BUCKLING OF ORTHOTROPIC WEBS IN PROCESS MACHINERY

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ABSTRACT

Many webs in web process machinery exhibit out-of-plane deformations, defined as troughs, in free web spans between rollers. In other cases when the troughs become severe the out-of-plane web deformations will begin to transcend rollers. Any out-of-plane web deformations that transcend rollers are defined as wrinkles. Troughs and wrinkles in webs are often undesirable as they can interfere with web processes such as coating, they can result in web breaks and thereby decreased productivity, or these deformations may become permanent and result in quality loss.

Many plastic film, paper, tissue and nonwoven webs are highly anisotropic either by design or just as a result of the process by which the web is made. The first objective of this paper is to show how anisotropic web properties affect the buckling and wrinkling tendencies of these webs. Previously algorithms have been developed that show how roller misalignment can induce troughs and wrinkles. The second objective of this paper is to demonstrate how web orthotrophy can affect the allowable roller misalignment in a web span and the web tension required to sustain a wrinkle upon a roller.

NOMENCLATURE

a	span length
A_{mn}	amplitude coefficient for buckled shape
A_s	area of a beam which reacts shear, $\frac{5}{6}bh$ for a web
b	web width
f_{yi}	lateral force in web at upstream roller
f_{yj}	lateral force in web at downstream roller
E_x	MD modulus
E_y	CMD modulus
G	shear modulus
h	web thickness

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ho	air film thickness
I	web area moment of inertia, $\frac{hb^3}{12}$ for a web
m	number of half waves in buckled shape in MD, always unity in this application.
M_i	bending moment in web at upstream roller
M_j	bending moment in web at downstream roller
n	number of half waves in buckled shape in CMD
N_x	MD surface traction
N_y	CMD surface traction
R	roller radius
Rq	equivalent RMS roughness
Rq, roller	RMS roughness of roller
Rq,web	RMS roughness of web
T	total web tension
T_w	web tension per unit width
v_i	lateral deflection in web at upstream roller
v_j	lateral deflection in web at downstream roller
V	web velocity
w	out of plane web deflection
x	coordinate aligned with MD
y	coordinate aligned with CMD
ε_x	MD strain
ε_y	CMD strain
γ_{xy}	shear strain
ν_{xy}	Poisson's ratio, relates an σ_x stress to an ε_y strain
ν_{yx}	Poisson's ratio, relates an σ_y stress to an ε_x strain
ϕ	shear parameter
λ	wavelength of buckled sector
μ_t	web/roller traction as affected by air entrainment
η	viscosity of air
σ_{md}	MD stress
σ_x	stress aligned with x direction, positive when induces compression
σ_y	stress aligned with y direction, positive when induces compression
σ_{ycr}	stress aligned with y direction required to induce troughs or wrinkles in the web
$\sigma_{y,max}$	maximum CMD stress which can be supported in web based upon web traction
θ_i	slope in web at upstream roller
θ_j	slope in web at downstream roller
θ_{cr}	slope required to trough the web
τ	shear stress in web

ANALYSIS OF WEB TROUGHS IN FREE SPANS

In this analysis we will assume an orthotropic web is subject to buckling. Previous studies by Good and Shelton have assumed isotropic material properties[1-3]. The deflection equation for an orthotropic plate including the effects of membrane forces is[4]:

$$D_1\frac{\partial^4 w}{\partial x^4}+2D_3\frac{\partial^4 w}{\partial x^2\partial y^2}+D_2\frac{\partial^4 w}{\partial y^4}+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}=0 \qquad \{1\}$$

where N_x and N_y are the membrane forces in units of load per unit length in the x and y directions per Figure 1 on a web span of length a and width b and D_1, D_2, and D_3 are:

$$D_1=\frac{E_x h^3}{12(1-\nu_{xy}\nu_{yx})} \quad D_2=\frac{E_y h^3}{12(1-\nu_{xy}\nu_{yx})} \quad D_3=D_1\nu_{xy}+2D_k \quad D_k=\frac{G h^3}{12} \qquad \{2\}$$

where E_x and E_y are the principal moduli of elasticity, G is the shear modulus of rigidity, and h is the web thickness. Perhaps the most concise manner for defining Poisson's ratios is through the constitutive relations between strain and stress:

$$\varepsilon_x=\frac{\sigma_x}{E_x}-\frac{\nu_{yx}\sigma_y}{E_y} \quad \varepsilon_y=\frac{\sigma_y}{E_y}-\frac{\nu_{xy}\sigma_x}{E_x} \quad \gamma_{xy}=\frac{1}{G}\tau_{xy} \qquad \{3\}$$

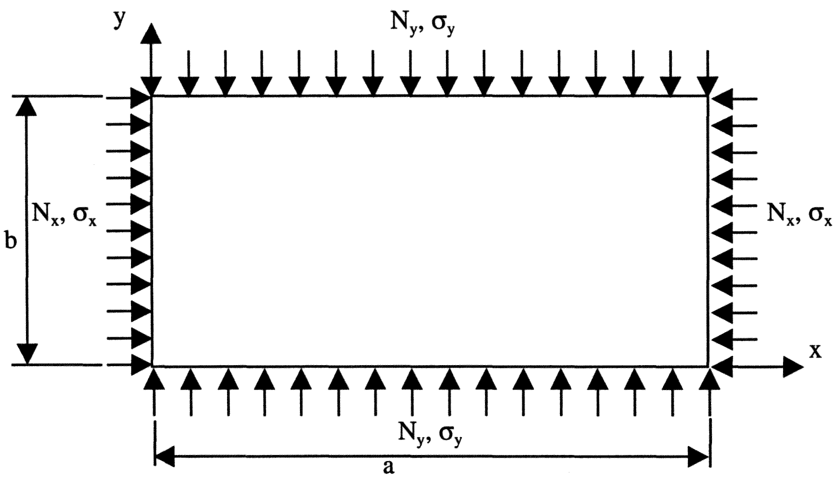

Figure 1 - A Plate Submitted to Uniform Edge Loadings

In this analysis it is assumed that the x direction is aligned with the Machine Direction (MD) and that the y direction is aligned with the Cross Machine Direction (CMD).

A solution is sought for the out-of-plane deformation w (in the z direction) of the form:

$$w=A_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad \{4\}$$

If expression {4} is substituted into expression {1} and divided by the web thickness (h) the following relation will result:

$$\sigma_x m^2+\sigma_y n^2\frac{a^2}{b^2}=\frac{\pi^2}{h}\left[\frac{m^4}{a^2}D_1+2\left[\frac{mn}{b}\right]^2 D_3+a^2\frac{n^4}{b^4}D_2\right] \qquad \{5\}$$

where σ_x and σ_y are the surface tractions in units of stress. In web processing machinery it is expected that the web will be transported at some tension in the x direction and thus wrinkles cannot form which are oriented in the y direction. Thus m will become unity in expression {5} and if an equivalent stress is introduced of the form:

$$\sigma_e = \frac{\pi^2 \sqrt{D_1 D_2}}{a^2 h} \qquad \{6\}$$

expression {5} becomes:

$$\sigma_x + \left(\frac{an}{b}\right)^2 \sigma_y = \sigma_e \left[\sqrt{\frac{D_1}{D_2}} + 2\left(\frac{an}{b}\right)^2 \frac{D_3}{\sqrt{D_1 D_2}} + \left(\frac{an}{b}\right)^4 \sqrt{\frac{D_2}{D_1}} \right] \qquad \{7\}$$

Expression {7} can be rearranged to yield the σ_y stress required to buckle the web into a number of n half waves:

$$\sigma_y = \left\{ \sigma_e \left[\sqrt{\frac{D_1}{D_2}} + 2\left(\frac{an}{b}\right)^2 \frac{D_3}{\sqrt{D_1 D_2}} + \left(\frac{an}{b}\right)^4 \sqrt{\frac{D_2}{D_1}} \right] - \sigma_x \right\} \left(\frac{b}{an}\right)^2 \qquad \{8\}$$

For a web with given thickness, width, and material properties and a known span length the only unknowns are σ_y and n. For a given half wave number (n) this expression will yield the CMD stress level σ_{ycr} required to buckle the web. For most instability problems it is assumed that the instability will occur at the lowest possible load or stress level unless constraints exist that would prevent the buckled shape associated with that lowest stress level to be assumed by the structure or web in this case. Since no such constraints exist in this case it will be assumed that the web will buckle at the lowest possible stress level. If it is assumed that expression {8} can be considered continuous with respect to the half wave number n, the value of n required to minimize σ_{ycr} can be obtained by setting the derivative equal to zero and solving for n.

$$\frac{2\left[\pi^2 a^4 n^4 D_2 + b^4 (h a^2 \sigma_x - \pi^2 D_1)\right]}{h b^2 n^3 a^4} = 0 \qquad \{9\}$$

There are four roots of this expression in n, three of which either lead to negative or imaginary wave numbers. The other root which is real and positive is:

$$n = \frac{b}{a\sqrt{\pi}} \sqrt[4]{\frac{\pi^2 D_1 - h a^2 \sigma_x}{D_2}} \qquad \{10\}$$

This root will always be positive for webs as σ_x is always a negative number due to the original sign convention chosen. Substitution of typical web values for h, a, σ_x, E_x, ν_{xy}, and ν_{yx} into {10} shows the $\pi^2 D_1$ term is two to three orders of magnitude smaller than the $ha^2\sigma_x$ term. Thus the first term is neglected and after substituting and rearranging expression {10} can be written as:

$$n = b\sqrt{\frac{2}{\pi a h}} \sqrt[4]{\frac{3(1 - \nu_{xy}\nu_{yx})\sigma_{md}}{E_y}} \qquad \{11\}$$

The positive σ_{md} stress is used to replace the $-\sigma_x$ term to reduce possible confusion in terms of sign. Substitution of expression {10} into expression {8} yields:

$$\sigma_{ycr} = \frac{2\pi}{a^2 h} \frac{\left(\pi^2 D_1 \sqrt{D_2} - ha^2 \sigma_x \sqrt{D_2} + \pi D_3 \sqrt{\pi^2 D_1 - ha^2 \sigma_x}\right)}{\sqrt{\pi^2 D_1 - ha^2 \sigma_x}} \qquad \{12\}$$

Again for typical web parameters it can be argued that $\pi^2 D_1 << ha^2 \sigma_x$ and if the three parameter groups in the numerator are compared it is found that the first and third are negligible compared to the second group. Simplifying and substituting the material properties that represent D_2 yields:

$$\sigma_{ycr} = \frac{\pi h}{\sqrt{3} a} \sqrt{\frac{-\sigma_x E_y}{1 - \nu_{xy} \nu_{yx}}} = \frac{\pi h}{\sqrt{3} a} \sqrt{\frac{\sigma_{md} E_y}{1 - \nu_{xy} \nu_{yx}}} \qquad \{13\}$$

Expressions {11} and {13} provide concise expressions for calculating the half wave number and the CMD stress required to buckle the web. Examples have been run with a broad range of material parameters, web thickness and aspect ratios that show good agreement with the more complex expressions {10} and {12} which are void of simplifying assumptions. It should be noted as well that expressions {11} and {13} will condense to the isotropic forms reported earlier if E_y becomes E and ν_{xy} is equal to ν_{yx}[3]. With increasing values of h and to a much lesser extent E_x the term $\pi^2 D_1$ may become more comparable to the $ha^2\sigma_x$ term and then expressions {10} and {12} should be used.

COMMENTS REGARDING INPUTS

Expressions {12} and {13} are functions of material parameters through the variables D_1, D_2, and D_3. Since the web is subject to plane stress conditions and is orthotropic there are five independent properties including E_x, E_y, ν_{xy}, ν_{yx}, and G per expression {3}. Maxwell's reciprocal theorem can be used to prove the Poisson's ratios and Young's moduli are related as follows:

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \qquad \{14\}$$

This reduces the number of independent properties to four. Some authors have presented expressions that relate the shear modulus to the remaining parameters. Szilard [5] for instance presents:

$$G = \frac{\sqrt{E_x E_y}}{2\left(1 + \sqrt{\nu_{xy} \nu_{yx}}\right)} \qquad \{15\}$$

Cheng et al [6] present a derivation which yields:

$$G = \frac{E_x E_y}{E_x\left(1 + \nu_{yx}\right) + E_y\left(1 + \nu_{xy}\right)} \qquad \{16\}$$

Whether such expressions are reasonable approximations is unknown, experimental verification does not appear to exist. Whether such expressions are reasonable for some subsets of web materials such as homogenous films and papers or possibly some nonwovens is unknown but worthy of further investigation. Thus only three independent parameters may fully define the properties of an orthotropic web material subject to plane stress conditions.

WRINKLING OF WEBS UPON ROLLERS

Webs deform into thin shell structures as they pass around rollers. The buckling stress of the web in shell form will be considerably larger than the buckling stress of the same web in a free span per expression {13}.

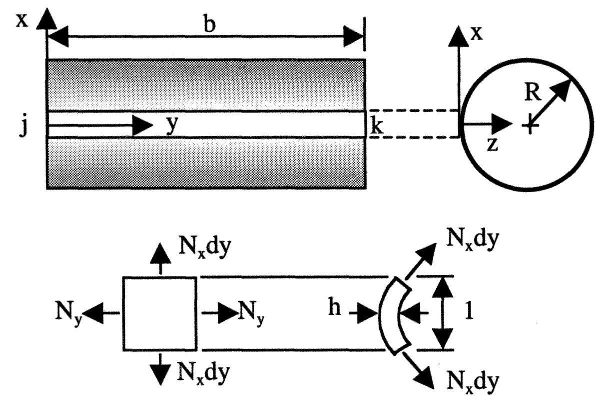

Figure 2 – Nomenclature for the Instability of a Cylinder

The critical buckling load per unit length of a sector of a cylinder has been shown by others to be identical to the buckling load per unit length of an entire cylinder. In Figure 2 an element of a thin cylindrical shell is shown.

The constitutive expressions in {3} can be written in terms of stresses and if multiplied by the shell thickness h will yield expressions relating the membrane forces to the strains. When an axisymmetric structure is subject to axisymmetric loading the circumferential strain ε_x becomes –w/R and thus the membrane forces can be written:

$$N_x = \frac{E_x h}{1-\nu_{xy}\nu_{yx}}\left[\varepsilon_x + \nu_{yx}\varepsilon_y\right] = \frac{E_x h}{1-\nu_{xy}\nu_{yx}}\left[-\frac{w}{R} + \nu_{yx}\varepsilon_y\right] \qquad \{17a\}$$

$$N_y = \frac{E_y h}{1-\nu_{xy}\nu_{yx}}\left[\varepsilon_y + \nu_{xy}\varepsilon_x\right] = \frac{E_y h}{1-\nu_{xy}\nu_{yx}}\left[\varepsilon_y - \nu_{xy}\frac{w}{R}\right] \qquad \{17b\}$$

If expressions {17} are compared and if expression {14} is employed it can be found that:

$$N_x = E_x\left[\frac{N_y \nu_{yx}}{E_y} - h\frac{w}{R}\right] = N_y \nu_{xy} - h E_x \frac{w}{R} \qquad \{18\}$$

Considering the strip jk in Figure 2 the forces {18} yield a component of force in the z direction, the magnitude of which per unit length is:

$$\frac{N_x}{R} = \frac{1}{R}\left[N_y \nu_{xy} - h E_x \frac{w}{R}\right] \qquad \{19\}$$

Summing all z direction loads per unit length of strip jk yields:

$$q + \frac{N_y \nu_{xy}}{R} - h E_x \frac{w}{R^2} + N_y \frac{d^2 w}{dy^2} \qquad \{20\}$$

where q is a potential pressure acting upon the element and the last term represents the component of transverse load due the membrane load N_y acting through the out-of-plane deformation w. Thus the differential equation for the bending of the strip jk is:

$$D_2 \frac{d^4 w}{dy^4} = q + \frac{N_y \nu_{xy}}{R} - h E_x \frac{w}{R^2} + N_y \frac{d^2 w}{dy^2} \qquad \{21\}$$

In applying this expression q is set to zero and w is now taken from the middle surface of the cylinder after the uniform compression N_y is applied. Thus w will be replaced by:

$$w \Rightarrow w + \frac{N_y \nu_{xy} R}{E_x h} \qquad \{22\}$$

and the differential equation becomes:

$$D_2 \frac{d^4 w}{dy^4} + N_y \frac{d^2 w}{dy^2} + E_x h \frac{w}{R^2} = 0 \qquad \{23\}$$

after assuming N_y is positive when compression results. The cylindrical shell is expected to buckle into axisymmetric modeshapes that can be represented by the waveform:

$$w = -A \sin \frac{n\pi y}{b} \qquad \{24\}$$

Inserting expression {24} into {23} and eliminating like terms yields:

$$\frac{R^2 h^3 n^4 \pi^4 E_y + 12 b^2 \left(h b^2 E_x - R^2 n^2 \pi^2 N_y\right)\left(1 - \nu_{xy}\nu_{yx}\right)}{12 R^2 b^4 \left(1 - \nu_{xy}\nu_{yx}\right)} = 0 \qquad \{25\}$$

Solving expression {25} for N_y yields:

$$N_y = \frac{h b^2 E_x}{R^2 n^2 \pi^2} + \frac{h^3 n^2 \pi^2 E_y}{12 b^2 \left(1 - \nu_{xy}\nu_{yx}\right)} \qquad \{26\}$$

Thus an expression has been produced that relates the axial load per unit length to the half wave number n. The goal is to determine the minimum value of N_y associated with any buckled shape. A new variable λ is substituted for $n\pi/b$ in expression {26}:

$$N_y = \frac{h E_x}{R^2 \lambda^2} + \frac{h^3 \lambda^2 E_y}{12\left(1 - \nu_{xy}\nu_{yx}\right)} \qquad \{27\}$$

Assuming λ is a continuous variable the minimum is found by equating the derivative of expression {27} with respect to λ equal to zero.

$$\frac{\partial N_y}{\partial \lambda} = -\frac{2 h E_x}{R^2 \lambda^3} + \frac{h^3 \lambda E_y}{6\left(1 - \nu_{xy}\nu_{yx}\right)} = 0 \qquad \{28\}$$

There are four roots to this equation, only one of which is real and positive which is:

$$\lambda = \frac{n\pi}{b} = \sqrt{\frac{2}{Rh}} \sqrt[4]{\frac{3 E_x \left(1 - \nu_{xy}\nu_{yx}\right)}{E_y}} \qquad \{29\}$$

Substituting this root back into expression {27} yields the critical buckling load:

$$N_{ycr} = \frac{h^2}{R} \sqrt{\frac{E_x E_y}{3\left(1 - \nu_{xy}\nu_{yx}\right)}} \qquad \{30\}$$

Thus the critical stress required to buckle an orthotropic cylinder is:

$$\sigma_{ycr} = \frac{h}{R} \sqrt{\frac{E_x E_y}{3\left(1 - \nu_{xy}\nu_{yx}\right)}} \qquad \{31\}$$

where the associated half wave number n can be found using expression {29}.

APPLICATION TO ISOLATED WEB SPANS WITH MISALIGNED ROLLERS

Now that a troughing and a wrinkling failure criterion for an orthotropic web have been developed they will be employed in the analysis of a web span that has been subjected to shear from a misaligned roller. Webs undergoing lateral deformation can be modeled as beams. A web span ratio (a/b) can take on a large variation depending on the particular industry. In the paper industry rollers are often mounted very close together with wide web widths and thus shear stiffness effects must be considered. In the film and metal strip industries long spans and narrow webs are not uncommon and thus the effect of web tension can be significant on the lateral deformations and shears within the web. Thus a thorough model should account for both shear and tension stiffening effects.

Przemieniecki [8] and others have developed stiffness matrices for beams stiffened by tension and shear effects. In Figure 3 a free body of such a beam is shown.

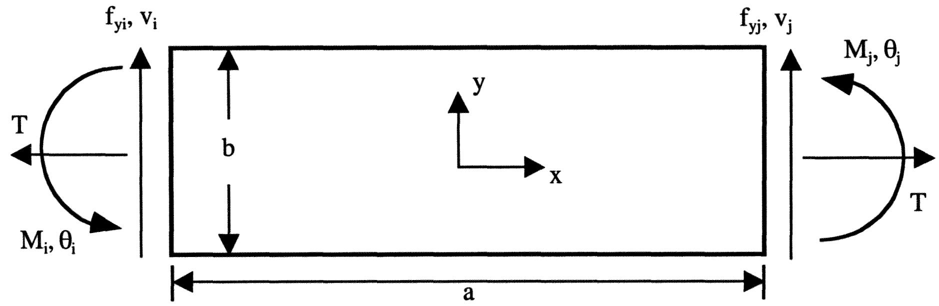

Figure 3 – Sign Convention for Positive Loads, Deformations and Rotations

The stiffness matrix for this beam is:

$$\begin{Bmatrix} f_{yi} \\ M_i \\ f_{yj} \\ M_j \end{Bmatrix} = \begin{bmatrix} \frac{12E_xI}{a^3(1+\phi)}+\frac{6T}{5a} & \frac{6E_xI}{a^2(1+\phi)}+\frac{T}{10} & -\frac{12E_xI}{a^3(1+\phi)}-\frac{6T}{5a} & \frac{6E_xI}{a^2(1+\phi)}+\frac{T}{10} \\ \frac{6E_xI}{a^2(1+\phi)}+\frac{T}{10} & \frac{(4+\phi)E_xI}{a(1+\phi)}+\frac{2Ta}{15} & -\frac{6E_xI}{a^2(1+\phi)}-\frac{T}{10} & \frac{(2-\phi)E_xI}{a(1+\phi)}-\frac{Ta}{30} \\ -\frac{12E_xI}{a^3(1+\phi)}-\frac{6T}{5a} & -\frac{6E_xI}{a^2(1+\phi)}-\frac{T}{10} & \frac{12E_xI}{a^3(1+\phi)}+\frac{6T}{5a} & -\frac{6E_xI}{a^2(1+\phi)}-\frac{T}{10} \\ \frac{6E_xI}{a^2(1+\phi)}+\frac{T}{10} & \frac{(2-\phi)E_xI}{a(1+\phi)}-\frac{Ta}{30} & -\frac{6E_xI}{a^2(1+\phi)}-\frac{T}{10} & \frac{(4+\phi)E_xI}{a(1+\phi)}+\frac{2Ta}{15} \end{bmatrix} \begin{Bmatrix} v_i \\ \theta_i \\ v_j \\ \theta_j \end{Bmatrix} \qquad \{32\}$$

Where ϕ is defined as the shear parameter:

$$\phi = \frac{12 E_x I}{G A_s a^2} \qquad \{33\}$$

and A_s is the area of the cross section subject to shear. For rectangular cross sections A_s=5bh/6. It is assumed the web is the beam and that it is supported by rollers at i and j in Figure 6 and that the web is traveling from left to right and that the roller at position j is misaligned to some degree θ_j. This will induce a shear force and therefore shear deformation into the span. At the upstream roller it will be arbitrarily assumed that v_i is zero without loss of generality. The rotation at i will be non-zero due to the shear deformation and equal to:

$$\theta_i = \frac{f_{yj}}{GA_s} \tag{34}$$

Shelton determined that the moment in the web just prior to a downstream roller is zero under steady-state conditions [9]. Thus with knowledge that M_j is zero the fourth equation in the stiffness matrix {32} can be solved for v_j as:

$$v_j = \frac{a\left[Ta^2\left(4A_sG\theta_j - f_{yj}\right)(1+\phi) + 30E_xI\left\{f_{yj}(2-\phi) + A_sG\theta_j(4+\phi)\right\}\right]}{3A_sG\left\{60E_xI + Ta^2(1+\phi)\right\}} \tag{35}$$

With v_i assumed zero and θ_i and v_j known the third expression in the stiffness matrix {32} yields an expression relating f_{yj}, and thereby the shear in the web F, to the misalignment of the downstream roller θ_j:

$$f_{yj} = F = \frac{A_sG\left[240E_x^2I^2 + 3T^2a^4(1+\phi) + 8E_xITa^2(13+3\phi)\right]}{240E_x^2I^2 + Ta^4(2A_sG+T)(1+\phi) + 8E_xIa^2\left[15A_sG - T(2-3\phi)\right]}\theta_j \tag{36}$$

Expression {36} can be used to determine the shear in a web span with a misaligned downstream roller and bending stiffness, shear stiffness, and web tension are accounted for. This expression has but one known bound which is that it is valid until the downstream roller is misaligned to the extent that web edge slackness occurs at the upstream roller.

Expression {36} can be used to determine the shear stress in the web. The average shear stress was determined by dividing the shear by the cross section area. The second principal stress will be negative (compressive) and can be determined using the expression:

$$\sigma_2 = \frac{\sigma_{md}}{2} - \sqrt{\left(\frac{\sigma_{md}}{2}\right)^2 + \tau^2} \tag{37}$$

If this principal stress is equated to expression {13} the critical rotation to induce troughs in the web can be determined:

$$\theta_{cr,\tau_{avg}} = \frac{2}{5Ga}\frac{5E_x^2h^2b^6 + Ta^4(3T+5Ghb)(1+\phi) + E_xha^2b^3\left[25Ghb + T(6\phi-4)\right]}{5E_x^2h^2b^6 + 9T^2a^4(1+\phi) + 2E_xhTa^2b^3(3\phi+13)} \times\sqrt{\frac{3\sqrt{3}\pi ha\sqrt{E_y}\,\sigma_{md}^{3/2}}{\sqrt{1-\nu_{xy}\nu_{yx}}} - \frac{3\pi^2h^2E_y\sigma_{md}}{\left(1-\nu_{xy}\nu_{yx}\right)}} \tag{38}$$

At this rotation of the downstream roller troughs would be expected across much of the web width. Troughs should first appear at the center of the web where the flexural shear stress is maximum, 1.5 times greater than the average value. If the maximum shear stress is inserted into expression {37}, the principal stress can again be equated to the critical buckling stress in expression {13} to provide an expression for the critical rotation to predict the onset of troughing:

$$\theta_{cr,\tau_{max}} = \frac{4}{5Ga}\frac{5E_x^2h^2b^6 + Ta^4(3T+5Ghb)(1+\phi) + E_xha^2b^3\left[25Ghb + T(6\phi-4)\right]}{5E_x^2h^2b^6 + 9T^2a^4(1+\phi) + 2E_xhTa^2b^3(3\phi+13)} \times\sqrt{\frac{\pi ha\sqrt{E_y}\,\sigma_{md}^{3/2}}{\sqrt{3\left(1-\nu_{xy}\nu_{yx}\right)}} - \frac{\pi^2h^2E_y\sigma_{md}}{3\left(1-\nu_{xy}\nu_{yx}\right)}} \quad \text{or} \quad \theta_{cr,\tau_{max}} = \frac{2}{3}\theta_{cr,\tau_{avg}} \tag{39}$$

VERIFICATION OF TROUGH MODEL

Expressions {38} and {39} provide a model for predicting the occurrence of troughs. A set of experiments were conducted to test the validity of the shear and tension stiffening assumptions and to verify that the web orthotrophy was included satisfactorily. In these experiments a downstream roller would be misaligned until a trough was produced. The misalignment would then be further increased until a wrinkle was produced on the misaligned roller. The experimental setup is shown in Figure 4. The web is traveling from left to right in this picture and the horizontal span is the test span, note the troughs in the web. Prior to entering the span the web just exited a web guide and a roller mounted on load cells such that the edge position of the web is maintained constant and the web tension is known. The downstream roller is mounted upon a yoke as shown. The rotation (θ_j) of the yoke is precisely adjusted using an end micrometer. The downstream rolls sit upon an adjustable table of a former lathe bed and thus the web span length (a) is easily manipulated.

The first tests were run on a polyester film that was 23.4 μm thick and 15.24 cm wide. This film was chosen as it was isotropic with a Young's modulus of 5 GPa. The isotropic Poisson's ratio will be assumed to be 0.3 temporarily which will result in a shear modulus of 1.92 GPa. In these tests web tension was set, the span length was varied, and the rotation required to induce troughs was recorded. Results from two experiments are shown in Figure 5 for a web tension of 1.84 N/cm. Results are also shown for expression {38} which are labeled "Avg Shear Stress" and for expression {39} which are labeled "Max Shear Stress" since these expressions relied upon assumptions of an average and maximum shear stress, respectively. Also shown is a result labeled "Beam Theory" which are results from an expression from a previous study in which shear and tension stiffening were not accounted for [4]. The previous study focused upon span ratios where neither the shear nor tension stiffening terms had a sizable impact on the results. The current tests show influences of both shear and tension stiffening. The influence of tension stiffening is seen above span ratios of 4.

The "Beam Theory" results from the previous study also assumed an average shear stress distribution. The effects of shear stiffening are not evident on the scale of Figure 5. The data in Figure 5 were plotted again but over smaller ranges of the abscissa and the ordinate in Figure 6. Note that the "Beam Theory" predicts that the critical angle approaches zero at small span ratios whilst expressions {38} and {39} yield results which show that the critical angle begins to increase with decreasing span ratio and this behavior is shown in the experimental data as well.

The results shown in Figure 7 are for the same polyester web but now the effect of an increase in web tension to 3.6 N/cm is shown. The experimental data appears to follow expression {38} which relied upon the average shear stress assumption.

In Figure 8 the results are shown for yet a higher web tension. If Figures 5, 7, and 8 are reviewed the trend appears that at low span ratios the experimental data appears to fit both expressions {38} and {39} reasonably well but at higher span ratios the data appears closest to expression {38}. Both expressions rely upon the failure criterion {13}, the development of which assumed the σ_y compressive stress was present across the entire web width (b). Now consider the half wave numbers per expression {11} as shown in Figure 9. At high span ratios there are only 8 half waves or 4 whole sinusoidal waves across the entire web width of 15.2 cm (a wavelength of about 3.8 cm). The flexural shear stress is maximum at the center of the web and decays to zero in parabolic form towards the web edges. In the development of algorithm {38} an average shear stress assumption was made and per expression {37} would lead to a uniform compressive σ_2 stress across the entire web width (neglecting the violation of surface equilibrium at the web edges). Thus expression {38} would theoretically predict that all 8 half waves would appear simultaneously when the critical angle is surpassed. Expression {39} calculates the rotation required to induce the critical stress level {13} at a point at the web center.

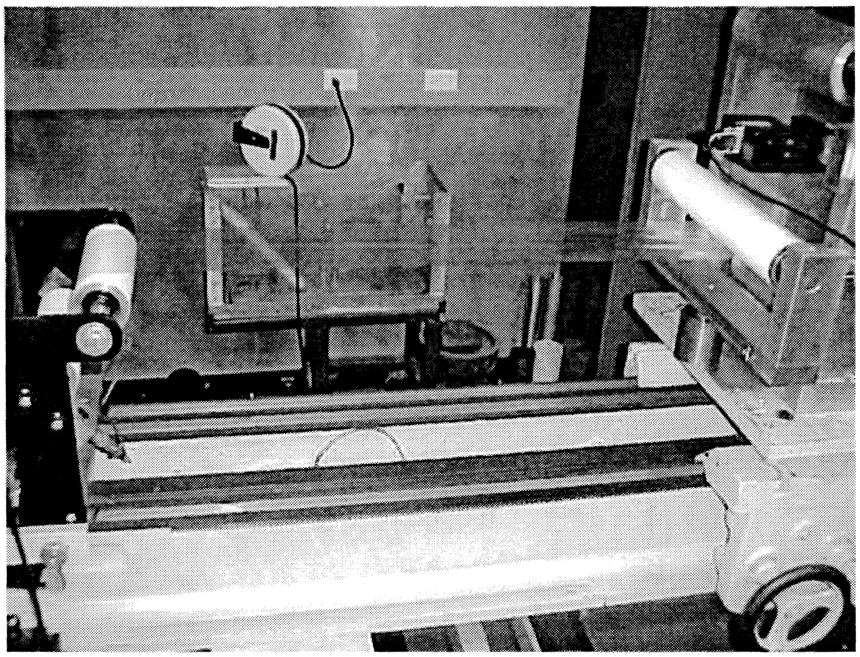

Figure 4 – Experimental Setup

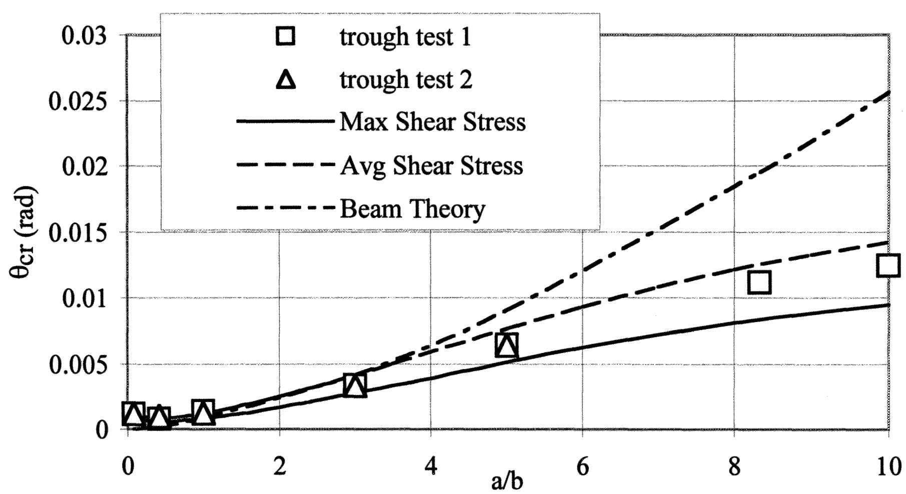

Figure 5 – Trough Tests for PET, T_w = 1.84 N/cm

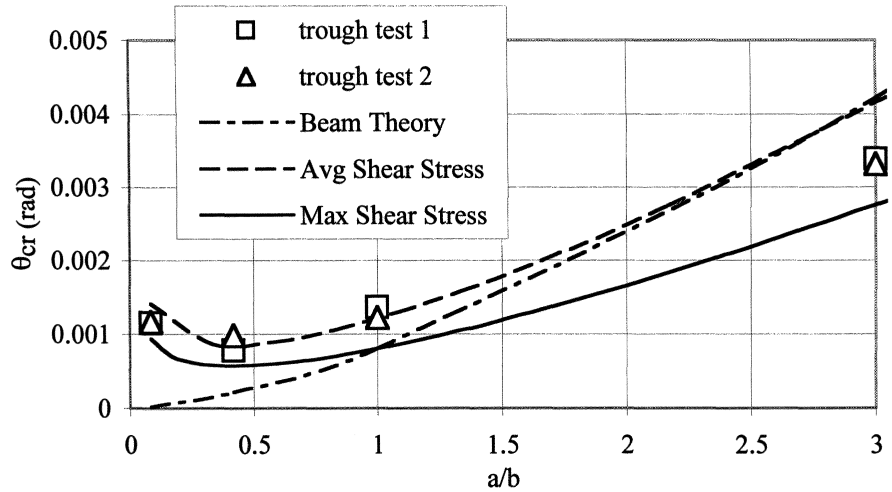

Figure 6 – Zoom of Trough Tests from Figure 5 for PET, T_w = 1.84 N/cm

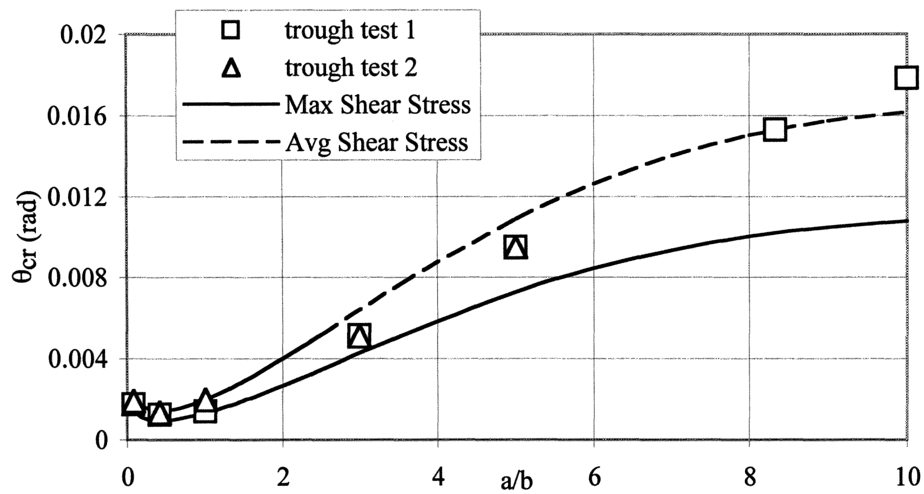

Figure 7 Trough Tests for PET, T_w = 3.6 N/cm

Thus additional rotation would be required to exert a compressive stress in excess of the critical value over one full wavelength. At the lowest span ratio the half wave numbers vary from 57 to 75, depending on web tension, or approximately 28 to 37 whole sinusoidal waves across the 15.2 cm span width. Since the length of one buckled wave has drastically been reduced in comparison to the large span ratio wavelength (about 1.38 cm when n is 28), expression {39} appears to fit the test data reasonably well at low span ratios.

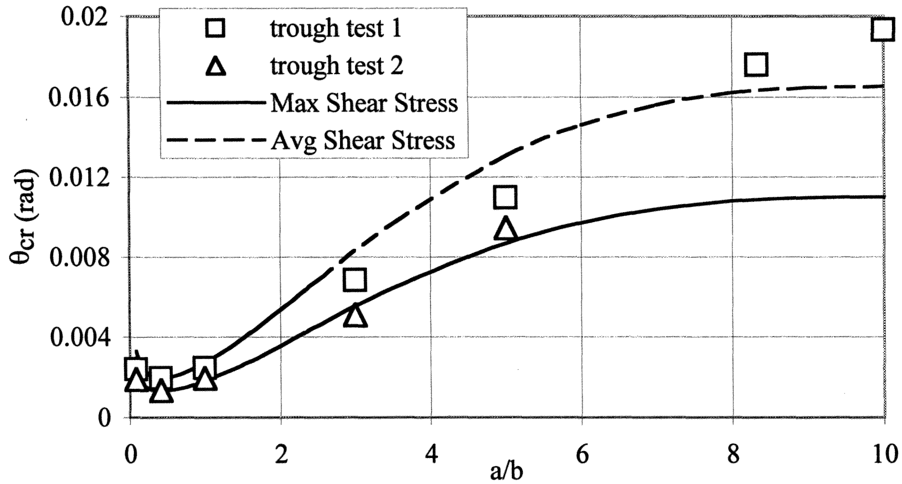

Figure 8 - Trough Tests for PET, Tw = 5.6 N/cm

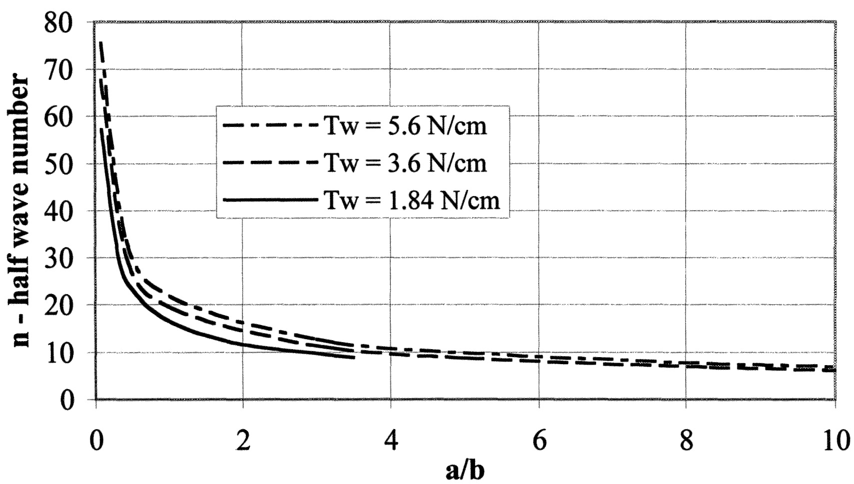

Figure 9 – Half Wave Numbers for PET at Web Tensions Tested

The impact of Poisson's ratio is shown to be minor in Figure 10 on the polyester web under study. Poisson's ratio appears to be an insensitive input to the expression and from hereon it shall be assumed that ν_{xy} is 0.3 and ν_{yx} will be calculated using expression {14}. This finding is important as it is difficult to measure the Poisson's ratio of thin webs since they often trough during extension making if difficult to measure the lateral contraction.

Newsprint was the second web to be tested, the results are shown in Figure 11. This web was 71 μm thick and 15.7 cm wide. The MD and CMD modulus (E_x and E_y) was tested and found to be 4.34 and 2.76 GPa, respectively. With ν_{xy} assumed as 0.3 expression {14} yields that ν_{yx} is 0.19 and expression {16} yields the shear modulus to be 1.43 GPa. The trough expression {38} based upon an average shear stress assumption appears to fit the test data the best over the range of span ratios tested.

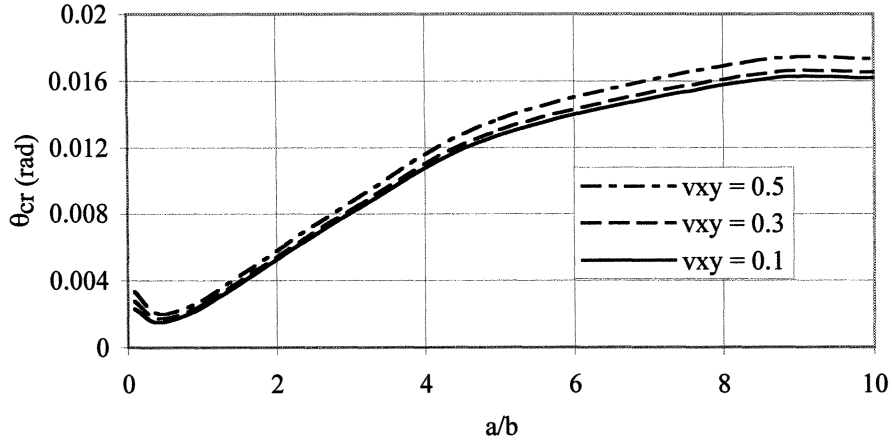

Figure 10 – Effect of Poisson's ratio per {38} for PET, $T_w = 5.6$ N/cm

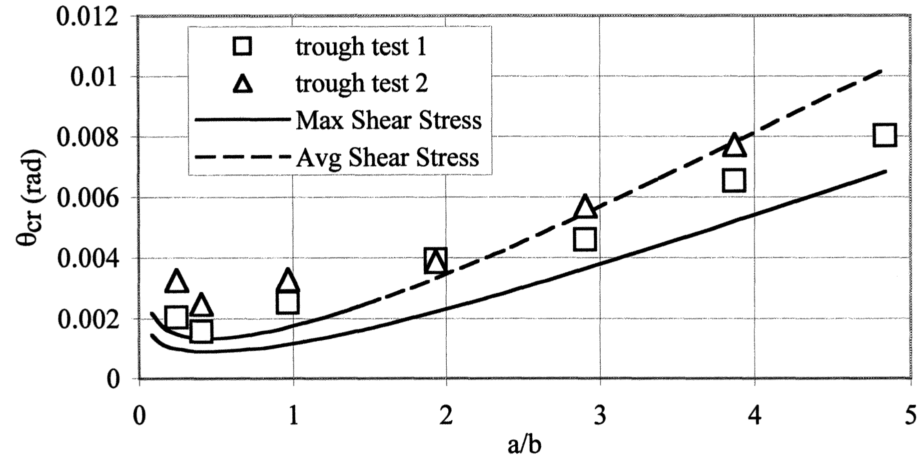

Figure 11 - Trough Tests for Newsprint, $T_w = 4.17$ N/cm

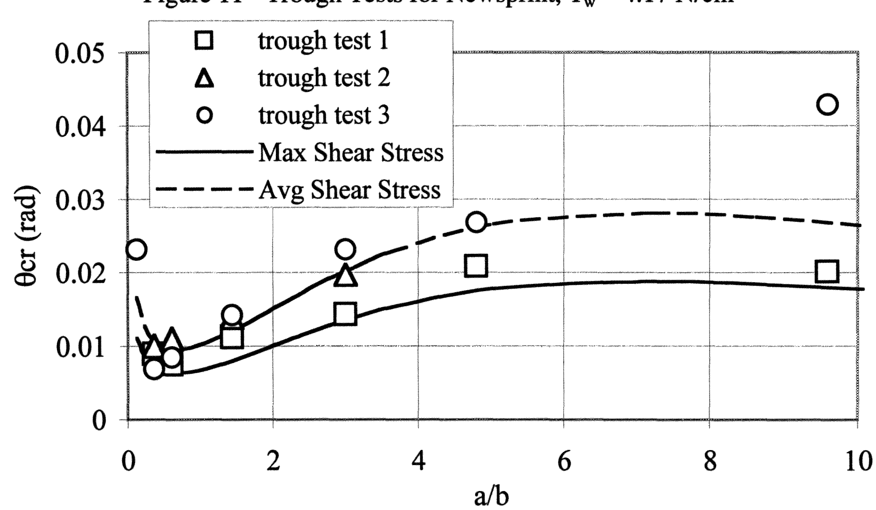

Figure 12 – Trough Tests for 22 gsm Spun-Bound Nonwoven Polypropylene
$T_w = 0.63$ N/cm

The final web studied was a spun-bond polypropylene non-woven, the results are shown in Figure 12. This web was 127 μm thick and 10.6 cm wide. The MD and CMD modulus (E_x and E_y) was tested and found to be 55.1 and 8.27 MPa, respectively. With ν_{xy} assumed as 0.3 expression {14} yields that ν_{yx} is 0.04 and expression {16} yields the shear modulus to be 6.67 MPa. The three sets of test data compare reasonably well with the results using expression {38}. Variations in density were visually apparent in this web and undoubtedly there were variations in modulus associated therewith.

THE MINIMUM TRACTION REQUIRED TO SUSTAIN WRINKLES

For a uniform web tension the maximum compressive stress which can be supported by the traction between a web and a roller, which occurs at the web center, on a per unit circumference basis is:

$$\sigma_{y,\max} = \frac{T_w \mu_t b}{2Rh} \tag{40}$$

where μ_t is the coefficient of traction, that may be affected by entrained air. If the maximum lateral compressive stress which can be supported by the traction predicted by expression {40} is less than the buckling stress predicted by expression {31}, a wrinkle cannot be sustained in the web upon the roller [2,3]. Wrinkles that attempt to form as the web enters the roller glide out upon the roller surface since the traction is unable to sustain them. Previous research has focused on how traction is reduced by entrained air[10]. A simple model that works well for ungrooved rollers and non-permeable webs is:

$$\begin{aligned}
&\mu_t = \mu_{st} \quad h_o \le R_q \\
&\mu_t = -\frac{\mu_{st}}{2R_q} h_o + \frac{3}{2}\mu_{st} \quad R_q \le h_o \le 3R_q \\
&\mu_t = 0 \quad h_o \ge 3R_q
\end{aligned} \tag{41}$$

where h_o is an air film layer due to hydrodynamic lubrication that was first shown to exist between moving webs and idler rollers by Knox and Sweeney [11] and μ_{st} is the static coefficient of friction between the web and roller surface. Those interested in the traction for permeable webs may refer to Ducotey and Good [12]. Knox and Sweeney verified the following relationship was applicable to webs moving over rollers:

$$h_o = 0.643R\left[\frac{12\eta V}{100\ T_w}\right]^{2/3} \tag{42}$$

where η is the viscosity of air ($3.08*10^{-7}$ N-min/m^2@ 27°C) and V is the web velocity (m/min), and T_w is the web tension (N/cm). R_q is the combined rms roughness of the web and roller surfaces in contact defined as:

$$R_q = \sqrt{R_{q,\text{roller}}^2 + R_{q,\text{web}}^2} \tag{43}$$

Expressions {31} and {40} can be equated and solved for the web tension (T_w) beneath which a wrinkle cannot be sustained upon the roller. The equated expressions cannot be solved directly for T_w due to dependence of μ_t on T_w and the piecewise definition of μ_t depending on the air layer thickness. However in just a few iterations the expressions can be solved.

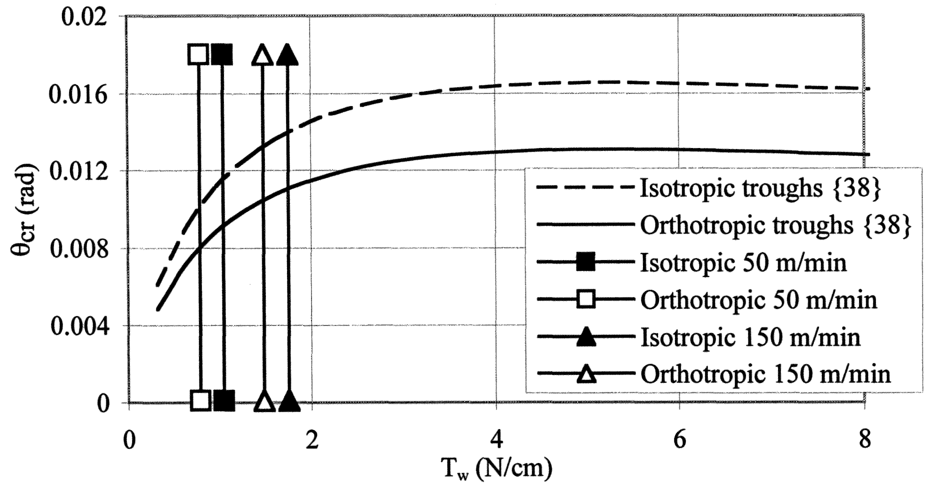

Figure 13 – The Effect of Orthotropic Web Properties on Troughs and the Minimum MD Tension to Sustain Wrinkling.

To exhibit the impact of orthotropic web properties an example comparison of isotropic and orthotropic webs will be used. For the isotropic web the properties stated earlier for the 15.24 cm wide, 23.4 μm thick polyester web will be used (i.e. E = 5 GPa, ν = 0.3, and G = 1.92 GPa). The orthotropic web will have the same width and thickness with moduli E_x and E_y of 5 and 2.07 GPa, respectively. The Poisson's ratio ν_{xy} will be assumed to be 0.3 and a ν_{yx} of 0.12 will result from expression {14} and a shear modulus of 1.24 GPa will result from expression {16}. Furthermore, assume R is 3.68 cm, μ_{st} is 0.26, and R_q is 2.44 μm. Velocities of 50 and 150 m/min will be studied. Assume the span length is 1.524 m, resulting in a span ratio of 10. The results of this comparison are shown in Figure 13. Expression {38} was used to generate the curves to predict troughs as shown. Beneath each curve the web would be planar at a given web tension starting at zero misalignment and proceeding up to a particular curve. Thus it is shown the orthotropic web has a smaller region of planarity than the isotropic web as might be expected due to the smaller CMD modulus (E_y). The vertical lines represent web tensions beneath which no wrinkle can be sustained upon the defined roller surface. The decrease in the CMD modulus amounts to a 58.6% reduction. At web velocities of 50 and 150 m/min the reduced modulus lead to a 24% and 19% reduction in the MD tension required to sustain a wrinkle on the roller surface, respectively.

CONCLUSIONS

The impact of web orthotrophy has been investigated by producing a trough {13} and a wrinkle {31} failure criterion. The wrinkle failure criterion is applicable to webs transgressing rollers and winding onto wound rolls.

The trough failure criterion was incorporated into a model developed for predicting the allowable misalignment of a downstream roller in isolated web spans. Expressions {38} and {39} incorporate:

- orthotropic web properties
- the effect of shear stiffness that can be important for short web spans
- the effect of tension stiffness which can be important for long web spans

These expressions were verified by tests of polyester, newsprint, and a spun bond polypropylene webs. In general it appears that expression {38}, which incorporates an assumption of a uniform, average shear stress is the more accurate solution over broad ranges of span ratio.

The impact of web orthotrophy was also examined on the MD tension required to sustain a wrinkle upon a roller surface. In the example studied it appears that the web orthotrophy produces only modest changes in the MD tension required.

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