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DIRECTIONAL EMITTANCE OF EMITTING, ABSORBING
AND SCATTERING MEDIA

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ABSTRACT

The directional emittance for three different geometries is analyzed for several varying optical conditions. The geometries are:

- (1) Semi-infinite slab with free boundaries.
- (2) Infinite medium (coating) with Fresnel reflecting boundaries.
- (3) Semi-infinite slab (coating) with Fresnel reflecting boundaries.

Monte Carlo techniques are used to obtain the solutions. Isotropic scattering is assumed for all conditions, and the effects of multiple scattering are included.

The semi-infinite medium with free boundaries is analyzed for two optical depths, 1.0 and 3.0, and two scattering/extinction ratios, 0.1 and 0.9. The emittance is presented in the form of curves at various edge and plane locations, from which three-dimensional views of the emittance may be obtained. Wherever possible, a comparison is made with previously published approximate analytical solutions. The emittance is found to be non-diffuse and dependent on optical depth, location, and absorption coefficient. The edge emittance is more directional than the plane surface emittance, and the higher optical depths were, in general, more diffuse.

The one-dimensional coating solutions are obtained to investigate the effects of refractive index, optical depth, and absorption coefficient

on the directional emittance. Curves are presented which will allow the directional emittance to be predicted for any optical depth from 1.0 to 6.0 and any refractive index between 1.4 and 1.8, for the two scattering/extinction ratios of 0.95 and 0.99. The directional emittance is found to increase with an increase in optical depth, absorption coefficient, and refractive index for the two scattering/extinction ratios considered. The computer programs for the one-dimensional solutions were much shorter than the two-dimensional programs; consequently, the one-dimensional results are used to extend the two-dimensional data to optical conditions not actually run.

The two-dimensional coating results are obtained for a refractive index of 1.4, optical depths of 3.0 and 5.0, and scattering/extinction ratios of 0.95 and 0.99. These optical conditions are representative of ceramic coatings. Using the one-dimensional coating results, the two-dimensional data is extended to include the directional emittance for optical depths of 1.0 to 6.0 and refractive indexes of 1.4, 1.6, and 1.8, for the two scattering/extinction ratios. The directional emittance may be obtained for any edge location or for any plane location up to one optical depth from the edge.

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LIST OF SYMBOLS

A	Generating area (SOL x L)
dx, dy, dz	Differential elements
D	Infinite optical dimension
E	Electrostatic intensity vector
f(), g(), h()	Density functions
F(), G(), H()	Cumulative distribution functions
H	Semi-infinite strip width
I	Intensity of thermal radiation
I_{bb}	Black body intensity
I_o	Reference intensity
L	Slab thickness
n	Refractive index
N	Total number of emissions
N'	Number of emissions through a selected area
P	Probability
R	Uniform random number
s	Path length
S	Surface area
SOL	Slab optical length
V	Volume
X, Y, Z	System coordinates

x, y, z	Emission coordinate system
y', z'	Translated coordinates or escape locations
β_0	Extinction coefficient
γ	Absorption to extinction ratio
ϵ	Emittance
θ	Polar angle
θ'	Leaving polar angle
θ''	Refracted polar angle
θ_c	Critical polar angle of incidence
θ_F	Fresnel polar angle
κ	Absorption coefficient
λ, ψ	Dummy variables
μ'	Cosine of escape polar angle
ρ	Reflectance
σ	Scattering to extinction ratio
σ'	Scattering coefficient
τ	Optical depth
φ	Azimuthal angle
φ'	Escape azimuthal angle
ω	Solid angle

DIRECTIONAL EMITTANCE OF EMITTING,
ABSORBING, AND SCATTERING MEDIA

CHAPTER I

INTRODUCTION

The area of emitting, absorbing, and scattering media in the field of radiative heat transfer has received a great deal of attention in recent years, primarily motivated by the new space age technology. Subjects such as light scattering by atmospheres, scattering by fogs or aerosols, and scattering by chemical suspensions, to name a few, have been studied for many years. Various aspects of these studies are still being analyzed; however, additional problems are arising which have to be considered. Examples are: the determination of the heat transfer rate from a hot rocket exhaust to its base, the analysis and development of ceramic coatings for thermal protection, the analysis of reentry heat transfer rates at high velocities, and the analysis and development of thermal coatings for spacecraft or satellite temperature control.

The specific areas covered by this work include the following:

- (1) directional emittance of a two-dimensional,
emitting, absorbing, and scattering medium
with free boundaries.
- (2) directional emittance of a one-dimensional

ceramic coating considering Fresnel
reflectances at the boundaries.

- (3) directional emittance of a two-dimensional
ceramic coating considering Fresnel
reflectances at the boundaries.

All solutions were obtained using Monte Carlo techniques. In general, the nomenclature and derivations from Love (1) will be used.

An approximate solution to the two-dimensional, free-boundary problem has been obtained previously by Bobco (2), using a diffusion approximation. His analytical technique produced closed-form results. The directional emittance solutions for the one- and two-dimensional ceramic coatings have not been obtained previously.

Monte Carlo methods have been known for many years and were applied to neutron diffusion problems as early as 1950. (See (1), Appendix VI.) Bhattacharjie and Skumanich (3) were among the first to apply Monte Carlo methods to radiative heat transfer problems when they determined reflected and transmitted radiation from a plane cloud illuminated by a pulsed point source on top of the cloud. Howell and Perlmutter used Monte Carlo to solve several different radiative heat transfer problems. References (4) and (5) were two of their earliest publications, dealing with heat transfer between surfaces separated by a participating medium. Reference (4) contains an excellent discussion of the Monte Carlo distribution functions applicable to radiative heat transfer analyses. Since these early publications, a number of papers have been published using Monte Carlo methods to solve many complex problems. Corlett (6) obtained configuration factors for radiative

exchange in a space environment using Monte Carlo. Campbell (7) solved a nonlinear, frequency dependent radiative heat transfer problem, and Stockham (8) determined radiative heat transfer rates from a hot exhaust cloud, both using Monte Carlo techniques. Scofield (9) applied Monte Carlo methods to determine heat transfer rates to various fogs and aerosols. Plass and Kattawar (10, 11, 12, 13, 14, and 15) applied Monte Carlo techniques to analyze various aspects of radiative heat transfer rates from atmospheric clouds.

Conduction and convection heat transfer analyses have also been made using Monte Carlo methods. Haji-Sheikh and Sparrow (16) solved steady state and transient conduction problems, and Chandler, et al (17) solved laminar convection problems. Emery and Carson (18) suggested a modification of the Monte Carlo method--the "Exodus" method, which can be applied to nodal-type problems encountered in conduction and convection analyses.

The application of Monte Carlo techniques is not restricted to the solution of neutron diffusion and heat transfer problems. The method may be used as a numerical-type integrator, and it has been successfully used by Skidmore and Penzo (19) to simulate midcourse guidance for lunar flights. Logsdon and Africano (20) used Monte Carlo methods to compute histograms for trajectory analysis, propellant burnout studies, etc., and then devised a modified Monte Carlo procedure which reduced computer time by a factor of 10.

In the specific area of coatings Monte Carlo methods have been used successfully to investigate various properties. Honeywell Systems and Research Center (21) analyzed a pigmented coating with an incident

intensity to determine the reflective properties. Researchers at the Illinois Institute of Technology (22) used Monte Carlo to investigate scattering in highly-pigmented coatings.

Coatings, in general, have received much attention because of their application in spacecraft temperature control, and most of the published literature is concerned with paint coatings containing selected pigments. The amount of information available on ceramic coatings is much less; and the principal references used in this work are the reports by Folweiler, Mallio, and Hobbs of Lexington Laboratories (23). Since the optical properties of these ceramic materials are essential to a thermal analysis, a summary of the material properties is included in Appendix A. Additional references are also cited.

The versatility of Monte Carlo techniques has been shown by the previous discussion of the various problems investigated and solved by Monte Carlo methods. The amount of computer time generally required to obtain an acceptable solution is quite large; therefore, if exact or nearly exact methods of solution are available, they would be preferred over Monte Carlo. The application of Monte Carlo, therefore, is to problems which are too complex to be solved by more conventional methods. Since exact closed-form solutions have not been obtained for the emitting, absorbing, and scattering media investigated herein, and since the validity of the available approximate methods of solution is somewhat questionable, a Monte Carlo solution seems justified. The following chapter discusses the statistical and probability methods required for a Monte Carlo analysis.

CHAPTER II

STATISTICS AND PROBABILITY METHODS

The usage of Monte Carlo techniques in obtaining an analytical solution to the directional emittance of a medium is based on probability theory. A particle is emitted within the medium and followed until it either escapes or is absorbed by the medium. Each event for which a decision is required is determined by probabilities.

Probability Theory

The following brief discussion is intended to cover the probability theory necessary to apply Monte Carlo techniques and is taken from Mood and Graybill (24). The type of variables dealt with are discrete random variables. These variables have a discrete or finite number of possible outcomes, and the distribution of outcomes is called a discrete probability distribution.

Consider a quality control problem where parts are selected at random and checked for size within a required tolerance. If the frequency of occurrence for given intervals is plotted, the graph might look like Figure 2-1. Assuming that this sample is representative of the entire population, the probability that a new part will lie within a certain interval, say 0 to 1, is given by the area of the rectangle between 0 and 1. Since the relative-frequency distribution includes all allowable

tolerances, the probability that a selected part will fall within the required tolerances is one, and conversely, the probability of a part falling outside the allowable tolerance is zero. If a smooth curve is drawn through the relative frequency points as in Figure 2-2, such that there is the same amount of area under the curve from point to point as for the discrete case, the probability could be found by integrating over the desired interval.

$$P(0 \leq x \leq 1) = \int_0^1 f(x) dx$$

P represents the probability over the given interval, and $f(x)$ is the smooth curve graph of the relative-history distribution. The smooth curve should be acceptable because the intervals chosen were arbitrary and could have been made sufficiently small to approximate more closely the smooth curve areas.

If the smooth curve is allowed to take on all possible outcomes on the x-axis, the probability of any event in the x-axis occurring is one.

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$f(x)$ is said to be a density function for the event if it satisfies the following:

$$\int_{-\infty}^{\infty} f(x) dx = 1,$$

and $f(x) \geq 0$ over the interval $-\infty < x < \infty$.

The probability for an event occurring in a given interval (a,b) is found by integrating over that interval.

$$P(a \leq x \leq b) = \int_a^b f(x) dx .$$

The cumulative distribution function of x is the probability that the value of the random variable will be equal to or less than x . It is defined by the integral

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt .$$

F is a nondecreasing, continuous function with the properties that $F(-\infty) = 0$ and $F(\infty) = 1$. The cumulative distribution function for f from Figure 2-2 is sketched in Figure 2-3. If the cumulative distribution function is known, the density function may be found by differentiating $F(x)$.

$$f(x) = \frac{dF(x)}{dx} .$$

The function f may involve one or more random variables, and they may be independent or dependent in the probability sense. $f(x,y)$ is known as a joint distribution function. If two random variables are independent, the outcome of one event is unaffected by any knowledge or probability of the other. If the probability that an event occurs within the interval $(a < x < b)$ and $(a' < y < b')$ for a joint distribution function $f(x,y)$, it would be found from

$$P(x,y) = \int_{a'}^{b'} \int_a^b f(x,y) dx dy .$$

If the random variables are independent in the probability sense, the joint distribution function may be rewritten as the product of two density functions.

FIGURE 2-1
RELATIVE-FREQUENCY HISTOGRAM

FIGURE 2-2
PROBABILITY DENSITY FUNCTION

FIGURE 2-3
CUMULATIVE DISTRIBUTION FUNCTION

$$f(x,y) = g(x) \cdot h(y) \quad .$$

The previous probability could then be written

$$P(x,y) = \int_{a'}^{b'} \int_a^b g(x) h(y) dx dy$$

$$\text{or } P(x,y) = \int_{a'}^{b'} h(y) dy \int_a^b g(x) dx \quad .$$

The joint density functions satisfy the same conditions of single density functions in that for events in x and y ,

$$f(x,y) \geq 0 \quad \text{and}$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dx dy = 1 \quad .$$

Density functions, independent random variables, joint density functions, and cumulative distribution functions all play important roles in obtaining solutions by Monte Carlo techniques. The next chapter applies these principles in developing an analytical model for the particular Monte Carlo analyses.

CHAPTER III

MODELS AND MONTE CARLO TECHNIQUES

Three different physical models are analyzed in this work. They are:

- Case 1: two-dimensional, absorbing, emitting,
and scattering media with free boundaries.
- Case 2: one-dimensional, absorbing, emitting,
and scattering media (coating) with
reflecting boundaries.
- Case 3: two-dimensional, absorbing, emitting,
and scattering media (coating) with
reflecting boundaries.

The geometry for each case is shown in Figure 3-1. The directional emittance of the infinite plane was determined for Case 2, and the emittance of both the edge and the plane for several locations was determined for Cases 1 and 3. The analytical models to be derived and used are as follows:

- (1) Particle emission from a volume element.
- (2) Coordinates of emission.
- (3) Particle path length.
- (4) Scattering model.

(a) Semi-infinite medium with free boundaries

(b) Infinite coating

(c) Semi-infinite coating

FIGURE 3-1
GEOMETRIES ANALYZED

Particle Emission from a Volume Element

The geometry for emission from a volume element is shown in Figure 3-2. The particles which are emitted from a differential volume source are assumed to be emitted isotropically, that is, all directions are equally probable. The number of particles N' which pass through the differential area on the surface of the sphere is directly proportional to the area of the differential surface. The ratio N' to N , the total number of emissions passing through the spherical surface, is given by

$$N'/N = \frac{\sin \theta \, d\theta \, d\varphi}{4\pi}$$

Since
$$\int_0^{2\pi} \int_0^{\pi} \frac{\sin \theta \, d\theta \, d\varphi}{4\pi} = 1$$

and
$$\frac{\sin \theta}{4\pi} \geq 0 \quad (0 \leq \theta \leq \pi) \quad ,$$

the requirements for a joint density function, $f(\theta, \varphi)$, are met. Further, the random variables are independent and may be written as the product of two density functions, $g(\theta)$ and $h(\varphi)$, where

$$g(\theta) = \frac{\sin \theta}{2} \quad \text{and} \quad h(\varphi) = \frac{1}{2\pi}$$

The cumulative distribution functions may now be found by integrating the above density functions.

$$G(\theta) = \int_0^{\theta} \frac{\sin \lambda \, d\lambda}{2} \quad \text{and} \quad H(\varphi) = \int_0^{\varphi} \frac{d\psi}{2\pi}$$

$$G(\theta) = \frac{1}{2} [1 - \cos \theta] \quad \text{and}$$

$$H(\varphi) = \frac{\varphi}{2\pi}$$

Since $G(\theta)$ and $H(\varphi)$ vary between 0 and 1, they may be set equal to random numbers (also varying between 0 and 1) for the Monte Carlo analysis.

$$R_\theta = \frac{1}{2} [1 - \cos \theta] \quad \text{and}$$

$$R_\varphi = \frac{\varphi}{2\pi}$$

The generating direction is now found to be

$$\cos \theta = 1 - 2R_\theta \quad (-1 \leq \cos \theta \leq +1)$$

$$\varphi = 2\pi R_\varphi \quad (0 \leq \varphi \leq 2\pi)$$

Coordinates of Emission

The geometry shown in Figure 3-3 is three-dimensional, but because of symmetry about the Y-axis, the problem reduces to two dimensions. In other words, the emission is independent of location along the X-axis. The problem then becomes to locate the particle within the rectangle SOL x L. The distance SOL stands for slab optical length and is a variable depending on the media optical conditions and will be discussed in detail later for each specific optical condition. The differential area for the generating slab is $dydz$, and the joint density function is

$$f(y, z) = \frac{dydz}{(SOL)(L)}$$

Again, the random variables are independent, and

$$g(y) = \frac{dy}{SOL}$$

$$h(z) = \frac{dz}{L}$$

Integrating the density functions as before and setting the cumulative distribution functions equal to different random numbers yields

FIGURE 3-2
EMISSION FROM A VOLUME ELEMENT

FIGURE 3-3
COORDINATE GEOMETRY

$$y = \text{SOL} \cdot R_y$$

$$z = L \cdot R_z$$

The particle emission location and direction have now been determined. The next problem is to determine the particle path length.

Particle Path Length

Once there is an emission from a given volume element in a given direction, the energy will continue to travel in a straight line until it escapes the media or is attenuated by absorption or by scattering. The probable path length is a function of the optical properties of the media and may be determined as follows. Consider a pencil of rays as shown in Figure 3-4. From Sparrow and Cess (25), the intensity of thermal radiation, I , is given by

$$\kappa I ds + \sigma' I ds = \beta_o I ds$$

where β_o = extinction coefficient and is equal to the sum of σ' , the scattering coefficient, and κ , the absorption coefficient.

$$dI = -\beta_o I ds \text{ or } I = I_o e^{-\beta_o s}$$

If I is substituted back into the equation,

$$dI = I_o \beta_o e^{-\beta_o s} ds, \text{ or}$$

$$\frac{dI}{I_o} = \beta_o e^{-\beta_o s} ds$$

dI represents the bundle of energy which is in the volume element ds compared to the total energy present at $s = 0$, and this is the form of a density function. Integrating the above equation yields

$$\frac{1}{I_o} \int dI = \beta_o \int_0^{\infty} e^{-\beta_o s} ds = e^{-\beta_o s} \Big|_0^{\infty} = 1$$

FIGURE 3-4
DEFINITION FOR ATTENUATION OF INTENSITY

Since $\beta_o e^{-\beta_o s} \geq 0$, the equation meets the requirements for a density function.

$$f(s) = \beta_o e^{-\beta_o s} .$$

As before the cumulative distribution function is found by integrating the density function.

$$F(s) = \int_0^s \beta_o e^{-\beta_o t} dt = 1 - e^{-\beta_o s} .$$

$$R'_s = 1 - e^{-\beta_o s}$$

and $1 - R'_s = e^{-\beta_o s} .$

Since $1 - R'_s$ is also a number randomly distributed between 0 and 1, $1 - R'_s$ becomes R_s . Taking the logarithm of both sides of the equation yields

$$\ln R_s = -(\beta_o s),$$

or $s = \left(-\frac{1}{\beta_o}\right) \ln R_s .$

The path length is seen to be inversely proportional to the extinction coefficient. If the particle path length is such that the particle does not escape, its history may be terminated by absorption, and its direction may be altered by scattering.

Determination of Scattering Direction

Scattering is defined as any change in the original direction of propagation of the energy. For the three cases analyzed here, the scattering is assumed to be caused by inhomogeneities in the media. These inhomogeneities may be solid particles suspended within the media, or in the case of a ceramic coating, could be voids in the solid matrix. The

scattering is assumed to be isotropic, that is, there is no preferred scattering direction. For a rigorous treatment of scattering, see Van de Hulst (26).

When a particle is scattered, the scattering is assumed to occur at the end of the probable path length, and new coordinates are determined.

$$y' = y + (s)(\sin \theta)(\sin \varphi)$$

$$z' = z + (s)(\cos \theta)$$

These translated coordinates are the origin of the particle, and a new direction and path length are calculated as before using the cumulative distribution functions. The computer program will follow each particle, including the effects of multiple scattering, until it is absorbed or it escapes the medium.

Collection Areas for Energy Escapes

Cases 1 and 3

The two-dimensional media surfaces were divided into infinite collection strips parallel to the X-axis as shown in Figure 3-5. The number of strips for the semi-infinite plane varied, depending on optical conditions, and ranged from eight to thirteen. The number of strips for the edge was nine for both cases. The escapes were captured into one of eight polar angles for case 1 and into one of ten locations for case 3. The azimuthal divisions are twelve.

Symmetry was utilized as much as possible. Escapes in the top half of the edge and in the top plane were catalogued into their respective locations on the bottom half of the edge and the plane. Further symmetry was utilized about the Y-axis for both the plane and

the edge for the azimuthal divisions. This reduced the azimuthal collection areas to seven.

Case 2

For the one-dimensional coating problem with real boundaries, the emission coordinates, direction, and path length were computed in the same manner as before, but the collection areas are different. Since the planes are infinite in extent, there is no azimuthal dependence. The particle was followed in three dimensions, but only the escape polar angle was used to determine the exit location. To simulate an infinite media, the collection area started at a distance D from the edge such that the area could not "see" the edge. The length D varied with the optical properties of the media. The collection areas are shown in Figure 3-6.

The polar divisions were 17, the first increment being 10° and all the others 5° . Reducing the problem to one dimension allowed a greater number of divisions to be taken.

CHAPTER IV

GEOMETRICAL CONSIDERATIONS FOR EACH MODEL

The geometry for all three cases is somewhat similar, but there are enough differences that each case will be discussed separately. The description will also include the methods of determining the escape polar and azimuthal angles for all three cases. The computer program includes the effects of absorption and multiple scattering, but for purposes of this discussion it will be assumed that the energy bundle in question does not interact with any inhomogeneities enroute to the boundary.

Two-Dimensional Medium with Free Boundaries

An energy bundle is emitted at a point y, z within the medium. The polar direction of emission, θ , may vary from 0° to 180° , and the azimuthal angle, ϕ , may vary from 0° to 360° . The exit may be out the top or bottom plane or the edge, depending on the combination of angles. Table 4-1 summarizes the possible combinations.

The geometry for each of the six combinations, (a) through (f) of Table 4-1 will be discussed separately.

(a) $0^\circ \leq \phi \leq 180^\circ$, $\theta < 90^\circ$: Top Escape

The determination of the position of exit and the escape angles are shown in Figure 4-1(a). The location at exit, y' , and escapes angles θ' and ϕ' are:

TABLE 4-1

SUMMARY OF EXIT DIRECTIONS

Condition	(a) $0^\circ \leq \varphi \leq 180^\circ$ $\theta < 90^\circ$	(b) $0^\circ \leq \varphi \leq 180^\circ$ $\theta > 90^\circ$	(c) $180^\circ < \varphi < 360^\circ$ $\theta < 90^\circ$
Escape Location	Top	Bottom	Top or Edge
Condition	(d) $180^\circ < \varphi < 360^\circ$ $\theta > 90^\circ$	(e) $0^\circ \leq \varphi \leq 180^\circ$ $\theta = 90^\circ$	(f) $180^\circ < \varphi < 360^\circ$ $\theta = 90^\circ$
Escape Location	Bottom or Edge	(will be scattered or absorbed)	Edge

Location: $y' = y + (1-z) \tan \theta \sin \varphi$

Polar Angle: $\theta' = \theta$

Azimuthal Angle: $\varphi' = \varphi$.

(b) $0^\circ \leq \varphi \leq 180^\circ$; $\theta > 90^\circ$: Bottom Escape

The determination of the position of exit and the escape angles are shown in Figure 4-1(b). The location at exit, y' , and escape angles θ' and φ' are:

Location: $y' = y - (z) \tan \theta \sin \varphi$

Polar Angle: $\theta' = 180^\circ - \theta$

Azimuthal Angle: $\varphi' = \varphi$.

(c) $180^\circ < \varphi < 360^\circ$, $\theta < 90^\circ$: Top or Edge Escape

If an energy bundle is emitted within this quarter sphere, it can escape out the edge or the top, depending on the combination of angles and location. The assumption is made that the exit will be out the top, and the projection length on the Y-axis is calculated and compared with the actual distance y from the edge. If the calculated distance, YT , is greater than y , the exit is out the edge. If not, the exit is out the top. This geometry is shown in Figure 4-1(c). The location and escape angles for each case are:

Top Escape

Same as (a) above .

Edge Escape

Location: $y = 0$

$$z' = z - y \cot \theta / \sin \varphi$$

$$\text{Polar Angle: } \theta' = \cos^{-1} [-\sin \theta \sin \varphi]$$

$$\text{Azimuthal Angles: } \varphi' = \tan^{-1} \left[\frac{-\cot \theta}{\cos \varphi} \right], 180^\circ < \varphi \leq 270^\circ$$

$$\varphi' = \frac{\pi}{2} + \tan^{-1} [\tan \theta \cos \varphi], 270^\circ < \varphi < 360^\circ.$$

(d) $180^\circ < \varphi < 360^\circ$, $\theta > 90^\circ$: Bottom or Edge Escape

Energy bundles emitted in this quarter sphere will escape out the edge or the bottom surface. Just as for the previous case, the assumption is made that the exit is out the plane surface (bottom), and the projection length on the Y-axis, YT , is calculated. This length is compared with the actual distance y from the edge, and if YT is greater than y , the exit is out the edge. If not, the exit is out the bottom. This

(a) Top escape: $0^\circ \leq \varphi \leq 180^\circ$, $\theta < 90^\circ$

(b) Bottom escape: $0^\circ \leq \varphi \leq 180^\circ$, $\theta > 90^\circ$

(c) Top or edge escape: $180^\circ < \varphi < 360^\circ$
 $\theta < 90^\circ$

(d) Bottom or edge escape: $180^\circ < \varphi < 360^\circ$
 $\theta > 90^\circ$

FIGURE 4-1

ESCAPE GEOMETRY FOR FREE BOUNDARY SOLUTION

geometry is shown in Figure 4-1(d). The exit location and angles are found by:

Bottom Escape

Same as (b) above.

Edge Escape

Location: $y = 0$

$$z' = z - y \cot \theta / \sin \varphi$$

$$\text{Polar Angle: } \theta' = \cos^{-1} [-\sin \theta \sin \varphi]$$

$$\text{Azimuthal Angles: } \varphi' = 3\pi/2 + \tan^{-1} [\tan \theta \cos \varphi], 180^\circ < \varphi \leq 270^\circ$$

$$\varphi' = \pi + \tan^{-1} \left[\frac{-\cot \theta}{\cos \varphi} \right], 270^\circ < \varphi < 360^\circ .$$

(e) $0 < \varphi < 180^\circ$, $\theta = 90^\circ$: Scattered or Absorbed Point

Energy emitted in this direction could not escape and would be scattered or absorbed at a distance s equal to the path length. The new coordinates and emission direction would be determined at that point.

(f) $180^\circ < \varphi < 360^\circ$, $\theta = 90^\circ$: Edge Escape

Energy which is emitted with this direction will strike the edge. The exit location is at the same vertical distance z , and the exit polar angle is found from (c) or (d). The azimuthal escape angle, φ' , is 0° if φ is equal to or less than 270° and is 180° if φ is greater than 270° .

Directional Emittance - Free Boundaries

The directional emittance of the two-dimensional plane slab is found using the basic concept of emittance.

$$e(\mu', \varphi') = \frac{\text{Emitted Intensity}}{\text{Black body intensity}} = \frac{I(\mu', \varphi')}{I_{bb}}$$

Intensity is defined as $\frac{\text{energy}}{\text{area} \times \text{time} \times \text{stearadian} \times \text{frequency}}$.

In terms of the Monte Carlo application the intensities are given by:

$$I_{bb} = \frac{N}{(4\pi)(\text{volume})\kappa}$$

$$\text{and } I(\mu', \varphi') = \frac{N(\mu', \varphi')}{(\text{Area}) \cos \theta_{\text{avg.}} \Delta \omega}$$

where N = Total number of emissions

μ' = Cosine of escape angle

κ = Absorption coefficient, $1/\text{Length}$

φ' = Escape azimuthal angle

$N(\mu', \varphi')$ = Number of emissions in the escape solid angle

$\Delta \omega$ = Escape solid angle

$\cos \theta_{\text{avg.}}$ = Cosine of the average angle in the escape interval

Because of symmetry the volume term reduces to an area and the area term reduces to the semi-infinite strip width. The complete directional emission is thus given by

$$\epsilon(\mu', \varphi') = \frac{4\pi A \kappa N(\mu', \varphi')}{\cos \theta_{\text{avg.}} H \Delta \omega N}$$

A is the generating area ($\text{SOL} \times L$) and H is the semi-infinite strip width.

The free boundary problem was somewhat easier to analyze because the energy bundle escaped if its path length exceeded the distance to the nearest boundary. This is not true, however, for the coatings of Cases 2 and 3. The energy will be reflected from the boundary if the angle of incidence is greater than some critical value, and it may be reflected in accordance with Fresnel's relations even if the angle of incidence is less than the critical angle. Energy which leaves the medium has its

exit angle changed in accordance with Snell's Law. Snell's Law and the Fresnel equations will now be discussed before considering the geometry of the coatings.

Snell's Law and the Fresnel Equations

The following is taken from Love (1). Electromagnetic plane waves such as thermal radiation will undergo certain predictable changes following an interaction with a dielectric material with index of refraction n . Figure 4-2 illustrates the geometry. The incident angle is θ , the reflected angle is θ' , and the refracted angle is θ'' . The interface is a smooth surface.

Snell's Law states that

$$n \sin \theta = n_1 \sin \theta'',$$

or for this case,

$$\sin \theta'' = \frac{1}{n_1} \sin \theta.$$

The reflected angle, θ' , is equal to θ from Snell's Law.

The amount of energy reflected and transmitted can be determined by the Fresnel equations.

$$E'_{\parallel} = E_{\parallel} \frac{\tan (\theta - \theta'')}{\tan (\theta + \theta'')}$$

$$E'_{\perp} = -E_{\perp} \frac{\sin (\theta - \theta'')}{\sin (\theta + \theta'')}$$

$$E''_{\parallel} = E_{\parallel} \frac{2 \sin \theta'' \cos \theta}{\sin (\theta + \theta'') \cos (\theta - \theta'')}$$

$$E''_{\perp} = E_{\perp} \frac{2 \sin \theta'' \cos \theta}{\sin (\theta + \theta'')}.$$

The reflectivity of the surface is proportional to the ratio of the square of the incident and reflected waves.

FIGURE 4-2
GEOMETRY FOR FRESNEL'S RELATIONS

$$\rho_{\perp} = \left(\frac{E'_{\perp}}{E_{\perp}} \right)^2, \quad \rho_{\parallel} = \left(\frac{E'_{\parallel}}{E_{\parallel}} \right)^2$$

The polarization of the incident light is usually not known, and, in practice, the reflectivity is found by averaging the perpendicular and parallel components.

$$\rho = \frac{1}{2} [\rho_{\perp} + \rho_{\parallel}]$$

$$\rho = \frac{1}{2} \left\{ \left[\frac{\sin(\theta - \theta'')}{\sin(\theta + \theta'')} \right]^2 + \left[\frac{\tan(\theta - \theta'')}{\tan(\theta + \theta'')} \right]^2 \right\}$$

For normal incidence the equations are not valid, and the reflectivity becomes

$$\rho = \left[\frac{(n-1)}{(n+1)} \right]^2$$

The Fresnel equations also apply if the wave is incident upon a medium with a lower index of refraction. There is, however, an angle beyond which all incident radiation is totally reflected. This critical angle is given by

$$\sin \theta_c = \frac{1}{n},$$

where n is the relative index of refraction. With these relationships now available the real-boundary geometries will be discussed.

Monte Carlo Decisions Affecting Fresnel Reflectivity

When a ray of intensity I traversing a dielectric is incident upon the dielectric-air interface within the critical angle, a portion will be reflected, and that remaining will be transmitted through the surface. The Fresnel reflectivity is shown in Figure 4-3 for a refractive index of 1.5. If the Fresnel equations are used for computing the

FIGURE 4-3

FRESNEL REFLECTIVITY - REFRACTIVE INDEX = 1.5

energy leaving the dielectric, then various fractions of the energy of the energy bundles would have to be calculated. The energy fraction would range from 0.04 to 0.99+ for an n of 1.5. This type of accounting is not pleasing and may be eliminated by using a Monte Carlo decision.

When an energy packet strikes the interface, the value of the Fresnel reflectivity is calculated for the particular angle of incidence. A random number is then computed and compared with the value of the reflectivity. If the random number exceeds the reflectivity, the energy escapes. If not, the energy is assumed to be totally reflected. The total energy escaping the dielectric is conserved when a large number of histories is considered. For instance, if 100 rays were incident on the interface at an angle near the normal, 96 would, on the average, be transmitted and 4 would be totally reflected. This greatly simplifies the "bookkeeping" and is consistent with the procedure followed when a path length is terminated by absorption or by scattering. A random number is computed and compared with the scattering/extinction ratio. If the random number is larger, the ray is absorbed. If not, the ray is scattered and a new direction is randomly chosen.

One-Dimensional Infinite Medium with Reflecting Boundaries

Since this problem can be reduced to one dimension, the geometry is much easier. The emission location and direction are determined as before, but the exit will be out the top or bottom plane, depending only on θ .

Top Escape	Bottom Escape	Scattered or Absorbed
(a) $0^\circ \leq \theta < 90^\circ$	(b) $90^\circ < \theta \leq 180^\circ$	(c) $\theta = 90^\circ$

(a) $0^\circ \leq \theta < 90^\circ$: Top Escape

The geometry is shown in Figure 4-4(a). θ' is the exit angle and θ_F is the angle between the surface normal and incident direction. θ is equal to θ_F for this case. If θ_F is greater than the critical angle, total reflection will occur off the top surface. If θ_F is less than the critical, the energy bundle may escape in accordance with Fresnel's relations, or it may also be totally reflected from the top, determined by a Monte Carlo decision. If the ray escapes, the angle θ' is determined from Snell's Law.

$$\sin \theta' = n \sin \theta_F$$

Since there is no azimuthal dependence, this is the only angle calculation required. The slab location, y' , is determined as before.

$$y' = y + (1-z) \tan \theta \sin \varphi$$

If a reflection occurs at the top surface, the location is:

$$y' = y + (1-z) \tan \theta \sin \varphi$$

$$z' = 1.0$$

$$\theta = 180^\circ - \theta_F$$

$$\varphi = \varphi$$

A new path length is generated for the energy with the above position and direction.

(b) $90^\circ < \theta \leq 180^\circ$: Bottom Escape

This geometry is shown in Figure 4-4(b). θ_F is equal to $180^\circ - \theta$, and total reflection occurs if θ_F is greater than the critical angle. If the photon escapes, θ' is again found by Snell's Law.

$$\sin \theta' = n \sin \theta_F$$

(a) Top escape: $0^\circ \leq \theta < 90^\circ$

(b) Bottom escape: $90^\circ < \theta \leq 180^\circ$

FIGURE 4-4

GEOMETRY FOR INFINITE COATING ESCAPES

If a reflection occurs at the bottom surface, the location is:

$$y' = y - z \tan \theta \sin \varphi$$

$$z' = 0$$

$$\theta = \theta_F$$

$$\varphi = \varphi$$

A path length is then generated for the new location and direction on the bottom surface.

(c) $\theta = 90^\circ$: Scattering or Absorption

If θ equals 90° , the ray cannot escape and will be absorbed or scattered at the end of the path length s . The coordinates and a new emission direction would be determined at that point.

Two-Dimensional Media with Reflecting Boundaries

The geometry for this case is essentially a combination of Cases 1 and 2 above with the addition of edge reflections. Table 4-1 and the discussion for each choice listed under Case 1 apply. If a reflection occurs from the top or bottom surface, the location and emission direction would be the same as for Case 2 above. Energy emitted in the direction of the edge will escape or be reflected depending on the angle of incidence and the Fresnel equations. The edge geometry is shown in Figure 4-5.

Edge Escape

$$0^\circ < \theta \leq 90^\circ$$

$$\cos \theta_F = - \sin \theta \sin \varphi$$

$$\sin \theta' = n \sin \theta_F \quad (\text{Escape Angle})$$

$$90^\circ < \theta \leq 180^\circ$$

(a) $0^\circ \leq \theta \leq 90^\circ$

(b) $90^\circ \leq \theta \leq 180^\circ$

FIGURE 4-5

GEOMETRY FOR COATING EDGE ESCAPES

$$\cos \theta_F = (-\sin \theta) \sin \varphi$$

$$\sin \theta' = n \sin \theta$$

For an edge escape φ' is found in the same manner as in Case 1.

Edge Reflection

If a ray is reflected off the edge, the position is determined, and a new path length is determined at that point.

$$z' = z - y \cot \theta / \sin \varphi$$

$$\theta = \theta$$

$$\varphi = 2\pi - \varphi \text{ previous}$$

Directional Emittance-Reflecting Boundaries

The directional emittance for the reflecting boundary problems is determined in the same manner as before.

$$\epsilon(\mu', \varphi') = \frac{I(\mu', \varphi')}{I_{bb}}$$

For these cases, however, the black body intensity within the coating is not equal to the black body intensity on the outside of the coating. The two intensities differ by a factor n^2 . This may be seen from the following from Love (1). Consider an enclosure with a dielectric material of index of refraction n in a vacuum. The isolated enclosure and interface geometry are shown in Figure 4-6(a) and (b) respectively. The intensity of thermal radiation in the vacuum is black body intensity, and in equilibrium, there cannot be a net exchange of radiation across the interface. The energy balance is

$$I_{bb} \rho \cos \theta \, d\omega + I(1-\rho') \cos \theta' \, d\omega' = I_{bb} \cos \theta \, d\omega.$$

(a) Isolated enclosure

(b) Interface energy balance

FIGURE 4-6

ISOLATED ENCLOSURE AND DIELECTRIC-VACUUM INTERFACE

From Snell's Law $n \sin \theta' = \sin \theta$

$$\text{and } n \cos \theta' d\theta' = \cos \theta d\theta .$$

$$d\omega = \sin \theta d\theta d\phi$$

$$d\omega' = \sin \theta' d\theta' d\phi' .$$

Substituting these relations into the energy balance equation yields

$$\begin{aligned} I_{bb} \cos \theta \rho \sin \theta d\theta d\phi + I (1-\rho') \frac{1}{n} \cos \theta d\theta \frac{\sin \theta d\phi'}{n} \\ = I_{bb} \cos \theta \sin \theta d\theta d\phi . \end{aligned}$$

$d\phi = d\phi'$, which yields

$$I_{bb} (1-\rho) = \frac{1}{n^2} I(1-\rho') .$$

From Fresnel's equations, $\rho' = \rho$; therefore, the intensity within the medium is related to the black body intensity by $I = n^2 I_{bb}$.

For the coating emittance

$$\epsilon(\mu', \varphi') = \frac{n^2}{\cos \theta} \frac{4\pi A_{\text{avg}} N(\mu', \varphi')}{H \Delta \omega N} .$$

The basic equations for determining the emittances, and the geometry for all three cases have been discussed. The next three chapters deal with the specific methods used in determining the results and with the directional emittances obtained.

CHAPTER V

TWO-DIMENSIONAL MEDIA WITH FREE BOUNDARIES

The four different media analyzed were optical depths of 1.0 and 3.0, and scattering/extinction ratios of 0.1 and 0.9. The general approach for all four media is the same; however, the number of collection strips and generating area lengths vary. For these reasons each of the particular problems is discussed separately.

$$\tau = 1.0, \sigma = 0.9$$

This was the first solution attempted, and two different data runs were made in order to determine what problems existed in obtaining the Monte Carlo solution. Preliminary runs were made to see how far the ray would travel in the Y-direction prior to escaping or being absorbed. Most of the time the energy escaped directly if the path length was long. For shorter path lengths a scattering usually occurred. The maximum distance traveled was a dimensionless distance of about 6.0, or 6 optical depths. From this information a generating length of 11.0 and a collection length of 4.0 were chosen. The y and z dimensions were non-dimensionalized by dividing by the medium thickness, L, which was assigned a value of one. A length of 11 thus corresponds to 11 edge thicknesses or 11 optical depths. The collection length was chosen using Bobco's (2) data as a guide. His data indicated little directional characteristics

beyond a dimensionless distance of $y/L=3.0$. Three data runs totaling 600,000 original emissions were made with $SOL = 11.0$. The results of this data substantiated Bobco's results by exhibiting little or no difference in directional emittance at $y/L = 3.0$.

Two problems were noted as a result of the runs with $SOL = 11.0$. First, too many losses were occurring outside the counting region, and second, the losses out the edge were very small. To increase the number of escapes from the edge and to increase the countable escapes/total emission ratio, the generating length was reduced to 8.0. The collecting area was reduced to 3.2 for all runs except one, which was 3.6. Table 5-1 is a summary of the tests and results. The normal emittance for the plane and edge is shown in Figure 5-1. Three-dimensional directional emittance curves for various locations from the edge are plotted in Figures 5-2 to 5-5. Edge directional emittance curves are shown in Figures 5-6 and 5-7.

$$\tau = 1.0, \sigma = 0.1$$

The data collected for these optical conditions was obtained with two different optical lengths, $SOL = 7.0$ and 5.0 . Since an energy bundle is much less likely to travel as far in this highly absorbing medium, this allowed the generating length to be reduced from 8.0 to 7.0. The smaller generating length was used to obtain edge data and more escapes near the corner. Table 5-2 summarizes the tests and results for all the runs. The normal emittance for the plane and edge is shown in Figure 5-1. The directional emittance of the plane at four locations is shown in Figures 5-8 to 5-11. Edge emittances are shown in Figures 5-12 and 5-13.

TABLE 5-1

SUMMARY OF TESTS AND RESULTS FOR $\tau = 1.0$, $\sigma = 0.9$

Generating Length, SOL	No. of Emissions	No. of Scatterings	No. of Absorptions	No. Edge Escapes	No. Plane Escapes	Collection Length
11.0	150,000	179,891	20,244	4,179	43,433	0 to 4.0
11.0	150,000	180,607	20,157	4,227	43,268	0 to 4.0
11.0	300,000	359,744	40,286	8,198	86,709	0 to 4.0
8.0	300,000	346,923	38,747	11,395	106,839	0 to 3.6
8.0	100,000	116,115	12,864	3,813	31,460	0 to 3.2
8.0	300,000	346,718	38,609	11,354	93,838	0 to 3.2
8.0	300,000	346,684	38,208	11,776	93,464	0 to 3.2

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TABLE 5-2

SUMMARY OF TESTS AND RESULTS FOR $\tau = 1.0$, $\sigma = 0.1$

Generating Length, SOL	No. of Emissions	No. of Scatterings	No. of Absorptions	No. Edge Escapes	No. Plane Escapes	Collection Length
7.0	600,000	38,083	341,604	14,841	108,354	0 to 3.2
7.0	600,000	37,356	341,330	14,699	108,419	0 to 3.2
7.0	600,000	37,793	341,568	14,685	107,944	0 to 3.2
7.0	600,000	37,497	341,988	14,602	108,213	0 to 3.2
7.0	600,000	38,100	340,887	14,740	108,827	0 to 3.2
5.0	600,000	37,721	338,039	20,767	52,203	0 to 1.2
5.0	400,000	25,079	225,476	13,877	34,861	0 to 1.2

$$\underline{\tau = 3.0, \sigma = 0.9}$$

The computer time required to converge to a solution increases with an increase in optical depth. Consequently, to use the same collection area used for $\tau = 1.0$ would almost double the computer time required. The saving feature is the fact that Bobco's results showed the medium to have little or no directional characteristics at a dimensionless distance of $y/L = 1.5$ from the edge. If the Monte Carlo program could substantiate this result, the collection area could be reduced to 1.6, thus allowing a further reduction in the generating length. The first runs were made with $SOL = 2.6$, approximately one-third of the length for $\tau = 1.0$, and the results agreed very well with Bobco's data for $y/L = 1.5$ as the emittance showed little directionality. The generating length of $SOL = 2.6$ was kept for most of the runs. Additional runs were made to obtain more edge escapes and data near the corner by reducing the generating length to 1.8. These results are tabulated in Table 5-3. The normal emittance for the edge and plane is shown in Figure 5-1. Directional emittance for the plane at three locations is shown in Figures 5-14 to 5-16. Edge emittance is shown in Figures 5-17 and 5-18.

$$\underline{\tau = 3.0, \sigma = 0.1}$$

This "strongly"-absorbing medium with a high optical depth required a much greater number of emissions to achieve the same total number of escapes. The mean free path is small, and the history usually terminated with an absorption. Most of the data was obtained with a generating length of 2.6 and a collection length of 1.6. There was a bigger disagreement with Bobco's results for this case, particularly for the edge,

TABLE 5-3

SUMMARY OF TESTS AND RESULTS FOR $\tau = 3.0$, $\sigma = 0.9$

Generating Length, SOL	No. of Emissions	No. of Scatterings	No. of Absorptions	No. Edge Escapes	No. Plane Escapes	Collection Length
2.6	250,000	697,740	77,387	20,502	84,372	0 to 1.6
2.6	250,000	694,893	77,587	20,311	84,058	0 to 1.6
2.6	250,000	698,398	77,188	20,650	84,148	0 to 1.6
2.6	250,000	693,957	77,165	20,553	84,554	0 to 1.6
1.8	300,000	815,050	90,467	35,430	44,454	0 to 0.6
1.8	150,000	405,906	45,174	17,882	22,181	0 to 0.6

44

TABLE 5-4

SUMMARY OF TESTS AND RESULTS FOR $\tau = 3.0$, $\sigma = 0.1$

Generating Length, SOL	No. of Emissions	No. of Scatterings	No. of Absorptions	No. Edge Escapes	No. Plane Escapes	Collection Length
2.6	500,000	43,694	397,950	14,564	51,889	0 to 1.6
2.6	750,000	66,253	597,153	21,993	78,062	0 to 1.6
2.6	750,000	66,157	597,142	22,088	78,030	0 to 1.6
2.6	750,000	66,529	597,302	22,029	78,061	0 to 1.6
2.6	750,000	66,341	597,391	21,934	78,140	0 to 1.6
1.6	1,000,000	86,191	779,487	47,901	58,949	0 to 0.6
1.6	1,000,000	86,796	779,699	48,006	58,908	0 to 0.6
1.0	1,000,000	83,652	757,359	76,153	26,464	0 to 0.2

and two additional generating lengths, $SOL = 1.6$ and 1.0 , were run in an attempt to resolve the differences. If the value of the emittance turned out to be related in any way to the generating areas, it should appear in these tests. The number of edge escapes was approximately the same for all three SOL 's, so that the convergence would be approximately equal. As expected, there was no dependence on generating length. A summary of all the tests run is shown in Table 5-4. The normal emittance of the plane and edge is shown in Figure 5-1. Directional emittance of the plane at three locations is shown in Figures 5-19 to 5-21. Edge emittance at two locations is shown in Figures 5-22 and 5-23.

On all curves where applicable, Bobco's analytical solutions are plotted. His data was obtained only for two orthogonal "view" planes, and this is the reason why it is shown on only two of the four azimuthal directions.

FIGURE 5-1
NORMAL EMITTANCE FOR EDGE AND PLANE

Optical depth $\tau=1.0$

Scattering/Extinction $\sigma=0.9$

Dimensionless distance from edge $y/L=0.1$

FIGURE 5-2

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 1.0$

Scattering/Extinction $\sigma = 0.9$

Dimensionless distance from edge $y/L = 0.5$

FIGURE 5-3

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 1.0$

Scattering/Extinction $\sigma = 0.9$

Dimensionless distance from edge $y/L = 1.0$

FIGURE 5-4
DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau=1.0$

Scattering/Extinction $\sigma = 0.9$

Dimensionless distance from edge $y/L = 3.0$

FIGURE 5-5

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau=1.0$

Scattering/Extinction $\sigma=0.9$

Dimensionless distance from bottom surface $z/L=0.22$

FIGURE 5-6

DIRECTIONAL EMITTANCE OF EDGE SURFACE

Optical depth $\tau = 1.0$

Scattering/Extinction $\sigma = 0.9$

Dimensionless distance from bottom surface $z/L = 0.5$

FIGURE 5-7

DIRECTIONAL EMITTANCE OF EDGE SURFACE

Optical depth $\tau=1.0$

Scattering/Extinction $\sigma=0.1$

Dimensionless distance from edge $y/L = 0.1$

FIGURE 5-8
DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 1.0$

Scattering/Extinction $\sigma = 0.1$

Dimensionless distance from edge $y/L = 0.5$

FIGURE 5-9
DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau=1.0$

Scattering/Extinction $\sigma=0.1$

Dimensionless distance from edge $y/L=1.0$

FIGURE 5-10

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 1.0$

Scattering/Extinction $\sigma = 0.1$

Dimensionless distance from edge $y/L = 3.0$

FIGURE 5-II

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau=1.0$

Scattering/Extinction $\sigma=0.1$

Dimensionless distance from bottom surface $z/L=0.22$

FIGURE 5-12

DIRECTIONAL EMITTANCE OF EDGE SURFACE

Optical depth $\tau=1.0$

Scattering/Extinction $\sigma=0.1$

Dimensionless distance from bottom surface $z/L = 0.5$

FIGURE 5-13

DIRECTIONAL EMITTANCE OF EDGE SURFACE

Optical depth $\tau = 3.0$

Scattering/Extinction $\sigma = 0.9$

Dimensionless distance from edge $y/L = 0.1$

FIGURE 5-14
DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 3.0$

Scattering/Extinction $\sigma = 0.9$

Dimensionless distance from edge $y/L = 0.5$

FIGURE 5-15

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 3.0$

Scattering/Extinction $\sigma = 0.9$

Dimensionless distance from edge $y/L = 1.5$

FIGURE 5-16
DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau=3.0$

Scattering/Extinction $\sigma=0.9$

Dimensionless distance from bottom surface $z/L=0.22$

FIGURE 5-17

DIRECTIONAL EMITTANCE OF EDGE SURFACE

Optical depth $\tau=3.0$

Scattering/Extinction $\sigma=0.9$

Dimensionless distance from bottom surface $z/L=0.5$

FIGURE 5-18

DIRECTIONAL EMITTANCE OF EDGE SURFACE

Optical depth $\tau = 3.0$

Scattering/Extinction $\sigma = 0.1$

Dimensionless distance from edge $y/L = 0.1$

FIGURE 5-19

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 3.0$

Scattering/Extinction $\sigma = 0.1$

Dimensionless distance from edge $y/L = 0.5$

FIGURE 5-20

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 3.0$

Scattering/Extinction $\sigma = 0.1$

Dimensionless distance from edge $y/L = 1.5$

FIGURE 5-21

DIRECTIONAL EMITTANCE OF PLANE SURFACE

Optical depth $\tau = 3.0$

Scattering/Extinction $\sigma = 0.1$

Dimensionless distance from bottom surface $z/L = 0.22$

FIGURE 5-22

DIRECTIONAL EMITTANCE OF EDGE SURFACE

Optical depth $\tau=3.0$

Scattering/Extinction $\sigma=0.1$

Dimensionless distance from bottom surface $z/L=0.5$

FIGURE 5-23

DIRECTIONAL EMITTANCE OF EDGE SURFACE

CHAPTER VI

ONE-DIMENSIONAL COATING RESULTS

There are three parameters which affect the directional emittance of a coating; the refractive index, the optical depth, and the absorption coefficient. The purpose of this chapter is to show the variation of the directional emittance as each of the above three parameters is varied. Table 6-1 summarizes the tests run and references the applicable figure where the results are shown.

All results were obtained using the geometry of Figure 3-6. SOL varied from 12 to 48 depending on the optical properties of the medium, and D is equal to SOL/4. D represents an optical distance of approximately ∞ to simulate an infinite medium. The data is obtained in terms of the scattering/extinction coefficient, σ , refractive index or optical depth.

The variation of directional emittance with change in optical depth is shown in Figure 6-1 for four different optical depths. The refractive index and scattering/extinction ratio were constant at 1.4 and 0.99, respectively, for all runs. The extinction coefficient is equal to 200/cm (making the absorption coefficient 2.0/cm), consistent with the material results shown in Appendix A. The four optical depths thus correspond to coating thicknesses of 0.002 in., 0.004 in., 0.006 in., and 0.010 in. Table 6-2 lists the data for the four different tests.

TABLE 6-1

SUMMARY OF TESTS FOR ONE-DIMENSIONAL COATING

Optical Depth	Scattering Extinction	Refractive Index	Figure Number	SOL	D
1.0	0.99	1.4	6-1	48.0	12.0
2.0	0.99	1.4	6-1	48.0	12.0
3.0	0.99	1.4	6-1,6-3	16.0	4.0
5.0	0.99	1.4	6-1,6-5	12.0	3.0
1.0	0.95	1.4	6-2	48.0	12.0
2.0	0.95	1.4	6-2	48.0	12.0
3.0	0.95	1.4	6-2,6-4	16.0	4.0
5.0	0.95	1.4	6-2,6-6	12.0	3.0
3.0	0.99	1.4	6-1,6-3	16.0	4.0
3.0	0.99	1.6	6-3	16.0	4.0
3.0	0.99	1.8	6-3	16.0	4.0
3.0	0.95	1.4	6-2,6-4	16.0	4.0
3.0	0.95	1.6	6-4	16.0	4.0
3.0	0.95	1.8	6-4	16.0	4.0
5.0	0.99	1.4	6-1,6-5	12.0	3.0
5.0	0.99	1.6	6-5	12.0	3.0
5.0	0.99	1.8	6-5	12.0	3.0
5.0	0.95	1.4	6-2,6-6	12.0	3.0
5.0	0.95	1.6	6-6	12.0	3.0
5.0	0.95	1.8	6-6	12.0	3.0

Refractive index $n = 1.4$

Scattering/Extinction $\sigma = 0.99$

FIGURE 6-1
DIRECTIONAL EMITTANCE OF INFINITE
COATING FOR VARIOUS OPTICAL DEPTHS

TABLE 6-2

RESULTS FOR $\sigma = 0.99$, $n = 1.4$

$\tau = 1.0$																	
No. of emissions = 15,000									No. of scatterings = 69,951								
No. of absorptions = 671									No. lost outside collection area = 7231								
No. escapes in each polar angle increment																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
20	122	231	367	474	532	589	641	660	599	605	551	506	444	339	233	185	

$\tau = 2.0$																	
No. of emissions = 15,000									No. of scatterings = 111,648								
No. of absorptions = 1168									No. lost outside collection area = 6936								
No. escapes in each polar angle increment																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
18	105	232	340	369	525	552	558	623	649	616	548	534	428	329	273	197	

$\tau = 3.0$																	
No. of emissions = 20,000									No. of scatterings = 204,786								
No. of absorptions = 2083									No. lost outside collection area = 9055								
No. escapes in each polar angle increment																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
26	122	250	409	524	616	699	752	766	823	787	746	667	590	476	359	250	

$\tau = 5.0$																	
No. of emissions = 17,500									No. of scatterings = 294,363								
No. of absorptions = 2940									No. lost outside collection area = 7319								
No. escapes in each polar angle increment																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
21	91	202	287	459	483	536	652	678	664	644	605	518	486	406	276	233	

The directional emittance curves increase with increasing optical depth and have approximately the same shape.

Figure 6-2 is also a plot of the change in directional emittance with changes in optical depth. The parameters are the same as for Figure 6-1, except the scattering/extinction ratio is 0.95. Table 6-3 lists the data for the four different optical depths. The curves are essentially the same shape and increase with increasing optical depth.

The effect of the coating refractive index on the directional emittance is shown in Figure 6-3 for three indexes of refraction, $n = 1.4$, 1.6 , and 1.8 . This range of 1.4 to 1.8 is representative of the total spread in refractive index for most ceramic coating materials. Contrary to the results for the infinite absorbing medium of Appendix B, an increase in the index of refraction is found to increase the directional emittance for the "strongly"-scattering medium. The optical depth is 3.0 , and the scattering/extinction ratio is 0.99 . Table 6-4 lists the data used in obtaining the curves.

Figure 6-4 shows the change in directional emittance with refractive index changes for a lower scattering/extinction ratio, 0.95 . The same curve characteristics are again observed. Table 6-5 summarizes the data for Figure 6-4.

The optical depths and scattering/extinction ratios run for the two-dimensional coating tests discussed in Chapter VII were $\tau = 3.0$ and 5.0 and $\sigma = 0.95$ and 0.99 . To gain some insight into the behavior of the emittance with the higher optical depth, additional data was obtained for the same three indexes of refraction $n = 1.4$, 1.6 , and 1.8 . These results are shown in Figures 6-5 and 6-6.

TABLE 6-3

RESULTS FOR $\sigma = 0.95$, $n = 1.4$

$\tau = 1.0$																
No. of emissions = 25,000								No. of scatterings = 92,933								
No. of absorptions = 5055								No. lost outside collection area = 10,135								
No. escapes in each polar angle increment																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
16	161	307	504	604	737	788	904	867	878	823	747	729	586	488	378	293

$\tau = 2.0$																
No. of emissions = 20,000								No. of scatterings = 110,153								
No. of absorptions = 5756								No. lost outside collection area = 7161								
No. escapes in each polar angle increment																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
23	91	226	298	414	493	563	626	603	667	638	607	537	453	365	267	212

$\tau = 3.0$																
No. of emissions = 35,000								No. of scatterings = 241,793								
No. of absorptions = 12,776								No. lost outside collection area = 6522								
No. escapes in each polar angle increment																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
31	159	331	498	658	767	898	965	963	983	956	854	798	691	551	405	324

$\tau = 5.0$																
No. of emissions = 30,000								No. of scatterings = 286,331								
No. of absorptions = 15,108								No. lost outside collection area = 7748								
No. escapes in each polar angle increment																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
16	80	233	304	417	471	589	591	633	708	654	647	516	427	383	257	218

FIGURE 6-3
VARIATION OF DIRECTIONAL EMITTANCE WITH CHANGE
IN REFRACTIVE INDEX-INFINITE COATING

TABLE 6-4

RESULTS FOR $\tau = 3.0$, $\sigma = 0.99$

n = 1.4																
(See Table 6-2)																
n = 1.6																
No. of emissions = 15,000								No. of scatterings = 199,682								
No. of absorptions = 2031								No. lost outside collection area = 6587								
No. escapes in each polar angle increment																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
15	69	158	294	373	429	543	570	586	576	621	521	462	421	303	241	200
n = 1.8																
No. of emissions = 20,000								No. of scatterings = 338,198								
No. of absorptions = 3329								No. lost outside collection area = 8395								
No. escapes in each polar angle increment																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
13	113	246	384	501	569	692	683	744	724	760	660	636	497	436	331	287

TABLE 6-5

RESULTS FOR $\tau = 3.0$, $\sigma = 0.95$

n = 1.4																
(See Table 6-3)																
n = 1.6																
No. of emissions = 35,000								No. of scatterings = 291,641								
No. of absorptions = 15,384								No. lost outside collection area = 10,208								
No. escapes in each polar angle increment																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
24	109	244	394	528	692	730	818	861	833	826	769	755	650	492	358	325
n = 1.8																
No. of emissions = 35,000								No. of scatterings = 331,692								
No. of absorptions = 17,421								No. lost outside collection area = 9284								
No. escapes in each polar angle increment																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
22	116	217	363	480	584	683	758	776	754	702	707	638	507	425	320	243

Optical depth $\tau = 5.0$

Scattering/Extinction $\sigma = 0.99$

FIGURE 6-5
VARIATION OF DIRECTIONAL EMITTANCE WITH CHANGE
IN REFRACTIVE INDEX—INFINITE COATING

FIGURE 6-6
VARIATION OF DIRECTIONAL EMITTANCE WITH CHANGE
IN REFRACTIVE INDEX—INFINITE COATING

Figure 6-5 is for scattering/extinction ratio of 0.99 and an optical depth of 5.0. Table 6-6 contains the data for these runs.

Figure 6-6 is for an optical depth of 5.0 and a scattering extinction ratio of 0.95. It is seen in Figure 6-6 that the increase in index of refraction from 1.6 to 1.8 effects only a small increase in the value of the directional emittance. The higher index of refraction causes more energy to be reflected, and this fact, combined with the greater number of absorptions in the higher optical thickness medium, limits the value of the emittance. A further increase in the index of refraction would eventually lead to a limiting maximum value of the directional emittance. Table 6-7 lists the data for Figure 6-6.

RESULTS FOR $\tau = 5.0$, $\sigma = 0.99$

$n = 1.4$ (See Table 6-2)																	
$n = 1.6$ No. of emissions = 17,000 No. of scatterings = 366,433 No. of absorptions = 3645 No. lost outside collection area = 7053 No. escapes in each polar angle increment 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 13 57 159 317 382 547 606 615 604 594 585 544 509 430 362 276 202																	
$n = 1.8$ No. of emissions = 12,500 No. of scatterings = 307,469 No. of absorptions = 3166 No. lost outside collection area = 4729 No. escapes in each polar angle increment 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 10 56 115 209 250 334 396 399 420 447 394 385 332 287 249 185 137																	

TABLE 6-7

RESULTS FOR $\tau = 5.0$, $\sigma = 0.95$

<p style="text-align: center;">$n = 1.4$</p> <p style="text-align: center;">(See Table 6-3)</p>																																																		
<p style="text-align: center;">$n = 1.6$</p> <p>No. of emissions = 30,000 No. of scatterings = 323,476</p> <p>No. of absorptions = 16,933 No. lost outside collection area = 6850</p> <p style="text-align: center;">No. escapes in each polar angle increment</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">1</td><td style="text-align: center;">2</td><td style="text-align: center;">3</td><td style="text-align: center;">4</td><td style="text-align: center;">5</td><td style="text-align: center;">6</td><td style="text-align: center;">7</td><td style="text-align: center;">8</td><td style="text-align: center;">9</td><td style="text-align: center;">10</td><td style="text-align: center;">11</td><td style="text-align: center;">12</td><td style="text-align: center;">13</td><td style="text-align: center;">14</td><td style="text-align: center;">15</td><td style="text-align: center;">16</td><td style="text-align: center;">17</td> </tr> <tr> <td style="text-align: center;">14</td><td style="text-align: center;">73</td><td style="text-align: center;">156</td><td style="text-align: center;">259</td><td style="text-align: center;">321</td><td style="text-align: center;">412</td><td style="text-align: center;">541</td><td style="text-align: center;">519</td><td style="text-align: center;">546</td><td style="text-align: center;">615</td><td style="text-align: center;">561</td><td style="text-align: center;">516</td><td style="text-align: center;">489</td><td style="text-align: center;">436</td><td style="text-align: center;">328</td><td style="text-align: center;">247</td><td style="text-align: center;">184</td> </tr> </table>																	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	14	73	156	259	321	412	541	519	546	615	561	516	489	436	328	247	184
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17																																		
14	73	156	259	321	412	541	519	546	615	561	516	489	436	328	247	184																																		
<p style="text-align: center;">$n = 1.8$</p> <p>No. of emissions = 30,000 No. of scatterings = 350,513</p> <p>No. of absorptions = 18,595 No. lost outside collection area = 6283</p> <p style="text-align: center;">No. escapes in each polar angle increment</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">1</td><td style="text-align: center;">2</td><td style="text-align: center;">3</td><td style="text-align: center;">4</td><td style="text-align: center;">5</td><td style="text-align: center;">6</td><td style="text-align: center;">7</td><td style="text-align: center;">8</td><td style="text-align: center;">9</td><td style="text-align: center;">10</td><td style="text-align: center;">11</td><td style="text-align: center;">12</td><td style="text-align: center;">13</td><td style="text-align: center;">14</td><td style="text-align: center;">15</td><td style="text-align: center;">16</td><td style="text-align: center;">17</td> </tr> <tr> <td style="text-align: center;">16</td><td style="text-align: center;">65</td><td style="text-align: center;">129</td><td style="text-align: center;">240</td><td style="text-align: center;">310</td><td style="text-align: center;">320</td><td style="text-align: center;">403</td><td style="text-align: center;">453</td><td style="text-align: center;">525</td><td style="text-align: center;">470</td><td style="text-align: center;">469</td><td style="text-align: center;">440</td><td style="text-align: center;">385</td><td style="text-align: center;">329</td><td style="text-align: center;">264</td><td style="text-align: center;">165</td><td style="text-align: center;">139</td> </tr> </table>																	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	16	65	129	240	310	320	403	453	525	470	469	440	385	329	264	165	139
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17																																		
16	65	129	240	310	320	403	453	525	470	469	440	385	329	264	165	139																																		

CHAPTER VII

TWO-DIMENSIONAL COATING RESULTS

The directional emittance results for the two-dimensional coating study are obtained for two optical depths, $\tau = 3$ and 5, and for two scattering/extinction ratios, $\sigma = 0.95$ and 0.99. The refractive index is 1.4 for all the two-dimensional results. In terms of physical dimensions an optical depth of 3 corresponds to an extinction coefficient of 200/cm and a coating thickness of slightly less than 0.006 in. An optical depth of 5 corresponds to a coating thickness of slightly less than 0.01 in. for the same extinction coefficient. The absorption coefficients were 2.0/cm and 10/cm for the scattering/extinction ratios of 0.99 and 0.95, respectively. Figure 3-3 illustrates the geometry used in obtaining results, and Table 7-1 summarizes the four tests conducted.

TABLE 7-1

SUMMARY OF TESTS FOR TWO-DIMENSIONAL COATING

Optical Conditions	Generating Length, SOL	Collection Length, Plane	No. of Strips Plane	No. of Strips Edge
$\tau=3, \sigma=0.99$	3.0	1.6	8	9
$\tau=3, \sigma=0.95$	3.0	1.6	8	9
$\tau=5, \sigma=0.99$	2.4	1.2*	6*	9
$\tau=5, \sigma=0.95$	2.4	1.2*	6*	9

* The collection length was 1.6, but the data was valid only through 1.2.

Because of the large amount of computer time required to complete one set of test conditions, the data was obtained by several shorter runs, just as for the two-dimensional, free-boundary study. The data from each run was added to obtain the final results. Each set of optical conditions will be discussed separately, including the results of each of the separate, shorter runs. Figure 7-1 shows the normal emittance of both the plane and the edge for all four optical conditions.

$$\tau = 3.0, \sigma = 0.99$$

As noted in Table 7-1, the collection length is 1.6, which allowed the emittance to be obtained at a maximum averaged distance of 1.5 from the edge. It was anticipated that this distance would be sufficient to obtain the "infinite" conditions for the coating, but it was not. As noted in Figure 7-1, the normal emittance is approximately 90 per cent of the normal emittance obtained from the one-dimensional results for the same optical conditions. Extrapolation of the normal emittance curve to the one-dimensional emittance value indicates that a collection length greater than 2.0 would have been required to reach the "infinite" emittance conditions. Nevertheless, sufficient data has been obtained to analyze the coating directional emittance properties.

The most significant (and unexpected) result is the azimuthal independence exhibited by the coating emittance. Even at the closest distance to the edge, y/L 0.1, there is no azimuthal dependence. The directional emittance thus becomes a function of location and polar angle only. These effects are shown in Figure 7-2 for various distances from the edge.

Edge emittance results are plotted in Figure 7-3. The edge normal emittance is very flat and does not approach the maximum value obtained for the plane emittance. The azimuthal independence is also observed for the edge directional emittance. Table 7-2 summarizes the results for both the plane and the edge for all runs.

TABLE 7-2

SUMMARY OF TESTS AND RESULTS FOR $\tau = 3$, $\sigma = 0.99$

Generating Length, SOL = 3.0 Collection Length = 1.6				
No. of emissions	No. of absorptions	No. of scatterings	No. of edge escapes	No. of plane escapes
50,000	4,354	434,706	5,319	18,848
70,000	6,220	611,012	7,534	26,319
70,000	6,257	608,630	7,192	26,677
70,000	6,127	607,067	7,493	26,623
70,000	6,195	611,769	7,429	26,458
100,000	8,597	867,925	10,557	38,000
200,000	17,479	1,746,416	21,191	75,485

 $\tau = 3.0$, $\sigma = 0.95$

The results for this set of optical conditions are shown in Table 7-3, and the directional emittance curves for the plane and the edge are shown in Figures 7-4 and 7-5, respectively. Again it was expected that a dimensionless distance of 1.5 from the edge would produce the emittance values for "infinite" conditions, but the normal emittance at 1.5 is slightly below the one-dimensional value. The difference is less than three per cent, which is within the expected accuracy of the results; however, assuming that there is still a locational dependence at 1.5, extrapolation to the one-dimensional value indicates a dimensionless distance of about 2.0 is required to reach "infinite" conditions.

The edge normal emittance is slightly flatter than the plane emittance. Just as for the plane emittance, the edge emittance did not exhibit an azimuthal dependence.

TABLE 7-3

SUMMARY OF TESTS AND RESULTS FOR $\tau = 3$, $\sigma = 0.95$

Generating Length, SOL = 3.0 Collection Length = 1.6				
No. of emissions	No. of absorptions	No. of scatterings	No. of edge escapes	No. of plane escapes
70,000	22,903	436,173	5,646	19,599
75,000	24,474	461,185	6,044	21,117
100,000	32,718	616,781	8,090	28,123
100,000	32,506	618,521	8,134	28,091
100,000	32,759	615,833	8,049	27,993
100,000	32,259	618,457	8,069	28,319
250,000	80,848	1,547,747	20,484	70,615

 $\tau = 5.0$, $\sigma = 0.99$

The directional emittance increased with the increased optical depth, as shown in Figures 7-6 and 7-7. Because of the greater optical depth, which in turn results in smaller particle path lengths, the generating area was reduced from 3.0 to 2.4. The data would be valid up to about $y/L = 1.2$, but it was anticipated that a distance of $y/L = 1.0$ would be sufficient to obtain the "infinite" conditions. As shown in Figure 7-1, the normal emittance has reached only about 90 per cent of the one-dimensional result at $y/L = 1.1$. If the results are extrapolated to the "infinite" value, a distance of about $y/L = 2.0$ is required to reach "infinity." The edge normal emittance is flatter than the plane normal emittance and reaches a maximum value somewhat less than the maximum of the plane. There is no azimuthal dependence for either the

edge or the plane for this optical depth. A total of 375,000 emissions was followed in five separate runs to obtain the data. These results are summarized in Table 7-4.

TABLE 7-4

SUMMARY OF TESTS AND RESULTS FOR $\tau = 5$, $\sigma = 0.99$

Generating Length, SOL = 2.4 Collection Length, = 1.2				
No. of emissions	No. of absorptions	No. of scatterings	No. of edge escapes	No. of plane escapes*
60,000	7,838	782,602	6,705	26,015
60,000	7,914	779,931	6,848	26,032
60,000	8,042	782,682	6,680	26,123
75,000	9,784	980,105	8,494	32,748
120,000	15,686	1,545,248	13,715	51,947

* Includes escapes to $y/L = 1.6$

$$\tau = 5.0, \sigma = 0.95$$

This was the only one of the four two-dimensional coatings analyzed which reached the "infinite" conditions within the collection area. The normal emittance of the plane converges exactly to the value obtained from the one-dimensional coating study, as shown in Figure 7-1. The edge and plane directional emittances are shown in Figures 7-8 and 7-9, respectively. Just as observed previously, there is no dependence on the azimuthal direction. Both the edge and the plane normal emittances depend strongly on location. The data was collected in a total of six runs following 655,000 histories as shown in Table 7-5.

TABLE 7-5

SUMMARY OF TESTS AND RESULTS FOR $\tau = 5$, $\sigma = 0.95$

Generating Length, SOL = 2.4 Collection Length = 1.2				
No. of emissions	No. of absorptions	No. of scatterings	No. of edge escapes	No. of plane escapes*
60,000	25,389	484,874	4,660	16,763
75,000	31,887	603,601	5,745	20,967
80,000	33,689	647,122	6,106	22,688
80,000	33,971	649,373	6,176	22,358
160,000	67,978	1,293,750	12,309	44,949
200,000	84,657	1,612,560	15,403	55,878

* Includes escapes to $y/L = 1.6$

FIGURE 7-1

COATING NORMAL EMITTANCE FOR EDGE AND PLANE

FIGURE 7-2
TWO-DIMENSIONAL COATING PLANE EMITTANCE

FIGURE 7-3
TWO-DIMENSIONAL COATING EDGE EMITTANCE

FIGURE 7-4

TWO-DIMENSIONAL COATING PLANE EMITTANCE

FIGURE 7-5

TWO-DIMENSIONAL COATING EDGE EMITTANCE

FIGURE 7-6
TWO-DIMENSIONAL COATING PLANE EMITTANCE

FIGURE 7-7
TWO-DIMENSIONAL COATING EDGE EMITTANCE

FIGURE 7-8
TWO-DIMENSIONAL COATING PLANE EMITTANCE

FIGURE 7-9
TWO-DIMENSIONAL COATING EDGE EMITTANCE

CHAPTER VIII

DISCUSSION OF RESULTS

Two-Dimensional, Free-Boundary Solutions

The two-dimensional free-boundary problem produced some varied directional emittance results. They may be summarized as follows:

- (1) The emittance was non-diffuse, and
the directionality depended on optical
depth, location, and absorption/extinction
ratio.
- (2) The edge emittance was generally more
directional than the plane emittance.
- (3) The emittance increased with both an
increase in optical depth and absorption
coefficient.
- (4) The "strongly"-absorbing medium was
more diffuse than the "weakly"-absorbing
medium.
- (5) The optical depth was the main feature
in determining the general curve shape.

As shown in Figures 5-1 to 5-23, the agreement with Bobco's (2) results is generally quite good. The directional emittances for the "weakly"-absorbing media are in good agreement for both optical depths

for both the edge and the plane, and the differences are small for the plane surfaces for "strongly"-absorbing media. Many of the differences are within the expected accuracy of the Monte Carlo solution, and the minor differences in curve shape are, in general, not significant.

Edge emittance values for the "strongly"-absorbing media did vary from Bobco's results. The curve shapes were very similar, and if the directional emittance curves are normalized by dividing by the normal emittance, the resulting curves (ϵ/ϵ_N) are in good agreement. It would appear from the results that Bobco's analytical solution may not be valid for "strongly"-absorbing media which view an infinite source. This, however, may not be an area of concern in the application of the analytical approximation. If one can generalize the results and conclude that the approximation is valid for surfaces which view a "finite" source, then the directional emittance of the conical dispersions also predicted by Bobco and Edwards (40) should be very accurate. This means, for instance, that once sufficient optical data is available from rocket exhaust plumes, an analytical solution may be available which could predict quite accurately the radiative heat transfer from the plume to the vehicle.

One-Dimensional Coating Solutions

The one-dimensional coating solutions were obtained for the purpose of correlating the results with the more complicated two-dimensional study, since the two-dimensional solutions required too much computer time to investigate all the effects of optical depth, refractive index, and absorption coefficient on the directional emittance. Some parameter was necessary to correlate the one-dimensional results, and this was

found to be the emittance divided by the absorption coefficient, ϵ/κ . A correlation also has to be obtained between the refractive index and emittance as a function of optical depth. Utilizing the one-dimensional data for refractive index, Figures 8-1 and 8-2 can be obtained. These two figures represent the increase in normal emittance over the value that would be obtained for a refractive index of 1.4. There is only a small change in the emittance ratio over the full range of optical depths studied, and the changes with n can be considered almost linear.

The results of Figures 8-1 and 8-2 and of the ϵ/κ data of Figures 5-1 and 5-2 are used to extend the directional emittance results for changes in refractive index to other optical depths not calculated. The results of this extension are shown in Figure 8-3 for $\sigma = 0.99$ and Figure 8-4 for $\sigma = 0.95$ for the one-dimensional coating. From these figures the normal emittance may be found for any optical depth between one and six, and for refractive indexes of 1.4, 1.6, and 1.8. Intermediate values of refractive index may be interpolated. Non-dimensional composite curves of ϵ/ϵ_N are shown in Figure 8-5 which can be used to find the directional emittance for the value of ϵ_N obtained from either Figure 8-3 or 8-4. The restrictions on this usage are, of course, that the scattering/extinction ratio must be either 0.95 or 0.99.

Two-Dimensional Coating Solution

The two-dimensional results were obtained for two optical depths and two scattering/extinction ratios. Using the one-dimensional results for the various values of optical depth and refractive index, the two-dimensional results can be extended to include these additional

FIGURE 8-1

INCREASE IN NORMAL EMITTANCE AS A FUNCTION OF
REFRACTIVE INDEX AND OPTICAL DEPTH

FIGURE 8-2

INCREASE IN NORMAL EMITTANCE AS A FUNCTION OF
REFRACTIVE INDEX AND OPTICAL DEPTH

FIGURE 8-3

ONE-DIMENSIONAL COATING NORMAL EMITTANCE AS A
FUNCTION OF OPTICAL DEPTH AND REFRACTIVE INDEX

FIGURE 8-4

ONE-DIMENSIONAL COATING NORMAL EMITTANCE AS A
FUNCTION OF OPTICAL DEPTH AND REFRACTIVE INDEX

(a) $\sigma = 0.99$ (b) $\sigma = 0.95$

FIGURE 8-5
COMPOSITE CURVES OF ϵ/ϵ_N FOR USE WITH
FIGURES 8-3 AND 8-4

conditions. The value of the directional emittance divided by the absorption coefficient is shown in Figure 8-6 for the two optical depths considered and two scattering/extinction ratios. It is noted that the curve differences are about the same up to a dimensionless distance of about $y/L=1.0$, and this difference corresponds roughly to the same amount observed in the one-dimensional problem for the same optical conditions. It is on this basis that the one-dimensional results are assumed directly applicable to the two-dimensional coating. The normal emittance is assumed to be increased by an increase in refractive index according to the one-dimensional results.

The results of the extended analysis are shown in Figures 8-7 through 8-18. Table 8-1 summarizes the optical conditions and notes the applicable Figure numbers. The curves for $\tau = 3$ and 5 in Figures 8-7

TABLE 8-1

SUMMARY OF THE TWO-DIMENSIONAL COATING ANALYSIS

Figure No.	Location	n	σ
8-7	edge	1.4	0.99
8-8	plane	1.4	0.99
8-9	edge	1.6	0.99
8-10	plane	1.6	0.99
8-11	edge	1.8	0.99
8-12	plane	1.8	0.99
8-13	edge	1.4	0.95
8-14	plane	1.4	0.95
8-15	edge	1.6	0.95
8-16	plane	1.6	0.95
8-17	edge	1.8	0.95
8-18	plane	1.8	0.95

and 8-8 and in Figures 8-13 and 8-14 were actually computed. All other curves were obtained from an extension of the one-dimensional data.

(a) $\sigma = 0.99$ (b) $\sigma = 0.95$

FIGURE 8-6

 ϵ_N/k FOR TWO-DIMENSIONAL COATING

FIGURE 8-7
NORMAL EMITTANCE OF EDGE FOR VARIOUS
OPTICAL DEPTHS — $n=1.4, \sigma=0.99$

FIGURE 8-8
NORMAL EMITTANCE OF PLANE FOR VARIOUS
OPTICAL DEPTHS — $n=1.4, \sigma=0.99$

FIGURE 8-9
NORMAL EMITTANCE OF EDGE FOR VARIOUS
OPTICAL DEPTHS — $n=1.6, \sigma=0.99$

FIGURE 8-10
NORMAL EMITTANCE OF PLANE FOR VARIOUS
OPTICAL DEPTHS — $n=1.6, \sigma=0.99$

FIGURE 8-11
NORMAL EMITTANCE OF EDGE FOR VARIOUS
OPTICAL DEPTHS — $n=1.8, \sigma=0.99$

FIGURE 8-12
NORMAL EMITTANCE OF PLANE FOR VARIOUS
OPTICAL DEPTHS — $n=1.8, \sigma=0.99$

FIGURE 8-13
NORMAL EMITTANCE OF EDGE FOR VARIOUS
OPTICAL DEPTHS— $n=1.4$, $\sigma=0.95$

FIGURE 8-14
NORMAL EMITTANCE OF PLANE FOR VARIOUS
OPTICAL DEPTHS— $n=1.4$, $\sigma=0.95$

FIGURE 8-15
NORMAL EMITTANCE OF EDGE FOR VARIOUS
OPTICAL DEPTHS— $n=1.6$, $\sigma=0.95$

FIGURE 8-16
NORMAL EMITTANCE OF PLANE FOR VARIOUS
OPTICAL DEPTHS— $n=1.6$, $\sigma=0.95$

FIGURE 8-17
NORMAL EMITTANCE OF EDGE FOR VARIOUS
OPTICAL DEPTHS — $n=1.8$, $\sigma = 0.95$

FIGURE 8-18
NORMAL EMITTANCE OF PLANE FOR VARIOUS
OPTICAL DEPTHS — $n=1.8$, $\sigma = 0.95$

(a) Plane

(b) Edge

FIGURE 8-19
COMPOSITE CURVES OF ϵ/ϵ_N FOR EDGE
AND PLANE EMITTANCE

Composite curves of ϵ/ϵ_N for the edge and plane surfaces are shown in Figure 8-19 for use with Figures 8-7 through 8-18. For a scattering/extinction ratio of either 0.95 or 0.99, the directional emittance may be obtained for any optical depth between one and six, for any refractive index between 1.4 and 1.8 (by interpolation between curves if the value is not either 1.4, 1.6, or 1.8), for any edge location and for any plane location up to one optical depth from the edge. If the directional emittance is desired beyond one optical depth from the edge, the normal emittance will be between the value at an optical depth of one and the infinite value obtained in the one-dimensional study. The normal emittance could be obtained by interpolation between these two values.

Error Analysis

An exact error analysis in a Monte Carlo program such as this one is exceedingly difficult. The collection strips are different widths on the edge surface than on the plane surfaces. Symmetry considerations allow more escapes (and consequently a better confidence in the results) in some of the azimuthal directions, and finally, the polar solid angles themselves are of varying sizes. This means that some exit locations are going to have more escapes than others and thus theoretically will not have the same relative error. To eliminate some of the inequities for the edge, smaller generating lengths were used (for the free-boundary problem), and only edge and near-corner escapes were catalogued. While this reduction in generating length added to the accuracy of the edge emittance values, it complicated the error analysis because it is easier

to escape out the edge for a smaller generating area. In order to obtain an estimate of the magnitude of the errors involved, one azimuthal section was chosen as "typical," and the approximate errors were calculated. Many of the individual exit locations will be more accurate statistically, and some will be less accurate.

There are several ways of analyzing the error. One way is the binomial approximation and the use of the normal density function used by Stockham (8) and discussed in (24). Another method is to utilize the Student "t" distribution, discussed in most statistics books such as Hahn and Shapiro (38), Uspensky (39), and Mood and Graybill (24). Both methods will be used for this particular analysis.

The binomial approximation assumes that each event is independent, and that a particular history either exits or fails to exit a particular location. For a very large number of emissions, the relative frequency of escapes in a particular location would approach a limit, the escape probability p .

$$p = \frac{N(\mu', \sigma')}{N} \quad \text{as } N \rightarrow \infty \quad .$$

Since each outcome is assumed to be an independent Bernoulli trial having either success or failure, the probability of success is p , and the probability of failure is $1-p$ or q . In the actual case neither p nor q is known, but by using the normal approximation to the binomial distribution, a confidence level for a particular error can be found using the approximate values for p and q .

$$\left| \frac{N(\mu', \varphi')}{N} - p \right| \geq \epsilon$$

For a given confidence level, say $(100-\alpha)$ per cent, the data would be expected to fall within the interval $(100-\alpha)$ per cent of the time and be outside the interval α per cent of the time. Confidence levels are commonly given as 0.90, 0.95, and 0.99 (38). For this work, a confidence level of 0.90 will be used. Expressed mathematically, using the normal approximation,

$$p(-y < z < +y) = \frac{1}{\sqrt{2\pi}} \int_{-y}^y e^{-1/2 z^2} dz = 0.90$$

$$\text{where } y = \epsilon \sqrt{\frac{n}{pq}}$$

The usual form of the limits for the normal approximation is (24)

$$y = \frac{\bar{x} - \mu}{\sigma} \sqrt{n},$$

where σ = variance (known)

μ = actual mean (known)

$\bar{x}$ = sample mean

n = number of trials.

For this binomial case represented here

$$\sigma = \sqrt{pq} \quad \text{and} \quad (\bar{x} - \mu) = \epsilon$$

The cumulative distribution for the normal density function is tabulated in most statistics books.

The Student "t" distribution may also be used for determining confidence intervals, providing n samples are taken from a given population. The Monte Carlo computer data was usually printed after a certain

number of emission histories, and the particular example used to illustrate the expected error is for 40 samples, each consisting of 25,000 histories, for a total of 1,000,000 histories. The method should be more accurate in predicting confidence levels since it does not involve any assumptions concerning the standard deviation or the mean. From (38):

$$t = \frac{\bar{x} - \mu}{\sqrt{\sum (x_i - \bar{x})^2 / n(n-1)}}$$

Where $\bar{x}$ = mean of the total sample

μ = true mean

n = number of samples .

For a 90 per cent confidence interval

$$P(-t_{.05} < t < t_{.05}) = \int_{-t_{.05}}^{t_{.05}} f(t; n-1) dt = 0.90$$

or

$$P\left[\bar{x} - t_{.05} \sqrt{\frac{\sum (x_i - \bar{x})^2}{n(n-1)}} < \mu < \bar{x} + t_{.05} \sqrt{\frac{\sum (x_i - \bar{x})^2}{n(n-1)}}\right] = 0.90.$$

The values of t are tabulated in most statistics books.

Table 8-2 shows the data to be used in the example. Recall that this data is for only one of seven azimuthal directions for $y/L = 0.4$ to 0.6 for $\tau = 3.0$ and $\gamma = 0.1$.

The binomial approximation shows the section with the most escapes to be the most accurate, and the one with the fewest escapes to be the least accurate, while the Student t distribution takes into consideration the spread of the data about the sample mean, i.e., the sample variance. For this reason, the Student t accuracies are not solely dependent on the

TABLE 8-2

EXAMPLE DATA FOR ERROR ANALYSIS

No. of Samples 40		Optical depth $\tau = 3.0$						
Emissions per sample 25,000		Scattering/Extinction $\sigma = 0.9$						
Azimuthal section No. 12		$y/L = 0.4$ to 0.6						
Polar Collection Area								
Run No.	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
1	6	11	22	23	36	23	24	13
2	3	13	28	31	26	24	25	26
3	4	16	27	26	28	29	29	27
4	4	13	27	26	31	26	24	25
5	6	10	22	32	34	27	26	16
6	7	13	28	29	37	30	13	13
7	5	12	37	39	38	29	27	21
8	4	15	19	29	36	33	30	29
9	5	17	21	28	34	30	30	21
10	4	15	17	29	36	36	26	23
11	5	20	25	32	39	33	27	24
12	2	14	20	25	23	32	26	20
13	9	19	27	41	26	29	22	24
14	2	10	24	30	37	32	26	24
15	4	7	21	30	33	33	22	25
16	3	14	21	29	29	31	25	22
17	4	14	27	24	26	30	28	19
18	3	15	25	32	28	27	22	18
19	7	11	28	26	33	34	23	13
20	11	12	31	26	22	27	21	18
21	5	22	23	29	29	33	23	22
22	1	16	26	29	26	37	29	23
23	3	19	27	34	37	24	26	18
24	6	17	17	21	33	35	27	22
25	5	15	20	20	30	35	30	23
26	3	13	36	23	25	31	26	24
27	7	17	25	36	40	36	27	19
28	6	11	26	36	27	26	18	19
29	5	17	16	27	38	33	18	24
30	5	15	23	26	27	29	22	23
31	5	18	24	34	34	31	22	19
32	3	18	41	27	24	35	18	13
33	3	16	32	24	37	34	29	23
34	1	11	24	24	37	25	22	22
35	5	12	22	25	33	28	27	19
36	2	13	22	25	33	27	25	9
37	5	13	31	18	35	23	33	17
38	4	16	28	33	36	31	28	22
39	4	16	29	28	41	25	27	28
40	<u>3</u>	<u>21</u>	<u>26</u>	<u>32</u>	<u>30</u>	<u>20</u>	<u>28</u>	<u>28</u>
Totals	179	557	1005	1138	1284	1193	1001	832

TABLE 8-3

SUMMARY OF RESULTS FOR CONFIDENCE LEVEL OF 0.90

Collection Area	Computed Emittance	Binomial Approximation Error Limits	Student t Error Limits	Smooth Curve Emittance Value
1	0.28	12.3%	9.5%	0.27
2	0.31	6.9%	4.7%	0.30
3	0.35	5.2%	4.1%	0.32
4	0.33	4.9%	3.5%	0.33
5	0.35	4.6%	3.3%	0.34
6	0.34	4.8%	2.9%	0.34
7	0.35	5.2%	3.3%	0.34
8	0.33	5.7%	5.0%	0.34

total number of escapes and should be more accurate than the binomial approximation. Both methods of calculation agree very well, however, and predict the final errors to be of the order of five per cent.

As discussed in Appendix B, the maximum errors for the one-dimensional program were expected to be about two per cent. The expected error for the two-dimensional coating problem should be of the same order as the two-dimensional, free-boundary solution. There were fewer histories followed, but because of the higher scattering/extinction ratios, there was a higher percentage of escapes.

Monte Carlo methods have proven to be a means of predicting directional emittances from emitting, absorbing, and scattering media. The method is not exact, but since the individual data points have as much chance of being high as low in the error interval, a "best" curve drawn through the data points should be reasonably accurate and should, in many cases, be more accurate than five per cent.

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APPENDIX A

PROPERTIES OF COATING MATERIALS

The determination of exact optical and radiative properties of ceramic materials is virtually impossible, thus making the correlation of data from two different experimenters exceedingly difficult. Material properties which affect the emittance and reflectance of a coating, such as density and porosity, are dependent on the manufacturing processes. They may vary from one sample to another within a batch and would certainly vary with different manufacturers. To say, then, that a particular coating is alumina, Al_2O_3 , would be insufficient information to determine most of the material properties. An additional complication is the wavelength dependence of the optical properties. The refractive index, scattering coefficient, and absorption coefficient vary with wavelength, which, in turn, affects the emittance. The purpose of this appendix is to discuss some of the results obtained from different investigators and to justify the selection of the constants used in this directional emittance study.

Folweiler (23) was the principal author of a report by Lexington Laboratories which presented both experimental and analytical data on the material and optical properties of the following ceramics:

- (1) Alumina (6 different types)
- (2) Strontium titanate

(3) Magnesium oxide

(4) Pyroceram (2 different types)

The experimental data from this report forms the basis for selection of the coating properties. The theoretical analysis conducted by Folweiler used Hamaker's (27) attenuated flux approach which has been shown by Love (1) to be inaccurate for some calculations. Because there was a large discrepancy between some of the analytical scattering coefficient results and experimental data (an order of magnitude in some cases), the analytical results will not be discussed.

Typical extinction coefficients as a function of wavelength are shown in Figures A-1 (a), (b), (c) and (d) taken from (23). As shown in the sketches, the extinction coefficient varies from about 20/cm in A-1 (d) to almost 500/cm in A-1 (b). The value of the extinction coefficient which best fits most of the data is approximately 200/cm, which was the value used for the directional emittance study. Lee and Kingery (28) at least partially support the selection of 200/cm for Al_2O_3 . Their data indicates a scattering coefficient of 200/cm for A-5 and from 60/cm to 15/cm for A-4 in the 1 to 5 micron wavelength range. Since the scattering coefficient is approximately equal to the extinction coefficient, the results are comparable. Cox (29) estimated the average extinction coefficient of zirconium oxide was about 90/cm in the temperature range of 3500°F to 4200°F and further stated that 200/cm would be a good room temperature estimate of the extinction coefficient.

The absorption coefficients for several ceramic oxide crystals were obtained by (23). The Al_2O_3 crystals absorbed as shown in Figure A-2 (a) with a minimum of 0.1/cm and a maximum of about 4.0/cm

in the range 1 to 5 microns. The SrTiO_3 crystals exhibited both a maximum and a minimum in the 1 to 5 micron wavelength range as shown in Figure A-2 (b). The minimum value was about 0.3/cm and the maximum value increased to about 3/cm. The absorption coefficient apparently continued to increase beyond 5 microns, but the data for these crystals was limited to that maximum wavelength. The temperature dependence of the crystals is also shown for SrTiO_3 . MgO crystals were tested to 8 microns by (23) and showed an absorption coefficient of about 10/cm at 8 microns.

Using the minimum and maximum values of absorption coefficients and the extinction coefficient found in (23) and cited above leads to scattering/extinction ratios of 0.9995 to 0.95. The upper limit of 0.9995 is too restrictive since it is associated with a very low emittance at the small optical depths found in coatings. Consequently, the decision was made to use 0.99 as the maximum scattering/extinction ratio. This corresponds to an absorption coefficient of 2.0/cm which is an approximate wavelength-averaged value. The lower limit of 0.95 was accepted as a minimum value for the higher wavelengths.

The remaining parameter to be determined is the refractive index. This property can be measured quite accurately, but there is both a wavelength dependence and a variation from one ceramic material to another. Table A-1 lists the refractive index for several ceramics.

With the exception of SrTiO_3 , all values of the refractive index between 0.5 and 3 microns are between 1.4 and 1.8. Extrapolating to 5 microns reduces the refractive index just to 1.3. Three of the six ceramics (excluding one of the two Pyroceram materials since they have

FIGURE A-2

ABSORPTION COEFFICIENTS FOR CERAMIC CRYSTALS

From REF (23), PART II

FIGURE A-3
SCHEMATIC OF NORMAL SPECTRAL EMITTANCE
FOR CERAMIC OXIDES

the same properties) have a wavelength-averaged value of approximately 1.4, two have an average value of about 1.7, and SrTiO_3 has an average value of approximately 2.25. Since 1.4 is the most common value for the refractive index, it was chosen for the two-dimensional directional emittance calculations. Values of $n = 1.6$ and 1.8 were also used for the one-dimensional coating emittance calculations to determine the effect of refractive index on the directional emittance.

TABLE A-1*

INDEX OF REFRACTION VARIATIONS FOR CERAMICS

Material	0.5 μ	1 μ	2 μ	3 μ	4 μ	5 μ
Al_2O_3	1.78	1.75	1.74	1.72	1.65	1.6
SrTiO_3	2.48	2.31	2.27	2.23	2.18	2.11
9606 (Pyroceram)	1.45	1.45	1.43	1.41	(1.35)	(1.30)
9608 (Pyroceram)	1.45	1.45	1.43	1.41	(1.35)	(1.30)
MgO	1.75	1.72	1.70	1.68	1.65	1.55
CaF_2	1.43	1.42	1.41	1.41	1.40	1.39
SiO_2	1.45	1.45	1.43	1.41	(1.35)	(1.30)

* From (23), Part II, Appendix I
(Parentheses indicate extrapolated values)

Several investigators have determined the emittance of different ceramic materials. Usually the normal spectral emittance was measured, but in some cases total normal and hemispherical emittances were also determined. Blair (30) measured the total emittance and spectral emittance at 0.64 and 1 micron of Frenchtown Alumina 4402, Coors Alumina AD99 and Corning 9608 (Pyroceram) at elevated temperatures. Weigandt and Olson (31) measured the normal emittance of Pyroceram 9608 at 1000°K from 1 to 12

microns. Bogannov, et al (32) measured the total emittance of thin samples of aluminum oxide, zirconium oxide, zircon, titanium oxide, and chromic oxide at temperatures between 700 and 1100°K. Grenis and Levitt (33) measured normal spectral emittance and total hemispherical emittance for aluminum oxide, chromic oxide, zirconium oxide, and boron nitride at 1300°K. The wavelength range for the spectral measurements was 1 to 10 microns. Hobbs and Folweiler (23) also measured the normal spectral emittance and calculated the hemispherical emittance for their ceramic samples. Some of the above data was taken on similar materials at approximately the same conditions, but for the most part, the results are for different isolated conditions.

The emission characteristics of cavities in ceramic surfaces were measured and analyzed by Cox (29) for several widely varying scattering albedos. Gannon and Linder (34) determined the effects of surface roughness and porosity (on the total normal emittance) of Norton Company El11 alumina in the temperature range 685-885° C. Almost no change in the emittance was observed as the porosity was varied from 9.7 to 30 per cent, and as the roughness was varied from 20 μ in. to 200 μ in. Torrance and Sparrow (35) determined the directional emittance of thick Al_2O_3 samples as a function of surface roughness and wavelength. Plots were given in the form of ϵ/ϵ_N , but no actual values were given for the emittance.

In most of the above studies a very wide range in emittance values is observed for the various ceramic materials. The normal spectral emittance data of Folweiler (23) and Grenis and Levitt (33) exhibited a general characteristic as shown in Figure A-3, but the

minimum values varied considerably. For instance (23) obtained emittances as low as 0.03 for Al_2O_3 at 4 microns, while other Al_2O_3 samples had minimum values of 0.5 for the emittance. Almost no definable minimum was observed for certain samples of MgO , since the minimum value of emittance was approximately 0.8. The maximum value of emittance varied from 0.95 to 0.99 in the 8 to 10 micron range.

APPENDIX B

MONTE CARLO SOLUTIONS FOR AN INFINITE, ABSORBING, EMITTING DIELECTRIC WITH REFLECTING BOUNDARIES

After obtaining solutions for the directional emittance of the two-dimensional, free boundary media, the Fresnel relations were to be added to the computer program to account for real surfaces. A one-dimensional program was written to check out the reflecting boundaries, and solutions were obtained for an infinite, absorbing, emitting medium for which exact analytical solutions had been previously obtained.

The Monte Carlo solutions were obtained for an optical depth of $\tau = 0.6$, and for two indexes of refraction, $n = 1.5$ and 3.5 . The optical depths for ceramic coatings would generally be greater than 0.6 , but this was the largest optical depth considered by Francis (36) and Francis and Love (37). Since most of the ceramic materials have indexes of refraction between 1.3 and 1.8 , the solution for $n = 3.5$ is primarily for the purpose of comparing the accuracy of the Monte Carlo solution with the exact analytical solution for a higher index of refraction.

The emission location, direction, and path length were determined as for the infinite scattering media. The generating areas are shown in Figure B-1, and Tables B-1 and B-2 summarize the data obtained for the two different solutions.

(a) $n=1.5$ (b) $n=3.5$

FIGURE B-1
GENERATING AND COLLECTION AREAS FOR
INFINITE, ABSORBING, EMITTING MEDIUM

TABLE B-1

SUMMARY OF RESULTS FOR $n = 1.5$

No. of emissions = 200,000										No. lost outside collection							
No. of absorptions = 123,150										area = 63,108							
No. of escapes in each polar angle increment																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
39	228	456	676	891	1052	1136	1185	1243	1213	1183	1065	951	837	671	505	411	

TABLE B-2

SUMMARY OF RESULTS FOR $n = 3.5$

No. of emissions = 600,000										No. lost outside collection							
No. of absorptions = 426,892										area = 165,698							
No. of escapes in each polar angle increment																	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
36	147	278	351	493	573	623	626	673	593	599	618	499	440	371	275	215	

The emittance for the infinite absorbing media is determined in the same manner as for the scattering media. The emittance is given by

$$\epsilon(\mu') = \frac{n^2}{\cos \theta} \frac{4\pi A_{\mu} N(\mu')}{\text{avg. } H \Delta \omega N} \quad .$$

The equations used to calculate ϵ for the 17 escape locations are shown in Table B-3.

The Monte Carlo results are shown in Figure B-2 along with the analytical solution of Francis and Love. The Monte Carlo solutions fit the exact analytical solutions very well. The maximum error is about

TABLE B-3

EQUATIONS FOR THE DETERMINATION OF EMITTANCE

n = 1.5 Location		n = 3.5 Location	
1	$\epsilon = 888.8 \text{ N}'/\text{N}$	1	$\epsilon = 3880 \text{ N}'/\text{N}$
2	$\epsilon = 299.7 \text{ N}'/\text{N}$	2	$\epsilon = 1303 \text{ N}'/\text{N}$
3	$\epsilon = 183. \text{ N}'/\text{N}$	3	$\epsilon = 798 \text{ N}'/\text{N}$
4	$\epsilon = 135. \text{ N}'/\text{N}$	4	$\epsilon = 588 \text{ N}'/\text{N}$
5	$\epsilon = 109.5 \text{ N}'/\text{N}$	5	$\epsilon = 476 \text{ N}'/\text{N}$
6	$\epsilon = 94.7 \text{ N}'/\text{N}$	6	$\epsilon = 412 \text{ N}'/\text{N}$
7	$\epsilon = 85.4 \text{ N}'/\text{N}$	7	$\epsilon = 371 \text{ N}'/\text{N}$
8	$\epsilon = 80.1 \text{ N}'/\text{N}$	8	$\epsilon = 348 \text{ N}'/\text{N}$
9	$\epsilon = 77.7 \text{ N}'/\text{N}$	9	$\epsilon = 339 \text{ N}'/\text{N}$
10	$\epsilon = 77.6 \text{ N}'/\text{N}$	10	$\epsilon = 338 \text{ N}'/\text{N}$
11	$\epsilon = 80.1 \text{ N}'/\text{N}$	11	$\epsilon = 348 \text{ N}'/\text{N}$
12	$\epsilon = 85.5 \text{ N}'/\text{N}$	12	$\epsilon = 372 \text{ N}'/\text{N}$
13	$\epsilon = 95.6 \text{ N}'/\text{N}$	13	$\epsilon = 416 \text{ N}'/\text{N}$
14	$\epsilon = 109.5 \text{ N}'/\text{N}$	14	$\epsilon = 476 \text{ N}'/\text{N}$
15	$\epsilon = 135.3 \text{ N}'/\text{N}$	15	$\epsilon = 590 \text{ N}'/\text{N}$
16	$\epsilon = 182.7 \text{ N}'/\text{N}$	16	$\epsilon = 796 \text{ N}'/\text{N}$
17	$\epsilon = 222.8 \text{ N}'/\text{N}$	17	$\epsilon = 970 \text{ N}'/\text{N}$

two per cent, occurring for the higher index of refraction. The errors for the one-dimensional medium considering scattering should be about the same.

○ ○ Ref. (36) Solution
 — Monte Carlo Solution

FIGURE B-2
DIRECTIONAL EMITTANCE OF INFINITE ABSORBING
MEDIUM, OPTICAL DEPTH $\tau = 0.6$

APPENDIX C

COMPUTER PROGRAMS

Two of the three computer programs used in this study are included in the appendix. They are the one- and two-dimensional coating programs. The two-dimensional, free-boundary computer program was modified by adding the Fresnel boundary conditions; therefore, to obtain the free-boundary program from the two-dimensional coating program, the Fresnel subroutine and Fresnel angles would have to be removed, and the correct escape angles substituted. All other relations are the same. The programs are written in Fortran IV language compatible with the IBM 360-40 installation.

All random numbers are generated by the scientific subroutine provided by IBM for use with the 360-40 computer. The random number generator is of the multiplicative congruence type and is not supposed to repeat a number in 2^{39} generations. The non-randomness of these "pseudo" random number generators has been severely questioned in recent years. The criticism of these uniform number generators is that successive pairs or higher orders of generated numbers may not be random even though the total properties of the generated set may be uniform. Coveyou and MacPherson (41) performed a Fourier analysis on several commonly-used number generators to determine how well they performed in the generation of random numbers in pairs, triplets, ..., up to ten. The authors found

that the random numbers generated by smaller generators (2^7 and 2^{17}) were generally not satisfactory, while those numbers obtained by larger generators usually were satisfactory. The IBM generator used herein was not among those tested, but the generator is 2^{31} , which falls into the larger generator category and should be satisfactory. The authors in (41) further stated that despite the difficulties found thus far, the multiplicative congruence generators were superior to all other methods of random number generation. To check out the IBM subroutine for this program, a few simple tests were performed similar to those conducted by Scofield (9), and the results were acceptable. The real proof of the randomness is in the results, and in every case where the operation could be checked, the results were good. For instance, if the scattering/extinction ratio was given as 0.9, the number of scatterings divided by the number of scatterings plus the number of absorptions (all determined by random number decisions) came out to be almost exactly 0.9 for the total run. Other such tests gave comparable results.

The amount of computer time required to obtain solutions was very large, particularly for the two-dimensional programs, where run times in excess of 20 hours were required to obtain a solution. Since this was a prohibitive amount of time for a single run, several smaller runs were made, usually between two and five hours in length. The one-dimensional coating solutions were much smaller, averaging less than 60 minutes for a complete solution. All 16 one-dimensional solutions required less total computer time than the shortest two-dimensional coating solution. Table C-1 summarizes the tests run and approximate run times.

The remainder of the computer time was used in running Monte

TABLE C-1

RUN TIMES FOR COMPUTER PROGRAMS

(a) Free-Boundary Solution			
Optical Condition	No. of Emissions	Approximate run time	
$\tau = 1, \sigma = 0.9$	1,600,000	13 hours	
$\tau = 1, \sigma = 0.1$	4,000,000	15 hours	
$\tau = 3, \sigma = 0.9$	1,450,000	18 hours	
$\tau = 3, \sigma = 0.1$	6,500,000	<u>24 hours</u>	
	Total	<u>70 hours</u>	

(b) Two-Dimensional Coating Solution			
Optical Condition	Refractive Index	No. of Emissions	Approximate run time
$\tau = 3, \sigma = 0.99$	1.4	630,000	22 hours
$\tau = 3, \sigma = 0.95$	1.4	795,000	20 hours
$\tau = 5, \sigma = 0.99$	1.4	375,000	17 hours
$\tau = 5, \sigma = 0.95$	1.4	655,000	<u>20 hours</u>
		Total	<u>79 hours</u>

(c) One-Dimensional Coating Solution			
Optical Condition	Refractive Index	No. of Emissions	Approximate run time
$\tau = 1, \sigma = 0.99$	1.4	15,000	19 min.
$\tau = 2, \sigma = 0.99$	1.4	15,000	26 min.
$\tau = 3, \sigma = 0.99$	1.4	20,000	47 min.
$\tau = 3, \sigma = 0.99$	1.6	15,000	57 min.
$\tau = 3, \sigma = 0.99$	1.8	20,000	67 min.
$\tau = 5, \sigma = 0.99$	1.4	17,500	64 min.
$\tau = 5, \sigma = 0.99$	1.6	17,500	65 min.
$\tau = 5, \sigma = 0.99$	1.8	12,500	63 min.
$\tau = 1, \sigma = 0.95$	1.4	25,000	25 min.
$\tau = 2, \sigma = 0.95$	1.4	20,000	26 min.
$\tau = 3, \sigma = 0.95$	1.4	35,000	52 min.
$\tau = 3, \sigma = 0.95$	1.6	35,000	64 min.
$\tau = 3, \sigma = 0.95$	1.8	35,000	70 min.
$\tau = 5, \sigma = 0.95$	1.4	30,000	63 min.
$\tau = 5, \sigma = 0.95$	1.6	30,000	68 min.
$\tau = 5, \sigma = 0.95$	1.8	30,000	<u>68 min.</u>
		Total	<u>14 hours 4 min.</u>

TABLE C-1 (CONTINUED)

(d) One-Dimensional Absorbing, Emitting Medium			
Optical Condition	Refractive Index	No. of Emissions	Approximate run time
$\tau = 0.6, \sigma = 0.0$	1.5	200,000	38 min.
$\tau = 0.6, \sigma = 0.0$	3.5	600,000	122 min.
Total			2 hours 40 min.
Total Productive Computer Time - Approximately 166 hours			

Carlo checkout programs, "de-bugging" time, and in writing and checking out a three-dimensional program. Originally it was intended to determine the emittance properties of a three-dimensional, free boundary medium, and a computer program was written for this purpose. This program was very lengthy, containing more than 1500 cards and requiring more than ten minutes of 360-40 run time just to compile. Several checkout runs were made in an optically-thin medium, and 20,000 histories required 35 minutes run time. Because of the extremely long run times required for a single solution, the three-dimensional program was abandoned in favor of the test program conducted.

The following list of symbols is used in the computer programs, and the symbols are listed in the order in which they are found in the program.

TWO-DIMENSIONAL COATING SYMBOL LIST

NE(i,j,k)	Integer for edge escapes	(i,j,k)=(location strip, azimuthal location, polar location)
NT(i,j,k)	Integer for top and bottom escapes	
PI	3.1416	
EL	edge thickness	

EL2	edge thickness/2.
PI3	$3(3.1416)/2$.
PI2	$2(3.1416)$
IX	Starting integer for random number program (any odd integer between four and eight digits in length was used)
INW	Printout number for emissions
NFW	Total number of emissions
RI	Refractive index
STE	Scattering to extinction ratio
TAU	Optical depth
SOL	Slab optical length (total generating length)
STC	Sine of the critical angle
NSUM	Sum of emissions (inside NEMIS summing)
NLOS	No. of histories lost outside of the collection area because the distance to the edge was too great
NEMIS	Sum of total emissions
ISCAT	No. of scatterings
ITMISS	No. of exits outside collection area
IS	No. of side escapes
NABS	No. of absorptions
JROF	No. of Fresnel reflections from top face
IROS	No. of Fresnel reflections from edge
JROB	No. of Fresnel reflections from bottom
IT	No. of top escapes
IB	No. of bottom escapes
RANDU is the random number generator subroutine	

Y,YP	Y coordinates of emission point
Z,ZP	Z coordinates of emission point
PHI	Azimuthal direction
CT	Cosine of the polar angle theta
ST	Sine of the polar angle theta
PL	Path length of the energy bundle
C	Distance to the nearest boundary in the direction of emission
H	Projection of C on the X-Y plane
YM	Projection of C on the Y-axis
STV	Sine of the escape angle in vacuum, θ'
CTV	Cosine of the escape angle in vacuum, θ'
FRES is the Fresnel reflectance subroutine	
G	Escape azimuthal angle, considering symmetry
LOCAT subroutine determines the y-location integer	
ANGL1 subroutine determines the azimuthal integer location	
ANGL2 subroutine determines the polar integer location	
B is CTV for subroutines	
YY,ZZ	Translation distance for new scattering coordinates
YT	Test distance to determine if exit is out edge or plane
CTF, STF	Cosine, sine of Fresnel angle, θ_F
ZM	Projection of C on Z-axis
XM	Projection of C on X-axis
LOCAE is the subroutine for locating the edge escape integer	

ONE-DIMENSIONAL COATING SYMBOL LIST

Many of the symbols are common to both the one- and two-dimensional

computer programs; therefore, only those symbols which are different will be defined below.

NT	No. of top escapes
NB	No. of bottom escapes
NWRTE	Print integer for emissions (equivalent to INW in previous program)
NSTOP	Total number of histories (equivalent to NFW in previous program)
ILTL	No. of histories escaping the top to the left of the collection area
ILTR	No. of histories escaping the top to the right of the collection area
ILBR	No. of histories escaping the bottom to the right of the collection area
ILBL	No. of histories escaping the bottom to the left of the collection area
ILOST	No. of histories whose Y-coordinate exceeded a predetermined distance
JLTRT	No. of Fresnel reflections from top
KLTRT	No. of Fresnel reflections from bottom
LOCTI subroutine locates exit strip integer. The exits were actually divided into two top strips and two bottom strips within the collection strip to see how the escapes were distributed. The data shown in Chapter VI reflects only total escapes in each polar angle increment.	
ANGL1 subroutine locates the polar angle integer	

The computer programs are listed on the following pages.

```

        DIMENSION NE(10,12,10),NT(20,12,10)
        COMMON PI,EL,EL2,PI3,PI2
        READ(1,750)IX,INW,NFW,RI
750  FORMAT(3I10,1F6.3)
        EL=1.0
        STE=0.99
        PI=3.1416
        TAU=5.0
        SOL=2.4
        PI3=3.*PI/2.0
        PI2=2.0*PI
        EL2=EL/2.0
        STC=1.0/RI
        NSUM=0
        NLOS=0
        NEMIS=0
        ISCAT=0
        ITMISS=0
        IS=0
        NABS=0
        JROF=0
        IROS=0
        JROB=0
        IT=0
        IB=0
        DO 310 I=1,5
        DO 310 J=1,12
        DO 310 K=1,10
        NE(I,J,K)=0
310  CONTINUE
        DO 311 I=1,8
        DO 311 J=1,12
        DO 311 K=1,10
        NT(I,J,K)=0
311  CONTINUE
C      GENERATE COORDINATES

```

```

1 CALL RANDU(IX,IY,YFL)
  IX=IY
  YP=YFL*SOL
  Y=YP
  CALL RANDU(IX,IY,YFL)
  IX=IY
  ZP=YFL*EL
  Z=ZP
C   GENERATE EMISSION DIRECTION
353 CALL RANDU(IX,IY,YFL)
  IX=IY
  PHI=YFL*6.2832
302 CALL RANDU(IX,IY,YFL)
  IX=IY
  CT=1.-2.*YFL
377 ST=SQRT(1.-CT*CT)
C   GENERATE PATH LENGTH
210 CALL RANDU(IX,IY,YFL)
  IX=IY
211 PL=(-1./TAU)*ALOG(YFL)
C   TEST ON THE ANGLE THETA
  IF(CT.LT.0.0.AND.PHI.LE.PI) GO TO 50
  IF(CT.LT.0.0.AND.PHI.GT.PI) GO TO 51
  IF(CT.GE.0.0.AND.PHI.LE.PI) GO TO 10
  GO TO 25
C   THIS IS FOR THETA LESS THAN 90 AND PHI LESS THAN 180
10 IF(CT.EQ.0.0) GO TO 12
  C=(EL-Z)/CT
  IF(PL-C)12,11,11
11 H=C*ST
54 YM=H*SIN(PHI)
  Y=Y+YM
  IF(Y.GT.2.4) GO TO 900
C   CHECK FOR TOTAL REFLECTION
  IF(ST.GT.STC) GO TO 450
C   THETA IS LESS THAN CRITICAL

```

```

      STV=RI*ST
      CTV=SQRT(1.0-STV*STV)
      CALL FRES(RI,ST,CT,STV,CTV,RHO)
      CALL RANDU(IX,IY,YFL)
      IX=IY
      IF(YFL.GE.RHO) GO TO 451
      JROF=JROF+1
      GO TO 450
C     TOTAL REFLECTION FROM TOP
450  Z=1.0
      CT=(-CT)
      GO TO 210
451  IF(Y.GT.1.6) GO TO 13
      IT=IT+1
972  B=CTV
      IF(PHI.LE.1.8326.OR.PHI.GE.4.4506) GO TO 81
      IF(PHI.GT.1.8326.AND.PHI.LE.PI) GO TO 82
      G=9.4248-PHI
      GO TO 83
81   G=PHI
      GO TO 83
82   G=PI-PHI
83   CALL LOCAT(Y,IL)
      CALL ANGL1(G,IJJ)
      CALL ANGL2(B,IKK)
      DO 14 I=1,8
        IF(I-IL)14,15,15
14   CONTINUE
15   I1=I
      DO 16 J=1,12
        IF(J-IJJ)16,17,17
16   CONTINUE
17   J1=J
      DO 18 K=1,10
        IF(K-IKK)18,19,19
18   CONTINUE

```



```

19 K1=K
   NT(I1,J1,K1)=NT(I1,J1,K1)+1
   GO TO 500
13 ITMISS=ITMISS+1
   GO TO 500
C   PHOTON DID NOT ESCAPE-WAS IT ABSORBED OR SCATTERED
12 CALL RANDU(IX,IY,YFL)
   IX=IY
   IF(YFL-STF)20,21,21
20 ISCAT=ISCAT+1
C   DETERMINE NEW COORDINATES
   YY=PL*ST*SIN(PHI)
   Y=Y+YY
   IF(Y.GT.2.4) GO TO 900
   ZZ=PL*CT
   Z=Z+ZZ
   GO TO 353
21 NABS=NABS+1
   GO TO 500
C   THIS IS FOR PHI GREATER THAN 180 AND THETA LESS THAN 90
C   ASSUME EXIT IS OUT TOP
25 IF(CT.EQ.0.0.AND.PHI.EQ.PI2) GO TO 12
   IF(CT.EQ.0.0.AND.PHI.LT.PI2) GO TO 30
   H=(EL-Z)*ST/CT
   TY=-H*SIN(PHI)
   IF(TY-Y)10,30,30
C   EXIT IS TOWARD SIDE-DETERMINE IF PHOTON ESCAPES
30 C=(-Y)/(ST*SIN(PHI))
   IF(PL-C)12,31,31
31 CTF=(-ST)*(SIN(PHI))
   STF=SQRT(1.0-CTF*CTF)
C   CHECK FOR REFLECTION OFF EDGE
   IF(STF.GT.STC) GO TO 700
C   THETA IS LESS THAN CRITICAL
   STV=RI*STF
   CTV=SQRT(1.0-STV*STV)

```

```

      CALL FRES(RI,STF,CTF,STV,CTV,RHO)
      CALL RANDU(IX,IY,YFL)
      IX=IY
      IF(YFL.GE.RHO) GO TO 701
      IROS=IROS+1
      GO TO 700
C     PHOTON IS TOTALLY REFLECTED
700  Y=0.0
      ZM=C*CT
      Z=Z+ZM
      PHI=2.0*PI-PHI
      GO TO 210
701  IS=IS+1
      ZM=C*CT
      Z=Z+ZM
      XM=(Y*COS(PHI))/SIN(PHI)
      XM=ABS(XM)
915  ZM=ABS(ZM)
      IF(Z.GT.EL2) GO TO 33
      GO TO 34
33   Z=1.-Z
      IF(CT.LT.0.0.AND.PHI.LE.PI3) GO TO 101
      IF(CT.LT.0.0.AND.PHI.GT.PI3) GO TO 102
      IF(CT.GE.0.0.AND.PHI.LT.PI3) GO TO 103
      G=PI+ATAN(ZM/XM)
      GO TO 104
101  G=ATAN(ZM/XM)
      GO TO 104
102  G=PI/2.0+ATAN(XM/ZM)
      GO TO 104
103  G=PI3+ATAN(XM/ZM)
      GO TO 104
34   IF(CT.GE.0.0.AND.PHI.LE.PI3)GO TO 105
      IF(CT.GE.0.0.AND.PHI.GT.PI3)GO TO 106
      IF(CT.LT.0.0.AND.PHI.GT.PI3)GO TO 107
      G=PI3+ATAN(XM/ZM)

```

```

      GO TO 104
105  G=ATAN(ZM/XM)
      GO TO 104
106  G=PI/2.0+ATAN(XM/ZM)
      GO TO 104
107  G=PI+ATAN(ZM/XM)
C    USE SYMMETRY PROPERTIES
104  IF(G.LE.1.8325.OR.G.GE.4.4507) GO TO 90
      IF(G.GT.1.8325.AND.G.LE.PI) GO TO 108
      G=9.4248-G
      GO TO 90
108  G=PI-G
      90 CALL LOCAE(I,Z,ISE)
      IF(ISE.EQ.5) GO TO 888
889  CALL ANGL1(G,IJJ)
      GO TO 890
888  IF(G.LE.1.8326.OR.G.GE.6.0214) GO TO 889
      IF(G.GT.1.8326.AND.G.LE.3.1416) GO TO 891
      IF(G.GT.3.1416.AND.G.LE.4.7124) GO TO 892
      G=6.2832-G
      GO TO 889
891  G=3.1416-G
      GO TO 889
892  G=G-3.1416
      GO TO 889
890  B=CTV
      CALL ANGL2(B,IKK)
      DO 35 I=1,5
      IF(I-ISE)35,36,36
35  CONTINUE
36  I2=I
      DO 37 J=1,12
      IF(J-IJJ)37,38,38
37  CONTINUE
38  J2=J
      DO 39 K=1,10

```

```

      IF(K-IKK)39,40,40
39  CONTINUE
40  K2=K
      NE(I2,J2,K2)=NE(I2,J2,K2) +1
      GO TO 500
C   PHOTON DIRECTION IS TOWARD BOTTOM
50  C=(-Z)/CT
      IF(PL-C)12,52,52
52  H=C*ST
      YM=H*SIN(PHI)
      Y=Y+YM
      IF(Y.GT.2.4) GO TO 900
C   CHECK FOR REFLECTION OFF BOTTOM
      IF(ST.GT.STC) GO TO 475
C   THETA IS LESS THAN CRITICAL
      STV=RI*ST
      CTV=SQRT(1.0-STV*STV)
      FT=(-CT)
      CALL FRES(RI,ST,FT,STV,CTV,RHO)
      CALL RANDU(IX,IY,YFL)
      IX=IY
      IF(YFL.GE.RHO) GO TO 476
      JROB=JROB+1
C   TOTAL REFLECTION OFF BOTTOM
475 Z=0.0
      CT=(-CT)
      GO TO 210
476 IF(Y.GT.1.6) GO TO 13
      IB=IB+1
      GO TO 972
C   PHI IS GREATER THAN 180 SO EXIT MAY BE OUT SIDE OR BOTTOM
C   ASSUME BOTTOM EXIT
51  H=(-Z)*ST/CT
      YT=(-H)*SIN(PHI)
      IF(YT-Y)50,30,30
900 NLOS=NLOS+1

```

```

500 NEMIS=NEMIS+1
    NSUM=NSUM+1
    IF(NSUM-INW)1,999,999
999 WRITE(3,416)TAU,STE
416 FORMAT(1H,'OPTICAL DEPTH=',F4.2,3X,'SCATTERING TO EXTINCTION RATIO= ',F4.2,/)
    WRITE(3,407)NEMIS,NABS,ISCAT
407 FORMAT(1H,'NO. OF EMISSIONS=',I8,2X,'NO. ABSORBED=',I8,2X,'NO. SCATTERED=',I8,/)
    WRITE(3,415)IS,IT,I8,ITMISS,NLOS
415 FORMAT(1H,'SIDE ESCAPES=',I5,2X,'TOP ESCAPES=',I6,2X,'BOTTOM ESCAPES=',I6,2X,'NO. MISSES=',I6,2X,'NO. LOST=',I6,/)
    WRITE(3,420)JROF,IROS,JROB
408 FORMAT(1H,'2013)
420 FORMAT(1H,'NO.REFL.FROM TOP=',I6,2X,'NO.REFL.FROM SIDE=',I6,2X,'NO.REFL.FROM BOT.=',I6,/)
    WRITE(3,409)
409 FORMAT(1H,'42X,'SIDE ESCAPES',/)
    WRITE(3,425)((NE(I2,J2,K2),K2=1,10),J2=1,12),I2=1,5)
    WRITE(2,425)((NE(I2,J2,K2),K2=1,10),J2=1,12),I2=1,5)
425 FORMAT(1H,'10I5)
    WRITE(3,410)
410 FORMAT(1H,'15X,'TOP AND BOTTOM ESCAPES',/)
    WRITE(3,408)((NT(I1,J1,K1),K1=1,10),J1=1,12),I1=1,8)
    WRITE(2,408)((NT(I1,J1,K1),K1=1,10),J1=1,12),I1=1,8)
    NSUM=0
    IF(NEMIS-NFW)1,430,430
430 CALL EXIT
    END

```

```

SUBROUTINE RANDU(IY,YFL)
  IY=IX*65539
  IF(IY)5,6,6
5  IY=IY+2147483647+1
6  YFL=IY
  YFL=YFL*.4656613E-9
  RETURN
  END

```

```

SUBROUTINE LOCAE(Z,ISS)
  COMMON PI,EL,EL2,PI3,PI2
  IF(Z.LE.0.1111) GO TO 1
  IF(Z.GT.0.1111.AND.Z.LE.0.2222) GO TO 2
  IF(Z.GT.0.2222.AND.Z.LE.0.3333) GO TO 3
  IF(Z.GT.0.3333.AND.Z.LE.0.4444) GO TO 4
  IF(Z.GT.0.4444.AND.Z.LE.0.5000) GO TO 5
1  ISS=1
  RETURN
2  ISS=2
  RETURN
3  ISS=3
  RETURN
4  ISS=4
  RETURN
5  ISS=5
  RETURN
  END

```

```

SUBROUTINE ANGL1(GAMMA,IM)
  COMMON PI,EL,EL2,PI3,PI2
  IF(GAMMA.GT.3.1416)GO TO 2
  IF(GAMMA.GE.2.8798.AND.GAMMA.LT.3.1416) GO TO 3

```

```

      IF(GAMMA.GE.2.3562.AND.GAMMA.LT.2.8798) GO TO 4
      IF(GAMMA.GE.1.8326.AND.GAMMA.LT.2.3562) GO TO 5
      IF(GAMMA.GE.1.3090.AND.GAMMA.LT.1.8326) GO TO 6
      IF(GAMMA.GE.0.7854.AND.GAMMA.LT.1.3090) GO TO 7
      IF(GAMMA.GE.0.2618.AND.GAMMA.LT.0.7854) GO TO 8
      IF(GAMMA.LT.0.2618.AND.GAMMA.GE.0.0) GO TO 9
      WRITE(3,1903)
1903 FORMAT(1H,'PROGRAM STOPPED BECAUSE OF ERROR NO 3')
      IM=12
      RETURN
      2 IF(GAMMA.LE.3.4034.AND.GAMMA.GT.3.1416) GO TO 3
      IF(GAMMA.LE.3.9270.AND.GAMMA.GT.3.4034) GO TO 10
      IF(GAMMA.LE.4.4506.AND.GAMMA.GT.3.9270) GO TO 11
      IF(GAMMA.LE.4.9742.AND.GAMMA.GT.4.4506) GO TO 12
      IF(GAMMA.LE.5.4978.AND.GAMMA.GT.4.9742) GO TO 13
      IF(GAMMA.LE.6.0214.AND.GAMMA.GT.5.4978) GO TO 14
      IF(GAMMA.GT.6.0214.AND.GAMMA.LE.6.2832) GO TO 9
      WRITE(3,1904)
1904 FORMAT(1H,'PROGRAM STOPPED BECAUSE OF ERROR NO 4')
      IM=12
      RETURN
      9 IM=12
      RETURN
      14 IM=11
      RETURN
      13 IM=10
      RETURN
      12 IM=9
      RETURN
      11 IM=8
      RETURN
      10 IM=7
      RETURN
      3 IM=6
      RETURN
      4 IM=5

```

```
      RETURN
5  IM=4
      RETURN
6  IM=3
      RETURN
7  IM=2
      RETURN
8  IM=1
      RETURN
      END
```

```
      SUBROUTINE ANGL2(BETA,IN)
      COMMON PI,EL,EL2,PI3,PI2
      IF(BETA.LE.1.0.AND.BETA.GE.0.9397) GO TO 24
      IF(BETA.GE.0.866) GO TO 23
      IF(BETA.GE.0.766) GO TO 22
      IF(BETA.GE.0.6428) GO TO 21
      IF(BETA.GE.0.500) GO TO 20
      IF(BETA.GE.0.3420) GO TO 19
      IF(BETA.GE.0.2588) GO TO 18
      IF(BETA.GE.0.1736) GO TO 17
      IF(BETA.GE.0.0872) GO TO 16
      GO TO 15
15  IN=1
      RETURN
16  IN=2
      RETURN
17  IN=3
      RETURN
18  IN=4
      RETURN
19  IN=5
      RETURN
20  IN=6
```



```

      RETURN
21  IN=7
      RETURN
22  IN=8
      RETURN
23  IN=9
      RETURN
24  IN=10
      RETURN
      END

```

```

      SUBROUTINE FRES(RI,ST,CT,STV,CTV,RHO)
      COMMON PI,EL,EL2,PI3,PI2
      IF(CTV.EQ.1.000) GO TO 10
      A=ST*CTV-CT*STV
      B=ST*CTV+CT*STV
      RS=(A/B)*(A/B)
      C=CT*CTV+ST*STV
      D=CT*CTV-ST*STV
      RP=((A*D)/(B*C))*((A*D)/(B*C))
      RHO=0.5*(RS+RP)
      RETURN
10  RHO=((RI-1.0)/(RI+1.0))**2.0
      RETURN
      END

```

```

      SUBROUTINE LOCAT(Y,IBT)
      COMMON PI,EL,EL2,PI3,PI2
      IF (Y.GT.2.0) GO TO 30
      IF(Y.GT.1.0) GO TO 50
      GO TO 60
50  IF(Y.LE.2.0.AND.Y.GE.1.80) GO TO 10

```

```

        IF(Y.LT.1.80.AND.Y.GE.1.60) GO TO 9
        IF(Y.LT.1.60.AND.Y.GE.1.40) GO TO 8
        IF(Y.LT.1.40.AND.Y.GE.1.20) GO TO 7
        IF(Y.LT.1.20.AND.Y.GT.1.0) GO TO 6
6   IBT=6
    RETURN
7   IBT=7
    RETURN
8   IBT=8
    RETURN
9   IBT=9
    RETURN
10  IBT=10
    RETURN
60  IF(Y.LE.1.0.AND.Y.GE.0.8) GO TO 5
    IF(Y.LT.0.8.AND.Y.GE.0.6) GO TO 4
    IF(Y.LT.0.6.AND.Y.GE.0.4) GO TO 3
    IF(Y.LT.0.4.AND.Y.GE.0.2) GO TO 2
    IBT=1
    RETURN
2   IBT=2
    RETURN
3   IBT=3
    RETURN
4   IBT=4
    RETURN
5   IBT=5
    RETURN
30  IF(Y.GT.2.0.AND.Y.LE.2.4) GO TO 11
    IF(Y.GT.2.4.AND.Y.LE.2.8) GO TO 12
    IF(Y.GT.2.8.AND.Y.LE.3.2) GO TO 13
    IF(Y.GT.3.2.AND.Y.LE.3.6) GO TO 14
    IF(Y.GT.3.6.AND.Y.LE.4.0) GO TO 15
11  IBT=11
    RETURN
12  IBT=12

```

RETURN
13 IBT=13
RETURN
14 IBT=14
RETURN
15 IBT=15
RETURN
END

```

C      EMISSION FROM INFINITE ABSORBING, EMITTING, AND SCATTERING MEDIA
      DIMENSION NT(2,17), NB(2,17)
      READ(1,600) RI, TAU, STE, NWRTE, NSTOP, IX
600    FORMAT(3F10.6, 3I10)
      EL=1.0
      PI=3.1416
      SOL=12.0
      ILTL=0
      ILTR=0
      ILBL=0
      ILBR=0
      ISCAT=0
      ILOST=0
      STC=1.0/RI
      JLTRT=0
      KLTRB=0
      NSUM=0
      NABS=0
      NEMIS=0
      IT=0
      IB=0
      DO 10 I=1,2
      DO 10 J=1,17
      NT(I,J)=0
10    CONTINUE
      DO 11 K=1,2
      DO 11 L=1,17
      NB(K,L)=0
11    CONTINUE
C      GENERATE COORDINATES
1    CALL RANDU(IX,IY,YFL)
      IX=IY
      Y=YFL*SOL
      CALL RANDU(IX,IY,YFL)
      IX=IY
      Z=YFL*EL

```

```

C   GENERATE EMISSION DIRECTION
2  CALL RANDU(IX,IY,YFL)
   IX=IY
   PHI=YFL*2.0*PI
   CALL RANDU(IX,IY,YFL)
   IX=IY
   CT=1.0-2.0*YFL
   ST=SQRT(1.0-CT*CT)
C   GENERATE PATH LENGTH
403 CALL RANDU(IX,IY,YFL)
   IX=IY
   PL=(-1./TAU)*ALOG(YFL)
C   TEST ON THE ANGLE THETA
   IF(CT.LT.0.0) GO TO 300
   IF(CT.EQ.0.0) GO TO 12
C   THIS IS FOR THETA LESS THAN 90 DEGREES
   C=(EL-Z)/CT
   IF(PL-C)12,13,13
13  H=C*ST
   Y=Y+H*SIN(PHI)
   IF(Y.LT.0.00.OR.Y.GT.12.0) GO TO 900
C   CHECK FOR TOTAL REFLECTION
   IF(ST.GT.STC) GO TO 18
C   THETA IS LESS THAN CRITICAL
   STV=RI*ST
   CTV=SQRT(1.0-STV*STV)
   CALL FRES(RI,ST,CT,STV,CTV,RHO)
   CALL RANDU(IX,IY,YFL)
   IX=IY
   IF(YFL.GE.RHO) GO TO 22
   JLTRT=JLTRT+1
   GO TO 18
C   IS PHOTON ABSORBED OR SCATTERED
12 CALL RANDU(IX,IY,YFL)
   IX=IY
   IF(YFL.LE.STE) GO TO 100

```

```

      NABS=NABS+1
      GO TO 500
100  ISCAT=ISCAT+1
      YY=PL*ST*SIN(PHI)
      Y=Y+YY
      IF(Y.LT.0.00.OR.Y.GT.12.0) GO TO 900
      ZZ=PL*CT
      Z=Z+ZZ
      GO TO 2
19  ILTL=ILTL+1
      GO TO 500
21  ILTR=ILTR+1
      GO TO 500
C   TOTAL REFLECTION FROM TOP
18  Z=1.0
      CT=(-CT)
      GO TO 403
C   PHOTON ESCAPES FROM TOP
22  IF(Y.LT.3.00) GO TO 19
      IF(Y.GT.9.00) GO TO 21
      IT=IT+1
      CALL LOCT1(Y,IJ)
      CALL ANGL1(CTV,IK)
      DO 23 I=1,2
        IF(I-IJ)23,20,20
23  CONTINUE
20  I1=I
24  DO 25 JK=1,17
      IF(JK-IK)25,26,26
25  CONTINUE
26  JK1=JK
      NT(I1,JK1)=NT(I1,JK1)+1
      GO TO 500
C   THETA IS TOWARD THE BOTTOM
300 C=(-Z/CT)
      IF(PL-C)12,31,31

```

```

31 H=C*ST
   Y=Y+H*SIN(PHI)
   IF(Y.LT.0.00.OR.Y.GT.12.0) GO TO 900
   CT=-CT
C   CHECK FOR TOTAL REFLECTION
   IF(ST.GT.STC) GO TO 34
   STV=RI*ST
   CTV=SQRT(1.0-STV*STV)
   CALL FRES(RI,ST,CT,STV,CTV,RHO)
   CALL RANDU(IX,IY,YFL)
   IX=IY
   IF(YFL.GE.RHO) GO TO 35
   KLTRB=KLTRB+1
   GO TO 34
C   PHOTON ESCAPES FROM BOTTOM
35 IF(Y.LT.3.00) GO TO 32
   IF(Y.GT.9.00) GO TO 33
   IB=IB+1
   CALL LOCT1(Y,IJ)
   CALL ANGL1(CTV,IK)
   DO 38 I=1,2
   IF(I-IJ)38,39,39
38 CONTINUE
39 J1=I
   DO 40 K=1,17
   IF(K-IK)40,41,41
40 CONTINUE
41 K1=K
   NB(J1,K1)=NB(J1,K1)+1
   GO TO 500
32 ILBL=ILBL+1
   GO TO 500
33 ILBR=ILBR+1
   GO TO 500
C   TOTAL REFLECTION FROM BOTTOM
34 Z=0.0

```

```

      GO TO 403
900  ILOST=ILOST+1
500  NEMIS=NEMIS+1
      NSUM=NSUM+1
      IF(NSUM-NWRTE)1,999,999
999  WRITE(3,501)RI,TAU
501  FORMAT(1H,'REFRACTIVE INDEX IS',F4.2,2X,'OPTICAL DEPTH IS',F4.2)
      WRITE(3,502)NEMIS,NABS,ISCAT
502  FORMAT(1H,'NO.OF EMISSIONS=',I6,2X,'NO.ABSORBED=',I6,2X,'NO.SCATT
      IERED=',I8,/)
      WRITE(3,503)IT,ILTL,ILTR,JLTR,ILOST
503  FORMAT(1H,'TOP ESCAPES',I6,2X,'TOP LOSSES-LEFT',I6,2X,'TOP LOSSES
      I-RIGHT',I6,2X,'REFLECTIONS-TOP',I6,2X,'NO.LOST=',I4,/)
      WRITE(3,504)IB,ILBL,ILBR,KLTRB
504  FORMAT(1H,'BOT.ESCAPES',I6,2X,'BOT.LOSSES-LEFT ',I5,2X,'BOT.LOSSE
      IS-RIGHT',I6,2X,'REFLECTIONS-BOT.',I6,/)
      WRITE(3,505)
505  FORMAT(1H,22X,'TOP ESCAPES',/)
      WRITE(3,506)((NT(I1,JK1),JK1=1,17),I1=1,2)
506  FORMAT(1H,17I4)
      WRITE(3,507)
507  FORMAT(1H,20X,'BOTTOM ESCAPES',/)
      WRITE(3,506)((NB(J1,K1),K1=1,17),J1=1,2)
      NSUM=0
      IF(NEMIS- NSTOP)1,998,998
998  CALL EXIT
      END

```

```

      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
5  IY=IY+2147483647+1
6  YFL=IY
      YFL=YFL*.4656613E-9
      RETURN
      END

```



```

SUBROUTINE LOCT1(Y,IJ)
  IF(Y.GE.3.00.AND.Y.LE.6.00) GO TO 1
  IF(Y.GT.6.00.AND.Y.LE.9.00) GO TO 2
  IF(Y.LT.3.00.OR.Y.GT.9.00) GO TO 3
1  IJ=1
  RETURN
2  IJ=2
  RETURN
3  WRITE(3,4)Y
4  FORMAT(1H ,'Y IS NOT WITHIN LIMITS IN LOCT1==',F10.4)
  IJ=2
  RETURN
END

```

```

SUBROUTINE ANGL1(CTV,IK)
  IF(CTV.GE.0.6428) GO TO 10
  IF(CTV.LT.0.0) GO TO 21
  IF(CTV.GE.0.0000.AND.CTV.LT.0.0872) GO TO 1
  IF(CTV.GE.0.0872.AND.CTV.LT.0.1737) GO TO 2
  IF(CTV.GE.0.1737.AND.CTV.LT.0.2588) GO TO 3
  IF(CTV.GE.0.2588.AND.CTV.LT.0.3420) GO TO 4
  IF(CTV.GE.0.3420.AND.CTV.LT.0.4226) GO TO 5
  IF(CTV.GE.0.4226.AND.CTV.LT.0.5000) GO TO 6
  IF(CTV.GE.0.5000.AND.CTV.LT.0.5736) GO TO 7
  IF(CTV.GE.0.5736.AND.CTV.LT.0.6428) GO TO 8
1  IK=1
  RETURN
2  IK=2
  RETURN
3  IK=3
  RETURN
4  IK=4
  RETURN
5  IK=5

```

```

      RETURN
6  IK=6
      RETURN
7  IK=7
      RETURN
8  IK=8
      RETURN
12 IK=9
      RETURN
13 IK=10
      RETURN
14 IK=11
      RETURN
15 IK=12
      RETURN
16 IK=13
      RETURN
17 IK=14
      RETURN
18 IK=15
      RETURN
19 IK=16
      RETURN
20 IK=17
      RETURN
10 IF(CTV.GE.0.6428.AND.CTV.LT.0.7071) GO TO 12
    IF(CTV.GE.0.7071.AND.CTV.LT.0.7660) GO TO 13
    IF(CTV.GE.0.7660.AND.CTV.LT.0.8192) GO TO 14
    IF(CTV.GE.0.8192.AND.CTV.LT.0.8660) GO TO 15
    IF(CTV.GE.0.8660.AND.CTV.LT.0.9063) GO TO 16
    IF(CTV.GE.0.9063.AND.CTV.LT.0.9391) GO TO 17
    IF(CTV.GE.0.9391.AND.CTV.LT.0.9659) GO TO 18
    IF(CTV.GE.0.9659.AND.CTV.LT.0.9848) GO TO 19
    IF(CTV.GE.0.9848.AND.CTV.LT.1.0000) GO TO 20
21 WRITE(3,22) CTV
22 FORMAT(1H ,'CTV IS NOT WITHIN LIMITS IN ANGL1--=',F10.6)

```

```

IK=1
RETURN
END

```

```

SUBROUTINE FRES(RI,ST,CT,STV,CTV,RHO)
IF(CTV.EQ.1.000) GO TO 10
A=ST*CTV-CT*STV
B=ST*CTV+CT*STV
RS=(A/B)*(A/B)
C=CT*CTV+ST*STV
D=CT*CTV-ST*STV
RP=((A*D)/(B*C))*((A*D)/(B*C))
RHO=0.5*(RS+RP)
RETURN
10 RHO=((RI-1.0)/(RI+1.0))*2.0
RETURN
END

```

1.800	5.00	0.990	2500	15000	1852379
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