APPLICATION OF DIMENSIONAL ANALYSIS TO ROLL-TO-ROLL MANUFACTURING SYSTEMS

By

Pramod R. Raul and Prabhakar R. Pagilla
Oklahoma State University
USA

ABSTRACT

In this paper, we investigate the application of dimensional analysis for R2R systems. First, dimensional form of governing equations for web tension and speed are transformed to non-dimensional form using dimensional analysis, and discussions are provided to highlight their usefulness in further analysis and design of R2R systems. Second, two specific cases are considered to show application of dimensional analysis to R2R systems: analysis of an accumulator and redesign for capacity scaling and scaling of process and controller parameters of an example R2R system for a change in the web material. In the first case, we provide results

from model computer simulations to evaluate the method, and in the second case, results from experiments conducted on a large R2R platform are presented and discussed. Further, frequency response experiments are conducted for a subsystem of web spans and rollers for different materials to also evaluate the results of applying dimensional analysis to R2R systems.

NOMENCLATURE

A	:	Area of cross-section of web
B_f	:	Coefficient of viscous friction
E	:	Modulus of elasticity of web material
F_d	:	Disturbance force on accumulator carriage
f	:	Resonant frequency
g	:	Gravitational constant
J_i	:	Inertia of i^{th} roller
K	:	Actuator gain
k_p	:	Proportional gain
k_i	:	Integral gain
L_i	:	Length of i^{th} web span
m	:	Mass inertia of roller
M_i	:	Driving Motor of i^{th} roller
M_c	:	Mass of accumulator carriage
N	:	Number of spans in accumulator
n_i	:	Gear ratio of i^{th} motor
Π	:	Dimensionless parameter
R_i	:	Radius of i^{th} roller
s	:	Laplace variable
t	:	Time parameter
t_c	:	Average web tension in accumulator spans
t_i	:	Actual web tension in i^{th} web span
u_c	:	Controlled force on accumulator
u_i	:	Torque input to i^{th} roller
v_i	:	Velocity of i^{th} roller
x_c	:	Position of accumulator carriage
ω	:	Cut-off frequency in a PI controller
ρ	:	Density of web material
τ	:	Dimensionless Time

Subscripts:

i	:	Span index, $i = 0, 1, 2, \ldots$
r	:	Reference value

INTRODUCTION

Two strategies are mainly used for tension control, load-cell based and dancer based feedback control systems. In a previous study, Raul (2010), controller normalization procedure is studied and used to determine the gains for the dancer position feedback based web tension control system evaluated from a well-tuned load cell based web tension feedback control system. The goal of controller normalization is to match controller gains based on the relation between the dancer position and the tension from load-cells. The controller normalization procedure further motivated us to study dimensional analysis for obtaining non-dimensional form of governing equations for key web transport variables, such as web tension and web velocity. The dimensional analysis technique is extensively used in fluid mechanics and can also be applied to interconnected systems such as robotics, R2R systems, etc.

Industrial R2R systems are large-scale interconnected systems. Dimensional analysis may be performed on a small-scale experimental platform and then scaled up to the actual large scale industrial setup. Dimensional analysis is an useful and effective technique to scale web components, as well as process and controller parameters. Dimensional analysis can also be applied to frequency response study to evaluate the resonant frequency for the scaled web system. It reduces the time required to design a new web transport line, and also helps in designing controllers for new industrial lines without much additional effort.

The concept of dimensional analysis and Buckingham Pi theorem to evaluate dimensionless "Pi" parameters are described in Szirtes (1998). In this paper, dimensional analysis technique is applied to the governing equations for the key process variables (web tension and web speed) of a web transport system. The Buckingham Pi theorem is used to convert the dimensional form of governing equations of web transport system into non-dimensional form. In particular, we illustrate the application of dimensional analysis to roll-to-roll systems using three different examples: (1) web accumulator, (2) web tension control system, and (3) resonant frequencies due to a system of idle rollers and spans.

Non-dimensional dynamics for an accumulator system is formed by using the Buckingham Pi theorem. Based on the non-dimensional dynamics, an accumulator can be effectively reconfigured, such as the scenario of doubling the accumulator capacity. Dimensional analysis can expedite the design procedure of web handling components in such scenarios.

The PI control strategy once tuned for an existing web system may not perform well with changes in web line configuration unless retuned. The dimensional analysis technique simplifies retuning of the PI controller in the event of changes in web materials, web flow path, etc. The Buckingham Pi theorem is applied to a basic web system in order to obtain and evaluate scaled controller parameters for the modified web system.

Since almost all web processing lines contain systems of idle rollers and spans between driven rollers, it is important to determine the resonant frequencies arising from this system of rollers and spans to better understand and control web transport behavior. A particular interest to a systems engineer is the determination of the minimum resonant frequency. The analytical solution for determining resonant frequencies of a web section with three or more idler rollers is involved. The dimensionless Pi theorem provides relations between resonant frequencies and the material and geometric parameters of a web transport system for any number of idler rollers. This simplified relation reduces the effort in terms of repeating the experiment or doing involved calculations to evaluate the resonant frequencies of the R2R system for different web configurations and any number of idle rollers. The application of dimensional analysis to scaling of a PI tension controller and scaling of the minimum resonant frequency is experimentally verified.

DIMENSIONAL ANALYSIS TECHNIQUE

Any physical quantity can be expressed in two forms: dimensional and dimensionless. Since fundamental quantities are related with each other by physical laws, a fundamental set of units is used to form units for all physical quantities. A physical quantity may have a variety of units but is always presented with the same dimensional convention.

A non-dimensional quantity is generally defined by a ratio or product of dimensional quantities. Most physical phenomena in the universe operate on groups of variables which are pure numbers, and called as non-dimensional quantities or parameters. For example, 'Mach number', an important phenomenon in fluid mechanics, is a non-dimensional quantity which characterizes the relative velocity of sound in a flow of fluid of velocity. Such non-dimensional parameters can be used to characterize dynamic systems. One key requirement for performing dimensional analysis is that the governing equations of the given physical system must be dimensionally homogeneous.

Two common techniques are used for dimensional analysis: the Buckingham Pi method and the Rayleigh method, as discussed in Szirtes (1998). These techniques require knowledge of variables which influence the physical system.

The Buckingham Pi theorem provides a basis for dimensional analysis. This is a more generalized and systematic approach for determining the minimum number of dimensionless variables that characterize the system.

Buckingham Pi Theorem: Let $f(V_1, V_2, ...V_n) = 0$ be a set of dimensional homogeneous equations with 'n_d' parameters and described by 'm_r' fundamental dimensions. The dimensional equation '$f(V)$' can be expressed by another set of equations '$F(\Pi)$' with '$n_d - m_r$' dimensionless parameters. The non-dimensional groups are referred to as 'Π' terms. The set of equations '$F(\Pi)$' can be written as $F(\Pi_1, \Pi_2, ...\Pi_n) = 0$ and the non-dimensional 'Π' terms characterize the system.

The Buckingham Pi theorem reduces the number of variables and presents the system equations in a simplified manner.

The scale factor for a particular parameter is defined as the ratio of the magnitude of that parameter for the candidate model to its magnitude for the base model. For example; the web span length scale factor is

$$S_L = \frac{L_c}{L_b} \qquad \{1\}$$

where L_c is a span length of scaled or candidate R2R system, L_b is a span length of base R2R system. The principle of similarity (similitude) or model law is a relation or set of relations between scale factors. It is used to relate physical systems of different sizes and is beneficial for scaling the physical systems. In many applications it is advantageous to perform experiments on a small prototype before building the actual system. The similitude ensures similar behavior between the scaled prototype and the actual system.

Dimensional analysis can be applied to dynamically equivalent systems with different materials, different tension feedback systems, and web configurations. The dynamic equivalence condition restricts the application of scaling factors to any general web transport system.

APPLICATION OF DIMENSIONAL ANALYSIS TO A WEB TRANSPORT SYSTEM

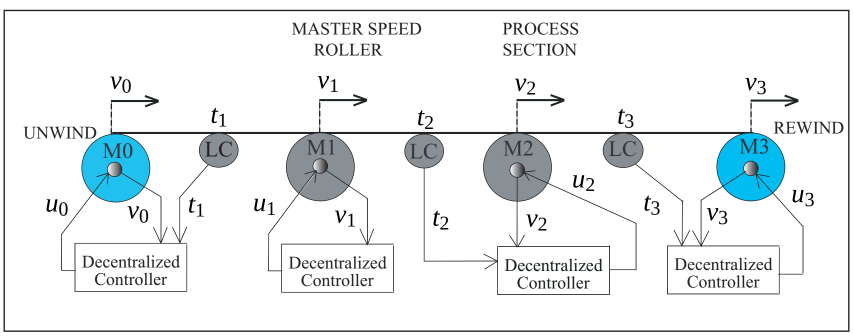

Figure 1: Web Line Sketch

A simplified three tension zone web line is shown in Figure 1 to illustrate the application of dimensional analysis. The simplified web line mimics many features of a general web transport system, and it is also a representation of the Euclid Web Line (EWL), an experimental platform available in the Web Handling Research Center (WHRC). The web line shown in Figure 1 is divided into four sections: unwind section, master speed roller, process section and rewind section. In Figure 1, M_i denotes driving motors, v_i denotes web transport velocity on the i^{th} roller, u_i denotes input torque from the i^{th} motor, and t_i denotes web tension between $(i-1)^{th}$ and i^{th} driven rollers. The master speed roller regulates reference speed for the web line and does not regulate tension in adjacent spans. The master speed roller operates under a pure speed control loop. The unwind/rewind roll and the pull roll are regulated by a cascaded tension and speed control system. For the purpose of illustrating dimensional analysis, the unwind section is considered. It is also possible to perform dimensional analysis for rewind and pull roll sections using the same procedure. Dimensional and non-dimensional dynamics of the unwind section is presented in the following subsections.

Unwind Section Dimensional Equations

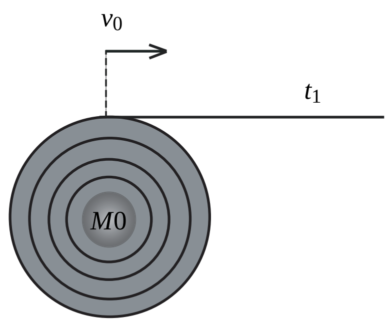

Figure 2: Unwind Roller

The unwind section governing equations are obtained by considering tension in the upstream span t_1 and web velocity of the unwind v_0. The governing equation for web tension neglecting the effect of radius and inertia change is given by

$$\dot{t}_1 = \frac{EA}{L_1}(v_1 - v_0) + \frac{t_0 v_0}{L_1} - \frac{t_1 v_1}{L_1}, \tag{2}$$

and the governing equation for web velocity of the unwind neglecting friction is given by

$$\dot{v}_0 = \frac{R_0^2}{J_0}(t_1 - t_0) + \frac{R_0}{J_0} n_0 u_0 \tag{3}$$

where t_1 is the web tension, t_0 is the web tension in the unwind roll, v_0 is the velocity of the unwind, v_1 is the velocity of the master speed roller, R_0 is the unwind radius, n_0 is the gear ratio, J_0 is the inertia of the unwind, E is the elastic modulus of the web material, A is the cross section area of the web, L_1 is the span length of the web, and u_0 is the torque input to the unwind.

Non-dimensional Governing Equations

The dimensional dynamic equations for the unwind section indicates that ten parameters are influenced in the selected web zone. Three fundamental dimensions, length, force and time are chosen to describe these parameters. The unit system used for the fundamental dimensions are feet (ft), pound-force (lbf), and seconds (s), respectively. Using the Buckingham Pi theorem, there are seven non-dimensional Pi parameters which characterize the dimensionless system.

$$\Pi_1 = tR_0\sqrt{\frac{EA}{L_1 J_0}} = \tau \qquad \Pi_2 = u_0\frac{L_1}{R_0^2 EA} = \overline{u}_0 \qquad \Pi_3 = \frac{v_0}{R_0^2}\sqrt{\frac{L_1 J_0}{EA}} = \overline{v}_0$$

$$\Pi_4 = t_0\frac{L_1}{R_0 EA} = \overline{t}_0 \qquad \Pi_5 = \frac{v_1}{R_0^2}\sqrt{\frac{L_1 J_0}{EA}} = \overline{v}_1 \qquad \Pi_6 = t_1\frac{L_1}{R_0 EA} = \overline{t}_1$$

$$\Pi_7 = \frac{R_0}{L_1} = \overline{c}$$

The dimensionless governing equations can be obtained by substituting the above formed Pi groups in dimensional equations {2} and {3}. The dimensionless equations for web tension and web speed are given by

$$\dot{\overline{t}}_1 = (\overline{v}_1 - \overline{v}_0) + \overline{c}(\overline{t}_0\overline{v}_0 - \overline{t}_1\overline{v}_1) \qquad \{4\}$$

$$\dot{\overline{v}}_0 = \overline{t}_1 - \overline{t}_0 + n_0\overline{u}_0 \qquad \{5\}$$

Clearly, the dimensionless tension and web velocity equations with a reduced number of variables are simple in representation compared to the dimensional model. This simple representation is expected to help in analysis and design of controllers.

DIMENSIONAL ANALYSIS OF AN ACCUMULATOR

The accumulator is an important primitive element in web transport systems and plays a key role in continuous operation of many web processing lines. The capacity of an accumulator may be increased by increasing the carriage height. Dimensional analysis of an accumulator is performed in terms of capacity scaling. A simplified dynamic model of the accumulator is taken into consideration, that includes accumulator carriage dynamics, average web tension dynamics in accumulator web spans, driven roller dynamics at entry and process sides. We consider accumulator carriage control in conjunction with entry and process driven rollers. A schematic of an accumulator is shown in Figure 3 which includes carriage, web spans and rollers.

The carriage dynamics of the accumulator is given by

$$M_c\frac{d^2x_c(t)}{dt^2} = u_c(t) - F_d(t) - M_c g - Nt_c(t) \qquad \{6\}$$

where u_c is the controlled force, F_d is the disturbance force due to friction in the hydraulic cylinder and rod seals, friction in carriage guides, and external forces on the carriage, M_c is the mass of carriage, t_c is the average tension in the accumulator spans, and N is the number of spans in the accumulator.

The average tension dynamics in accumulator web spans is given by

$$\frac{dt_c}{dt} = \frac{AE}{x_c}\frac{1}{N}(v_p(t) - v_u(t)) + \frac{AE}{x_c}\dot{x}_c(t) \qquad \{7\}$$

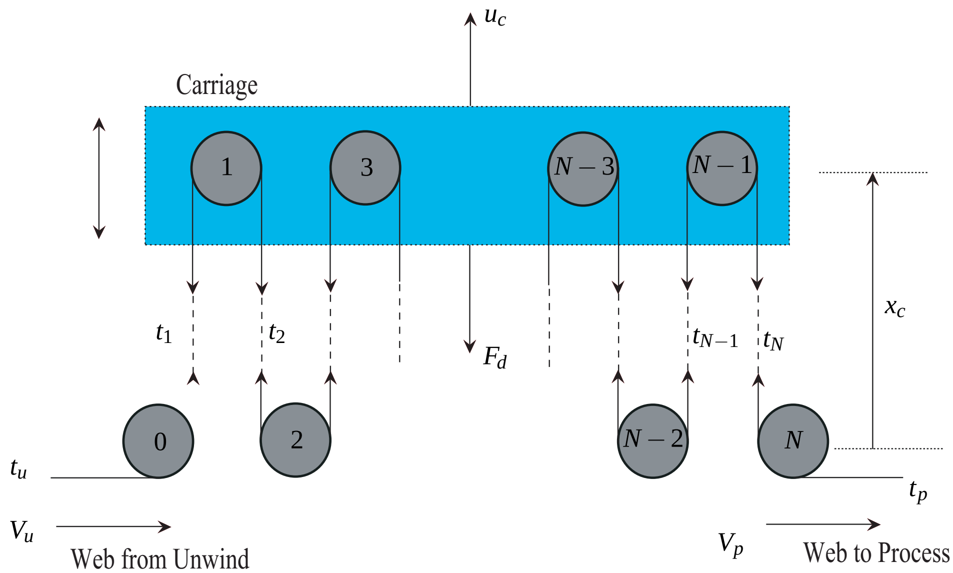

Figure 3: Schematic of an Entry Accumulator

Assuming that there is no slip between the web and the roller, roller angular velocities and web velocities are related by: $v_u = R\omega_0$ and $v_p = R\omega_N$, where R is the roller radius.

The entry and process web velocities are given by

$$\dot{v}_u = \frac{1}{J}(-B_{fu}v_u(t) + R^2(t_c(t) - t_r) + RK_u u_u(t)) \qquad \{8\}$$

$$\dot{v}_p = \frac{1}{J}(-B_{fp}v_p(t) + R^2(t_r - t_c(t)) + RK_p u_p(t)) \qquad \{9\}$$

where B_{fp} and B_{fu} are coefficients of viscous friction, t_r is reference tension in entry and process web span, i.e., $t_r = t_u = t_p$, u_p and u_u are the control inputs to driven roller actuators, and K_p and K_u are the actuators gains.

Non-dimensional Parameters

For the described system, application of Buckingham Pi theorem results in the following dimensionless groups:

$$\Pi_{1A} = t\sqrt{\frac{AER}{J}} \qquad \Pi_{2A} = \frac{u_p}{AER} \qquad \Pi_{3A} = K_p$$

$$\Pi_{4A} = \frac{u_u}{AER} \qquad \Pi_{5A} = K_u \qquad \Pi_{6A} = \frac{u_c}{AE}$$

$$\Pi_{7A} = \frac{M_c}{J}R^2 \qquad \Pi_{8A} = \frac{F_d}{AE} \qquad \Pi_{9A} = \frac{gJ}{AER^2}$$

$$\Pi_{10A} = \frac{t_c}{AE} \qquad \Pi_{11A} = \frac{x_c}{R} \qquad \Pi_{12A} = \frac{v_u}{R}\sqrt{\frac{J}{AER}}$$

$$\Pi_{13A} = \frac{v_p}{R}\sqrt{\frac{J}{AER}} \qquad \Pi_{14A} = \frac{t_u}{AE} \qquad \Pi_{15A} = \frac{t_p}{AE}$$

$$\Pi_{16A} = \frac{B_{fu}}{\sqrt{JAER}} \qquad \Pi_{17A} = \frac{B_{fp}}{\sqrt{JAER}} \qquad \Pi_{18A} = N$$

Dimensionless equations may be formed with the derived Pi parameters for the accumulator system.

Scaling laws establish relations between the parameters of the model and the prototype systems. The model law applied to the accumulator equations gives the scale factors

$$S_t = S_{AE}^{-0.5} S_R^{-0.5} S_J^{0.5} \qquad S_{K_u} = 1 \qquad S_{u_p} = S_{AE} S_R$$

$$S_{u_c} = S_{AE} \qquad S_{K_p} = 1 \qquad S_{M_c} = S_J S_R^{-2}$$

$$S_{u_u} = S_{AE} S_R \qquad S_{F_d} = S_{AE} \qquad S_g = S_{AE} S_R^2 S_J^{-1}$$

$$S_{t_c} = S_{AE} \qquad S_{x_c} = S_R \qquad S_{v_u} = S_R^{1.5} S_{AE}^{0.5} S_J^{-0.5}$$

$$S_{v_p} = S_R^{1.5} S_{AE}^{0.5} S_J^{-0.5} \qquad S_{t_u} = S_{AE} \qquad S_{t_p} = S_{AE}$$

$$S_{B_{fu}} = S_J^{0.5} S_{AE}^{0.5} S_R^{0.5} \qquad S_{B_{fp}} = S_J^{0.5} S_{AE}^{0.5} S_R^{0.5} \qquad S_N = 1$$

The model law is used to evaluate the dynamic equivalence conditions. The model law helps in calculating controller gains for the scaled up accumulator. In this scenario, the accumulator height is doubled to scale up the capacity and the controller gains for the modified system may be obtained based on the original accumulator system.

Model Simulation Results For Accumulator Scaling

The original accumulator system has the following parameter values: $M_c = 7310\ kgs$, $A = 3.27 \times 10^{-4}\ m^2$, $E = 6.9 \times 10^{10}\ N/m^2$, $N = 34$, $v_f = 35.037 \times 10^5\ N-s/m$, $R = 0.1524\ m$, $J = 2.1542\ kg-m^2$, $B_f = 0.02$, $t_r = 5180\ N$. The desired process speed is 3.3 m/s. A sinusoidal disturbance (amplitude 0.25 m/s^2 and frequency 0.5 Hz) is used as the disturbance force on the carriage. A typical entry speed scenario during unwind roll change is shown in Figure 4. The desired profile for carriage velocity is given by

$$v_c^d = \frac{v_u^d - v_p^d}{N} \qquad \{10\}$$

The control objective is to track the desired trajectory for the carriage position, entry velocity, and process velocity while maintaining the average web tension; a PI controller is employed for this purpose. Simulations are performed with actuator gain values $K_p = K_u = 10$ and driven roller proportional and integral gain values $k_{pu} = 396$, $k_{iu} = 19.2$, $k_{pp} = 387$, $k_{ip} = 8.3$. The controlled carriage position response is shown in Figure 4.

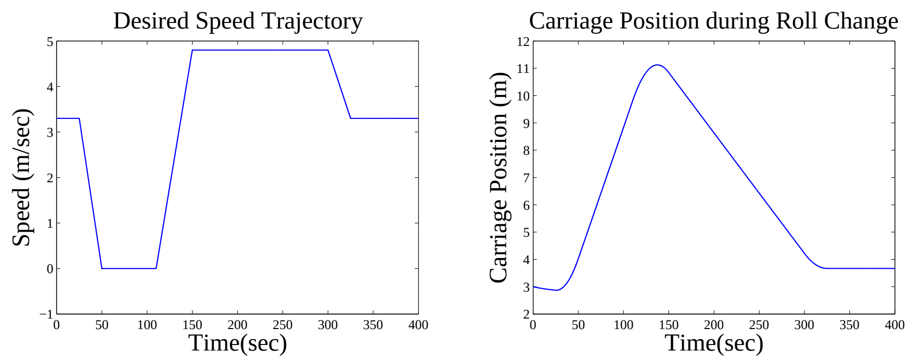

Figure 4: Left: Desired Entry Speed Profile for Unwind Roll Change; Right: Carriage Position During Roll Change

A scaled system is designed with twice the capacity of the original system. For the scaled up accumulator, roller radius is increased to twice the value used for the original system to maintain dynamical equivalence and while other system parameters are kept constant. The scale factors with the modified system are given in Table 1.

Scale Factor	Value	Scale Factor	Value
S_t	$\sqrt{2}$	S_{u_p}	2
S_{K_p}	1	S_{u_u}	2
S_{K_u}	1	S_{u_c}	1
S_{M_c}	1	S_{F_d}	1
S_g	1	S_{t_c}	1
S_{xc}	2	S_{v_u}	$\sqrt{2}$
S_{t_u}	1	S_{t_c}	1
$S_{B_{fu}}$	$2\sqrt{2}$	$S_{B_{fp}}$	$2\sqrt{2}$
S_N	1	S_{v_p}	$\sqrt{2}$

Table 1: Scale Factors for Accumulator with Double Capacity

The model law indicates that the control effort is scaled by a factor of 2. In the case of PI control law, proportional gains change by a factor of $\sqrt{2}$ and integral gains for both systems remain the same. The scaled accumulator system design is simulated with scaled controller gains. The plots shown in Figure 5 indicate scaled

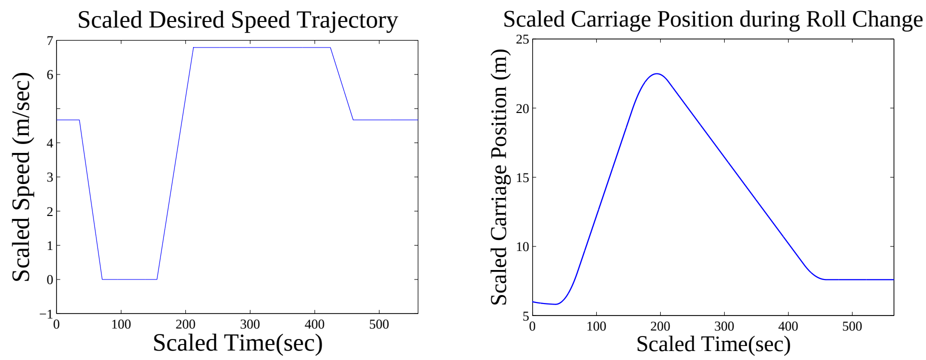

Figure 5: Left: Scaled Desired Entry Speed Trajectory; Right: Scaled Carriage Position during Roll change

desired entry speed and scaled carriage position during unwind/rewind roll change. The scaled capacity is twice that of the original system. The scaled control scheme achieved the desired position by matching the original system performance.

DIMENSIONAL ANALYSIS OF A WEB TENSION CONTROL SYSTEM

A scaling law for a PI tension control strategy will be determined in this section. Controller scaling can be extended to any type of controller with its characteristic parameters. The tension control strategy for the unwind section is shown in Figure 6. The control signal for the outer loop is based on tension feedback and controller scaling laws are applied to the tension loop. The tension

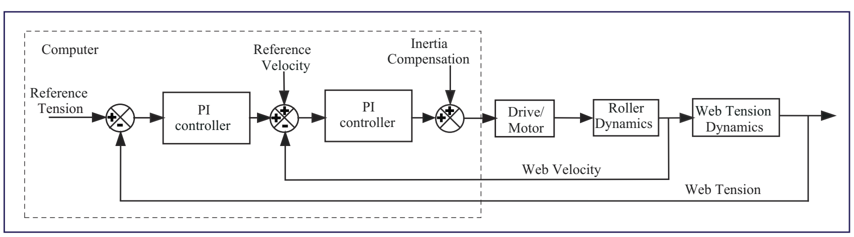

Figure 6: Unwind Roller Control Strategy

loop PI controller can be expressed in the frequency domain as:

$$C(s) = k_p \frac{s+\omega}{s} \qquad \{11\}$$

where k_p is the proportional gain (ft), ω is the zero crossover frequency (rad/sec), and s is the frequency domain parameter (1/sec).

Hence, by applying the Buckingham theorem to the closed loop system we get the following Pi parameters

$$\Pi_8 = \frac{k_p}{R_0} = \overline{k}_p \qquad \Pi_9 = s\frac{1}{R_0}\sqrt{\frac{L_1 J_0}{EA}} = \overline{s} \qquad \Pi_{10} = \omega\frac{1}{R_0}\sqrt{\frac{L_1 J_0}{EA}} = \overline{\omega}$$

These Pi parameters indicate that the proportional gain is inversely proportional to the roller radius. The Pi parameter corresponding to controller frequency (Π_{10}) depends on the modulus parameter EA and parameters R, J, and L. The non-dimensional form of the controller is given by

$$\overline{C}(\overline{s}) = \overline{k}_p \frac{\overline{s} + \overline{\omega}}{\overline{s}} \qquad \{12\}$$

The model law applied to the non-dimensional parameters, gives the following scale factors:

$$S_t = S_{L_1}^{0.5} S_{J_0}^{0.5} S_{R_0}^{-1} S_{EA}^{-0.5} \qquad S_{u_0} = S_{R_0}^{2} S_{EA} S_{L_1}^{-1} \qquad S_{v_0} = S_{R_0}^{2} S_{EA}^{0.5} S_{L_1}^{-0.5} S_{J_0}^{-0.5}$$
$$S_{t_0} = S_{R_0} S_{EA} S_{L_1}^{-1} \qquad S_{v_1} = S_{R_0}^{2} S_{EA}^{0.5} S_{L_1}^{-0.5} S_{J_0}^{-0.5} \qquad S_{t_1} = S_{R_0} S_{EA} S_{L_1}^{-1}$$
$$S_{R_0} = S_{L_1} \qquad S_{k_p} = S_{R_0} \qquad S_s = S_{R_0} S_{EA}^{0.5} S_{L_1}^{-0.5} S_{J_0}^{-0.5}$$
$$S_\omega = S_{R_0} S_{EA}^{0.5} S_{L_1}^{-0.5} S_{J_0}^{-0.5}$$

Transporting a different web material through the same web line results in change in parameters, such as modulus of elasticity (E), density (ρ), etc. With different web material and the dynamic equivalence conditions given by the scale factors S_{v_0} and S_{t_0} implies that the operating web speed and tension must be scaled. Similarly, the scale factors S_{k_p}, S_s, and S_ω indicate that, for the same web platform, i.e., geometrical similar systems, controller gains change only with a change in web material property values. Hence, controller gains can be scaled based on the basis of material properties for the same line. Controller scaling may expedite the process of tuning controller parameters.

DIMENSIONAL ANALYSIS OF A SYSTEM OF IDLE ROLLERS AND SPANS FOR RESONANT FREQUENCIES

An important aspect to consider in web transport systems is the effect of the idle rollers and spans between two driven rollers. Since the system idle rollers and spans introduce mechanical resonance, it is beneficial to know the resonant frequencies of this system as they influence the tension signal and how the control systems are tuned. Of particular importance is the minimum resonant frequency due to idle rollers and spans. In this subsection, we will show how dimensional analysis could be used to relate the minimum resonant frequencies due to running of different materials through the same system of idle rollers and spans. An one idler two-span system is shown in Figure 7. The minimum resonant frequency for the one idler web system is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{EA}{m}}\sqrt{\frac{1}{L_1} + \frac{1}{L_2}} \qquad \{13\}$$

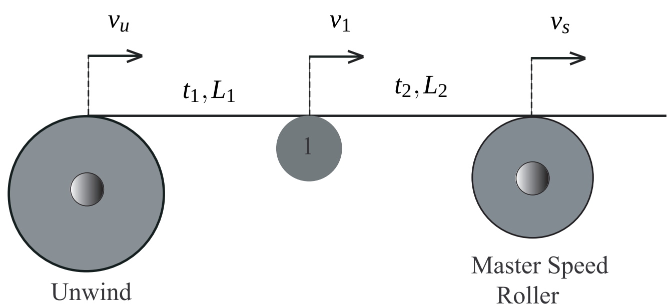

Figure 7: One-Idler Web System

where L_1 and L_2 are the web span lengths, EA is the modulus constant of web material, and m is the mass inertia of the idle roller.

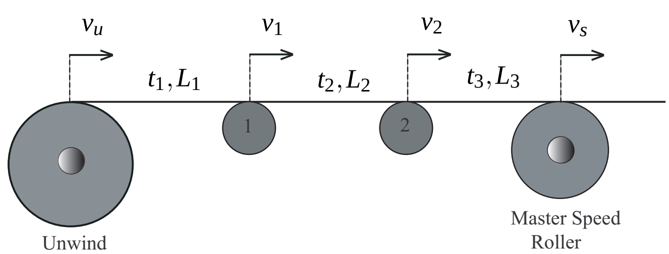

Figure 8: Two-Idler Web System

A two web idler system is shown in Figure 8. The minimum resonant frequency for the two idler system is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{EA}{m}}\sqrt{\frac{-\sqrt{L_2^2(L_3-L_1)^2+4(L_1L_3)^2}+L_1L_2+2L_1L_2+L_2L_3}{2L_1L_2L_3}} \qquad \{14\}$$

where L_1, L_2, and L_3 are the web span lengths.

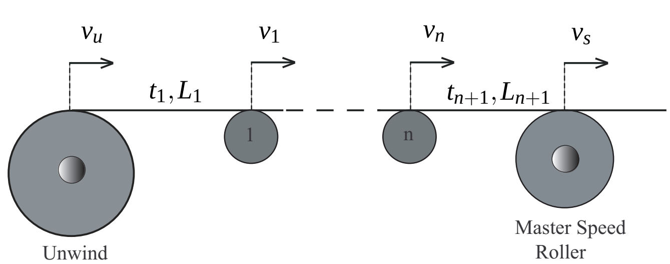

Figure 9: n-Idler Web System

It is difficult to obtain analytical expressions for the resonant frequencies of systems containing more than two idle rollers. further, if one is able to perform laborious algebra to obtain expressions for them, it is difficult to analyze these expressions to provide useful information. Often numerical analysis is employed for higher order systems. In the following we will show how dimensional analysis may be used to relate resonant frequencies of a system of idle rollers and spans containing n-idle rollers (see Figure 9) when transporting different web materials through this system.

Resonant frequencies are functions of system parameters such as modulus parameter (EA), idle roller equivalent mass (m), and length of web spans within the system. Three fundamental dimensions, length, force and time are chosen to describe these parameters. The unit system used for the fundamental dimensions are feet (ft), pound-force (lbf), and seconds (s), respectively. Using the Buckingham Pi theorem, there are two non-dimensional Pi parameters which characterize the dimensionless system shown in Figure 7.

$$\Pi_1 = f\sqrt{\frac{L_1 m}{EA}} \qquad \{15\}$$

$$\Pi_2 = \frac{L_2}{L_1}$$

For the one-idler case, the dimensionless resonant frequency equation may be formed by substituting the above formed Pi groups (Π_1, Π_2) in the dimensional equation {13}, which is given by

$$\Pi_1 = \frac{1}{2\pi}\sqrt{\frac{1}{\Pi_2} + 1} \qquad \{16\}$$

For the two-idler system, in addition to the Pi groups (Π_1 and Π_2), there is an additional Pi group which is

$$\Pi_3 = \frac{L_3}{L_1}$$

For the n-idler system, there are $(n+1)$ dimensionless Pi parameters. The parameter Π_1 is the same as given in equation {15}. The other n Pi parameters are given by

$$\Pi_i = \frac{L_i}{L_1}$$

where $i = 2, ..., n+1$.

The model law is applied to Pi parameters of the idle roller system gives the following scale factor:

$$S_f = S_{EA}^{0.5} S_{L_1}^{-0.5} S_m^{-0.5}$$

where S_f is the frequency scale factor, S_{EA} is the modulus scale factor, and S_{L_1} is the length scale factor. Similarly, the model law applied to length Pi parameters gives the scale factors $S_{L_2} = S_{L_1}$, $S_{L_3} = S_{L_1}$,...., $S_{L_n} = S_{L_1}$.

The scale factors indicates that the resonant frequency is affected by a change in the material. If the resonant frequencies of one material through the idle roller system is known, then one can find the resonant frequencies for a different material by using the scale factor relation. The scale factors also simplify the analytical calculation of resonant frequencies for multiple idle roller systems.

EXPERIMENTAL RESULTS

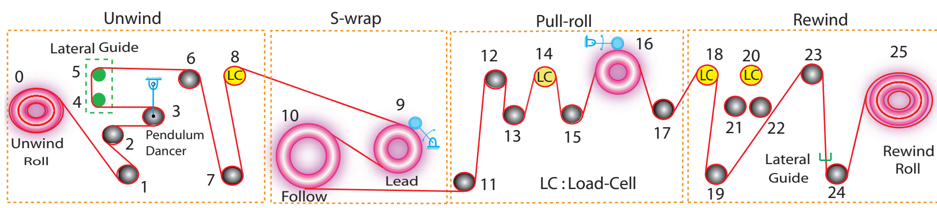

Figure 10: Euclid Web Line Sketch

The unwind section of the Euclid Web Line (EWL) shown in Figure 10, which contains a pendulum dancer and several load cell (LC) rollers, is used to conduct dimensional analysis experiments. The EWL consists of four sections, unwind, s-wrap (master speed section), pull roll (process section), and rewind section.

Experimental Verification of Scaled Web Tension Controller

The application of dimensional analysis to controller scaling may be illustrated by taking into account a web transport system (EWL) which is capable of processing different types of materials. The unwind roll section of the EWL with a speed-based tension control system (outer tension loop and inner velocity loop) is considered for this study. In this application, the web material is transported on the same platform, and that implies geometric similarity. The kinematic and dynamic similarity can be achieved by maintaining equal dimensionless Pi parameters between the original and scaled system. The different types of web materials chosen in this experiment are Tyvek ($EA = 2800$ lbf) and Polyester ($EA = 7400$ lbf). A PI controller is tuned for Tyvek material and then scaled for Polyester web system by scale factors. The controller scaling laws are given by the scale factors S_{k_p}, S_s, and S_ω.

For the original web system with Tyvek material, the system parameters are $EA = 2800$ lbf, $L_i = 2.7$ ft, $t_r = 10$ lbf, $v_r = 150$ fpm, and $R_i = 0.125$ ft. Consider the system with Polyester with $EA = 7400$ lbf. All other parameters related to the geometry of web line for both systems are the same. The objective is to design a control system for the Polyester material by scaling the tuned controller for Tyvek material. The model law provides the scale factor values for transporting

Polyester material, which are given in Table 2.

Scale Factor	Value	Scale Factor	Value
S_t	0.6151	S_{u_2}	2.64
S_{v_1}	1.6256	S_{t_3}	2.64
S_{R_2}	1	S_{k_p}	1
S_s	1.6256	S_ω	1.6256

Table 2: Scale Factors for Polyester System

The scaled process parameters for transporting Polyester are $t_r = 26.4$ lbf, $V_r = 243.84$ fpm. The scale factor values indicate that the proportional gain for both materials remains the same. The integral gain changes by a factor of 1.6256. The controller gain values tuned for Tyvek on EWL unwind section are proportional gain $k_p = 24$ and integral gain $k_i = 0.5$. So, the scaled controller gain values for transporting Polyester material are proportional gain $k_p = 24$ and integral gain $k_i = 0.8128$.

Experiments are performed for the two web materials on EWL. For transporting Polyester material the web line is operated with scaled reference tension and scaled controller gains derived by dimensional analysis. Experimental results for both materials are shown in Figure 11. The plots show that the scaled controller

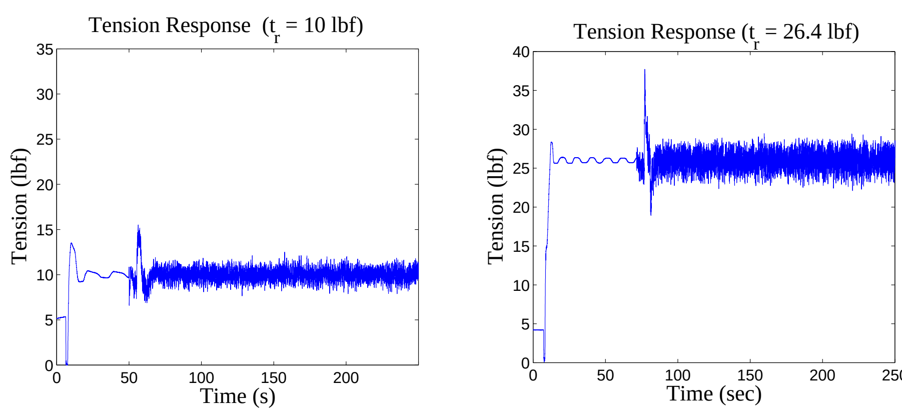

Figure 11: Web Tension Response with PI Controller; Left: Tyvek web system; Right: Polyester scaled web system.

gives similar performance for transporting Polyester when compared to that of Tyvek.

Experimental Verification of Scaled Resonant Frequencies

The resonant frequencies of a system of idle rollers and web spans were analyzed and experimentally evaluated in Diao (2008). The primary goal of this frequency response study was to determine the minimum resonant frequency. The minimum resonant frequency is a function of the number of idle rollers, web span lengths, geometric parameters of idle rollers, such as inertia and radius, and web material properties, such as modulus of elasticity and area of cross section. Proper selection of system parameters, such as web span lengths, number of idler rollers, and web paths, results in maximizing the minimum resonant frequency.

For the purpose of illustrating dimensional analysis, a simple idler system is considered. First, a state space model is constructed for a system of idle rollers and spans based on the linearized tension and web velocity equations. Analytical solutions for the resonant frequencies are obtained from the linearized state space model. Dimensional analysis is performed on those analytical solutions. Different web system configurations, such as change in number of idle rollers, web materials, web span lengths, and geometric and material properties of rollers are investigated. Frequency response experiments are conducted for the scenario of changing web materials.

The minimum resonant frequency is a function of the modulus parameter (EA). Application of dimensional analysis can be illustrated by scaling the minimum resonant frequencies of a web transport system for the scenario of transporting different kinds of web materials. Figure 12 shows the effect of web material modulus on the minimum resonant frequency for the unwind section of EWL.

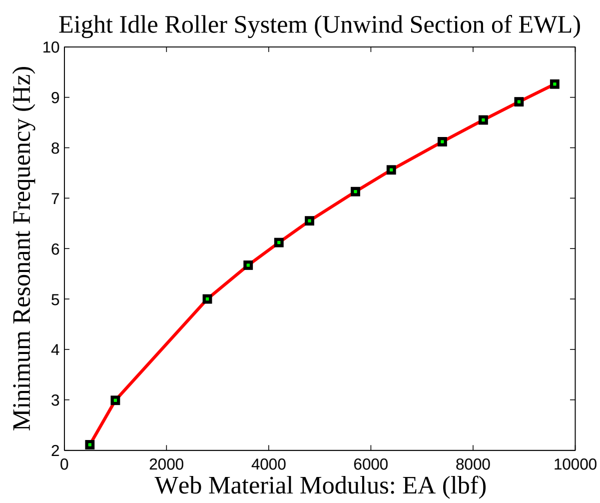

Figure 12: Effect of EA on the Minimum Resonant frequency

The scale factor S_f indicates that, on the same web platform, i.e., a geometrically similar system, a change in web material results in change in the minimum resonant frequency. Two web materials are chosen, Tyvek and Polyester. The scale factor values for Polyester material are given in Table 3.

Scale Factor	Value
S_{f_1}	1.6257
S_{L_1}	1
S_m	1
S_{EA}	2.6429

Table 3: Scale Factors for Minimum Resonant Frequency

The minimum resonant frequency is experimentally derived for transporting Tyvek and Polyester web materials on the EWL. Experimental results indicate that the resonant frequency is changed by a scale factor value (S_f). Experimentally obtained minimum resonant frequency of the unwind section idler system with Tyvek material is 5 Hz. Hence, the scaled minimum resonant frequency for the Polyester material calculated using dimensional analysis is 8.12 Hz. Frequency response experiments are conducted with Polyester material to determine the minimum resonant frequency. A value of 8.2 Hz is obtained from the experiment, which is close to the calculated value of 8.12 Hz. A Fast Fourier Transform (FFT) of web tension experimental data and the resonant frequencies for both Tyvek and Polyester materials are shown in Figure 13.

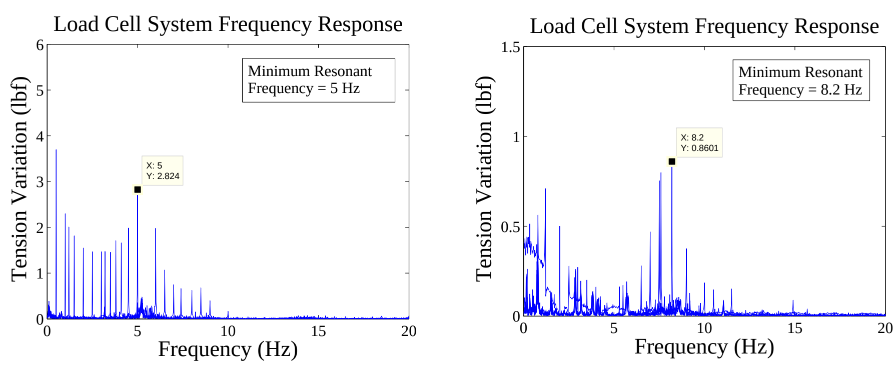

Figure 13: FFT of Idle Roller System with: Left-Tyvek Web Material; Right-Polyester Web Material

Effect of Change in Idle Roller Mass and Span Length

The scale factor (S_f) also provides the relation between the minimum resonant frequency and idle roller mass. The scale factor S_f indicates that, for the same web material and same span length configuration, the minimum resonant frequency is an inverse square root function of idler roller mass. An increase in roller mass causes a decrease in minimum resonant frequency. The scale factors S_{L_i} indicate that in order to maintain dimensional equivalence, a change in a span length of original web system leads to a change in span length of the scaled web system.

SUMMARY AND CONCLUSION

Application of dimensional analysis to web processing lines was discussed in this paper. The benefits of dimensional analysis are shown by considering several case studies. Dimensional analysis provides a systematic method to scale-up primitive elements as well as in determining key controller gains and values of reference variables when a different material is transported through the same line, that is, it facilitates a systematic transition from the old configuration to the new configuration by mimicking the features and performance of the old configuration to the new configuration.

ACKNOWLEDGEMENTS

This work was supported by the Web Handling Research Center, Oklahoma State University, Stillwater, Oklahoma.

REFERENCES

[1] Shelton, J. J., "Limitations to sensing of web tension by means of roller reaction forces," Proceedings of the Fifth Intl. Conf. on Handling, June 1999.

[2] Shelton, J. J., "Dynamics of web tension control with velocity or torque control," Proceedings of the American Control Conference, June 1986, pp. 1423–1427.

[3] Pagilla, P. R., and Diao, Y., "Resonant Frequencies in Web Process Lines Due to Idle Rollers and Spans," ASME Journal of Dynamic Systems, Measurement and Control, vol. 133, no.5, November 2011.

[4] Pagilla, P. R., Garimella, S. S., Dreinhoefer, L. H., and King, E. O., "Dynamics and control of accumulators in continuous strip processing lines," IEEE Transactions on Industry Applications, vol. 37, pp. 934-940, June 2001.

[5] Szirtes, T., Applied Dimensional Analysis and Modeling. McGraw-Hill, New York, 2nd ed., 1998.

[6] Brennan S., "On Size and control: The use of dimensional analysis in controller design," Phd thesis, University of Illinois, Urbana-Champaign, November 2002.

[7] Diao, Y., "Resonant frequencies in web lines and filtering of web tension," Master's thesis, Oklahoma State Univeristy, Stillwater, December 2008.

[8] Raul, P. R., "Frequency repsonse of web systems containing load cells and dancers, dimensional analysis, and model reference adaptive schemes for tension control," Master's thesis, Oklahoma State Univeristy, Stillwater, July 2010.

[9] Pagilla, P. R., Dwivedula, R. V., and Siraskar, N. B., "A decentralized model reference adaptive controller for large-scale systems," IEEE Transactions on Mechatronics, vol. 12, pp. 154-163, April 2007.

[10] Dwivedula, R. V., "Modeling the effects of belt compliance, backlash, and slip on web tension and new methods for decentralized control of web processing lines," Phd thesis, Oklahoma State University, Stillwater, OK, December 2005.

[11] Pagilla, P. R., Siraskar, N. B., and Dwivedula, R. V., "Decentralized control of web processing lines," IEEE Transactions on Control Systems Technology, vol. 15, pp. 106-117, January 2007.