ANALYSIS OF PINNED-END FRAMES WITH BENT MEMBERS BY THE STRING POLYGON METHOD

By

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PREFACE

The material presented in this thesis is the outgrowth of seminar lectures delivered by Professor Jan J. Tuma in the spring of 1959 and 1960. The literature survey and the general theory of the string polygon for straight and bent members were prepared by Professor Tuma (1).

The application of the string polygon to the calculation of beam constants was reported by Chu (2). Maydayag (3) developed the string polygon calculations for deflection of airplane wings and programmed this procedure for the IBM 650 electronic computer. Harvey (4) developed a string polygon application to the analysis of column beams.

The application of the String Polygon Method to the analysis of single span pinned-end frames with members of linear variation in cross-section is presented in this thesis. Two other graduate students are extending the application of this theory to frames with members of parabolic variation in cross-section (5) and to frames with sudden change in cross-section (6).

Finally, the general theory of the string polygon in terms of the energy due to shearing forces, normal forces, and bending moments is developed by Wu (7).

I wish to express my indebtedness and gratitude to the

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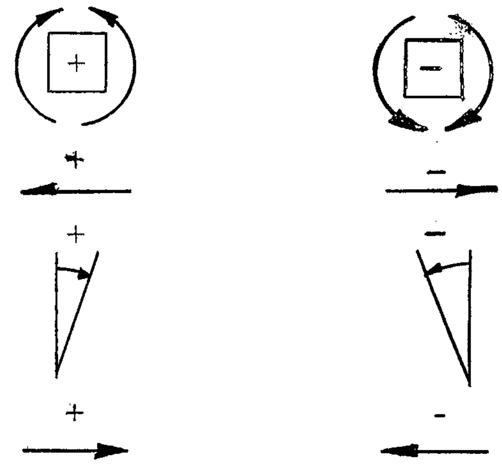
SIGN CONVENTION

Moment

Horizontal Displacement

Angular Rotation

Horizontal Thrust



NOMENCLATURE

b . . . . Width of Beam.

d_j . . . . Length of Straight Member $\overline{ij}$.

d_{jx} . . . . Horizontal Projection of Member $\overline{ij}$.

d_{jy} . . . . Vertical Projection of Member $\overline{ij}$.

$d\triangle_x$ . . . . Differential Horizontal Displacement Due to $d\phi_m$.

$d\phi_m$ . . . . Differential Change in Slope at m.

h_o . . . . Minimum Haunch Depth.

q . . . . Specific Weight of Material.

u, u', v, v' . . . . . . Co-ordinates of Cross-Sections.

w . . . . Intensity of Uniform Load.

x, x' . . . . Position Co-ordinates of Unit Load.

BM. . . . . Bending Moment Due to Loads.

F's, G's, $\mathcal{T}$'s . . . . . . Angular Functions.

H . . . . Horizontal Thrust Redundant.

I_o . . . . Minimum Moment of Inertia.

L . . . . Length of Span.

L_i, L_j, L_k . . . . Length of Spans $\overline{hi}$, $\overline{ij}$, and $\overline{jk}$, respectively.

M_i, M_j, M_k . . . . Bending Moments at i, j, and k, respectively.

$\bar{P}_j^{(L)}$ . . . . Elastic Weight at j Due to Loads.

$\bar{P}_j^{(H)}$ . . . . Elastic Weight at j Due to H.

$\bar{P}_m$ . . . . Elastic Weight at Any Point m.

$\bar{R}$	Reaction of Conjugate Beam.
S'_m, S_m.	Position Co-ordinates of Elastic Weight.
Y_i, Y_j, Y_k.	Moment Arms From Transformed Axis to Point of Application of Elastic Weight.
Z	Distance Between Points of Application of Elastic Weights.
β, ω	Dimension Coefficients of Length and Depth, respectively.
δ.	Haunch Coefficient Which Is Used in Tables of Beam Constants.
Δ_i, Δ_j, Δ_k	Vertical Displacement at i, j, and k, respectively.
$\Delta\beta$, $\Delta\omega$.	Increments by Which β and ω are Increased for Each Beam Size Until They Reach a Maximum.
Θ.	Angle Which the Line of Axis of the Hinges Makes With the Horizontal.
ϕ_j	Angular Deviation Adjacent String Lines of String Polygon at j.
π_i	Angle Which the Axis of a Bent Member ij Makes With the Horizontal.
ρ_j	Angular Deviation of Two Adjacent Bent Members at j.
ω_{jk}.	Angle Between the String Line of a Polygon and the Horizontal at j.

PART I

INTRODUCTION

The representation of the elastic curve of a straight beam as a differential string polygon was introduced by Mohr (8) in connection with his concept of elastic weights and of conjugate beams.

Müller-Breslau developed the idea of joint loads (knoten lasten) for straight members (9) and bent members (10). In his definition of joint loads the influence of the load on the element was neglected and only the effects of the moments, shears, and normal forces were considered.

The restatement of the formulation of joint loads may be found in recent publications (11, 12, 13, 14).

The idea of angle change traverses is discussed by Cross (15) and Michalos (16) and is essentially the idea of the string polygon as formulated by Mohr and Müller-Breslau.

By adding the angular load function "τ", Tuma (1) generalized the String Polygon and related it to the three moment equation, thus making possible the application of beam constants now available (17). This function accounts for loads at the intermediate points between vertices of the polygon and yields exact results.

The analysis of pinned-end frames of variable cross-section is a very commonplace problem in structural analysis.

Utilizing the string polygon theory and the analogy of elastic weights, the analysis of such frames is a simple problem is statics. When the tables of beam constants are used to compute the slopes, computation time is brought to a minimum.

The subsequent study is divided into eight parts. In the second part the string polygon theory and the analogy of elastic weights is presented. The elastic weight is a three-moment equation stating the change in slope of two adjacent string lines of the polygon. The third part deals with the expansion of this theory to include bent members and the formulation of relationships needed to compute load functions necessary in the evaluation of elastic weights. In the fourth part the String Polygon Method is applied to pinned-end frames.

The fifth part involves the development of an IBM computer program which tabulates numerical coefficients for computation of end slopes for beams with a linear variation in cross-section. This program is an outgrowth of similar work done by Lassley (18) for members of parabolic variation in cross-section. Tables of beam constants for straight haunched members are explained and tabulated in part six.

The seventh part gives the procedure of analysis and follows this procedure through two numerical examples. Finally, the thesis is summarized and conclusions are drawn.

PART II

THEORY OF THE STRING POLYGON

1. Definition.

The string polygon is an analogy based upon several elementary structural relationships. A simple beam of variable cross-section acted on by a general system of transverse loads is considered (Fig. 2-1).

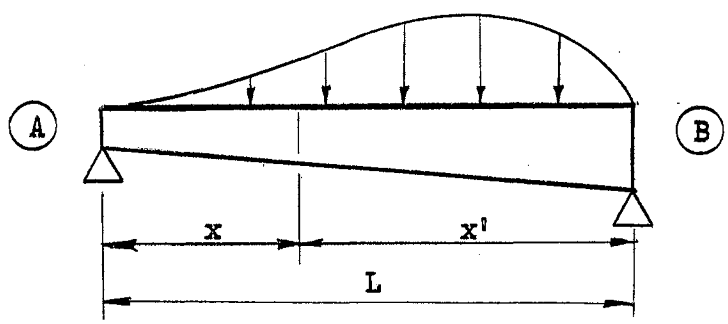


Fig. 2-1

Simple Beam

A finite number of points (i, j, k) along the beam are considered. The corresponding points on the elastic curve (i', j', k'), joined by straight lines form a string polygon (Fig. 2-2).

The relationship between the geometry of the string polygon and the deformations of the beam is investigated.

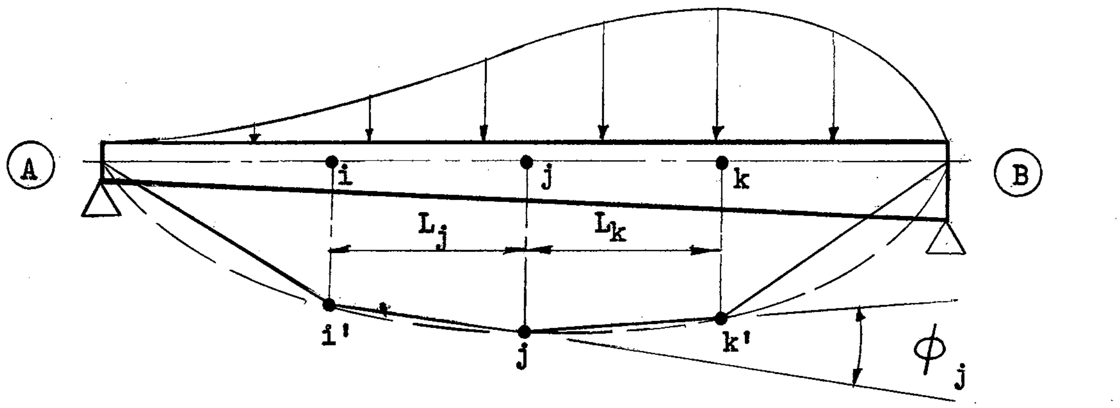


Fig. 2-2

String Polygon

2. Three Moment Equation.

Let (i, j, k) represent three consecutive points on the beam. The lengths L_j and L_k are the distances $\overline{ij}$ and $\overline{jk}$, respectively (Fig. 2-2). Segments $\overline{ij}$ and $\overline{jk}$ are isolated and from the conditions of static equilibrium the end moments and shears are calculated (Figs. 2-3,4). The variation of the bending moment in the first segment is given by Eq. (2-1).

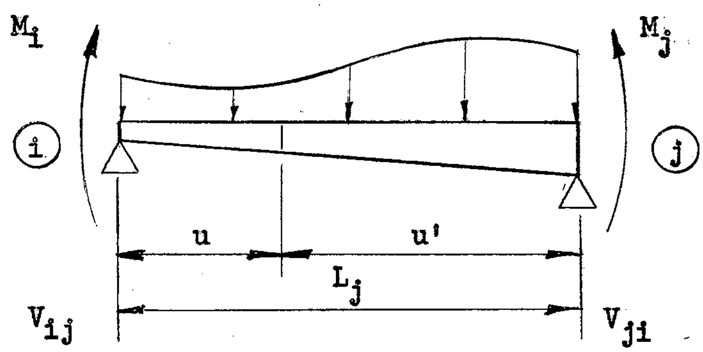


Fig. 2-3

Segment $\overline{ij}$

$$M_u^{(i)} = o \rightarrow L_j = BM_u + M_i \frac{u'}{L_j} + M_j \frac{u}{L_j} \qquad (2\text{-}1)$$

The ordinates u and u' are measured from i and j respectively. Similarly

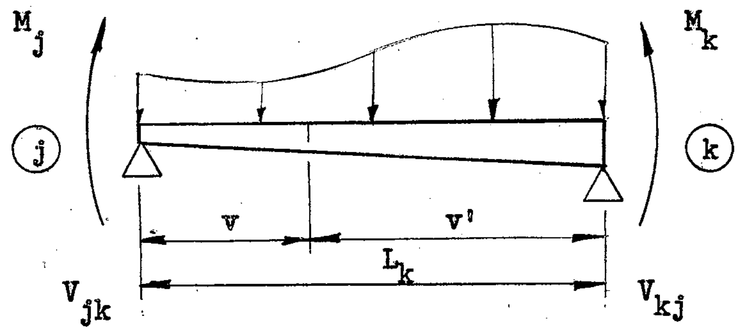


Fig. 2-4

Segment $\overline{jk}$

$$M_v^{(j)} = o \rightarrow L_k = BM_v + M_j \frac{v'}{L_k} + M_k \frac{v}{L_k} \qquad (2\text{-}2)$$

The strain energy of $\overline{ijk}$ due to bending is:

$$U_{ijk} = \int_i^j \frac{M_u^2 \, du}{2EI_u} + \int_j^k \frac{M_v^2 \, dv}{2EI_v}. \qquad (2\text{-}3)$$

The change in slope of the string polygon at j (ϕ_j) Fig. 2-2 may be computed by various methods. Castigiliano's theorem is applied.

$$\frac{\partial U_{ijk}}{\partial M_j} = \phi_j = \int_i^j \frac{2M_u \frac{\partial M_u}{\partial M_j} du}{2EI_u} + \int_j^k \frac{2M_v \frac{\partial M_v}{\partial M_j} dv}{2EI_v} \qquad (2\text{-}4)$$

In terms of bending moments, defined by Eq.'s (2-1,2), the change in slope is:

$$\phi_j = \int_i^j \frac{BM_u \, udu}{L_j EI_u} + M_i \int_i^j \frac{u\,u'\,du}{L_j^2\,EI_u} + M_j \int_i^j \frac{u^2\,du}{L_j^2\,EI_u}$$

$$+ \int_j^k \frac{BM_v \, v'dv}{L_k EI_v} + M_j \int_j^k \frac{v'^2\,dv}{L_k^2\,EI_v} + M_k \int_j^k \frac{v\,v'\,dv}{L_k^2\,EI_v} \qquad (2\text{-}5)$$

Eq. (2-5) with new equivalents is:

$$\phi_j = M_i\,G_{ij} + M_j \sum F_j + M_k\,G_{kj} + \sum \tau_j \qquad (2\text{-}6)$$

The right side of Eq. (2-6) takes the form of the general three moment equation in terms of angular flexibilities (F's), angular carry-over values (G's), angular load functions (τ's) and bending moments.(19) The definitions of these functions are stated in the following section.

3. Angular Functions.

A. The Angular Flexibility. F_{ji} (or F_{jk}) is the end slope of the simple beam $\overline{ij}$ (or $\overline{jk}$) at j, due to a unit moment applied at that end. (Fig. 2-5).

$$F_{ji} = \int_i^j \frac{u^2\,du}{L_j^2\,EI_u} \qquad\qquad F_{jk} = \int_j^k \frac{v'^2\,dv}{L_k^2\,EI_v} \qquad (2\text{-}7)$$

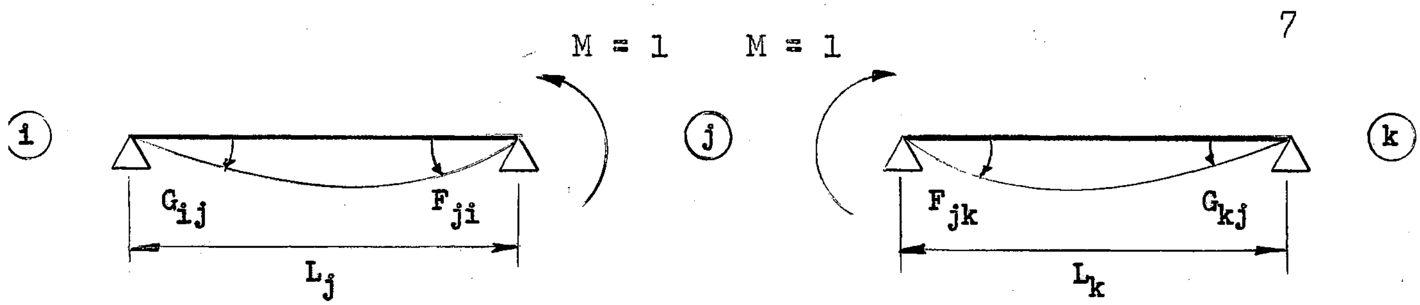


Fig. 2-5

Angular Flexibilities and Carry-Over Values

B. The Carry-Over Value. G_{ij} (or G_{kj}) is the end slope of the simple beam $\overline{ij}$ (or $\overline{jk}$) at i (or k) due to a unit moment applied at the far end j. (Fig. 2-5).

$$G_{ij} = \int_i^j \frac{u\,u'\,du}{L_j^2\,EI_u} \qquad G_{kj} = \int_j^k \frac{v\,v'\,dv}{L_k^2\,EI_v} \qquad (2\text{-}8)$$

C. The Angular Load Function. τ_{ji} (or τ_{jk}) is the end slope of the simple beam $\overline{ij}$ (or $\overline{jk}$) at j, due to loads, (Fig. 2-6).

$$\tau_{ji} = \int_i^j \frac{BM_u\,u\,du}{L_j EI_u} \qquad \tau_{jk} = \int_j^k \frac{BM_v\,v'dv}{L_k EI_v} \qquad (2\text{-}9)$$

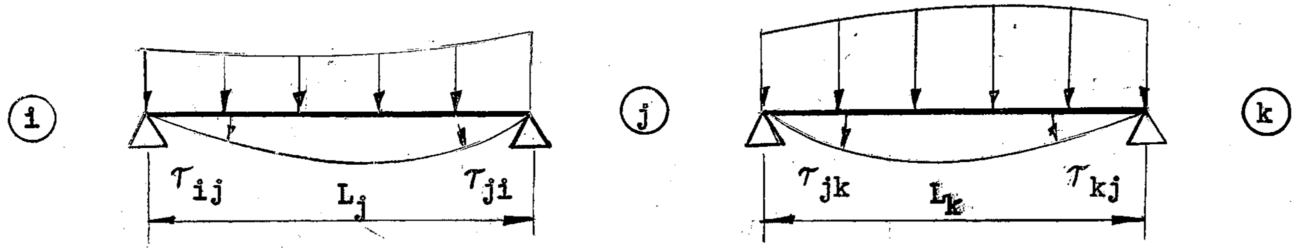


Fig. 2-6

Angular Load Functions

4. Conjugate Beam and Elastic Weights.

The equation for the change in slope of adjoining string lines of the polygon (Eq. 2-6) is perfectly general and may be written for any point of the polygon, $\overline{A_{ijk}B}$ (Fig. 2-7). If the slopes of the polygon strings are denoted as ω_{ai}, ω_{ij}, ω_{jk}, ω_{kB}, their relationships to the changes in slope are (Fig. 2-7)

$$\left.\begin{aligned} \omega_{ij} &= \omega_{Ai} - \phi_i \\ \omega_{jk} &= \omega_{Ai} - \phi_i - \phi_j \\ \omega_{kB} &= \omega_{Ai} - \phi_i - \phi_j - \phi_k \end{aligned}\right] \qquad (2\text{-}10a)$$

With notation

$$\left.\begin{aligned} &\omega_A = \omega_{Ai} \qquad \omega_B = -\omega_{kB} \\ &\omega_A - \omega_B = \phi_i + \phi_j + \phi_k \end{aligned}\right] \qquad (2\text{-}10b)$$

the end slopes of the polygon (Fig. 2-7) are

$$\omega_A = \frac{1}{L}\left[\begin{aligned} &\phi_i\,(L_j + L_k + L_B) \\ &+ \phi_j\,(L_k + L_B) \\ &+ \phi_k\,(L_B) \end{aligned}\right] \qquad (2\text{-}11a)$$

$$\omega_B = \frac{1}{L}\left[\begin{aligned} &\phi_i\,(L_i) \\ &+ \phi_j\,(L_i + L_j) \\ &+ \phi_k\,(L_i + L_j + L_k) \end{aligned}\right] \qquad (2\text{-}11b)$$

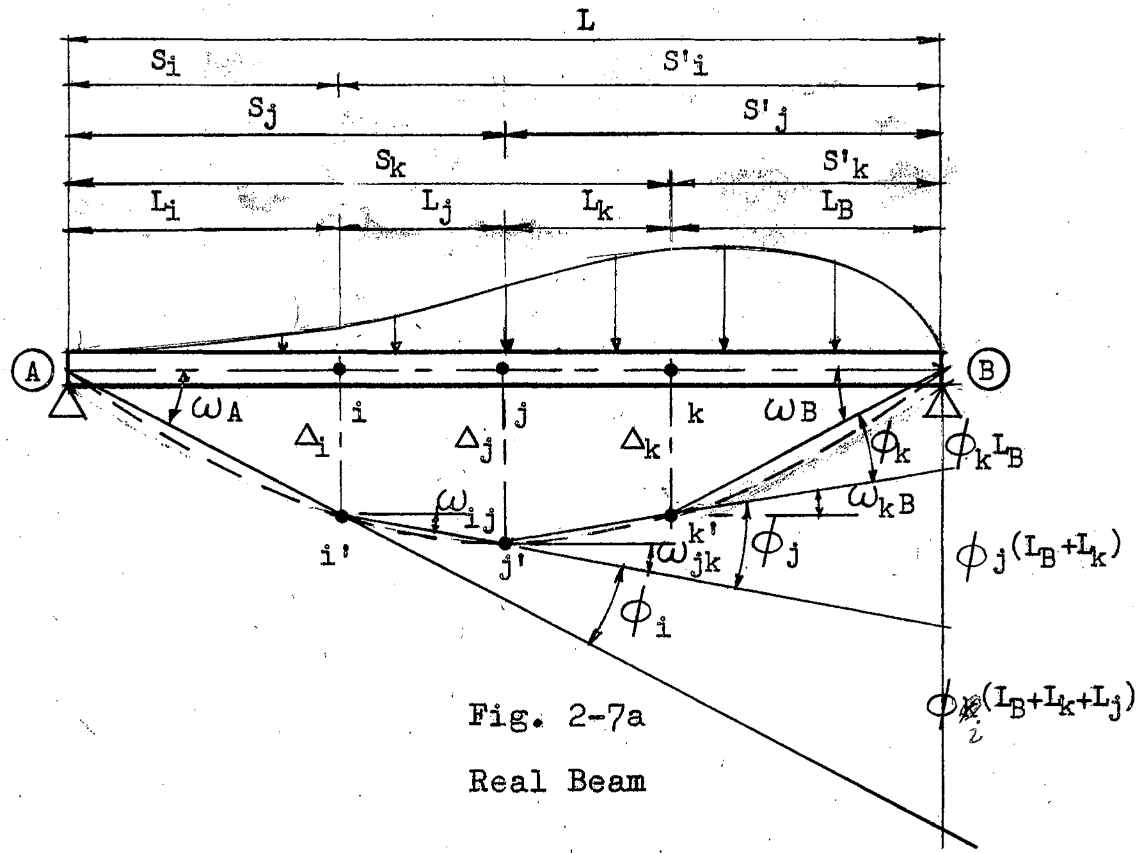


Fig. 2-7a

Real Beam

Geometry of String Polygon

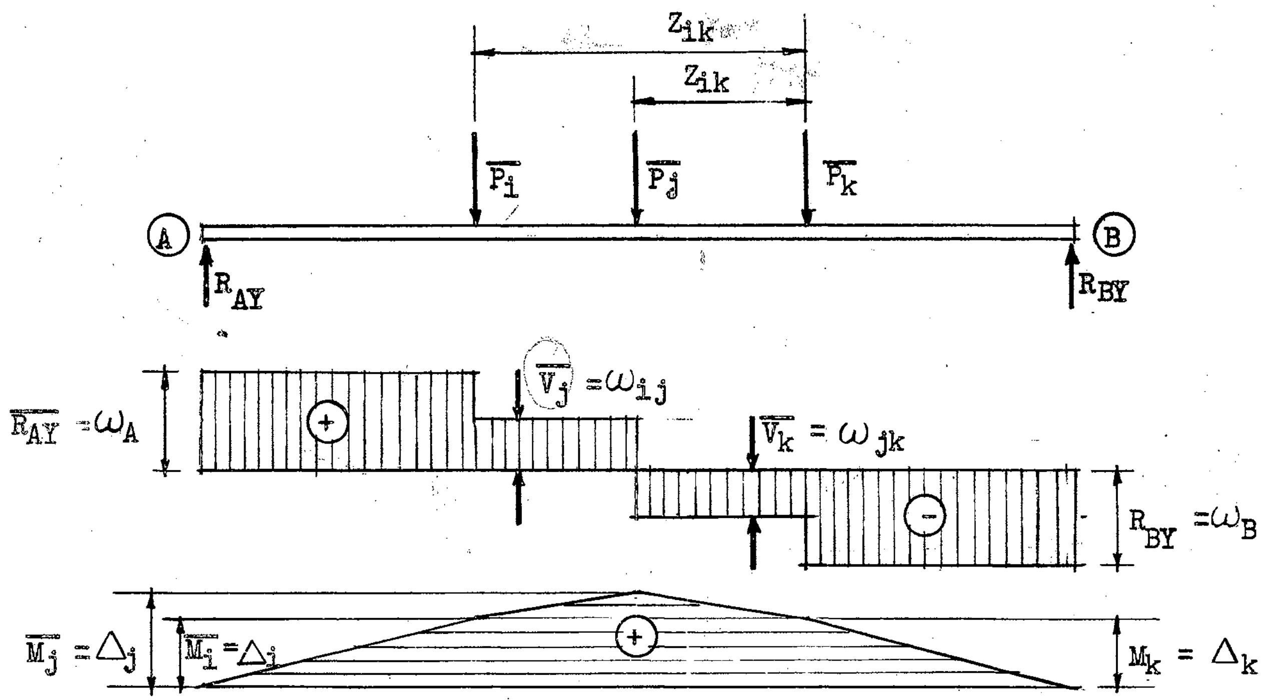


Fig. 2-7b

Conjugate Beam

Function of String Polygon

These relationships are based on the assumption that the elastic curve is almost flat and all slopes are small. The equalities will be

$$\left.\begin{array}{llllll} \tan\omega \doteq & \sin\omega \doteq \omega & & & \\ \overline{Ai} & \doteq \overline{A_i'} & & \overline{jk} & \doteq & \overline{j'k'} \\ \overline{ij} & \doteq \overline{i'j'} & & \overline{kB} & \doteq & \overline{k'B'} \\ \tan\phi \doteq & \sin\phi \doteq \phi & & & \end{array}\right\} \qquad (2\text{-}12)$$

With these simplifications the deflections in terms of ϕ's become

$$\left.\begin{array}{lll} \Delta_i & = & \omega_A\,(L_i) \\ \Delta_j & = & \omega_A\,(L_i + L_j) - \phi_i\,(L_j) \\ \Delta_k & = & \omega_A\,(L_i + L_j + L_k) \\ & & - \phi_i\,(L_j + L_k) - \phi_j\,(L_k) \end{array}\right\} \qquad (2\text{-}13)$$

From closer inspection of Eq.'s (2-10, 11, 13) it may be observed that:

1. The end slopes given by Eq. (2-11) are functionally similar to the reactions of a simple beam of the same length (conjugate beam) loaded by ϕ's (elastic weights).
2. The slopes of the strings (Eq.'s 2-10a, 10b) are functionally similar to the shear of a simple beam of the same length loaded by ϕ's.
3. The deflections of the real beam at the points of the string polygon are equal to the bending moments

of a simple beam of the same length loaded by ϕ's.

From these three observations the analogy between the deformation of the real beam and the statical functions of the conjugate beam is established. To make the similarity more perceptible, new nomenclature is introduced:

$\overline{Pm} = \phi m$ = The change in slope at point m' of two adjacent string lines.

Sm = The horizontal distance from the left end of the beam to the point of application of $\overline{P_m}$.

$S'm$ = The horizontal distance from the right end of the beam to the point of application of $\overline{P_j}$.

Zm_j = The horizontal distance from the point of application of $\overline{P_m}$ to the point of application of $\overline{P_j}$.

$\overline{R_{AY}} = \omega_A$ = The reaction of the conjugate beam or the slope of the real beam at A.

$\overline{R_{BY}} = \omega_B$ = The reaction of the conjugate beam or the slope of the real beam at B.

$\overline{V_j} = \omega_{jk}$ = The shear of the conjugate beam or the slope of the real beam at j.

$\overline{M_j} = \triangle_j$ = The moment of the conjugate beam or the deformation of the real beam at j.

In terms of these notations the governing equations of the string polygon are:

$$\overline{R_{AY}} = \sum_{A}^{B} \frac{\overline{P}_m S'_m}{L} \qquad \overline{R_{BY}} = \sum_{A}^{B} \frac{\overline{P}_m S_m}{L} \quad (2\text{-}14a)$$

$$\left.\begin{aligned} \overline{V_j} &= \overline{R_{AY}} - \sum_{A}^{i} \overline{P_m} \\ \overline{M_j} &= \overline{R_{BY}}\, S_j - \sum_{A}^{i} \overline{P_m}\, Z_{mj} \end{aligned}\right\} \quad (2\text{-}14b)$$

The analogy is shown in Fig. (2-7).

PART III

STRING POLYGON FOR BENT MEMBERS

1. Basic Relationships.

The theory of the string polygon can be very easily applied to inclined and bent members. A bent member $\overline{ijk}$ of variable cross-section acted on by a general system of transverse loads is considered. The slopes of segments $\overline{ij}$ and $\overline{jk}$, are π_j and π_k, respectively (Fig. 3-1).

If segments $\overline{ij}$ and $\overline{jk}$ are isolated into two free bodies, the bending moments are obtained in similar form as in Part II, Section 2.

$$\left.\begin{aligned} M_u &= M_i \frac{u'}{d_j} + M_j \frac{u}{d_j} + BM_u \\ M_v &= M_j \frac{v'}{d_k} + M_k \frac{v}{d_k} + BM_v \end{aligned}\right] \qquad (3\text{-}1)$$

The relative angular displacements (ϕ_j) are somewhat more difficult to understand for the bent member than for the straight member. For clarity it should be remembered that if the change in slope of the real member at j (before deformation) is ρ_j, then the change in slope of the string polygon at j (after deformation) is $\rho_j + \phi_j$, and the difference of these two changes is the change in slope of the polygon due to deformation.

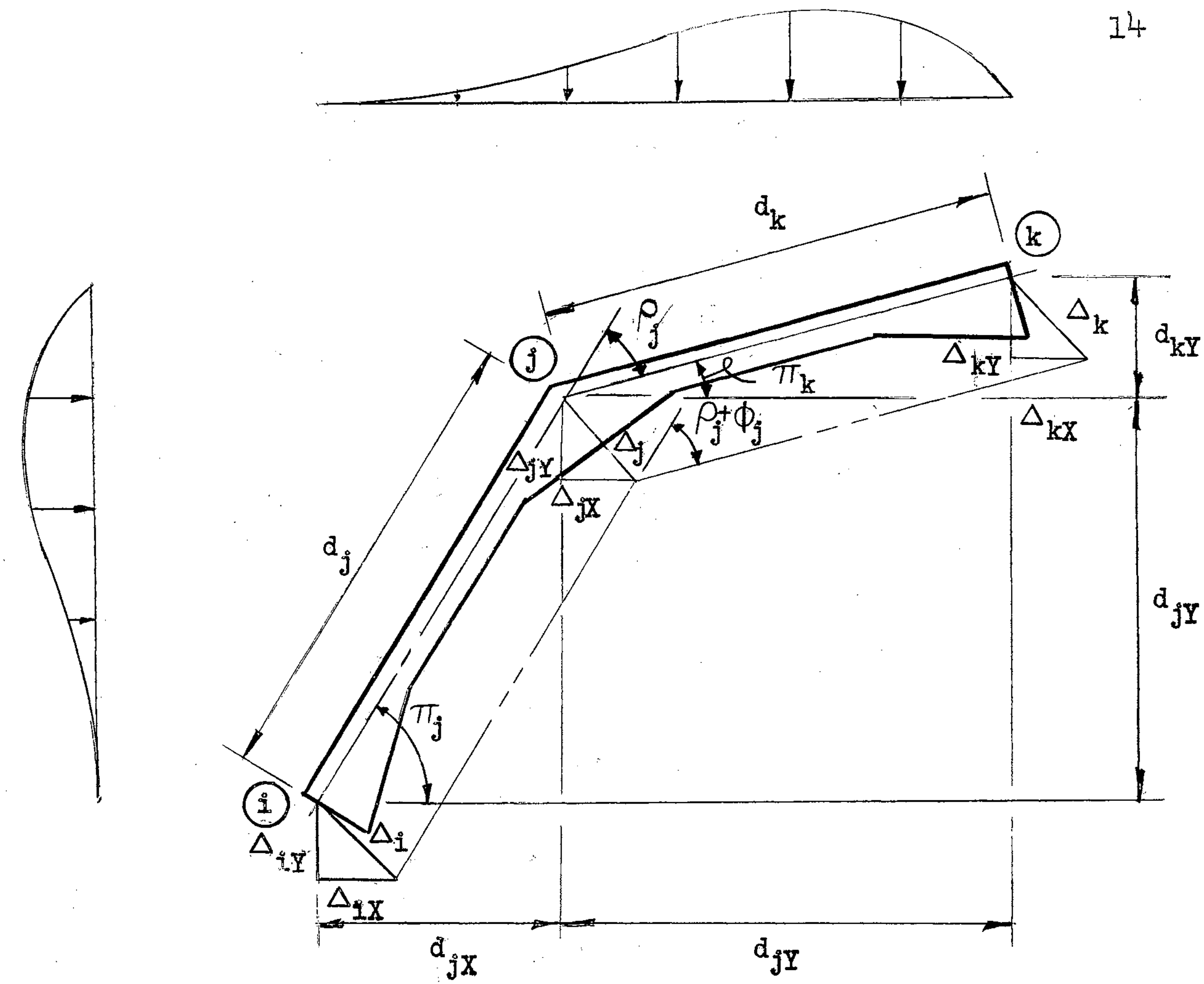


Fig. 3-1

Bent Member $\overline{ijk}$

For simplicity:

ρ_j = Change in slope due to geometry

ϕ_j = Change in slope due to deformation

$\rho_j + \phi_j$ = Total change in slope.

For the computation of deformations both values (ρ_j and ϕ_j) are of extreme importance.

2. Three Moment Equation.

The strain energy for a bent member $\overline{ijk}$ due to bending is:

$$U_{ijk} = \int_i^j \frac{M_u^2 \, du}{2 \, EI_u} + \int_j^k \frac{M_v^2 \, dv}{2 \, EI_v} \qquad (3\text{-}2)$$

Castigiliano's theorem is applied, and the change in slope (ϕ_j), in terms of bending moments (Eq. 3-1) is:

$$\left.\begin{aligned} \phi_j = & \int_i^j \frac{BM_u \; udu}{d_j \; EI_u} + M_i \int_i^j \frac{u \; u' \; du}{d_j^2 \; EI_u} + M_j \int_i^j \frac{u^2 \; du}{d_j^2 \; EI_u} \\ & + \int_j^k \frac{BM_v \; v'dv}{d_k \; EI_v} + M_j \int_j^k \frac{v'^2 \; dv}{d_k^2 \; EI_v} + M_k \int_j^k \frac{v \; v' \; dv}{d_k^2 \; EI_v} \end{aligned}\right] \qquad (3\text{-}3)$$

Eq. (3-3) with new equivalents (Part II, Section 3) is:

$$\phi_j = M_i \, G_{ij} + M_j \sum F_j + M_k \, G_{kj} + \sum \tau_j \qquad (3\text{-}4)$$

The general three moment equation is again obtained. The only difference between Eq. (2-6) and Eq. (3-4) appears in the angular load function due to the inclination of the bent member.

3. Angular Load Functions.

The angular load function τ_{ji} (or τ_{jk}) of a bent member due to vertical loads is the end slope of a simple beam $\overline{i'j'}$ at j, having as its length the horizontal projection d_{jx} (or d_{kx}) or the bent member, multiplied by a trigonometric function of the slope (Fig. 3-2)

The angular load function for vertical loads is:

$$\tau_{ji} = \int_i^j \frac{BM_u \; udu}{d_j \; EI_u} \qquad (3\text{-}5a)$$

$$\tau_{ji} = \frac{1}{\cos \pi_j} \int_i^j \frac{BM_x \, xdx}{d_{jx} \, EI_x} = \frac{1}{\cos \pi_j} \tau_{jix} \quad (3\text{-}5b)$$

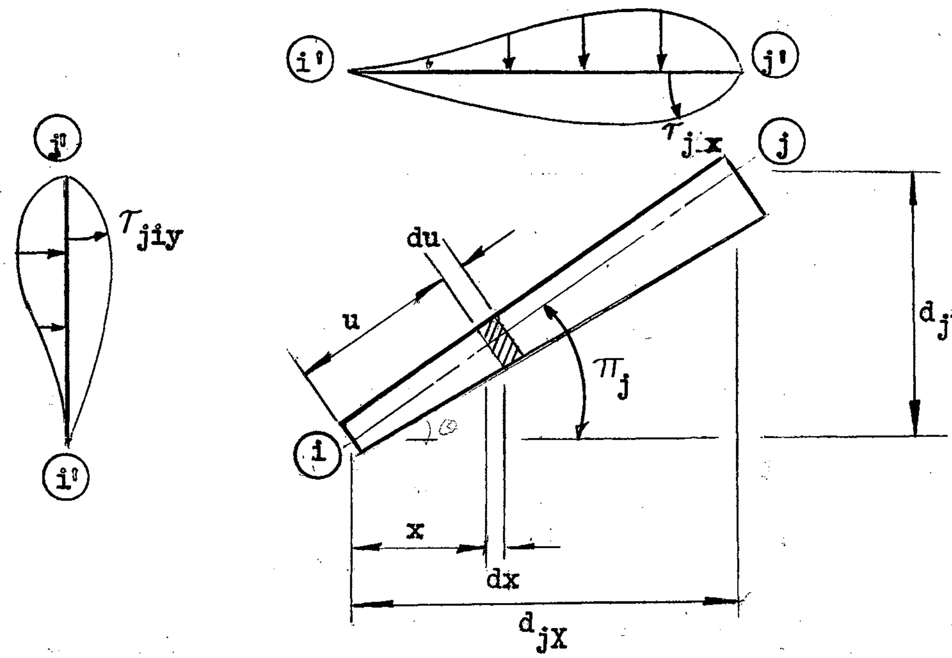


Fig. 3-2

τ_{ji} Due to Applied Loads

The change in slope at j (elastic weight) due to vertical loads on bent member $\overline{ij}$ (Fig 3-2) is:

$$\left.\begin{aligned} \phi_j &= M_i G_{ij} + M_j \sum F_j + M_k G_{kj} \\ &+ \frac{1}{\cos \pi_j} \tau_{jix} + \frac{1}{\cos \pi_k} \tau_{jkx} \end{aligned}\right] \quad (3\text{-}6)$$

Similarly the angular load function τ_{ji} (or τ_{jk}) of a bent member $\overline{ij}$ due to horizontal loads (Fig. 3-2) is:

$$\tau_{ji} = \int_i^j \frac{BM_u \, udu}{d_j \, EI_u} \quad (3\text{-}7a)$$

$$\tau_{ji} = \frac{1}{\sin \pi_j} \int_i^j \frac{BM_y \quad ydy}{d_{jy} \quad EI_x} = \frac{1}{\sin \pi_j} \tau_{jiy} \qquad (3\text{-}7b)$$

The change in slope at j (elastic weight) due to horizontal loads on bent member $\overline{ij}$ (Fig. 3-3) is:

$$\left.\begin{aligned} \phi_j &= M_i G_{ij} + M_j \sum F_j + M_k G_{kj} \\ &+ \frac{1}{\sin \pi_j} \tau_{jiy} + \frac{1}{\sin \pi_j} \tau_{jky} \end{aligned}\right] \qquad (3\text{-}8)$$

PART IV

STRING POLYGON FOR PINNED-END FRAMES

1. Pinned-End Frames With Hinges at the Same Level.

The theory of the string polygon presents a very simple approach to obtain both the horizontal displacement and the horizontal thrust redundant in the analysis of statically indeterminate pinned-end frames. A pinned-end frame with supports on the same level, of variable cross-section, and acted on by a general system of horizontal and vertical transverse loads is considered (Fig.'s 4-1, 2).

A. Horizontal Displacement.

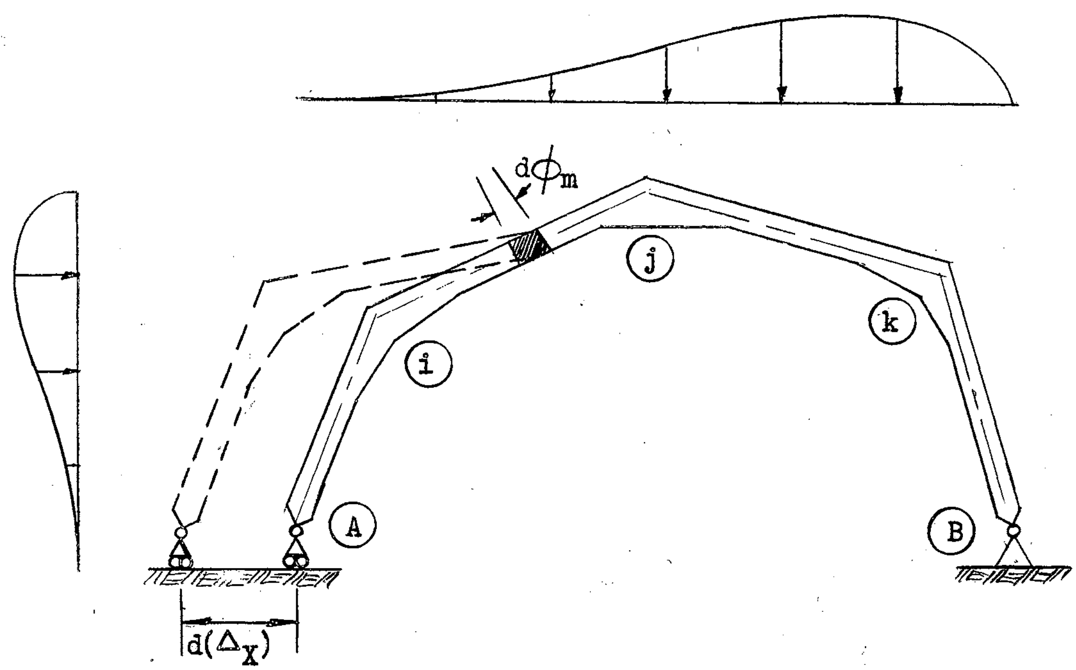


Fig. 4-1

Differential Horizontal Displacement at A

The horizontal displacement $d(\Delta_x)$ of the frame $\overline{AijkB}$ is the static moment of $d\phi_m$ (elastic weight at m) about the axis of the hinges.

$$d(\Delta_x) = (d\phi)\, Y_m \quad . \tag{4-1}$$

The total horizontal displacement is:

$$\Delta_{Ax} = \sum_A^B \phi_m Y_m \quad . \tag{4-2}$$

Similarly

$$\Delta_{Ax} = \sum_A^B \overline{P}_m Y_m \quad . \tag{4-3}$$

The physical interpretation of Eq.'s (4-2) and (4-3) is: The horizontal displacement is equal to the static moment of the elastic weights about $\overline{AB}$.

B. Horizontal Thrust Redundant.

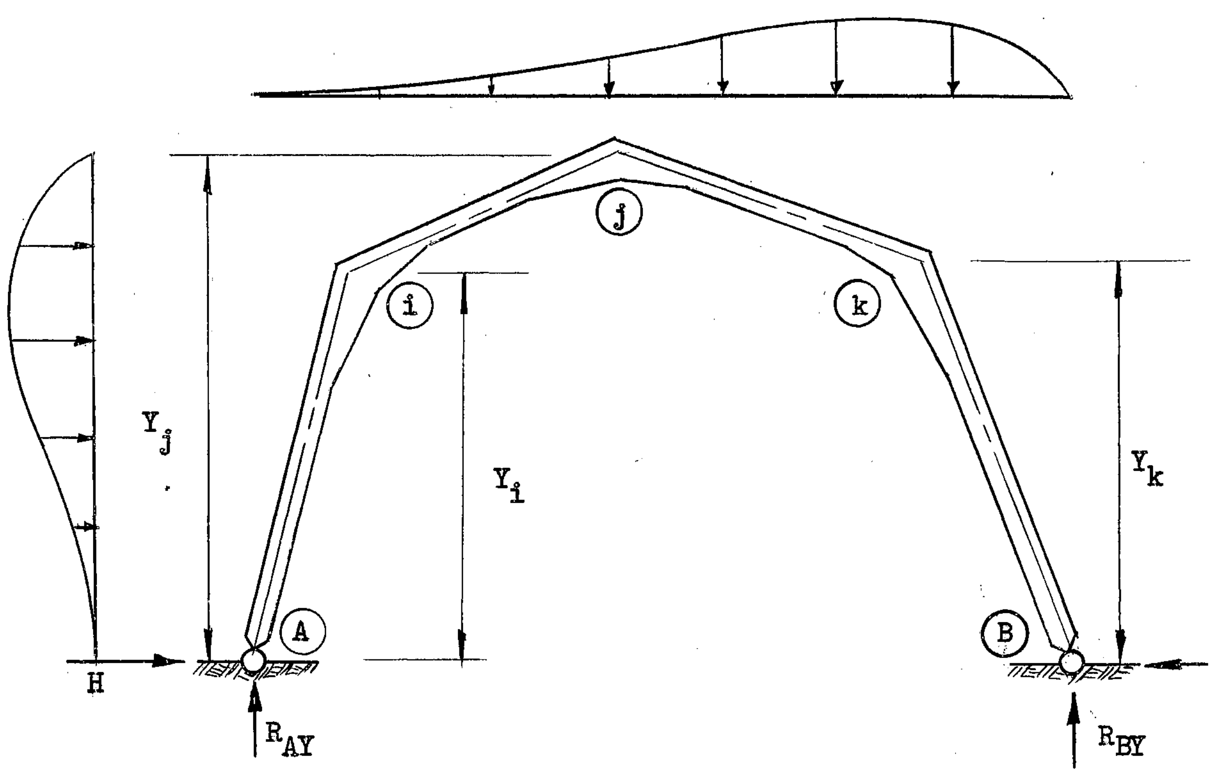


Fig. 4-2

Pinned-End Frame with Hinges at the Same Level

Referring to Eq. 3-6 and Eq. 3-8, the elastic weight at point j (any point) due to loads and redundant H is:

$$\left.\begin{aligned}\overline{P}_j &= M_i G_{ij} + M_j \sum F_j + M_k G_{kj} \\ &+ \tau_{jix} \frac{1}{\mathrm{Cos}\ \pi_j} + \tau_{jiy} \frac{1}{\mathrm{Sin}\ \pi_j} \\ &+ \tau_{jkx} \frac{1}{\mathrm{Cos}\ \pi_k} + \tau_{jky} \frac{1}{\mathrm{Sin}\ \pi_k} .\end{aligned}\right] \qquad (4\text{-}4)$$

The bending moments at joints of the frame are functions of the loads and H.

$$\left.\begin{aligned}M_i &= BM_i + H\, Y_i \\ M_j &= BM_j + H\, Y_j \\ M_k &= BM_k + H\, Y_k\end{aligned}\right] \qquad (4\text{-}5a)$$

Thus

$$\overline{P}_j = \overline{P}_j^{(L)} + \overline{P}_j^{(H)} . \qquad (4\text{-}5b)$$

Therefore the elastic weight due to loads is:

$$\left.\begin{aligned}\overline{P}_j^{(L)} &= M_i\, G_{ij} + M_j \sum F_j + M_k\, G_{kj} \\ &+ \tau_{jix} \frac{1}{\mathrm{Cos}\ \pi_j} + \tau_{jky} \frac{1}{\mathrm{Sin}\ \pi_j} \\ &+ \tau_{ikx} \frac{1}{\mathrm{Cos}\ \pi_k} + \tau_{jky} \frac{1}{\mathrm{Sin}\ \pi_k}\end{aligned}\right] \qquad (4\text{-}5c)$$

and the elastic weight due to H is:

$$\overline{P}_j^{(H)} = H\left[Y_i\, G_{ij} + Y_j \sum F_j + Y_k\, G_{kj} \right]. \qquad (4\text{-}5d)$$

From Eq. 4-3 the static moment of the elastic weights about the axis $\overline{AB}$ is:

$$M_x = 0 = \sum \overline{P}_m^{(L)} Y_m + \sum \overline{P}_m^{(H)} Y_m . \qquad (4\text{-}6a)$$

or

$$M_x = 0 = \sum \overline{P}_m^{(L)} Y_m + H \sum \overline{P}_m^{(H=1)} Y_m . \qquad (4\text{-}6b)$$

From these investigations the equation for the horizontal thrust redundant is obtained.

$$H = - \frac{\sum \overline{P}_m^{(L)} Y_m}{\sum \overline{P}_m^{(H=1)} Y_m} . \qquad (4\text{-}7)$$

2. Pinned-End Frames With Hinges at Different Levels.

A pinned-end frame with supports at different levels, of variable cross-section, and acted on by a general system of horizontal and vertical transverse loads is considered (Fig. 4-2).

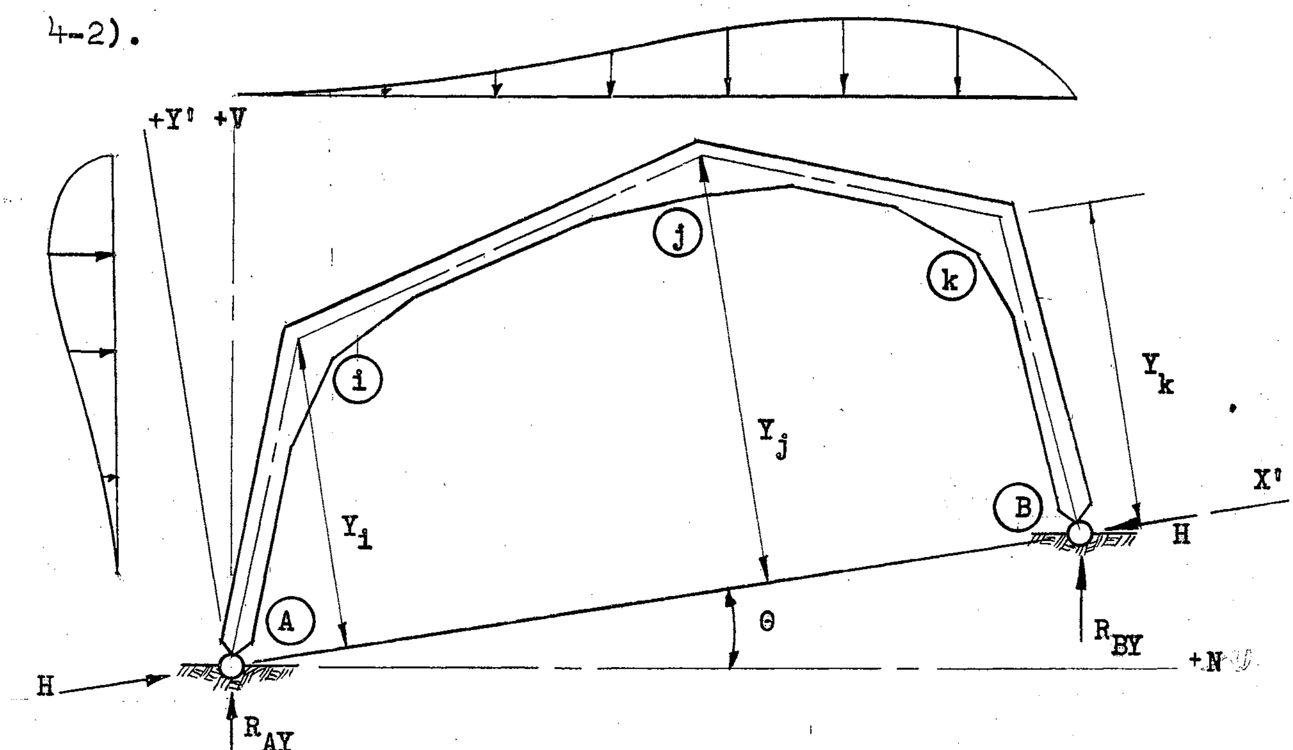


Fig. 4-2

Pinned-End Frame With Hinges at Different Levels

The frame would be relatively difficult to analyze if the computations would take into consideration the orientation of the frame with the horizontal and vertical axes. The analysis of the thrust redundant would then involve consideration of its vertical and horizontal components. This tedious process is eliminated if the axes are rotated at an angle Θ so the transformed horizontal axis (x' axis) has its line of action coincide with the line of action of the thrust redundant (Fig. 4-2).

The angular load functions do not change when the axis is rotated. The value of the load function for a bent member is due to the orientation of the member and not the frame.

The distances, from the transformed axis to the point of application of elastic loads, are easily computed from trigonometric functions of the angle of rotation (Θ). Point j in Fig. (4-3) is considered.

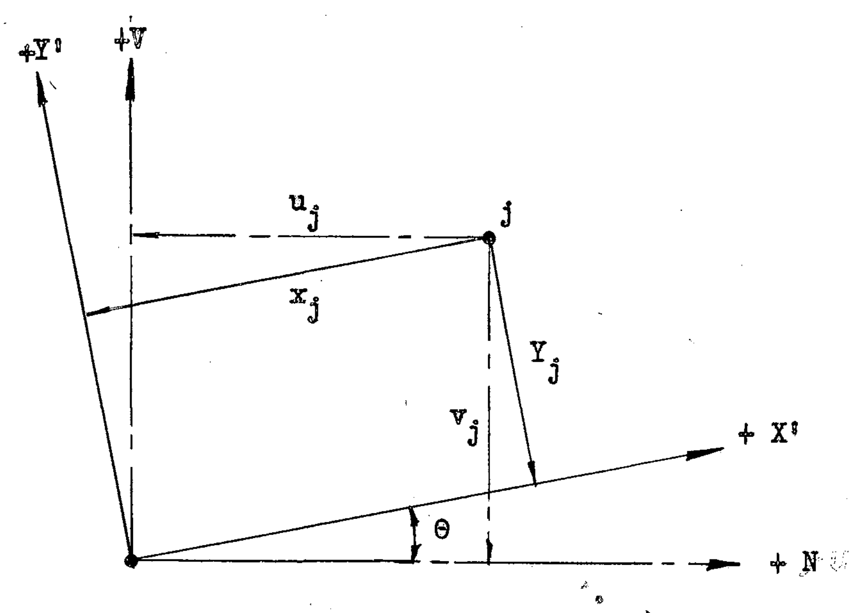


Fig. 4-3
Orientation of Point j

The geometry of Fig. (4-3) is considered and the distance Y_j is:

$$Y_j = v_j \cos \Theta - u_j \sin \Theta \qquad (4\text{-}8)$$

The only difference between the calculation of pinned-end frames with hinges on one level and pinned-end frames with hinges on different levels is the transformation of the axes. When the transformed axes are considered, calculations of Part IV, Section 1 apply and Eq. (4-7) can be used.

PART V

I. B. M. 650 PROGRAM FOR DETERMINATION OF END SLOPES FOR BEAMS WITH STRAIGHT HAUNCHES

1. General.

Lassley (18), in his M. S. Thesis at Oklahoma State University, developed a computer program for determination of end slopes for beams with parabolic haunches. The author has revised this work by changing the mathematical expressions of the haunch from parabolic to linear values.

A program is a precise sequence of coded instructions which an electronic computer interprets to solve a particular problem.

The programming of any problem on an electronic computer is accomplished in two steps. First, a schematic drawing or flow chart is made showing each phase and sequence of operations. Next, from this flow chart, a series of instructions for the computer is established.

The program in this section was compiled through the facilities of the computer center at Oklahoma State University. The coding form used is that of I. B. M.'s Symbolic Optimal Assembly Program, Type II. (20) For high-speed processing immediate access storage is utilized.(21)

2. Input Data Format.

The description of the beam, for which constants are desired, is introduced in the computer with seven words, (Fig. 5-1)

Word	Card Columns	Data
1	1-10	ω
2	11-20	β
3	21-30	$\Delta\omega$
4	31-40	$\Delta\beta$
5	41-50	ω_{max}
6	51-60	β_{max}
7	61-70	Beam Type
8	71-80	Zeros

Fig. 5-1

Input Data Card

The dimension coefficients of length and depth are ω and β, respectively. The symbols $\Delta\omega$ and $\Delta\beta$ are the increments by which the dimension coefficients are to be increased. Words five and six are the maximum values which the dimension coefficients may attain. The beam type number is zero for unsymmetrical beams and one for symmetrical beams. Floating

Decimal Arithmetic is used for words one through six.

A general data card for unsymmetrical beams will be (Fig. 5-2).

ω = 0.1 → 2.0 in increments of 0.1

β = 0.1 → 0.5 in increments of 0.1

Word	Data Entered
1	1000000050
2	1000000050
3	1000000050
4	1000000050
5	2000000051
6	1000000051
7	0000000000
8	(Not Used)

Fig. 5-2

General Input Data Card for Unsymmetrical Beams

3. Output Card Format.

The angular function coefficients will be in floating decimal form on either three or four cards, depending upon the type of beam.

The beam identification number

05 003 00 001

will appear on the first putput card for the unsymmetrical beam for which

$$\omega = 0.5$$

$$\beta = 0.3$$

and the number

05 003 09 001

will appear on the card containing influence coefficients for

$$n = 0.7, 0.8, 0.9.$$

The first output card will be arranged as follows (Fig. 5-3).

Word	Information
1	Identification
2	f_{BA}
3	g
4	f_{AB}
5	$t_{BA}^{(UL)}$
6	$t_{AB}^{(UL)}$
7	$t_{BA}^{(DL)}$
8	$t_{AB}^{(DL)}$

Fig. 5-3

First Output Card

The angular live-load coefficients will appear as follows (Fig. 5-4).

Word	Information	Position of Load
1	Identification	
2	$t_{BA}^{(LL)}$	n-2
3	$t_{AB}^{(LL)}$	n-2
4	$t_{BA}^{(LL)}$	n-1
5	$t_{AB}^{(LL)}$	n-1
6	$t_{BA}^{(LL)}$	n
7	$t_{AB}^{(LL)}$	n
8	(Not Used)	

Fig. 5-4

Output Card for Live-load Coefficients

4. Flow Chart.

The flow chart (Fig. 5-5) was prepared as an aid to setting up the sequence of instructions for computation of end slope coefficients for symmetrical and unsymmetrical straight haunched beams.

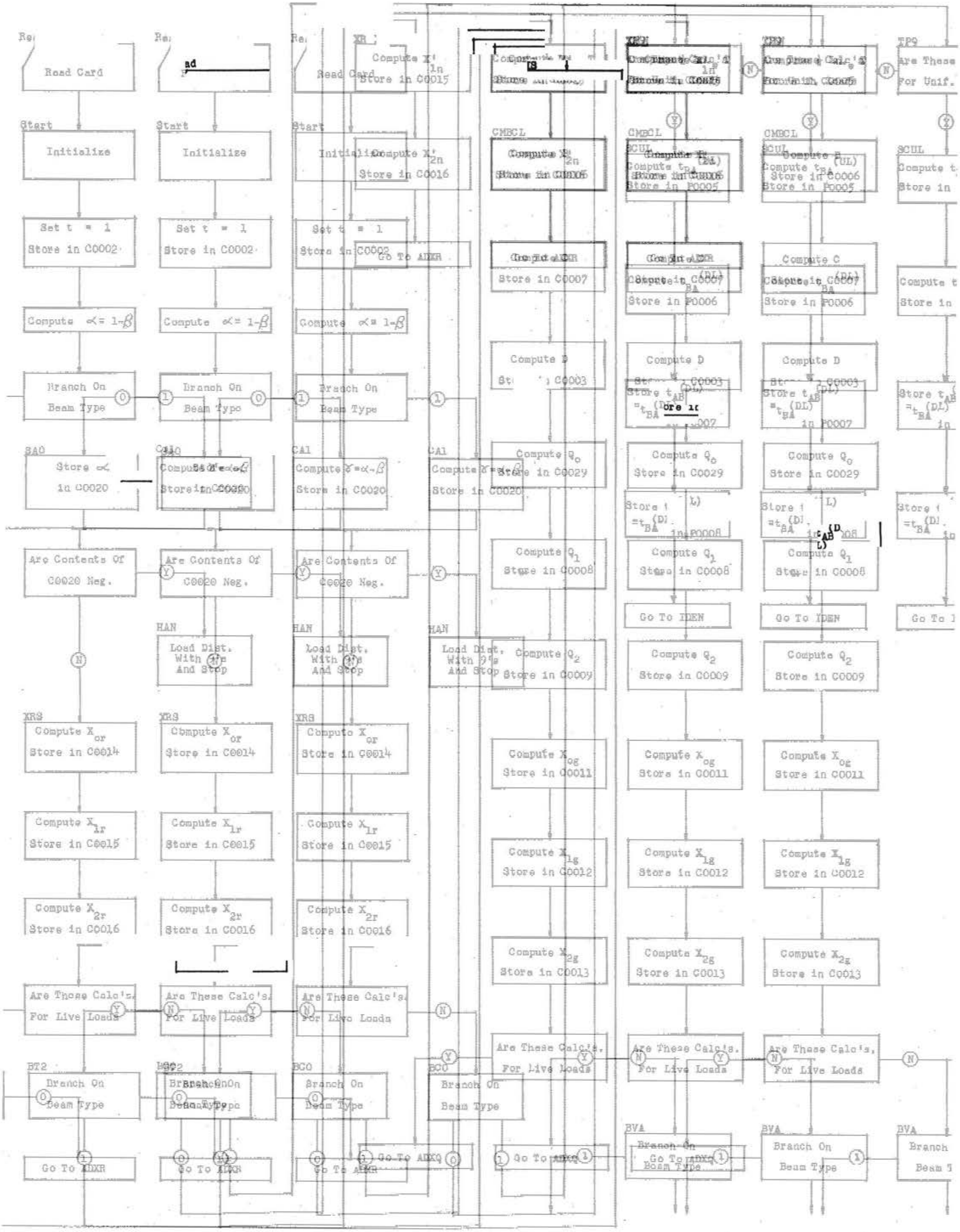


Fig. 5-5 Flow Chart

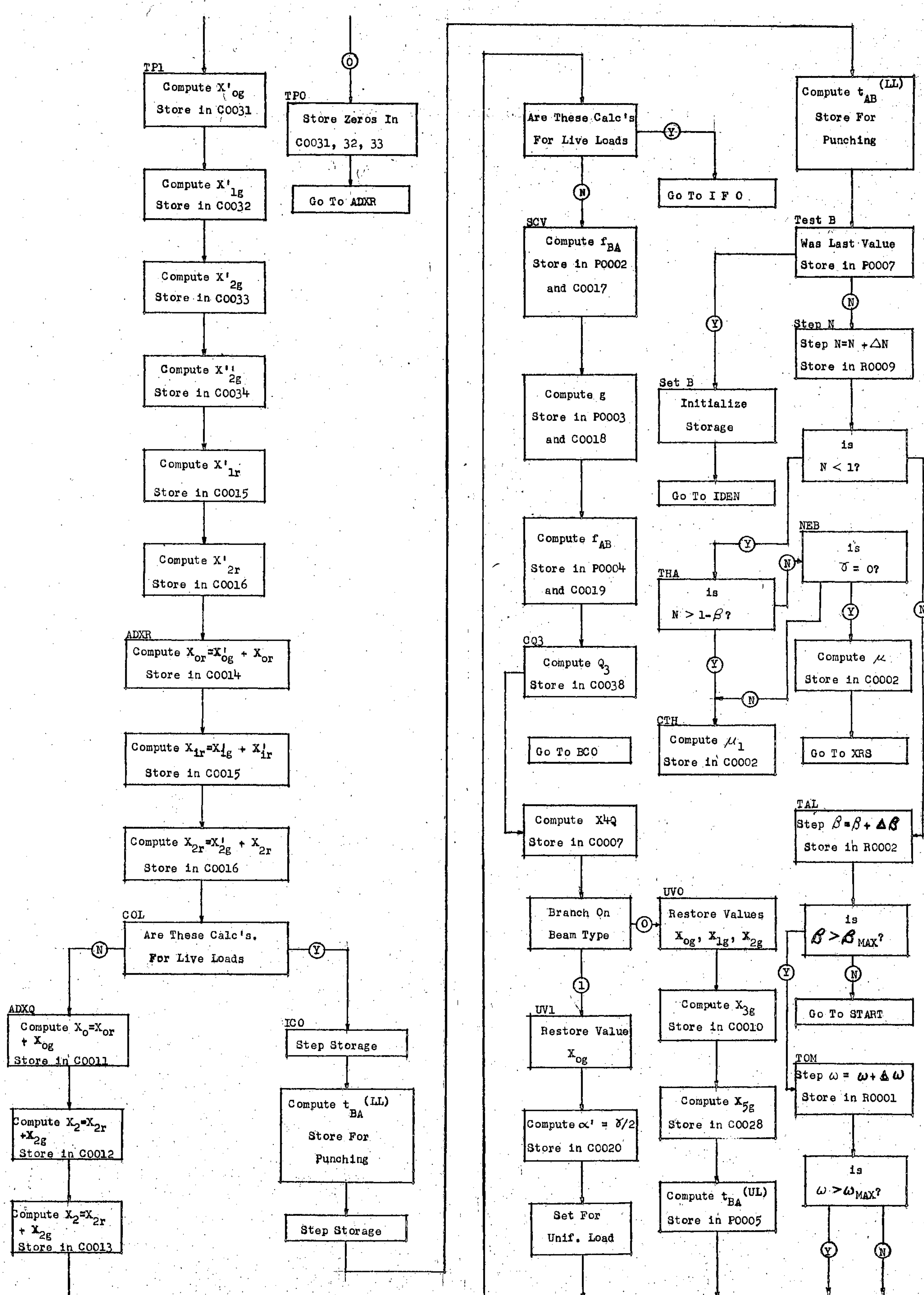


Fig. 5-5 (Con't.) Flow Chart

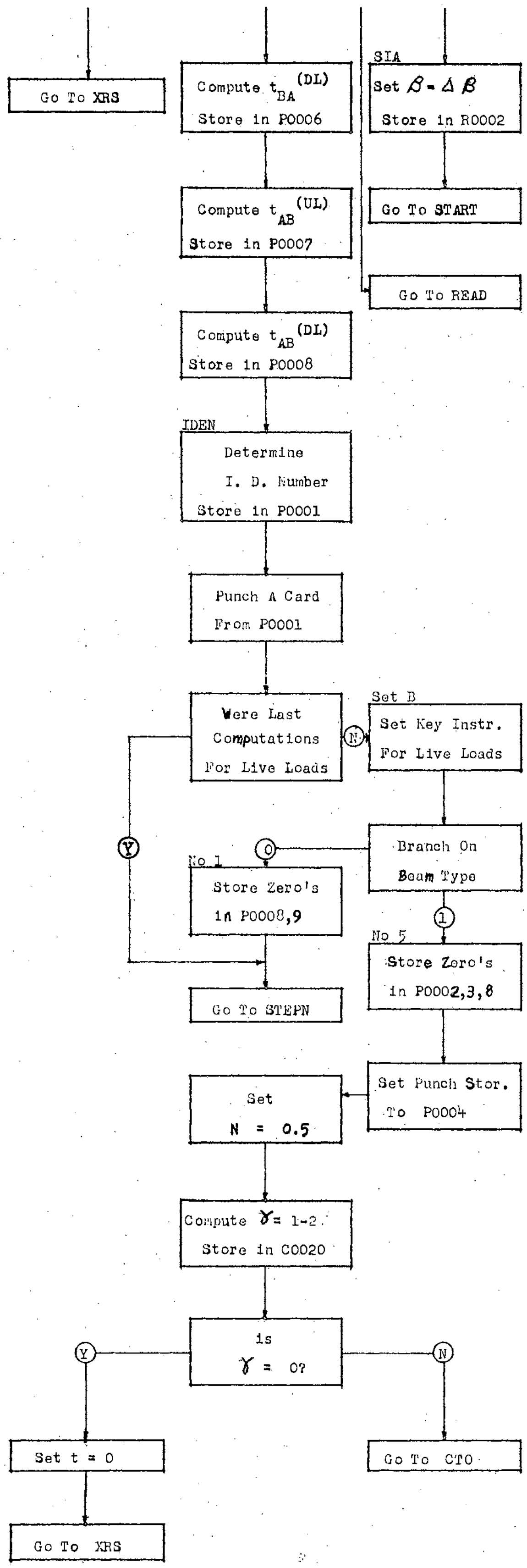


Fig. 5-5 (Con't) Flow Chart

5. Statement of Program.

The following program, assembled in I.B.M. Soap II, will compile coefficients for calculation of end slopes for symmetrical and unsymmetrical straight haunched beams. The calculations include ten-point influence coefficients for beams of either type for which β is expressed as a multiple of one-tenth. No provision is made for beams of constant moment of inertia and the entry of ω or β equal to zero will result in an attempt to divide by zero. The computer will stop if α or γ become minus, as would be the case if β for a symmetrical beam would be entered as 0.6. No other stops are incorporated.

The statement of the program or sequence of instructions are as follows (Fig. 5-6).

T P	S N	LOCA- TION	OPER CODE	DATA ADDRESS	T A G	ADDRESS	T A G	REMARKS
1		END SLO	PES	FOR ST	R	AIGHT		
1		HAUNCHED	BEAMS					
1								
1		M S THE	SIS					
1								
1		HENRY C	BOECK	ER				
1								
1		AUGUST	1960					
1								
			SYN	READ		0000		BEGIN 0000
			BIR	1600		1670		
			REG	R9000		9009		READ AREA
			REG	C9010		9049		CALC AREA
			REG	P9050		9059		PCH AREA
		READ	RD1	R0001		START		READ CARD
		START	LDD	ZERO				INITIALIZE
			STD	R0009				N
			STD	P0009				NO DL OP
			RAA	R0007				BM TYPE
			RAB	0000				NO LL OP
			RAU	ONE				SET T TO
			STU	C0002				ONE AND
			FSB	R0002				COMPUTE
			NZA	CA1		SAO		ALPHA OR
		CA1	FSB	R0002		SAO		GAMMA
		SAO	STU	C0020				AND STORE
			BMI	HAN		XRS		IF NEG
		HAN	LDD	NINES				STOP
			HLT					
		XRS	RAU	C0020				COMPUTE
			FMP	C0002				
			STU	C0014				XOR
			FMP	C0014				
			FMP	HALF				
			STU	C0015				X1R
			FMP	C0014				
			FMP	FRAC				AND
			STU	C0016				X2R
			NZB	BT2		BCO		
		BCO	NZA	TP9		CLN		IS LN
		TP9	RAU	P0009				REQUIRED
			NZU	SCUL		CLN		
		CLN	RAU	R0001				
			FMP	C0002				
			FAD	ONE				ONE PLUS
			STU	C0004				OMEGA T
			RAC	0008				SET TERMS
			LDD	ZERO				INITALIZE
			STD	C0005				FOR LOG
			LDD	ONE				SERIES
			STD	C0023				
			STD	C0024				
			RAU	C0004				COMPUTE
			FAD	ONE				RATIO FOR
			STU	C0025				SERIES
			FSB	TWO				
			FDV	C0025				
			STU	C0026				
			FMP	C0026				
			STU	C0028		LOOP		
		LOOP	RAU	C0024				COMPUTE
			FMP	C0026				SUM OF
			FAD	C0005				TERMS
			STU	C0005				STORE LOG
			BMC	CMBCL		STA		
		STA	RAU	C0023				STEP FOR
			FAD	TWO				NEXT TERM
			STU	C0027				AND
			RAU	C0023				REPEAT
			FMP	C0024				
			FMP	C0028				
			FDV	C0027				
			STU	C0024				
			LDD	C0027				

T P	S N	LOCA- TION	OPER CODE	DATA ADDRESS	T A G	INST. ADDRESS	T A G	REMARKS
			STD	C0023				
			SXC	0001		LOOP		
		CMBCL	RAU	C0002				COMPUTE
			FDV	TWO				AND STORE
			FDV	C0004				
			FDV	C0004				
			STU	C0006				B COMPUTED
			FMP	C0004				
			STU	C0007				C COMPUTED
			FAD	C0006				COMPT AND
			STU	C0029				STORE QO
			FSB	C0007				
			FMP	C0002				
			STU	C0008				Q1
			RAU	R0001				
			FMP	R0001				
			FMP	R0001				
			STU	C0023				STORE WWW
			RAU	THREE				
			FMP	C0007				
			FSB	C0006				
			FMP	R0001				
			STU	C0003				
			RAU	TWO				
			FMP	C0005				
			FSB	C0003				
			FDV	C0023				AND
			STU	C0009				Q2
			RAU	C0029				COMPUTE
			FMP	R0002				AND STORE
			STU	C0011				XOQ
			RAU	C0008				
			FSB	C0029				
			FMP	R0002				
			FMP	R0002				
			FAD	C0011				AND
			STU	C0012				X1Q
			RAU	R0002				COMPUTE
			FMP	R0002				AND STORE
			FMP	R0002				3 POWERS
			STU	P0010				BETA
			RAU	C0029				COMPUTE
			FAD	C0009				AND STORE
			FSB	C0008				
			FSB	C0008				
			FMP	P0010				
			FAD	C0012				
			FAD	C0012				
			FSB	C0011				
			STU	C0013				X2Q
			NZB	ADXQ		BVA		BRN LL OP
		BVA	NZA	TP1		TPO		BRN BM TYP
		TPO	LDD	ZERO				LT HAUNCH
			STD	C0031				XOQ
			STD	C0032				X1Q AND
			STD	C0033		ADXR		X2Q
		TP1	LDD	C0011				LT HAUNCH
			STD	C0031				XOQ
			RAU	C0011				
			FSB	C0012				
			STU	C0032				X1Q
			RAU	C0013				
			FAD	C0011				
			FSB	C0012				
			FSB	C0012				
			STU	C0033				X2Q
			RAU	C0011				
			FDV	FOUR				
			FAD	C0013				
			FSB	C0012				AND MOD
			STU	C0034		XRIS		X2Q
		XRIS	RAU	R0002				COMPUTE

Fig. 5-6

Statement of Program

T P	S N	LOCA-TION	OPER CODE	DATA ADDRESS	T A G	INST. ADDRESS	T A G	REMARKS
			FMP	C0014				AND STORE
			FAD	C0015				FOR TYPE 1
			STU	C0015				X1R
			FAD	C0015				
			FDV	R0002				
			FSB	C0014				
			FMP	R0002				
			FMP	R0002				
			FAD	C0016				AND
			STU	C0016		ADXR		X2R
		ADXR	RAU	C0031				SUM LT XKQ
			FAD	C0014				AND XKR
			STU	C0014				X0R
			RAU	C0032				
			FAD	C0015				
			STU	C0015				X1R
			RAU	C0033				
			FAD	C0016				AND
			STU	C0016		COL		X2R
		BT2	NZA	XR1S		ADXR		
		COL	NZB	IC0		ADXQ		BRN LL OP
		ADXQ	RAU	C0011				FUM XKR
			FAD	C0014				ANDXKQ
			STU	C0011				X0Q
			RAU	C0012				
			FAD	C0015				
			STU	C0012				X1Q
			RAU	C0013				
			FAD	C0016				AND
			STU	C0013				X2Q
			NZB	IF0		SCV		BRN LL OP
		SCV	LDD	C0013				COMPUTE
			STD	P0002				AND STORE
			STD	C0017				FBA
			RAU	C0012				
			FSB	C0017				
			STU	P0003				
			STU	C0018				G
			RAU	C0011				
			FSB	C0018				
			FSB	C0018				
			FSB	C0017				
			STU	P0004				AND
			STU	C0019		CQ3		FAB
		CQ3	RAU	C0009				
			FMP	THREE				
			STU	C0039				COMPUTE
			RAU	C0006				AND STORE
			FMP	TWO				
			FSB	C0039				
			FDV	R0001				
			STU	C0038				Q3
			RAU	P0010				
			FMP	R0002				4 POWERS
			STU	P0010				OF BETA
			FMP	C0038				
			STU	C0030				X4Q
			NZA	UV1		UVO		BRN BM TYP
		UVO	RAU	C0011				RESTORE
			FSB	C0014				
			STU	C0011				X0Q
			RAU	C0012				
			FSB	C0015				
			STU	C0012				X1Q
			RAU	C0013				
			FSB	C0016				
			STU	C0013				X2Q
			RAU	C0008				COMPUTE
			FSB	C0009				AND
			FMP	THREE				STORE
			FAD	C0038				
			FSB	C0029				
			FMP	P0010				
			STU	C0010				

T P	S N	LOCA-TION	OPER CODE	DATA ADDRESS	T A G	INST. ADDRESS	T A G	REMARKS
			RAU	C0013				
			FSB	C0012				
			FMP	THREE				
			FAD	C0010				
			FAD	C0011				
			STU	C0010				X3Q
			RAU	SIX				
			FMP	C0009				
			STU	C0037				
			RAU	C0007				
			FDV	FIVE				
			FSB	C0006				
			FMP	FIVE				
			FAD	C0037				
			FDV	R0001				
			FDV	R0001				Q 4
			FSB	C0038				
			FMP	P0010				
			FMP	R0002				
			FAD	C0030				
			STU	C0028				X5Q
			RAU	C0020				BRN IS
			NZU	ANO		AEQO		ALPH EQ O
		AEQO	RAU	C0017		SX4R		DO NOT
		ANO	RAU	C0017				DIVIDE
			FDV	C0014				BY ZERO
			FDV	DEC				IN THESE
			FSB	C0016				CALCS
			FMP	C0014				
			FMP	DEC		SX4R		
		SX4R	FSB	C0010				COMPUTE
			FMP	HALF				AND STORE
			STU	P0005				UL TBA
			RAU	P0010				
			FDV	R0002				
			STU	P0010				
			FMP	C0017				
			FSB	C0028				
			FMP	R0001				
			FDV	SIX				
			FDV	R0002				
			FAD	P0005				
			STU	P0006				DL TBA
			RAU	C0018				
			FMP	HALF				
			FSB	P0005				
			STU	P0007				UL TAB
			RAU	C0017				
			FAD	C0018				
			FMP	P0010				
			FSB	C0030				
			FMP	R0001				
			FDV	SIX				
			FDV	R0002				
			FAD	P0007				
			STU	P0008		IDEN		DL TAB
		UV1	RAU	C0011				RESTORE
			FSB	C0014				
			STU	C0011				X0Q
			RAU	C0020				COMPUTE
			FMP	HALF				GAMMA AND
			STU	C0020				SET FOR
			RAU	ONE				DL OP
			STU	P0009		XRS		
		SCUL	RAU	C0014				COMPUTE
			FAD	C0011				AND STORE
			FDV	FOUR				
			FSB	C0016				
			FSB	C0034				
			FMP	HALF				
			STU	P0005				UL TBA
			RAU	C0014				
			FAD	C0011				
			FMP	P0010				

Fig. 5-6 (Con't.)

Statement of Program

T/P	S/N	LOCA-TION	OPER CODE	DATA ADDRESS	TAG	INST. ADDRESS	TAG	REMARKS
			FDV	HALF				
			FSB	C0030				
			FMP	R0001				
			FDV	SIX				
			FDV	R0002				
			FAD	P0005				
			STU	P0006				DL TBA
			LDD	P0005				
			STD	P0007				UL TAB
			LDD	P0006				
			STD	P0008				DL TAB
			LDD	ZERO				SET FOR
			STD	P0009		IDEN		NO DO OP
		IDEN	RAU	R0001				STORE
			FSB	ONE				FOR
			BMI	SFT1		SFT2		IDENTIF
		SFT1	RAL	R0001				
			SRT	0003				
			SLT	0002				
			STL	C0035		IDB		OMEGA
		SFT2	RAL	R0001				
			SRT	0002				
			SLT	0002				OR
			STL	C0035		IDB		OMEGA
		IDB	RAU	R0002				
			FSB	ONE				
			BMI	SFT3		SFT4		
		SFT3	RAL	R0002				
			SRT	0008				
			SLT	0004				
			STL	C0036		IDN		BETA
		SFT4	RAL	R0002				
			SRT	0008				
			SLT	0005				OR
			STL	C0036		IDN		BETA
		IDN	RAL	R0009				COMPOSE
			SRT	0006				AND STORE
			ALO	8005				
			ALO	C0035				
			ALO	C0036				
			STL	P0001				
			WR1	P0001				ID NUMBER
			NZB	STEPN		SETB		BRN LL OP
		SETB	RAB	0001				INIT LL OP
			NZA	NO5		NO1		BRN BM TP
		NO1	LDD	ZERO				
			STD	P0008				TYPE 0 LL
			STD	R0009		STEPN		N TO ZERO
		NO5	LDD	ZERO				TYPE 1 LL
			STD	P0002				INITIAL
			STD	P0003				VALUES
			STD	P0008				STORAGE
			AXB	0002				
			LDD	HALF				AND
			STD	R0009				N IS HALF
			RAU	ONE				COMPUTE
			FSB	R0002				GAMMA
			FSB	R0002				
			STU	C0020				
			NZU	CTO		GAO		IF ZERO
		GAO	STU	C0002		XRS		LIM IS ZER
		CTO	RAU	R0009				COMPUTE
			NZA	SR2		CTAL		AND STORE
		SR2	FSB	R0002		CTAL		UPPER LIM
		CTAL	FDV	C0020				FOR CONST
			STU	C0002		XRS		I
		STEPN	RAU	R0009				STEP N
			FAD	DELTN				AND TEST
			STU	R0009				
			FSB	ONE				IF LESS
			BMI	THA		TAL		THAN ONE
		THA	RAU	ONE				AND GRTR
			FSB	R0002				THAN ONE

T/P	S/N	LOCA-TION	OPER CODE	DATA ADDRESS	TAG	INST. ADDRESS	TAG	REMARKS
			FSB	R0009				MIN BETA
			BMI	CTH		NEB		
		NEB	RAU	C0020				
			NZU	CTO		CTH		
		CTH	RAU	R0002				COMPUTE
			FSB	ONE				AND STORE
			FAD	R0009				UPPER LIM
			FDV	R0002				FOR VAR
			STU	C0002		BCO		I
		ICO	AXB	0001				COMPUTE
			RAU	C0018				AND
			FSB	C0015				
			FMP	R0009				STORE
			FAD	C0016				
			STU	P0000 B				LL TBA
			RAU	C0018				
			FAD	C0019				
			FSB	C0014				
			FMP	R0009				
			FAD	C0015				
			FSB	P0000 B				
			AXB	0001				AND
			STU	P0000 B		TESTB		LL TAB
		TESTB	RAU	8006				IF LAST
			SUP	SVEN				VALUE IN
			BMI	STEPN		AD1		P0007
		AD1	RAB	0001		IDEN		INIT B PCH
		IFO	AXB	0001				COMPUTE
			RAU	C0018				FOR VAR I
			FSB	C0012				AND STORE
			FMP	R0009				
			FAD	C0013				
			STU	P0000 B				LL TBA
			RAU	C0018				
			FAD	C0019				
			FSB	C0011				
			FMP	R0009				
			FAD	C0012				
			FSB	P0000 B				
			AXB	0001				AND
			STU	P0000 B		TESTB		LL TAB
		TAL	RAU	R0002				TEST IF
			FAD	R0004				BETA IS
			STU	R0002				MAX IF NO
			RAU	R0006				STEP IF
			FSB	R0002				YES
			BMI	TOM		START		
		TOM	RAU	R0001				TEST IF
			FAD	R0003				OMEGA IS
			STU	R0001				MAX AND
			RAU	R0005				
			FSB	R0001				
			BMI	READ		SIA		READ OR
		SIA	RAU	R0004				STEP BETA
			STU	R0002		START		
1 1 1			C	ONSTA		NTS		
		FIVE	50	0000		0051		
		ZERO						
		ONE	10	0000		0051		
		HALF	50	0000		0050		
		NINES	99	9999		9999		
		FOUR	40	0000		0051		
		THREE	30	0000		0051		
		TWO	20	0000		0051		
		OCT	80	0000		0051		
		DEC	75	0000		0050		
		TWELV	12	0000		0052		
		DELTN	10	0000		0050		
		SVEN				7		
		FRAC	66	6666		6750		
		SIX	60	0000		0051		

Fig. 5-6 (Con't.)

Statement of Program

PART VI

TABLES OF BEAM CONSTANTS

1. Background.

Tables of coefficients for the calculation of angular functions, for parabolic haunch beams, have been compiled by Tuma, French, and Lassley. (18) Their work includes all coefficients for symmetrical beams and the haunch end coefficients for unsymmetrical beams. Oden (5) has expended this work to include coefficients for computation of angular functions for the small end of unsymmetrical, parabolic haunch beams.

The tables compiled in this chapter are all for straight haunched beams. A specific beam is located, in the tables, by the ratio of the haunch length to the total length and the ratio of the minimum and maximum cross-section (Fig. 6-1).

In order to provide numerical results which cover the range of beams usually encountered in engineering practice, combinations of the following ratios are used.

$$\beta = 0, .1, .2 \cdots .8, .9, 1.0$$

$$\delta = 1.1, 1.2, 1.3 \cdots 2.8, 2.9, 3.0.$$

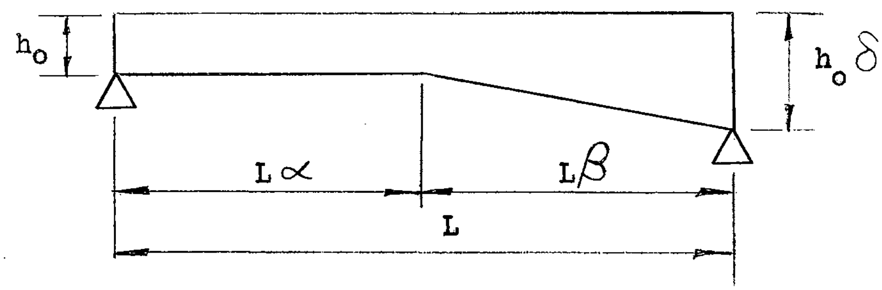


Fig. 6-1

Unsymmetrical Beam With Straight Haunch

2. Types of Tables.

From the I. B. M. 650 program in chapter IV approximately 7000 numerical values were calculated and recorded in three types of tables.

A. Constant Depth Beams (Table A-0). The coefficients for a prismatic beam of constant cross-section are recorded in Table A-0. Formulas for calculation of the angular functions are respectively

1. Angular Flexibilities

$$F_{AB} = F_{BA} = F = \frac{L}{3\ E\ Io}$$

2. Angular Carry-over Values.

$$G_{AB} = G_{BA} = G = \frac{L}{6\ E\ Io}$$

3. Angular Live-load Functions.

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EIo} \qquad \tau_{BA}^{(LL)} = t_2 \frac{L^2}{EIo}$$

t_1 = left end slope coefficient due to unit load at L_n.

t_2 = right end slope coefficient due to unit load at L_n.

Influence values of these coefficients for 100 positions of unit load are shown in the table.

4. Angular Dead Load Functions.

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = \frac{b\ h_o\ g\ L^3}{24\ E\ I_o}$$

b = width of the beam

h_o = constant depth of the beam

g = specific weight of the beam.

5. Angular Functions Due to Uniformly Distributed Load.

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = \frac{w\ L^3}{24\ E\ I_o}$$

w = intensity of the load.

B. Unsymmetrical Beams (Tables A-1, 2, • • • 9, 10).

The coefficients for a prismatic beam with one straight haunch are recorded in Tables A-1, 2, • • • 9, 10. The geometry of the beam is defined by the sketch and parameters:

L = length of the haunch

h_A = h_o = minimum depth

h_B = h_o = maximum depth.

Angular functions are respectively:

1. Angular Flexibilities.

$$F_{AB} = f_1 \frac{L}{EI_o} \qquad F_{BA} = f_2 \frac{L}{EI_o}$$

f_1 = left end angular flexibility coefficient

f_2 = right end angular flexibility coefficient.

2. Angular Carry-over Values.

$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

g = angular carry-over coefficient.

3. Angular Live-load Functions.

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o} \qquad \tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

t_1 = left end slope coefficient due to unit load at L_n.

t_2 = right end slope coefficient due to unit load at L_n.

Influence values of this coefficient for nine positions of unit load are shown in each table.

4. Angular Dead Load Functions.

$$\tau_{AB}^{(DL)} = t_3 \frac{b\ h_o\ g\ L^3}{E\ I_o} \qquad \tau_{BA}^{(DL)} = t_4 \frac{b\ h_o\ g\ L^3}{E\ I_o}$$

t_3 = left end slope coefficient due to dead load of the beam.

t_4 = right end slope coefficient due to dead load of the beam.

5. Angular Functions Due to Uniformly Distributed Load.

$$\tau_{AB}^{(UL)} = t_5 \frac{w\ L^3}{E\ I_o} \qquad \tau_{BA}^{(UL)} = t_6 \frac{w\ L^3}{E\ I_o}$$

t_5 = left end slope coefficient due to uniformly distributed load.

t_6 = right end slope coefficient due to uniformly distributed load.

C. Symmetrical Beams (Tables B-1, 2, 3, 4, 5). The coefficients for a prismatic beam with two symmetrical parabolic haunches are recorded in tables B-1, 2, 3, 4, 5. The geometry of the beam is defined by the sketch and parameters:

$L\beta$ = length of the haunch

$h_A = h_B = h_o\delta$ = maximum depth

$h_C = h_o$ = minimum depth.

Angular functions are respectively

1. Angular Flexibilities.

$$F_{AB} = F_{BA} = F = f \frac{L}{E\ I_o}$$

f = angular flexibility coefficient.

2. Angular Carry-over Values.

$$G_{AB} = G_{BA} = G = g \frac{L}{E\ I_o}$$

g = angular carry-over coefficient.

3. Angular Live-load Functions.

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E\ I_o} \quad \Big| \quad \tau_{BA}^{(LL)} = t_2 \frac{L^2}{E\ I_o}$$

t_1 = left end slope coefficient due to unit load at L_n.

t_2 = right end slope coefficient due to unit load at L_n.

From the symmetry of the beam

t_1 = due to unit load at L_n =

t_2 = due to unit load at $L(1-n)$.

Thus, from one set of coefficients, influence values for t_1 and t_2 are available as shown in each Table B. Influence of nine positions of unit load is recorded.

4. <u>Angular Dead-load Functions.</u>

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \frac{b\ h_o\ g\ L^3}{E\ I_o}$$

t_3 = end slope coefficient due to dead load.

5. <u>Angular Functions Due to Uniformly Distributed Load.</u>

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_5 \frac{w\ L^3}{E\ I_o}$$

t_5 = end slope coefficient due to uniformly distributed load.

3. Members ith Haunches of Varying Depths.

Very often in the design of frames with varying cross-section, the depth of the haunches of one particular member will vary (Fig. 6-2).

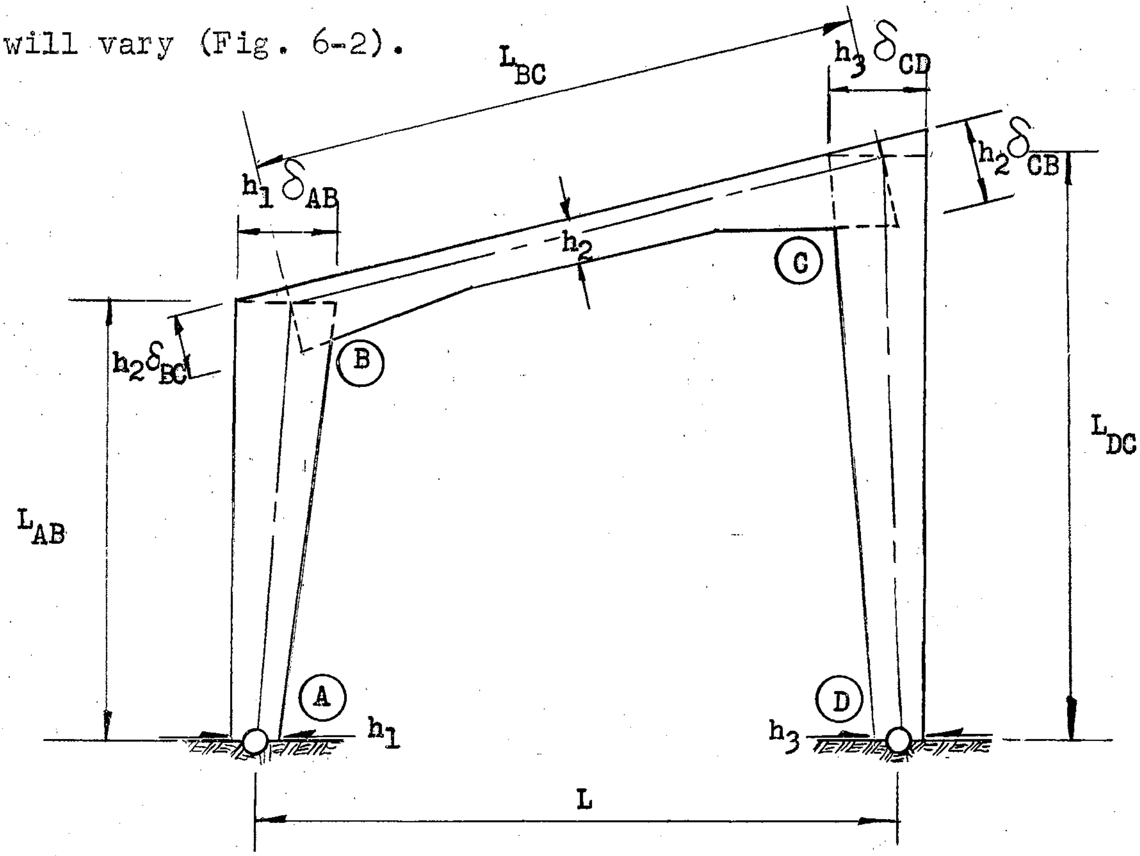


Fig. 6-2
Frame With Varying Haunch Depths

The length of the segments of the frame (Fig. 6-2) will be considered to be the distance between the intersection of their axes. The depth of the haunches are taken as the perpendicular distance, at the end of the segment, from the top of the member to the continuation of the haunch line. (Fig. 6-2).

With the convenience of the tables presented in this chapter the analysis of a frame with members having varying haunch depths is very simple (Fig. 6-3). Leontovich (22) and Guldan (23) have presented methods for superposition of elastic constants and by simple arithmetic obtain the required constants. The procedure is:

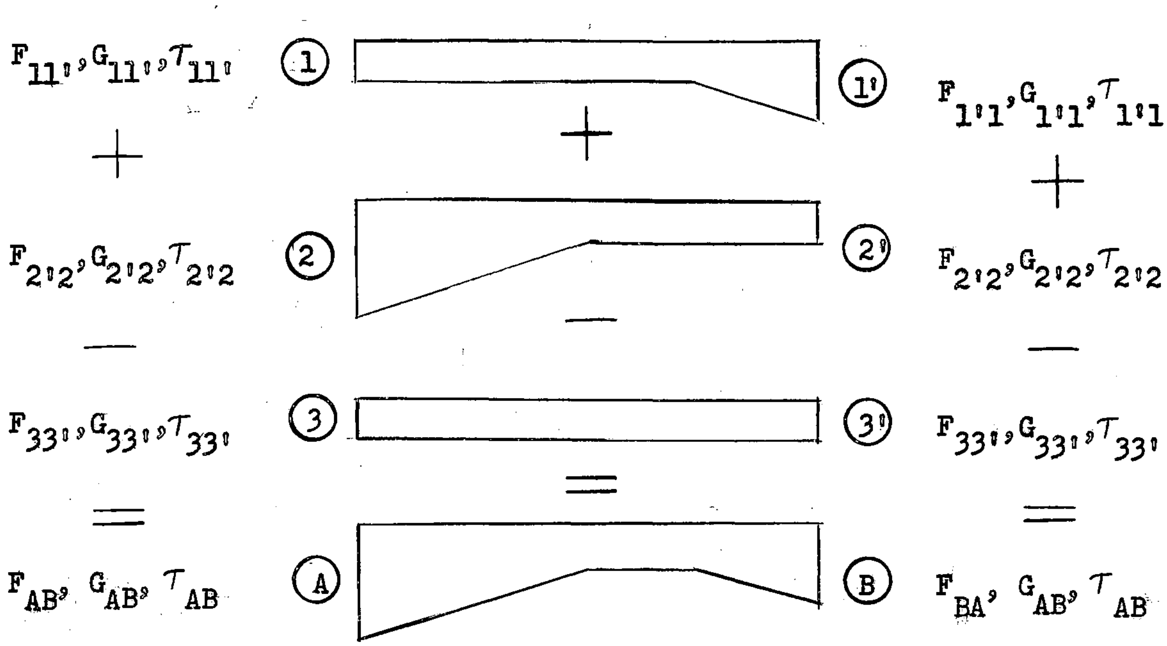
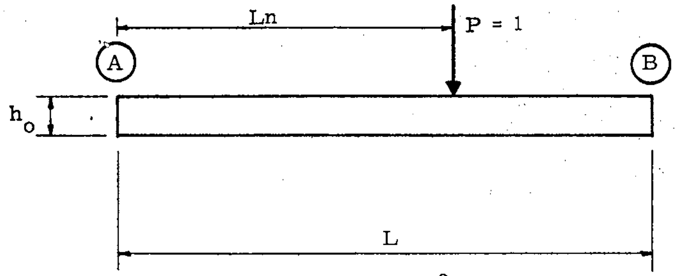


Fig. 6-3

Superposition of Angular Functions

TABLE A-0

$\beta = 0.0$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{AB} = F_{BA} = \frac{L}{3EI_o}$$

$$G_{AB} = G_{BA} = \frac{L}{6EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = \frac{bh_o qL^3}{24EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = \frac{wL^3}{24EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

Influence Coefficients t_1

n	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0033	.0065	.0096	.0125	.0154	.0182	.0209	.0236	.0261
0.1	.0285	.0309	.0331	.0353	.0373	.0393	.0412	.0430	.0448	.0464
0.2	.0480	.0495	.0509	.0522	.0535	.0547	.0558	.0568	.0578	.0587
0.3	.0595	.0603	.0609	.0615	.0621	.0626	.0630	.0633	.0636	.0638
0.4	.0640	.0641	.0642	.0641	.0641	.0639	.0638	.0635	.0632	.0629
0.5	.0625	.0621	.0616	.0610	.0604	.0598	.0591	.0584	.0577	.0569
0.6	.0560	.0551	.0542	.0532	.0522	.0512	.0501	.0490	.0479	.0467
0.7	.0455	.0443	.0430	.0417	.0404	.0391	.0377	.0363	.0349	.0335
0.8	.0320	.0305	.0290	.0275	.0260	.0244	.0229	.0213	.0197	.0181
0.9	.0165	.0149	.0133	.0116	.0100	.0083	.0067	.0050	.0033	.0017

Influence Coefficients t_2

n	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0017	.0033	.0050	.0067	.0083	.0100	.0116	.0133	.0149
0.1	.0165	.0181	.0197	.0213	.0229	.0244	.0260	.0275	.0290	.0305
0.2	.0320	.0335	.0349	.0363	.0377	.0391	.0404	.0417	.0430	.0443
0.3	.0455	.0467	.0479	.0490	.0501	.0512	.0522	.0532	.0542	.0551
0.4	.0560	.0569	.0577	.0584	.0591	.0598	.0604	.0610	.0616	.0621
0.5	.0625	.0629	.0632	.0635	.0638	.0639	.0641	.0641	.0642	.0641
0.6	.0640	.0638	.0636	.0633	.0630	.0626	.0621	.0615	.0609	.0603
0.7	.0595	.0587	.0578	.0568	.0558	.0547	.0535	.0522	.0509	.0495
0.8	.0480	.0464	.0448	.0430	.0412	.0393	.0373	.0353	.0331	.0309
0.9	.0285	.0261	.0236	.0209	.0182	.0154	.0125	.0096	.0065	.0033

TABLE A-I
$\beta = 0.1$

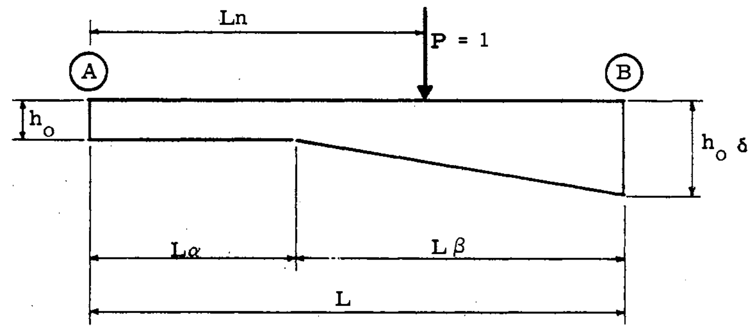
$$F_{AB} = f_1 \frac{L}{EI_o}$$

$$G_{AB} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$$

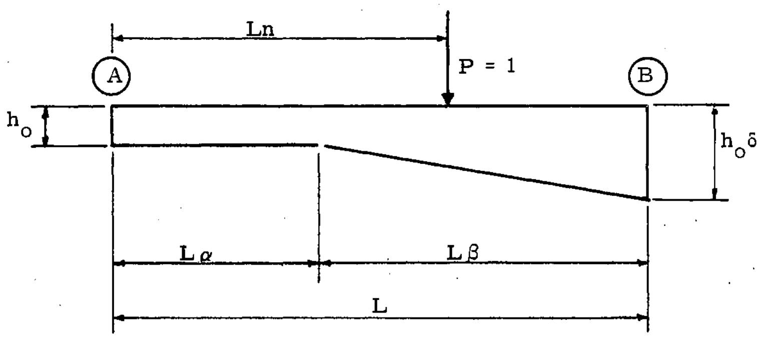


$$I_o = \frac{bh_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

δ \ n	Influence Coefficients t_1									f_1	g	t_3	t_5
	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1662	.0417	.0416
1.2	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0165	.3333	.1659	.0418	.0416
1.3	.0285	.0480	.0595	.0640	.0625	.0560	.0455	.0320	.0164	.3333	.1656	.0419	.0416
1.4	.0285	.0480	.0595	.0640	.0625	.0560	.0454	.0320	.0164	.3333	.1653	.0419	.0416
1.5	.0285	.0480	.0595	.0640	.0625	.0560	.0454	.0320	.0164	.3332	.1651	.0420	.0416
1.6	.0285	.0480	.0595	.0640	.0624	.0559	.0454	.0319	.0164	.3332	.1649	.0421	.0416
1.7	.0285	.0480	.0595	.0640	.0624	.0559	.0454	.0319	.0164	.3332	.1647	.0421	.0416
1.8	.0285	.0480	.0595	.0640	.0624	.0559	.0454	.0319	.0164	.3332	.1646	.0422	.0416
1.9	.0285	.0480	.0595	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1644	.0423	.0416
2.0	.0285	.0480	.0595	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1643	.0423	.0416
2.1	.0285	.0480	.0595	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1642	.0424	.0416
2.2	.0285	.0480	.0595	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1641	.0425	.0416
2.3	.0285	.0480	.0595	.0639	.0624	.0559	.0454	.0319	.0164	.3332	.1640	.0425	.0416
2.4	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0319	.0163	.3332	.1639	.0426	.0416
2.5	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0319	.0163	.3332	.1638	.0427	.0416
2.6	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0319	.0163	.3332	.1638	.0427	.0416
2.7	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0319	.0163	.3332	.1637	.0428	.0416
2.8	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0318	.0163	.3331	.1636	.0429	.0416
2.9	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0318	.0163	.3331	.1636	.0429	.0416
3.0	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0318	.0163	.3331	.1635	.0430	.0416

TABLE A-I
β = 0.1



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0165	.0319	.0454	.0558	.0623	.0637	.0592	.0477	.0281	.3210	.1662	.0415	.0415
1.2	.0164	.0318	.0453	.0557	.0621	.0635	.0589	.0473	.0278	.3113	.1659	.0414	.0413
1.3	.0164	.0318	.0452	.0556	.0620	.0633	.0587	.0471	.0275	.3036	.1656	.0413	.0411
1.4	.0164	.0317	.0451	.0555	.0618	.0632	.0586	.0470	.0273	.2973	.1653	.0412	.0410
1.5	.0163	.0317	.0450	.0554	.0617	.0631	.0584	.0467	.0271	.2921	.1651	.0412	.0409
1.6	.0163	.0316	.0450	.0553	.0616	.0629	.0583	.0466	.0269	.2878	.1649	.0411	.0408
1.7	.0163	.0316	.0449	.0552	.0615	.0628	.0581	.0464	.0267	.2841	.1647	.0411	.0407
1.8	.0163	.0316	.0449	.0552	.0615	.0627	.0580	.0463	.0266	.2809	.1646	.0410	.0407
1.9	.0163	.0316	.0448	.0551	.0614	.0627	.0580	.0462	.0265	.2781	.1644	.0410	.0406
2.0	.0163	.0315	.0448	.0551	.0613	.0626	.0578	.0461	.0264	.2757	.1643	.0410	.0406
2.1	.0163	.0315	.0448	.0550	.0613	.0625	.0578	.0460	.0263	.2736	.1642	.0410	.0405
2.2	.0162	.0315	.0447	.0550	.0612	.0625	.0577	.0459	.0262	.2717	.1641	.0410	.0405
2.3	.0162	.0315	.0447	.0549	.0612	.0624	.0576	.0459	.0261	.2700	.1640	.0410	.0404
2.4	.0162	.0315	.0447	.0549	.0611	.0624	.0575	.0458	.0260	.2685	.1639	.0410	.0404
2.5	.0162	.0314	.0447	.0549	.0611	.0623	.0575	.0457	.0260	.2672	.1638	.0410	.0403
2.6	.0162	.0314	.0446	.0548	.0611	.0623	.0575	.0457	.0259	.2659	.1638	.0410	.0403
2.7	.0162	.0314	.0446	.0548	.0610	.0622	.0574	.0456	.0258	.2648	.1637	.0410	.0403
2.8	.0162	.0314	.0446	.0548	.0610	.0622	.0574	.0456	.0258	.2638	.1636	.0410	.0402
2.9	.0162	.0314	.0446	.0548	.0610	.0621	.0573	.0455	.0257	.2629	.1636	.0410	.0402
3.0	.0162	.0314	.0446	.0547	.0609	.0621	.0573	.0455	.0257	.2620	.1635	.0411	.0402

TABLE A-2
β = 0.2

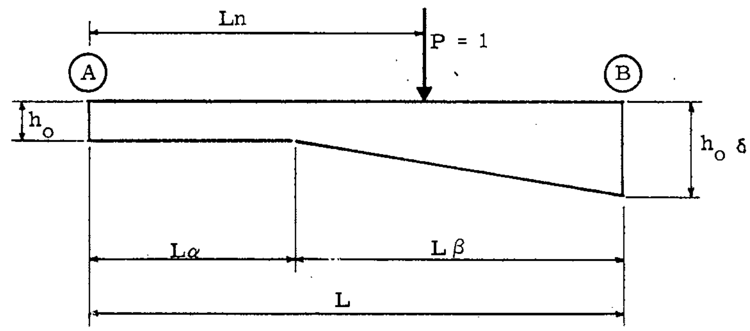
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$

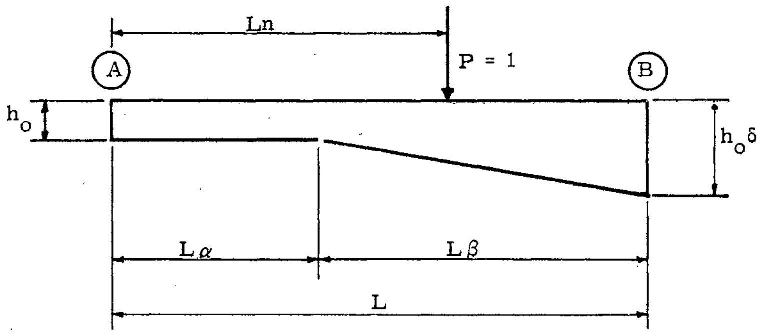


$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0285	.0480	.0594	.0639	.0624	.0559	.0454	.0319	.0164	.3331	.1650	.0419	.0416
1.2	.0285	.0480	.0594	.0639	.0623	.0558	.0453	.0317	.0163	.3330	.1637	.0421	.0415
1.3	.0285	.0479	.0594	.0638	.0623	.0557	.0452	.0316	.0162	.3328	.1625	.0423	.0415
1.4	.0284	.0479	.0593	.0638	.0622	.0556	.0451	.0315	.0161	.3327	.1615	.0425	.0414
1.5	.0284	.0479	.0593	.0637	.0621	.0556	.0450	.0314	.0160	.3326	.1607	.0427	.0414
1.6	.0284	.0478	.0592	.0637	.0621	.0556	.0450	.0313	.0160	.3325	.1600	.0429	.0413
1.7	.0284	.0478	.0592	.0636	.0620	.0555	.0449	.0313	.0159	.3324	.1593	.0431	.0413
1.8	.0284	.0478	.0592	.0636	.0620	.0554	.0448	.0312	.0158	.3324	.1587	.0433	.0412
1.9	.0284	.0478	.0592	.0636	.0620	.0554	.0448	.0312	.0158	.3323	.1582	.0435	.0412
2.0	.0284	.0478	.0592	.0636	.0619	.0553	.0447	.0311	.0157	.3322	.1578	.0437	.0412
2.1	.0284	.0478	.0591	.0635	.0619	.0553	.0447	.0311	.0157	.3322	.1574	.0439	.0412
2.2	.0284	.0478	.0591	.0635	.0619	.0553	.0446	.0310	.0157	.3321	.1570	.0441	.0411
2.3	.0284	.0477	.0591	.0635	.0619	.0552	.0446	.0310	.0156	.3320	.1567	.0443	.0411
2.4	.0284	.0477	.0591	.0635	.0618	.0552	.0446	.0309	.0156	.3320	.1563	.0445	.0411
2.5	.0284	.0477	.0591	.0634	.0618	.0552	.0445	.0309	.0156	.3319	.1561	.0447	.0411
2.6	.0284	.0477	.0591	.0634	.0618	.0551	.0445	.0309	.0156	.3319	.1558	.0449	.0410
2.7	.0284	.0477	.0591	.0634	.0618	.0551	.0445	.0308	.0155	.3319	.1555	.0451	.0410
2.8	.0283	.0477	.0590	.0634	.0617	.0551	.0444	.0308	.0155	.3318	.1553	.0453	.0410
2.9	.0283	.0477	.0590	.0634	.0617	.0551	.0444	.0308	.0155	.3318	.1551	.0455	.0410
3.0	.0283	.0477	.0590	.0634	.0617	.0551	.0444	.0307	.0155	.3318	.1549	.0457	.0410

TABLE A-2
β = 0.2



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0163	.0317	.0450	.0553	.0617	.0630	.0584	.0467	.0272	.3103	.1650	.0411	.0409
1.2	.0162	.0314	.0446	.0548	.0610	.0622	.0574	.0456	.0262	.2924	.1637	.0407	.0403
1.3	.0161	.0312	.0443	.0543	.0604	.0615	.0566	.0447	.0253	.2781	.1625	.0403	.0398
1.4	.0160	.0310	.0440	.0540	.0600	.0610	.0559	.0439	.0246	.2666	.1615	.0400	.0394
1.5	.0159	.0308	.0437	.0536	.0595	.0604	.0553	.0432	.0240	.2571	.1607	.0398	.0390
1.6	.0158	.0307	.0435	.0533	.0592	.0600	.0548	.0427	.0235	.2490	.1600	.0396	.0387
1.7	.0158	.0305	.0433	.0531	.0588	.0596	.0544	.0421	.0230	.2423	.1593	.0395	.0384
1.8	.0157	.0304	.0431	.0528	.0585	.0593	.0540	.0417	.0226	.2365	.1587	.0393	.0381
1.9	.0157	.0303	.0430	.0526	.0583	.0589	.0536	.0412	.0222	.2316	.1582	.0392	.0379
2.0	.0156	.0302	.0428	.0524	.0581	.0587	.0533	.0409	.0220	.2272	.1578	.0392	.0377
2.1	.0156	.0301	.0427	.0523	.0579	.0584	.0530	.0406	.0216	.2234	.1574	.0391	.0375
2.2	.0155	.0301	.0426	.0521	.0577	.0582	.0527	.0403	.0214	.2200	.1570	.0391	.0374
2.3	.0155	.0300	.0425	.0520	.0575	.0580	.0525	.0400	.0212	.2170	.1567	.0390	.0372
2.4	.0155	.0300	.0424	.0519	.0573	.0578	.0523	.0397	.0209	.2144	.1563	.0390	.0371
2.5	.0154	.0299	.0423	.0518	.0572	.0576	.0521	.0395	.0208	.2119	.1561	.0390	.0370
2.6	.0154	.0298	.0422	.0516	.0571	.0575	.0519	.0393	.0206	.2098	.1558	.0390	.0368
2.7	.0154	.0298	.0422	.0515	.0569	.0573	.0517	.0391	.0204	.2078	.1555	.0390	.0367
2.8	.0154	.0297	.0421	.0515	.0568	.0572	.0516	.0389	.0203	.2060	.1553	.0391	.0366
2.9	.0153	.0297	.0420	.0514	.0567	.0571	.0514	.0387	.0201	.2044	.1551	.0391	.0365
3.0	.0153	.0296	.0420	.0513	.0566	.0569	.0513	.0386	.0200	.2029	.1549	.0391	.0365

TABLE A-3
β = 0.3

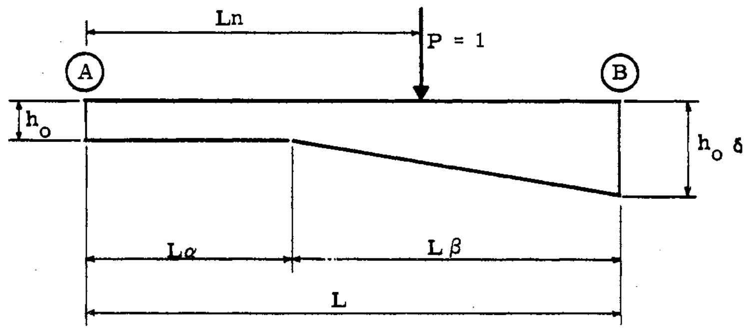
$$F_{AB} = f_1 \frac{L}{EI_o}$$

$$G_{AB} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$$

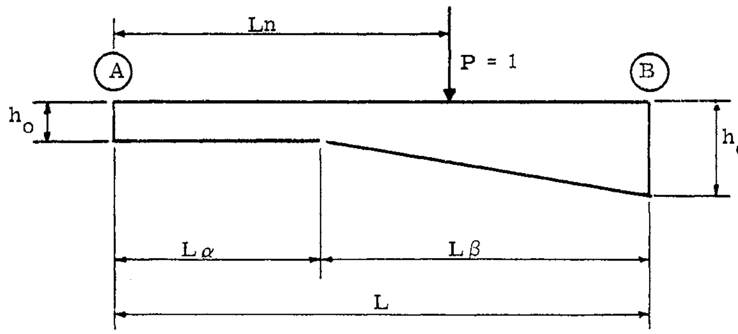


$$I_o = \frac{bh_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0284	.0479	.0593	.0637	.0622	.0556	.0451	.0315	.0162	.3327	.1632	.0420	.0414
1.2	.0284	.0478	.0592	.0635	.0620	.0553	.0447	.0311	.0160	.3322	.1603	.0424	.0412
1.3	.0283	.0477	.0590	.0633	.0617	.0550	.0443	.0308	.0157	.3317	.1579	.0427	.0410
1.4	.0283	.0476	.0589	.0632	.0615	.0548	.0441	.0305	.0155	.3313	.1559	.0430	.0408
1.5	.0283	.0475	.0588	.0630	.0613	.0545	.0438	.0302	.0153	.3309	.1541	.0433	.0407
1.6	.0282	.0475	.0587	.0629	.0611	.0544	.0436	.0300	.0152	.3306	.1525	.0437	.0406
1.7	.0282	.0474	.0586	.0628	.0610	.0542	.0434	.0298	.0151	.3303	.1512	.0440	.0404
1.8	.0282	.0473	.0585	.0627	.0608	.0540	.0432	.0296	.0150	.3300	.1500	.0443	.0403
1.9	.0281	.0473	.0584	.0626	.0607	.0539	.0430	.0294	.0149	.3298	.1489	.0447	.0402
2.0	.0281	.0472	.0584	.0625	.0606	.0537	.0429	.0293	.0148	.3295	.1480	.0450	.0401
2.1	.0281	.0472	.0583	.0624	.0605	.0536	.0427	.0291	.0147	.3293	.1471	.0453	.0401
2.2	.0281	.0472	.0582	.0623	.0604	.0535	.0426	.0290	.0146	.3291	.1463	.0456	.0400
2.3	.0281	.0471	.0582	.0623	.0603	.0534	.0425	.0289	.0145	.3290	.1456	.0460	.0399
2.4	.0280	.0471	.0581	.0622	.0602	.0533	.0423	.0288	.0145	.3288	.1450	.0463	.0398
2.5	.0280	.0471	.0581	.0621	.0602	.0532	.0422	.0287	.0144	.3287	.1443	.0466	.0398
2.6	.0280	.0470	.0581	.0621	.0601	.0531	.0421	.0286	.0144	.3285	.1438	.0470	.0397
2.7	.0280	.0470	.0580	.0620	.0600	.0530	.0420	.0285	.0143	.3284	.1433	.0473	.0397
2.8	.0280	.0470	.0580	.0620	.0600	.0530	.0420	.0284	.0142	.3283	.1428	.0476	.0396
2.9	.0280	.0470	.0580	.0620	.0600	.0529	.0419	.0283	.0142	.3282	.1424	.0480	.0396
3.0	.0280	.0470	.0580	.0619	.0599	.0528	.0418	.0283	.0142	.3280	.1420	.0483	.0396

TABLE A-3
β = 0.3



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0162	.0313	.0445	.0546	.0608	.0620	.0571	.0453	.0264	.3012	.1632	.0406	.0402
1.2	.0159	.0307	.0436	.0535	.0593	.0602	.0551	.0432	.0247	.2763	.1603	.0397	.0390
1.3	.0156	.0303	.0429	.0525	.0581	.0588	.0534	.0413	.0233	.2566	.1579	.0389	.0380
1.4	.0154	.0298	.0423	.0517	.0570	.0575	.0519	.0398	.0221	.2407	.1559	.0384	.0371
1.5	.0152	.0295	.0417	.0510	.0562	.0565	.0507	.0385	.0212	.2276	.1541	.0379	.0364
1.6	.0151	.0292	.0413	.0503	.0554	.0555	.0496	.0373	.0203	.2167	.1525	.0375	.0357
1.7	.0150	.0289	.0409	.0498	.0548	.0547	.0487	.0363	.0196	.2075	.1512	.0371	.0352
1.8	.0148	.0287	.0405	.0493	.0542	.0540	.0478	.0354	.0190	.1996	.1500	.0369	.0347
1.9	.0147	.0284	.0402	.0489	.0536	.0533	.0471	.0347	.0185	.1929	.1489	.0366	.0342
2.0	.0146	.0283	.0399	.0485	.0531	.0528	.0464	.0340	.0180	.1870	.1480	.0364	.0338
2.1	.0145	.0281	.0396	.0482	.0527	.0523	.0458	.0333	.0176	.1819	.1471	.0363	.0335
2.2	.0145	.0279	.0394	.0479	.0523	.0518	.0452	.0328	.0172	.1774	.1463	.0362	.0332
2.3	.0144	.0278	.0392	.0476	.0520	.0514	.0447	.0323	.0169	.1734	.1456	.0361	.0329
2.4	.0143	.0277	.0390	.0473	.0516	.0510	.0443	.0318	.0166	.1700	.1450	.0360	.0326
2.5	.0143	.0275	.0388	.0471	.0513	.0506	.0439	.0314	.0163	.1667	.1443	.0360	.0324
2.6	.0142	.0274	.0386	.0468	.0511	.0503	.0435	.0310	.0160	.1638	.1438	.0359	.0322
2.7	.0142	.0273	.0385	.0466	.0508	.0500	.0431	.0306	.0158	.1612	.1433	.0359	.0319
2.8	.0141	.0272	.0383	.0465	.0506	.0497	.0428	.0303	.0156	.1588	.1428	.0359	.0318
2.9	.0141	.0271	.0382	.0463	.0504	.0494	.0425	.0300	.0154	.1567	.1424	.0359	.0316
3.0	.0140	.0271	.0381	.0461	.0501	.0492	.0422	.0297	.0152	.1547	.1420	.0359	.0314

TABLE A-4
β= 0.4

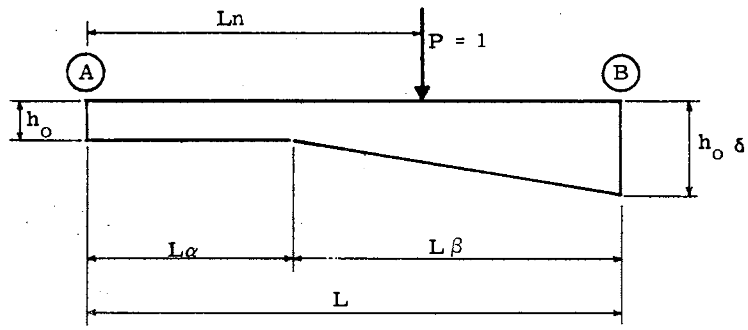
$$F_{AB} = f_1 \frac{L}{E\,I_o}$$

$$G_{AB} = g \frac{L}{E\,I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E\,I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E\,I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E\,I_o}$$

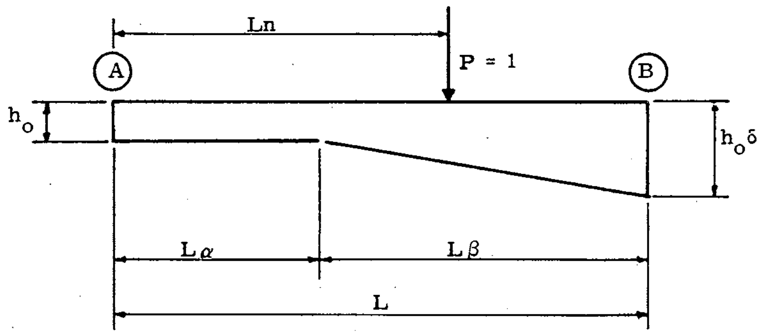


$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0284	.0477	.0591	.0634	.0618	.0551	.0445	.0311	.0160	.3319	.1609	.0421	.0411
1.2	.0282	.0474	.0587	.0629	.0611	.0543	.0437	.0303	.0155	.3306	.1561	.0425	.0406
1.3	.0281	.0472	.0583	.0624	.0606	.0537	.0429	.0297	.0151	.3295	.1521	.0429	.0402
1.4	.0280	.0470	.0580	.0621	.0601	.0531	.0423	.0291	.0148	.3285	.1487	.0433	.0400
1.5	.0279	.0469	.0578	.0617	.0596	.0526	.0417	.0286	.0145	.3276	.1457	.0437	.0395
1.6	.0278	.0467	.0575	.0614	.0592	.0521	.0412	.0282	.0143	.3268	.1432	.0441	.0392
1.7	.0278	.0466	.0573	.0611	.0589	.0517	.0407	.0278	.0141	.3261	.1409	.0444	.0387
1.8	.0277	.0464	.0571	.0609	.0586	.0512	.0403	.0275	.0139	.3255	.1390	.0448	.0387
1.9	.0277	.0463	.0570	.0606	.0583	.0509	.0399	.0272	.0137	.3249	.1372	.0452	.0385
2.0	.0276	.0462	.0568	.0604	.0580	.0506	.0396	.0269	.0135	.3244	.1356	.0456	.0383
2.1	.0276	.0461	.0567	.0602	.0578	.0503	.0393	.0266	.0134	.3239	.1342	.0460	.0382
2.2	.0275	.0460	.0565	.0600	.0575	.0501	.0390	.0264	.0133	.3234	.1329	.0464	.0380
2.3	.0275	.0459	.0564	.0599	.0573	.0498	.0387	.0262	.0132	.3230	.1318	.0468	.0379
2.4	.0274	.0459	.0563	.0597	.0571	.0496	.0384	.0260	.0131	.3226	.1307	.0472	.0377
2.5	.0274	.0458	.0562	.0596	.0570	.0494	.0382	.0258	.0130	.3223	.1297	.0476	.0376
2.6	.0274	.0457	.0561	.0594	.0568	.0492	.0380	.0256	.0129	.3219	.1288	.0480	.0375
2.7	.0273	.0457	.0560	.0593	.0566	.0490	.0378	.0255	.0128	.3216	.1280	.0484	.0374
2.8	.0273	.0456	.0559	.0592	.0565	.0488	.0376	.0253	.0127	.3213	.1272	.0488	.0373
2.9	.0273	.0455	.0558	.0591	.0564	.0486	.0375	.0252	.0126	.3210	.1265	.0492	.0372
3.0	.0272	.0455	.0557	.0590	.0562	.0485	.0373	.0251	.0126	.3208	.1259	.0497	.0371

TABLE A-4
β = 0.4



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0159	.0308	.0438	.0537	.0597	.0605	.0555	.0440	.0256	.2935	.1609	.0399	.0393
1.2	.0154	.0299	.0423	.0518	.0572	.0577	.0523	.0408	.0234	.2628	.1561	.0385	.0374
1.3	.0150	.0291	.0411	.0502	.0552	.0552	.0495	.0382	.0215	.2386	.1521	.0374	.0358
1.4	.0147	.0284	.0401	.0488	.0535	.0532	.0472	.0360	.0200	.2191	.1487	.0364	.0345
1.5	.0144	.0278	.0392	.0476	.0520	.0514	.0452	.0340	.0188	.2032	.1457	.0356	.0333
1.6	.0142	.0273	.0385	.0466	.0508	.0499	.0435	.0324	.0177	.1899	.1432	.0349	.0324
1.7	.0139	.0269	.0378	.0457	.0496	.0486	.0420	.0310	.0168	.1788	.1409	.0344	.0315
1.8	.0137	.0265	.0372	.0449	.0487	.0474	.0407	.0298	.0160	.1694	.1390	.0339	.0307
1.9	.0136	.0261	.0367	.0442	.0478	.0463	.0395	.0287	.0153	.1613	.1372	.0335	.0301
2.0	.0134	.0258	.0362	.0436	.0470	.0454	.0384	.0278	.0147	.1543	.1356	.0332	.0295
2.1	.0133	.0255	.0358	.0430	.0463	.0445	.0375	.0270	.0142	.1482	.1342	.0329	.0289
2.2	.0131	.0253	.0354	.0425	.0456	.0438	.0366	.0262	.0138	.1429	.1329	.0326	.0285
2.3	.0130	.0250	.0350	.0420	.0451	.0431	.0359	.0255	.0134	.1382	.1317	.0324	.0280
2.4	.0129	.0248	.0347	.0416	.0445	.0424	.0352	.0249	.0130	.1340	.1307	.0322	.0276
2.5	.0128	.0246	.0344	.0412	.0440	.0418	.0345	.0244	.0127	.1302	.1297	.0321	.0273
2.6	.0127	.0244	.0342	.0409	.0436	.0413	.0339	.0239	.0124	.1269	.1288	.0320	.0269
2.7	.0126	.0243	.0339	.0405	.0432	.0408	.0334	.0234	.0121	.1239	.1280	.0319	.0266
2.8	.0126	.0241	.0337	.0402	.0428	.0403	.0329	.0230	.0119	.1211	.1272	.0318	.0264
2.9	.0125	.0240	.0335	.0399	.0424	.0399	.0325	.0226	.0116	.1186	.1265	.0318	.0261
3.0	.0124	.0238	.0333	.0397	.0421	.0395	.0320	.0222	.0114	.1163	.1258	.0318	.0259

TABLE A-5
β = 0.5

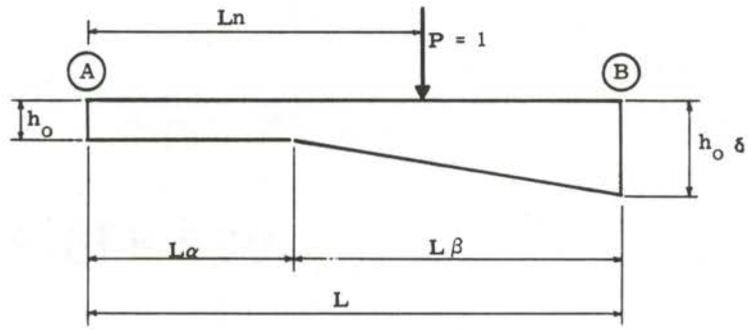
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$

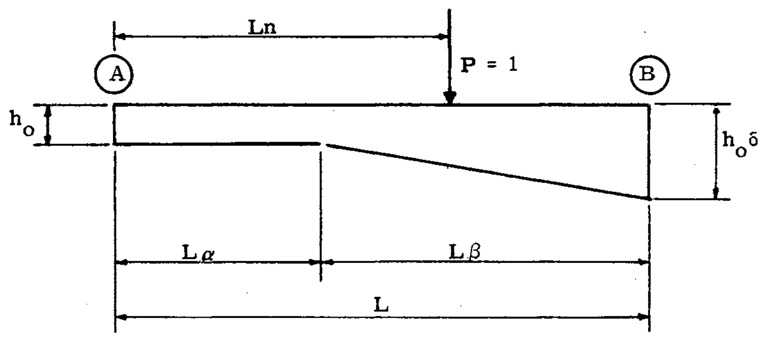


$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0282	.0474	.0586	.0628	.0611	.0543	.0438	.0306	.0157	.3304	.1582	.0421	.0407
1.2	.0280	.0469	.0579	.0618	.0598	.0529	.0423	.0294	.0150	.3279	.1512	.0425	.0398
1.3	.0277	.0465	.0572	.0610	.0587	.0516	.0411	.0284	.0145	.3258	.1454	.0428	.0391
1.4	.0276	.0461	.0567	.0602	.0578	.0505	.0400	.0275	.0140	.3238	.1404	.0432	.0382
1.5	.0274	.0458	.0561	.0595	.0569	.0495	.0390	.0268	.0136	.3221	.1362	.0435	.0378
1.6	.0272	.0455	.0557	.0589	.0561	.0486	.0382	.0261	.0132	.3206	.1325	.0438	.0373
1.7	.0271	.0452	.0553	.0584	.0554	.0478	.0374	.0255	.0129	.3192	.1293	.0442	.0369
1.8	.0270	.0449	.0549	.0579	.0548	.0471	.0367	.0250	.0126	.3180	.1265	.0445	.0365
1.9	.0269	.0447	.0546	.0574	.0543	.0465	.0361	.0245	.0124	.3168	.1239	.0449	.0361
2.0	.0267	.0445	.0542	.0570	.0537	.0459	.0356	.0241	.0121	.3158	.1217	.0452	.0358
2.1	.0267	.0443	.0540	.0566	.0533	.0453	.0351	.0237	.0119	.3149	.1197	.0458	.0354
2.2	.0266	.0441	.0537	.0563	.0528	.0448	.0346	.0234	.0118	.3140	.1178	.0459	.0352
2.3	.0265	.0440	.0535	.0559	.0524	.0444	.0342	.0231	.0116	.3132	.1162	.0463	.0349
2.4	.0264	.0438	.0532	.0556	.0520	.0439	.0338	.0228	.0115	.3124	.1147	.0467	.0347
2.5	.0263	.0437	.0530	.0554	.0517	.0436	.0334	.0225	.0113	.3117	.1133	.0471	.0344
2.6	.0263	.0435	.0528	.0551	.0514	.0432	.0331	.0223	.0112	.3111	.1120	.0475	.0342
2.7	.0262	.0434	.0526	.0548	.0511	.0428	.0328	.0221	.0111	.3104	.1108	.0479	.0340
2.8	.0262	.0433	.0525	.0546	.0508	.0425	.0325	.0219	.0110	.3099	.1098	.0483	.0339
2.9	.0261	.0432	.0523	.0544	.0505	.0422	.0322	.0217	.0109	.3093	.1088	.0487	.0337
3.0	.0260	.0431	.0521	.0542	.0502	.0419	.0320	.0215	.0108	.3088	.1078	.0491	.0335

TABLE A-5
β = 0.5



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0157	.0303	.0430	.0526	.0583	.0590	.0540	.0429	.0250	.2870	.1582	.0393	.0384
1.2	.0150	.0289	.0409	.0498	.0548	.0548	.0496	.0388	.0223	.2516	.1512	.0373	.0358
1.3	.0144	.0277	.0391	.0475	.0519	.0514	.0459	.0354	.0201	.2237	.1454	.0357	.0336
1.4	.0139	.0268	.0376	.0455	.0494	.0484	.0428	.0326	.0183	.2014	.1405	.0344	.0318
1.5	.0135	.0259	.0364	.0438	.0473	.0460	.0402	.0303	.0168	.1832	.1362	.0332	.0307
1.6	.0131	.0252	.0353	.0423	.0454	.0438	.0379	.0283	.0155	.1683	.1325	.0323	.0289
1.7	.0128	.0245	.0343	.0411	.0438	.0419	.0359	.0266	.0145	.1557	.1293	.0314	.0278
1.8	.0125	.0240	.0334	.0399	.0424	.0403	.0342	.0252	.0136	.1451	.1265	.0307	.0268
1.9	.0122	.0235	.0327	.0389	.0411	.0388	.0327	.0239	.0128	.1361	.1239	.0301	.0259
2.0	.0120	.0230	.0320	.0380	.0400	.0375	.0314	.0228	.0122	.1283	.1217	.0296	.0251
2.1	.0118	.0226	.0314	.0372	.0390	.0363	.0302	.0218	.0116	.1215	.1197	.0292	.0244
2.2	.0116	.0222	.0309	.0365	.0381	.0352	.0292	.0209	.0111	.1156	.1178	.0288	.0238
2.3	.0115	.0219	.0304	.0358	.0373	.0343	.0282	.0201	.0106	.1104	.1162	.0284	.0232
2.4	.0113	.0216	.0299	.0352	.0365	.0334	.0273	.0194	.0102	.1058	.1147	.0281	.0227
2.5	.0112	.0213	.0295	.0346	.0358	.0326	.0266	.0188	.0098	.1017	.1133	.0279	.0222
2.6	.0110	.0211	.0291	.0341	.0352	.0319	.0259	.0182	.0095	.0980	.1120	.0276	.0218
2.7	.0109	.0208	.0288	.0337	.0346	.0312	.0252	.0177	.0092	.0947	.1108	.0274	.0214
2.8	.0108	.0206	.0284	.0332	.0340	.0306	.0246	.0172	.0089	.0918	.1098	.0273	.0210
2.9	.0107	.0204	.0281	.0328	.0335	.0300	.0241	.0168	.0087	.0891	.1088	.0271	.0207
3.0	.0106	.0202	.0279	.0325	.0331	.0295	.0236	.0164	.0085	.0866	.1078	.0270	.0204

TABLE A-6
β = 0.6

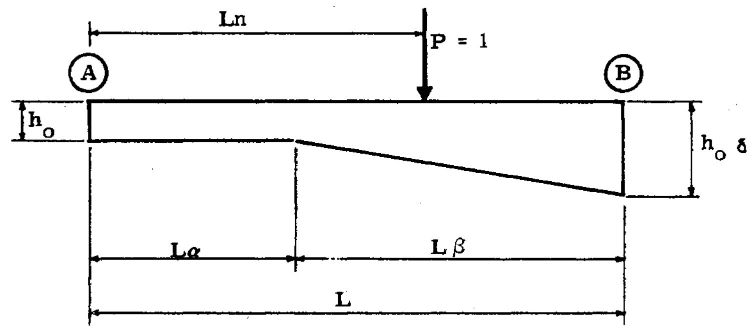
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$

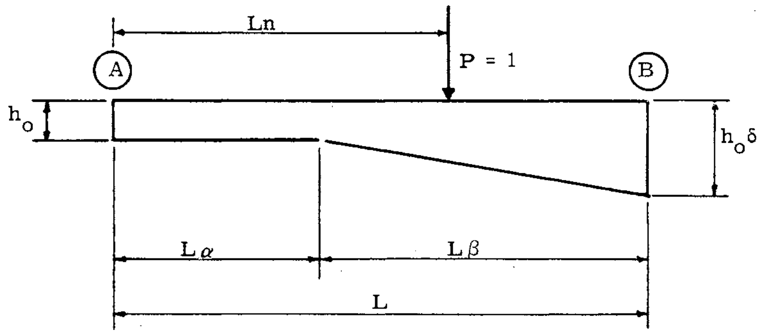


$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0280	.0470	.0580	.0620	.0600	.0533	.0430	.0300	.0154	.3283	.1553	.0420	.0401
1.2	.0276	.0461	.0567	.0603	.0579	.0510	.0408	.0284	.0145	.3240	.1460	.0422	.0387
1.3	.0272	.0454	.0556	.0588	.0561	.0491	.0390	.0270	.0137	.3202	.1382	.0424	.0376
1.4	.0269	.0448	.0546	.0574	.0545	.0473	.0374	.0258	.0131	.3169	.1316	.0426	.0365
1.5	.0266	.0441	.0537	.0563	.0530	.0458	.0361	.0247	.0125	.3140	.1260	.0428	.0357
1.6	.0263	.0436	.0529	.0552	.0518	.0445	.0349	.0238	.0121	.3113	.1212	.0429	.0349
1.7	.0261	.0431	.0522	.0543	.0506	.0433	.0338	.0230	.0117	.3090	.1169	.0431	.0342
1.8	.0258	.0427	.0515	.0534	.0500	.0422	.0328	.0223	.0113	.3068	.1132	.0433	.0335
1.9	.0257	.0423	.0510	.0526	.0486	.0412	.0320	.0217	.0110	.3048	.1099	.0435	.0329
2.0	.0255	.0419	.0504	.0519	.0478	.0404	.0312	.0212	.0107	.3031	.1069	.0437	.0324
2.1	.0253	.0416	.0499	.0512	.0470	.0396	.0305	.0207	.0104	.3014	.1043	.0439	.0320
2.2	.0252	.0413	.0495	.0506	.0462	.0388	.0299	.0202	.0102	.3000	.1019	.0441	.0315
2.3	.0250	.0410	.0490	.0501	.0456	.0382	.0293	.0198	.0100	.2985	.0998	.0444	.0311
2.4	.0249	.0408	.0487	.0495	.0449	.0375	.0288	.0194	.0098	.2972	.0978	.0446	.0308
2.5	.0248	.0405	.0483	.0491	.0444	.0370	.0283	.0191	.0096	.2960	.0960	.0449	.0304
2.6	.0247	.0403	.0480	.0486	.0438	.0364	.0279	.0188	.0094	.2948	.0944	.0451	.0301
2.7	.0245	.0401	.0476	.0482	.0433	.0360	.0275	.0185	.0093	.2938	.0929	.0454	.0298
2.8	.0244	.0399	.0473	.0478	.0429	.0355	.0271	.0182	.0091	.2928	.0915	.0457	.0295
2.9	.0244	.0397	.0471	.0474	.0424	.0351	.0267	.0180	.0090	.2919	.0902	.0460	.0293
3.0	.0243	.0395	.0468	.0471	.0420	.0347	.0264	.0177	.0089	.2910	.0890	.0463	.0291

TABLE A-6
$\beta = 0.6$



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0154	.0297	.0421	.0515	.0569	.0575	.0527	.0419	.0245	.2817	.1553	.0386	.0376
1.2	.0144	.0279	.0393	.0477	.0522	.0522	.0473	.0371	.0214	.2423	.1460	.0362	.0343
1.3	.0137	.0263	.0370	.0446	.0484	.0478	.0428	.0331	.0189	.2116	.1382	.0341	.0315
1.4	.0130	.0250	.0350	.0420	.0451	.0442	.0391	.0299	.0169	.1871	.1316	.0324	.0293
1.5	.0124	.0239	.0333	.0397	.0424	.0410	.0359	.0273	.0152	.1673	.1260	.0309	.0274
1.6	.0119	.0229	.0318	.0378	.0400	.0384	.0333	.0250	.0138	.1510	.1212	.0297	.0257
1.7	.0115	.0221	.0306	.0361	.0379	.0360	.0310	.0231	.0127	.1375	.1169	.0286	.0243
1.8	.0112	.0213	.0295	.0346	.0360	.0340	.0290	.0215	.0117	.1261	.1132	.0277	.0231
1.9	.0108	.0206	.0285	.0333	.0344	.0322	.0273	.0201	.0108	.1164	.1099	.0269	.0220
2.0	.0105	.0201	.0276	.0321	.0330	.0306	.0258	.0188	.0101	.1081	.1069	.0261	.0210
2.1	.0103	.0195	.0268	.0311	.0317	.0292	.0244	.0177	.0095	.1009	.1043	.0255	.0202
2.2	.0100	.0191	.0261	.0301	.0305	.0280	.0232	.0168	.0090	.0946	.1019	.0249	.0194
2.3	.0098	.0186	.0254	.0292	.0295	.0268	.0222	.0159	.0085	.0891	.0998	.0245	.0188
2.4	.0096	.0182	.0248	.0285	.0285	.0258	.0212	.0152	.0080	.0843	.0978	.0240	.0181
2.5	.0094	.0179	.0243	.0277	.0276	.0249	.0204	.0145	.0077	.0800	.0960	.0236	.0176
2.6	.0093	.0175	.0238	.0271	.0268	.0241	.0196	.0139	.0073	.0761	.0944	.0233	.0171
2.7	.0091	.0172	.0234	.0265	.0261	.0233	.0189	.0134	.0070	.0727	.0929	.0230	.0166
2.8	.0090	.0170	.0230	.0259	.0254	.0226	.0182	.0129	.0067	.0696	.0915	.0227	.0162
2.9	.0089	.0167	.0226	.0254	.0248	.0219	.0176	.0124	.0065	.0669	.0902	.0224	.0158
3.0	.0087	.0165	.0222	.0249	.0242	.0213	.0171	.0120	.0062	.0643	.0890	.0222	.0154

TABLE A-7
β = 0.7

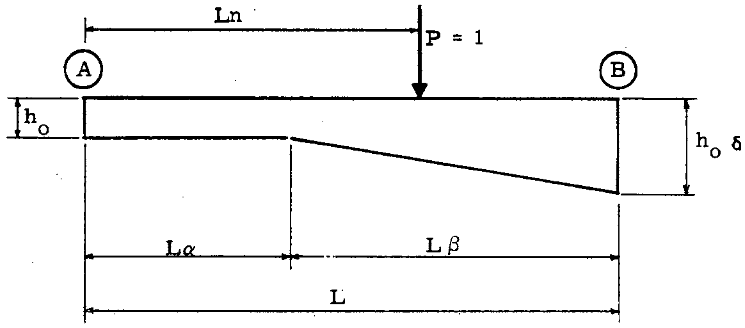
$F_{AB} = f_1 \frac{L}{EI_o}$

$G_{AB} = g \frac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$

$\tau_{AB}^{(DL)} = t_3 \frac{bh_o q L^3}{EI_o}$

$\tau_{AB}^{(UL)} = t_5 \frac{wL^3}{EI_o}$

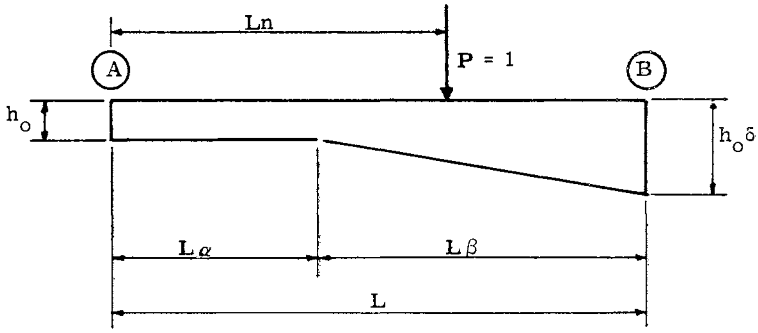


$I_o = \frac{bh_o^3}{12}$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0277	.0464	.0571	.0609	.0589	.0523	.0421	.0294	.0151	.3254	.1523	.0418	.0394
1.2	.0270	.0450	.0551	.0582	.0557	.0491	.0393	.0273	.0140	.3185	.1406	.0417	.0374
1.3	.0264	.0438	.0533	.0558	.0531	.0464	.0369	.0255	.0130	.3126	.1309	.0417	.0358
1.4	.0259	.0428	.0517	.0538	.0507	.0440	.0348	.0240	.0122	.3073	.1227	.0416	.0343
1.5	.0254	.0419	.0503	.0519	.0486	.0420	.0330	.0227	.0115	.3026	.1157	.0415	.0331
1.6	.0250	.0410	.0490	.0503	.0468	.0402	.0315	.0216	.0109	.2984	.1097	.0414	.0320
1.7	.0246	.0403	.0479	.0488	.0452	.0386	.0301	.0206	.0104	.2946	.1045	.0413	.0310
1.8	.0243	.0396	.0469	.0475	.0437	.0371	.0289	.0197	.0100	.2912	.0999	.0412	.0301
1.9	.0240	.0390	.0460	.0463	.0424	.0359	.0278	.0189	.0096	.2881	.0959	.0411	.0293
2.0	.0237	.0384	.0451	.0452	.0412	.0347	.0269	.0182	.0092	.2852	.0923	.0411	.0286
2.1	.0234	.0379	.0443	.0442	.0400	.0337	.0260	.0176	.0089	.2826	.0890	.0410	.0280
2.2	.0232	.0374	.0436	.0432	.0390	.0327	.0252	.0171	.0086	.2802	.0861	.0410	.0273
2.3	.0230	.0369	.0429	.0424	.0381	.0319	.0245	.0166	.0083	.2780	.0835	.0410	.0268
2.4	.0228	.0365	.0423	.0416	.0373	.0311	.0239	.0161	.0081	.2759	.0811	.0410	.0262
2.5	.0226	.0361	.0417	.0408	.0365	.0303	.0232	.0157	.0079	.2740	.0790	.0411	.0258
2.6	.0224	.0358	.0412	.0402	.0358	.0297	.0228	.0153	.0077	.2722	.0770	.0411	.0253
2.7	.0222	.0354	.0407	.0395	.0351	.0290	.0222	.0149	.0075	.2705	.0752	.0412	.0250
2.8	.0221	.0351	.0402	.0389	.0344	.0285	.0217	.0146	.0073	.2690	.0735	.0413	.0246
2.9	.0219	.0348	.0397	.0384	.0339	.0279	.0213	.0143	.0072	.2675	.0720	.0414	.0242
3.0	.0218	.0346	.0393	.0378	.0333	.0274	.0209	.0140	.0070	.2661	.0706	.0415	.0239

TABLE A-7
β= 0.7



$$I_o = \frac{b h_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{E I_o}$$

$$G_{BA} = g \frac{L}{E I_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{E I_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{w L^3}{E I_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0151	.0291	.0412	.0503	.0555	.0562	.0516	.0411	.0241	.2774	.1523	.0381	.0368
1.2	.0139	.0268	.0377	.0456	.0499	.0500	.0453	.0357	.0206	.2349	.1406	.0351	.0329
1.3	.0129	.0248	.0348	.0418	.0452	.0448	.0403	.0313	.0179	.2020	.1309	.0327	.0297
1.4	.0121	.0232	.0323	.0385	.0413	.0406	.0361	.0278	.0158	.1759	.1227	.0306	.0270
1.5	.0114	.0218	.0302	.0357	.0380	.0370	.0326	.0249	.0140	.1549	.1157	.0289	.0248
1.6	.0108	.0206	.0284	.0334	.0352	.0339	.0296	.0224	.0125	.1376	.1097	.0274	.0229
1.7	.0103	.0196	.0268	.0313	.0327	.0313	.0271	.0204	.0113	.1234	.1045	.0261	.0213
1.8	.0098	.0186	.0255	.0295	.0306	.0290	.0250	.0186	.0103	.1115	.0999	.0249	.0199
1.9	.0094	.0178	.0243	.0279	.0287	.0270	.0231	.0171	.0094	.1014	.0959	.0239	.0186
2.0	.0091	.0171	.0232	.0265	.0270	.0253	.0215	.0158	.0086	.0927	.0923	.0230	.0176
2.1	.0087	.0165	.0222	.0252	.0256	.0238	.0200	.0147	.0080	.0853	.0890	.0222	.0166
2.2	.0084	.0159	.0213	.0240	.0242	.0224	.0188	.0137	.0074	.0789	.0861	.0215	.0158
2.3	.0082	.0154	.0206	.0230	.0231	.0212	.0177	.0128	.0069	.0733	.0835	.0209	.0150
2.4	.0079	.0149	.0198	.0221	.0220	.0201	.0167	.0121	.0065	.0684	.0811	.0203	.0143
2.5	.0077	.0145	.0192	.0212	.0210	.0191	.0158	.0114	.0061	.0640	.0790	.0198	.0137
2.6	.0075	.0141	.0186	.0205	.0201	.0182	.0150	.0107	.0057	.0601	.0770	.0193	.0132
2.7	.0074	.0137	.0181	.0198	.0193	.0174	.0142	.0102	.0054	.0567	.0752	.0189	.0127
2.8	.0072	.0134	.0176	.0191	.0186	.0166	.0136	.0097	.0051	.0536	.0735	.0185	.0122
2.9	.0070	.0131	.0171	.0185	.0179	.0160	.0130	.0092	.0049	.0508	.0720	.0181	.0118
3.0	.0069	.0128	.0167	.0179	.0173	.0153	.0124	.0088	.0046	.0483	.0706	.0178	.0114

TABLE A-8
β = 0.8

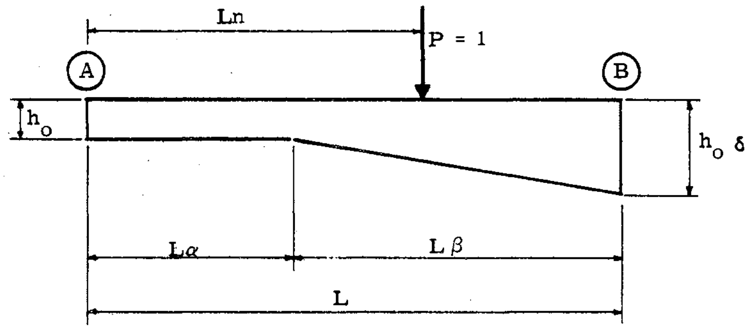
$F_{AB} = f_1 \frac{L}{E I_o}$

$G_{AB} = g \frac{L}{E I_o}$

$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$

$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$

$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$

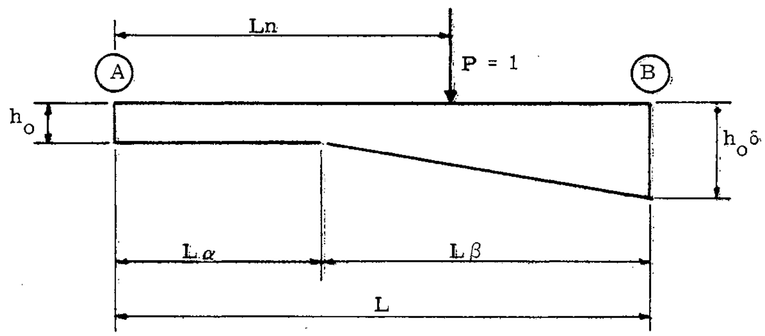


$I_o = \frac{b h_o^3}{12}$

Coefficients For Angular Functions Per Unit Width Of Slab

δ \ n	Influence Coefficients t_1 .1	.2	.3	.4	.5	.6	.7	.8	.9	f_1	g	t_3	t_5
1.1	.0273	.0456	.0560	.0596	.0576	.0512	.0413	.0288	.0148	.3215	.1494	.0415	.0387
1.2	.0263	.0436	.0530	.0558	.0535	.0471	.0378	.0263	.0134	.3112	.1354	.0411	.0361
1.3	.0254	.0418	.0503	.0526	.0500	.0437	.0348	.0241	.0123	.3023	.1238	.0406	.0339
1.4	.0246	.0402	.0480	.0497	.0469	.0408	.0323	.0223	.0113	.2944	.1141	.0402	.0320
1.5	.0239	.0388	.0460	.0472	.0442	.0382	.0301	.0207	.0105	.2895	.1059	.0400	.0303
1.6	.0233	.0376	.0441	.0450	.0419	.0360	.0283	.0194	.0098	.2812	.0988	.0392	.0288
1.7	.0227	.0364	.0425	.0430	.0398	.0340	.0266	.0182	.0092	.2755	.0927	.0388	.0276
1.8	.0222	.0354	.0410	.0412	.0379	.0323	.0252	.0172	.0087	.2705	.0873	.0384	.0264
1.9	.0217	.0345	.0396	.0396	.0362	.0307	.0239	.0163	.0082	.2658	.0826	.0380	.0254
2.0	.0213	.0336	.0384	.0381	.0347	.0293	.0228	.0155	.0078	.2616	.0784	.0376	.0244
2.1	.0209	.0329	.0373	.0368	.0333	.0281	.0217	.0148	.0075	.2577	.0747	.0373	.0236
2.2	.0206	.0321	.0362	.0355	.0321	.0270	.0208	.0141	.0071	.2541	.0714	.0370	.0228
2.3	.0202	.0315	.0352	.0344	.0310	.0260	.0200	.0135	.0068	.2507	.0684	.0367	.0221
2.4	.0199	.0309	.0344	.0334	.0299	.0250	.0192	.0130	.0066	.2477	.0657	.0364	.0215
2.5	.0196	.0303	.0335	.0324	.0290	.0241	.0185	.0125	.0063	.2448	.0632	.0361	.0209
2.6	.0194	.0298	.0328	.0315	.0281	.0233	.0179	.0121	.0061	.2421	.0610	.0359	.0203
2.7	.0191	.0293	.0320	.0307	.0273	.0226	.0173	.0117	.0059	.2396	.0590	.0357	.0198
2.8	.0189	.0288	.0314	.0300	.0265	.0220	.0168	.0113	.0057	.2373	.0570	.0355	.0194
2.9	.0187	.0283	.0307	.0292	.0258	.0213	.0163	.0110	.0055	.2351	.0553	.0353	.0189
3.0	.0185	.0280	.0301	.0286	.0251	.0208	.0159	.0107	.0054	.2330	.0537	.0352	.0185

TABLE A-8
β = 0.8



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0148	.0286	.0403	.0493	.0545	.0552	.0507	.0404	.0237	.2739	.1494	.0376	.0360
1.2	.0134	.0258	.0362	.0438	.0480	.0481	.0438	.0345	.0200	.2290	.1354	.0343	.0317
1.3	.0122	.0234	.0327	.0393	.0427	.0424	.0382	.0299	.0172	.1944	.1238	.0315	.0281
1.4	.0112	.0215	.0298	.0355	.0383	.0377	.0337	.0261	.0149	.1671	.1141	.0292	.0251
1.5	.0104	.0198	.0273	.0323	.0346	.0338	.0300	.0230	.0130	.1452	.1059	.0271	.0226
1.6	.0097	.0184	.0252	.0296	.0314	.0305	.0268	.0205	.0115	.1274	.0988	.0254	.0206
1.7	.0091	.0172	.0234	.0273	.0287	.0277	.0242	.0183	.0102	.1128	.0927	.0239	.0188
1.8	.0086	.0161	.0218	.0253	.0264	.0252	.0220	.0165	.0092	.1006	.0873	.0226	.0173
1.9	.0081	.0152	.0204	.0234	.0244	.0232	.0200	.0150	.0083	.0903	.0826	.0214	.0159
2.0	.0077	.0144	.0192	.0219	.0226	.0213	.0183	.0137	.0075	.0816	.0784	.0204	.0148
2.1	.0073	.0136	.0181	.0205	.0210	.0197	.0168	.0125	.0068	.0741	.0747	.0195	.0138
2.2	.0070	.0129	.0171	.0193	.0196	.0183	.0156	.0115	.0063	.0677	.0714	.0186	.0129
2.3	.0067	.0123	.0162	.0181	.0184	.0171	.0144	.0106	.0058	.0620	.0684	.0179	.0121
2.4	.0064	.0118	.0154	.0171	.0172	.0160	.0134	.0098	.0053	.0571	.0657	.0172	.0114
2.5	.0062	.0113	.0147	.0162	.0162	.0150	.0125	.0092	.0049	.0528	.0632	.0166	.0107
2.6	.0059	.0109	.0140	.0154	.0153	.0141	.0118	.0085	.0046	.0490	.0610	.0160	.0101
2.7	.0057	.0104	.0134	.0146	.0145	.0132	.0110	.0080	.0043	.0456	.0590	.0155	.0096
2.8	.0055	.0101	.0128	.0140	.0138	.0125	.0104	.0075	.0031	.0426	.0570	.0150	.0091
2.9	.0054	.0097	.0123	.0133	.0131	.0118	.0098	.0071	.0038	.0399	.0553	.0146	.0087
3.0	.0052	.0094	.0119	.0128	.0125	.0112	.0093	.0067	.0035	.0374	.0537	.0142	.0083

TABLE A-9
β = 0.9

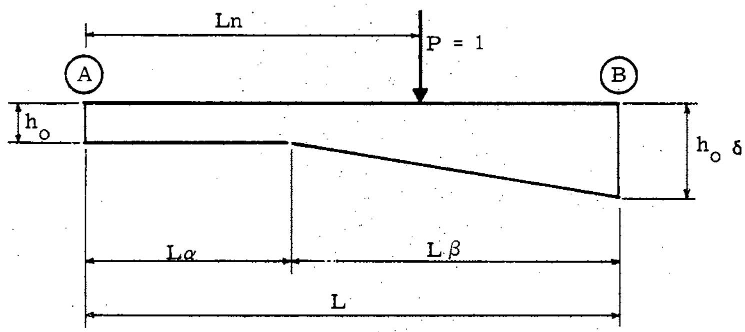
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$

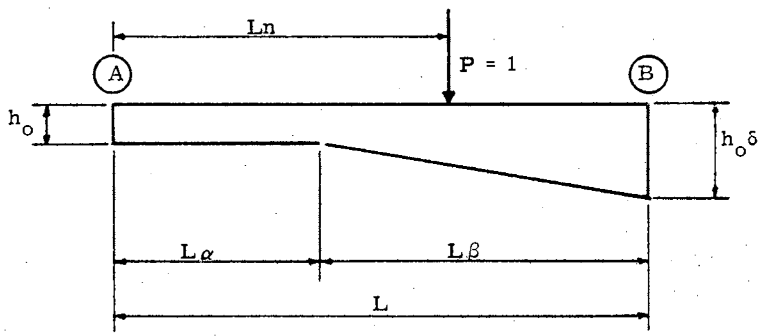


$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0268	.0447	.0548	.0584	.0565	.0501	.0405	.0283	.0145	.3164	.1467	.0411	.0379
1.2	.0254	.0418	.0508	.0535	.0514	.0453	.0364	.0253	.0130	.3019	.1306	.0403	.0346
1.3	.0241	.0393	.0473	.0494	.0471	.0412	.0329	.0228	.0117	.2892	.1174	.0395	.0319
1.4	.0230	.0371	.0442	.0458	.0433	.0378	.0300	.0207	.0106	.2780	.1063	.0386	.0296
1.5	.0220	.0352	.0415	.0427	.0401	.0348	.0275	.0190	.0096	.2680	.0970	.0377	.0275
1.6	.0211	.0334	.0391	.0400	.0373	.0322	.0254	.0174	.0089	.2591	.0890	.0369	.0258
1.7	.0203	.0319	.0370	.0375	.0349	.0299	.0235	.0161	.0082	.2511	.0822	.0361	.0242
1.8	.0195	.0304	.0351	.0354	.0327	.0279	.0219	.0150	.0076	.2438	.0762	.0353	.0228
1.9	.0189	.0292	.0333	.0334	.0307	.0262	.0204	.0140	.0071	.2372	.0710	.0345	.0216
2.0	.0182	.0280	.0318	.0317	.0290	.0246	.0192	.0131	.0066	.2311	.0664	.0338	.0205
2.1	.0177	.0269	.0303	.0301	.0274	.0232	.0181	.0123	.0062	.2256	.0623	.0331	.0195
2.2	.0172	.0259	.0290	.0286	.0260	.0219	.0170	.0116	.0058	.2205	.0586	.0325	.0185
2.3	.0167	.0250	.0278	.0273	.0247	.0208	.0161	.0109	.0055	.2157	.0554	.0319	.0177
2.4	.0163	.0242	.0267	.0261	.0235	.0198	.0153	.0104	.0052	.2113	.0524	.0313	.0170
2.5	.0159	.0234	.0257	.0250	.0225	.0188	.0145	.0098	.0050	.2073	.0497	.0308	.0163
2.6	.0155	.0227	.0248	.0240	.0215	.0180	.0138	.0094	.0047	.2034	.0473	.0303	.0156
2.7	.0152	.0220	.0239	.0230	.0206	.0172	.0132	.0089	.0045	.1999	.0451	.0298	.0150
2.8	.0148	.0214	.0231	.0222	.0197	.0164	.0126	.0085	.0043	.1965	.0431	.0293	.0145
2.9	.0145	.0208	.0223	.0213	.0190	.0158	.0121	.0082	.0041	.1934	.0413	.0289	.0140
3.0	.0142	.0202	.0216	.0206	.0183	.0152	.0116	.0078	.0039	.1904	.0396	.0285	.0135

TABLE A-9
β = 0.9



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0145	.0280	.0396	.0484	.0535	.0544	.0500	.0399	.0235	.2711	.1467	.0372	.0354
1.2	.0129	.0248	.0349	.0423	.0464	.0467	.0426	.0337	.0200	.2244	.1306	.0336	.0307
1.3	.0116	.0222	.0310	.0373	.0406	.0405	.0367	.0288	.0166	.1885	.1174	.0305	.0268
1.4	.0105	.0200	.0277	.0332	.0359	.0355	.0319	.0248	.0142	.1604	.1063	.0280	.0236
1.5	.0095	.0181	.0250	.0297	.0319	.0314	.0280	.0216	.0123	.1380	.0970	.0258	.0210
1.6	.0087	.0165	.0227	.0268	.0286	.0280	.0247	.0190	.0108	.1199	.0890	.0239	.0187
1.7	.0080	.0152	.0207	.0243	.0257	.0250	.0220	.0168	.0095	.1050	.0822	.0223	.0169
1.8	.0075	.0140	.0190	.0221	.0233	.0225	.0197	.0150	.0084	.0927	.0762	.0208	.0153
1.9	.0069	.0129	.0174	.0202	.0212	.0204	.0178	.0134	.0075	.0824	.0710	.0195	.0139
2.0	.0065	.0120	.0161	.0186	.0194	.0185	.0161	.0121	.0067	.0736	.0664	.0184	.0127
2.1	.0061	.0112	.0150	.0172	.0178	.0169	.0146	.0110	.0061	.0662	.0623	.0174	.0117
2.2	.0057	.0105	.0139	.0159	.0164	.0155	.0133	.0100	.0055	.0598	.0586	.0165	.0108
2.3	.0054	.0098	.0130	.0147	.0151	.0143	.0122	.0091	.0050	.0543	.0554	.0156	.0100
2.4	.0051	.0093	.0121	.0137	.0140	.0132	.0112	.0083	.0046	.0495	.0524	.0149	.0092
2.5	.0048	.0087	.0114	.0128	.0130	.0122	.0104	.0077	.0042	.0453	.0497	.0142	.0086
2.6	.0046	.0082	.0107	.0120	.0122	.0113	.0096	.0071	.0039	.0415	.0473	.0136	.0080
2.7	.0043	.0078	.0101	.0112	.0114	.0105	.0089	.0066	.0036	.0383	.0451	.0130	.0075
2.8	.0041	.0074	.0095	.0106	.0106	.0098	.0083	.0061	.0033	.0354	.0431	.0125	.0070
2.9	.0040	.0070	.0090	.0100	.0100	.0092	.0077	.0057	.0031	.0328	.0413	.0120	.0066
3.0	.0038	.0067	.0086	.0094	.0094	.0086	.0073	.0053	.0029	.0304	.0396	.0115	.0062

TABLE A-10
β = 1.0

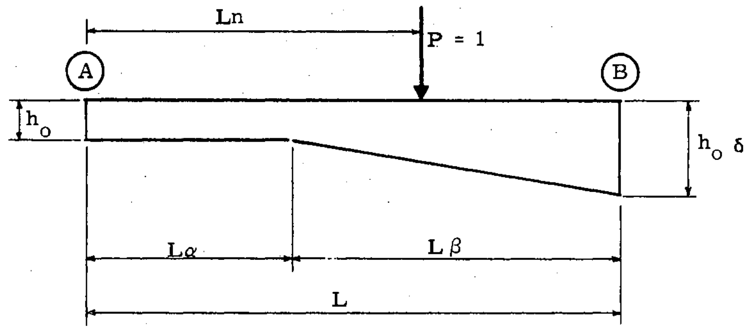
$$F_{AB} = f_1 \frac{L}{E I_o}$$

$$G_{AB} = g \frac{L}{E I_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{E I_o}$$

$$\tau_{AB}^{(DL)} = t_3 \frac{b h_o q L^3}{E I_o}$$

$$\tau_{AB}^{(UL)} = t_5 \frac{w L^3}{E I_o}$$

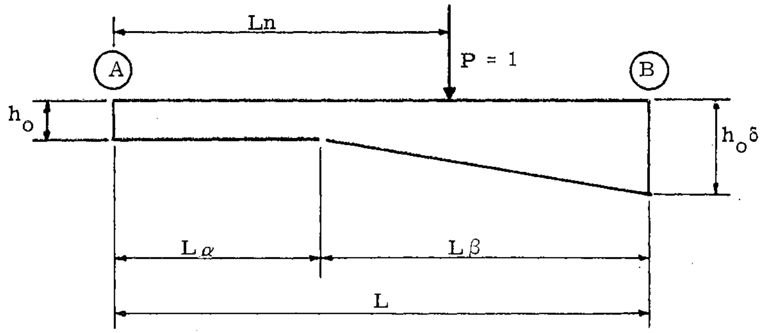


$$I_o = \frac{b h_o^3}{12}$$

Coefficients For Angular Functions Per Unit Width Of Slab

	Influence Coefficients t_1												
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9	f_1	g	t_3	t_5
1.1	.0262	.0437	.0537	.0572	.0554	.0493	.0398	.0278	.0143	.3102	.1444	.0408	.0372
1.2	.0243	.0401	.0487	.0513	.0495	.0437	.0351	.0245	.0125	.2902	.1265	.0396	.0333
1.3	.0226	.0369	.0445	.0467	.0445	.0391	.0313	.0217	.0111	.2728	.1119	.0383	.0302
1.4	.0211	.0341	.0408	.0425	.0403	.0352	.0281	.0194	.0099	.2574	.0998	.0371	.0275
1.5	.0198	.0317	.0376	.0389	.0367	.0319	.0253	.0175	.0089	.2437	.0896	.0359	.0251
1.6	.0186	.0296	.0348	.0358	.0336	.0291	.0230	.0158	.0081	.2315	.0810	.0347	.0231
1.7	.0175	.0276	.0323	.0330	.0308	.0266	.0210	.0144	.0073	.2205	.0736	.0335	.0213
1.8	.0166	.0259	.0301	.0306	.0284	.0245	.0192	.0132	.0067	.2105	.0673	.0324	.0198
1.9	.0157	.0244	.0281	.0284	.0263	.0226	.0177	.0121	.0061	.2014	.0617	.0314	.0184
2.0	.0149	.0230	.0263	.0265	.0244	.0209	.0163	.0112	.0057	.1931	.0569	.0304	.0171
2.1	.0142	.0217	.0247	.0248	.0228	.0194	.0151	.0103	.0052	.1855	.0526	.0294	.0160
2.2	.0135	.0205	.0233	.0232	.0213	.0181	.0141	.0096	.0049	.1785	.0488	.0285	.0150
2.3	.0129	.0195	.0220	.0218	.0199	.0169	.0131	.0089	.0045	.1720	.0454	.0277	.0141
2.4	.0123	.0185	.0207	.0205	.0187	.0158	.0123	.0084	.0042	.1660	.0423	.0269	.0133
2.5	.0118	.0176	.0196	.0193	.0176	.0148	.0115	.0078	.0040	.1604	.0396	.0261	.0126
2.6	.0113	.0168	.0186	.0183	.0165	.0139	.0108	.0073	.0037	.1552	.0372	.0254	.0119
2.7	.0109	.0160	.0177	.0173	.0156	.0131	.0102	.0069	.0035	.1503	.0349	.0247	.0113
2.8	.0105	.0153	.0168	.0164	.0148	.0124	.0096	.0065	.0033	.1457	.0329	.0240	.0107
2.9	.0101	.0146	.0160	.0156	.0140	.0117	.0091	.0061	.0031	.1414	.0310	.0234	.0102
3.0	.0097	.0140	.0153	.0148	.0133	.0111	.0086	.0058	.0029	.1373	.0293	.0228	.0097

TABLE A-10
β = 1.0



$$I_o = \frac{bh_o^3}{12}$$

$$F_{BA} = f_2 \frac{L}{EI_o}$$

$$G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(DL)} = t_4 \frac{bh_o q L^3}{EI_o}$$

$$\tau_{BA}^{(UL)} = t_6 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0143	.0276	.0390	.0477	.0528	.0536	.0494	.0395	.0232	.2688	.1444	.0369	.0349
1.2	.0125	.0240	.0338	.0411	.0452	.0455	.0416	.0330	.0192	.2208	.1265	.0330	.0299
1.3	.0110	.0212	.0296	.0358	.0391	.0391	.0355	.0279	.0161	.1840	.1119	.0298	.0258
1.4	.0098	.0188	.0261	.0314	.0341	.0339	.0305	.0239	.0137	.1553	.0998	.0271	.0224
1.5	.0088	.0168	.0232	.0278	.0300	.0296	.0265	.0206	.0118	.1326	.0896	.0248	.0197
1.6	.0079	.0151	.0208	.0247	.0265	.0261	.0232	.0179	.0102	.1143	.0810	.0228	.0174
1.7	.0072	.0136	.0187	.0221	.0236	.0231	.0248	.0157	.0089	.0994	.0736	.0211	.0155
1.8	.0066	.0124	.0169	.0199	.0212	.0206	.0182	.0139	.0078	.0871	.0673	.0195	.0139
1.9	.0060	.0113	.0154	.0180	.0190	.0185	.0162	.0124	.0069	.0768	.0617	.0182	.0125
2.0	.0055	.0104	.0140	.0164	.0172	.0166	.0145	.0110	.0062	.0681	.0569	.0170	.0113
2.1	.0051	.0095	.0129	.0149	.0156	.0150	.0131	.0099	.0055	.0608	.0526	.0159	.0103
2.2	.0047	.0088	.0118	.0137	.0143	.0136	.0119	.0089	.0050	.0545	.0488	.0150	.0094
2.3	.0044	.0081	.0109	.0125	.0130	.0124	.0108	.0081	.0045	.0491	.0454	.0141	.0086
2.4	.0041	.0076	.0101	.0115	.0120	.0114	.0098	.0074	.0041	.0445	.0423	.0133	.0079
2.5	.0038	.0070	.0093	.0107	.0110	.0104	.0090	.0067	.0037	.0404	.0396	.0126	.0072
2.6	.0036	.0066	.0087	.0099	.0102	.0096	.0082	.0062	.0034	.0368	.0372	.0120	.0067
2.7	.0034	.0061	.0081	.0092	.0094	.0089	.0076	.0057	.0031	.0337	.0349	.0114	.0062
2.8	.0032	.0057	.0075	.0085	.0087	.0082	.0070	.0052	.0029	.0309	.0329	.0108	.0058
2.9	.0030	.0054	.0071	.0080	.0081	.0076	.0065	.0048	.0026	.0284	.0310	.0103	.0054
3.0	.0028	.0051	.0066	.0074	.0076	.0071	.0060	.0044	.0024	.0262	.0293	.0099	.0050

TABLE B-1
$\beta = 0.1$

$F_{AB} = F_{BA} = f \dfrac{L}{EI_o}$

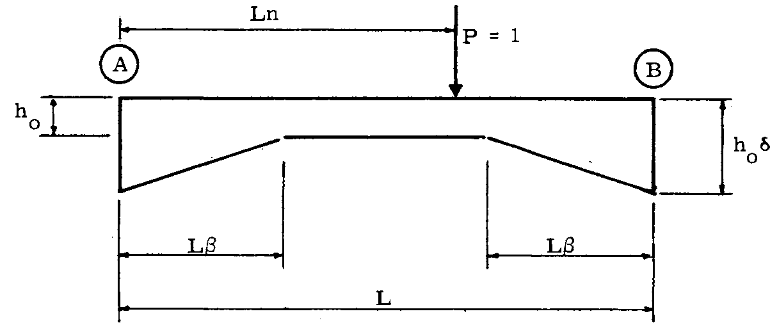
$G_{AB} = G_{BA} = g \dfrac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \dfrac{L^2}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \dfrac{L^2}{EI_o}$

$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \dfrac{bh_o qL^3}{EI_o}$

$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \dfrac{wL^3}{EI_o}$

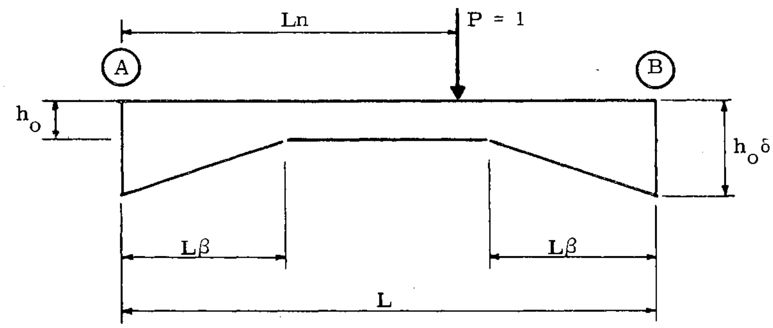


$I_o = \dfrac{bh_o^3}{12}$

Coefficients for Angular Functions Per Unit Width of Slab

Influence Coefficient t_1										f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0281	.0477	.0592	.0637	.0623	.0558	.0454	.0319	.0164	.3210	.1658	.0415	.0415
1.2	.0278	.0474	.0589	.0635	.0621	.0557	.0452	.0318	.0164	.3113	.1651	.0413	.0413
1.3	.0275	.0471	.0587	.0633	.0619	.0555	.0451	.0317	.0163	.3036	.1645	.0412	.0411
1.4	.0273	.0469	.0585	.0632	.0618	.0554	.0450	.0317	.0163	.2973	.1640	.0410	.0410
1.5	.0271	.0467	.0584	.0630	.0617	.0553	.0450	.0316	.0163	.2920	.1635	.0410	.0409
1.6	.0269	.0466	.0582	.0629	.0616	.0552	.0449	.0316	.0162	.2877	.1631	.0409	.0408
1.7	.0267	.0464	.0581	.0628	.0615	.0552	.0448	.0315	.0162	.2840	.1628	.0408	.0407
1.8	.0266	.0463	.0580	.0627	.0614	.0551	.0448	.0315	.0162	.2807	.1625	.0407	.0406
1.9	.0265	.0462	.0579	.0626	.0613	.0550	.0447	.0314	.0162	.2780	.1622	.0407	.0405
2.0	.0264	.0461	.0578	.0625	.0613	.0550	.0447	.0314	.0161	.2756	.1619	.0406	.0405
2.1	.0263	.0460	.0577	.0625	.0612	.0549	.0447	.0314	.0161	.2739	.1617	.0406	.0404
2.2	.0262	.0459	.0577	.0624	.0611	.0549	.0446	.0314	.0161	.2715	.1615	.0405	.0404
2.3	.0261	.0458	.0576	.0623	.0611	.0548	.0446	.0313	.0161	.2699	.1613	.0405	.0403
2.4	.0260	.0458	.0575	.0623	.0610	.0548	.0446	.0313	.0161	.2683	.1612	.0405	.0403
2.5	.0259	.0457	.0575	.0622	.0610	.0548	.0445	.0313	.0161	.2670	.1610	.0405	.0403
2.6	.0259	.0456	.0574	.0622	.0610	.0547	.0445	.0313	.0160	.2658	.1609	.0404	.0402
2.7	.0258	.0456	.0574	.0621	.0609	.0547	.0445	.0313	.0160	.2646	.1607	.0404	.0402
2.8	.0258	.0455	.0573	.0621	.0609	.0547	.0445	.0312	.0160	.2636	.1606	.0404	.0402
2.9	.0257	.0455	.0573	.0621	.0609	.0547	.0444	.0312	.0160	.2627	.1605	.0404	.0401
3.0	.0257	.0455	.0572	.0620	.0608	.0546	.0444	.0312	.0160	.2618	.1604	.0404	.0401
δ \ n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
Influence Coefficients t_2													

TABLE B-2
β = 0.2



$$I_o = \frac{bh_o^3}{12}$$

$$F_{AB} = F_{BA} = f \frac{L}{EI_o}$$

$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \frac{bh_o qL^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficient t_1									f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0272	.0467	.0583	.0629	.0616	.0552	.0449	.0315	.0162	.3102	.1634	.0409	.0409
1.2	.0262	.0455	.0573	.0621	.0608	.0546	.0444	.0311	.0160	.2921	.1607	.0404	.0402
1.3	.0253	.0446	.0565	.0613	.0602	.0541	.0439	.0308	.0157	.2777	.1584	.0399	.0396
1.4	.0246	.0438	.0557	.0607	.0596	.0536	.0435	.0305	.0156	.2660	.1565	.0395	.0391
1.5	.0239	.0431	.0551	.0601	.0592	.0532	.0432	.0302	.0154	.2563	.1548	.0392	.0387
1.6	.0234	.0425	.0546	.0597	.0588	.0528	.0429	.0300	.0153	.2483	.1533	.0389	.0383
1.7	.0229	.0420	.0541	.0592	.0584	.0525	.0427	.0298	.0152	.2414	.1520	.0387	.0380
1.8	.0225	.0415	.0537	.0589	.0581	.0522	.0424	.0296	.0150	.2356	.1509	.0385	.0377
1.9	.0221	.0411	.0533	.0585	.0578	.0520	.0422	.0295	.0150	.2305	.1498	.0383	.0375
2.0	.0218	.0407	.0529	.0582	.0575	.0518	.0421	.0293	.0149	.2261	.1489	.0382	.0372
2.1	.0215	.0403	.0526	.0580	.0573	.0516	.0419	.0292	.0148	.2222	.1481	.0380	.0370
2.2	.0213	.0400	.0524	.0577	.0570	.0514	.0417	.0291	.0147	.2188	.1473	.0379	.0368
2.3	.0210	.0397	.0521	.0575	.0568	.0512	.0416	.0290	.0146	.2157	.1466	.0379	.0367
2.4	.0208	.0395	.0519	.0573	.0567	.0511	.0415	.0289	.0146	.2130	.1460	.0378	.0365
2.5	.0206	.0392	.0517	.0571	.0565	.0509	.0413	.0288	.0145	.2106	.1454	.0377	.0364
2.6	.0204	.0390	.0515	.0569	.0563	.0508	.0412	.0287	.0145	.2084	.1449	.0377	.0362
2.7	.0203	.0388	.0513	.0567	.0562	.0507	.0411	.0286	.0144	.2063	.1444	.0376	.0361
2.8	.0201	.0386	.0511	.0566	.0561	.0506	.0410	.0285	.0143	.2045	.1440	.0376	.0360
2.9	.0200	.0384	.0509	.0564	.0560	.0505	.0410	.0285	.0143	.2028	.1435	.0376	.0360
3.0	.0199	.0383	.0508	.0563	.0558	.0505	.0409	.0284	.0143	.2013	.1431	.0376	.0358
δ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
	Influence Coefficients t_2												

TABLE B-3
β = 0.3

$$F_{AB} = F_{BA} = f \frac{L}{EI_o}$$

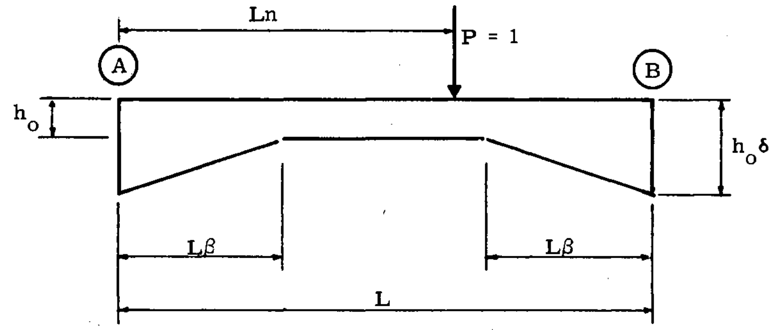
$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \frac{bh_o qL^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$$

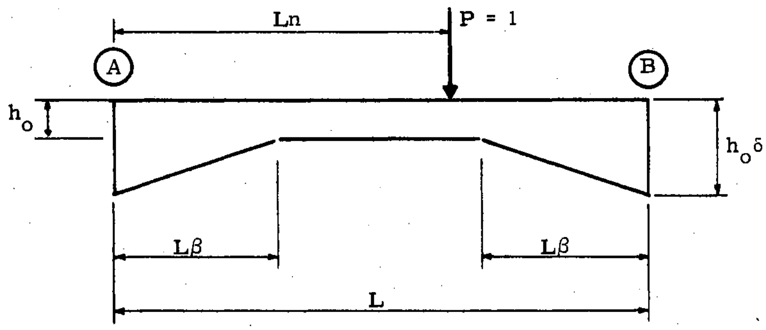


$$I_o = \frac{bh_o^3}{12}$$

Coefficients for Angular Functions Per Unit Width of Slab

	Influence Coefficient t_1									f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0263	.0452	.0570	.0617	.0605	.0542	.0440	.0308	.0158	.3006	.1597	.0403	.0399
1.2	.0246	.0430	.0547	.0597	.0588	.0528	.0428	.0299	.0152	.2752	.1540	.0391	.0385
1.3	.0231	.0410	.0529	.0581	.0573	.0515	.0417	.0291	.0148	.2550	.1492	.0382	.0373
1.4	.0219	.0394	.0513	.0567	.0561	.0504	.0408	.0283	.0144	.2386	.1451	.0374	.0363
1.5	.0209	.0380	.0500	.0555	.0550	.0495	.0400	.0277	.0141	.2252	.1415	.0368	.0354
1.6	.0200	.0368	.0488	.0544	.0541	.0487	.0393	.0272	.0138	.2139	.1384	.0363	.0346
1.7	.0193	.0357	.0477	.0535	.0532	.0480	.0387	.0267	.0135	.2044	.1357	.0359	.0339
1.8	.0187	.0348	.0468	.0527	.0525	.0473	.0382	.0263	.0133	.1963	.1333	.0355	.0333
1.9	.0181	.0339	.0460	.0519	.0518	.0468	.0377	.0259	.0131	.1893	.1312	.0352	.0328
2.0	.0176	.0332	.0453	.0513	.0513	.0462	.0372	.0255	.0129	.1833	.1292	.0349	.0323
2.1	.0172	.0325	.0446	.0507	.0507	.0458	.0368	.0252	.0127	.1779	.1275	.0347	.0319
2.2	.0168	.0319	.0440	.0501	.0502	.0453	.0365	.0249	.0126	.1732	.1259	.0345	.0315
2.3	.0164	.0314	.0434	.0496	.0498	.0450	.0361	.0247	.0124	.1691	.1245	.0344	.0311
2.4	.0161	.0309	.0429	.0492	.0494	.0446	.0358	.0244	.0123	.1653	.1232	.0343	.0308
2.5	.0158	.0304	.0425	.0487	.0490	.0443	.0355	.0242	.0122	.1620	.1220	.0342	.0305
2.6	.0156	.0300	.0420	.0483	.0487	.0440	.0353	.0240	.0121	.1590	.1209	.0341	.0302
2.7	.0153	.0296	.0416	.0480	.0483	.0437	.0350	.0238	.0120	.1562	.1199	.0340	.0300
2.8	.0151	.0293	.0413	.0477	.0480	.0434	.0348	.0236	.0119	.1538	.1189	.0340	.0297
2.9	.0149	.0290	.0409	.0473	.0478	.0432	.0346	.0235	.0118	.1515	.1181	.0340	.0292
3.0	.0147	.0287	.0406	.0471	.0475	.0430	.0344	.0233	.0117	.1494	.1173	.0340	.0293
δ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
	Influence Coefficients t_2												

TABLE B-4
β = 0.4



$$I_o = \frac{bh_o^3}{12}$$

$$F_{AB} = F_{BA} = f \frac{L}{EI_o}$$

$$G_{AB} = G_{BA} = g \frac{L}{EI_o}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_o}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_o}$$

$$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \frac{bh_o qL^3}{EI_o}$$

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_o}$$

Coefficients for Angular Functions Per Unit Width of Slab

Influence Coefficient t_1										f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0255	0437	.0551	.0599	.0589	.0528	.0428	.0299	.0154	.2920	.1551	.0395	.0388
1.2	.0231	0403	.0514	.0566	.0558	.0501	.0405	.0282	.0144	.2777	.1455	.0378	.0364
1.3	.0211	0374	.0484	.0537	.0533	.0478	.0385	.0268	.0137	.2347	.1375	.0363	.0344
1.4	.0195	0350	.0457	.0513	.0511	.0458	.0369	.0255	.0130	.2142	.1307	.0351	.0327
1.5	.0182	0329	.0435	.0491	.0492	.0442	.0354	.0244	.0124	.1974	.1248	.0341	.0312
1.6	.0171	0311	.0415	.0473	.0475	.0427	.0341	.0235	.0119	.1834	.1197	.0333	.0299
1.7	.0161	0295	.0398	.0457	.0460	.0414	.0330	.0227	.0115	.1716	.1152	.0326	.0288
1.8	.0152	0282	.0383	.0442	.0447	.0402	.0320	.0219	.0111	.1616	.1113	.0320	.0278
1.9	.0145	0271	.0369	.0430	.0436	.0392	.0311	.0213	.0107	.1529	.1078	.0314	.0269
2.0	.0139	0260	.0357	.0418	.0425	.0382	.0303	.0207	.0104	.1454	.1046	.0310	.0262
2.1	.0133	0251	.0346	.0407	.0415	.0373	.0295	.0201	.0102	.1388	.1018	.0306	.0254
2.2	.0128	0242	.0337	.0398	.0407	.0366	.0289	.0196	.0100	.1330	.0992	.0303	.0248
2.3	.0123	0235	.0328	.0389	.0399	.0358	.0282	.0191	.0097	.1279	.0969	.0300	.0242
2.4	.0119	0228	.0320	.0381	.0392	.0352	.0277	.0188	.0095	.1233	.0948	.0298	.0237
2.5	.0116	0222	.0312	.0374	.0385	.0346	.0272	.0184	.0093	.1192	.0928	.0296	.0232
2.6	.0112	0216	.0305	.0367	.0379	.0340	.0267	.0181	.0091	.1155	.0910	.0294	.0228
2.7	.0109	0211	.0299	.0361	.0373	.0335	.0262	.0178	.0089	.1121	.0894	.0293	.0223
2.8	.0106	0206	.0293	.0355	.0368	.0330	.0258	.0175	.0088	.1091	.0878	.0291	.0220
2.9	.0104	0201	.0288	.0350	.0363	.0326	.0254	.0172	.0086	.1063	.0864	.0291	.0216
3.0	.0102	0197	.0283	.0345	.0358	.0322	.0251	.0169	.0085	.1038	.0851	.0290	.0213
δ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
Influence Coefficients t_2													

TABLE B-5
β= 0.5

$F_{AB} = F_{BA} = f \dfrac{L}{EI_o}$

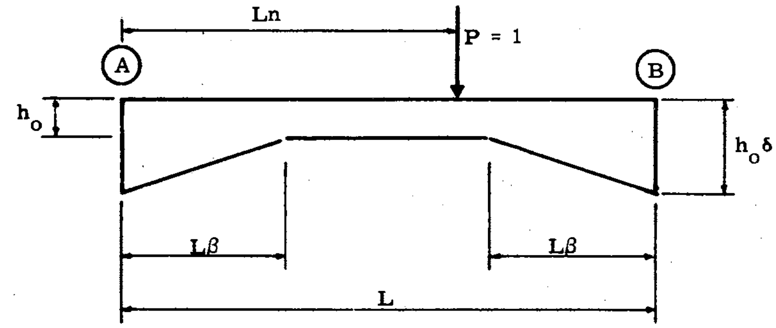
$G_{AB} = G_{BA} = g \dfrac{L}{EI_o}$

$\tau_{AB}^{(LL)} = t_1 \dfrac{L^2}{EI_o}$

$\tau_{BA}^{(LL)} = t_2 \dfrac{L^2}{EI_o}$

$\tau_{AB}^{(DL)} = \tau_{BA}^{(DL)} = t_3 \dfrac{bh_o qL^3}{EI_o}$

$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = t_4 \dfrac{wL^3}{EI_o}$



$I_o = \dfrac{bh_o^3}{12}$

Coefficients for Angular Functions Per Unit Width of Slab

Influence Coefficient t_1										f	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.1	.0247	.0423	.0532	.0578	.0568	.0509	.0412	.0289	.0148	.2842	.1497	.0388	.0374
1.2	.0217	.0377	.0480	.0527	.0521	.0467	.0377	.0263	.0135	.2462	.1358	.0365	.0339
1.3	.0193	.0339	.0436	.0484	.0481	.0431	.0347	.0241	.0123	.2161	.1241	.0345	.0310
1.4	.0173	.0307	.0399	.0447	.0446	.0400	.0321	.0223	.0114	.1919	.1142	.0327	.0286
1.5	.0157	.0281	.0368	.0415	.0417	.0373	.0299	.0207	.0105	.1720	.1057	.0312	.0264
1.6	.0143	.0258	.0341	.0387	.0391	.0349	.0279	.0193	.0098	.1556	.0984	.0299	.0246
1.7	.0131	.0238	.0317	.0363	.0368	.0329	.0262	.0180	.0092	.1416	.0919	.0288	.0230
1.8	.0121	.0221	.0296	.0341	.0347	.0310	.0247	.0169	.0086	.1298	.0863	.0278	.0216
1.9	.0112	.0206	.0278	.0322	.0329	.0294	.0233	.0160	.0081	.1196	.0812	.0269	.0203
2.0	.0104	.0193	.0262	.0305	.0313	.0279	.0221	.0151	.0076	.1108	.0767	.0260	.0192
2.1	.0097	.0181	.0247	.0289	.0298	.0265	.0210	.0143	.0072	.1031	.0727	.0253	.0182
2.2	.0091	.0170	.0234	.0275	.0284	.0253	.0199	.0136	.0069	.0963	.0690	.0246	.0173
2.3	.0086	.0161	.0222	.0262	.0272	.0242	.0190	.0130	.0066	.0903	.0657	.0240	.0164
2.4	.0081	.0152	.0211	.0250	.0260	.0231	.0182	.0124	.0063	.0849	.0627	.0234	.0157
2.5	.0077	.0145	.0201	.0240	.0250	.0222	.0174	.0119	.0060	.0801	.0600	.0229	.0150
2.6	.0073	.0138	.0192	.0230	.0240	.0213	.0167	.0114	.0057	.0758	.0574	.0224	.0143
2.7	.0069	.0131	.0183	.0220	.0231	.0205	.0160	.0190	.0055	.0719	.0550	.0220	.0138
2.8	.0066	.0125	.0176	.0212	.0223	.0198	.0154	.0104	.0053	.0683	.0529	.0216	.0132
2.9	.0063	.0120	.0169	.0204	.0215	.0191	.0149	.0101	.0051	.0651	.0509	.0212	.0127
3.0	.0060	.0115	.0162	.0197	.0208	.0184	.0143	.0097	.0049	.0621	.0490	.0208	.0123
δ / n	.9	.8	.7	.6	.5	.4	.3	.2	.1	f	g	t_3	t_5
Influence Coefficients t_2													

PART VII

PROCEDURE OF ANALYSIS AND EXAMPLES

The analysis of pinned-end frames, with linear variation in cross-section, by means of the string polygon theory and the application of numerical coefficients developed and tabulated in this study is illustrated by two numerical examples. All values are given in feet, kips or kip-feet, except where otherwise noted.

1. Example One.

A symmetrical trapezodial pinned-end frame of variable cross-section acted on by a uniform horizontal load is considered (Fig. 7-1). The bending moments at A and B are calculated. Results are compared with those found by Leontovich (22).

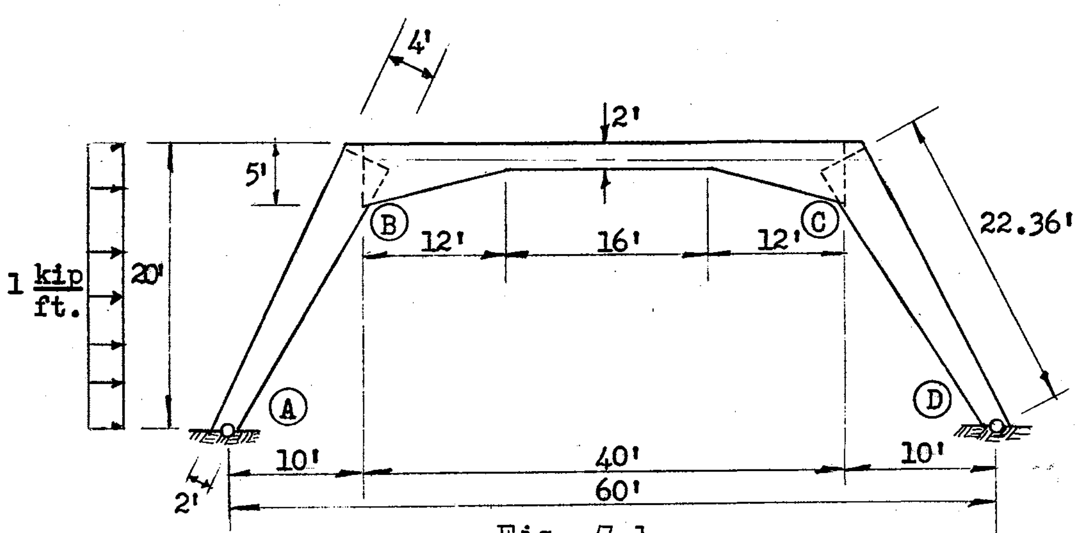


Fig. 7-1

Symmetrical Trapezodial Pinned-End Frame With Straight Haunches.

A. Calculation of Angular Functions.

1. Dimension Coefficients β's and δ's:

Spans $\overline{AB}$ and $\overline{CD}$

$$\beta = \frac{L_{AB}\,\beta}{L_{AB}} = \frac{22.36}{22.36} = 1 \qquad \delta = \frac{h_A}{h_o} = \frac{4}{2} = 2$$

Span $\overline{BC}$

$$\beta = \frac{L_{BC}\,\beta}{L_{BC}} = \frac{12}{40} = .3 \qquad \delta = \frac{h_C}{h_o} = \frac{5}{2} = 2.5$$

2. Angular Flexibilities (Eq. 2-7).

$$F_{BA} = F_{CD} = f_{BA}\frac{L_{AB}}{EI_o} = (.0681)\,\frac{(22.36)}{EI_o}$$

$$= \frac{1.523}{EI_o} \qquad \text{(Table A-10)}$$

$$F_{BC} = F_{CD} = f_{BC}\frac{L_{BC}}{EI_o} = (.1620)\,\frac{(40)}{EI_o}$$

$$= \frac{6.48}{EI_o} \qquad \text{(Table B-3)}$$

$$\sum F_B = \sum F_B = \frac{8.003}{EI_o}$$

3. Angular Carry-Over Values (Eq. 3-8).

$$G_{BC} = G_{CB} = g_{BC}\frac{L_{BC}}{EI_o} = (.1220)\,\frac{(40)}{EI_o}$$

$$= \frac{4.88}{EI_o} \qquad \text{(Table B-3)}$$

4. Angular Load Function (Eq. 2-9).

$$\tau_{BA}(UL) = t_{BA}(UL)\,\frac{wL^3_{BA}}{EI_o}\cdot\frac{1}{\sin\pi_A} =$$

$$= \frac{(.0113)(1)(22.36)^3}{EI_o} \quad \frac{(22.36)}{30} = \frac{141.2}{EI_o} \text{ (Table A-10).}$$

B. Calculation of Moments Due to Loads and Redundants.

Consider the basic structure (Fig. 7-2).

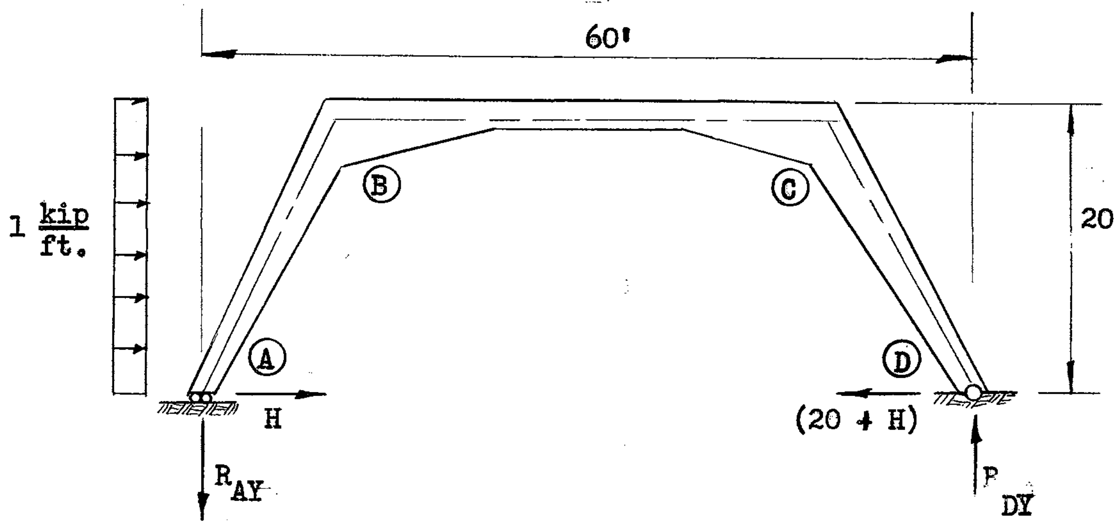


Fig. 7-2

Basic Structure

The reactions and moments are:

1. Reactions.

$$\sum M_C = \frac{wL_{AB}^2}{2} - 60\ R_{AY}$$

$$= \frac{(1)(40)^2}{2} - 60\ R_{AY} = 0$$

$R_{AY} = 3.33$ kips ↑

$R_{DY} = 3.33$ kips ↓

2. Moments Due to Loads.

$$M_B^{(L)} = \frac{wL_{AB}^2}{2} + 10\ R_{AY}$$

$$= 200 + 33.3 = 233.3 \text{ kip - ft.}$$

$$M_C(L) = \frac{w L_{AB}^2}{2} - 50 R_{AY}$$

$$= 200 + 166.65 = 366.65 \text{ kip - ft.}$$

3. Moments Due to Redundants.

$$M_B(H) = (20)(H)$$

$$M_C(H) = (20)(H)$$

C. Calculation of Elastic Weights. (Eq.'s 4-5c, 5d).

1. Elastic Weights Due to Loads.

$$\bar{P}_B(L) = M_B(L) \sum F_B + M_C(L)\, G_{CB} + \sum \tau_B$$

$$= \frac{1}{EI_o}\Big[-(233.3)(8.003) - (366.65)(4.88) + 141.2\Big]$$

$$= -\frac{3515.38}{EI_o}$$

$$\bar{P}_C(L) = M_B(L)\, G_{CB} + M_C(L) \sum F_B$$

$$= \frac{1}{EI_o}\Big[-(233.3)(4.88) - (366.65)(8.003)\Big]$$

$$= -\frac{4072.95}{EI_o}$$

2. Elastic Weights Due to Redundant.

$$\bar{P}_C(H) = \bar{P}_B(H) = M_B(H) \sum F_B + M_C(H)\, G_{CB}$$

$$= \frac{1}{EI_o}\Big[-(20)(H)(8.003) - (+H)(20)(4.88)\Big]$$

$$= -\frac{(257.66)(H)}{EI_o}$$

D. Calculation of Redundant. (Eq. 4-7).

$$H = \frac{\bar{P}_B^{(L)}\ Y_B + \bar{P}_C^{(L)}\ Y_C}{\bar{P}_B^{(H=1)}\ Y_B + \bar{P}_C^{(H=1)}\ Y_C}$$

$$H = \frac{(3515.38 + 4072.95)}{(257.66 + 257.66)}$$

$$H = -\ 14.73$$

$$H = 14.73 \text{ kip}$$

E. Comparison of Results. The final moments compared with those found by Leontovich are (Fig. 7-3):

Unknown	String Polygon	Leontovich
H	14.73	14.71
M_B	61.3	60.9
M_C	71.5	72.3

Fig. 7-3

Comparison of Results

2. Example Two.

A pinned-end bridge frame of variable cross-section with hinges at different levels is considered (Fig. 7-4). The magnitude of the thrust redundant for ten positions of unit live-load is calculated.

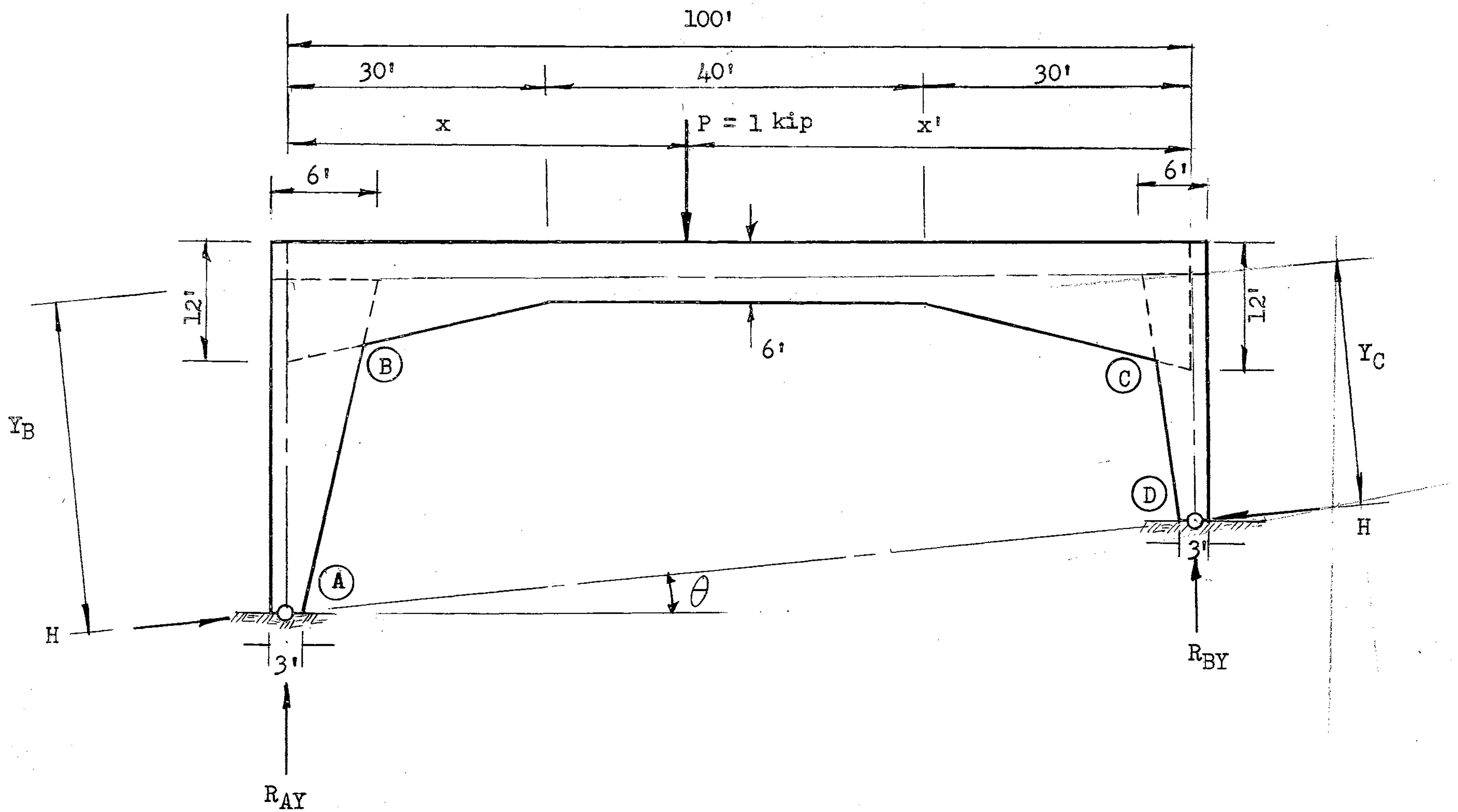


Fig. 7-4

Bridge Frame With Hinges at Different Levels

A. Calculation of Angular Functions.

1. Dimension Coefficients β's and δ's.

Spans $\overline{AB}$ and $\overline{CD}$

$$\beta = \frac{L_{AB}\,\beta_{AB}}{L_{AB}} = \frac{40}{40} = 1 \quad \Big| \quad \delta = \frac{h_B}{h_A} = \frac{6}{3} = 2$$

Span $\overline{B\text{-}C}$

$$\beta = \frac{L_{BC}\,\beta_{BC}}{L_{BC}} = \frac{30}{100} = .3 \quad \Big| \quad \delta = \frac{h_B}{h_o} = \frac{12}{6} = 2$$

2. Moment of Inertia.

$$I_A = I_o^{(AB)} = I_o$$

$$I_o^{(BC)} = \frac{h_o^{(BC)3}}{h_o^{(BA)3}} = \frac{216}{27} = 8I_o^{(AB)} = 8I_o$$

3. Angular Flexibilities (Eq. 2-7)

$$F_{BA} = F_{CD} = f_{BA}\frac{L_{AB}}{EI_o} = (.0681)\,\frac{(40)}{EI_o}$$

$$= \frac{2.72}{EI_o} \qquad \text{(Table A-10)}$$

$$F_{BC} = F_{CB} = f_{BC}\frac{L_{BC}}{EI_o} = (.1833)\frac{(100)}{8EI_o}$$

$$= \frac{2.29}{EI_o} \qquad \text{(Table B-3)}$$

$$\sum F_B = \sum F_C = \frac{5.01}{EI_o}$$

4. <u>Angular Carry-Over Values (Eq. 2-8).</u>

$$G_{BC} = G_{CB} = g_{BC} \frac{L_{BC}}{8EI_o} = \frac{(.1292)(100)}{8EI_o}$$

$$= \frac{1.62}{EI_o} \qquad \text{(Table B-3)}$$

5. <u>Angular Load Function (Eq. 2-9).</u>

$$\mathcal{T}_{BC}^{(LL)} = t_{BC} \frac{L^2}{8EI_o} = t_{BC} \frac{1250}{EI_o} \qquad \text{(Table A-10)}$$

The load function will vary for each position of loading (Fig. 7-5).

Position	t_{BC}	$f_{BC}^{(LL)}$	$f_{CB}^{(LL)}$
0	0	0	0
1	.0176	22.0	16.13
2	.0332	41.5	31.88
3	.0453	56.63	46.5
4	.0513	64.13	57.75
5	.0513	64.13	64.13
6	.0462	57.75	64.13
7	.0372	46.5	56.63
8	.0255	31.88	41.5
9	.0129	16.13	22.0
10	0	0	0

Fig. 7-5

Angular Load Functions

B. Calculation of Moments Due to Loads and Redundant.

1. Moment Arms (Eq. 4-8).

$\text{Sin}\ \Theta = \frac{10}{100.5} \quad \text{Cos}\ \Theta = \frac{100}{100.5}$

$Y_B = v_B \text{ Cos } \Theta - u_B \text{ Sin } \Theta$

$= (40)(.996)$

$= 39.8$ ft.

$Y_C = v_C \text{ Cos } \Theta - u_B \text{ Sin } \Theta$

$= (39.8) - (.0996)(100)$

$= 29.84$ ft.

2. Moments Due to Loads.

The moments due to applied loads are zero at A and B (Fig. 7-6).

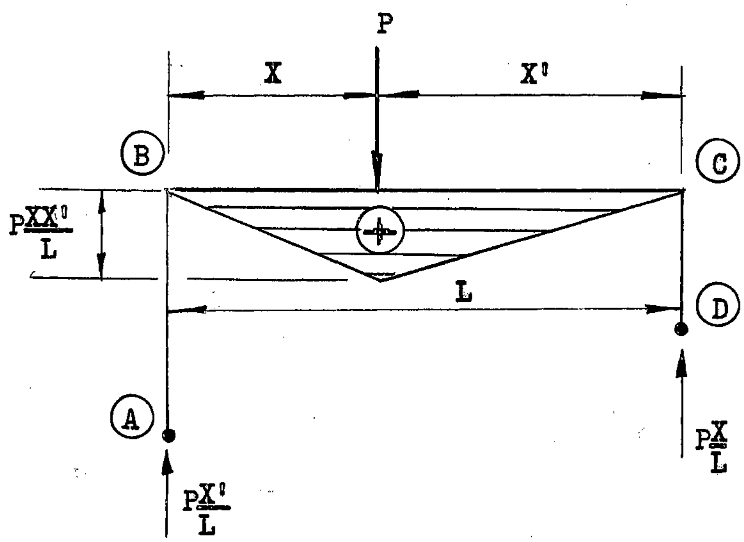


Fig. 7-6

Moments Due to Loads

3. Moments Due to Redundant.

The moments due to the redundant are (Fig. 7-7).

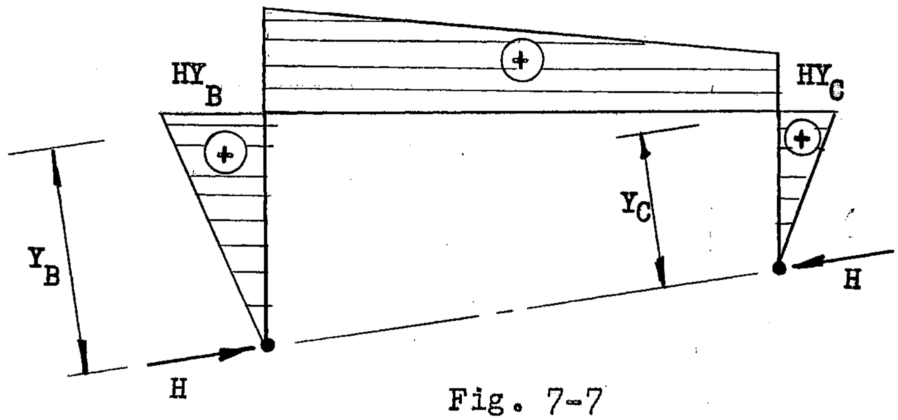


Fig. 7-7

Moments Due to Redundant

$$M_B = -(H)(39.8)$$
$$M_C = -(H)(29.84)$$

C. <u>Calculation of Elastic Weights (Eq.'s 4-5c, 5d)</u>

1. <u>Elastic Weights Due to Loads.</u>

$$\bar{\bar{P}}_B^{(L)} = \sum \mathcal{T}_B = \mathcal{T}_{BC}^{(LL)}$$

$$\bar{\bar{P}}_C^{(L)} = \sum \mathcal{T}_C = \mathcal{T}_{CB}^{(LL)}$$

Refer to Fig. 7-5.

2. <u>Elastic Weights Due to Redundant.</u>

$$\bar{\bar{P}}_B^{(H)} = M_B \sum F_B + M_C\, G_{CB}$$
$$= -(H)(39.8)(5.01) - (H)(29.84)(1.62)$$
$$= -(H)(247.74)$$

$$\bar{\bar{P}}_C^{(H)} = M_B\, G_{BC} + M_C \sum F_C$$
$$= -(H)(29.8)(1.62) - (H)(29.84)(5.01)$$
$$= -(H)(213.98)$$

D. <u>Calculation of Redundant (Eq.4-7).</u> The redundant is calculated for each position of the unit live load (Fig. 7-8).

$$H = \frac{\overline{P}_B^{(L)} Y_B + \overline{P}_C^{(L)} Y_C}{\overline{P}_B^{(H=1)} Y_B + \overline{P}_C^{(H=1)} Y_C}$$

$$H = \frac{\overline{P}_B^{(L)} Y_B + \overline{P}_C^{(L)} Y_C}{16245.21}$$

Position	$\overline{P}_B^{(L)} Y_B$	$\overline{P}_C^{(L)} Y_C$	$\sum \overline{P}_j^{(L)} Y_j$	H
0	0	0	0	0
1	875.6	481.32	1356.92	.084
2	1651.7	951.30	2603.0	.160
3	2254.67	1387.56	3642.43	.224
4	2552.37	1732.36	4284.73	.264
5	2552.37	1913.64	4466.01	.275
6	2298.45	1913.64	4212.09	.259
7	1850.7	1689.84	3540.54	.218
8	1268.82	1238.36	2507.18	.154
9	641.97	656.48	1298.45	.080
10	0	0	0	0

Fig. 7-8

Values of Redundant for Different

Positions of Unit Live Load

PART VIII

SUMMARY AND CONCLUSIONS

The primary objective of this study is to develop a simplified, easy-to-follow method for the analysis of pinned-end frames.

The elastic curve of any straight or bent member of constant or variable cross-section may be divided into a finite number of segments or string polygon. Each segment can be considered as a simple beam. Using the angular functions, G, F, and τ (end slopes of the simple beams), the change in angle of two adjacent string lines may be expressed in terms of a three-moment equation and used as an elastic weight on the conjugate structure.

The reaction of the conjugate structure is the slope of the real structure and the moment of the conjugate structure is the deflection of the real structure. With this in mind, an expression for the horizontal thrust redundant is easily obtained.

Using the program presented, the computer will evaluate constants for beams with either one straight haunch or two symmetrical straight haunches for which β is expressed as a multiple of one-tenth and ω does not exceed two. Tables of beam constants obtained by the results of this program minimize the time of computation of the elastic weights. The constants, being perfectly general, may be used with methods other than the string polygon for many kinds of structures.

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