

# A simulation approach to PERT network analysis

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*This paper presents simulation as a useful analytical tool for project network analysis. Simulation is a powerful tool for evaluating many of the decision parameters involved in project management. A computer program, named STARC, is used to illustrate the effectiveness of computer simulation for project planning. STATGRAPHICS software is used to illustrate some of the post-simulation statistical analyses that can be conducted.*

## Introduction

A project network is an example of a problem that lends itself very well to simulation applications. At any given time, only a small segment of a project network will be available (or current) for direct observation and analysis. The major portion of the project will either have been in the past or expected in the future. Such non-observable parts of the project can best be studied by simulation. A computer program, named STARC (Badiru 1991), is used to illustrate several simulation analyses for project planning.

Using the historical information from previous or completed parts of the project and the prevailing events in the project environment, projections can be made about the future of the project. Outputs of simulation can alert management to impending and potential problems. The advance alert will facilitate better management decisions. Most of the available commercial project management software packages take a static view of the project network. Those that consider probabilistic activity durations do so by using the conventional PERT (Program Evaluation and Review Technique) estimates to generate static time estimates. The use of these static time estimates robs the software package of its potential for robustness. Simulation can preserve and enhance the advantages of PERT and CPM (Critical Path Method) by retaining the probabilistic view professed by PERT. The approach of this paper involves the four steps presented below in the use of simulation in project analysis:

1. PERT Activity Time Modeling
2. Simulation of Project Network Using STARC
3. Managerial Decisions based on the Simulation Output
4. Statistical Analysis of the Simulation Output

The methodology for activity time modeling used by STARC is presented in the Appendix of this paper. Interested readers should refer to Badiru (1991) for further details on the methodology.

### Project Simulation Network

STARC is a project planning aid. It was developed to simulate project networks and perform what-if analysis of projects involving probabilistic activity times and resource constraints. The program is useful as a training tool for project management analysts and students.

The simulated schedules generated by the program are designed to serve as decision aids for project planners. The effects of different activity time estimates and resource allocation options can be studied with STARC prior to operational implementations in an actual project.

STARC is a menu-driven program written in BASIC programming language for the IBM PC and compatibles. The program is easy to run and provides a quick view of the effects of variabilities in the project environment. An illustration of the type of pre-project analysis that can be performed with STARC is presented in this section.

The simulation run time for large networks will typically be very long. To expedite the analysis of large networks, the project analyst may use WBS (Work Breakdown Structure) to partition the network into small subprojects which can be quickly analyzed. By using the project work breakdown structure (WBS), the results for the subprojects can be integrated to obtain an overall project plan.

### Running STARC

STARC is available in a compiled executable form. Its main menu includes the following options:

1. RUN STARC SIMULATION USING DISK DATAFILE
2. CREATE NEW DATAFILE
3. EDIT A DATAFILE
4. PRINT A DATAFILE
5. PRINT AN OUTPUT FILE
6. TERMINATE PROGRAM

The following input items are needed to run a simulation of a project network from option one in the main menu.

1. Name of the data file in which the project data has been previously saved.
2. Number of simulation runs desired. The larger the number of simulation runs, the better the histogram of the project duration in the printed output. For large projects, memory requirement limitations and long simulation times may prevent using very large numbers of simulation runs.
3. The number of project completion deadlines to be evaluated. STARC calculates the probability of completing a project within a given deadline. Up to 10 deadlines can be evaluated during one simulation run.
4. Resource allocation weighting factor ( $w$ ). The user inputs a desired value of  $w$  between 0.0 and 1.0. The same value of  $w$  is applicable for all activities during the simulation experiment. The weighting factor is used in calculating the resource allocation priority measure for each project activity. The priority measure used by STARC is the Composite Allocation Factor (CAF) developed by Badiru (1988a). The measure takes into consideration both the resource requirements of an activity and the time duration variability of the activity. The weighting factor is used to give relative weights to these two components of CAF. When activities compete for resources, the weighted values of CAF are used in prioritizing the activities for resource allocation. Higher CAF values yield higher priority ratings. Badiru (1988a) presents a detailed account of how CAF is used in allocating resources to activities. To assure the validity of the CAF assumption (the larger the CAF, the higher the priority), Badiru (1988b) presents a favorable comparison of CAF heuristic with other activity scheduling heuristics.
5. Duration risk coverage factor. This is a percentage factor for simulating the duration of each activity in the project. It provides a risk coverage for the imprecision in the three PERT time estimates for each activity. A risk factor of 10%, for example, extends the range  $[a,b]$  of an activity's duration by ten percent over the specified PERT time interval. An example of the extension of the activity duration interval is shown in Figure 1. In the Figure, the range of activity duration is extended by 10 percent. The extension of the range ensures that there is some probability (greater than zero) of generating activity times below the optimistic estimate or above the pessimistic estimate.

The complete procedure for effecting the PERT time interval extension is presented by (Badiru and Whitehouse 1989). If a risk coverage of 0% is specified, there is no interval dilation and no adjustments are made to any of the PERT estimates. On the other hand, a coverage of 100% will yield a dilated interval that is twice as wide as the original PERT interval. While a

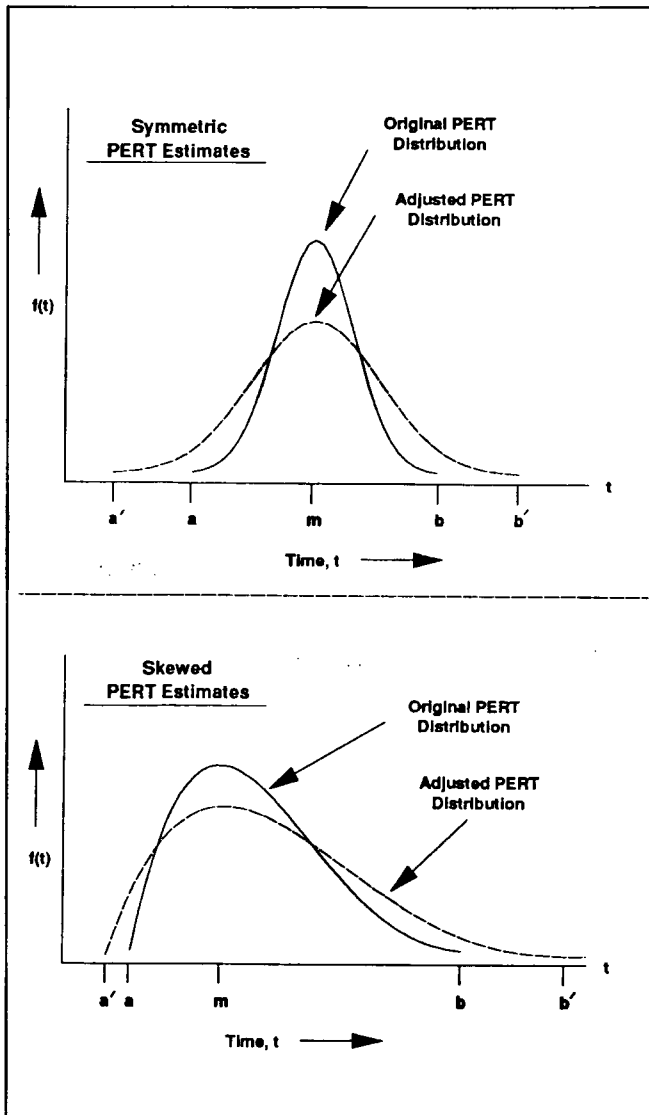


Figure 1. Risk Coverage Adjustment for PERT Estimates

large extension of the time range may be desirable for more elastic activity times, it does result in a high variance for the activity times.

The other options available in the submenus provided by STARC include the following:

1. Add New Activities
2. Delete Activity
3. Change Individual Activity Data
4. Change Resource Availability
5. Change Project Data File Description
6. Print the File Being Edited
7. End Editing and Save Data File
8. Return to Main Menu

If option number 4 is selected for resource changes, the following options are provided:

1. Change Units Available
2. Add New Resource Type
3. Delete Resource Type
4. Return to Editing Menu

With the above data editing facilities, changes can quickly be made in the project data and the effects of the changes can be studied interactively by repeating the simulation of the project network. The resource menu offers various options for conducting "what-if" analysis of resource availability. The simulation output can be printed in any of the following three options:

1. Print Current Output on Screen
2. Print Current Output on Line Printer
3. Store Current Output in an Output File

The output is presented in a simple and organized format with the appropriate level of detail for quick managerial decisions. STARC simulates a project schedule based on the following assumptions:

1. Resource availability is in whole units.
2. No partial assignment of resources.
3. No splitting of activities.
4. Activity preemption is not allowed.
5. Total resource units required must be available before an activity can start.
6. All predecessors must be finished before an activity can start.

The small project presented in Table 1 is used to illustrate a project network simulation analysis using STARC. The project network is shown in Figure 2. The PERT three time estimates (optimistic time, most likely time, and pessimistic time) for each activity are shown below the activity label in the network. The sample project contains seven activities and one resource type (say machine operator). There are ten units of the same resource available at the beginning of the project.

Table 1. Sample Project Data With Resource Constraint

Activity	Activity No.	Predecessor	(DAYS)			Resource Units	
			a	m	b	Required	CAF
A	1	-	1	2	4	3	55.4
B	2	-	5	6	7	5	100.0
C	3	-	2	4	5	4	72.6
D	4	A	1	3	4	2	54.0
E	5	C	4	5	7	4	88.0
F	6	A	3	4	5	2	66.6
G	7	B, D, E	1	2	3	6	75.3

Total resource units available = 10 units

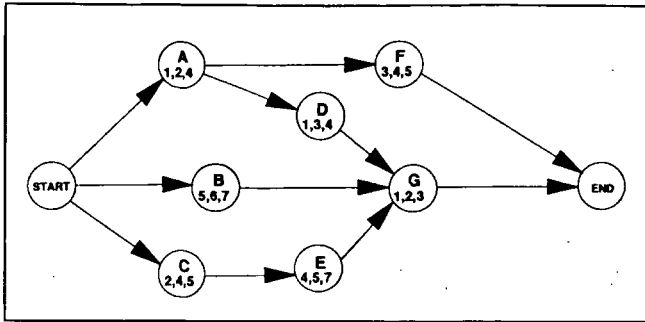


Figure 2. Project Network Example

### Sample Simulation Output

The sample network in Figure 2 was simulated with a sample size of 100. As a part of the input, a request was made to analyze 10 potential deadlines for the project. Partial listings of some of the simulation outputs are shown in Figures 3 through 9. The network activities originally labelled A, B, C, D, and so on, have been renamed as activities 1, 2, 3, 4, and so on in the data file to facilitate computational manipulations.

Figure 3 shows the output heading indicating the project name, number of activities, number of resources, resource allocation weighting factor, risk coverage factor, and initial resource availability. Figure 4 shows the output of the conventional PERT analysis without resource limitation. The expected duration (DUR), earliest start (ES), earliest completion (EC), latest start (LS), latest completion (LC), total slack (TS), free slack (FS), and the indicator for being on critical path (CRIT) are presented for each activity. The output shows that the PERT time without resource constraint is 11 days. It is seen that activities 3, 5, and 7 are on the critical path.

STARC PROJECT SCHEDULING SIMULATION OUTPUT	
PROJECT NAME: - EXAMPLE	
NUMBER OF ACTIVITIES = 7	
NUMBER OF RESOURCES = 1	
CAF weighting factor, w = .5	
Duration risk coverage factor, q = .15	
RESOURCE AVAILABILITY	
RESOURCE #	UNITS AVAILABLE
1	10

Figure 3. STARC Output Heading.

Figure 5 shows the simulated sample averages of the variables associated with each activity. The average project duration in 100 simulation runs is 12.96 days. With the resource constraints, the criticality indices are 0.96, 0.0, 0.04, 0.74, 0.04, 0.22, and 0.78, respectively for activities 1 through 7. The criticality index of an activity is the probability that the activity will fall on the critical path. In other words, the criticality index indicates the

UNCONSTRAINED PERT SCHEDULE								
ACT.	DUR.	ES	EC	LS	LC	TS	FS	CRIT
1	2.17	0.00	2.17	4.00	6.17	4.00	0.00	0.000
2	6.00	0.00	6.00	3.00	9.00	3.00	3.00	0.000
3	3.83	0.00	3.83	0.00	3.83	0.00	0.00	1.000
4	2.83	2.17	5.00	6.17	9.00	4.00	4.00	0.00
5	5.17	3.83	9.00	3.83	9.00	0.00	0.00	1.000
6	4.00	2.17	6.17	7.00	11.00	4.83	0.00	0.000
7	2.00	9.00	11.00	9.00	11.00	0.00	0.00	1.000

UNCONSTRAINED PERT PROJECT COMPLETION TIME = 11

Figure 4. Conventional PERT Analysis

SIMULATED SAMPLE AVERAGES								
ACT. #	MEAN DUR.	MEAN ES	MEAN EC	MEAN LS	MEAN LC	MEAN TS	MEAN FS	CRIT INDEX
1	2.16	6.01	8.17	6.01	8.18	0.06	0.00	0.960
2	6.01	0.00	6.01	4.94	10.95	4.94	4.86	0.000
3	3.87	0.00	3.87	1.86	5.73	1.86	0.00	0.040
4	2.67	8.17	10.85	8.28	10.95	0.16	0.08	0.740
5	5.22	3.87	9.09	5.73	10.95	1.86	1.78	0.040
6	4.00	8.17	12.17	8.96	12.96	0.79	0.00	0.220
7	2.00	10.89	12.89	10.96	12.96	0.08	0.00	0.780

AVERAGE PROJECT DURATION = 12.96228

Figure 5. Simulated Sample Averages.

long-run tendencies of an activity. It is seen in the simulation result that Activity 1 is critical most of the time (probability of 0.96) whereas Activity 2 is never on the critical path. The average observed durations of the activities are also presented in the figure. It should be noted that the ES, EC, LS, LC, TS, and FS values in the sample average output will not necessarily match conventional PERT network calculations. In other words, we cannot draw a PERT network based on the output in Figure 5. This is because each average value (e.g. MEAN EC) is computed from the results from the 100 independent simulation runs. However, for each given simulation run, all the values observed will conform to the conventional PERT computational procedure.

Figure 6 shows a deadline analysis for a set of selected project deadlines. The second column in the figure presents the probabilities calculated analytically based on the Central Limit Theorem (i.e. normally distributed project completion times). The third column presents the sample probabilities based on the observations in the simulated sample. The analytical probabilities in the second column are presented as a validation measure for the observed probabilities. The larger the number of simulation runs, the closer both probability values would be. Suppose we are considering a contract deadline of 14 days, we might like to know the probability of finishing the project in that time frame. The simulation output indicates a calculated probability of

PROJECT DEADLINE ANALYSIS		
DEADLINE	CALCULATED PROBABILITY	OBSERVED PROBABILITY
10.00	0.0003	0.0000
11.00	0.0114	0.0000
12.00	0.1321	0.1300
13.00	0.5174	0.5600
14.00	0.8858	0.9100
15.00	0.9910	0.9900
16.00	1.0000	1.0000
17.00	1.0000	1.0000
18.00	1.0000	1.0000
19.00	1.0000	1.0000

Figure 6. Project Deadline Analysis

0.8858 and a simulated probability of 0.9100. So, there seems to be a good chance of finishing the project in 14 days. We can, thus, proceed with the contract. Even though the conventional PERT duration for the project is 11 days, it is seen that there is a very low probability (0.0114 calculated and 0.0 observed) of finishing the project in 11 days when we consider resource limitations.

Figure 7 shows the shortest simulated schedule for the example. The shortest observed project duration in a sample of 100 simulation runs is 11.01 days. If plotted on a Gantt chart, this schedule can serve as an operational schedule for the project. In this schedule, only two activities (1 and 6) are on the critical path.

SHORTEST SIMULATED SCHEDULED								
ACT.	DUR.	ES	EC	LS	LC	TS	FS	CRIT
1	1.83	5.16	6.99	5.16	6.99	-0.00	0.00	1.000
2	5.16	0.00	5.16	3.74	8.89	3.74	3.64	0.000
3	3.11	0.00	3.11	0.10	3.21	0.10	0.00	0.000
4	1.14	6.99	8.13	7.75	8.89	0.76	0.66	0.000
5	5.68	3.11	8.79	3.21	8.89	0.10	0.00	0.000
6	4.03	6.99	11.01	6.99	11.01	0.00	0.00	1.000
7	2.12	8.79	10.92	8.89	11.01	0.10	0.00	0.000

SHORTEST SIMULATED PROJECT DURATION = 11.01316

Figure 7. Best Simulated Project Schedule.

Other portions of the simulation output (not shown) present the sample duration variances, sample duration ranges, and the parameters of the fitted beta distributions for the activities in the project. The variances might be needed for statistical analysis and other analytical purposes. The sample ranges might be useful for analyses such as control charts for activity durations and resource loading diagrams. An echo of the initial PERT data for the project is shown in Figure 8. The scaled priority measure (CAF) for allocating resources to competing activities are presented for each activity in the last column in the figure. It is seen that Activity 2 has the highest priority for resource allocation when activities compete for units of the available resources.

ACTIVITY	PROJECT ACTIVITIES DATA						
	A	M	B	MEAN	VAR.	RANGE	CAF
1	1.0	2.0	4.0	2.2	0.3	3.0	55.4
2	5.0	6.0	7.0	6.0	0.1	2.0	100.0
3	2.0	4.0	5.0	3.8	0.3	3.0	72.6
4	1.0	3.0	4.0	2.8	0.3	3.0	54.0
5	4.0	5.0	7.0	5.2	0.3	3.0	88.0
6	3.0	4.0	5.0	4.0	0.1	2.0	66.6
7	1.0	2.0	3.0	2.0	0.1	2.0	75.3

Figure 8. Project Data for Simulation.

A frequency distribution histogram for the project duration based on the simulated sample is presented in Figure 9. Also, the 95% confidence interval, [12.79, 13.13], for the project duration is computed by STARC. So, there is a 95% confidence that the project can be completed between 12.79 and 13.13 days.

FREQUENCY DISTRIBUTION HISTOGRAM FOR PROJECT DURATION		
CLASS	INTERVAL	ELEMENTS
1	11.01 to 11.24	3 *****
2	11.24 to 11.46	0
3	11.46 to 11.68	5 *****
4	11.68 to 11.90	4 *****
5	11.90 to 12.12	3 *****
6	12.12 to 12.35	7 *****
7	12.35 to 12.57	9 *****
8	12.57 to 12.79	11 *****
9	12.79 to 13.01	14 *****
10	13.01 to 13.23	7 *****
11	13.23 to 13.46	11 *****
12	13.46 to 13.68	8 *****
13	13.68 to 13.90	6 *****
14	13.90 to 14.12	4 *****
15	14.12 to 14.35	1 ***
16	14.35 to 14.57	2 *****
17	14.57 to 14.79	2 *****
18	14.79 to 15.01	2 *****
19	15.01 to 15.23	0
20	15.23 to 15.46	1 ***

Figure 9. Histogram of Project Duration.

### Resource Allocation Heuristic

During the simulation, STARC uses the composite allocation factor (CAF) to prioritize activities for resource allocation (Badiru 1988b). The resource allocation process takes into account both the resource requirements and the variabilities in activity times. For the sample network, the ranking of activities for resource allocation is shown in the last column of Figure 8. Activities with higher values of CAF are given priority during the resource allocation process. For each activity  $i$ , CAF is computed as a weighted and scaled sum of two priority measures: Resource Allocation Factor (RAF) and Stochastic Activity Duration Factor (SAF).



The computations are performed as presented below:

$$CAF_i = (w)RAF_i + (1 - w)SAF_i$$

where  $w$  is a weighting factor between 0 and 1. RAF is defined for each activity  $i$  as:

$$RAF_i = \frac{1}{t_i} \sum_{j=1}^R \frac{x_{ij}}{y_j}$$

where

$X_{ij}$  = number of resource type  $j$  units required by activity  $i$

$y_j$  = maximum units of resource type  $j$  required by any activity in the project

$t_i$  = the expected duration of activity  $i$

$R$  = the number of resource types involved

RAF is a measure of the relative expected resource consumption per unit time. A scaling procedure is used in such a way that the differences among the units of resource types are eliminated to obtain real dimensionless numbers that are amenable to ordinary addition. The set of RAF values is itself scaled from 0 to 100. This helps to eliminate the time-based unit. Thus, the RAF measure is reduced to a dimensionless real number. Resource-intensive activities have larger magnitudes of RAF and, as such, require a greater attention in the scheduling process. To incorporate the stochastic nature of activity times in a project schedule, SAF is defined for each activity  $i$  as:

$$SAF_i = t_i + \frac{s_i}{t_i}$$

where,

$t_i$  = expected duration for activity  $i$ ,

$s_i$  = standard deviation of duration for activity  $i$ , and

$s/t_i$  = coefficient of variation of the duration of activity  $i$ .

The SAF values are also scaled to eliminate the discrepancy in the units of the terms in the mathematical expression. It is on the basis of the magnitudes of CAF that an activity is assigned a priority for resource allocation in the project schedule. An activity that lasts longer, consumes more resources, and varies more in duration will have a larger magnitude of CAF. Such an activity is given priority for resources during the scheduling process. The weighting factor,  $w$ , is used to vary the relative weights assigned to the RAF and SAF priority measures in the resource allocation process. Consequently, STARC gives a project analyst the option of assigning more weight to the resource requirement aspects of a project and less to the probabilistic time aspects and vice versa. A simulation experiment may be

conducted to find out the best value of  $w$  for a given project. Such an experiment is described later in this paper.

## Project Decision Analysis

Microcomputers offer a friendly and easily accessible medium for conducting simulation experiments for managerial decision analysis (Carroll 1987). A review of the output of STARC may indicate what type of what-if analysis may be performed. For example, a revision of the project data (resource availability, time estimates, predecessors, and resource requirements) was effected in the project data presented earlier. The number of available units of resource was increased from 10 to 15. With the additional resource allocation and the other data changes, the average project duration was reduced from 12.96 days to 11.09 days. The deadline analysis shows that the probability of finishing the project in 14 days has increased to 1.0 both by analytical calculation and sample estimate. There is now a good chance that the contract of 14 days can be satisfied. Even a duration of 13 days has a high probability of being achieved. The new 95% confidence interval for the project duration is [10.86, 11.32]. Based on the revised simulation output, management can make effective planning decisions accordingly.

A second revision of the original project data was also analyzed. In this revision, only the resource availability is changed from 10 units to 7 units. It turns out that decreasing the initial resource availability by three units caused the average project duration to increase from 12.96 to 17.56 days. Under the data revision, the revised deadline analysis indicates that even a generous deadline of 17 days has a low probability of being accomplished (0.28 calculated and 0.32 observed). With this type of information, a project analyst can study the sensitivity of project completion times to changes in resource availability. The third what-if analysis shows that the 95% confidence interval for the project duration has changed to [17.29, 17.82].

## Statistical Experiment and Analysis

With "what-if" analyses similar to those presented in the preceding sections, a project analyst can study the potential effects of decisions prior to making actual resource and time commitments. STARC can help the analyst to determine appropriate project inputs in the "safe environment" of simulation. The information acquired with the aid of simulation can then serve as the input for developing operational project schedules. If desired, the simulation output can be used to conduct additional statistical analyses that can further enhance project decisions. Such analyses are discussed below.

STATGRAPHICS software package was used to

perform a series of statistical analyses for the sample project data presented earlier in Table 1. An additional resource type was added to the project data to obtain the revised project data shown in Table 2. Several simulation experiments were conducted on the sample project. The weighting factor,  $w$ , and risk coverage factor,  $q$ , were used as decision variables in the simulation trials. Several combinations of  $w$  and  $q$  were used and the average project durations were recorded for simulation sample sizes of 100. The simulation outputs are tabulated as shown in Table 3. Values of  $w$  range from 0.0 to 1.0 while values of  $q$  are 0, 0.1, 0.15, and 0.2. The observed average project durations are given inside the body of the Table.

**Table 2.** Project Data With One Additional Resource Type

Activity	Predecessor	a	m	b	Resource Units	
					Type 1	Type 2
A	-	1	2	4	3	0
B	-	5	6	7	5	4
C	-	2	4	5	4	1
D	A	1	3	4	2	0
E	C	4	5	7	4	3
F	A	3	4	5	2	7
G	B,D,E	1	2	3	6	2

Units of resource type 1 available initially = 10  
 Units of resource type 2 available initially = 15

**Table 3.** Output of Simulation Experiments

w	Average Project Duration			
	q = 0.0	q = 0.1	q = 0.15	q = 0.2
0.0	12.98	13.06	13.12	12.60
0.1	13.56	12.88	13.33	13.05
0.2	13.56	12.96	13.03	13.30
0.3	13.48	13.18	12.90	13.03
0.4	13.33	13.08	13.13	13.02
0.5	12.69	13.34	12.51	13.63
0.6	12.76	13.12	13.11	12.91
0.7	13.33	12.10	12.65	12.50
0.8	13.01	13.09	13.45	13.19
0.9	13.25	13.42	13.04	13.23
1.0	16.89	16.77	16.71	17.03

A multiple X-Y scatter plot of the data in Table 3 was developed using the STATGRAPHICS software package. The plot is shown in Figure 10. It is observed that

there is not much difference between the simulation results for risk coverage levels ( $q$ ) of 0%, 10%, 15%, and 20%. So, for the particular project involved in this experiment, an analyst can infer that the project duration is insensitive to risk coverage levels less than or equal to 20%. This preliminary conclusion was later confirmed by a formal statistical test.

Based on the plot in Figure 10, there do seem to be differences between the simulation results for different weighting levels ( $w$ ) between 0.0 and 1.0. In fact, the increase in the project durations for values of  $w$  greater than 0.9 seems to be particularly pronounced. So, the project duration appears to be sensitive to changes in  $w$ . This observation was also later confirmed by a formal statistical test. However, a prudent project analyst might want to conduct follow-up detailed simulation experiments for values of  $w$  between 0.9 and 1.0.

A multifactor analysis of variance (ANOVA) of the simulation results was conducted with STATGRAPHICS. Two replicates of the simulation experiment were used in the analysis. The data shown in Table 3 is for the first replicate. A screen dump of the resulting ANOVA table is presented in Figure 11. The significance level (last column) of the ANOVA table shows that the effect of  $w$  on the project duration is significant at the 95% confidence level ( $\alpha = 0.05$ ) while the effect of  $q$  is not significant. This confirms our earlier observation based on the scatter plot in Figure 11. It is also noted that the interaction effect of  $w$  and  $q$  is significant at the 95% confidence level. Even though  $q$  does not seem to have a direct effect on the project duration, it interacts with  $w$  to contribute to the observed differences in the project duration.

To further analyze the differences in the levels of the factors involved in the experimental study of the project duration, Duncan's multiple range analysis was conducted. The analysis shows that all the four levels of  $q$  fall in the same homogeneous group at the 95% confidence level. So, the project analyst can conclude that there is no significant difference between the levels of  $q$  for this particular project. This again agrees with the earlier observation in the scatter plot. The multiple range analysis also shows that there are significant differences between the levels of  $w$  for the project.

To a project analyst, the results above indicate that discriminating attention should be given to resource and time variability aspects of scheduling this project. Thus, the project manager can determine where to direct most of his or her control actions. Since  $w = 1.0$  yielded the longest average project duration, the priority measure defined as

$$CAF = (w)RAF + (1 - w)SAF$$

suggests that this particular project duration elongates when total emphasis is placed on the resource aspects alone.

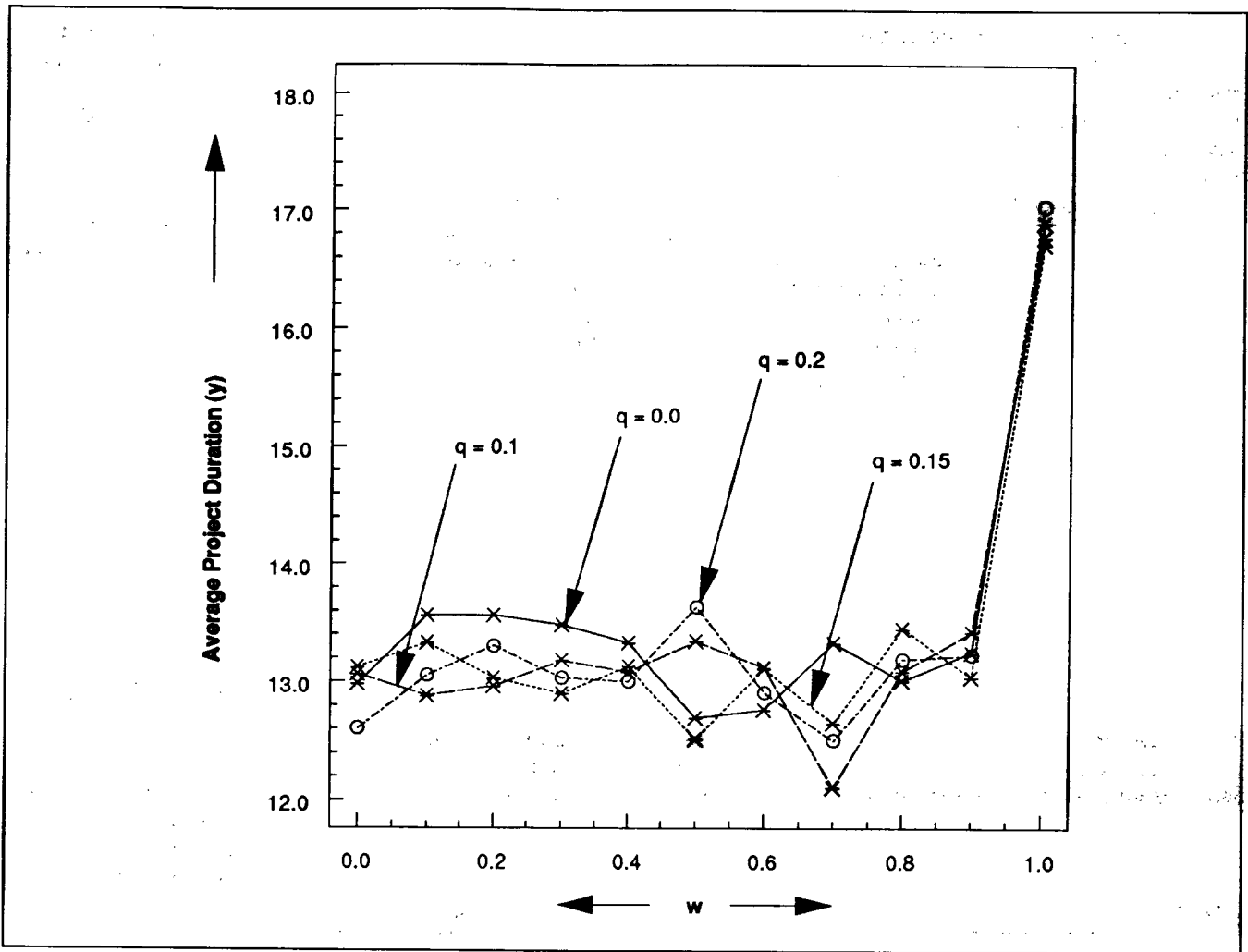


Figure 10. Multiple X-Y Plot of Simulation Results.

Analysis of Variance for STARC.dur

Source of variation	Sum of Squares	d.f.	Mean square	F-ration	Sig. level
MAIN EFFECTS	107.56827	13	8.274483	167.434	.0000
STARC.w	107.36331	10	10.736331	217.250	.0000
STARC.q	.20497	3	.068322	1.383	.2606
2-FACTOR INTERACTIONS	5.2261205	30	.1742040	3.525	.0001
STARC.w STARC.q	5.2261205	30	.1742040	3.525	.0001
RESIDUAL	2.1744500	44	.0494193		
TOTAL (CORR.)	114.96884	87			

0 missing values have been excluded.

Figure 11. Multifactor ANOVA Table.



An inquisitive project analyst might want to study the regression relationship between the average project duration,  $y$ , and  $w$ . Since there are 11 levels of  $w$  in the experiment, a regression model could be attempted. The simulation results for  $q$  of 15% was selected from Table 3 as the data for the regression modeling process. Thus,  $q$  was fixed at 15% for the regression study. STATGRAPHICS was used to fit a linear regression function for the selected data. The result is shown in Figure 12. The fitted model is:

$$y = 12.565 + 1.5936w.$$

This model does not represent a good fit because R-squared is only 21.40% and the ANOVA result indicates that the model does not significantly account for variability in the project duration. Other regression models that were investigated include exponential, reciprocal, and multiplicative models. None of these fared any better than the simple linear model. For other project configurations, an analyst might be able to develop a reliable regression model that can be used for prediction purposes. The variable  $q$  was not included as a variable in the regression model because it was found earlier that it does not significantly affect the project duration in this particular sample network. However, for general analysis, the project analyst may consider a multiple nonlinear regression model of the form:

$$y = \beta_0 + \beta_1 q^{\alpha_1} + \beta_2 w^{\alpha_2} + \epsilon$$

where  $\beta_i$  and  $\alpha_i$  are appropriate model parameters and  $\epsilon$  is the error term.

In the absence of any other tool for predicting the project duration, the fitted linear model, even with its noted deficiencies, could still be useful for project planning purposes. As long as the decision maker exercises caution in invoking the model, a dim glimpse at the future of a project should be better than no glimpse at all.

## Conclusion

This paper has presented a simulation approach to PERT network analysis. Simulation is an effective analytic tool that can greatly enhance the planning and control functions in project management. The simulation analysis discussed uses a BASIC program to simulate project networks. The results of the simulation exercises can give management an opportunity to examine many of the ramifications of a project before actual implementations are undertaken. A detailed statistical analysis was conducted with STATGRAPHICS software using the outputs of the simulation experiments. This provides a basis for studying the interrelationships among the variables involved in a project. The procedures and what-if analyses presented are very valuable to project analysts who must deal with uncertainty in project environments.

Regression Analysis — Linear model: $Y = a + bX$					
Dependent variable: STARC.average			Independent variable: STARC.weight		
Parameter	Estimate	Standard Error	T Value	Prob. Level	
Intercept	12.565	0.602278	20.8625	.00000	
Slope	1.59364	1.01804	1.5654	.15193	
Analysis of Variance					
Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	2.7936445	1	2.7936445	2.450489	.15193
Error	10.260319	9	1.140035		
Total (Corr.)	13.053964	10			
Correlation Coefficient = 0.462609			R-squared = 21.40 percent		
Std. Error of Est. = 1.06772					

Figure 12. Simple Linear Regression Model for Project Duration.

## Appendix

### PERT Activity Time Modeling

A key component of simulating a project network is the statistical modeling of activity durations. Previous attempts to model activity durations have considered several probability density functions including normal distribution, uniform distribution, lognormal distribution, and beta distribution. But the beta distribution still remains the favorite mainly because of its connection with the popular PERT technique (McBride and McClelland 1967). Several researchers have addressed several of the conceptual and managerial aspects of modeling activity times (Farnum and Stanton 1987; Golenko 1988; MacCrimmon and Ryavec 1964; Moder and Rodgers 1968). Several alternatives have also been proposed for the analytical modeling a probability density function for PERT activity times (Gallagher 1987; Littlefield and Randolph 1987; Sasieni 1986; Welsh 1965). In the methodology used by STARC (Badiru 1988a 1991), a beta distribution is fitted for the three time estimates,  $a$  (optimistic time),  $m$  (most likely time), and  $b$  (pessimistic time). For a simulation analysis, activity times are generated at random from the fitted beta probability density function. The development that follows shows how the three PERT time estimates are used to fit beta shape parameters for each activity. The PERT approximation of the mean activity duration is given by:

$$t_e = \frac{a + 4m + b}{6}$$

while the theoretical mean of the general beta distribution is given by

$$\mu_y = a + (b - a) \frac{\alpha}{\alpha + \beta}$$

where  $\alpha$  and  $\beta$  are the shape parameters of the beta distribution. Equating the above expressions yields

$$\frac{a + 4m + b}{6} = a + (b - a) \frac{\alpha}{\alpha + \beta} \quad (1)$$

Also, the PERT approximation of the variance of activity time is given by

$$S_e^2 = \frac{(b - a)^2}{36}$$

while the theoretical variance is given by

$$\sigma_y^2 = \frac{(\alpha\beta)(b - a)^2}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

The two variance expressions yield

$$(b - a)^2 \frac{(\alpha\beta)}{(\alpha + \beta)^2(\alpha + \beta + 1)} = \frac{(b - a)^2}{36} \quad (2)$$

Now, solving for the unknown variables in the equations above yields

$$\alpha = \frac{(5a - 4m - b)\beta}{(a + 4m - 5b)} \quad (3)$$

For simplification, the above expression is written as

$$\alpha = \phi\beta \quad (4)$$

where  $\phi$  is a constant given by

$$\phi = \frac{(5a - 4m - b)}{(a + 4m - 5b)} \quad (5)$$

From equation (4) and the fact that the shape parameters are nonzero, it is obvious that [EQN "phi"] is also nonzero. Consequently,

$$5a - 4m - b \neq 0$$

That is,

$$a \neq \frac{(4m + b)}{5},$$

which is logical since equality would imply that

$$a \geq m,$$

which is a contradiction of the PERT procedure which normally requires that "a" be less than "m" in the duration estimates. In fact, since the shape parameters are strictly positive, the constant  $\phi$  is also positive. Substituting equation (4) into equation (2) yields

$$(b - a)^2 \frac{(\phi\beta)\beta}{(\phi\beta + \beta)^2(\phi\beta + \beta + 1)} = \frac{(b - a)^2}{36},$$

which simplifies to

$$\beta^2(\phi + 1)^2(\phi\beta + \beta + 1) = 36\phi\beta^2$$

The expression above yields

$$\beta = \frac{-(\phi^2 - 34\phi + 1)}{(\phi + 1)^3} \quad (6)$$

which implies that

$$\phi \neq -1,$$

which in turn implies that

$$a \neq b$$

as required by the PERT procedure. Now, substituting equation (6) into equation (4) and simplifying yields

$$\alpha = \frac{(37\phi^2 + 2\phi + 1)}{(\phi + 1)^3} - 1 \quad (7)$$

Consequently, given three distinct PERT time estimates,  $a, m, b$ , equations (5), (6) and (7) can be used to approximate the shape parameters of an appropriate beta distribution. It is from the fitted beta distributions that random execution times are generated for activities during the simulation of a project network. The trade-off in the estimation process is a slight error in the mode of the fitted beta distribution. The ideal situation will be for the fitted mode to be equal to  $m$ . This occurs when the PERT estimates are symmetric about  $m$ . A complete analysis of the estimation error has been addressed elsewhere (Badiru 1988b).

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