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LONGITUDINAL MAGNETIC FIELDS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

degree of

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BY

JAMES EDWARD NICHOLSON

Norman, Oklahoma

1972

FLUX MOTION IN LEAD-INDIUM WIRES WITH
LONGITUDINAL MAGNETIC FIELDS

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FLUX MOTION IN LEAD-INDIUM WIRES WITH
LONGITUDINAL MAGNETIC FIELDS

CHAPTER I

INTRODUCTION

Type I vs. Type II Superconductivity

Approximately 50 years after the discovery of superconductivity in 1911 by Onnes, superconductors were separated into two types according to their magnetic characteristics.

The first group of superconductors is composed mainly of elements, each exhibiting the well-known Meissner effect¹ until it is driven normal at some critical field H_c . These materials are labeled "type I".

Another group of superconductors tends to be composed of alloys with high current-carrying abilities that exhibit the Meissner effect up to a field H_{c1} where magnetic flux penetrates the sample and remains until at some higher field H_{c2} the superconductor becomes normal, i.e., resistance returns. These are "type II" superconductors and the region between H_{c1} and H_{c2} is called the "mixed state".

While in the mixed state, a superconductor is neither purely superconducting nor purely normal but is composed of both superconducting and normal regions. One possible configuration is for the superconductor to be threaded by cylinders of normal material parallel to the magnetic field

$H_{c1} < H < H_{c2}$. These cylinders or normal cores are referred to as flux lines and are represented in Fig. 1.² (Notice the analogy between flux lines in a superconductor and vortex lines in rotating helium.)

In order to understand flux lines, consider a slab of superconducting material (as shown in Fig. 1) which is in the mixed state. (It is worthwhile to note that the field at which flux begins to penetrate into a slab is much less than the field H_{c1} as defined for a wire in a longitudinal magnetic field. The following arguments are still qualitatively valid.) The bulk of the material is diamagnetic so that the flux due to the applied magnetic field is opposed by diamagnetic currents i_d which circulate on the outer surface of the slab. The diamagnetic material of Fig. 1 is threaded by normal cores which are parallel to the applied field and contain magnetic flux in the direction of the applied field. The flux within each core is generated by persistent currents i_p circulating in the opposite ("paramagnetic") direction to the diamagnetic currents.²

For a type II superconductor it is energetically more favorable for the flux lines to penetrate the sample and thread it with normal regions than to be held out by surface currents as mentioned above. This is because of energy associated with the formation of the normal-superconducting interface--the surface energy--which for a type II superconductor is negative.

The surface energy (Δ) is related to the difference between the penetration depth (λ) and the coherence length (ξ) as

$$\Delta \propto (\xi - \lambda) \quad .$$

Figure 1. The Mixed State Showing Normal Cores and Encircling Supercurrent Vortices

The penetration depth λ is a distance in which a field decays to $1/e$ of its value at the surface and is typically 5×10^{-6} cm for metals. The coherence length ξ is a measure of the distance over which the superconducting wave function is allowed to change significantly and is about 10^{-4} cm. One can think of the coherence length as the radius of the vortex core and the penetration depth as the distance over which the magnetic flux associated with each core spreads into the surrounding material. In the Ginzburg-Landau theory the ratio of λ and ξ is defined as the Ginzburg-Landau parameter κ ,¹

$$\kappa = \lambda/\xi .$$

This allows one to view superconductivity from the viewpoint of surface energy. For a type I superconductor, $\xi > \lambda$ or $\kappa \leq 1/\sqrt{2}$ so that the surface energy is positive. It is more favorable for a sample to expel lines than to have the lines thread it. For a type II superconductor, $\xi < \lambda$ so that $\kappa \geq 1/\sqrt{2}$. It is energetically favorable to expel lines of flux up to $H = H_{c1}$ or $\kappa = 1/\sqrt{2}$. Above H_{c1} , $\kappa > 1/\sqrt{2}$ so that energy is released in forming the normal-superconducting interface and the surface energy is negative. Thus for $H_{c1} < H < H_{c2}$ it is energetically favorable to allow flux lines to enter the superconductor.

The differences in the behavior of type I and type II superconductors are shown in the magnetization curves in Fig. 2. Fig. 2a shows no flux penetration until $H = H_c$ while Fig. 2b shows flux penetration occurring at $H = H_{c1}$ and continuing until $H = H_{c2}$ at which time the sample is completely penetrated by flux and is normal.

Figure 2a. Magnetization Curve for a Type I Superconductor

Figure 2b. Magnetization Curve for a Type II Superconductor

Flux Motion

Whenever the magnetic field on a type II superconductor is less than H_{c1} , the superconductor behaves like a type I superconductor with a critical current determined very much like Silsbee's rule for a type I superconductor but with H_c replaced by H_{c1} .² Above H_{c1} the type II superconductor is driven into the mixed state but can carry some resistanceless currents.

For a type II superconducting wire in the mixed state, a voltage appears along its length when a large enough transport current is supplied. As long as the current is subcritical, no voltage appears. At its critical value I_c a voltage appears and increases with increasing current until the sample is driven out of the mixed state and into the normal state by the normal current I_n .

The voltage appearing across the sample is due to the motion of magnetic flux throughout the bulk of the superconductor in the mixed state. In cylindrical geometry with an external longitudinal magnetic field and with a transport current in the wire, flux lines appear in the form of helices. These helices are forced through a surface barrier by a Lorentz force set up by the transport current. The Lorentz force is given as,

$$F_L = J \Phi_0 \sin \theta$$

where J is the current density,

Φ_0 is the amount of flux enclosed by each vortex line, and

θ is the angle between the direction of the total magnetic

field (external and current fields) and of the transport current.

The cores containing flux are pinned in the sample for various reasons. The surface is irregular and vortex lines are held up at imperfections on the surface. There are bulk dislocations which trap the flux line in the interior of the sample. The transport current tends to flow in a manner caused by the balancing of circulation currents of neighboring vortices. In other words, there is interaction between vortices. A barrier exists at the edges of the sample due to an image force. Pinning arises from variations in the chemical potential due to irregularities in the lattice of the sample.³ Whatever the nature of the pinning force, however, the Lorentz force must overcome it in order to produce viscous flow of the flux lines.

Kim,^{4,5} Farrell,⁶ and several others have concentrated on flux motion in both transverse and longitudinal fields. They have restricted their investigations to slab geometry because of added difficulties that arise both in theoretical and experimental considerations in cylindrical geometry. Very little understandable work has been carried out for the case of a cylindrical wire in a longitudinal magnetic field. Interpretation of data has been rather incomplete.

It is the purpose of this dissertation to investigate flux motion in cylindrical geometry and to explain at least qualitatively the behavior of flux motion in a lead-indium wire. A theory of flux flow proposed by Dr. Paul T. Sikora, visiting assistant professor of physics at the University of Oklahoma, will be presented and the data will be analyzed according to the models presented.

CHAPTER II

THEORY

Flux Flow

The dominant mechanism leading to the motion of flux is the Lorentz force (or more properly the Magnus force)

$$\vec{F}_L = \vec{J} \times \vec{B}$$

or
$$F_L = J B \sin \theta \quad (2.1)$$

where J is the current density,

B is the magnetic induction, and

θ is the angle between $\vec{J}$ and $\vec{B}$.

One can think of this motion of flux arising because of interaction between moving charges of the transport current J and the magnetic field B . Flux will move then in a direction that is perpendicular both to the transport current and to the magnetic field.

Much work has been done on flux motion in "slab geometry" with the external magnetic field normal to the plane of the sample.⁴⁻⁹ This geometry leads to a simplification in the Lorentz force since the field and current are at right angles. (The "wire field" contributed by the transport current does not affect the total field appreciably. Note also

that one does not need to assume the direction of current flow so long as the current flows parallel to the plane of the slab. This always provides $\vec{J} \perp \vec{B}$. Because of the very thin samples used, the angle between $\vec{J}$ and $\vec{B}$ is nearly always 90° even if the flow is not in the plane of the slab.) The flux moves in the plane of the sample and can have components in the direction of the transport current as well as transverse to it.

Kim, et al.⁴⁻⁷ have studied this geometric configuration in detail and have arrived at values for flux flow resistivity. Their data indicate the flux flow resistivity, ρ_f , is a constant for a given applied field and is easily obtained from a plot of V vs. I which is linear in the upper current range.

Alekseevskii, et al.¹⁰ have studied flux motion in a PbIn wire in a transverse magnetic field. Unlike the linear relation that one obtains for slab geometry on a V vs. I plot, they obtain a "quasi-linear section". No attempt is made to explain the results in great detail although a qualitative argument is presented.

In this dissertation we study the motion of flux through a cylindrical wire which is placed in an external longitudinal magnetic field. This geometry introduces difficulties not encountered in the slab geometry. It is clear that there will be two components of the total magnetic field (the longitudinal applied field and the field of the wire due to the transport current). The total field is then the vector sum of the fields,

$$|\vec{B}_{\text{tot}}| = (B_{\text{app}}^2 + B_w^2)^{1/2} \quad (2.2)$$

and the direction is given by the angle

$$\theta = \tan^{-1} (B_w / B_{app}) \quad (2.3)$$

where B_{tot} is the total field,

B_{app} is the longitudinal applied field, and

B_w is the field due to the wire.

One can visualize the resulting total field as a helix which becomes more tightly wound as the transport current is increased. Note that we are assuming that the transport current flows purely longitudinally, or along the length of the wire. This is probably true for small currents and for a homogeneous wire. If the current does seek a path along another direction, such as one-half the angle of the resultant field, then eq. (2.2) and eq. (2.3) are not valid and must be modified. However, for most of our current ranges the angle between the wire axis and the total field is very small so that our equations are valid to a very good approximation.

The helical shaped flux lines are acted on by the transport current. When the transport current exceeds a certain value, then a magnetic pressure difference which is established across the wire allows flux lines to penetrate the sample. Because flux motion takes place perpendicular to both $\vec{J}$ and $\vec{B}_{tot}$, the motion of flux is radially inward.

In order to understand how this flux motion occurs, consider the simpler case of a wire carrying a transport current in the absence of any externally applied field. The flux lines produced by the transport current are rings which close in on the sample and migrate to the center of the wire under the influence of the Lorentz force. For a large enough transport current the sample is driven completely normal due to the catastrophic collapse of the flux lines into the sample.

For the type II superconductor in a longitudinal external field, the helical fluxoids will probably enter the sample on the average along the direction of the total magnetic field. The flux line if uninfluenced by any other agents will enter the bulk of the wire where it experiences a force. This force tries to stretch or twist the helix, thereby lengthening the flux line and reducing its energy. (A similar process occurs for superfluid helium flowing in narrow channels.) If the helix is broken, snapped, or otherwise destroyed, energy is released and dissipation occurs. This appears to be a complicated process which we hope can be answered by experimentation.

Suppose now that the wire sample is not ideal but has surface irregularities. Then the flux line may have difficulties in breaking through the surface due to a "surface energy barrier". There are two types of surface barriers---one due to the physical condition of the wire and the other due to the image force.

For an irregular surface with physical or chemical defects, the surface energy may vary along the length of the wire. The points of low energy could then possibly act as nucleation sites for the flux lines to enter the sample. (This is what is thought to happen in superfluid helium flowing in narrow channels whenever channel irregularities introduce vortices.) Thus the flux lines may not enter uniformly along the length of the sample.

Once the flux line has entered the bulk of the sample it may be attracted back to the surface due to an image force, much like what occurs in liquid helium. This force is due to the current distribution surrounding the vortex line. The flux line must break away from this force before it can propagate through the sample.

Melville¹¹ found that the penetration of magnetic fields (flux lines) into a sample is very much dependent on the surface condition. If the surface of the sample is very rough, e.g., longitudinal scratches appear along the length of the sample, then the orientation of the magnetic field is important. For a magnetic field applied across rather than down the trough, the magnetic field is enhanced. This is due to the entry of flux at lower fields in the ridges. When the magnetic field is parallel to the scratches, the field enhancement is much less.

The flux line once inside and no longer affected by the surface can experience other mechanisms which may hinder its motion. Chemical or physical defects in the sample give rise to "pinning sites" which tend to impede the flux motion and pin the flux lines. Fig. 3 illustrates the pinning of a flux line by a defect.¹² The flux lines tend to congregate in the region of defect due to the difference of chemical potential.³ As time passes, the flux line becomes more and more curved until it escapes from its pinning center. The unpinned vortex line moves on and another vortex becomes pinned in its place.⁷ Thus the pinning is an average effect, so that the pinning force is an average per unit volume per unit time. The observed flux motion is a resultant of the pinning and unpinning motion of all the flux lines. This motion has been confirmed by Melville.¹³

Melville mentions the possibility of flux being shielded out by currents carried in a "surface sheath" near the surface (surface superconductivity). Of course Meissner currents can circulate on the surface of the sample to shield out flux and the amount of current supported depends on the surface condition. Melville attempted to reduce the effect

Figure 3. Pinning of Flux Lines at Pinning Sites

of the surface sheath by plating the surface of the sample. He found at low fields there was very little difference between plated and nonplated samples while at fields near H_{c2} there was a noticeable difference. This tends to indicate the surface sheath may be important only near H_{c2} .¹¹ We have also found that copperplating makes little difference on results taken at low fields. Our discussions shall be limited to fields near H_{c1} where the surface sheath will be neglected.

Bulk pinning can be reduced to some extent by annealing. Stresses and strains introduced in the extrusion of the wire sample can be annealed out by allowing the wire to sit at room temperature or by applying a temperature less than needed for melting. This process will not eliminate chemical defects which arise because of foreign atoms in the sample or will it change the irregularity in the crystal structure.

Finally, vortices may be pinned by interaction with other vortices. This type of pinning occurs in combination with one of the other pinning mechanisms mentioned previously.³ For example, flux lines clustered near a physical defect experience forces from the other flux lines due to the inhomogeneous structure of the array of flux lines in the bulk of the sample.

In summary, the driving force (Lorentz force) must be large enough to overcome surface barriers and any pinning effects to give rise to a viscous flow of flux. The effect is an average one of vortices moving through the sample and being dissipated within the bulk, thus giving rise to a voltage drop across the sample.

Model I: Purely Longitudinal,
Uniform Current Density Model

The dominant force in flux motion, the Lorentz force F_L , competes against the pinning force F_p in order to bring about viscous flow of flux. There are three possible regions of interest which arise. The first region involves $F_L < F_p$. This implies that the driving force is not sufficiently large enough to completely dominate the flux lines. This process is thought to be dominated by thermal activation, giving rise to some exponential current-voltage characteristic. This is denoted as "flux creep" and will not concern us. Beasley,¹⁴ Anderson,¹⁵ and Kim¹⁶ have given considerable thought to this region.

The second region occurs when $F_L \approx F_p$. This is a mixed region in which a transition is occurring between thermally activated creep and current dominated flow. This region is probably almost impossible to unscramble and will not be studied.

The third region is the one of interest for us and involves $F_L > F_p$. In this region motion is assumed to be viscous flow until the sample is driven into the normal state.

In the viscous region, the excess driving force is related linearly to the flux migration velocity by a proportionality factor, η , known as the viscosity coefficient, so that per unit length

$$F_L - F_p = F_{\text{excess}} = \eta \bar{v}_{\text{flux}} \quad (2.4)$$

Eq. (2.4) is true for any geometrical configuration. The viscosity coefficient is dependent on the material used and does not depend on the purity

of the sample.² The term $\bar{v}_{\text{flux}}$ is the average flow velocity during flux flow, and is possibly sample, temperature, and/or field dependent.⁷

Let us assume that the forces are per unit volume, so that the Lorentz force becomes

$$F_L = J \phi_o n \sin \theta$$

where n is the flux line density inside the wire (# lines/area) and is defined as

$$n = B_{\text{int}} / \phi_o$$

and B_{int} is the magnetic induction inside the wire where

$$\phi_o = 2.07 \times 10^{-7} \text{ gauss-cm}^2 ,$$

the unit of a fluxoid. The quantity B_{int} is unknown except that it is probably less than the total external field B_{tot} , and B_{int} is probably a function of B_{tot} . The Lorentz force becomes

$$F_L = J B_{\text{int}} \sin \theta .$$

From eq. (2.3) the angle is

$$\tan \theta = B_w / B_{\text{app}} .$$

For small angles, i.e., for $B_w / B_{\text{app}} \ll 1$, one can write

$$\sin \theta \approx \tan \theta$$

$$\sin \theta \approx B_w / B_{\text{app}} .$$

Thus

$$F_L \approx J B_{\text{int}} B_w / B_{\text{app}} . \quad (2.5)$$

The voltage arising along the length of the wire sample whenever the transport current exceeds its critical value is due to an induced e.m.f. generated by the motion of the magnetic flux in the moving cores. The dissipative motion of flux then produces a voltage and the sample shows resistance.

Because of viscous drag on the flux lines moving through the metal, energy is required to maintain the core motion. This can only come from the transport current which must do work in moving the flux. We can describe the current-voltage relation by equating the power input per unit volume in which the dissipation is occurring to the work done per unit time and volume by the driving force in moving a flux line a certain distance in the direction of the driving force. The power input is $I \cdot V$ and the dissipation volume is Ω . (We know very little about Ω except that rough calculations from some of our data show it is very small. Of course Ω can never be greater than the volume of the wire sample. Problems can possibly arise in estimating a value for Ω . For the derivation we will assume that Ω is independent of current or field.) The driving force per unit volume is the Lorentz force and the distance per time is the average flux line velocity. We can then write

$$\frac{I \cdot V}{\Omega} = \vec{F}_L \cdot \vec{v}_f \quad . \quad (2.6)$$

From eq. (2.4) and taking forces per unit volume so that η is per unit length, then we can write

$$v_f = \frac{F_L - F_P}{n \eta} \quad .$$

Substituting into eq. (2.6) and using eq. (2.5) with the definition of n ,

$$V = \frac{\Omega}{\eta} \frac{1}{I} \frac{\Phi_o}{B_{int}} \left[\left(J B_{int} \frac{B_w}{B_{app}} \right)^2 - F_P J B_{int} \frac{B_w}{B_{app}} \right] .$$

The assumption basic to this model is a uniform current flowing throughout the volume of the wire, so that the current density becomes

$$J = \frac{I}{A}$$

where A is the cross sectional area of the wire. The field due to the current in the wire is simplified by beginning with the expression from elementary electricity and magnetism

$$B_w = \frac{\mu_o I}{2\pi a}$$

and writing

$$B_w = c I$$

where

$$c \equiv \frac{\mu_o}{2\pi a} .$$

Thus

$$V = \frac{\Omega}{\eta} \frac{\Phi_o}{B_{app}} \left[\left(\frac{c}{A} \right)^2 \beta I^3 - \frac{c}{A} F_P I \right] \quad (2.7)$$

where we have defined the field ratio β as

$$\beta \equiv \frac{B_{int}}{B_{app}}$$

or

$$B_{int} \equiv \beta B_{app} .$$

Thus the voltage is cubic in current unlike the linear dependence of current and voltage in slab geometry with a transverse field.

Data are analyzed by plotting V/I vs. I^2 to achieve a straight line with

$$\beta = \frac{m B_{app}}{\Phi_0} \left(\frac{A}{c} \right)^2 \left(\frac{\eta}{\Omega} \right) \quad (2.8)$$

$$F_P = \frac{-b B_{app}}{\Phi_0} \left(\frac{A}{c} \right) \left(\frac{\eta}{\Omega} \right) \quad (2.9)$$

where m is the slope from the V/I vs. I^2 plot, and

b is the intercept.

The quantities β and F_P are defined in terms of (η/Ω) and can be difficult to obtain. We postpone discussion of the viscosity coefficient-volume ratio until later.

Several important things can be learned from eq. (2.8) and from eq. (2.9). Notice that β is dependent only on the slope of the data obtained from the V/I vs. I^2 plot and on constants depending upon the wire diameter. Thus the larger slopes indicate larger field penetration. For B_{app} very near H_{c1} we expect that B_{int} is quite different from B_{app} (or B_{tot}). Since this is the region of interest and the range of fields investigated is much smaller than the difference $H_{c2} - H_{c1}$, we expect the slopes to be fairly constant for each sample. Thus the field penetration ratio will be approximately the same for all fields. The ratio may vary, however, from sample to sample due to various surface and bulk differences.

One of the striking features of the pinning force is the need for a negative intercept, i.e., b must be negative. Now I_p is found from the extended slope at $V/I = 0$ and is defined from eq. (2.7) as the squared pinning current

$$I_p^2 = \frac{-b}{m} .$$

Eq. (2.9) indicates the pinning force is strongly dependent on the intercept and weakly dependent on the slope. This implies that the intercept is a measure of the pinning strength. For slopes that are nearly field independent, F_p increases with increasing intercept. This necessarily implies that a large pinning force is associated with a large pinning current, i.e., $F_p \propto I_p^2$. Thus F_p is field dependent and increases with decreasing field. (These results are also obtained from the slab geometry.⁴⁻⁶) If the excess force is constant for a given sample and F_p decreases with decreasing field, then from eq. (2.4) F_L decreases. Flux flow can then occur at lower currents for higher applied fields.

One can now use eq. (2.7) to obtain a value for the flux flow resistivity ρ_f and compare its value to that obtained for other geometries. This may be difficult because of the cylindrical geometry.

For the slab geometry, the flux flow resistance is defined as the slope of the linear region on a V vs. I plot at constant applied field. For the slab geometry in a transverse field and from eq. (2.6)

$$V = \frac{1}{n} \frac{\Omega}{\eta} \frac{1}{I} \left(F_L^2 - F_L F_P \right)$$

where now the Lorentz force is given as

$$F_L = J B$$

and B is to a good approximation $B_{\text{int}} = B_{\text{tot}} = B_{\text{app}}$. So we have

$$V = \frac{\Omega}{\eta} \frac{1}{n} \left[\frac{I}{A^2} B_{\text{tot}}^2 - \frac{F_P B_{\text{tot}}}{A} \right]$$

$$\therefore \frac{dV}{dI} = \frac{\Omega}{\eta} \frac{B_{\text{tot}}^2}{n A^2} \equiv R_f \quad . \quad (2.10)$$

Now differentiate eq. (2.7) for the cylindrical wire in a longitudinal field:

$$\left(\frac{dV}{dI} \right)_{\text{long.}} = \frac{\Omega}{\eta} \frac{1}{n} \left[\left(\frac{c}{A} \right)^2 \beta^2 3 I^2 - \left(\frac{c}{A} \right) F_P \beta \right] \quad .$$

Using eq. (2.10) from the slab geometry to define R_f , then

$$\left(\frac{dV}{dI} \right)_{\text{long.}} = 3 c^2 \frac{\beta^2}{B_{\text{tot}}^2} R_f I^2 - \frac{\Omega}{\eta} \frac{c}{A} F_P \beta \quad . \quad (2.11)$$

As expected, the derivative is parabolic in I. The constant term should not vary appreciably within a given sample if $\beta \ll 1$.

One can take eq. (2.11), solve for R_f , and convert to flow resistivity ρ_f ,

$$R_f = \frac{B_{\text{tot}}^2}{3 c^2 \beta^2 I^2} \left(\frac{dV}{dI} \right) + \frac{\frac{\Omega}{\eta} \frac{c}{A} F_P B_{\text{tot}}^2}{3 c^2 \beta I^2} \quad .$$

Assuming small angles so that $B_{\text{tot}} \approx B_{\text{app}}$, then

$$R_f = \frac{B_{\text{app}}^2}{3 c^2 \beta^2 I^2} \left(\frac{\frac{dV}{dI}}{1 - \frac{F_P}{3 F_L}} \right) \quad .$$

In the region of interest $F_L \gg F_P$ so that

$$R_f \approx \frac{B_{app}^2 \frac{dV}{dI}}{3 c^2 \beta^2 I^2} \quad (2.12)$$

The resistivity is then

$$\rho_f = R_f \frac{A}{L}$$

where L is the sample length, or

$$\rho_f = \frac{B_{app}^2 \frac{dV}{dI}}{3 c^2 \beta^2 \frac{L}{A} I^2} \quad (2.13)$$

Let us make some comments about the flow resistivity. Recall from eq. (2.8) that the field ratio β involves the term (Ω/η) . This leads to an unsolvable problem for ρ_f as an exact number, i.e.,

$$\rho_f = \text{constant} \times B_{app}^2 (\Omega/\eta)^2$$

Expressing ρ_f in this form is confusing since the ambiguity is with β and not really (Ω/η) . The easiest way out of the dilemma is to run the experiment on slab geometry where this ratio does not appear and solve for (Ω/η) in eq. (2.13) by using the ρ_f vs. B_{app} data from the slab. One finds that (Ω/η) is constant for all fields on one sample and is nearly constant for all samples. Since the viscosity coefficient is sample independent and is characteristic of the material used, then this implies that the dissipation volume term is nearly constant for all samples. The dissipation is occurring in about the same volume for all samples.

Farrell, et al.⁶ found that plotting $V/I_P R_f$ vs. I/I_P forced all his various data to coalesce into a narrow band and thereby made the plot independent of temperature, composition, etc. Very little satisfying explanation is given as to why the coalescence arises.⁶

To continue the analogy with slab geometry we attempt to obtain coalescence for the various samples with their different fields. By writing eq. (2.7) in slope-intercept form and using the pinning force in terms of pinning current, then

$$\frac{V/I}{\frac{\Omega}{\eta} \frac{\Phi_o}{B_{app}} \left(\frac{c}{A}\right)^2 \beta I_P^2} = \left(\frac{I}{I_P}\right)^2 - 1 \quad (2.14)$$

We define

$$K \equiv \left[\frac{\Omega}{\eta} \frac{\Phi_o}{B_{app}} \left(\frac{c}{A}\right)^2 \beta I_P^2 \right]^{-1} \quad (2.15)$$

as the normalization factor for V/I and I_P as the normalization factor for I . The factor K is $-1/b$ and represents a shifting of curves to force all the intercepts to be the same.

If we make a plot of KV/I vs. $(I/I_P)^2$ then we should obtain a straight line with a slope of unity and an intercept of negative unity. Chandrasekhar has forced his slope and intercept to be these values also by his method of plotted reduced variables.

Thus, the only real significance of making such a reduced plot is to check on the linearity of an equation. There will be scatter if the form of the equation is really

$$V = c_1 I^3 + c_2 I + f(I)$$

where $f(I)$ is some function of I . These extra terms of $f(I)$ may explain the sometimes gentle curving of the linear portion at large currents when the small angle approximation is becoming invalid.

Model II: Constant Lorentz Force Model

One of the shortcomings of the purely longitudinal, uniform current density model is the indeterminate dissipation volume Ω . This volume is questionable since it is not determined from experimental data and becomes an adjustable parameter to match resistivity measurements.

The concept of dissipation occurring in a narrow sheath is questionable from the persistatron behavior. Suppose that we have two parallel portions of the wire, both carrying a current. The model suggests that each segment is carrying the same amount of current, but one segment is supporting a voltage drop due to dissipation while the other is not. From the persistatron theory¹⁷ one finds this is an unstable or possibly metastable configuration which decays toward a configuration in which dissipation is occurring over an entire cross section.

Suppose that we now propose a model that does not involve a dissipation volume, but instead consider the dissipation occurring equally over the entire volume of the sample, except possibly a small region near the wire axis which must be considered normal. Measurements of $B(r)$ by Sikora, Makiej, and Trojnar¹⁸ for a type II wire in the mixed state indicate that the current density increases to a maximum at the wire axis. They present curves that show the typical variation of the magnetic induction as a function from the center of the wire. Data by Sikora, et al. support the concept of nonuniform current density.

The concept of nonuniform current density leads us to the requirement of a constant Lorentz force, independent of the radial distance off the axis for a homogeneous wire. Recall that the Lorentz force per unit length of flux line is given by

$$F_L = J \phi_o \sin \theta$$

and the current is assumed purely longitudinal. From elementary electricity and magnetism the field at a distance r from a wire of radius a is given by

$$B_w(r) = \frac{\mu_o i(r)}{2\pi r} \quad r \geq a$$

$$B_w(r) = \frac{\mu_o i}{2\pi r} \frac{\pi r^2}{\pi a^2} \quad r \leq a$$

Since in the previous model we assumed $J \neq J(r)$ and $B_w(r) \propto r$, then clearly $F_L \propto B J \propto r$ and we see that the Lorentz force decreases as we move inward toward the wire axis. Thus a current distribution is needed that increases with decreasing r to make the Lorentz force independent of position in the wire. This modification of $J(r)$ to produce such a Lorentz force will probably not be very much since this modification also results in a modification of $B_w(r)$.

Suppose we search for $J = J(r)$ in the small angle approximation by recalling that per unit length,

$$F_L \approx J \phi_o \frac{B_w}{B_{app}}$$

and

$$B_w(r) = \frac{\mu_o i(r)}{2\pi r}$$

$$B_w(r) = \frac{\mu_o}{2\pi r} \int_A \vec{J}(r) \cdot d\vec{A} \quad .$$

For cylindrical geometry of azimuthal symmetry, we write

$$B_w(r) = \frac{\mu_o}{2\pi r} \int_0^r J(r') 2\pi r' dr'$$

$$F_L = \frac{\phi_o \mu_o}{B_{app}} \frac{J(r)}{r} \int_0^r J(r') r' dr'$$

$$K \equiv \frac{J(r)}{r} \int_0^r J(r') r' dr' \quad . \quad (2.16)$$

In order to solve eq. (2.16) assume a power law form for $J(r)$, i.e.,

$$J(r) = J_o \left(\frac{a}{r} \right)^n \quad . \quad (2.17)$$

Substituting eq. (2.17) into eq. (2.16) one finds that

$$K = \frac{J_o^2 a^{2n}}{2-n} r^{-2n+1} \quad .$$

For this to be a constant we choose $n = \frac{1}{2}$,

$$K = \frac{2}{3} J_o^2 a \quad .$$

Thus eq. (2.16) is indeed a constant if we select $n = \frac{1}{2}$ so that eq. (2.17) becomes

$$J(r) = J_0 \left(\frac{a}{r} \right)^{\frac{1}{2}} . \quad (2.18)$$

It is interesting to note that Sikora, et al.¹⁸ have curves of $B(r)$ which make it possible to find $J(r)$. One of their curves (Fig. 2, curve C) was analyzed by computer and we found from their data that $B(r) \propto r^{\frac{1}{2}}$ so that $J(r) = dB/dr \propto r^{-\frac{1}{2}}$. An "instantaneous slope" method in conjunction with the computer yielded similar results that $J(r) \propto r^{-\frac{1}{2}}$.

Figure 4. Current Distribution Inside a Superconductor

We can make a sketch of $J(r)$ vs. r to get some idea of its physical variation throughout the wire. Fig. 4 shows a square root singularity at the origin that can be eliminated by assuming that the core of the wire is normal so that $J(r) = J_c$ for $0 \leq r \leq r_c$, which is

determined by requiring an equal voltage drop between sample ends. The current thus falls to its critical value J_c at the core, and the square root singularity is removed.

Even though the Lorentz force is not a function of r and the dissipation is occurring uniformly throughout the volume of the wire, the dissipation rate is still not fixed at a constant because the flux lines can possibly "pack" as they penetrate the wire. Eq. (2.4) suggests that for constant F_L and F_P the flux line velocity v_f should also be constant. This implies an inhomogeneous flux density as a function of r and consequently a dissipation volume rate which is largest at points near the axis of the wire in steady state.

If v_f is a constant, independent of r except for $r \leq r_c$, then the flux density must increase inversely as r . We expect this to occur when the flux density is low everywhere and the interaction of flux lines is almost negligible. On the other hand, if the flux line velocity increases as the radius decreases, then the rate of increase of the flux density will be less than r^{-1} . This would result in a build-up of lines, or an increase in the flux density near the center of the wire. The interaction among flux lines will be significant and could possibly revise the picture of flux motion. We would also expect some modification to occur when the spacing of the flux lines becomes comparable with the penetration depth. One finds that the assumption of flux conservation cannot be maintained, i.e., flux lines must dissipate at the core radius.

An appropriate choice of flux density $n(r)$ makes the dissipation density constant. This is found by considering the dissipation due to the driving Lorentz force. One begins with eq. (2.4) and writes it

per unit volume. Looking just at the Lorentz force terms we can determine if the dissipation rate is uniform.

$$F_L \cdot v_f = \frac{F_L^2}{n \eta}$$

$$F_L \cdot v_f = \frac{1}{\eta n(r)} \left[J(r) \phi_o n(r) \frac{B_w}{B_{app}} \right]^2$$

$$F_L \cdot v_f = \frac{1}{\eta} \left[\frac{\phi_o J_o a^{\frac{1}{2}} \mu_o}{B_{app}} \frac{1}{n(r)} \frac{1}{r^{3/2}} J_o a^{\frac{1}{2}} \int_0^r r^{\frac{1}{2}} dr \right]^2$$

$$F_L \cdot v_f = \frac{1}{\eta} \left[\frac{2\phi_o J_o^2 a^2 \mu_o}{3 B_{app}} \right]^2 \frac{1}{n(r)^2}$$

Thus if $F_L \cdot v_f$ is a constant, then this implies that $n(r)$ is also a constant. Therefore,

$$n(r) = \frac{B_{int}}{\phi_o} . \quad (2.19)$$

We have constructed a model with two very important features:

- (1) the Lorentz force per unit volume is a constant throughout the wire,
- (2) the dissipation is occurring uniformly throughout the wire.

The same techniques employed in Model I are used to find the current-voltage characteristics for this model. The power dissipation in the wire is equated to the power supplied by the driving force,

$$I \cdot V = \int F_L \cdot v_f d^3r . \quad (2.20)$$

Recalling that the Lorentz force per unit volume is given by

$$F_L = J n \phi_o \sin \theta$$

and the excess force by

$$F_L - F_P = n \eta v_f ,$$

then the power dissipated by the driving force on the flux line is

$$F_L \cdot v_f = \frac{1}{n \eta} \left(F_L^2 - F_L F_P \right) .$$

Using the small angle approximation so that $\sin \theta \approx B_w / B_{app}$ and using the flux density condition stated by eq. (2.19), the condition of constant dissipation per unit volume is satisfied and we can write

$$I \cdot V = \frac{\phi_o}{B_{int} \eta} \int \left\{ \left[B_{int} \frac{B_w}{B_{app}} J(r) \right]^2 - J(r) B_{int} \frac{B_w}{B_{app}} F_P \right\} d^3r$$

$$I \cdot V = \frac{\phi_o \pi a^2 L}{B_{int} \eta} \left\{ \left[\beta J(r) B_w \right]^2 - \beta F_P J(r) B_w \right\}$$

$$I \cdot V = \frac{\phi_o \pi a^2 L}{\beta \eta B_{app}} \left\{ \beta^2 \left[J_o \left(\frac{a}{r} \right)^{\frac{1}{2}} \frac{\mu_o}{2\pi r} \int_0^r J(r') 2\pi r' dr' \right]^2 - \beta F_P J_o \left(\frac{a}{r} \right)^{\frac{1}{2}} \frac{\mu_o}{2\pi r} \int_0^r J(r') 2\pi r' dr' \right\}$$

$$I \cdot V = \frac{\phi_0 \pi a^2 L}{\eta B_{app}} \left\{ \beta \left(J_0^2 \frac{2\mu_0 a}{3} \right)^2 - F_P \left(J_0^2 \frac{2\mu_0 a}{3} \right) \right\} . \quad (2.21)$$

Let us evaluate J_0 , which is $J_{surface}$. Now $J(r)$ is fixed by the condition

$$I = \int_0^a J(r) 2\pi r dr$$

for cylindrical symmetry. Considering the core volume and the other volume separately

$$I = 2\pi \int_0^{r_c} J_c r' dr' + 2\pi \int_{r_c}^a J(r') r' dr' .$$

Substituting for $J(r')$ from eq. (2.18) and integrating,

$$I = \frac{4\pi}{3} J_0 a^{\frac{1}{2}} (a^{3/2} - r_c^{3/2}) + \pi r_c^2 J_c .$$

The current J_c is an intrinsic function of the alloy and r_c is the critical radius at which the current J_c takes place. As long as $r_c \ll a$ and the core contribution is by volume, then the contribution of the core is negligible and we can write approximately,

$$J_0 = \frac{3I}{4\pi a^2} . \quad (2.22)$$

Comparison of the current density to one of a uniform distribution yields

$$J = \frac{I}{\pi a^2} .$$

The surface current density is thus down by 25% from the uniform current density case due to the factor 3/4.

In order to find the voltage produced by the moving flux lines, substitute for J_o from eq. (2.22) into eq. (2.21)

$$V = \frac{\pi a^2 L \phi_o}{\eta B_{app}} \frac{1}{I} \left\{ \beta \left[\frac{9 I^2}{16 \pi^2 a^4} \frac{2 \mu_o a}{3} \right]^2 - F_P \left[\frac{9 I^2}{16 \pi^2 a^4} \frac{2 \mu_o a}{3} \right] \right\}$$

$$V = \frac{L \phi_o}{\eta B_{app}} \left[\beta \left(\frac{3}{8} \mu_o a \right)^2 \left(\frac{I}{A} \right)^3 - F_P \left(\frac{3}{8} \mu_o a \right) \frac{I}{A} \right] .$$

Define

$$k \equiv \frac{3 \mu_o a}{8} ,$$

$$\therefore V = \frac{L \phi_o}{\eta B_{app}} \left[\beta k^2 \left(\frac{I}{A} \right)^3 - F_P k \frac{I}{A} \right] . \quad (2.23)$$

Thus the current-voltage characteristic for this model is cubic in current, just like the uniform current density model.

If eq. (2.23) is to be analyzed, data must be plotted in the same way as the first model. Defining the slope as m and the intercept as b on a plot of V/I vs. I^2 , one can solve for the field penetration ratio as well as the pinning force in terms of known quantities

$$\beta = \frac{m A^3}{L \phi_o k^2} B_{app} \eta \quad (2.24)$$

$$F_p = \frac{-b}{L} \frac{A}{\Phi_0 k} B_{app} \eta \quad . \quad (2.25)$$

Notice that the slope is an indication of the field penetration and the intercept is a measure of the pinning force. As was encountered in the first model, β and F_p are defined in terms of the unknown viscosity coefficient η . It may not be possible to obtain numbers for β and F_p without relying on other data. This was also the case encountered in the first model.

To find the resistivity differentiate eq. (2.23). First rewrite the equation and consider just the first term, since the second term is unimportant in resistance measurements. The second term contains the pinning force which should not affect the flow resistivity. Thus

$$V \sim \frac{L \Phi_0}{\eta B_{app}} \beta \left(\frac{3}{8} \mu_0 a \right)^2 \left(\frac{I}{A} \right)^3$$

$$\frac{dV}{dI} = \frac{3 L \Phi_0}{\eta A B_{app}} \beta \left(\frac{3 \mu_0 I}{8 \pi a} \right)^2 \quad . \quad (2.26)$$

We identify the term in parenthesis by considering the "average" magnetic induction in the wire. We assume that

$$\overline{J B} = \overline{J} \overline{B}$$

$$\overline{J B} \equiv \frac{\mu_0}{2\pi r} J(r) \int_0^r J(r') 2\pi r' dr' \quad .$$

Substituting for $J(r)$ from eq. (2.18), we can then write the "average"

magnetic induction in the wire as

$$\overline{J B} = \frac{2}{3} \mu_o J_o^2 a \quad . \quad (2.27)$$

Using eq. (2.22) and the definition of $\overline{B}$ we can write

$$\overline{J B} = \frac{3 \mu_o I^2}{8 \pi^2 a^3}$$

$$\overline{B}_w = \frac{\overline{J B}}{\overline{J}}$$

$$\overline{B}_w = \frac{3 \mu_o I}{8 \pi a} \quad . \quad (2.28)$$

The term in eq. (2.26) is an average magnetic volume induction. This is similar to the magnetic induction at the surface of a wire carrying a uniform current density

$$B_w = \frac{\mu_o I}{2 \pi a} \quad ,$$

the only difference is a factor "3/8" rather than "1/2". Substituting eq. (2.28) into eq. (2.26)

$$\frac{dV}{dI} = \frac{3 L \phi_o}{A \eta B_{app}} \beta \overline{B}_w^2$$

$$\frac{dV}{dI} = \frac{3 L \phi_o \beta \tan^2 \bar{\phi}_c B_{app}}{A \eta} \quad (2.29)$$

where

$$\tan \bar{\phi}_c \equiv \frac{\overline{B}_w}{B_{app}} \quad .$$

Now the problem arises as how to define a resistance from eq. (2.29). Consider the slab in a transverse field by beginning with eq. (2.20) and realizing that the Lorentz force is

$$F_L = J \phi_o n .$$

$$F_L \cdot v_f = \frac{1}{n \eta} \left[F_L^2 - F_L F_P \right]$$

$$I \cdot V = \int F_L \cdot v_f d^3r$$

$$I \cdot V = \frac{A L}{n \eta} \left[(J n \phi_o)^2 - (J n \phi_o) F_P \right]$$

$$V = \frac{L}{\eta} \left[\frac{n \phi_o^2}{A} I - \phi_o F_P \right]$$

$$\frac{dV}{dI} = \frac{L}{\eta} \frac{n \phi_o^2}{A} .$$

There is no question about resistance in the linear region,

$$R_f \equiv \frac{dV}{dI}$$

$$R_f = \frac{L B_{app}^2}{n A \eta} . \quad (2.30)$$

Comparing with eq. (2.29),

$$\frac{1}{\beta \tan^2 \phi_c} \frac{dV}{dI} = \frac{3 L \phi_o B_{app}}{A \eta}$$

$$\frac{1}{3 \beta^2 \tan^2 \bar{\phi}_c} \frac{dV}{dI} = \frac{L B_{app}^2}{n A \eta} \equiv R_f . \quad (2.31)$$

And thus the resistivity is given by

$$\rho_f = \frac{\frac{dV}{dI} \frac{A}{L}}{3 \beta^2 \tan^2 \bar{\phi}_c} = \frac{B_{app}^2}{n \eta} . \quad (2.32)$$

One can either use the right hand side or the left hand side to calculate the resistivity.

Notice from eq. (2.32) that one encounters problems similar to those of eq. (2.13) of the first model. The resistivity can be determined only to within the constant β or η . This means that in order to obtain a pure number for ρ_f , these quantities must be evaluated by using data from another experiment. This problem does not arise in slab geometry with transverse field since $\beta=1$ and all the quantities of the left hand side of eq. (2.32) are determined from experiment. The number for ρ_f can be equated with the right hand side to find the viscosity coefficient.

Comparison of Models

One finds striking similarities between the two models. Recall that the first model assumes a uniform current density and a nonuniform Lorentz force with dissipation occurring in a small volume of the wire, the dissipation volume Ω . The second model, on the other hand, results in the assumption of nonuniform current density and a uniform, or constant, Lorentz force with dissipation occurring uniformly throughout the volume of the wire. Both models equate the power dissipated to the power

supplied by the driving force, F_L . The satisfying result is that both models give a current-voltage characteristic showing that the voltage produced during flux flow is proportional to the third power of the transport current.

We will now show that both models reduce to the same results (within a constant) and thus give the same resistivity and comparable field ratios and pinning forces. Suppose we force the resistivities to be the same by both models, which they hopefully should be. (Later we will show that the resistivities are exactly the same by both models.) Assume that

$$\rho_I = \rho_{II} \quad . \quad (2.33)$$

Equating eq. (2.13) of Model I with eq. (2.32) of Model II gives

$$\frac{B_{app}^2 \frac{dV}{dI} \frac{A}{L}}{3 \beta_I^2 B_w^2} = \frac{B_{app}^2 \frac{dV}{dI} \frac{A}{L}}{3 \beta_{II}^2 \bar{B}_w^2}$$

$$\beta_I^2 B_w^2 = \beta_{II}^2 \bar{B}_w^2 \quad .$$

Substituting for B_w from eq. (2.28a) and for $\bar{B}_w$ from eq. (2.28) one obtains

$$\beta_I = \frac{3}{4} \beta_{II} \quad (2.34a)$$

or

$$\beta_{II} = \frac{4}{3} \beta_I \quad . \quad (2.34b)$$

Let us find the relationship between β_I from eq. (2.8) and β_{II} from eq. (2.24),

$$\beta_I^2 = \frac{m A^2}{n c^2} \frac{\eta}{\Omega} \quad (2.8)$$

$$\beta_{II} = \frac{m A^3}{L \phi_o k} B_{app} \eta \quad (2.24)$$

First rewrite β_{II} by recalling that $n\phi_o = B_{int}$,

$$\beta_{II}^2 = \frac{m A^3}{L k^2} \frac{n \eta}{\Omega} \quad .$$

Forming the ratio of β_I to β_{II} and assuming that the dissipation volume Ω is the sample area times the sample length which cancels with the flux line density, then

$$\frac{\beta_I^2}{\beta_{II}^2} = \frac{k^2}{c^2 A^2} \quad .$$

Using the definitions of c from eq. (2.7) and k from eq. (2.23) gives

$$\frac{\beta_I^2}{\beta_{II}^2} = \left(\frac{3}{4} \right)^2$$

$$\therefore \beta_I = \frac{3}{4} \beta_{II} \quad (2.34a)$$

$$\text{or} \quad \beta_{II} = \frac{4}{3} \beta_I \quad (2.34b)$$

It is very satisfying that the formulation of eq. (2.34) results in the resistivity by both models giving the same result, as was the assumption of eq. (2.33). This is what one naturally hopes for. The factor "4/3" turns out to be significant in another way, as seen from comparing B_w and $\bar{B}_w$ from eq. (2.28a) and eq. (2.28).

$$\frac{\bar{B}_w}{B_w} = \frac{3 \mu_o I}{8 \pi a} \div \frac{\mu_o I}{2 \pi a}$$

$$\bar{B}_w = \frac{3}{4} B_w \quad (2.35a)$$

or
$$B_w = \frac{4}{3} \bar{B}_w . \quad (2.35b)$$

Thus the difference in β between models arises because of the difference in form of the field due to the transport current, i.e., because of the assumption of the current distribution in the wire.

Other Models

Both Model I and Model II were proposed on the assumption of purely longitudinal current flow. This is known to be incorrect from numerous experiments. The consequence of total axial flow is a maximum dissipation since any paramagnetic spiral of the current reduces the angle between the transport current and the helical magnetic field and thus reduces the magnitude of the Lorentz force, as seen by eq. (2.1).

Several other models have been devised to try to explain the effects of flux motion in a longitudinal field. LeBlanc, Belanger, and Fielding¹⁹ report on two models, one of which is compatible with our geometry. The Force-Free Model assumes that the current density is

directly related to the field as

$$\vec{J}(r) = f(B,r) \vec{B}(r) .$$

The assumption is made that

$$f(B,r) = \alpha(H_T)$$

where H_T is the total field at the surface of the wire and α is a constant. Thus, for this model, $\vec{J} \times \vec{B} = 0$, so that the Lorentz force is zero and no flux flow can occur. Agreement between data and theory is limited to an ideal type II superconductor.¹⁹ A reversible type II superconductor can carry much higher currents in longitudinal rather than transverse fields due to the zero Lorentz force.²⁰

The Meissner Model (Nearly Force-Free Model) of LeBlanc, et al.¹⁹ gives good qualitative agreement with our models. Their model assumes a uniform current density j_z and makes use of Silsbee's rule to find the total field. The model requires the magnetic pressure gradient to be zero throughout the specimen, i.e.,

$$P_m \equiv \nabla B^2 = 0 ,$$

and the Lorentz force to be nonuniform and inward. The consequence of the model is that the current spirals at approximately one-half the pitch of the magnetic field, i.e.,

$$\tan \phi_c = \frac{1}{2} \tan \phi_f$$

where ϕ_c is the angle between axis and current flow and

ϕ_f is the angle between axis and total magnetic induction B .

This is similar to our models, except our current is assumed to flow along the direction of the axis. The factor " $\frac{1}{2}$ " due to the field and current spiral is introduced in the Lorentz force calculation, so that their Lorentz forces are one-half of our values.¹⁹

LeBlanc²¹ proposed another model in which the Lorentz force is a constant and is defined by the usual equation, such as eq. (2.1). The current density then adjusts itself to allow for this to occur. The current density is the vector sum of axial and azimuthal current densities,

$$|\vec{j}| \equiv (j_z^2 + j_\theta^2)^{\frac{1}{2}} = j_c \quad .$$

LeBlanc adds the additional restriction that the longitudinal current density $|\vec{j}_z|$ is uniform and he allows $|\vec{j}_\theta|$ to change in magnitude to provide for a constant Lorentz force. Starting with Maxwell's equation,

$$\frac{4\pi}{10} j_\theta = - \frac{dB_z}{dr} \quad ,$$

LeBlanc derives an equation for $B_z(r)$. In order to compare these results with either of our models, we would have to decide, for example, on the average value of j_z for LeBlanc's model to solve for some current-voltage relation.

London and Walmsley²⁰ note that flux flow occurs when $\vec{j} \times \vec{B} \neq 0$. They found that often the flux flow phenomenon is not clearly identifiable at large fields and do not offer an explanation.

CHAPTER III

THE EXPERIMENT

Experimental Design

In order to investigate flux motion in a type II superconductor, a suitable sample should be selected. An alloy, if chosen, should be a good type II superconductor, should be easy to prepare, should be very homogeneous, and should be easy to work with.²² Due to these reasons as well as others, lead-indium alloys were selected for investigation. It is well-known that lead-indium alloys are very homogeneous in the range from 35 atomic percent to 95 atomic percent lead. This is evident from the phase diagram in Fig. 5.²³ An alloy of 83 atomic percent lead and 17 atomic percent indium was arbitrarily chosen since published data exist on magnetization and flux flow for slab geometry in longitudinal and transverse magnetic fields.⁴⁻⁶ These results may be helpful in analyzing flux flow data of cylindrical geometry in a longitudinal magnetic field.

Sample Preparation

The starting materials employed were 99.9999% pure lead wireⁱ of 4 mm diameter, 99.9999% pure indium shot,ⁱⁱ and 99.999% pure indium wireⁱ

ⁱThe lead and indium wires are supplied by Electronic Space Products Inc., Los Angeles.

ⁱⁱThe indium shot is supplied by Cominco America Inc., Spokane, Washington.

Figure 5. Lead-Indium Phase Diagram

of 0.0254 cm diameter. The lead wire was first cleaned with an etching solution of 4 parts glacial acetic acid and 1 part 30% hydrogen peroxide before any measurements were taken.²⁴ The etching solution seemed to react very quickly with PbIn, so the solution was diluted to 8 parts acetic acid and 1 part hydrogen peroxide.

The atomic percentages were converted to weight percentages so that the lead and indium could be weighed directly on an analytical balance.²³ This means now that one needs 89.82% by weight of Pb and 10.18% by weight of In in order to obtain a sample of $\text{Pb}_{83}\text{In}_{17}$ (where the subscripts denote atomic percentages).

By considering the dimensions of the sample of Farrell, et al.⁶ in the slab geometry, it was decided to make a comparable sample of cylindrical wire of diameter 0.0508 cm. After careful discussions with Mr. Gene Scott (shop foreman at the University of Oklahoma), it was ascertained that an extrusion die could be made to work in conjunction with a 30 ton hydraulic press of Dr. Stanley Babb, Jr. (Professor of Physics). The extrusion apparatus was designed as shown in Fig. 6.

After the extrusion apparatus was constructed, the proper weights of lead and indium were placed in a Pyrex glass holder to be mixed and cast into cylindrical form to fit the extruder. The tube was flushed with argon, evacuated and sealed at 5 microns of pressure, and placed in a furnace to make the final casting. The glass holder containing the sample was rotated for 30 minutes at a rate of 30 revolutions per minute, allowed to remain at 350° C for 1½ hours, shook for 30 minutes, and then air quenched.

The PbIn slug was cleaned with the etching solution before being placed in the apparatus for extrusion. The slug and extruder were placed

(ALL MEASUREMENTS
ARE GIVEN IN CENTIMETERS)

Figure 6. Extrusion Apparatus

under the 30 ton press and pressure was applied very slowly, allowing the sample to come to thermal equilibrium and flow uniformly. At 5 tons of pressure the wire began extruding and continued for a few minutes until approximately 5 feet of wire were obtained and the pressure had dropped to 3 tons. During this time, a constant force was maintained on the handle operating the hydraulic press in an attempt to maintain a constant pressure on the sample being extruded. As the extrusion process continued, the 0.0508 cm wire was wound onto a metal spool of 2 inch diameter. The spool and wire were stored in liquid nitrogen to prevent annealing of the wire.

Magnet Design

One must design a suitable sample holder and a magnet to be used to apply an external longitudinal magnetic field. We required a linearity of 0.1% over the 2 cm length of the sample. The first magnet used in this experiment was 10 inches long and was wound to a linearity of 0.1% over the 7 cm sample. The main drawback of the magnet is its length which requires 5 liters of liquid helium to cover it in the helium Dewar used. This long magnet also requires long samples and consequently considerable amounts of PbIn wire. The magnet was eliminated in favor of a shorter one which was used to take final data.

The superconducting magnet is linear over one inch to 0.1%. The magnet itself, as shown in Fig. 7, is 4 inches long and is wound, with the aid of a variable speed lathe, on a hollow phenolic core of one-half inch diameter. Calculations were made to determine the number of layers of NbZr wire needed to give 0.1% linearity over one inch with an approximate

Figure 7. Superconducting Magnet

rating of 2,000 gauss at 5 amps.ⁱ The magnet needs a turn density per layer of 28.1 turns/cm for 13 layers to meet these specifications. A calculation was made using the formula for a finite solenoid to determine a theoretical coil constant. The magnet was later experimentally calibrated using a Hall probe.²⁵

Sample Holder

There was some question concerning the material to be used as a sample holder. We found that the sample cannot be annealed in the holder due to an interaction between PbIn and PbSn solder. The sample holder will not have to withstand large annealing temperatures, so phenolic can be used as starting material.

Phenolic has the advantage of being very easy to work with. Fig. 8 shows two types of phenolic holders employed in the experiment. The holders are very easily machined and can be constructed in less than a day. The holder is obtained from a solid cylinder of one-half inch diameter phenolic that is split lengthwise to fit the core of the magnet.

Voltage Leads

A four-probe technique is employed in data taking. This implies that voltage leads are attached in some manner to the sample. The obvious method is to solder the voltage lead (#34 gauge copper wire) directly to the sample. This is a standard technique used by experimenters measuring thermomagnetic effects.^{26,27} They use point-soldering and an indium-base

ⁱThe alloy wire is available from Norton-Supercon Division, Natick, Mass., and has a diameter of 0.0127 cm with a layer of 0.0051 cm of formvar insulation.

HOLDER WITH GROOVES

HOLDER WITH CLAMPS

HOLDER WITH REMOVABLE SECTION

Figure 8. Sample Holders

solder with a Variac to vary the heat. Another experimenter in flux flow studies obtained best results when the voltage leads were wrapped around the sample and then soldered with Woods Metal.²⁸

Several soldering techniques were attempted using a "Research Solder Kit"²⁹ containing 15 indium-based solders and 5 fluxes. Merely reheating of the solder at the point of attach of voltage leads to the sample results in a drastic change in current-voltage data of the sample.

Silver paint, which is nearly inert and requires no heat to apply, is also inadequate as an agent for connecting voltage leads. Paint additives increase the paint resistance and introduce contact resistance which varies from sample to sample.

It appears that the voltage leads should be attached to the sample without introduction of another element. The only immediate alternative is to make mechanical connections, such as clamping the sample and voltage leads together as shown in Fig. 8. Small, thin copper strips are placed over the sample so that by applying pressure to the strips, the voltage lead connection is made. It is not certain whether the leads should be wrapped around the sample and then clamped, laid on top of the sample and then clamped, or placed between the phenolic holder and copper bar and clamped with the sample.

A test was made to check the possibility of these setups. A sample of 99.9999% pure 0.0508 cm diameter lead wire was prepared. It is a well annealed type I superconductor. We do not worry about inhomogeneities in the sample; therefore, all effects measured are due to the superconductor and not to the experimental techniques. It is assumed that any voltage variation from run to run is due to the voltage leads

rather than to the sample of lead wire. Three different setups of voltage leads were made: (1) the leads were wrapped around the sample several times and then securely clamped, (2) the clamps were loosened from the first sample and then tightened again, (3) the voltage leads were laid under the bar and clamped. The copper bars are clean and flat to insure good contact. Brass screws rather than steel ones are used in the clamps because the screws do not exhibit magnetic properties and have about the same thermal properties as copper.

For this lead sample the critical field is 550 gauss at 4.2° K. The sample was subjected to fields of 400 gauss and 500 gauss to determine the critical current, I_c , and the normal current, I_n . The critical current is arbitrarily selected as the point of first detectable change of voltage on the 1μ V scale. The three samples give comparable results at both fields. We assume that the voltage lead problem is solved. It appears that the voltage across the sample is independent of how tightly the clamp is secured and of how many times the wire is clamped and unclamped. One should make sure, though, that there is good contact between the copper bar, the sample, and the voltage lead. After each clamping, the PbIn sample becomes flatter and flatter, so that one must be careful that the sample does not become flush with or go below the top of the phenolic holder. If this situation occurs, then the voltage wire should be placed on top of the PbIn sample and clamped securely. One should avoid unnecessary flattening of the sample or cutting of the sample by the voltage lead when carrying out this technique. As far as we can determine, the voltage across the sample is independent of the mode of attachment of the voltage lead.

Annealing Properties

After the voltage lead problem is solved, one can prepare to take data on the sample. At first it was not certain how fast $\text{Pb}_{83}\text{In}_{17}$ anneals at room temperature or how the surface and bulk features affect the flux flow resistivity. Since a lot is unknown about annealing properties, it was decided to stay with the two extreme cases--highly strained and perfectly annealed. In order to retain the wire in the strained state, it is stored in liquid nitrogen.

We are lead to believe by tests conducted on samples at various stages of annealing that the annealing process for $\text{Pb}_{83}\text{In}_{17}$ is rather fast. Thus it is imperative that one prepare a fresh sample as quickly as possible. It requires one hour to prepare the sample for data taking during which time much annealing occurs.

Surface Effects

The extrusion of the PbIn wire through the 0.0508 cm die results in the introduction of surface irregularities. Due to the inability to maintain a constant pressure on the driving piston, the wire has a tendency to extrude at different rates, and as a result the wire diameter varies about 0.0013 cm. This is only a slight effect (2.5%) so the wire diameter is measured for each sample at several places and an average diameter is quoted.

Etching the wire for about one minute removes most of the scratch marks and renders the surface shiny. The wire diameter after etching usually varies between 0.0432 and 0.0457 cm, depending on the amount of etching.

Small dents and craters as well as small bumps are often on the surface of the wire. The etching solution shows no preference to certain parts of the wire and tends to shrink the defects and the wire at the same rate.

It is pointed out in the theory of flux flow that the flux lines entering the sample make an angle with the axis of the wire as given by

$$\tan \theta = \frac{B_{\text{wire}}}{B_{\text{applied}}} .$$

Longitudinal scratches are introduced by the extrusion process along the surface of the wire. They can be utilized to possibly reduce the surface barrier by allowing the flux lines to enter the wire at the angle of the scratch. The wire is twisted to incline the scratches at an angle corresponding to a typical value of θ . A one degree inclination along the surface of a wire of length 3.8 cm and diameter of 0.0254 cm requires a 150° rotation of one end of the wire relative to the other.

In order to obtain the one degree scratch on the wire, one end of the sample was placed in a stationary vise and the other one in a chuck vise on which was mounted a protractor. A $1\frac{1}{2}$ inch sample was rotated through 150° over a period of time to prevent breaking of the wire. A rotation of 20° was made every 5 minutes until 100° was reached. The wire was held at 100° for 30 minutes to allow the strains to anneal out. By this time the wire was quite soft and it was easy to turn through the last 50° . The wire was then clamped at 150° and allowed to sit and anneal for 7 hours.

Under a 50X microscope the spiraling scratch is clearly visible. For a one inch sample a scratch comes into view on one edge of the wire

at one end and goes out of view on the opposite edge on the other end. The angle of spiral, unfortunately, cannot be measured with our microscope. Data taken on the twisted sample were inconclusive. It appears that the strains introduced by twisting overshadow the effects of the spiraling surface scratch.

One may be concerned about the surface of the wire giving rise to surface superconductivity which persists up to a field

$$H_{c3} = 1.695 H_{c2} \quad .$$

Surface currents may be eliminated by coating the sample with a normal metal. A standard technique of coating is copperplating. The plating bath contains a compound of copper, such as CuSO_4 , that is to be deposited on the sample. The sample is the cathode and a strip of copper is the anode. The time and the current required to plate a selected thickness of copper are found by using the electrochemical equivalent of copper (3037 coulombs/gram).

In order to obtain meaningful data, the same sample should be run both unplated and plated with few other effects introduced. Therefore, the sample should be plated in the holder. A sample holder with a removable section allows the sample to be completely immersed in the CuSO_4 for plating. If desired, the anode and cathode can be interchanged and the copper can be stripped from the surface of the wire.

Experimental Procedure

The procedure for carrying out a complete experiment is quite involved and often requires the aid of an extra person. Extreme care is

taken when inserting the cryostat into the Dewar to prevent breakage of the delicate voltage leads and bumping of the sample. If the Dewar is cold, an added precaution prevents damage of the Hall probe.²⁵

The resistance of the copper coated NbZr magnet serves as a crude resistance thermometer during cooling of the sample. During this period the voltage on the sample is followed on a Keithley 147 Nanovolt Null Detector. Sometimes the expansion of the holder and clamps open circuits the voltage leads and the voltage on the Keithley 147 exceeds the 100 mV scale indicating either an infinite or very large resistance. The only solution to the open circuit is to remove the sample from the cryostat and to tighten all connections and make sure the PbIn is not damaged in any way. It may be necessary to flatten and sandpaper the voltage clamps to insure a good contact with the sample.

The standard techniques of helium transfer are employed with a vent hole of 1/4 inch diameter at the top of the cryostat allowing helium vapors to escape during transfer. A mechanical pump reduces the temperature by reducing the vapor pressure over the helium bath. A system of parallel valves of 1/4 inch and 1/8 inch diameters is used to regulate the pumping speed, and the temperature can be stabilized to within 0.1° K. A mercury manometer reads pressure differences over the helium bath and the corresponding temperature is then obtained from tables.³⁰

Before data are taken, certain preliminary adjustments are made. The residual voltage of the Hall probe is nulled.²⁵ Voltage measurements are taken indirectly from the Keithley 147 by using the voltage output from the Keithley 147 to drive a Keithley 370 Recorder. Adjustments are made to insure complete scale agreement between the two instruments. A

mechanical adjustment on the chart allows adjustment of the base line and a voltage divider across the Keithley 147 output allows scale adjustment of the chart recorder. Two 10,000 μ f capacitors in parallel with the chart input stabilize noise generated by the equipment. This does not affect the accuracy of the data but it lengthens the time constant of data taking. After approximately 15 seconds the reading on the chart is about 95% of its final value. A block diagram of the equipment is shown in Fig. 9.

The superconducting magnet used to apply the external longitudinal magnetic field is not completely reversible and contains some hysteresis. The current through the magnet is monitored by a digital voltmeter and a standard resistor of 0.1 ohm. The field is monitored by a Hall probe.²⁵

The magnet is cycled 4 times up to 2500 gauss and back to zero in order to reduce hysteresis. On the fifth cycle the field is raised to the appropriate value by using the Hall constant given in gauss/mV and by reading the voltage across the Hall probe on the 1 mV scale of a Keithley 153 Microvolt-Ammeter.

If the desired field corresponds to a voltage greater than 1 mV, a nulling process is used. The current through the magnet is increased so that the Hall probe gives full scale deflection on the 1 mV scale of the Keithley 153. A nulling circuit supplies a back voltage to null the Hall voltage to zero. This incrementing and nulling procedure is continued until the proper field (corresponding to the amount of voltage nulled out) is obtained. The voltmeter is set on "center zero" to detect voltage changes of 0.05 mV or field changes of approximately 2 gauss. A slight adjustment of the current supply to the magnet brings the external

Figure 9. Block Diagram of Equipment

field back into alignment. One should check for field variations caused by changes in the control current to the Hall probe. Adjustments of both control current and magnet current may be necessary throughout the experiment.

If the operating current range is larger than 10 amps, a shunt is employed to divert large currents from the normal sample. The shunt in this experiment consists of copper wire and alumel wire in series to achieve a resistance slightly smaller than the normal resistance of the sample of PbIn.

The shunt is calibrated once in the configuration that is run with the sample. The cryostat is removed from the Dewar and the sample is cut causing the current to flow only through the shunt and not the sample. This insures one that the resistance of leads, etc., are taken into account in the calibration. Calibration requires a standard resistor to determine the current through the shunt and a potentiometer or voltmeter to measure the voltage across the shunt. A least squares program gives the best value of shunt resistance R_s from the current-voltage data. The shunt resistance is fairly independent of small temperature variations and of the exact contact configuration. Thus, R_s is determined once and taken as a constant for all data interpretation.

The value of current through the sample is found by considering the sample and shunt as parallel resistors. The current splits,

$$I = I_w + I_s$$

where I is the total current,

I_w is the current in the wire sample, and

I_s is the current in the shunt.

From Ohm's Law,

$$I_s = \frac{V}{R_s} .$$

$$I_w = I - I_s$$

$$= I - \frac{V}{R_s}$$

$$I_w = \frac{I R_s - V}{R_s} .$$

The current in the wire is determined by the current from the power supply, the voltage across the parallel circuit, and the resistance of the shunt.

Once the external field is set, measurements are begun on the 1 μ V scale of the Keithley 147. The current through the sample, which is supplied by a Hewlett-Packard current regulated power supply, is slowly increased until the first detectable reading is visible on the chart recorder. This value of current is noted as I_c , the critical current. This is the current necessary to instigate flux motion. The current is then raised to give a reading of approximately 0.1 μ V on the 1 μ V scale, and the current is thereafter methodically increased to give about 0.1 scale change per reading, i.e., about 0.1 μ V change on the 1 μ V scale. Current values are noted on the chart at each voltage reading. Near 80% or 90% full scale deflection, the scale is raised on the Keithley 147 and consequently on the Keithley 370 Recorder. The current is continually increased until a large, sudden voltage change is detected by the Keithley

Nanovoltmeter. This value of current is taken as I_n , the normal current. The current is immediately reduced to zero to prevent damage to the sample.

The I-V data come from the chart tape on which the current values were noted during the experiment. The corresponding voltages are read off the chart tape to an accuracy of 0.155% by the use of a ruler having as its smallest division 1/32 inch. The values of I_w are found from R_s and corresponding I-V data from the chart. The I_w -V data are then analyzed according to theory.

CHAPTER IV

RESULTS AND DISCUSSIONS

Introduction

We have seen in Chapter II that the I-V relations of either Model I or Model II are the same and the results are qualitatively the same. This allows us to choose either of the two methods for an analysis with the understanding that the relations between the quantities in the two models are

$$\beta_{II} = \frac{4}{3} \beta_I \quad (4.1)$$

and

$$\rho_{II} = \rho_I \quad (4.2)$$

Model I is arbitrarily chosen for quantitative analysis and corresponding quantities in Model II are calculated using eq. (4.1) and eq. (4.2). In order to eliminate the ambiguity in Ω/η (or η), flux flow resistivity data are used from Kim, et al.⁴ and from Farrell, et al.⁶ Error can result here since data are not published in the form of flow resistivity, but are merely indicated graphically on V vs. I plots. We extract flow resistivity by using the slopes of the curves in the upper linear portions along with the samples' dimensions. The sample dimensions for Kim's sample are not known exactly so there may be a very slight error in the resistivity for his data.

Table 1 illustrates flux flow resistivities for PbIn slab in a transverse magnetic field. One notes that the concentration percentages are not exactly the same and the temperatures vary slightly. Even so, one cannot help but notice that the resistivities calculated from Farrell and those from Kim disagree by a factor of three or four. This possibly implies that flow resistivity depends appreciably on the sample itself, and may explain the lack of tables of this quantity.

If we are going to extract Ω/η (or η) from our data using published data, then we must decide which to use. Since Kim's data are so large (and this may be due to the uncertainty in sample dimensions), we find that resistivity calculations on our data will result in resistivities three times larger than by Farrell and in field penetration ratios one-third of those by Farrell. We will therefore choose Farrell's data because of the certainty in sample dimensions and because of the greater number of fields from which to calculate resistivities.

Various sized wires of $\text{Pb}_{83}\text{In}_{17}$ are used to study the effect of pinning on viscous flow. The extrusion apparatus produces a wire of 0.0508 cm diameter, but longitudinal scratches are present. In order to reduce surface irregularities, wires are etched down to 0.0483 cm, 0.0457 cm, 0.0432 cm, and 0.0406 cm. If the scratches and thus the energy barriers due to the surface are reduced, then the field penetration should vary. Data for the sample to be analyzed are given in Table 2.

Another way of studying pinning effects is to look at wires in various stages of anneal. Since we are able to calculate pinning forces

TABLE 1

FLUX FLOW RESISTIVITY FOR LEAD-INDIUM SLAB
GEOMETRY IN A TRANSVERSE MAGNETIC FIELD

Sample Concentration (Atomic Percentages)	Sample Dimensions (cm)	Temp. (°K)	B _{app} (Gauss)	ρ_f ($\mu\Omega$ -cm)
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Farrell^a

Pb ₈₀ In ₂₀	1.0 × 0.20 × 0.02	1.8	1100	0.76
Pb ₈₀ In ₂₀	1.0 × 0.20 × 0.02	1.8	1750	0.96
Pb ₈₀ In ₂₀	1.0 × 0.20 × 0.02	1.8	2100	1.26
Pb ₈₀ In ₂₀	1.0 × 0.20 × 0.02	1.8	2800	1.92

Kim^b

Pb ₈₃ In ₁₇	1.5 × 0.25 × 0.011	2.0	500	0.92
Pb ₈₃ In ₁₇	1.5 × 0.25 × 0.011	2.0	1000	3.30
Pb ₈₃ In ₁₇	1.5 × 0.25 × 0.011	2.0	2000	3.66

^aD. E. Farrell, I. Dinewitz, and B. S. Chandrasekhar, Phys. Rev. Lett., 16, 91 (1966).

^bY. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev., 139, A1163 (1965).

TABLE 2

PHYSICAL CHARACTERISTICS OF SAMPLES

Sample	Surface Condition	Bulk Condition	Wire Diameter a (cm)	Wire Constant c (gauss/amp)	Wire Constant k (gauss-cm ² /amp)	Cross-Sectional Area (cm ²)	Temp. (°K)
46A	Smooth	Annealed	0.0432	9.264	1.02×10^{-2}	1.464×10^{-2}	1.8
47A	Smooth	Annealed	0.0483	8.288	1.14×10^{-2}	1.829×10^{-2}	1.8
48A	Smooth	Annealed	0.0457	8.749	1.08×10^{-2}	1.642×10^{-2}	1.8
49A	Rough	Strained	0.0508	7.874	1.20×10^{-2}	2.027×10^{-2}	1.8
51A	Rough	Annealed	0.0508	7.874	1.20×10^{-2}	2.027×10^{-2}	$\begin{bmatrix} 4.2 \\ 1.8 \end{bmatrix}$

(to within Ω/η), then we can see if the magnitude of the pinning forces are consistent with the stages of anneal.

Sample #49A

This sample of wire is freshly extruded (~one hour old) with a diameter of 0.0508 cm and is run at 1.8°K in parallel with a shunt of 0.01962 ohms. The surface of the wire is untouched with chemicals so that this sample hopefully demonstrates the effects of large surface pinning as well as large bulk pinning.

Fig. 10 shows the I-V characteristics of sample #49A on a plot of V/I vs. I^2 . The linear portion in the large current regime indicates a cubic dependence of current as predicted by eq. (2.7) or by eq. (2.23). The lower current range is taken to be creep and creep-flow mixtures and will not be explained quantitatively. The shunt allowed the sample to carry large currents to drive the sample normal, but the large voltages restricted data to low currents.

All the curves in the linear region are parallel. Since the slope is a measure of the field penetration ratio, as shown in eq. (2.8) and eq. (2.24), then we do not expect the field ratio to vary much from curve to curve. This occurs since data for this sample of $\text{Pb}_{83}\text{In}_{17}$ ($T_c = 7.0^\circ\text{K}$) are taken at 1.8°K where there is a very large mixed state with $H_{c1} = 260$ gauss and $H_{c2} = 5000$ gauss.⁵ Clearly now both β and B_{int} are functions of B_{app} . We are sampling only a small segment of the mixed state (~10%) so that we expect the field ratio β to be approximately constant for a given sample. On the phase diagram shown in Fig. 11 we are possibly in the shaded area indicated. It has been

Figure 10. Sample #49A

Figure 11. Phase Diagram for Lead-Indium

suggested by London and Walmsley²⁰ that some areas of the phase diagram may not support flux flow. We have chosen a region in which flux flow is definitely occurring.

One can set an upper and a lower boundary on β by realizing that below H_{c1} there cannot be any field inside the wire if it is perfectly diamagnetic. Above H_{c2} (and neglecting surface superconductivity) the wire should be normal with the field inside the same as the applied field. Thus

$$0 \leq \beta \leq 1 \quad .$$

We are not certain of the exact meaning of β since it is an average field penetration. We do expect that the field varies continuously

Figure 12. Magnetic Induction Inside a Superconductor

from just outside to just inside the surface of the wire. Fig. 12 shows suitable as well as unsuitable configurations for the magnetic induction B across the surface and inside the sample. There cannot be an abrupt change (or physical discontinuity) in the magnetic induction.

Sikora, et al.¹⁸ have demonstrated the suitable field variation by measuring the internal field with a bismuth probe. Our range of values for β ($\beta \approx 0.1 - 0.6$) is justified by considering the four samples measured by Sikora, et al. and by observing that the field penetration is not exactly the same for each sample. Fig. 1 of Sikora, et al. illustrates field ratios of about this magnitude when averaged over the sample.

Sikora³¹ has suggested a way for checking on the magnitude of β . The method involves using magnetization curves similar to the one in Fig. 2b to find B_{int} and then to form approximate ratios of B_{int} to B_{app} . We can get a general idea of how to make the calculation by qualitative analysis from Campbell, Evetts, and Dew-Hughes.³² They make the assumption that there is a simple inverse proportionality between pinning force

and B_{int} . This allows Campbell, et al. to explain magnetization curves for various pinning conditions. They arrive at four typical magnetization curves as shown in Fig. 13. Illustrations of effects of weak pinning forces and of strong pinning forces are shown respectively in Fig. 13a and in Fig. 13b. Fig. 13c shows what is thought to occur if the pinning centers are very closely packed, while Fig. 13d shows the effect of mixing strong, widely spaced pinning centers and weak, closely spaced centers. It appears almost hopeless to find sufficient data to give an accurate check on our values of β .

One can place an upper experimental limit on the value of β by considering both reversible and irreversible magnetization curves as shown in Fig. 14.² For an "ideal" type II superconductor, flux is held out until H_{c1} (see Chapter I), at which time flux begins penetrating. When the externally applied field reaches H_{c2} , $B_{int} = B_{app}$. However, for "real" type II superconductors, defects affect the penetration of flux, as shown in Fig. 14b, so that it is difficult for flux to enter the sample at H_{c1} . A reduction in field from above H_{c2} to zero may result in trapped flux.

Dubeck, Aston, and Rothwarf³³ indicate a nearly reversible magnetization curve for $Pb_{84.5}In_{15.5}$ at $2.0^{\circ}K$ in Fig. 1. We can use this data to carry out Sikora's suggestion³¹ and calculate β_{max} . For the sample of Dubeck, et al. the quantities H_{c1} , H_{c2} , and T_c are very nearly the same as for our samples, so that β_{max} will be approximately the same. One finds that for the sample of Dubeck, et al. at 1000 gauss, $\beta_{max} = 0.9$ and at 1500 gauss, $\beta_{max} = 0.94$. These values of β_{max} are much larger than our largest value ($\beta=0.6$) and represent a maximum value of β for an "ideal" type II superconducting PbIn sample.

Figure 13. Magnetization Curves for Various Types of Pinning

(a) "Ideal" Superconductor

(b) "Real" Superconductor

Figure 14. Magnetization Curves for Type II Superconductors

If we make a calculation of β using eq. (2.7), we reach a dilemma. Due to the indeterminable constant η/Ω , β can only be determined to within a constant. We can substitute

$$\beta = \text{constant} \times \frac{\eta}{\Omega}$$

into eq. (2.13) and arrive at

$$\rho_f = \text{constant} \times B_{\text{app}}^2 \times \left(\frac{\Omega}{\eta}\right)^2.$$

By selecting resistivity values along with their appropriate field values from Table 1, one can find Ω/η and then can substitute back into the formula for β to obtain a value for β and thus a value for ρ_f . Table 3 illustrates calculations performed in this way.

Another method of analysis is to plot the data in another manner. One must obtain two more distinctly different equations in order to have four equations with four unknowns-- Ω , η , β , and F_p . We are led to identities by other methods of plotting, so it appears that the best way of further analysis is to use Table 1.

If one calculates β for each field using Table 1, he finds that β is the same (since the slopes are the same) and of course the constant Ω/η is the same. We would hope that even if the ratio β is not constant, the quantity Ω/η is constant. This will be illustrated in later samples and discussed further then. The value of $\beta = 0.41$ indicates that the field inside is on the average about one-half the applied field. This may seem large since our field range is one-tenth of the mixed state, but one must consider the physical condition of the wire. Since this

TABLE 3

DATA FOR SAMPLE #49A FROM FIG. 10

Field (Gauss)	Slope ($\mu\text{V}/\text{amps}^3$)	Intercept ($\mu\text{V}/\text{amp}$)	I_P^2 (amps^2)	I_{break}^2 (amps^2)	$F_L @ I_b^2$ (nt/m^3)	F_P (nt/m^3)	F_{excess} (nt/m^3)
1200	3.1×10^{-3}	-1.44	580	600	9.55×10^5	7.0×10^5	2.55×10^5
1400	3.4×10^{-3}	-0.85	360	420	6.67×10^5	4.4×10^5	2.27×10^5
1500	3.6×10^{-3}	-0.65	280	350	5.66×10^5	3.4×10^5	2.16×10^5
1600	3.55×10^{-3}	-0.33	220	300(?)	4.76×10^5	2.7×10^5	2.06×10^5

RESISTIVITY FOR SAMPLE #49A

Field (Gauss)	(Ω/η) ($\text{m}^7/\text{nt-sec}$)	η ($\text{nt-sec}/\text{m}^2$)	β_I	β_{II}	ρ_I ($\mu\Omega\text{-cm}$)	ρ_{II} ($\mu\Omega\text{-cm}$)
1200	12.25×10^{-10}	1.05×10^{-7}	0.41	0.55	0.43	0.43
1400	12.25×10^{-10}	0.82×10^{-7}	0.41	0.55	0.65	0.64
1500	12.25×10^{-10}	0.73×10^{-7}	0.41	0.55	0.79	0.78
1600	12.25×10^{-10}	0.69×10^{-7}	0.41	0.55	0.88	0.87

sample has a very rough surface, it is possible that large currents cannot be supported so that Meissner currents will be smaller than expected. Thus one can imply that the Meissner currents cannot shield out very much flux.

We may expect an enhancement of field due to a spiraling field cutting a rough surface. Melville¹¹ found this to be true. The field may then be greater at the rough "peak" due to the reduced curvature of the wire and could allow flux penetration as illustrated schematically in Fig. 15. The reduced energy barrier could allow flux to enter easier

Figure 15. Field Enhancement at a Rough Surface

"in spots" than for a smooth surface where there would be no preferential entry of flux.

We can calculate the pinning force per unit volume from eq. (2.9). Notice that the pinning force and the field penetration ratio are functionally dependent on B_{app} in inverse fashion. Now

$$\beta \propto m$$

$$\beta \propto B_{app}$$

and

$$F_P \propto b$$

$$F_P \propto \frac{1}{B_{app}} \quad .$$

This occurs since the intercept is decreasing as we go to larger and

larger fields. In other words, lower fields have larger pinning currents and larger pinning forces. Because of the equal, parallel spacing of the curves in sample #49A, each pinning force could be a multiple of a force for a lower curve.

If one does a calculation, he finds F_p decreases with increasing field as illustrated in Table 3. The significance of this result is explained by eq. (2.4),

$$F_L - F_p = F_{\text{excess}} = \eta v_f .$$

The excess force is the factor which is producing a flux flow velocity v_f . For low fields with large pinning forces we must have large Lorentz forces to produce F_{excess} . As we go to higher fields the pinning force decreases; therefore, the Lorentz force can be smaller and still give the same excess force.

Fig. 10 illustrates the excess force concept quite dramatically. Notice that there are definite points for which linearity sets in and are indicated as I_{break}^2 in Table 3. If one takes this as an indication of the point at which the Lorentz force has completely dominated the pinning force, then he can get an estimate of the excess force driving the flux lines. One can make a calculation of F_L at I_b^2 and compare with F_p . We use eq. (2.5) and substitute I_b^2 for I^2 . The tabulated values of F_{excess} for the four curves obtained indicate that this is what the break means physically. It appears that whenever the excess force is $\sim 2 \times 10^5 \text{ nt/m}^3$ the flux flow process is well defined. Below I_b^2 the flux motion is taken to be mixtures of thermal- and current-dominated processes.

Sample #48A

Sample #48A is a well annealed wire which was run in parallel with a shunt of 0.01962 ohms at 1.8°K . The surface of the wire was etched with a solution of hydrogen peroxide and acetic acid twice for one minute each until the wire diameter was 0.0457 cm. Etching gives a polished surface that is shiny to the unaided eye but appears slightly irregular when viewed under a 50X microscope. It is hoped that etching reduces the surface roughness and thus reduces the surface barrier caused by irregular pinning sites along the surface. The smooth surface now may be able to support a larger surface current than sample #49A so that more shielding of the flux lines can take place. This sample should be the extreme opposite of #49A, since the sample should support large surface currents and should have very small pinning forces.

A plot of V/I vs. I^2 in Fig. 16 illustrates the current-voltage characteristics for sample #48A; the data are summarized in Table 4. Because of the shunt used, the entire curve was obtained for each field without risk of burning out the sample. One can see a definite trend in the slopes of the various curves. As a result, the field penetration ratio should, and does, vary noticeably with field. The field ratio varies directly with field and changes by a factor of two over the range of field of 1000 gauss to 1700 gauss. This is much more variation than illustrated by sample #49A and is evidently due to the difference in the physical condition of the two wires. It appears that the smooth surface has allowed surface currents to shield out the flux more than in sample #49A.

One feature of this sample is the lack of breaks indicating the transition to pure viscous flow. If one assumes that $I_b^2 \approx I_p^2$, i.e.,

Figure 16. Sample #48A

TABLE 4

DATA FOR SAMPLE #48A FROM FIG. 16

Field (Gauss)	Slope ($\mu\text{V}/\text{amps}^3$)	Intercept ($\mu\text{V}/\text{amp}$)	I_P^2 (amps^2)	I_{break}^2 (amps^2)	$F_L @ I_b^2$ (nt/m^3)	F_P (nt/m^3)	F_{excess} (nt/m^3)
1000	1.38×10^{-4}	-0.0550	400	400	1.42×10^5	1.42×10^5	~ 0
1200	2.29×10^{-4}	-0.0298	130	130	0.60×10^5	0.60×10^5	~ 0
1400	3.49×10^{-4}	-0.0279	80	80	0.45×10^5	0.45×10^5	~ 0
1500	4.54×10^{-4}	-0.0295	65	65	0.42×10^5	0.42×10^5	~ 0
1600	5.84×10^{-4}	-0.0350	60	60	0.44×10^5	0.44×10^5	~ 0
1700	6.56×10^{-4}	-0.0328	50	50	0.39×10^5	0.39×10^5	~ 0

RESISTIVITY FOR SAMPLE #48A

Field (Gauss)	Ω/η ($\text{m}^7/\text{nt-sec}$)	η ($\text{nt-sec}/\text{m}^2$)	β_I	β_{II}	ρ_I ($\mu\Omega\text{-cm}$)	ρ_{II} ($\mu\Omega\text{-cm}$)
1000	10.9×10^{-10}	7.00×10^{-7}	0.067	0.089	0.33	0.33
1200	10.9×10^{-10}	4.55×10^{-7}	0.086	0.115	0.48	0.48
1400	10.9×10^{-10}	3.13×10^{-7}	0.106	0.141	0.65	0.66
1500	10.9×10^{-10}	2.57×10^{-7}	0.121	0.161	0.75	0.75
1600	10.9×10^{-10}	2.14×10^{-7}	0.138	0.184	0.84	0.84
1700	10.9×10^{-10}	1.90×10^{-7}	0.146	0.195	0.95	0.95

the creep region is vanishingly small, then he finds that the excess force is zero. This is not surprising if one considers Model I, eq. (2.5) and substitutes $I = I_p$ to find that

$$F_L = \beta \frac{c}{A} I_p^2 .$$

Using eq. (2.8) and the definition of I_p^2 , one finds that the above equation is the same as the pinning force from eq. (2.9). This is consistent and says that for $I < I_p$ the flux lines are pinned very strongly, and for $I = I_p$ the flux lines are in a metastable state, and for $I > I_p$ the Lorentz force is "completely beginning to dominate" the pinning force.

The slopes vary by a factor of five and the ratio of the dissipation volume to the viscosity coefficient is a constant. If the viscosity coefficient is a constant for the material,² then the volume in which dissipation is occurring is also a constant.

Sample #47A

This sample, as well as #48A, is well annealed and is run at 1.8°K. The diameter was reduced by chemical etching to 0.0483cm. Unlike the first two samples, this wire is run without a shunt. This resulted in low currents with an uncertainty in the amount of data lacking on the complete curve for each field.

Data for sample #47A are illustrated in Fig. 17 and results tabulated in Table 5. As in sample #48A which was chemically etched, the field penetration ratios are extremely small and there is little variation in β .

Figure 17. Sample #47A

TABLE 5

DATA FOR SAMPLE #47A FROM FIG. 17

Field (Gauss)	Slope ($\mu\text{V}/\text{amps}^3$)	Intercept ($\mu\text{V}/\text{amp}$)	I_p^2 (amps^2)	I_{break}^2 (amps^2)	$F_L @ I_b^2$ (nt/m^3)	F_p (nt/m^3)	F_{excess} (nt/m^3)
1300	1.56×10^{-4}	-0.0603	385	500	1.83×10^5	1.42×10^5	0.41×10^5
1500	1.48×10^{-4}	-0.0326	220	400	1.43×10^5	0.79×10^5	0.64×10^5
1600	1.46×10^{-4}	-0.0241	165	360	1.27×10^5	0.59×10^5	0.68×10^5
1700	1.31×10^{-4}	-0.0124	95	325	1.09×10^5	0.32×10^5	0.77×10^5
2500	0.99×10^{-4}	+0.0276	---	---	----	----	----

RESISTIVITY FOR SAMPLE #47A

Field (Gauss)	Ω/η ($\text{m}^7/\text{nt-sec}$)	η ($\text{nt-sec}/\text{m}^2$)	β_I	β_{II}	ρ_I ($\mu\Omega\text{-cm}$)	ρ_{II} ($\mu\Omega\text{-cm}$)
1300	11.55×10^{-10}	4.64×10^{-7}	0.081	0.108	0.53	0.54
1500	11.55×10^{-10}	4.14×10^{-7}	0.079	0.105	0.71	0.71
1600	11.55×10^{-10}	3.90×10^{-7}	0.078	0.104	0.82	0.82
1700	11.55×10^{-10}	3.89×10^{-7}	0.074	0.099	0.92	0.91
2500	11.55×10^{-10}	0.03×10^{-5}	0.065	0.087	----	----

A curve was obtained midway between H_{c1} and H_{c2} -- 2500 gauss. One can get a value for the field penetration ratio but cannot calculate a pinning force. Extrapolation yields a negative I_p^2 or an imaginary pinning current. This is inconsistent with what we know physically. The 2500 gauss curve is concave downward and the curves for 1300 gauss to 1700 gauss are all concave upward. It appears that the low field curves are more clearly pronounced in curvature and decrease in curvature with an increasing field until the concavity changes to downward. The positive curvature is shown also in sample #49A and the linear curve with no curvature is shown in #48A.

The low value of β that one obtains from a straightforward calculation for 2500 gauss indicates that the slope is not large enough. The curve should "S" and have a positive curvature going into another straight line. Sikora³¹ indicates this as the trend of an exponential (in current) when plotted on a V/I vs. I^2 plot. It is then entirely possible that at large fields the motion of flux is almost entirely creep and creep-flow mixtures. London and Walmsley²⁰ find that above a certain field flux flow is nonexistent. The currents necessary to produce flux flow at large fields such as 2500 gauss may be large enough to drive the sample completely normal so that flux flow will not be seen. We are limited in our current, both by the power supply and by the lack of a shunt.

An analysis of the low current portion of the 2500 gauss curve is lacking in great detail. A rough plot on semilog paper tends to indicate some exponential behavior in the lower portion of the data and suggests the possibility of flux creep. It is clear from the V/I vs. I^2 plot that any calculation of β or ρ_f is meaningless.

One finds breaks occurring in the other curves similar to sample #49A. The excess force is not as constant for this sample as for #49A, however. Since β appears in the calculation of F_L , an incorrect value of β will give an incorrect Lorentz force. Further analysis of this sample reveals that the slopes are varying in the wrong manner expected, i.e., the slopes are going down with increasing field. Even though the slopes vary inversely to what is expected, the field ratio β is fairly constant. A larger field ratio for 1700 gauss and a lower one for 1300 gauss would tend to increase F_{excess} at 1300 gauss and to decrease F_{excess} at 1700 gauss in order to smooth out the variations in F_{excess} for all fields.

Sample #46A

This is one of the first samples on which reliable data were obtained. The sample was well annealed and the surface was very highly polished until the wire diameter was 0.0432 cm. Unfortunately, at the time data were taken no shunt was incorporated so that we were restricted to small currents ($I \leq 15$ amps).

The data as shown in Fig. 18 and illustrated in Table 6 is ambiguous at first glance. One notices that the 1500 gauss curve is reported twice, with distinctively different characteristics. This sample is the only sample in which the field was varied in two directions, i.e., the field was decreased from 1500 gauss by the nulling procedure of Chapter III to 1000 gauss and then increased back to 1500 gauss. The field procedure now is to begin at 1000 gauss and increase to higher values of field. By this method no two curves are reproduced. It is not exactly certain

Figure 18. Sample #46A

TABLE 6

DATA FOR SAMPLE #46A FROM FIG. 18

Field (Gauss)	Slope ($\mu\text{V}/\text{amps}^3$)	Intercept ($\mu\text{V}/\text{amp}$)	I_p^2 (amps^2)	I_{break}^2 (amps^2)	$F_L @ I_b^2$ (nt/m^3)	F_P (nt/m^3)	F_{excess} (nt/m^3)
1000	1.67×10^{-3}	-0.375	225	225	3.13×10^5	3.06×10^5	~0
1300	2.67×10^{-3}	-0.169	63	63	1.07×10^5	1.09×10^5	~0

RESISTIVITY FOR SAMPLE #46A

Field (Gauss)	Ω/η ($\text{m}^7/\text{nt}\cdot\text{sec}$)	η ($\text{nt}\cdot\text{sec}/\text{m}^2$)	β_I	β_{II}	ρ_I ($\mu\Omega\text{-cm}$)	ρ_{II} ($\mu\Omega\text{-cm}$)
1000	9.0×10^{-10}	2.37×10^{-7}	0.22	0.29	0.29	0.30
1300	9.0×10^{-10}	1.42×10^{-7}	0.27	0.36	0.53	0.53

whether the differences in the two 1500 gauss curves are due to an inaccuracy in setting the field or whether the error is physical and represents hysteresis.

We tend to attribute the field error to inaccuracy in absolute field and not to hysteresis. The reason for this is justified by observation of the reproducibility of data at one field and temperature. For any given field, data have been reproduced as many as four times prior to changing this field value. If hysteresis is occurring, then after the sample is driven completely normal, flux is "frozen in" and all remaining cycles of the same curve give approximately the same results, which are different from the first cycle. This does not occur. In fact, after the sample is driven into the normal state, one finds that a sudden reduction in the transport current to zero gives rise to a negative voltage drop across the sample. We attribute this phenomenon to the expulsion of flux from the sample as the sample returns to the superconducting state. When equilibrium is reached, there is no detectable shift in the zero line voltage.

The superconducting magnet is also cycled to freeze the flux and to reduce flux jumps that could occur in the magnet while data are being taken. Only on very rare occasions has the magnet given problems.

We assume that from Fig. 18, 1500 G (2) is probably 1600 G; 1600 G is probably 1700 G; etc. The Hall probe which is rather insensitive to large changes in fields makes it necessary to monitor small field changes and not the absolute values of fields. The error in field will propagate with each adjustment and could conceivably give an absolute error in field of ± 100 gauss if one is not careful.

A very disturbing thing about the data of sample #46A is the concave downward curves for $B_{app} \geq 1500$ gauss. We notice this behavior in sample #47A also. The two lower curves of 1000 gauss and 1300 gauss are very linear without breaks and give values of β which seem reasonable. The natural tendency is to extend all the other curves into straight lines, all mutually parallel. The problem is again the same as encountered in #47A--imaginary pinning currents.

The only way out of the dilemma is to prepare another sample with the exact physical conditions (which may be almost impossible) and introduce a shunt so larger current can be used to drive the sample well into the flow regime. Hopefully, an "S" will occur in the I-V characteristics (as suggested by Sikora³¹ on page 82 of this dissertation). Such a sample was prepared and data were taken. The results appear to be inconclusive since at currents large enough to produce pure viscous flow, the ratio B_w/B_{app} is no longer small and the small angle approximation breaks down. This means that analysis is almost impossible due to the complexity of the I-V equation.

One reason for the breakdown in the small angle approximation is the wire diameter. This sample, which is the smallest of all samples analyzed, has the largest wire field constant and thus gives substantially higher wire field values at any given current. The wire constant varies inversely with wire diameter and is about 1.2 times the wire constant for the largest wire analyzed. There may be a critical radius for our wires so that below that value analysis will be inconclusive (due to the lack of the small angle approximation).

Sample #51A

The last sample to be discussed is a well annealed wire of 0.0508 cm diameter and is identical to sample #49A except data were taken at 4.2°K. One field was taken at 1.8°K in order to check on the similarities between this sample and #49A. Data are shown in Fig. 19 and illustrated in Table 7.

One finds that $V \propto I^3$ at 4.2°K as well as at 1.8°K. This is expected since the theory does not mention temperature in any way. Since data used to calculate ρ_f (or β) is taken at 1.8°K or at 2.0°K, it is not exactly correct to assume that the flux flow resistivity is independent of temperature. Kim, et al.⁴ illustrate in Fig. 4 of their paper that resistivity is not a simple function of applied field at various temperatures and thus is not a simple function of temperature for constant applied field. They find that $\rho_f/\rho_n = H/H_{c2}$ as $t = T/T_c \rightarrow 0$. Hopefully if we choose applied field values within the range of corresponding field values at 1.8°K, then we can use data from Farrell, et al.⁶ as has been done in all the other samples to find β .

From Farrell and Chandrasekhar²² one finds at 4.2°K, $H_{c1} = 200$ G and $H_{c2} = 3000$ G. We can calculate the percent of B_{app} above H_{c1} for 500 G and for 600 G at 4.2°K and compare with a similar calculation for 1000 G and 1600 G at 1.8°K. At 4.2°K and for 500 G we can write

$$\%H = \frac{500 - 200}{3000 - 200} \times 100\%$$

$$\%H = 11\%$$

At 4.2°K, 600 G one finds $\%H = 14\%$. Similarly for 1.8°K, 1000 G one

Figure 19. Sample #51A

TABLE 7

DATA FOR SAMPLE #51A FROM FIG. 19

Field (Gauss)	Slope ($\mu\text{V}/\text{amps}^3$)	Intercept ($\mu\text{V}/\text{amp}$)	I_P^2 (amps^2)	I_{break}^2 (amps^2)	$F_L @ I_b^2$ (nt/m^3)	F_P (nt/m^3)	F_{excess} (nt/m^3)
500*	7.86×10^{-3}	-4.16	520	725(?)	11.25×10^5	3.30×10^5	7.95×10^5
600*	8.85×10^{-3}	-4.01	455	650(?)	12.35×10^5	4.94×10^5	7.41×10^5
1000†	2.64×10^{-3}	-1.70	645	740(?)	10.90×10^5	9.46×10^5	1.44×10^5

*4.2°K
†1.8°K

06

RESISTIVITY FOR SAMPLE #51A

Field (Gauss)	Ω/η ($\text{m}^7/\text{nt-sec}$)	η ($\text{nt-sec}/\text{m}^2$)	β_I	β_{II}	ρ_I ($\mu\Omega\text{-cm}$)	ρ_{II} ($\mu\Omega\text{-cm}$)
500*	12.25×10^{-10}	0.96×10^{-7}	0.40	0.53	0.20	0.20
600*	12.25×10^{-10}	0.87×10^{-7}	0.49	0.65	0.22	0.22
1000†	12.25×10^{-10}	1.38×10^{-7}	0.38	0.51	0.30	0.29

*4.2°K
†1.8°K

finds %H = 16% and for 1600 G, %H = 28%. Since we are the same depth in the mixed state at 4.2°K as at 1.8°K, then we assume that we can use the resistivity values in Table 1 to find β . Such a calculation gives field penetration ratios only slightly larger than those encountered for sample #49A at 1.8°K.

In order to determine the consistency of the data with theory, a 1000 gauss curve was taken at 1.8°K to compare with data of sample #49A. One finds with satisfaction that the field ratios β are almost identical and the dissipation volume - viscosity coefficient ratio is exactly the same. This at least says that sample #49A and #51A are compatible. Excess forces between samples vary by a factor of one-half. The pinning force in #51A is larger than in #49A, but so is its Lorentz force.

An analysis of F_{excess} at 4.2°K yields a result that is not quite understood. The excess force at 4.2°K is about 5 times that at 1.8°K ! This can imply that the viscosity coefficient is temperature dependent since

$$F_L - F_P = F_{\text{excess}} = \eta v_f$$

$$\eta = \frac{F_{\text{excess}}}{v_f}$$

or that the flux line velocity is temperature dependent, or both. If $v_f \neq v_f(T)$, then

$$\eta_{4.2} = 5 \eta_{1.8} \quad .$$

If $\eta \neq \eta(T)$,

$$\frac{F_{\text{excess}(4.2)}}{v_f(4.2)} = \frac{F_{\text{excess}(1.8)}}{v_f(1.8)}$$

$$\frac{5}{v_f(4.2)} = \frac{1}{v_f(1.8)}$$

$$v_f(4.2) = 5 v_f(1.8) \quad .$$

The equation involving η implies that the medium is more viscous at 4.2°K than at 1.8°K. The lower temperature then acts to reduce the viscosity. The equation involving v_f indicates that the flux flow velocity is larger at 4.2°K than at 1.8°K. We are not certain as to which of the ideas are valid, if either.

One of the very important findings of this sample was the behavior of flux motion for zero applied field, both at 4.2°K and at 1.8°K. According to several authors³⁴ the critical current for zero applied field is less than the critical current at some small field. The critical current then decreases for increasing field. We are using the term critical current very loosely and should be referring to normal current or the current the sample is able to support. At 4.2°K and at zero field the flux motion is very erratic and nonreproducible. The normal current was 32 amps (± 0.5 amps). For 500 gauss the normal current was 40.5 amps and for 600 gauss it was 38 amps. Thus for a small longitudinally applied field the wire can actually carry more current than it can in zero field, up to a certain field. LeBlanc and Kiggins³⁵ as well as Bergeron³⁶ found a similar maximum in I_c vs. $H_{//}$ curves and attribute

the effect to an assumed helical pattern of current flow and to the dependence of pinning forces upon the magnetic induction. London and Walmsley²⁰ conclude that the lower current carrying ability at zero field is due to a lack of Lorentz force, so that flux motion enhances the normal current.

Comparison of Samples

We have talked about each sample individually; let us now compare and contrast the samples to get a total picture of the flux flow process occurring. Our five samples vary over four different diameters and are subject to five different physical conditions: (1) Sample #49A - rough surface, highly strained, (2) Sample #48A - moderately polished surface, well annealed, (3) Sample #47A - slightly polished surface, well annealed, (4) Sample #46A - highly polished surface, well annealed, (5) Sample #51A - rough surface, well annealed.

If one places Figs. 10, 16-19 next to one another in order of decreasing diameter (Table 2), he will immediately see what appear to be several trends of the data. As the wire decreases in diameter, the slopes vary from constant, to almost constant, to definitely changing, to indeterminable. One notices that along with the decrease in diameter goes a decrease in pinning current. We will see later that this does not necessarily imply a drastic variation in pinning force. The decreased pinning current is probably a function of the pinning effects as well as of the wire diameters as seen in eq. (2.29). But we know that just a decrease in wire diameter results in an increase in the field produced at the surface of the wire by a constant transport current. That necessarily implies that the ratio B_w/B_{app} increases as the wire diameter

decreases for a constant current. This suggests then that the Lorentz force, which is proportional to the cross-product of J and B increases with decreasing diameter. The excess force necessary to maintain viscous flow occurs for lower current values at reduced diameter wires.

The breaks in linearity at which $F_{\text{excess}} = \text{constant}$ occur, become less pronounced and then indeterminable for the smaller wire samples. It is difficult to ascertain, however, if the more important mechanism is the pinning force (which is not a function of diameter) or the Lorentz force. One finds that Lorentz forces in our geometry are about one-tenth the value from the slab geometry of flux flow.^{4,6}

The only other effect that appears to be diameter dependent is the dissipation volume - viscosity coefficient ratio (Ω/η) from Model I. One finds that the ratio decreases with decreasing diameter, i.e., with decreasing wire volume. It is gratifying that $\Omega/\eta \propto \text{Wire Volume}$ and not, say, inversely proportional.

We are not so lucky with the evaluation of η from Model II. One can see that there is a tremendous variation of η with field and with sample. The scatter is so large, in fact, that it is hopeless to try and make any sense of η . We intended to determine η by Model II and then to use Model I to find the dissipation volume from Ω/η . It appears that we cannot find the viscosity coefficient exactly so that we cannot determine the dissipation volume.

The remaining discussions will center about the physical condition of the wire. Let us compare β between samples. Since β is the ratio of the field inside the wire to the field outside, we expect β to be very

dependent on both surface and bulk pinning effects. Sample #49A has the roughest surface and is the most highly strained of all samples. This indicates two things: (1) the rough surface cannot support very much surface current and will allow much more flux to penetrate than in smoother surfaces, (2) the highly strained state will give rise to more pinning sites and will result in larger pinning currents and larger pinning forces than for highly annealed wires. One finds that this is true from the tables. Sample #49A has a larger field penetration ratio and larger pinning forces than any other sample. If one compares #49A with #51A which also has a rough surface but is highly annealed, β is the same in both cases and F_p is approximately the same. This tends to indicate that bulk defects play a smaller role in flux flow than surface irregularities. If this is true, then the values of β and F_p are dependent mainly on the surface condition.

One sample was constructed to test the effect of the longitudinal scratches introduced by the extrusion apparatus on the surface of the wire. The theory behind this idea is that field lines in our geometrical configuration are helices and if the lines are wrapped at the same pitch angle as the scratches, then there will be a lowering in the energy barrier and the lines would suddenly break through the surface. A typical pitch angle of 1° was introduced in the wire by twisting one end relative to the other. Data were taken on the sample with results indicating very little flux flow before the wire was driven completely normal. The pinning currents were very large because of bulk defects and the flow regime encompassed less than $1\mu V$ whereas typical data encompass $100\mu V$. It appears that bulk pinning due to the tremendous dislocations introduced by twisting has overshadowed any surface pinning in this case.

If one looks at the magnitude of pinning forces among samples (neglecting #51A) he finds that #49A, the highly strained sample, gives larger pinning forces than the other samples which are well annealed. This is what one expects if the volume pinning is the more important effect. Sample #51A has a rough surface and is well annealed. The pinning forces encountered are as large as for #49A. This must indicate that the surface has a definitely strong effect in pinning of flux lines. For our data we cannot determine which is the more important--surface effects or bulk effects. This problem arises because of the rapid annealing of our samples. It is impossible for us to obtain a sample in its virgin state of anneal since it requires about one hour to assemble the sample in its holder and cool it to liquid helium temperature.

One way of answering the question of dominant pinning effect is to take data for a rough surface, highly strained sample in a holder with removable section. Then one takes data on the sample after it has annealed to find bulk effects, and removes the section of the holder and chemically polishes to find the effects of the surface. This procedure was not undertaken and so the question is still unanswered.

One can see that the general shape of the curves vary for different amounts of polishing (different diameters). As previously mentioned, the pinning currents go down with polishing, and the currents at which voltage first appears (critical currents) are down. Sample #49A and #46A which represent extremes in the physical conditions illustrate an interesting point. For the highly strained sample (#49A) there are nothing but linear regions with upward bending; for the well annealed and highly polished sample (#46A) there are few linear regions with most

curves concave downward. Considering the small currents used for #46A this could possibly indicate that #46A represents the "toe" regions of #49A. On the other samples this region is not as pronounced as with #46A. Possibly if we expand the "toe" regions of each curve then we would see the appearance of such a concave downward trend and thus obtain the "S" previously mentioned.

Data Coalescence

In Chapter II we saw that linearity of data can be checked by plotting in reduced variables. We plot eq. (2.14) using eq. (2.15). Using Tables 3-7 one can tabulate the normalization factors for the plot V/I vs. I^2 as shown in Table 8 and can make a plot like Fig. 20. The dotted line represents a purely linear equation so that one can get an idea of how well the data fit the equation

$$\frac{V}{I} = m I^2 + b \quad .$$

One finds more scatter in the upper region than in the lower region. One reason for the cluster at the lower end is the mathematical forcing of all intercepts to occur at -1 . This is the effect of plotting $(I/I_p)^2$. The scatter at the upper end of the graph could be due to one of several factors. If the data are not very linear in the upper region, then there will be deviations. This will occur if the small angle approximation is breaking down and nonlinear terms arising. A poor choice of I_p will give improper scaling along the horizontal axis and will introduce scatter. Deviations could occur because of an inaccurate scaling factor K . This factor tends to slide the line of unit slope along the

TABLE 8
COALESCENT DATA

Sample	Field (Gauss)	I_P^2 (Amps ²)	K (Amp/Volt)	Sample	Field (Gauss)	I_P^2 (Amps ²)	K (Amp/Volt)
#46A	1000	225	2.52	#47A	1300	385	16.6
	1300	63	6.30		1500	220	30.6
#48A	1000	400	18.2		1600	165	41.9
	1200	130	33.8		1700	95	80.7
	1400	80	36.2	#49A	1200	580	0.56
	1500	65	34.2		1400	360	0.90
	1600	60	28.5		1500	280	1.16
	1700	50	30.6		1600	220	1.48

Figure 20. Coalescent Data

vertical axis. An erroneous calculation in any of the five quantities affecting $K(\Omega/\eta, c, A, \beta, I_p)$ will result in an inaccurate scaling factor with points deviating away from the dotted line more as we proceed to larger values of $(I/I_p)^2$. The deviations will be either above the line or below the line depending upon whether the value of K is too high or too low.

One finds that generally there is more deviation for high fields than for low fields. This may be merely incidental since there is no reason to expect more deviations at one field than at others, unless the small angle approximation is breaking down.

Not all data give a good fit to the line on Fig. 20. Data that have a lot of fluctuation in flux flow voltages and are not very good straight lines on V/I vs. I^2 plots give tremendous scatter at both ends of the coalescent plot. Sample #50A, which is hopefully identical to #46A, gave unexplainable results on a V/I vs. I^2 plot. The sample showed several linear portions and characteristic breaks for each field. The sample was driven completely normal, so that each curve was complete, but the small angle approximation broke down over one-half the length of each curve.

Several dozen other samples were run at 1.8°K and at 2.0°K at high fields (~ 2500 G - 3500 G). They do not appear to produce $V \propto I^3$ nor do they appear to be purely exponential. Consequently, we have not explained this region which we hope is the creep regime.

CHAPTER V

CONCLUSIONS

We saw in both Chapter II and Chapter IV that analysis by either Model I or Model II leads to identical values of resistivity. The difficulty in the cylindrical geometry configuration is the parameter Ω/η or η which arises because of the uncertainty in β . The only way these parameters can be determined is by the use of slab geometry data where they can be experimentally determined.

Several additional models from other experimenters¹⁹⁻²¹ gave $V \propto I^3$, as our two models which equated the power input from the current supply to the power dissipated by moving flux. This current-voltage relation holds only for the region in which the Lorentz force dominates all other forces, $F_L > F_p$ --the flow regime. Regions of $F_L \approx F_p$ and of $F_L < F_p$ are more complicated and do not satisfy $V \propto I^3$. Work was not conducted in these two regions.

Five different samples of various diameters and physical conditions were studied. We found several striking manifestations of flux flow. Almost all curves on a V/I vs. I^2 plot exhibited some bending in the lower current region and were either concave upward or downward. A relatively few curves bending downward tended to be unexplained, since in most cases they gave imaginary pinning currents. The upward bending curves tended to have definite breaks at which linearity set in. The break is interpreted as an indication of the Lorentz force dominating

the pinning force. Tables 3-7 point out this occurrence quite well. One notes, however, that the excess force is not a constant among the various samples, but is fairly constant for a particular sample.

The pinning forces vary significantly from sample to sample and appear to be largest for the highest strained sample with the roughest surface (#49A). From our experiments we could not determine the major pinning effect--surface or volume. We conclude from experiments that volume annealing is quite rapid. The surface of the wire seems to affect the field penetration considerably. Smooth surfaces can support shielding currents which can keep out magnetic flux. Rough surfaces cannot support much current and thus give a larger value of β than for smooth surfaces.

We can devise a tentative procedure to check out the importance of these two pinning effects. The sample holder with removable section must be used so that measurements can be made on one specific sample. One should begin with the rough surface and highly strained sample and should proceed in steps to arrive at a very smooth and well annealed sample. The procedure is as follows: (1) assemble and take data on rough, highly strained sample, (2) anneal the sample as completely as possible and repeat data, (3) etch the surface in increments of 0.002 cm and repeat data, (4) copperplate the sample in increments of 0.002 cm thicknesses and repeat data. Step (2) should show bulk effects and give an ideal of the magnitude of surface pinning over total pinning. Step (3) could possibly indicate surface improvement as the surface becomes smoother and can support larger currents. Step (4) should provide normal material so that surface currents should decay. This may result in

very low pinning and possibly an almost negligible flow region due to the catastrophic collapse of flux lines into the sample.

Another way of studying the surface is by looking again at the angle at which helical flux lines enter the sample. Previously, the sample was twisted; now, one should mechanically scratch the smooth surface of a well annealed wire by some means so that the pitch angle can be controlled to within one-half degree. This will allow one to determine if the energy is reduced somehow when the flux lines align with the scratches.

The slopes of the V/I vs. I^2 plots in the upper current regime are indications of the field inside the wire, as the intercepts are indications of the pinning forces. The slopes were fairly constant for various fields of a given sample but varied from sample to sample. This resulted in a fluctuation of β from sample to sample. However, β was approximately constant for a given sample, but did increase slightly with applied field. From the I-V characteristics of our data we found resistivities were determined to within β . The ambiguity in β was eliminated by comparison of resistivities to slab geometry values obtained by Kim⁴ and by Farrell.⁶ We found that $0.1 \leq \beta \leq 0.6$.

A maximum estimate of β for $\text{Pb}_{84.5}\text{In}_{15.5}$ from Dubeck, et al.³³ using reversible magnetization data yields $0.9 \leq \beta \leq 0.94$. However, their sample percentages are different than ours and give a mixed state region about 0.8 of ours. As mentioned in Chapter II, the magnetization data taken under equilibrium conditions could possibly account for the small penetration ratio we obtain under the nonequilibrium conditions of flux flow.

A similar calculation of $\beta_{\max}$ was made from $B(r)$ vs. r data for $\text{Pb}_6\text{In}_{94}$ of Sikora, et al.¹⁸ Computer analysis showed that $B(r) \propto r^{1/2}$, and integration of $B(r)$ over the sample volume to find $\overline{B_{\text{int}}}$ gave values $0.85 \leq \beta \leq 0.87$. This method of calculation of β is probably closer to our situation since a small transport current $I=26$ amps ($I/I_{c1} = 1.1$) is applied while B_{int} measurements are made. They do not have thermodynamic equilibrium.

These two additional calculations of β suggest that the resistivity in cylindrical geometry can be determined without ambiguity once β is determined from another experiment. If one can make simultaneous magnetization and flux flow resistivity measurements on each sample, then B_{int} and thus β can be found. Substitution of β into the resistivity formulas of eq. (2.13) and eq. (2.32) will give a value of ρ_f directly without the use of slab geometry data.

Once the ambiguity in ρ_f (or β) has been eliminated, one can concentrate on studying the effects of sample composition and temperature on flux motion. Along with this study could go theoretical and experimental exploration of the creep regime. Sikora³¹ has suggested a possible method of theoretical explanation of the creep region much like given in papers by Beasley,¹⁴ Anderson,¹⁵ and Kim.¹⁶

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