication and networking are found in Kleinrok [9], and Kobayashi [13] and in most of the references of this research.

A queuing system can be described in terms of customers arriving for service. A customer will be served immediately if the server is idle, or will wait in line (queue) if the server is busy serving another customer. The term "customer" is used in a general sense and does not imply necessarily a human. For example a customer could be a machine to be repaired, an airplane waiting in line to takeoff, or a packet waiting in a buffer to be transmitted on a computer network. In this research the customers will
be packets, which will arrive either singly or in bursts. Figure V.1 shows a single server queuing system.


Figure V.1. A single server queuing system.

A standard notation called Kendall notation (after its originator) is used to describe queuing systems with a single queue and one or more parallel servers. This notation has the following format:
$A D / S D / s / K / P$, where

- AD represents the inter-arrival time distribution
- SD represents the service time distribution
- s is the number of servers
- K is the maximum number of customers allowed in the system. In this research it will represent the capacity of the buffers, if K is infinite it will be omitted from the notation.
- $P$ is the number of customers available at the source. If $P$ is infinite, it is omitted from the notation.

In this research the number of customers (frames) is considered infinite.

Among the symbols commonly used to describe distributions are

- D constant (deterministic) inter-arrival or service time
- M exponential distribution
- $E_{k} k$-Erlang distribution
- $H_{k}$ k-stage hyperexponential distribution
- G general (arbitrary) distribution

The simplest and most interesting queuing model is the M/M/1 queue, where both Ms stand for exponential (or Markovian) distribution, and 1 for the number of servers. For M/M/1 the arrival rate and the service rate follow a Poisson distribution or, equivalently, the interarrival times and the service times obey the exponential distribution. It is shown in [7] that assuming the number of occurrences in some time interval to be a Poisson random variable is equivalent to assuming the time between successive occurrences to be an exponentially distributed variable. This means that the intrarrival times between events (packets) $\left\{\tau_{\mathrm{i}}: 1 \leq \mathrm{i}<\infty\right\}$ and the service times $\left\{S_{i}: 1 \leq i<\infty\right\}$ satisfy the following conditions. 1) The $\left\{\tau_{\mathrm{i}}\right\}$ are independently and identically distributed (iid)
random variables with $F(t)=\operatorname{Pr}\left\{\tau_{i} \leq t\right\}=1-e^{-\lambda t}$ $(0 \leq t<\infty), \lambda$ is the average arrival rate. 2) The service times $\left\{S_{i}\right\}$ are iid with $G(t)=\operatorname{Pr}\left\{S_{i} \leq t\right\}=1-e-\mu t$ $(0 \leq t<\infty), \mu$ is the average service rate. 3) The service and arrival processes are independent. To define a system, a queue discipline must be specified, the simplest discipline is the FIFO "first-in-first-out". The state of the system at time $t$ is the random process $X=\left\{X_{t}: 0 \leq t<\infty\right\}$ where $X_{t}$ is the number of customers (packets) either in service or waiting for service. Because of the memoryless property of the exponential distribution, this description of the state of the system defines a Markov process.

The definition and classification of the stochastic processes and the Markov chains and their use in queuing theory are widely discussed in the literature [7-9]. Following is some of the definitions of the processes and models used in this research.
a) Markov Process. A process is said to be Markovian if given the "present" condition of the process, the "future" is independent of the "past". The process is thus "memoryless". Mathematically, a discrete-parameter stochastic process $\{X(t), t=0,1,2, \ldots\}$ or a continuous stochastic process $\{X(t), t>0\}$ is said to be a Markov process if, for any set of $n$ time points $t_{1}<t_{2}<\ldots<t_{n}$ in the
index set or time range of the process, the conditional distribution of $X\left(t_{n}\right)$, given the values of $X\left(t_{1}\right), X\left(t_{2}\right)$, $X\left(t_{3}\right), \ldots, X\left(t_{n-1}\right)$, depends only on $X\left(t_{n-1}\right)$, the immediately preceding value; more precisely, for any real numbers $x_{1}, x_{2}, \ldots, x_{n}$,

$$
\begin{aligned}
\operatorname{Pr}\left\{X\left(t_{n}\right)\right. & \left.\leq x_{n} \mid X\left(t_{1}\right)=x_{1}, \ldots, X\left(t_{n-1}\right)=x_{n-1}\right\} \\
& =\operatorname{Pr}\left\{X\left(t_{n}\right) \leq x_{n} \mid X\left(t_{n-1}\right)=x_{n-1}\right\} .
\end{aligned}
$$

b) Imbedded Markov Chains. The Markov property imposes strong restrictions on the kind of processes that could be considered. The discrete-time Markov chain has the property that at every unit interval on the time axis, the process is required to make a transition from the current state to some other state (possibly back to the same state). The transition probabilities (the probability that the system goes from one state to the next state) were completely arbitrary; however, the requirement that a transition be made at every unit time leads to the fact that the time spent in a state is geometrically distributed [8]. To relax the restrictions, namely, to permit an arbitrary distribution of time the process may remain in a state, the times between state transitions are also allowed to obey an arbitrary probability distribution. At the instants of state transitions, the process behaves just like an ordinary Markov chain, and the process is called an imbedded Markov
process.
For the continuous-time Markov process the transitions are permitted at any instant in time. However, as opposed to a Markov process which requires an exponentially distributed time in state, the imbedded Markov process permits an arbitrarily distributed time in state. Now, truly continuous-parameter queuing processes could be studied as imbedded discrete-parameter Markov chain queuing processes. This affords much greater generality.
c) The M/G/1 Queue. The M/G/1 queue is a single-server system with Poisson arrivals and arbitrary service-time distribution (or general service time distribution). Because the M/G/1 queue is driven by a non-Markovian stochastic process, the imbedded Markov chain method was employed for the analysis of this system. An extremely well-known formula for the average number of customers in an M/G/1 system is the Pollaczek-Khinchin formula [7-9]. This average depends only upon the mean and the variance of the service time distribution, this result will be used later in this research.

## V.1. Modeling a Frame Relay Network Node

Recall that a Frame Relay node includes a Port Connection (access switch) and a PVC assigned to that port. A port connection can have many PVCs assigned to it. Figure
V. 2 shows a port connection with four input arrivals and four output PVCs. An input can be an external input from a T1 or a fractional T1 [1].


Figure V.2. A Frame Relay port connection

A data frame will be first processed at the port connection, then it will be dispatched to the destination PVC where it will be transmitted to the destination node. Therefore, the data frame will contend first for the port connection with other frames from different logical input channels. This will be modeled via an input queue. The data frame may encounter its first delay at the input queue. Then the data will be taken to the output port buffer which represents the
trunk buffer designated to the destination PVC. For each PVC there is a buffer on the provider trunk where the data will wait and contend with other data from other PVCs for the


Figure V.3. Input output queuing in packet-switching.
available bandwidth [28]. The data frame will encounter its second delay at the output queue.

Figure V. 3 shows a classical approach for input output queuing in packet switching [5] [12] [14] [25]. A buffer is provided at each input port and each output port of the port connection. If the output server (PVC) is not fast enough,
this approach can have a performance drawback, which is referred to as head-of-line blocking (HOL). A packet waiting at the head of an input queue for access to a specific output queue may block other packets waiting in the same queue for different output queues which could be idle. In Figure V. 4 the packets $j$ are waiting to be transmitted to the destination output port $j$ which is busy, and are blocking the line for packets $v$ waiting for the output line v which is idle.


Figure V.4. Head-of-line blocking (HOL).

An alternative is the multiple queuing approach [30], which uses both input and output queuing and always permits
routing for packets whose destination output queues are empty. In the multiple queuing approach, each input queue is split into $M$ separate queues, one for each possible output link (PVC) (see Figure V.5), this solves the head-of-line (HOL) problem. The packet whose destination output port is not empty is stored in the input queue reserved for that output port, and it will not block the line for other packets. This method is more complex and expensive but provides better performance [5][30].


Figure V. 5 Multiple queuing in packet switching.

We assume that the head-of-line problem is solved by using the multiple queuing method and that there is no head-of-line delay. Each Frame Relay node is modelled as an $N * M$ packet switch with $N$ inputs and $M$ outputs.

Next, to simplify the analysis of the model, we assume a homogeneous system where the traffic intensity is balanced and is the same to each input, the traffic is also uniformly distributed to each output. The balanced arrival process leads to a worst case system in terms of delay performance, since we have sustained arrival rates [34]. But the balanced traffic will permit us to represent $N$ inputs with arrival rate $\lambda / N$ by one input with arrival rate $\lambda$, in bursts/unit time, and $M$ outputs with service rate $\mu / M$ by an output with service rate $\mu$, usually in packets/unit time. Now the system can be represented by two queues in tandem (see Figures V. 6 and V.7). The input buffer is considered infinite and is used for the contention for switch processor at the input of the switch. The output buffer is used for the contention for the output link (PVC) at the output queue. The output buffer is considered infinite for the case of "delay not discard" policy, and finite for the case "discard not delay" policy [1].

Two queues in tandem means that the output process of the input queue feeds into the output queue. Therefore the interarrival time of the output queue will depend on the
interdeparture time of the input queue. This dependence is a well known obstacle in the modelling of packet network systems as queuing networks. But it was found by Burke [33] that a queue with Poisson arrival and exponential service time generates a Poisson process for departure. This result is referred to as Burke's theorem. Burke's theorem means that two queues in tandem can by studied independently, as long as the arrival process and the service time of the first queue are Markovians. In fact Burke's theorem tells us that we may connect many multiple server nodes (each server with exponential service time) together and still preserve a node-by-node decomposition. Kleinrok [8] studied this problem through simulation and came to the conclusion that in many packet networks, the dependence is diluted due to the mixing of traffic streams from different directions. Burke's results were restricted to a Poisson arrivals and exponential service time under the first-come first-served (FCFS) policy. This restriction was enlarged later to include a general service time distribution under different processing policies [9]. In this research, while we have the hurdles of both bursty traffic and bursty bandwidth, the input and output queues were modeled to keep a Markovian character. This allows the use of Burke's theorem and the independence assumption, and will permit, in the following sections, the delay at the Frame Relay node to be calculated


Figure V.6. Balanced traffic input.


Figure V.7. Two queues in tandem.
as the sum of the delays at the input queue and the output queue.

## V.2. Input Queue

The switch or port connection is represented by an input queue, with one server with exponentially distributed service time and bulk (or batch) arrivals.

The Poisson process as in M/M/1 has been widely used for many years, and because of its simplicity and usefulness for finding closed form analytical results, is still used with some specific systems. But traffic characteristics have changed and the Poisson process may no longer be suitable for describing network traffic.

Most of the research done on bursty traffic was targeted at integrated networks, such as ATM [3] [4]. Integrated networks, are expected to support bursty multimedia information such as voice, video, and data. It is often difficult for a network to acquire complete statistics for input traffic. Some of traffic measurements and their implications are outlined in [32]. In the absence of a complete assessment of the traffic characteristics, network designers must make a performance estimate based on some approximate arrival process, such as geometric processes, some processes derived from Poisson process such as Markov modulated Poisson process [35], or interrupted Poisson
process (IPP) [3]. Often the arrival processes are analyzed as a superposition of different processes [3] [4] [31].

For this work the traffic is assumed to be generated by mutually independent sources. To cover the burstiness of the traffic we model the arrivals by batch arrivals. The batch arrivals are assumed to form a Poisson process, with the batch size (the number of packets) a random variable which may take on any positive value with probability $c_{n}$. This type of queue is called $M^{[x]} / M / 1$ [7]. This is a multiple or compound Poisson process, which is composed from the overlap of a set of Poisson processes with different rates ( $\lambda_{x}, \quad x=1,2, \ldots$ ). $M^{[x]} / M / 1$ is considered Markovian, a characteristic that we wanted to use for the queues in tandem, so we can apply the independence assumption. Batch arrivals were used in [4] in superposition with other processes, in [10] batch arrivals were used, but the time between arrivals of successive batches was taken to be geometrically distributed.

## V.2.1. The Input Queue Delay

This section provides the derivation from [7] of the generating function $P(z)$ of $p_{n}$, the probability of $n$ packets in the system, as a function of $C(z)$ the generating function of $c_{n}$, the probability of having $n$ packets in one burst.

Then $P(z)$ is used to calculate $L$ the average number of packets in the system at any time. The delay at the input queue is then calculated using Little's formula, $L=A W$. Little's formula states that the average number of packets in a queuing system is equal to the average arrival rate $A$ of packets to that system, times the average time $W$ spent by a packet in that system.

The Chapman-Kolmogorof equations can be derived for the $M^{[x]} / M / 1$ queue [7], and the following equations are obtained

$$
\begin{aligned}
& \frac{d p_{n}(t)}{d t}=-(\lambda+\mu) p_{n}(t)+\mu p_{n+1}+\lambda \sum_{k=1}^{n} p_{n-k}(t) c_{k} \quad(n \geq 1) \\
& \frac{d p_{0}(t)}{d t}=-\lambda p_{0}(t)+\mu p_{1}(t)
\end{aligned}
$$

The last term in the equation for $d_{n} / d t$ comes from the fact that a total of $n$ packets in the system at time $t+\Delta t$ can arise from $n-k$ at time $t$ with a burst of size $k$ arriving in the subsequent interval of length $\Delta t$.

If the steady state is assumed, the $M^{[x]} / M / 1$ stationary equations are obtained:

$$
\begin{align*}
& 0=-(\lambda+\mu) p_{n}+\mu p_{n+1}+\lambda \sum_{k=1}^{n} p_{n-k} c_{k} \quad(n \geq 1)  \tag{V.1}\\
& 0=-\lambda p_{0}+\mu p_{1}
\end{align*}
$$

To solve the system of equations given by (V.1), a generating-function approach is used:

$$
P(z)=\sum_{n=0}^{\infty} p_{n} Z^{n} \quad(|z| \leq 1)
$$

and

$$
\begin{aligned}
C(z) & =\sum_{n=0}^{\infty} c_{n} z^{n} \quad(|z| \leq 1) \\
& =\sum_{n=1}^{\infty} c_{n} z^{n}
\end{aligned}
$$

will be the generating function of the steady state-state probabilities $\left\{p_{n}\right\}$ and the burst size distribution $\left\{C_{n}\right\}$, respectively. $c_{0}$ is omitted from $C(z)$ because the probability of having a burst with 0 packets is equal to 0 , a burst will always have at least one packet.

If each equation of (V.1) is then multiplied by the appropriate $z^{n}$ to find the generating function

$$
\begin{aligned}
& 0=-(\lambda+\mu) \sum_{n=1}^{\infty} p_{n} z^{n}+\mu \sum_{n=1}^{\infty} p_{n+1} z^{n}+\lambda \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} p_{n-k} c_{k} z \\
& 0=-\lambda p_{0}+\mu p_{1},
\end{aligned}
$$

and the two equations are summed

$$
\begin{aligned}
& 0=-\lambda\left(\sum_{n=1}^{\infty} p_{n} z^{n}+p_{0}\right)+\mu \sum_{n=1}^{\infty} p_{n} z^{n} \\
& +\mu\left(\sum_{n=1}^{\infty} p_{n+1} z^{n}+p_{1}\right)+\lambda \sum_{n=1}^{\infty} \sum_{k=1}^{n} p_{n-k} c_{k} z^{n} \\
& 0=-\lambda \sum_{n=0}^{\infty} p_{n} z^{n}+\mu \sum_{n=1}^{\infty} p_{n} z^{n} \\
& +\frac{\mu}{z}\left(\sum_{n=1}^{\infty} p_{n+1} z^{n+1}+p_{1} z\right)+\lambda \sum_{n=1}^{\infty} \sum_{k=1}^{n} p_{n-k} c_{k} z^{n} \\
& 0=-\lambda \sum_{n=0}^{\infty} p_{n} z^{n}+\mu \sum_{n=1}^{\infty} p_{n} z^{n} \\
& +\frac{\mu}{z} \sum_{n=0}^{\infty} p_{n+1} z^{n+1}+\lambda \sum_{n=1}^{\infty} \sum_{k=1}^{n} p_{n-k} c_{k} z^{n} .
\end{aligned}
$$

Finally

$$
\begin{align*}
0= & -\lambda \sum_{n=0}^{\infty} p_{n} z^{N}+\mu \sum_{n=1}^{\infty} p_{n} z^{n}+\frac{\mu}{z} \sum_{n=1}^{\infty} p_{n} z^{n}  \tag{V.2}\\
& +\lambda \sum_{n=1}^{\infty} \sum_{k=1}^{n} p_{n-k} c_{k} z^{n}
\end{align*}
$$

We observe that

$$
\sum_{k=1}^{n} p_{n-k} c_{k}
$$

is the probability function for the sum of the steady-state system size and batch size, since this is merely a convolution formula for discrete random variable. It can be shown that the generating function of this sum is the product of the respective generating functions (a basic
property of all generating functions), namely,

$$
\begin{aligned}
\sum_{n=1}^{\infty} \sum_{k=1}^{n} p_{n-k} c_{k} z^{n} & =\sum_{k=1}^{\infty} c_{k} z^{k} \sum_{n=k}^{\infty} p_{n-k} z^{n-k} \\
& =C(z) P(z)
\end{aligned}
$$

Hence (V.2) may be written as

$$
\begin{aligned}
0 & =-\lambda P(z)-\mu\left[P(z)-p_{0}\right]+\frac{\mu}{z}\left[P(z)-p_{0}\right]+\lambda C(z) P(z) \\
& =P(z)[-\lambda z-\mu z+\mu \lambda z C(z)]+\mu z p_{0}-\mu p_{0},
\end{aligned}
$$

and thus

$$
\begin{equation*}
P(z)=\frac{\mu p_{0}(1-z)}{\mu(1-z)-\lambda z[1-C(z)]} \quad(|z| \leq 1) \tag{V.3}
\end{equation*}
$$

To calculate $P(z)$ we need to find $p_{0}$ and $C(z)$. Using the condition $P(1)=1$, we obtain $p_{0}$

$$
1=\lim _{z \rightarrow 1} P(z)=\lim _{z \rightarrow 1}\left[\frac{\mu p_{0}(1-z)}{\mu(1-z)-\lambda z[1-C(z)]}\right]
$$

But since the numerator and denominator are both 0 , we use L'Hôpital's rule and find

$$
\lim _{z \rightarrow 1} P(z)=\lim _{z \rightarrow 1}\left[\frac{-\mu p_{0}}{-\mu-\lambda+\lambda C(z)+\lambda z d C(z) / d z}\right]
$$

Since

$$
\frac{d C(z)}{d z}=\sum_{n=1}^{\infty} n C_{n} z^{n-1}
$$

when evaluated at $\mathrm{Z}=1$

$$
\frac{d C(1)}{d z}=\sum_{n=1}^{\infty} n C_{n}
$$

is the mean burst size, we can write

$$
\lim _{z \rightarrow 1}(z)=\frac{-\mu p_{0}}{-\mu+\lambda E[X=\text { burstsize }]}
$$

Hence, $p_{0}$, the probability there are no bursts in the system, may be written as

$$
\begin{align*}
p_{0} & =1-\frac{\lambda E[X]}{\mu}  \tag{V.4}\\
& =1-\rho \quad(\rho \equiv \lambda E[X] / \mu) .
\end{align*}
$$

where $\rho$ is the system load.
Now, all we need to find the generating function of the steady-state probabilities $P(z)$ is the generating function $C(z)$ of the distribution of the batch size, and then use (V.3).

We model the batch size by a geometric distribution. The geometric distribution is widely used to represent the
number of packets in a burst when there is a lack of more precision [8] [34] [36]. As an example, in [15] the geometric distribution was used to model the number of cells in a burst of an ATM switching network. We model the number of packets in a burst as follows, the probability that a burst contains $n$ packets is equal to $\alpha^{n-1}(1-\alpha),(n \geq 1)$. Each arriving burst comes to an end with probability (1- $-\alpha$ or submits another packet with probability $\alpha$, each burst contains at least one packet,

$$
\begin{aligned}
C(z) & =(1-\alpha) \sum_{n=1}^{\infty} \alpha^{n-1} z^{n} \quad(|z| \leq 1) \\
& =\frac{z(1-\alpha)}{1-\alpha z} \quad(|\alpha z|<1)
\end{aligned}
$$

Thus from equation (V.3), we find that

$$
P(z)=\frac{\mu p_{0}(1-z)}{\mu(1-z)-\lambda z[1-z(1-\alpha) /(1-\alpha z)]}
$$

where

$$
p_{0}=1-\frac{\lambda E[X]}{\mu} .
$$

Since $E[X]$, the mean burst size, is the mean of the geometric distribution which is equal to $1 /(1-\alpha), p_{o}$ becomes

$$
p_{0}=1-\frac{\lambda}{\mu(1-\alpha)} .
$$

The load (system utilization) $\rho$, also becomes

$$
\begin{aligned}
\rho & =\frac{\lambda E[X]}{\mu} \\
& =\frac{\lambda}{\mu(1-\alpha)} .
\end{aligned}
$$

$P(z)$ can then be calculated as a function of $\alpha$, the burst size factor, and the load $\rho$

$$
\begin{aligned}
P(z) & =\frac{\mu p_{o}(1-z)}{\mu(1-z)-\lambda z(1-z) /(1-\alpha z)} \\
& =\frac{\mu(1-\rho)(1-\alpha z)}{\mu(1-\alpha z)-\lambda z} \\
& =\frac{(1-\rho)(1-\alpha z)}{(1-\alpha z)-(\lambda / \mu) z} \\
& =\frac{(1-\rho)(1-\alpha z)}{(1-\alpha z)-\rho(1-\alpha) z} \\
& -\frac{(1-\rho)(1-\alpha z)}{1-[\alpha+(1-\alpha) \rho] z} \\
& =(1-\rho)\left\{\frac{1}{1-[\alpha+(1-\alpha) \rho] z}\right. \\
& \left.-\frac{\alpha z}{1-[\alpha+(1-\alpha) \rho] z}\right\}
\end{aligned}
$$

To calculate $L$, the average number of packets in the input queue we need to find $P^{\prime}(z)$ for $z=1$,

$$
\begin{aligned}
L & =P^{\prime}(1) \\
& =(1-\rho)\left\{\frac{\rho-\alpha \rho}{(1-\alpha-\rho+\alpha \rho)^{2}}\right\} \\
& =\frac{\rho}{(1-\alpha)(1-\rho)} .
\end{aligned}
$$

Then using Little's formula, $L=A W$, we find the average waiting time $W_{i}$, at the input queue for a packet in time units per packet. The average packet arrival rate $A$, is replaced by $\lambda E[X]$, the average burst arrival rate times the average number of packets per burst.

$$
\begin{align*}
W_{i} & =\frac{L}{\lambda E[X]} \quad(E[X]=1 /(1-\alpha))  \tag{V.5}\\
& =\frac{L(1-\alpha)}{\lambda} \\
& =\frac{\rho}{(1-\rho) \lambda} .
\end{align*}
$$

## V.2.2 Numerical results

Figure V. 8 shows plots of the average delay of a packet at the input queue as a function of the average burst arrivals and the average burst size. Formula V. 5 was used to calculate the average waiting time for a mean burst sizes 1.1 and 1. The service rate $\mu$, of the port connection was set equal to one packet/time unit. Recall that the input queue represents the port connection. Note that when the burst size is equal to $1, \rho$ is equal to $\lambda / \mu$ and $V .5$ becomes

$$
W_{i}=\frac{1 / \mu}{1-\rho}
$$

which is the average waiting time of the $M / M / 1$ queue.
simulation results were also provided, a discrete-event
simulation program called SMPL [37], implemented in C language was used. SMPL uses three kinds of entities: facilities, tokens, and events.

Facilities represent some work-performing resources of the system being modeled. In this model the facility represents the port connection and its queue. SMPL provides functions to define facilities and reserve, release, and interrogate their status.

Tokens represent the data packets, which are the active entities of the system. The dynamic behavior of the system is modeled by the movement of tokens through the facility. Events represent a change of state of any system entity. SMPL includes an event list scheduling mechanism, which comprises a procedure for scheduling events, a procedure for selecting--"causing"-- the next event, a variable representing the current value of simulation time, and the event list data structure itself. This model includes three events. Event 1 represents a customer arrival, it schedules the server (port connection) request of the arriving customer for immediate occurrence, and schedules the next arrival. Event 2 is a server request. If the server is free, a function schedules the customer service completion. If the server is busy another function queues the request. Event 3 represents a service completion: a function is called to release the server, then if a
request has been queued, it dequeues it and event 2 is initiated.

SMPL provides several routines to generate random variates for the service time and the interarrival time distributions, such as the exponential, the Erlang, the hyperexponential for two bandwidths, the uniform, and the normal distributions. We developed routines to generate Poisson distributed burst arrivals with geometrically distributed burst size and included them in SMPL.

SMPL uses an algorithm to compute the average number of customers in the system, and the average waiting time is then computed using Little's Law.

In figure $V .8$ nine simulation runs were made with 200,000 packet generated at each run.

In Figure V.9, curves for different average burst sizes are plotted. We can see how the delay changes drastically for the average burst size of 3 packets. For all curves a port connection service rate $\mu=1$ packet/time unit was used.


Figure v.8. Input queue delay


Figure V.9. Input delay for different burst sizes.

## V.3. Output queue

For the output queue we concentrate our analysis on the variable service rate. Recall that the Frame Relay node is modeled by two queues in tandem. An input queue representing the port connection, and an output queue representing a PVC with bursty bandwidth (or variable service rate). In this section the output queue delay for the "delay not discard" policy is derived. Numerical results are provided for different sets of bandwidth burstiness using the analytical formulas and simulation. The system delay, which is the sum of the input queue delay and the output queue delay, is also calculated.

For the policy "discard not delay" the blocking probability is derived, and numerical results are provided with comparison of different sets of bandwidth burstiness. The blocking probability is calculated as a function of buffer capacity of the output queue.

```
V.3.1. Case A: The Output Queue Delay for "Delay not
        Discard" Policy
Most of the papers in the literature [16-24] consider
the queuing model for variable service rate as state dependent, which means the service rate depends on the state of the queue itself. A set of rules is preset, and is based on the decision of increasing or decreasing the service rate
```

whenever the queue size attains certain values. Yadin [16] used a doctrine where customers arrive at a service station in a steady Poisson stream, and a station is capable of rendering service at different controllable rates. An increase of the number of customers waiting in line brings about a higher service capacity, whereas a drop in the number of customers slackens the rate. He considered a sequence of feasible service rates $\left\{\mu_{0}, \mu_{1}, \ldots, \mu_{\mathrm{k}}, \ldots\right\}$ such that $\mu_{k+1}>\mu_{k}$ and $\mu_{0}=0$ and two sequences of integers representing the queue size are used in the management doctrine, these being

$$
\left\{R_{1}, R_{2}, \ldots, R_{k}, \ldots, \ldots\right\} \text { and }\left\{S_{0}, S_{1}, S_{2}, \ldots, S_{k}, \ldots\right\}
$$

such that $R_{k+1}>R_{k}, S_{k+1}>S_{k}, R_{k+1}>S_{k}, S_{0}=0$. Whenever the queue size reaches a value $\mathrm{R}_{\mathrm{k}}$ (from below) and service rate equals $\mu_{k-1}$, it is increased to $\mu_{\mathrm{k}}$. Whenever queue size drops to a value $S_{k}$ (from above) and service rate equals $\mu_{k+1}$, it is decreased to $\mu_{k}$. The capacity parameter $\mu_{k}$ is the reciprocal expectation of an exponentially distributed service time. Yadin denoted the (stationary) joint probability of the system being in phase $k$ and the number of customers in queue being $i$ by $p(k, i)$. The steady state equations and the conditional expected queue size for each phase were derived. After some manipulation the overall unconditional expected queue size was obtained.

Federgruen et al [17] also investigated a queuing model in which the service rate can be varied depending on the queue length. Each customer is served by using one of a finite number of available service types. He considered two possible service types. The service rate is characterized by two switch-over levels $R_{1}$ and $R_{2}$ where $R_{1}$ and $R_{2}$ are given integers with $0 \leq R_{2} \leq R_{1}$. The server switches from service type 1 to service type 2 only when the number of customers present is larger than $R_{1}$ and switches from service type 2 to service type 1 only when the number of customers present is smaller than or equal to $R_{2}$. A computational method was developed and the stationary probabilities of the queue size were recursively computed for an M/G/1 queuing system with two service rates.

Three different examples with state-dependent service times and bulk-arrival were investigated in [22]. Some results that characterize queue behavior were obtained using the imbedded Markov chain approach. The basic structure of the M/G/1 imbedded semi-Markov process was preserved, but the transition matrix of the chain was not as quite as simple. In the first example two types of service time with different exponential distribution were considered. The second example was quite similar except that one service time had one-parameter gamma distribution. For the final
example, service times for customers beginning service when there are $n$ in the system are exponential with mean $1 / n \mu$. The last corresponds to a situation in which the service rate increases linearly with the queue length. Conditions for the existence of steady-state probabilities for the imbedded chains were derived for the general model, and for the three examples. Expressions were also found for steady state probabilities of each imbedded chain, but the expressions of the expected queue size were found in closed form for the first two examples, it was found only numerically for the third example with $n$ exponential service time distributions with mean $1 / \mathrm{n} \mu$.

While in our model the service rate is variable, it does not depend on the queue size of the selected PVC. Therefore the service rate is considered state-independent. The server (PVC) of the output queue is part of a fiber pipe (or a trunk) of a carrier's backbone network. The service rate of a PVC depends on the intensity of the traffic on the carrier's fiber pipe. Recall that although the capacity of a PVC is reserved in advance, the available bandwidth could exceed the initially reserved bandwidth. This is called the burstiness capability as mentioned earlier. The burst bandwidth depends on the activity of the other PVCs on the carrier's trunk.

Because of the fluctuations of the available bandwidth,
the service times as seen by the end users are variable and cannot be modeled by an exponential distribution. However, for queue networks (queues in tandem), if we don't apply the Markovian property, the traffic process between nodes, and between the input and output queue at this point, becomes extremely complicated. To keep the output process Markovian, we use "the method of stages" [6] [8], which was conceived by Erlang. He used the notion of decomposing the service time distribution into a collection of structured exponential distributions. The stages can be arranged in series or in parallel. We use the parallel arrangement. The situation may be represented by the service structure shown in Figure V.10. A packet enters into the facility and proceeds to service stage 1 with probability $\alpha_{1}$, or will proceed to service stage 2 with probability $\alpha_{2}$, where $\alpha_{1}+\alpha_{2}=1$. It will then spend an exponentially distributed interval of time in the ith such stage whose mean is $1 / \mu_{\mathrm{i}}$. After that interval the customer departs and only then is a new packet allowed into the service facility. For the more general case, see Figure v.11. The model is called an $\mathrm{M} / \mathrm{H}_{\mathrm{k}} / 1$ queue, k is the number of stages. A packet is processed by a server which has a service time that is exponentially distributed with the service rate $\mu_{j}(j=1$, $2,3, \ldots k)$ and the probability that $\mu_{j}$ is available is $\alpha_{j}(j=$
$1,2,3, \ldots k$ ). At most one customer (one packet) is in the service mechanism at any time.

The service time distribution is given as

$$
\begin{equation*}
b(t)=\sum_{j=1}^{k} \alpha_{j} \mu_{J} \exp \left(-\mu_{J} t\right) \tag{V.6}
\end{equation*}
$$

where the mean service time is

$$
\begin{equation*}
E(t)=\sum_{J=1}^{k} \frac{\alpha_{j}}{\mu_{J}}, \tag{V.7}
\end{equation*}
$$

and the variance of the service time is [6]

$$
\begin{equation*}
V(t)=\sum_{j=1}^{k} \frac{2 \alpha_{j}}{\mu_{j}^{2}}-E^{2}(t) \tag{V.8}
\end{equation*}
$$

The density function given by (V.6) is referred to as hyperexponential distribution.

If $k=2$, then $b(t)$ reduces to

$$
\begin{equation*}
b(t)=\alpha \mu_{1} \exp \left(-\mu_{1} t\right)+(1-\alpha) \mu_{2} \exp \left(-\mu_{2} t\right) . \tag{V.9}
\end{equation*}
$$

For $k=2$, the hyperexponential has a closed form for the delay, but it can be seen from the derivation provided in [6] that lengthy and cumbersome steps were taken to reach the closed form solution. In this research we are interested in $k>2$, and to the best knowledge of the author, there are


Figure V.10. A two-stage parallel server.


Figure V.ll. A k-stage parallel server.
no closed form solutions reported in the literature for $\mathrm{k}>2$.

It is interesting to mention that A. K. Erlang developed the "method of stages" early in this century long before the tools of modern probability theory and queuing theory were available. To derive the expected delay at the output queue we use the results of the $M / G / 1$ queue, specifically the Pollaczec-Khintchine (P-K) formula [7] [8]. Recall that in M/G/1, G stands for general (or arbitrary) distribution, which means that $M / G / 1$ can represent an $M / H_{k} / 1$ queue.

The Pollaczek-Khintchine formula permits us to calculate the expected steady-state system size $\mathrm{L}_{\mathrm{o}}$ of the output queue at an arbitrary point of time via

$$
\begin{equation*}
L_{0}=\rho_{o}+\frac{\lambda_{0}^{2} V(t)+\rho_{o}^{2}}{2\left(1-\rho_{0}\right)} . \tag{V.10}
\end{equation*}
$$

All we need to use the Pollaczek-Khintchine formula is the variance $v(t)$ of the service time-distribution. For the hyperexponential distribution $V(t)$ is given by (V.8), which in turn requires the mean $E(t)$ of the hyperexponential distribution given by (V.7), and the service rates $\mu_{j}$ 's with their probabilities of occurrence $\alpha_{j}$ 's.

The system load $\rho_{0}$ is given as

$$
\rho_{0}=\sum_{j=1}^{k} \frac{\alpha_{j} \lambda_{0}}{\mu_{j}}
$$

where $\lambda_{0}$ is the packets arrival rates, which is equal to the average burst arrival rate times the burst mean size.

We then use Little's Formula to calculate the average waiting time for a packet in the output queue

$$
\begin{gather*}
W_{0}=\frac{L_{0}}{\lambda_{0}} \\
W_{0}=\frac{\rho_{0}}{\lambda_{0}}+\frac{\lambda_{0} V(t)+\rho_{0}^{2} / \lambda_{0}}{2\left(1-\rho_{0}\right)} \tag{V.11}
\end{gather*}
$$

## V.3.2. Numerical Results

The model developed in this research allows the user the flexibility to model the input queue according to the traffic characteristics, as long as the traffic fed to the output queue from the input queue is Poisson. Note that in general, for most random input processes injected into the input queue, the output process of that input queue will tend to be Poisson [8].

To implement this Frame Relay model, it is assumed that a mechanism to measure the availability of burst bandwidth exists. But if it doesn't exist, the following parameters should be known:

- the capacity of a virtual path
- the number (N) of the PVCs on the virtual path
- the probability $\beta$ that a PVC is active.

The probability that $n$ PVCs are active on the trunk can be considered to be Binomial and can be found via

$$
\begin{equation*}
\operatorname{Pr}(n)=\binom{N}{n} \beta^{n}(1-\beta)^{N-n} . \tag{V.12}
\end{equation*}
$$

This information can be used by the network provider to calculate thresholds for switching PVC burst bandwidth. Unfortunately these thresholds are usually unknown to the user, but given the variability of the offered services and the increasing competition, the policies and the thresholds of the burst bandwidth are more likely to be accessible to the user in the future. The proposed model could also be used by the carriers, who are the ones who set the thresholds, to compare different strategies or policies, and to set the thresholds.

For example a carrier could throttle the bandwidth to $100 \%$ of the CIR if more than $50 \%$ of the PVCs are active, or allow 200\% of the CIR if less than half of the PVCs are idle. In the latter case, if we have a trunk with $N=100$ PVCs, the probability of using a bandwidth of $200 \%$ of the CIR is equal to the probability $\alpha_{1}$ of having less than 50

PVCs active. Given $\beta$, V. 12 will be used to calculate $\alpha_{1}$ by setting $N=100$ and summing the probabilities from $n=1$ to 49. Figure V. 12 shows the two parallel stages, equivalent to the system of this example.


Figure V.12. Example of a two-stage parallel system.

For the numerical examples of our model, the percentage of the CIR with probabilities of occurrences need to be specified. The first experiment used a first set of percentage of the CIR $\mu_{0}=\{100 \%$, 125\%, 150\%, 175\%, 200\%\} with the respective probabilities of occurrence $\alpha=\{0.4$, $0.2,0.2,0.1,0.1\}$ for the case, when the CIR is guaranteed. The mean and the variance of the service time
distribution are calculated, and using formula (V.11), the average waiting time, Wo, is computed for this case. In Figure V.13, results from a comparison of this bursty channel and a fixed bandwidth channel are provided. From the curves we can see the benefit from bursty extra bandwidth. The waiting time is reduced and the system is able to handle a greater load than a fixed bandwidth channel.

Simulation results were also presented, nine runs of 200,000 packets were made. Each run had a different random number stream. The average waiting time was calculated as the average of the nine runs. The hyperexponential distribution was simulated by randomly choosing the available bandwidths. For all the experiments and the simulations the PVC service time Ts was set equal to 1 time unit, $T s=1 / \mu_{0}$.

In Figure V. 14 a more optimistic second set of percentages of the CIR is used, $\{100 \%, 125 \%, 200 \%, 300 \%$, 500\%\} with the respective probability of occurrence $\{0.6$, $0.1,0.1,0.1,01\}$. The gains are more visible in this example. We can see how the load can be increased. These examples show that bursty bandwidth services such as Frame Relay could be very convenient when a user is not sure of the intensity of the burstiness and the traffic of his network. Another example is provided in Figure V.15, this
time we considered a case where the CIR is not guaranteed and the bandwidth can be throttled even below the CIR, the third set of percentages of the CIR is \{100\%, 125\%, 80\%, 50\%, 200\%\} with the respective probability of occurrence $\{0.6,0.1,0.1,0.1,0.1\}$. We kept the mean of the service rate close to the CIR (1.06). Some providers don't guarantee the CIR, but they claim that the throughput will be equal to the CIR. In this case the gains are not very important as can be seen in Figure V.15.


Figure V.13. Output queue delay. Comparison of a bursty channel and fixed bandwidth channel (first set of burstiness).

Comparison of a Bursty Channel and a Fixed Bandwidth Channel


Burst \% \{100\%, 125\%, 200\%, 300\%, 500\%\}
and Respective Prob $\{.6, .1, .1, .1,1\}$
Service Time Ts=1 time unlt

Figure V.14. Output queue delay. Comparison of a bursty channel and a fixed bandwidth channel (second set of burstiness).

## Comparison of a Bursty Channel and a Fixed Bandwidth Channel



Burst \% \{100\%, 125\%, 80\%, 50\%, 200\%\}
and Respective Prob $\{.6, .1, .1, .1,1\}$
Service Time Ts=1 time unit

Figure V.15. Output queue delay. Comparison of a bursty channel and a fixed bandwidth channel (third set of burstiness).

## V.3.3. Bystem Delay

For the one node case we define $W$ as the system delay. $W$ is the sum of delays experienced by a packet while passing through a node. It includes the delay at the port connection which is represented by an input queue delay $W_{i}$, the delay $W_{0}$ of the output queue, and the propagation delay $P$,

$$
\begin{equation*}
W=W_{i}+W_{0}+P \tag{V.13}
\end{equation*}
$$

Both $W_{i}$ and $W_{o}$ include the waiting time at the queues (virtual waiting time) and the processing times. The processing time of the input queue depends on the port connection service rate, and the processing time of the output queue depends on the bandwidth of the PVC. P depends on the geographical length of the link. It is equal to the speed of light over fiber divided by the length of the link, plus deterministic cross-connect delays which occur when specific bits on the fiber trunk are switched (at switching nodes) to different fiber trunks. The average delay of a packet on the network is calculated as

$$
\begin{equation*}
W=\sum_{j=1}^{M}\left(W_{i j}+W_{o j}\right)+P_{j}+D \tag{V.14}
\end{equation*}
$$

$W$ depends on the number of nodes $M$ between the source
port connection and destination port connection. D is a switching delay calculated using the fixed bandwidth Kleinrock model for an M/M/1 queue. It is added to the total average delay because packets pass through one more access port than they do PVCs in their travels through the network. Some numerical results are provided for one node case, $P$ and $D$ are set equal to zero for the examples of Figures V.16.a and V.16.b. The formula (V.14) becomes

$$
\begin{equation*}
W=W_{i}+W_{0} \tag{V.15}
\end{equation*}
$$

For the examples of Figures V.16.a and V.16.b the set of burst bandwidth as a percentage of the CIR \{100\%, 125\%, 150\%, 175\%, 200\%\} with the respective probabilities of occurrence $\{0.4,0.2,0.2,0.1,0.1\}$ was used. The port connection service time Ts was set equal to 1 , and the PVC service time $T p$ was set equal to 2 ( $T p=1 / C I R$ ), which means the access speed was twice faster than the PVC service rate. We can see also how the waiting time changes as a function of the mean burst length $x$ of the input traffic. Figures V.16.b compares a bursty channel and a fixed bandwidth channel, the load is much higher for the bursty channel.


Figure V.16.a. System delay for one node case for different mean burst sizes of the input traffic.


Figure V.16.b. System delay comparison. Bursty channel and fixed bandwidth channel.
V.3.4. Case B: The Blocking Probability for "Discard not Delay" Policy

For the "discard not delay" policy, we consider an infinite capacity buffer for the input queue and a finite capacity buffer for the output queue, therefore the blocking of packets occurs only at the output queue. An infinite capacity buffer for the input queue could mean the switch processor is fast enough not to let the queue grow over a certain size. In this section a model is developed to calculate the blocking probability for the output queue. We will use the results of the M/G/1/K queue provided in [7] and apply them to the $M / H_{k} / 1 / K$ queue system. $K$ represents the buffer capacity.

First let's write the departure point transition probability matrix of the M/G/1 queue [7]:

$$
P=\left[p_{i j}\right]=\left[\begin{array}{cccc}
k_{0} & k_{1} & k_{2} & \cdots \\
k_{0} & k_{1} & k_{2} & \cdots \\
0 & k_{0} & k_{1} & \cdots \\
0 & 0 & k_{0} & \cdots \\
. & \cdot & \cdot & \\
. & \cdot & . & \cdots \\
. & . & . &
\end{array}\right]
$$

where $p_{i j}$ is the single-step transition probability of going from state $i$ to state $j$. It can also be written as

$$
\mathrm{p}_{\mathrm{ij}}=\operatorname{Pr}\{\text { the number of packets in the system }
$$

$$
\begin{aligned}
& \text { immediately after a departure point is } j \mid \text { the } \\
& \text { number of packets in the system after previous } \\
& \text { departure was } i\} \\
& =\operatorname{Pr}\left\{X_{n+1}=j \mid X_{n}=i\right\}
\end{aligned}
$$

and

$$
\begin{aligned}
k_{n} & =P r\{n \text { arrivals during a service time } S=t\} \\
& =\int_{0}^{\infty} \frac{e^{-\lambda_{0} t}\left(\lambda_{0} t\right)^{n}}{n!} d B(t)
\end{aligned}
$$

where
$B(t)$ is the cumulative distribution function of service times. $d B(t) / d t=b(t)$, and $b(t)$ is the service time distribution given by (V.6).

The steady-state probability vector, $\pi=\left\{\pi_{\mathrm{n}}\right\}$, can be found as the solution to the stationary equation

$$
\begin{equation*}
\pi P=\pi \tag{V.18}
\end{equation*}
$$

$\pi_{\mathrm{n}}$ is the steady-state probability of n packets in the system at a departure point, which means that after a long time, or after the system behavior "settles down", the probability that a departing packet leaves behind it $n$ packets in the system is $\pi_{n}$. Solving (V.18) [7] yields

$$
\begin{equation*}
\pi_{n}=\pi_{0} k_{n}+\sum_{j=1}^{n+1} \pi_{j} k_{n-j+1} \quad(n=0,1,2, \ldots) . \tag{V.19}
\end{equation*}
$$

Now let's find the steady-state probabilities for the M/G/1/K queue. The expected number of arrivals during a service period must be conditioned on the finite capacity of the system, and the transition matrix must be truncated at K-1 ( it is not $K$ since we are observing just after a departure)

$$
P=\left[\begin{array}{ccccc}
k_{0} & k_{1} & k_{2} & \ldots & 1-\sum_{n=0}^{x-2} k_{n}  \tag{V.20}\\
k_{0} & k_{1} & k_{2} & \ldots & 1-\sum_{n=0}^{x-2} k_{n} \\
0 & k_{0} & k_{1} & \ldots & 1-\sum_{n=0}^{x-3} k_{n} \\
0 & 0 & k_{0} & \ldots & 1-\sum_{n=0}^{x-4} k_{n} \\
& & & & \\
& & & & \\
0 & 0 & 0 & \cdots & 1-k_{0}
\end{array}\right]
$$

which gives the stationary equation

$$
\begin{array}{ll}
\pi_{n}=\pi_{0} k_{n}+\sum_{j=1}^{n+1} \pi_{f} k_{n-j+1} & \text { for }(n=0,1,2, \ldots, k-2) \\
\pi_{n}=1-\sum_{n=0}^{k-2} \pi_{n} & \text { for }(n=k-1) \tag{V.21}
\end{array}
$$

These $K$ equations and $k$ unknowns can be solved for all the
probabilities, but we follow the approach taken in [7]. Harris [7] noticed that the first portion of the stationary equation is identical to (V.19), the stationary equation to the unlimited $M / G / 1$, and therefore deduced that the respective stationary probabilities must be at worst proportional for $n \leq K-1$; that is

$$
\begin{equation*}
\pi_{n}=C \pi_{n}^{*} \quad(n=0,1, \ldots, K-1) \tag{V.22}
\end{equation*}
$$

with $\left\{\pi_{n}\right\}$ for the $M / G / 1 / K$ and $\left\{\pi_{n}{ }^{*}\right\}$ for the $M / G / 1 / \infty$. $c$ is found by the condition

$$
\sum_{n=0}^{k-1} \pi_{n}=1=C \sum_{n=0}^{k-1} \pi_{n}^{*}
$$

or

$$
\begin{equation*}
C=\frac{1}{\sum_{n=0}^{k-1} \pi_{n}^{*}} \tag{V.23}
\end{equation*}
$$

$\left\{\pi_{n}\right\}$ was obtained by using the transition matrix (V.20), which was written for an imbedded Markov chain at the departure point. To calculate the packet blocking probability, we need to find the probability of the number in the system encountered by an arrival, and it will be different from $\left\{\pi_{n}\right\}$, since now we need to expand the state space to include $K$. If we denote by $q_{n}$ the probability that
an arriving packet finds $n$ packets in the system, then $q_{n}$ was derived and given in [7] as

$$
\begin{equation*}
q_{n}=\frac{\pi_{n}}{\pi_{0}+\rho_{0}} \tag{V.24}
\end{equation*}
$$

Since an arriving packet that finds $K$ packets in the system will be discarded, $q_{k}$ will be the packet blocking probability for the $M / G / 1 / K$. Using the following relations given also in [7]

$$
\begin{aligned}
g_{K} & =\frac{\rho_{0}-1+p_{0}}{\rho_{0}} \\
p_{0} & =\frac{\pi_{0}}{\pi_{0}+\rho_{0}}
\end{aligned}
$$

Where $p_{0}$ is the probabability of 0 packets in the system. Recall that $\pi_{n}$ is the steady-state probability of $n$ packets in the system at a departure point, $q_{n}$ is the steady-state probability of $n$ packets in the system at an arrival point, and $p_{n}$ is the steady-state probability of $n$ packets in the system at an arbitrary point in time. The blocking probability is finally found

$$
\begin{equation*}
q_{K}=\frac{\left(\pi_{0}+\rho_{0}-1\right)}{\left(\pi_{0}+\rho_{0}\right)} . \tag{V.25}
\end{equation*}
$$

The system load $\rho$ is given as

$$
\rho_{0}=\sum_{j=1}^{k} \frac{\alpha_{j} \lambda_{0}}{\mu_{j}} .
$$

To calculate $\pi_{0}$ for the $M / G / 1 / K$ queue we need to find $\left\{\pi_{\mathrm{n}}{ }^{*}\right\}$ for the $\mathrm{M} / \mathrm{G} / 1$ queue and then use (V.22) and (V.23). First let's find $k_{n}$. Recall that $k_{n}$ was given by (V.17) as

$$
k_{n}=\int_{0}^{\infty} \frac{e^{-\lambda_{0} t}\left(\lambda_{0} t\right)^{n}}{n!} d B(t)
$$

since $d B(t) / d t=b(t)$, where $b(t)$ is the hyperexponential distribution of the service times given by

$$
b(t)=\sum_{j=1}^{k} \alpha_{j} \mu_{j} \exp \left(-\mu_{j} t\right)
$$

we can write

$$
\begin{aligned}
k_{n} & =\int_{0}^{\infty} \frac{e^{-\lambda_{0} t}\left(\lambda_{0} t\right)^{n}}{n!} b(t) d t \\
& =\int_{0}^{\infty} \frac{e^{-\lambda_{0} t}\left(\lambda_{0} t\right)^{n}}{n!} \sum_{j=1}^{k} \alpha_{j} \mu_{j} e^{-\mu, t} d t \\
& =\sum_{j=1}^{k} \alpha_{j} \int_{0}^{\infty} \frac{e^{-\lambda_{0} t}\left(\lambda_{0} t\right)^{n}}{n!} \mu_{j} e^{-\mu, t} d t \\
& =\sum_{j=1}^{k} \alpha_{j} \frac{\lambda_{0}^{n} \mu_{j}}{n!} \int_{0}^{\infty} e^{-\left(\lambda_{0}+\mu_{j}\right) t} t^{n} d t
\end{aligned}
$$

and integrating by parts

$$
\int_{0}^{\infty} e^{-\left(\lambda_{0}+\mu_{j}\right) t} t^{n} d t
$$

we find

$$
\begin{equation*}
k_{n}=\sum_{j=1}^{k} \alpha_{j} \frac{\lambda_{o \mu_{j}}^{n}}{\left(\lambda_{o}+\mu_{j}\right)^{n+1}} . \tag{V.26}
\end{equation*}
$$

Using (V.19) we find $\left\{\pi_{n}^{*}\right\}$ for the $M / G / 1$

$$
\begin{align*}
& \pi_{0}^{*}=\pi_{0}^{*} k_{0}+\pi_{1}^{*} k_{0} \\
& \pi_{1}^{*}=\pi_{0}^{*} k_{0}+\pi_{1}^{*} k_{1}+\pi_{2}^{*} k_{0} \\
& \pi_{2}^{*}=\pi_{0}^{*} k_{2}+\pi_{1}^{*} k_{2}+\pi_{2}^{*} k_{1}+\pi_{3}^{*} k_{0}  \tag{V.27}\\
& \pi_{3}^{*}=\pi_{0}^{*} k_{3}+\pi_{1}^{*} k_{3}+\pi_{2}^{*} k_{2}+\pi_{3}^{*} k_{1}+\pi_{4}^{*} k_{0} \\
& \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\end{align*}
$$

Hence, from the first equation of (V.27),

$$
\pi_{1}^{*}=\frac{\pi_{0}^{*}-\pi_{0}^{*} k_{0}}{k_{0}}
$$

$\pi_{0}{ }^{*}$ for the $M / G / 1$ queue is equal to $1-\rho_{0}$. From the second and third equations of (V.27),

$$
\pi_{2}^{*}=\frac{\pi_{1}^{*}-\pi_{0}^{*} k_{1}-\pi_{1}^{*} k_{1}}{k_{0}}
$$

$$
\pi_{3}^{*}=\frac{\pi_{2}^{*}-\pi_{0}^{*} k_{2}-\pi_{1}^{*} k_{2}-\pi_{2}^{*} k_{1}}{k_{0}}
$$

And for ( $2 \leq n \leq K-1$ ) the following general expression was determined

$$
\pi_{n}^{*}=\frac{\pi_{n-1}^{*}-\pi_{0}^{*} k_{n-1}-\sum_{j=1}^{n-1} \pi_{j}^{*} k_{n-j}}{k_{0}}
$$

## V.3.5. Numerical and simulation Results

A program was written to calculate the $\left\{\pi_{n}{ }^{*}\right\}$ and $q_{k}$ for different buffer capacities and for different burstiness capabilities.

Figure V. 17 represents the packet blocking probability as a function of the offered load for different buffer capacity K for a bursty channel. If K decreases, the blocking probability increases. A simulation model was developed to validate the analytical results of the blocking probability. Upon each packet arrival the queue length of the simulated PVC is checked and if it is greater than $K$ the packet is considered lost. Figures V.18-V. 19 show the results from the first set of burstiness, the CIR percentages of $\{100 \%, 125 \%, 150 \%, 175 \%, 200 \%\}$ with the probabilities of occurrence $\{0.4,0.2,0.2,0.1,0.1\}$.

Figures V.20-V. 21 show the results of the second set of burstiness, the CIR percentages of \{100\%, 125\%, 50\%, 80\%, 200\%\} with the probabilities of occurrence $\{0.4,0.2,0.2$, $0.1,0.1\}$. The results of the simulation model are consistent with the results of the analytical model and as was expected the second set of burstiness gives a higher blocking probability.

## Blocking Probability Vs. Load

Blocking Probability


Burst \% \{ 100\%, 125\%, 150\%, 175\%, 200\%\}
and Respective Prob $\{.4, .2, .2, .1,1\}$

Figure V.17. Blocking probability vs. load for different buffer capacities for the $M \backslash H_{k} \backslash 1 \backslash K$

## Blocking Probability

Analytical and Simulation Results


Burst \% \{100\%, 125\%, 150\%, 175\%, 200\% $\}$
and Respective Prob $\{.4, .2, .2, .1, .1\}$

Figure V.18. Blocking probability simulation results for the first set of burstiness ( $K=5, K=10$ ).


Figure V.19. Blocking probability simulation results for the first set of burstiness
( $\mathrm{K}=15, \mathrm{~K}=20$ ) .

## Blocking Probability

 Analytical and Simulation Results

Burst \% [100\%, 125\%, B0\%, 50\%, 200\%\}
and Respective Prob $\{.4, .2, .2, .1, .1\}$

Figure V.20. Blocking probability simulation results for the second set of burstiness
( $K=5, K=10$ ).


Figure. V.21. Blocking probability simulation results for the second set of burstiness ( $K=15$ ).

In Figures V.22-V. 25 comparisons between a bursty channel and a fixed bandwidth channel are provided. In most cases, when the CIR is guaranteed and bursty capability is available, we can see how the bursty Channel outperforms the fixed bandwidth channel. But in the examples of Figures V.24-V.25, where the CIR is not guaranteed, the blocking probability is better for a fixed bandwidth channel. For the fixed bandwidth, the blocking probability formula for the M/M/1/K was used [8]

$$
q_{K}=\frac{\left(1-\rho_{0}\right) \rho_{0}^{K}}{1-\rho_{0}^{K+1}} .
$$



Figure V.22. Blocking probability comparison for the first set of burstiness ( $\mathrm{K}=5, \mathrm{~K}=10$ ).


Figure V.23. Blocking probability comparison for the first set of burstiness ( $\mathrm{K}=15, \mathrm{~K}=20$ ).


Figure V.24. Blocking probability comparison for the second set of burstiness ( $K=5, K=10$ ).


Figure V.25. Blocking probability comparison for the second set of burstiness ( $\mathrm{K}=15, \mathrm{~K}=20$ ).

## VI. REAL FRAME RELAY NETWORR GIMULATION

The simulation model developed in the preceding chapters was based on the end user's point of view. A PVC was considered as a single physical medium between two points with variable bandwidth. In addition, the probabilities of using different percentages of the CIR were considered to be known and were the same for both the analytical and the simulation models. In this chapter a simulation model of a simplified Frame Relay network is developed from the network provider's view, Figure VI.1. A PVC shares a high bandwidth physical medium with other PVCs. The realizable PVC bandwidth at any point in time will be determined during the simulation. The purpose of this chapter is to show how the developed model mimics the real world Frame Relay network.

In the real world Frame Relay network, multiple user logical data streams can be multiplexed and demultiplixed within the same provider trunk. Frame Relay users are identified to the network through logical connections called data link connection identifiers (DLCIs). The multiplexed users are assembled into frames and transmitted across the network trunks. The frames retain their order of transmission and reception. Figure VI. 2 shows various logical channels attempting to transmit. Frame Relay traffic


Figure VI.1. Portion of an example of Frame Relay network.


Figure VI.2. Frame Relay Multiplexing
traffic from an input line to an output line. The end user is exchanged between the network users by mapping DLCI is responsible for building the Frame Relay frames and placing a DLCI value in the frame address field.

## VI. 1 PVC Transmission Delay

The input delay at the port connection that was determined in section V.2.1 is considered to accurately model a real Frame Relay Network and will not be investigated in this chapter. While the users access the Frame Relay networks through high speed links such as T1 or fractional T1, it is the access port connection speed along with the rate of the traffic generated by the users sources that defines the input delay. These two factors are negotiated by the user and the provider and were used to develop the input delay model of section V.2.1. In general the users use high speed access links to facilitate the upgrade when more bandwidth is needed.

A simulation model was developed to examine the queuing and transmission delay through a PVC. This delay was represented by the output queue delay of the model in section V.3.1 for the "delay not discard policy". Figure V. 3 shows a simulation model of a trunk of $N$ PVCs. Each PVC is represented by a queue and a server with a variable bandwidth. The extra bandwidth that a PVC can use depends on


Figure VI. 3 Simulation Model.
the bandwidth of the provider trunk, the activity of the other PVCs, and the subscription policy.

The following three elements are used to set the subscription policy on Frame Relay networks:

* The CIR

The average rate (in bit/s) at which the network guarantees to transfer information units over a measurement interval TC.

* Committed Burst Size (BC)

The committed burst size (Bc) is the maximum amount of data (expressed in bits) that can be transmitted during interval TC.

* Excess Burst Size (Be)

The excess burst size ( Be ) describes the maximum uncommitted amount of data (in bits) that can be transmitted during the interval Tc. The Be can be subject to a lower probability of delivery than Bc.

CCITT Recommendation I. 370 provides a useful diagram and explanation of the relationships of $\mathrm{BC}, \mathrm{Be}$, and the CIR over measurement period Tc. Figure VI. 4 shows these relationships. In Figure VI.4(a) frames 1 and 2 are sent within the Bc agreement and should have a guaranteed delivery. In Figure VI.4(b) frame 3 (with frames 1 and 2) is greater than Bc but does not exceed Bc plus Be. Frame 3 would most likely be delivered, although it could be marked


Figure VI.4. Relationship of the Frame Relay parameters.

Discard Eligible (DE). In Figure VI. $4(\mathrm{C})$, frame 4 (and the accumulation of frames 1, 2, and 3) violates the Bc plus Be provision and could be marked for probable discard or could be discarded at the entrance to the network. This will depend on the policy and the available bandwidth of the provider's network.

Based on the elements described above, it's clear that the user data can be transmitted with a variable bandwidth which can exceed the CIR. In the developed model these fractions of TC are represented by the equivalent probabilities of exceeding the CIR with certain percentages during the interval Tc. These probabilities depend on the policy and the available bandwidth on the provider's network. The providers use thresholds of the activity on the trunk to set the limits to the excess bandwidth that the users can use. These thresholds depend on the trunk rate.

An experiment with a simulation of a trunk of 100 PVCs is conducted, each PVC has a CIR of one packet/time unit equivalent to a service time $T$ s of one time unit. The port connection speed is considered to be equal to $200 \%$ of the CIR. For each PVC the following thresholds and policy were used:

- if the number of busy PVCs is less than 50 , the user's bandwidth is equal to $200 \%$ of the CIR.
- if the number of busy PVCs is less than 70 , the
user's bandwidth is equal to $150 \%$ of the CIR.
- if the number of busy PVCs is greater than 70, the user's bandwidth is throttled to $100 \%$ of the CIR.

For each PVC, before a service request, a routine checks the status of all PVCs of the simulated trunk and returns the number of the active ones. Depending on this number the corresponding percentage of the CIR is assigned to the PVC in service. The results of the simulation were compared to the results obtained using the analytical model. For the analytical model the same thresholds were used, but the number of busy PVCs was calculated using the binomial distribution, formula (V.12), and the method described in section V.3.2. As an example, for this case, the probability of using a bandwidth of $200 \%$ of the CIR is equal to the probability of having less than 50 busy PVCs. The graphs show the transmission delay of a PVC as a function of its arrival rate and the activity coefficient "a" of the other PVCs on the trunk. The activity coefficient "a" of a PVC is equal to it's load.

The simulation experiments were run long enough to reach the steady state and the results are shown in Figures VI.5-VI.8. The results of the simulation model of the real world Frame Relay network are consistent with the results obtained using the analytical model.


Figure VI.5. Model accuracy for a PVC transmission delay ( $a=0.1$ ).


Figure VI.6. Model accuracy for a PVC transmission delay ( $a=0.4$ ).


Figure VI.7. Model accuracy for a PVC transmission delay ( $a=0.6$ ).


Figure VI.8. Model accuracy for a PVC transmission delay ( $a=0.7$ ).

## VI. 2 Bystem Delay

The system delay is, as was defined in section V.3., The sum of the input delay experienced at the access port and the PVC transmission delay. For Figures VI.9-VI.12, the results of section V.2.2 and section VI.1 were used for the input delay and the PVC transmission delay respectively. For the overall time delay, the results of the developed model agree with the results of the simulation model of the real world Frame Relay network.


Figure VI.9. Model accuracy for the system delay ( $a=0.1$ ).


Figure VI.10. Model accuracy for the system delay (a=0.4).


Figure VI.11. Model accuracy for the system delay ( $a=0.6$ ).


Figure VI.12. Model accuracy for the system delay ( $a=0.7$ ).
VI. 3 simulation of the Blocking Probability.

The same approach as in section VI.l was used to conduct the experiments for the blocking probability. A trunk of 100 PVCs was simulated. A routine checks the status of each PVC and return the number of active PVCs. To decrease the simulation time, an experiment with higher blocking probability was chosen. The available bandwidth is considered equal to the port connection speed, but in the case of a higher activity on the trunk it is throttled under the following policy and thresholds:

- If the number of the busy PVCs is less than 30 the bandwidth is equal to $100 \%$ of port connection speed.
- If the number of the busy PVCs is greater than 30 and less than 60 the bandwidth is throttled to $80 \%$ of port connection speed.
- If the Number of the busy PVCs is greater than 60 the bandwidth is throttled to $60 \%$ of the port connection speed.

One PVC was considered for the measurement and a routine was added to check its current queue length. Upon each arrival if the current queue length is greater than the buffer size K , the packet is considered lost. Figures VI.13VI. 14 compare the results of the analytical model and simulation model with a $95 \%$ confidence interval. For the analytical model the number of busy PVCs was calculated using the binomial distribution. The results show that the
developed model accurately approximates the blocking probability when compared to the results of the simulation model for the buffer size $K=5$, but it gives a slightly higher results for $\mathrm{K}=10$.


Figure VI.12. Model accuracy for the blocking probability (a=0.3, K=5).


Figure VI.13. Model accuracy for the blocking probability (a=0.3, $\mathrm{K}=10$ ).

## VII. CONCLUSIONS AND FUTURE RESEARCH.

## VII. 1 summary and Conclusions

We have investigated the performance characteristics of Frame Relay, one of the must rapidly growing types of packet switched networks. In chapter. III we reviewed the evolution of packet switched networks from X. 25 , with the access speed of $56 / 64 \mathrm{Kbps}$, to ATM with the access speed up to 600 Mbps . Frame Relay with access speeds up to 2 Mbps fills the technology gap between X. 25 and $A T M$, and will be in competition with ATM as an interface to a service.

The concept and subscription policies for Frame Relay were discussed in chapter IV. One of the must attractive characteristics of Frame Relay is the burstiness capability. The burstiness capability provides additional bandwidth for application with bursty or delay sensitive traffic. Some problems with congestion management under different subscription policies when bursty bandwidth is allowed were also noted. The carriers allow the users to exceed their reserved bandwidth, but could also throttle the bandwidth in case of congestion, and the realizable bandwidth could be, in some cases, less than the reserved bandwidth.

In chapter $V$, we proposed a method and a queuing model to evaluate the performance characteristics of Frame Relay with bursty bandwidth capability. While in the literature
there are many works on bursty input traffic, there are no works on the evaluation of systems with bursty bandwidth. The developed model covers both bursty input traffic and bursty bandwidth, and could be used with any system with state-independent service rate. Given the complexity of both the input traffic and the service rate, we modeled an access node to a Frame Relay network node by a system of two queues in tandem. An input queue was modeled by an $M^{[x]} / M / 1$ queue, with burst arrivals and exponential service time, exact analytical results had been found by previous researches for the waiting time at the input queue. A geometrically distributed burst size was used. For the output queue we used the "method of stages" and modeled the output process by an $M / H_{k} / 1$, where $H_{k}$ stands for a hyperexponential service time distribution. The results of the M/G/1 queue were used to derive the average waiting time at the output queue. Computer simulations verified the analytical results and showed that the hyperexponential distribution can successfully be approximated by the M/G/1 queue. The examples provided showed that different subscription policies can affect the waiting time. Using this model, a provider network designer could develop subscription strategies that could lead to substantial cost savings. We then calculated the average system delay for the one node
case for the "delay not discard policy". We also showed how the system waiting time changes as a function of the mean burst length.

For the "discard not delay" policy, we considered an output queue with a finite capacity buffer, and used the result of the $M / G / 1 / K$ queue to derive the blocking probability for the $M / H_{k} / 1 / K$. Different examples were provided for different subscription policies for different buffer capacities. We showed that some subscriptions, specifically subscriptions with no guaranteed reserved bandwidth (CIR), do not outperform a fixed bandwidth channel.

In chapter $V$ the results were verified by a simulation of one PVC with variable bandwidth which is expressed through different percentages of the CIR. The probabilities of occurences of different percentages of the CIR were considered to be known and were the same for the analytical and the simulation models. In chapter VI a real world Frame Relay network simulation model was developed. A trunk of N PVCs was simulated. This simulation model included a routine to determine the number of busy pVCs on the trunk and assign percentages of the CIR based on a given subscription policy. The results were compared to the results of the analytical model which used the same policy but used the binomial
distribution to calculate the probability of the number of busy pVCs on the trunk. It was shown that for PVC transmission delay, the overall system delay, and the blocking probability the results of the analytical model were consistent with the results of the simulation model.

## VII.2. Future Research

In the future, applications with more heterogenous mix of traffic such as voice or video is expected to use Frame Relay bandwidth which is anticipated to reach 45 Mbps in the near future. The input traffic model needs to be extended to cover a wider range of traffic flow characteristics.

The developed model was limited to one node of Frame Relay network and is useful to assess the performances of fully meshed networks, but to be used for networks that include links with multiple Frame Relay switches, further research needs to be done to extend the model.

## GLOSSARY

Asynchronous Transfer Mode - A high-speed connectionoriented data transmission method that provides bandwidth on demand through packet-switching techniques using fixed-size cells.

American National standards Institute (ANSI) - A private, non-governmental, non-profit national organization which serves as the primary coordinator of standards within the United States.

Backward Explicit Congestion Notification (BECN) Convention in Frame Relay for network device to notify the user (source) device that network congestion has occurred. bandwidth - The amount of transport resource available to pass information (passband), measured in Hz for analog and bps (bits/second) for digital carriers.
broadband - While broadband once represented bandwidth in excess of the voice channel ( 3 kHz ), or in basic data communications using analog, modulated signals, it now refers to channels supporting rates in excess of T1 or El. CCITT - International Telegraphy and Telephone Consultative Committee.
cell - A fixed-length transmission unit used in ATM transmission; similar to a packet.
circuit switching - Switching based on dedicated paths
between terminating devices, and adding no intelligence to the transmission. Bandwidth is dedicated and delay is minimal.
codec - coder/decoder. A device used to digitize analog television signal.
congestion - The condition where network resources (bandwidth) are exceeded and additional information cannot be passed.

Data Link Connection Identifier (DLCI) - a Frame Relay address designator for each virtual circuit termination point (port).
delay-insensitive - Traffic types whose data is not affected by small delays during transmission.
delay-sensitive - Traffic types whose data is affected by small delays during transmission and cannot tolerate this delay (e.g., voice, video, real-time data).

Discard Eligibility (DE) bit - Used in Frame Relay, this bit signals (when set to one) that the particular frame is eligible for discard during congestion conditions. distributed processing - Sharing of applications, data, and the tasks operating among several small or mid-range processing devices, as opposed to a single mainframe in centralized processing.

Distributed Queue Dual Bus (DQDB) - The IEEE 802.6 MAN architecture standard for providing both circuit-switched
(isochronous) and packet-switched services.
E1 - The European T1 standard operating at 2.048 Mbps .
fast packet - The generic term used for advanced packet technologies such as Frame Relay, SMDS, and ATM.

Fiber Distributed Data Interface (FDDI) - Fiber optic LAN operating at 1000 Mbps.
fiber optics - Plastic or glass fibers which transmit high data rates through optical signals.

Forward Explicit Congestion Notification (FECN) - Convention in Frame Relay for a network device to notify the user (destination) device that network congestion is occurring. fractional TI (FTI) - The transmission of a fraction of a T1 channel, usually based in 64 Kbps increments but not less 64 Kbps total.
frame - a unit of transmission whose length is defined by flags at the beginning and end.

Frame Relay - An ANSI and CCITT defined LAN/WAN networking standard for switching frames in a packet mode similar to X.25, but at higher speeds and with less nodal processing. Frame Relay uses fiber optic links). full-duplex - The simultaneous bidirectional transmission of information over a common medium.
gateway - a device that interconnects dissimilar LANs that employ different high-level protocols.
half-duplex - The bidirectional transmission of information
over a common medium, but where information may only travel in one direction at any one time.
internetwork - A master network made up of multiple smaller networks.
isochronous - The circuit-switched transmission service offered in DQDB. This allows a consistent timed access of network bandwidth for delay-sensitive transmission of voice and video traffic.
local area network (LAN) - A computer communications network confined to short geographic distances, usually in an office, building, or campus environment. multiplexing - The technique of combining multiple single channels onto a single aggregate channel for sharing facilities and bandwidth.
mux - Multiplexer.
network - A system of autonomous devices, links, and subsystems which provide a platform for communications. packet-switching - A method of switching which segments the data into fixed or variable units of maximum size called packets.

Permanent Virtual Circuit (PVC) - A logical dedicated circuit between two user ports in a point-to-point configuration.
private network - A network providing interorganizational connectivity only.
private branch exchange (PBX) - a customer-site telephone switch, with some capability to integrate data.
protocol - The rules and guidelines by which information is exchanged and understood between two devices.
public data network (PDN) - A network designed to provide data transmission value-added services to the public. router - A LAN/WAN interconnection device. Distinguished from bridges by its capability to switch and route data based upon nertwork protocols such as IP.
simplex - One way transmission of information on a medium. subnetwork - The smaller units of LANs (called LAN segments) which can be more easily managed than the entire LAN/MAN/WAN .

Switched MultiMegabit Data Service (SMDS) - A MAN service offered at present over the IEEE DQDB bus.
switched virtual circuit (svc) - Virtual circuits similar to PVCs, but established on a call-by-call basis.

Synchronous Optical Network (SONET) - A United States highspeed fiber optic transport standard for a fiber optic digital hierarchy (speeds range from 51.48 Mbps to 2.5 Gbps).
synchronous transmission - The transmission of frames which are managed through a common clock between transmitter and receiver.

Synchronous Transfer Mode (STM) - The T1 carrier method of
assigning time slots as channels within a Tl or El circuit. T1 or T-1 - A circuit operating at 1.544 Mbps. T1 carrier - The TDM digital T1 hierarchy used in North America and Japan, with 24 voice channels constituting a single 1.544 Mbps Tl trunk.

T3 or T-3 - A circuit operating at 45 Mbps .
telecommunication - The transmission of voice, video, data, and images through the use of both computers and a communications medium.
time division multiplexing (TDM) - The method of aggregating multiple simultaneous transmissions (circuits) over a single high speed channel by using individual time slots (periods) for each circuit.

Transmission Control Protocol/Internet Protocol (TCP/IP) The combination of a network and transport protocol developed by ARPANET for internetworking IP-based networks. virtual circuit - A virtual connection established through the network from origination to destination, where packets, frames, or cells are routed over the same path for the duration of the call. These connections seem like dedicated paths to the users, but are actually network resources shared by all users. Bandwidth on virtual circuit is not allocated until it is used.
wide area network (WAN) - A network which operates over a large region and commonly uses carrier facilities and
services.
X. 25 - CCITT recommendation that specifies how user data terminal equipment (DTE) should interface with data circuitterminating equipment (DCE) for packet-switched networks. Parts of this glossary were taken from [26].

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