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GRADUATE COLLEGE

**HORIZONTAL WELL PRODUCTIVITY AND WELLBORE PRESSURE
BEHAVIOR INCORPORATING WELLBORE HYDRAULICS**

A Dissertation

SUBMITTED TO THE GRADUATE FACULTY

In partial fulfillment of the requirements for the

degree of

Doctor of Philosophy

By
Elizabeth G. Anklaam
Norman, Oklahoma
2001

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HORIZONTAL WELL PRODUCTIVITY AND WELLBORE PRESSURE
BEHAVIOR INCORPORATING WELLBORE HYDRAULICS

A Dissertation APPROVED FOR THE
MEWBOURNE SCHOOL OF PETROLEUM AND GEOLOGICAL ENGINEERING

By

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In memory of:

**Dorothy H. Anklam
July 26, 1937 – April 13, 2001**

Table of Contents

LIST OF FIGURES	VIII
LIST OF TABLES.....	X
ABSTRACT	XI
CHAPTER 1 INTRODUCTION.....	1
1.1 History of Horizontal Wells	1
1.2 Review of Previous Research on Horizontal Wells.....	2
1.3 Problem Statement	3
1.4 Research Objectives.....	5
CHAPTER 2 LITERATURE REVIEW	9
2.1 Vertical Inflow Performance Equations.....	9
2.1.1 The Productivity Index.....	10
2.1.2 Inflow Performance Relationships (IPR).....	10
2.2 Horizontal Inflow Performance Equations	11
2.2.1 Horizontal Productivity Index (J_h)	12
2.2.2 Multiphase Horizontal Well Performance	17
2.3 Wellbore Hydraulics in the Horizontal Well	19
2.3.1 Horizontal Pipe Flow Equations	20
2.3.2 Frictional effects on wellbore pressure.....	21
2.3.3 Additional effects on wellbore pressure	24
CHAPTER 3 MODEL DEVELOPMENT.....	29
3.1 The continuity equation	29
3.2 The energy equation	30
3.2 The Reservoir.....	31
3.4 The System of Equations	32

CHAPTER 4 A NEW PRODUCTIVITY INDEX METHOD WITH WELLBORE HYDRAULICS	37
4.1 Discussion of the Program.....	37
4.2 Comparison to theoretical and experimental results.....	38
4.2.1 Examination of data set using Novy.....	39
4.2.2 Comparison to maximum wellbore pressure drop	39
4.2.3 Comparison to Yuan and Asheim	40
4.2.4 Comparison to Yalniz and Ozkan	41
4.3 Sensitivity study	42
4.3.1 Varying the fluid density.....	43
4.3.2 Varying the fluid viscosity	43
4.3.3 Varying the reservoir permeability ratio, $\sqrt{k_h/k_v}$	44
4.3.4 Varying influx at the toe.....	44
4.3.5 Varying the horizontal well length.....	45
4.3.6 Varying the well diameter.....	45
4.3.7 Varying the pipe roughness	45
4.3.8 Varying the influx loss coefficient.....	45
4.3.9 The productivity index	46
4.4 Summary.....	46
CHAPTER 5 SUMMARY AND CONCLUSIONS	84
REFERENCES.....	86
NOMENCLATURE	90
APPENDIX A SUMMARY OF VERTICAL WELL PERFORMANCE RELATIONSHIPS	93

List of Figures

Figure 1.	Schematic of a vertical well	7
Figure 2.	Schematic of a horizontal well	8
Figure 3.	Schematic for the horizontal well problem	34
Figure 4.	The control volume	35
Figure 5.	The typical geometries used for a horizontal well	36
Figure 6.	The input screen of the program	48
Figure 7.	Curves that bound regions where friction is negligible in oil recovery, by Novy ³⁷	49
Figure 8.	Comparison of the maximum pressure drop and the program's pressure drop	50
Figure 9.	Comparison to Yalniz's experimental data, no influx	51
Figure 10.	Comparison to Yalniz's experimental data, influx to main rate, $q_{in}/Q = 0.02$	52
Figure 11.	Comparison to Yalniz's experimental data, $q_{in}/Q = 0.107$	53
Figure 12.	Comparison to Yalniz's experimental data, $q_{in}/Q = 0.217$	54
Figure 13.	Dimensionless pressure versus length for changing density	55
Figure 14.	Reynold's number versus friction factor for changing density	56
Figure 15.	Dimensionless rate versus dimensionless length for changing density ..	57
Figure 16.	Dimensionless pressure versus length for changing viscosity	58
Figure 17.	Reynold's number versus friction factor for changing viscosity	59
Figure 18.	Dimensionless rate versus dimensionless length for changing viscosity	60
Figure 19.	Dimensionless rate versus length for changing permeability ratio	61
Figure 20.	Reynold's number versus friction factor for changing permeability ratio	62
Figure 21.	Dimensionless rate versus length for changing permeability ratio	63
Figure 22.	Dimensionless pressure versus length for changing influx at toe	64
Figure 23.	Reynold's number versus friction factor for changing influx at toe	65
Figure 24.	Dimensionless rate versus length for changing influx at toe	66
Figure 25.	Dimensionless pressure versus length for changing well length	67
Figure 26.	Reynold's number versus friction factor for changing well length	68
Figure 27.	Dimensionless rate versus length for changing well length	69
Figure 28.	Dimensionless pressure versus length for changing well diameter	70
Figure 29.	Reynold's number versus friction factor for changing well diameter	71
Figure 30.	Dimensionless rate versus length for changing well diameter	72
Figure 31.	Dimensionless pressure versus length for changing pipe roughness	73
Figure 32.	Reynold's number versus friction factor for changing pipe roughness .	74
Figure 33.	Dimensionless rate versus length for changing pipe roughness	75
Figure 34.	Dimensionless pressure versus length for changing influx loss coefficient	76

Figure 35.	Reynold's number versus friction factor, changing influx loss coefficient	77
Figure 36.	Dimensionless rate versus length for changing influx loss coefficient ..	78
Figure 37.	Productivity index versus well length	79
Figure 38.	Reservoir drawdown versus flow rate	80

List of Tables

Table 2.1.	Horizontal Well Performance Equations	27
Table 2.2.	Example Reservoir, Fluid, and Well Data	28
Table 2.3.	Comparison of Horizontal Productivity Index Relations	28
Table 4.1.	Program Sections	81
Table 4.2.	Example Data Set	81
Table 4.3.	Comparison of Pressure Drop with Different Friction Factor	82
Table 4.4.	Wall Friction Factor Correlations	82
Table 4.5.	Percent Difference to Yalniz results	82
Table 4.6.	List of Variables and their Ranges	83

Abstract

Horizontal well performance is typically estimated using a steady-state productivity index equation similar to Joshi's productivity index, which assumes constant wellbore pressure. The wellbore pressure in a horizontal well is not constant, and the pressure drop in the horizontal section can approach the reservoir drawdown. Other authors have looked at the effects friction and influx have on the wellbore pressure by developing apparent friction factors. However, there has not been a method presented that incorporates the wellbore hydraulic effects in the productivity estimate.

This research reviews the equations used to determine horizontal well productivity, the models used for wellbore pressure, and develops a new method to estimate horizontal well productivity incorporating the effects of friction, acceleration, gravity, and fluid influx. The method is implemented in a program that is simple and quick to use. The new method calculates the wellbore pressure behavior and the apparent productivity, and compares the calculated productivity to three horizontal well productivity estimates. The proposed method reproduces the experimental results of two authors. Sensitivity analysis of the model shows the horizontal wellbore pressure has a characteristic shape on a dimensionless pressure-length graph and that fluid and well properties have significant effects on the estimate of friction factor.

Chapter 1 Introduction

Horizontal wells are drilled for the basic reason of producing more oil or gas than a vertical well. When an engineer is in the process of deciding whether to drill a horizontal or vertical well, one of the first thing he looks at is the ratio of horizontal productivity to vertical productivity. Besides being a function of the reservoir and well properties, these ratios have the underlying assumption that the wellbore pressure is constant. These ratios can lead the engineer to believe the horizontal well will produce two or more times the production of a vertical well. More often than not, the performance of the horizontal well never meets the expected productivity. Even after limiting the length of the horizontal section to reduce the frictional losses that cause a decrease in productivity, the horizontal well productivity is still based on the constant wellbore pressure assumption. Therefore, to adequately estimate horizontal well productivity, it is necessary to more accurately describe the wellbore pressure distribution along the horizontal section of the well.

1.1 History of Horizontal Wells

Over the past three decades, interest in horizontal wells has increased significantly due primarily to improvements in drilling and completion technologies necessary to successfully develop a horizontal well. During the 1920's, people began to notice that wells were not truly vertical, but were unintentionally deviated as a result

of the drilling process. As the drilling technology improved, wells were deliberately deviated in specific directions in the attempt to contact more of the reservoir. By the 1950's, the first truly horizontal wells were being drilled in Russia. Throughout the 1960's, 1970's, and 1980's, horizontal wells became very popular with the petroleum industry, due primarily to the improvements in drilling technology, the development of the positive displacement motor, and the development of measurement-while-drilling and logging-while-drilling tools and techniques. By the late 1990's, horizontal wells were being drilled routinely, in practically every location around the world.*

Horizontal wells are generally drilled for use 1) in thin, layered reservoirs, 2) in fractured reservoirs, 3) in offshore environments, and 4) in certain enhanced recovery processes, like steam flooding. They are drilled in these areas because they can contact more of the reservoir, they can reduce the pressure drawdown required to meet a specific flow rate, and they can be located in the formation such that water or gas coning are limited.¹

1.2 Review of Previous Research on Horizontal Wells

Until recently, the majority of research in horizontal wells was in the drilling and completion areas. Researchers were looking at ways to improve drilling and completion techniques, to drill longer wells, to drill multiple wells, to limit formation damage, etc. Research into the reservoir engineering aspects of horizontal wells was basically limited to computer simulations of specific reservoirs and wells due to the

* Chuck Henkes, Sperry Sun Drilling Services: oral presentation for the Society of Petroleum Engineers, University of Oklahoma Student Chapter, Norman, OK, January 20, 2000.

complexity of flow in and around the horizontal well. For the production aspects, research focused on ways to improve the performance of the horizontal well.

The general definition of well performance is the analysis of the relationship between the flow rate and the pressure drawdown between a reservoir and wellbore. As applied to horizontal wells, these analyses have been either relatively simple analytical and combination analytical/numerical solutions that could be used for any horizontal well or complex three-dimensional reservoir simulations, developed for specific wells and/or reservoirs.

Neglecting the three-dimensional simulations, the primary differences in the various horizontal well performance relations are 1) the fluid flow is either single phase (oil) or two phase (oil and gas), 2) the shape of the reservoir flow geometry ranges from simple to complex, and 3) the wellbore pressure is either assumed constant at a specific point along the wellbore, usually the midpoint of the horizontal section (uniform flux), or constant along the entire length of the wellbore (infinite conductivity). Of these differences, this research is focused on the way wellbore pressure is handled.

1.3 Problem Statement

Historically, the wellbore pressure has been defined as the pressure, either flowing or shut-in, located at the middle of the zone of interest, and constant over the entire zone, as shown in Figure 1. For a vertical well, this assumption is valid, since the perforated interval is very short compared to the length the fluid has to flow through to

reach the surface. In other words, the pressure drop, due to gravity, friction and other effects, over the perforated interval is negligible compared to the pressure drop that occurs over the tubing length, and to the drawdown between the reservoir and the wellbore. With this definition, all types of analyses were developed, like stabilized flow equations, pressure transient analysis, and well performance equations.

For a horizontal well, this definition of wellbore pressure is not appropriate. The length of the horizontal section is much greater than the thickness of the zone, and can approach the length of the vertical section, as shown in Figure 2. As fluid flows from the toe, the end or “bottom” of the horizontal well, to the heel, the start of the horizontal section, several things occur including frictional losses due to flow, kinetic losses, phase changes, gravity changes, and momentum changes from influx. These all cause changes in the pressure distribution within the horizontal section over the entire length. Therefore, the pressure in the wellbore cannot be assumed to be constant over the length of the horizontal section.

From basic fluid mechanics, in order to have flow, there has to be a change in the energy of the system, i.e. a pressure drop.² Therefore, for flow to occur from the toe to the heel of the horizontal section, the pressure must be decreasing from toe to heel. As the pressure is decreasing, the pressure difference between the reservoir and the wellbore at the influx points along the wellbore is increasing, implying an increase in the specific influx per unit length from the toe to the heel.

To adequately determine the productivity of a horizontal well, the changing wellbore pressure and influx along the length of the wellbore need to be estimated.

This research is focused on how the wellbore hydraulics affects the wellbore pressure profile and the overall horizontal well productivity.

1.4 Research Objectives

The objective of this research is to develop a simple method to estimate the horizontal productivity without relying on the unrealistic assumption of constant wellbore pressure. This is accomplished by developing a wellbore pressure equation that models the effects of wellbore hydraulics on the wellbore pressure distribution.

The new method will more accurately represent the wellbore by incorporating the effects of wellbore hydraulics to account for the changing wellbore pressure. Wellbore hydraulics is the term for the phenomena that cause changes in the flowing pressure. These phenomena include friction, acceleration, influx, and gravity. In vertical wells, reservoir fluid enters the wellbore at one point, the bottom of the well. In order to separate wellbore hydraulics effects from the reservoir performance of the well, engineers break the system at the bottom of the well at the midpoint of the perforations. As such, the wellbore hydraulic effects are considered to only occur in the tubing and do not affect the flow of fluid from the reservoir into the well. For horizontal wells, wellbore hydraulics need to be considered in the productivity equations, due to influx from the reservoir at multiple points along the length of the horizontal section.

Once developed, the new wellbore equation will be coupled to a single-phase reservoir model to allow the determination of horizontal well performance. This new

method will aid in understanding the flow and pressure behavior inside a horizontal well, predict more accurately the performance of the horizontal well, and allow comparison with current horizontal well performance equations. Knowledge of the wellbore pressure distribution, without having to physically measure it, and a well performance equation that incorporates it, will be useful in many areas: deciding whether to drill a horizontal well; determining the optimum well length; drilling, stimulation, and completion practices; and determining productivity or injectivity.

Figure 1. Schematic of a vertical well

Chapter 2 Literature Review

Well performance is an engineering analysis of the relationship between the flow rate and the pressure behavior. The ability to adequately determine the performance of a well is needed any time an engineer wants to predict flow rates or assess changes made in the production system. A well performance equation is basically a function for the flow rate in terms of the pressure difference between the reservoir and the wellbore.

$$q = f(p_r, p_{wf}) \dots\dots\dots 1$$

This chapter reviews the most commonly used well performance equations and wellbore hydraulic models.

2.1 Vertical Inflow Performance Equations

Figure 1 shows a typical representation of the production system for a vertical well. When analyzing the production system for the purpose of determining inflow performance, the system is broken at the wellbore. The pressure at the midpoint of the section open to flow is defined as the wellbore pressure. The well performance for a vertical well is dependent on the properties and characteristics of the reservoir. Wellbore hydraulics, i.e. friction, acceleration, gravity and such, are taken into account in the tubing portion of the system, and do not affect the reservoir performance of the well.

2.1.1 The Productivity Index

The simplest inflow performance equation is the productivity index (J), or straight-line inflow performance relation (IPR).³ This equation is for single-phase liquid flow, and can be written for steady state and semi-steady state flow in the reservoir. The productivity index is defined as the ratio of the flow rate to the pressure drop. Using the radial geometry, semi-steady state form of the stabilized flow equation for oil, J is defined as:

$$J = \frac{q_o}{P_r - P_{wf}} = \frac{kh}{141.2 \mu B \left[\ln \left(\frac{r_e}{r_w} \right) - \frac{3}{4} + s \right]} \dots\dots\dots 2$$

This relation only describes the performance of a well when the reservoir fluid is above the bubble point of the fluid, which is generally only early in the development, or for water wells. There is a similar form for single-phase gas flow.

2.1.2 Inflow Performance Relationships (IPR)

For multiphase flow, oil and gas, and oil, gas, and water, the straight-line relationship between flow rate and pressure drawdown does not hold. Evinger and Muskat⁴ were the first to show this non-linear behavior based on their multiphase flow equations.

$$q_o = C \int_{P_{wf}}^{P_r} \frac{k_{ro}}{\mu_o B_o} dp \dots\dots\dots 3$$

$$q_w = C \int_{P_{wf}}^{P_r} \frac{k_{rw}}{\mu_w B_w} dp \dots\dots\dots 4$$

$$q_g = C \int_{p_w}^{p_r} \left[\frac{k_{rg}}{\mu_g B_g} + \frac{k_{ro} R_s}{\mu_o B_o} + \frac{k_{rw} R_{sw}}{\mu_w B_w} \right] dp \dots\dots\dots 5$$

These equations are the principle descriptions for fluid flow in most of the computer simulations in use. The early difficulty in integrating these equations due to the pressure dependence of the fluid mobility and the uncertainty in the relative permeability values led to the development of several empirical relationships.⁴⁻⁹

Appendix A summarizes the commonly used vertical relationships.

2.2 Horizontal Inflow Performance Equations

For horizontal wells, the simplest way to model the well is to assume it behaves like a vertical well lying on its side. With this assumption, the equations derived for vertical wells can be applied to a horizontal well. However, there are limitations to this assumption. The most important one is the fluid flow must be radial. Therefore, the well must fully penetrate the reservoir, implying the well length and reservoir thickness must be very large. As a result, the majority of the efforts in understanding horizontal well performance emphasized various boundary conditions¹¹, or improvements in simulation models. Later, as the understanding increased, research began in formation damage, non-Darcy flow, arbitrary completions, and transient pressure theory. Table 2.1 lists the commonly used well performance equations for horizontal wells, which are discussed in the following subsections.

2.2.1 Horizontal Productivity Index (J_h)

The first to look at how the performance of a horizontal well differed from a vertical well was Borisov,¹² in 1964. He developed a theoretical model for the productivity of a horizontal well for the following conditions: steady state flow; single-phase, incompressible fluid; isotropic, homogenous reservoir with no formation damage; centered within the formation thickness; and an elliptical drainage area.

Borisov's equation is

$$J_h = \frac{q_h}{p_r - p_{wf}} = \frac{2\pi k_h h}{\mu_o B_o \left[\ln\left(\frac{4r_{eh}}{L}\right) + \frac{h}{L} \ln\left(\frac{h}{2\pi r_w}\right) \right]} \dots\dots\dots 6$$

This relationship assumed the well was like an infinite-acting fracture, i.e. constant wellbore pressure. However, one problem was in determining the horizontal drainage radius, r_{eh} . For an elliptical drainage area, the area was:

$$A = \pi a b \dots\dots\dots 7$$

where a and b were the major and minor half axes of the ellipse, respectively. Both a and b were functions of the drainage radius of a vertical well, r_{ev} , that could be drilled in the same lease, and r_{ev} was a function of the number of vertical wells. A second problem, which was not really identified until 1989, was the length of the horizontal section. Theoretically, the productivity index could become infinitely large as the length becomes infinite. Physically, there should be a limiting length, above which the productivity decreases due to friction and other pressure losses occurring in the wellbore.

In 1983, Giger¹²⁻¹⁴ developed a productivity index for a horizontal well centered in the formation. Other assumptions were steady state flow, single phase, slightly compressible fluid, isotropic, homogeneous reservoir, no formation damage, and an elliptical drainage area. Giger, Reiss, and Jourdan,¹⁴ in 1984, showed that the equation could be used for anisotropic reservoirs with the substitution of an equivalent permeability. Giger's equation is

$$\frac{q_o}{P_e - P_{wf}} = \frac{2\pi k_h L}{\mu_o B_o \left[\frac{L}{h} \ln \left(\frac{1 + \sqrt{1 - \left[\frac{L}{2r_{eh}} \right]^2}}{\frac{L}{2r_{eh}}} \right) + \ln \left(\frac{h}{2\pi r_w} \right) \right]} \dots\dots\dots 8$$

and the equivalent permeability is

$$k_h' = (k_h k_v)^{\frac{1}{2}} \dots\dots\dots 9$$

This equation has the same problems as Borisov's related to defining r_{ev} for use in r_{eh} , not recognizing a limiting length, and constant wellbore pressure.

Joshi^{12,15,16} in 1986, developed an equation for horizontal well productivity. Joshi's equation was for steady state flow of a single phase, slightly compressible fluid in a homogeneous, anisotropic reservoir, with no formation damage. Pressures at the outer boundary and the wellbore are constant, and the horizontal section is a known distance from the top boundary. Joshi states that this equation is only mathematically approximate and that the results obtained only approximate the true well performance. Joshi's equations for oil wells and the corresponding one for gas wells are two of the

most commonly used equations for the determination of horizontal well performance.

Joshi's horizontal oil well equation is

$$\frac{q_o}{p_e - p_{wf}} = \frac{2\pi \beta k_h h}{\mu_o B_o \left[\ln \left(\frac{a + \sqrt{a^2 - \left[\frac{L}{2} \right]^2}}{\frac{L}{2}} \right) + \beta \frac{h}{L} \ln \left(\frac{\left(\frac{\beta h}{2} \right)^2 + \beta^2 \delta^2}{\frac{\beta h r_w}{2}} \right) \right]} \quad \dots\dots\dots 10$$

This equation has several supporting equations, which need to be determined.

$$a = \frac{L}{2} \left[0.5 + \sqrt{0.25 + \left(\frac{2r_{eh}}{L} \right)^4} \right]^{0.5} \quad \dots\dots\dots 11$$

$$\beta = \sqrt{\frac{k_h}{k_v}} \quad \dots\dots\dots 12$$

$$\delta = \frac{h}{2} - d \quad \dots\dots\dots 13$$

where d is the distance from the top boundary to the center of the wellbore. This equation has the same limitations as Borisov's and Giger's; however, it is the only one to not require the well to be centered within the formation.

In 1988, Babu and Odeh^{17, 18, 19} developed a horizontal well performance equation for semi-steady state flow and a uniform flux assumption at the wellbore instead of infinite conductivity. Their other assumptions were a box shaped drainage volume, homogeneous, anisotropic reservoir with sealed boundaries, single-phase, slightly compressible fluid. To simplify the uniform flux assumption, they stated that the pressure at the midpoint of the horizontal well is representative of the well. To

utilize their equation requires the determination of two parameters, a fairly simple geometric shape factor and a complicated skin factor due to partial penetration of the formation and formation damage.

$$\frac{q_o}{P_r - P_{wf}} = \frac{0.00708 b \sqrt{k_x k_z}}{\mu_o B_o \left[\ln \left(\frac{\sqrt{a h}}{r_w} \right) + \ln(C_H) - 0.75 + S_R \right]} \dots\dots\dots 14$$

This equation is the first to attempt to deal with a changing wellbore pressure, by using the uniform flux assumption, i.e. constant influx rate. Because it was solved for the midpoint of the well length, the original partial differential equations must be solved for any other position. The other major drawbacks are the very complicated skin factor determination and estimation of the shape factor.

In 1990, Renard and Dupuy²⁰ attempted to generalize Joshi's and Giger's well productivity equations for a single-phase, incompressible fluid under steady state flow in an anisotropic homogeneous reservoir with and without damage. Their equation can only be used for circular, ellipsoidal, or rectangular drainage shapes.

$$\frac{q_o}{P_r - P_{wf}} = \frac{2 \pi k_h h}{\mu_o B_o \left[\cosh^{-1}(X) + \beta \frac{h}{L} \ln \left(\frac{h}{2 \pi r_w} \right) + \beta \frac{h}{L} s \right]} \dots\dots\dots 15$$

This equation has the same advantages and disadvantages as Joshi's and Giger's equations, as well as difficulty in determining the inverse hyperbolic cosine of X, which is a function of the drainage shape.

In 1996, Elgaghad, Osisanya and Tiab²¹ developed a simplified productivity index based on a more complex drainage area than a simple ellipse or rectangle. They

assumed the reservoir could be considered as a stacked series consisting of a semicircle and two rectangles. Their equation for single-phase, incompressible, fluid under steady state flow in an isotropic homogeneous reservoir with no formation damage is simpler than the others in terms of the auxiliary equations, and does not require the determination of the horizontal drainage radius, r_{eh} . Their equation only requires the determination of a parameter C, which is based strictly on the length of the horizontal section.

$$\frac{q_o}{P_r - P_{wf}} = \frac{2\pi k_h h}{\mu_o B_o \left[\frac{\ln\left(\frac{h}{2r_w}\right)}{\frac{L}{h}} + \left(0.25 + \frac{C}{L}\right)\left(\frac{1}{r_w} - \frac{2}{h}\right) \right]} \dots\dots\dots 16$$

To compare the relationships, productivity indices were calculated for each method using the reservoir and well data shown in Table 2.2. Table 2.3 shows the resulting comparison between the four horizontal well productivity relations of Joshi, Giger, Borisov, and Elgaghad. As can be seen, there is a fairly wide range in the productivity index depending on which model is used. All of the papers compared their results to the results of one or two other relations, as in Table 2.3. There were no examples of the relations applied to actual field data or compared to actual performance to see how the equations predicted the actual productivity. The lack of a specific productivity index comparison to actual well performance makes it difficult to judge the claims of how “better” one equation is compared to another.

Finally, except for Babu and Odeh, all of the horizontal productivity equations assumed the wellbore pressure was constant over the entire length of the horizontal

section. The primary reason given for this assumption was the wellbore pressure drop was negligible compared to the reservoir drawdown. Only Babu and Odeh acknowledge this assumption was in error

2.2.2 Multiphase Horizontal Well Performance

This research is primarily interested in single-phase fluid flow in the reservoir and wellbore. However, the same principles can be applied to multiphase flow in the wellbore or in both the wellbore and reservoir to determine how wellbore pressure behavior affects the reservoir performance. A short summary of the multiphase horizontal well performance methods is presented.

There has been work to develop Vogel-type IPR equations for two-phase flow of oil and gas. These equations are empirical correlations based on the results generated from many simulator runs, and are applicable for many fluid, reservoir, and well properties.

Bendakhlia and Aziz²² were the first to develop a Vogel-type IPR for a horizontal well in 1989. They assumed the reservoir pressure was below the bubble point pressure, the well was centrally located and fully penetrating, the drainage volume was box shaped, the reservoir was isotropic and homogeneous, flow was steady state, and the wellbore pressure was constant the entire length of the wellbore. Their equation is simple to use and is not sensitive to changes in fluid or well properties. It is valid for any known stage of depletion below the bubble point, and requires the determination of two parameters, V and n , which are functions of the stage of depletion.

$$\frac{q_o}{q_{o,max}} = \left(1 - V \frac{p_{wf}}{p_r} - (1-V) \left(\frac{p_{wf}}{p_r} \right)^2 \right)^n \dots\dots\dots 17$$

Cheng²³ developed a series of Vogel-type IPR equations based on the regression analysis of full three-dimensional simulation of wells ranging from vertical to horizontal. The simulation was developed for two-phase, steady state flow, in a homogeneous, isotropic reservoir with a constant outer and inner boundary pressures, and a centrally located well. Unlike Bendakhlia and Aziz's equation, Cheng's equation does not require the determination of parameters based on depletion and recovery factors.

$$\frac{q_o}{q_{o,max}} = 0.9885 + 0.2055 \left(\frac{p_{wf}}{p_r} \right) - 1.1818 \left(\frac{p_{wf}}{p_r} \right)^2 \dots\dots\dots 18$$

Wang²⁴ developed two IPR equations for horizontal well performance, one based on the depletion stage and one general for an unknown depletion stage. His assumptions were a centrally located, fully penetrating horizontal well, a rectangular drainage area, and heterogeneous, anisotropic reservoir with boundary dominated flow regimes. Wang's depletion based equation gives similar results as Bendakhlia and Aziz's, but the general IPR may overestimate the performance because it does not require knowledge of the depletion stage.

$$\frac{q_o}{q_{o,max}} = 1 - d \left(\frac{p_{wf}}{p_r} \right) - (1-d) \left(\frac{p_{wf}}{p_r} \right)^2 \dots \text{depletion} \dots\dots\dots 19$$

$$\frac{q_o}{q_{o,max}} = 1 - 0.4533 \left(\frac{p_{wf}}{p_r} \right) - 0.5467 \left(\frac{p_{wf}}{p_r} \right)^2 \dots \text{general} \dots\dots\dots 20$$

In addition to the two phase IPRs, there has been extensive research in site-specific full three-dimensional reservoir simulations in order to determine horizontal well performance. The results from the simulators are reasonably good, however, they require a significant amount of preparatory work and computation. As a result they are not very “portable” and do not provide “quick” determination of well performance. The major problems with the simulations are how the wellbore is treated and how the reservoir is coupled to the wellbore.

In addition to the single well performance relations, there has been research into estimating the well performance of more complex conditions, like heterogeneous reservoirs and multi-lateral horizontal wells.

2.3 Wellbore Hydraulics in the Horizontal Well

The generally accepted definition of wellbore flowing pressure, p_{wf} , is the pressure at the midpoint of the zone open to flow and is essentially constant over the entire zone. This definition is appropriate for vertical wells because the length of the zone open to flow is relatively short and the effects of friction, acceleration, and gravity are negligible. The length of the zone open to flow for a horizontal well can extend up to thousands of feet, especially for open-hole and slotted liner completions, nor is the well perfectly horizontal. As a result, the effects of friction, acceleration, and gravity are not negligible and the assumption of constant wellbore pressure is incorrect.

2.3.1 Horizontal Pipe Flow Equations

For fluid to flow in any type of system, there must be a change in the energy of the system. This change in energy can be from a change in pressure, velocity, elevation, frictional losses or other losses, and is evident in the energy equation. The following is a form of the energy equation:

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 + \frac{f L V^2}{D 2g} + h_{\text{additional}} \dots\dots\dots 21$$

which contains the pressure changes, kinetic changes, potential changes, frictional losses, and additional losses due to other effects.

For a horizontal well to maintain flow from the toe of the well to the heel, there must be some kind of pressure drop. Most models for horizontal wells assume the horizontal well to be like an infinite conductivity fracture, meaning the pressure drop inside the fracture is so small it can be assumed to be negligible. However, depending on the length and diameter of the wellbore, the type of fluid, the reservoir permeability, and the flow rate, the frictional and other pressure losses within the wellbore can become significant, resulting in pressure drops in the wellbore approaching that of the reservoir drawdown.¹²

Modeling of the horizontal wellbore began with the assumption the horizontal wellbore could be represented by a horizontal pipe. With this assumption, the basic pipe flow equation can be derived from the equations of continuity, momentum, and energy, as

$$\frac{dp}{dl} = \left(\frac{dp}{dl} \right)_{\text{gravity}} + \left(\frac{dp}{dl} \right)_{\text{acceleration}} + \left(\frac{dp}{dl} \right)_{\text{friction}} \dots\dots\dots 22$$

Research started first by assuming gravity and acceleration effects were negligible. Later works began adding in other effects, like influx losses.

2.3.2 Frictional effects on wellbore pressure

Dikken³⁴ first developed an analytical model for single phase, turbulent, steady flow in a horizontal well. He showed the flow in the horizontal wellbore is usually in transient or turbulent flow, and as such, the infinite conductivity assumption should not be considered. Using a volumetric balance to couple the wellbore to the reservoir and the Blasius equation for turbulent flow, Dikken developed the following equation:

$$\frac{d^2 q_w(x)}{dx^2} = c_r c_w q_w(x)^{2-\alpha} \dots\dots\dots 23$$

where c_r is the productivity index per unit length of horizontal wellbore, c_w is the flow resistivity inside the horizontal wellbore, and α is the Blasius parameter. He concluded that frictional losses would result in total production leveling off as well length increases. This is a direct contradiction of the productivity indices theory that productivity will continue to increase as length increases.

Brice³⁵ simulated Dikken's model and confirmed the presence of the pressure drop. He concluded that simulators do not predict the pressure drop accurately unless they are corrected by a pseudo-diameter, which is a function of the perforations, in order to represent the actual flow conditions.

Novy³⁶ generalized Dikken's work for the purpose of developing guidelines for the quick determination of the horizontal length and production for which frictional losses would not be significant. Novy developed a quick graphical check based on flow rate, well length and pipe diameter that would allow the engineer to determine if

friction would affect production. He concluded that if the ratio of wellbore pressure drop to reservoir drawdown at the heel is greater than 10% then frictional losses will reduce production. Two problems with Dikken and Novy are the methods and conclusions were developed for a specific drainage volume and well geometry, and the unconventional form of the wellbore pressure model makes incorporation into a reservoir model difficult.

Sharma et al.³⁷ rewrote the mechanical energy equation for flow in the wellbore to more easily couple it to the reservoir flow equations. They assumed friction and gravity significantly affect the pressure drop, while kinetic effects were negligible. They defined an “effective wellbore porosity,” ϕ_{we} , and an “effective wellbore permeability,” k_{we} , which were functions of the well’s geometry and flow regime, respectively. Using these two definitions, they developed the following equation that could be incorporated easily into a reservoir simulator, as it has the same form as Darcy’s law.

$$q_{well} = - \frac{k_{we} A_{well} \Delta\Phi}{\mu \Delta x} \dots\dots\dots 24$$

in which k_{we} was defined with any Fanning friction factor correlation. For laminar flow, the equation simplified to the Hagan-Poiseuille equation for laminar flow in pipes. They compared their results to those for an equivalent source/sink model and concluded that friction and gravity effects do result in significant variations in production rates. Because this equation was developed to represent the wellbore in a

reservoir simulator, it is difficult to define k_{we} and ϕ_{we} for use in an analytical/numerical solution, like a productivity index.

Ozkan³⁸ developed a general semi-analytical model that coupled wellbore hydraulics to reservoir flow. They developed their model for use with any regular pipe friction factor correlation, and made no assumptions as to how the fluid entered the wellbore, except for assuming no axial flow and no flow at the toe. They developed a model for the wellbore pressure that is a function of the dimensionless Reynold's number, well conductivity, and flux. The Reynold's number is a function of the location and flux distribution, and the well conductivity is a function of the well geometry and reservoir permeability and height. After numerically solving the equation, they concluded that from a qualitative standpoint, the horizontal well responses can be correlated through the distinct shapes of different values of well conductivity for given values of dimensionless length, wellbore radius, distance from the bottom boundary, and Reynold's number on graphs of dimensionless wellbore pressure versus dimensionless time. Quantitative determination of the effect of wellbore hydraulics, due to the complexity of the interaction between the wellbore and the reservoir, is only possible with the use of a model for the specific problem.

Joshi¹² presented an equation to determine the pressure drop in the wellbore for a single-phase fluid, flowing through a smooth liner and assuming all fluid enters the toe. With this assumption, the equation

$$\Delta p = \frac{(1.14644 \times 10^{-5}) f \rho q^2 L}{d^5} \dots\dots\dots 25$$

determined the maximum pressure drop for a given flow rate and well length. Joshi stated that influx along the wellbore will result in a lower pressure drop but he did not give any indication of the magnitude.

2.3.3 Additional effects on wellbore pressure

Sarica et al.³⁹ developed a semi-analytical model for gas flow in horizontal wellbores based on the work of Okzan.³⁸ They incorporated the effects of friction and acceleration, and showed that their model provides results that have qualitative agreement with field observations.

Ouyang et al.⁴⁰ developed a model that includes the effects of friction, acceleration and gravity on the horizontal wellbore pressure. They concluded that acceleration might be as important as friction on the wellbore pressure drop, depending on the pipe geometry, fluid properties, and flow conditions.

Asheim et al.⁴¹ developed an “effective friction factor” correlation, which incorporates wall friction and influx through perforations to be substituted for the friction factor in the frictional pressure drop equation. The new friction factor is the sum of a wall friction factor and an additional factor, which is a function of the influx rate per perforation and the overall pipe flow rate.

$$f_t = f_w + f_p = 0.16 R_*^{0.19} + 4D \frac{q_L}{q} \dots\dots\dots 26$$

This equation uses the Blasius correlation for a smooth pipe and was empirically derived from experiments for influx through a perforation. Asheim never shows the theoretical or empirical basis for the addition of the two friction factors.

Su and Gudmundsson⁴² determined experimentally the various factors that contribute to the pressure drop in a perforated pipe, but did not develop a relationship. They concluded that the total pressure drop in a perforated pipe is the sum of the pressure drop due to wall friction, acceleration due to fluid flow, perforation roughness, and fluid mixing. They state that the effect of the pressure drop due to perforations and fluid mixing is an irreversible pressure drop and will cause a maximum reduction in the wellbore pressure of about 3.4% of the ordinary frictional pressure drop.

Yuan⁴⁴ and Yuan et al.⁴⁵ looked at the apparent friction factor for friction and influx effects for a single perforation. They experimentally developed a friction factor equation, similar to Asheim's.

$$f_t = f_w + f_p = 80.45 R_e^{-0.146} + 2 \left(\frac{d}{\Delta x} \right) \left(\frac{q_{in}}{Q_1 + q_{in}} \right) \dots\dots\dots 27$$

They state their friction factor correlation matches with Blasius's correlation for a smooth pipe when there was no influx, but this is only true at large Reynold's numbers. They extended the single perforation concept to a multiple perforations by a series of single perforation elements. The problem with this is that, in order to be similar to the experimental conditions, the influx to main flow rate ratio needs to be constant, which implies the well length can be infinite. Yuan⁴⁶ did the same experiment with a slotted liner, and developed a similar equation with a different wall friction factor relation. Like Asheim, Yuan does not show the friction factors can be combined to an apparent friction factor.

Yalniz and Ozkan⁴⁷ looked at an apparent friction factor for influx through one perforation. Using Asheim's⁴¹ suggested apparent friction factor format and the basic energy equation, they developed new correlations to compute wall and perforation friction factors. They assumed the wall friction factor was the apparent friction factor when no influx was present and did not change with influx, and they do not show the theoretical proof of the apparent friction factor.

Table 2.1 Horizontal Well Performance Equations

Borisov	$J_h = \frac{q_h}{p_r - p_{wf}} = \frac{2\pi k_h h}{\mu_o B_o \left[\ln\left(\frac{4r_{eh}}{L}\right) + \frac{h}{L} \ln\left(\frac{h}{2\pi r_w}\right) \right]}$
Giger	$\frac{q_o}{p_r - p_{wf}} = \frac{2\pi k_h L}{\mu_o B_o \left[\frac{L}{h} \ln \left(\frac{1 + \sqrt{1 - \left[\frac{L}{2r_{eh}} \right]^2}}{\frac{L}{2r_{eh}}} \right) + \ln\left(\frac{h}{2\pi r_w}\right) \right]}$
Joshi	$\frac{q_o}{p_r - p_{wf}} = \frac{2\pi \beta k_h h}{\mu_o B_o \left[\ln \left(\frac{a + \sqrt{a^2 - \left[\frac{L}{2} \right]^2}}{\frac{L}{2}} \right) + \beta \frac{h}{L} \ln \left(\frac{\left(\frac{\beta h}{2} \right)^2 + \beta^2 \delta^2}{\frac{\beta h r_w}{2}} \right) \right]}$
Babu and Odeh	$\frac{q_o}{p_r - p_{wf}} = \frac{0.00708 b \sqrt{k_x k_z}}{\mu_o B_o \left[\ln\left(\frac{\sqrt{a} h}{r_w}\right) + \ln(C_H) - 0.75 + S_R \right]}$
Renard and Dupuy	$\frac{q_o}{p_r - p_{wf}} = \frac{2\pi k_h h}{\mu_o B_o \left[\cosh^{-1}(X) + \beta \frac{h}{L} \ln\left(\frac{h}{2\pi r_w}\right) + \beta \frac{h}{L} s \right]}$
Elgaghad, Osisanya and Tiab	$\frac{q_o}{p_r - p_{wf}} = \frac{2\pi k_h h}{\mu_o B_o \left[\frac{\ln\left(\frac{h}{2r_w}\right)}{\frac{L}{h}} + \left(0.25 + \frac{C}{L}\right) \left(\frac{1}{r_w} - \frac{2}{h}\right) \right]}$

Table 2.2. Example Reservoir, Fluid, and Well Data

Reservoir Data	Fluid Data	Well Data
$k_h = 75$ md	$\mu_o = 0.62$ cp	$L = 1000$ ft
$k_v = 75$ md	$B_o = 1.35$ rb/STB	$r_w = 0.365$ ft
$H = 160$ ft		$r_{ch} = 1053$ ft
		$A = 80$ acres

Table 2.3. Comparison of the Horizontal Productivity Index Relations

model	J STB/day-psi	% difference
Joshi	44.41	-
Borisov	48.29	8.74
Giger	49.74	12.0
Elgaghad et al	44.83	0.946

Chapter 3 Model Development

The purpose of this research is to develop a new method to determine the productivity of a horizontal well incorporating the wellbore hydraulics effects. In developing this method, there are three elements to the problem that must be examined: the wellbore, the reservoir, and how to couple the two together. Figure 3 shows a sketch of the overall problem. The model consists of a horizontal wellbore of length, L , and diameter, D . It is centered within a homogeneous, anisotropic reservoir of thickness, h , vertical permeability, k_v , horizontal permeability, k_h , and average reservoir pressure, p_{bar} . The wellbore has the potential for influx along the entire length plus the toe of the section.

3.1 The continuity equation

Two components of the model, the reservoir and the wellbore, are coupled together through the continuity equation. Figure 4 shows the control volume for the problem. This equation states that what flows into the control volume must equal what flows out plus what is accumulated in the control volume. Assuming steady-state flow of an incompressible fluid, and no accumulation within the control volume, the continuity equation becomes

$$\sum \rho \vec{V} \cdot \vec{A} = 0 \dots\dots\dots 28$$

For an incompressible fluid, this mass balance becomes:

$$-\rho V_1 A_1 + \rho_2 V_2 A_2 = \rho V_3 A_3 \dots\dots\dots 29$$

Assuming $A_1 = A_2 = \pi r_w^2$, $A_3 = 2\pi r_w x$, and $\rho_1 = \rho_2 = \rho_3$, this equation simplifies to:

$$V_{in} = \frac{r_w}{2} \frac{dV}{dx} \dots\dots\dots 30$$

3.2 The energy equation

The basis for the majority of fluid flow equations is the general energy equation, which states that the energy entering a control volume plus the work done on or by the fluid, plus the heat added or taken away, plus the energy accumulated in the control volume must equal the energy leaving the control volume.^{2,43} Assuming steady state flow and a single-phase, incompressible fluid under irreversible flow, the derivation goes as follows:

$$E_{in} = E_{out} + E_{lost} \dots\dots\dots 31$$

$$\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \right) + \left(\frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \right) = \left(\frac{p}{\gamma} + \frac{V^2}{2g} + z \right) + \frac{f x \bar{V}^2}{D 2g} \dots\dots\dots 32$$

Defining the potential as:

$$\Phi = p + \gamma z \dots\dots\dots 33$$

and taking the derivative with respect to x, Eq. 32 becomes

$$\frac{d}{dx} \left(\frac{\Phi_1}{\gamma} + \frac{V_1^2}{2g} \right) + \frac{d}{dx} \left(\frac{\Phi_3}{\gamma} + \frac{V_3^2}{2g} \right) = \frac{d}{dx} \left(\frac{\Phi}{\gamma} + \frac{V^2}{2g} \right) + \frac{d}{dx} \left(\frac{f x \bar{V}^2}{D 2g} \right) \dots\dots\dots 34$$

Fixing the values at position 1 as constants, and assuming V_3 has no axial component and Φ_3 represents the average reservoir potential, the left-hand side of Eq. 34 is zero.

$$\frac{d}{dx} \left(\frac{\Phi}{\gamma} + \frac{V^2}{2g} \right) + \frac{d}{dx} \left(\frac{f x \bar{V}^2}{D 2g} \right) = 0 \dots\dots\dots 35$$

Defining the average velocity as:

$$\bar{V} = \frac{V_1 + V}{2} \dots\dots\dots 36$$

Eq. 35 simplifies to

$$\frac{d}{dx} \left(\frac{\Phi}{\gamma} + \frac{V^2}{2g} \right) + \frac{f x \bar{V}}{2Dg} \frac{dV}{dx} + \frac{f \bar{V}^2}{2Dg} = 0 \dots\dots\dots 37$$

Substituting Eq. 30 into Eq. 37 and converting back to pressure, Eq. 37 becomes:

$$\frac{dp}{dx} = -\rho V \frac{dV}{dx} - \rho g \sin \theta - \frac{f \rho \bar{V}^2}{2D} - \frac{K \rho V_{in}^2}{2} \dots\dots\dots 38$$

where K is the influx loss coefficient, dp/dx is the pressure gradient, VdV/dx is the kinetic (or acceleration) term, $\rho g \sin \theta$ is the gravity term, and the last two terms are the frictional losses and influx losses, respectively.

Assumptions inherent in Eq. 38 are a single-phase, incompressible fluid, and steady state flow.

3.2 The Reservoir

In the reservoir, the fluid is assumed to be single phase, incompressible, and under steady state laminar flow. The reservoir is assumed to be homogeneous, with

anisotropic permeability in the horizontal and vertical directions. Flow in the reservoir is modeled after Darcy's law, which states:

$$q_{res} = -\frac{k A}{\mu} \frac{dp}{dr} \dots\dots\dots 39$$

When Darcy's law is solved for a single-phase fluid on a per length basis, it becomes

$$q_{res} = J_s (\bar{p} - p_w) \dots\dots\dots 40$$

where J_s is the productivity index per unit length of the wellbore,

$$J_s = \frac{J}{L} \dots\dots\dots 41$$

and is a function of the fluid properties, the reservoir properties, the wellbore properties, and the geometry of the system. The various forms of the productivity index (J) for a horizontal well were shown in Table 2.1. The basic difference between the various productivity indices was the geometry of the system, being cylindrical, ellipsoidal, or composite, as shown in Figure 5.

3.4 The System of Equations

The horizontal well performance problem consists of three equations: a wellbore equation, a reservoir equation, and a volume balance. Eqs. 30, 38, and 40 can be combined into a second order, non-linear partial differential equation.

$$\frac{dp}{dx} = -\frac{2\rho V J_s}{r_w A_{in}} (\bar{p} - p) - \rho g \sin \theta - \frac{f \rho \bar{V}^2}{2D} - \frac{K \rho}{2} \left(\frac{J_s}{A_{in}} \right)^2 (\bar{p} - p)^2 \dots\dots\dots 42$$

The system of equations is solved simultaneously using a Runge-Kutta fourth order method in the following order:

$$\frac{dp}{dx} = -\rho V \frac{dV}{dx} - \rho g \sin \theta - \frac{f \rho \bar{V}^2}{2D} - \frac{K \rho V_{in}^2}{2} \dots\dots\dots 43$$

$$\frac{dV}{dx} = \frac{2V_{res}}{r_w} \dots\dots\dots 44$$

and

$$V_{res} = \frac{q_{res}}{A_{in}} = \frac{J_s}{A_{in}} (\bar{p} - p) \dots\dots\dots 45$$

In summary, this chapter has developed a system of equations, consisting of a reservoir component, a wellbore component, and a volume balance. The wellbore component determines the pressure behavior in the wellbore, and considers the friction, acceleration, gravity, and influx effects on the pressure drop. With this system of equations, the pressure behavior of the wellbore can be determined, the productivity of the horizontal well can be determined, and the effects of wellbore hydraulics on the horizontal well productivity can be analyzed.

Figure 3. The schematic for the horizontal well problem.

Figure 4. The control volume.

Figure 5. The typical geometries used for a horizontal well.

Chapter 4 A New Productivity Index Method with Wellbore Hydraulics

The development of the wellbore pressure model was presented in the previous chapter. The system of equations to be solved for wellbore flowing pressure and flow rate was shown. This system of equations was solved with a Runge-Kutta method and programmed using Microsoft Visual Basic6. This chapter discusses the various aspects of the program, presents the results and sensitivity study of the various parameters.

4.1 Discussion of the Program

The system of equations to determine wellbore pressure and flow rate was derived in Chapter 3. These equations are:

$$\frac{dp}{dx} = -\rho V \frac{dV}{dx} - \rho g \sin \theta - \frac{f \rho \bar{V}^2}{2D} - \frac{K \rho V_{in}^2}{2} \dots\dots\dots 43$$

$$\frac{dV}{dx} = \frac{2V_{res}}{r_w} \dots\dots\dots 44$$

and

$$V_{res} = \frac{q_{res}}{A_{in}} = \frac{J_s}{A_{in}} (\bar{p} - p) \dots\dots\dots 45$$

Using a Runge-Kutta fourth order numerical method to solve this system, the solution was programmed in Visual Basic6. The program has four principal sections: the input section, the preliminary calculations, the wellbore pressure calculations, and the well performance calculations. In the input section, the reservoir and well properties are declared. Figure 6 shows the primary program screen. The preliminary calculations

section calculates the auxiliary values needed by three of the horizontal well productivity index equations presented in Table 2.1. Eqs. 43, 44, and 45 are the groundwork for the wellbore pressure section. This section calculates the pressure in the wellbore, the velocity of the fluid in the wellbore, the influx from the reservoir at each point along the wellbore, the specific productivity for each segment of the well, and the apparent productivity of the horizontal well. The well performance section calculates the productivity index for three of the most commonly used horizontal well productivity equations, Joshi, Borisov, and Elgaghad, in order to compare their results with the apparent well productivity as determined by the wellbore part of the program. Table 4.1 summarizes what each section of the program computes, and the input data required.

4.2 Comparison to theoretical and experimental results

Brekke⁴⁸ presented an example field data set, from which almost every paper that has looked at wellbore hydraulics has used as their application example. The data set is a thin oil rim located between a large gas cap and an active aquifer with an average reservoir pressure of 2300 psia. The production is restricted to a maximum reservoir drawdown of 15-20 psi in order to prevent gas coning. Table 4.2 shows the data set, which serves as the base case. Unless otherwise stated in a subsection, each of the following comparisons uses this data set.

4.2.1 Examination of data set using Novy

First, the data set should be checked to ensure that the wellbore hydraulics would have a significant effect on the productivity. This is done using Novy's figures. Novy³⁶ stated that the friction loss would start to hinder production when the wellbore pressure drop was 10% or more of the reservoir drawdown at the producing end (the heel). Figure 7 shows Novy's graphical solution for oil wells, which is used to determine where friction is negligible. For the given flow rate of 2700 rb/day and well length of 2625 feet, in order for frictional losses to have negligible effect on the productivity, the well diameter needs to be greater than 6 inches. The data set's well diameter is 2.5 inches. Therefore, wellbore hydraulics is expected to hinder production.

4.2.2 Comparison to maximum wellbore pressure drop

Figure 8 shows the wellbore pressure drop along the length of the wellbore, as determined by the program and Joshi's pressure drop equation, Eq. 25. Joshi's pressure drop equation assumes all of the fluid enters at the toe of the horizontal section and flows through the entire well length. The calculated wellbore pressure drop is the estimated maximum value. Joshi¹² continually stated that for influx along the wellbore, the pressure drop would be less, but failed to present how much less. Both lines are for a flow rate leaving the horizontal section of the well at 2700 rb/day. Equation 25 estimates the maximum pressure drop in the horizontal section for 2700 rb/day through a smooth pipe is 39 psi (for commercial grade steel pipe, the pressure drop is almost four times the smooth pipe value). This 39 psi pressure drop is nearly

double the data set's maximum reservoir drawdown of 20 psi. With influx along the entire wellbore, the program calculates a pressure drop of 7.15 psi and a reservoir drawdown at the heel of 9 psi.

4.2.3 Comparison to Yuan and Asheim

Table 4.3 shows the pressure drop Yuan^{44, 45} obtained with his apparent friction factor and the pressure drops as determined by Asheim's equation,⁴¹ a regular pipe correlation, and the program. Yuan^{44, 45} estimates a pressure drop of 5.25 psi and the program calculates a pressure drop of 7.15 psi, which is within the range of the other values.

Yuan^{44,45} stated that because the differences between the results of the three correlations he used were so great, it was important to use the proper total friction factor correlation, but did not indicate how to determine the proper one. Yuan⁴⁴ and Asheim⁴¹ use the same format for their apparent friction factors, a wall friction factor plus a perforation friction factor. The wall friction factor is some function of Reynold's number,

$$f_w = a R_e^b \dots\dots\dots 46$$

and the perforation friction factor is a constant multiplied by the ratio of influx to main flow rate, Equations 26 and 27. When examining the two perforation friction factors carefully, it becomes apparent the only difference between the two is a factor of two. Therefore, the wall friction factor is more important than they indicate. To show this, Table 4.4 presents pressure drops estimated using the friction factor correlations of Yuan, Asheim, Blasius, and Jain, and compares the results by using the same

correlations for the wall friction factor. When the same wall friction factor correlations are used, the differences in the estimated pressure drops are not as significant. Therefore, it is important to choose the proper wall friction factor correlation whose experimental conditions best coincide with the given flow and pipe conditions.

4.2.4 Comparison to Yalniz and Ozkan

Yalniz and Ozkan⁴⁷ presented their experimental results for flow through a single perforation. Figures 9, 10, 11, and 12 show the pressure drop, defined as:

$$\Delta p = p(x) - p_{heel} \dots\dots\dots 47$$

from the toe to the heel for the test section for different influx to main flow rate ratios. Figure 9 is a no influx case. Yalniz and Ozkan assumed a smooth pipe assumption when they developed their wall friction factor correlation from their observations for the no influx case. Assuming smooth pipe in the program, the program's results do not match the experimental results. This is because the pipe is not truly smooth, but has been altered by the perforation and the pressure ports. Even the authors' conclusion that the experimental results reasonably match the Blasius correlation is only correct for high Reynold's numbers ($Re > 20000$). When the roughness is considered, the program predicts the experimental results reasonably well, Table 4.5.

Figure 10 is for a low influx to main flow rate ratio and shows a no influx and influx rate case. For this low rate ratio, the differences between the no influx and influx cases are small, and the pressure drop across the perforation is not noticeable. Figure 11 is for a higher influx to main flow rate ratio and Figure 12 is for a high influx to main flow rate ratio. As the influx to main flow rate ratio increases, the pressure drop

at the perforation has a sudden change and the program matches that experimental observation. One problem with Yalniz's experimental results is that the three cases cannot really be compared to each other due to the way Yalniz had the experiment set up. In order to increase the influx to main flow rate ratio, he lowered the main flow rate coming into the test section, leaving the influx rate constant. As a result, the main flow rate for Figure 12 is lower than the flow rate for Figure 10, which is seen in the total pressure drop in the test section for both no influx cases. However, the purpose for comparing to Yalniz's data was to determine if the program correctly calculates the pressure at an influx point, which the figures confirm.

4.3 Sensitivity study

To evaluate the effects of each variable on the proposed model, a sensitivity analysis was conducted. Each variable was changed from its example value and the results plotted as dimensionless pressure versus dimensionless length, friction factor versus Reynold's number, and dimensionless rate versus dimensionless length. From dimensional analysis, six groups were determined to be important: dimensionless pressure or pressure coefficient, c_p , Reynold's number, N_{Re} , the friction factor, f , the influx loss coefficient, K , the influx to main flow rate ratio, q_w/q and a dimensionless length, L_d , where

$$c_p = \frac{\Delta p}{\frac{1}{2} \rho V^2} \dots\dots\dots 48$$

$$N_{Re} = \frac{\rho V d}{\mu} \dots\dots\dots 49$$

and

$$L_d = \frac{L}{Nd} \dots\dots\dots 50$$

Table 4.6 shows the range of values for each variable used in the sensitivity analysis.

4.3.1 Varying the fluid density

Figures 13, 14, and 15 show the dimensionless plots for varying fluid density. In Figure 13, the density does not appear to significantly affect the pressure drop near the toe, where there is low reservoir drawdown and influx, but as the heel is approached, there is a noticeable difference in the dimensionless pressure as the density increases. Figure 14 is a plot of Reynold's number versus friction factor. Density does not cause an alteration in the calculation of the Moody friction factor. Figure 15 is a plot of the influx to main flow rate ratio versus length. This ratio is an indication of the influx at a point relative to what is flowing in the wellbore at that point. There is more fluid coming into the wellbore at the ends of the horizontal section than the middle. As with the dimensionless pressure, density begins to affect the influx to main flow rate ratio near the heel, where there is higher reservoir drawdown and larger influx rates. Overall, the fluid density does not appear to have a significant effect on the wellbore pressure and total flow rate.

4.3.2 Varying the fluid viscosity

Figures 16, 17, and 18 present the results of varying fluid viscosity. Figure 16 has the same basic shape as Figure 13, and shows that fluid viscosity has a significant effect on the wellbore pressure. As the viscosity increases, the pressure drop increases. It also appears to remain in laminar flow near the toe for longer lengths. Figure 17 is a

plot of Reynold's number versus friction factor. Each line has the same relative roughness as used to determine the Moody friction factor. The viscosity has the effect similar to altering the relative roughness of the pipe. Increasing viscosity increases the influx to main flow rate ratio near the wellbore as shown in Figure 18. The fluid's viscosity appears to have a greater influence on the wellbore pressure and rate than the fluid density.

4.3.3 Varying the reservoir permeability ratio, $\sqrt{k_h/k_v}$

Figures 19, 20, and 21 present the results for varying the permeability ratio. As the permeability changes from high horizontal permeability, low vertical permeability to low horizontal permeability, high vertical permeability, the dimensionless pressure drop increases and the characteristic shape of dimensionless pressure flattens out as shown in Figure 19. Figure 20 shows the permeability ratio has the same result of effectively altering the relative roughness. The high permeability ratio has a large effect on the influx to main flow rate ratio, while the low permeability ratio causes the influx to main flow rate ratio to decrease near the heel as demonstrated in Figure 21.

4.3.4 Varying influx at the toe

Figures 22, 23, and 24 present the results for varying the influx into the wellbore, axially, from the toe. As the influx at the toe increases, the dimensionless pressure drop at the heel decreases, Figure 22. As the influx at the toe increases, it has the same effect as decreasing the relative roughness of the pipe, Figure 23, and increases the influx to main flow rate ratio at the heel, Figure 24.

4.3.5 Varying the horizontal well length

Figures 25, 26, and 27 present the results for varying the well length. As the well length increases, the dimensionless pressure drop increases then decreases, Figure 25. Figure 26 shows changing well length has the same effect as altering the relative roughness of the pipe. In Figure 27, as the well length increases more of the total influx comes from near the heel.

4.3.6 Varying the well diameter

Figures 28, 29, and 30 present the results for varying the well diameter. As the diameter increases from 2.5 inches to 12 inches, the dimensionless pressure drop in the well bore decreases as seen in Figure 28. As the well diameter increases, laminar flow becomes more apparent, and the diameter changes the relative roughness of the pipe, as relative roughness is a function of the diameter, Figure 29. The well diameter appears to not influence the influx to main flow rate ratio significantly as shown in Figure 30.

4.3.7 Varying the pipe roughness

Figures 31, 32, and 33 show the results for varying the pipe roughness. As the pipe roughness decreases, the effect on the dimensionless pressure is more apparent near the toe of the well, Figure 31. There is no unexpected effect on the friction factor, Figure 32, and there is not much change to the influx to main flow rate ratio, Figure 33.

4.3.8 Varying the influx loss coefficient

Figures 34, 35, and 36 present the results for varying the influx loss coefficient. Changing the influx loss coefficient appears to not have any affect on the dimensionless

pressure or influx to main flow rate ratio, Figure 34, 36. It has the effect of altering the relative roughness on the Reynold's number versus friction factor plot, Figure 35.

4.3.9 The productivity index

Figure 37 compares the apparent productivity as calculated by the program to the productivity indices of Joshi, Borisov, and Elgaghad et al.. The apparent productivity is the flow rate divided by the reservoir drawdown at the heel. Including the effects of friction and influx, the productivity index has a maximum for a well length of just under 1000 feet, and then decreases to zero at a well length just over 4000 feet.

Figure 38 is a plot of the reservoir drawdown versus flow rate. Joshi's equation predicts, for the example data set, the flow rate to be 30,000 STB/day for the maximum drawdown pressure of 20 psi. When considering the wellbore hydraulics, the flow rate at the maximum reservoir drawdown is significantly less, almost 3000 STB/day. This significant difference is because the Joshi equation was derived for a constant wellbore pressure throughout the entire length of the wellbore. This assumed negligible wellbore pressure drop has been shown to be invalid.

4.4 Summary

In summary, a new method for determining the productivity of a horizontal well has been developed. A model was developed by coupling the wellbore and the reservoir through a mass balance, the energy equation and Darcy's law. This model incorporates the effects of friction, acceleration, gravity, and fluid influx. The model was solved using a Runge-Kutta numerical solution and programmed in Visual Basic6.

This method, for given fluid, reservoir, and well data, calculates the wellbore pressure behavior, the influx along the wellbore, the total flow rate, and the apparent productivity of the well. The model shows good agreement in calculation of wellbore pressure to experimental pressure drop data that has been presented by Yuan, Asheim, and Yalniz in the literature.

From the sensitivity analysis, the following observations were made. Fluid density, pipe roughness, and influx loss coefficient generally have minor effects on the wellbore pressure behavior and productivity. The fluid viscosity, permeability ratio, influx at the toe, well length, and well diameter have significant effects on the wellbore pressure and productivity. There is a characteristic shape to the dimensionless pressure-dimensionless length curve that may develop into a type curve analysis that could be used to identify certain characteristics of the horizontal well. Fluid viscosity, permeability ratio, influx at the toe, well length, and influx effectively alter the relative roughness of the pipe, which is used in the Moody friction factor plot. This conclusion explains why the apparent friction factors developed by Yuan and Asheim have significantly different correlations for wall friction factor.

Finally, wellbore hydraulics has significant effects on the productivity of horizontal wells. This new method provides a quick, simple way to determine the wellbore pressure behavior and well productivity using fundamental principles of fluid mechanic and without having to resort to complicated “apparent” friction factors.

Figure 7. Curves that bound regions where friction is negligible in oil recovery, by

Novy³⁷

Figure 8. Comparison of the maximum pressure drop and the program's pressure drop

Figure 9. Comparison to Yalniz's experimental data, no influx

Figure 10. Comparison to Yalniz's experimental data, influx to main rate, $q_{in}/Q = 0.02$

Figure 11. Comparison to Yalniz's experimental data, $q_{in}/Q = 0.107$

Figure 12. Comparison to Yalniz's experimental data, $q_{in}/Q = 0.217$

Figure 13. Dimensionless pressure versus length for changing density

Figure 14. Reynold's number versus friction factor for changing density

Figure 15. Dimensionless rate versus dimensionless length for changing density

Figure 16. Dimensionless pressure versus length for changing viscosity

Figure 17. Reynold's number versus friction factor for changing viscosity

Figure 18. Dimensionless rate versus dimensionless length for changing viscosity

Figure 19. Dimensionless pressure versus dimensionless length for changing permeability ratio

Figure 20. Reynold's number versus friction factor for changing permeability ratio

Figure 21. Dimensionless rate versus dimensionless length for changing permeability ratio

Figure 22. Dimensionless pressure versus dimensionless length for changing influx at toe

Figure 23. Reynold's number versus friction factor for changing influx at toe

Figure 24. Dimensionless rate versus dimensionless length for changing influx at toe

Figure 25. Dimensionless pressure versus dimensionless length for changing well length

Figure 26. Reynold's number versus friction factor for changing well length

Figure 27. Dimensionless rate versus dimensionless length for changing well length

Figure 28. Dimensionless pressure versus dimensionless length for changing well diameter

Figure 29. Reynold's number versus fiction factor for changing well diameter

Figure 30. Dimensionless rate versus dimensionless length for changing well diameter

Figure 31. Dimensionless pressure versus dimensionless length for changing pipe roughness

Figure 32. Reynold's numbers versus friction factor for changing pipe roughness

Figure 33. Dimensionless rate versus dimensionless length for changing pipe roughness

Figure 34. Dimensionless pressure versus dimensionless length for changing influx loss coefficient

Figure 35. Reynold's number versus friction factor for changing influx loss coefficient

Figure 36. Dimensionless rate versus dimensionless length for changing influx loss coefficient

Figure 37. Productivity index versus well length

Figure 38. Reservoir drawdown versus flow rate

Table 4.1 Program sections

Input section	Preliminary calculations	Wellbore pressure	Well performance
Horizontal Permeability Vertical Permeability Thickness Viscosity Specific weight Formation volume factor Area Length Wellbore radius distance from top average reservoir pressure	Drainage radius, vertical well Drainage radius, horizontal Joshi parameter, a Permeability ratio, β	Wellbore pressure Wellbore velocity Reservoir influx Specific productivity Flow rate Productivity	Productivity by Borisov Productivity by Joshi Productivity by Elgaghad

Table 4.2 Example Data Set

$k_h = 8500$ md
$k_v = 1500$ md
$H = 72$ ft
Viscosity = 1.43 cp
$B_o = 1.16$ bbl/STB
Well spacing = 556 acres
Well length = 2625 ft
Well radius = 0.1045 ft
Distance from top boundary = 60.5 ft
Flow rate at toe = 233.4 STB/day
Number of influx points = 1334
Average Reservoir Pressure = 2300 psia
Drawdown at the toe = 2 psia
Maximum Drawdown = 20 psia

Table 4.3 Comparison of Pressure Drop with Different Friction Factor Correlations

Friction factor correlation used	Yuan	Asheim (altered)	Unidentified pipe correlation	Program (Jain)
Wellbore pressure drop	5.25 psi	7.06 psi	6.87 psi	7.15 psi

Table 4.4 Wall friction factor correlations

Correlation	Yuan	Asheim	Smooth pipe	Program
Yuan $f_w = 80.45 \text{Re}^{-.846}$	5.26 psi	5.29 psi	4.98 psi	N/a
Asheim $f_w = 0.16 \text{Re}^{-.19}$	9.54 psi	9.57 psi	9.26 psi	N/a
Blasius $f_w = 0.316 \text{Re}^{-.25}$	10.02 psi	10.05 psi	9.73 psi	N/a
Jain $f_w = (1.14 - 2\log(e/d + 21.25/\text{Re}^9))^{-2}$	N/a	9.83 psi	9.52 psi	7.15 psi

Table 4.5 Percent Difference of program to Yalniz results

Ratio of influx to main rate, q_{in}/Q	No influx case	Influx
0	-0.269 %	-0.123 %
0.02	-0.448 %	-0.139 %
0.107	1.98 %	-1.33 %
0.217	2.71 %	-7.15 %

Table 4.6 List of Variables and their Ranges

Specific Weight of the fluid, lb_f/ft³	55, 50, 40, 62.4, 70
Pipe Roughness, ft	0.01, 0.005, 0.002, 0.001, 0.0001
Influx Coefficient, K	1.8, 10, 100, 1000, 10000, 100000
Influx at toe, STB/day	0.53, 0.4, 0.6, 0.7, 0.8, 0.9, 1.0
Wellbore Diameter, inches	2.5, 3, 6, 12, 24
Well Length, ft	2625, 50, 500, 1000, 2000, 3000, 4000
Viscosity, cp	1.43, .8, 1, 1.2, 1.6, 1.8, 2, 5
Permeability Ratio (k_h/k_v)³	2.38, 4, 1, 0.75, 0.5, 0.1

Chapter 5 Summary and Conclusions

This research was concerned with the effects of wellbore hydraulics on horizontal well productivity. Methods used to estimate horizontal well productivity and the frictional pressure losses were reviewed. A model coupling Darcy's law and the basic energy equation was developed and solved with a Runge-Kutta numerical solution in a Visual Basic6 program. The proposed method reproduced experimental data for influx through a single perforation presented in the literature, and a sensitivity analysis on the individual variables was conducted. The following conclusions are presented.

1. A new method to determine the productivity of a horizontal well has been developed that incorporates friction, acceleration and influx losses. The new method does not require complex reservoir simulation or the use of an apparent friction factor.
2. Results obtained with the new method are comparable with the methods as proposed by Yuan and Asheim, but do not require specific knowledge of the ratio of influx to main flow rate to determine an apparent friction factor.
3. Influx along the wellbore length has the effect similar to altering the pipe roughness, which explains the uncertainty in Yuan's and Asheim's wall friction factor correlations used in their respective apparent friction factors.

There were a variety of assumptions made during the development of the new method. For the reservoir component of the model, the fluid was assumed to be single-

phase and incompressible, flowing under steady-state conditions and in the laminar region. For the wellbore component of the model, the fluid was assumed to be represented by a single-phase, incompressible mixture, and was flowing under steady-state conditions. The well was assumed to be truly horizontal, so there were no gravity effects, or liquid hold-up, and the fluid moved as a complete unit so there was no slippage. The program was developed for a producing horizontal well. Therefore, the following recommendations are suggested.

1. The model needs to be modified as follows. The wellbore component should be altered to include liquid holdup and gas slippage, as well as single-phase compressible flow. The reservoir component needs to be altered to include single-phase compressible flow, and two-phase flow. The model needs to be reversed so that it can be applied to horizontal injection wells.
2. The effects gravity has on a more representative sinusoidal well or a deviated well need to be analyzed.
3. More work needs to be performed to determine how the influx loss coefficient, K , is related to the influx area, and more case runs need to be made with different fluid and reservoir properties.

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Nomenclature

A	Area
a	major half axis
a	Joshi parameter
a	Babu and Odeh parameter
α	Blasius parameter
α	kinetic turbulence factor
B	formation volume factor
b	minor half axis
b	Babu and Odeh parameter
β	permeability ratio
C	Muskat arbitrary constant
C	Elgaghad et al parameter
C_H	Babu and Odeh shape factor
c_r	productivity index / unit length
c_w	flow resistivity
δ	Eccentricity factor
d	distance from top boundary to center of wellbore
D, d	pipe diameter
Δx	distance between perforations
f	friction factor
f_p	perforation friction factor
f_t	apparent friction factor
f_w	wall friction factor
γ	specific weight of the fluid
g	acceleration due to gravity
h	thickness
$h_{\text{additional}}$	additional head losses
J	productivity index
J_s	specific productivity index
k	Permeability
K	influx coefficient
K	loss coefficient
k_h'	equivalent permeability

k_r	relative permeability
L	length of horizontal section
μ	viscosity
n	Bendakhlia and Aziz parameter
p_r	average reservoir pressure
p_{wf}	wellbore pressure
q	flow rate
q	main flow rate
θ	angle of inclination of the horizontal section
Q_1	main flow rate upstream of perforation
q_{in}	influx through perforation
q_L	influx /unit length
r	fluid density
r_e	drainage radius
R_e	Reynold's number
r_{eh}	horizontal drainage radius
R_s	solution gas oil ratio
R_{sw}	solution gas water ratio
r_w	wellbore radius
r_w'	equivalent wellbore radius
s	skin factor
S_r	Babu and Odeh skin factor
V	Bendakhlia and Aziz parameter
V	velocity of fluid
V_{in}	influx velocity
X	Renard and Dupuy area
z	elevation

Subscripts

1, 2	points in flow stream
e	external boundary
h	horizontal
max	maximum
o, g, w	oil, water, gas

p	perforation, influx point
res	Reservoir
v	Vertical
x	horizontal axis
z	vertical axis

Appendix A Summary of Vertical Well Performance Relationships

This appendix contains additional tables that provide background or additional information.

Table A.1. Vertical Well Performance Equations

Straight-line IPR	$J = \frac{q_o}{p_r - p_{wf}} = \frac{kh}{141.2 \mu B \left[\ln \left(\frac{r_e}{r_w} \right) - \frac{3}{4} + s \right]}$	Single-phase, slightly compressible (oil), semi-steady state, radial flow
Vogel	$\frac{q_o}{q_{o,max}} = 1 - 0.2 \frac{p_{wf}}{p_r} - 0.8 \left(\frac{p_{wf}}{p_r} \right)^2$	Two-phase, oil and gas,
Fetkovich	$\frac{q_o}{q_{o,max}} = \left(1 - \left(\frac{p_{wf}}{p_r} \right)^2 \right)^n$	Two-phase
Jones, Blount, & Glaze	$p_r - p_{wf} = C q_o + D q_o^2$	Two-phase flow, non-Darcy flow effects
Wiggins	$\frac{q_o}{q_{o,max}} = 1 - 0.52 \frac{p_{wf}}{p_r} - 0.48 \left(\frac{p_{wf}}{p_r} \right)^2$ $\frac{q_w}{q_{w,max}} = 1 - 0.72 \frac{p_{wf}}{p_r} - 0.28 \left(\frac{p_{wf}}{p_r} \right)^2$	Three-phase
Backpressure	$q_s = C (p_r^2 - p_{wf}^2)^n$	Single-phase, compressible (gas)

Table A.2. Multiphase horizontal well performance equations

Bendakhlia and Aziz	$\frac{q_o}{q_{o,max}} = \left(1 - V \frac{p_{wf}}{p_r} - (1 - V) \left(\frac{p_{wf}}{p_r} \right)^2 \right)^n$
Cheng	$\frac{q_o}{q_{o,max}} = 0.9885 + 0.2055 \left(\frac{p_{wf}}{p_r} \right) - 1.1818 \left(\frac{p_{wf}}{p_r} \right)^2$
Wang	$\frac{q_o}{q_{o,max}} = 1 - 0.4533 \left(\frac{p_{wf}}{p_r} \right) - 0.5467 \left(\frac{p_{wf}}{p_r} \right)^2 \dots general$ $\frac{q_o}{q_{o,max}} = 1 - d \left(\frac{p_{wf}}{p_r} \right) - (1 - d) \left(\frac{p_{wf}}{p_r} \right)^2 \dots depletion$