ANALYSIS OF THE DISTRIBUTION PROPERTIES OF COST VARIANCES AND THEIR EFFECTS ON THE COST VARIANCE INVESTIGATION DECISION

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PREFACE

This study is concerned with the effects of distribution properties on the cost variance investigation decision. The primary objective was to examine the effect of making an incorrect assumption regarding distribution properties. Simulation and numerical analysis techniques were used to explore this issue.

I wish to express my sincere appreciation to Dr. Amy Lau, my committee chairperson, for her advice and assistance throughout this project. I am thankful for the patience and understanding exhibited by her. I am also grateful to my other committee members, Dr. Janet Kimbrell, Dr. Maryanne Mowen, and Dr. David Weeks, for their valuable contributions during the course of this work. Special thanks are due Dr. Hon-Shiang Lau for providing technical assistance which made this project possible. I am also thankful to Dr. Prem Prakash of the University of Pittsburgh for making his dynamic programming model available for my use.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

THE RESEARCH PROBLEM

1.1 Introduction

Surveys of firms indicate two of the most important uses of standard costs are cost control and performance evaluation ([Caplan, 1971] and [Cress and Pettijohn, 1985]). Actual results of operations are compared to the expected costs which are estimated using standard costs. The difference between the actual and expected cost is called a cost variance. When an actual cost exceeds (is less than) the expected cost, the resulting cost variance is unfavorable (favorable).

A cost variance may be indicative of a correctable inefficiency in the underlying production process; the process is then considered as "out-of-control." Under a management by exception philosophy, attention is flagged to this process. However, a cost variance may also arise as a mere result of random fluctuation from an "in-control" process. If an investigation prompted by an unfavorable variance reveals that the process is indeed out-of-control, the source of inefficiency can be corrected. But if the investigation reveals that the process is actually in-control, the investigation cost is wasted. Therefore, the manager needs a decision rule that enables him to distinguish those cost variances that warrant investigations from those that do not, developing such a rule is what constitutes the "cost variance investigation decision" or CVID problem. To formulate this problem statistically, an observed cost variance may come from either the in-control cost variance distribution or the out-of-control cost variance distribution, and the CVID involves determining from which distribution the observed variance comes.

1.2 Brief Description of CVID Models

Kaplan [1982] discusses various models which could be used as an aid by managers to determine which cost variances to investigate. One such model is based on rules of thumb. Two examples of this approach are materiality significance and statistical significance. The materiality significance rule investigates all variances which exceed the standard by an arbitrarily fixed percentage, say 10 percent. The statistical significance rule considers the variability of cost variances and recognizes some random fluctuation is expected. This rule investigates all variances which exceed the standard by a fixed number of standard deviations. A second model is based on control charts. The use of this model requires plotting the cost variances on a chart. Upper and lower statistical limits are placed on the chart. Control charts not only indicate whether variances are statistically 'significant,' but they allow managers to see the pattern of variances. A third model is based on Bayesian statistics. An advantage of this model is that it incorporates the costs and benefits of investigation. A disadvantage is the extensive data required to implement the model.

1.3 Distributional Assumptions of the Models

All of the previously mentioned models except for materiality significance assume cost variances are normally distributed. Kaplan [1982] suggests the use of a nonnormal distribution when specific knowledge indicates variances are not normally distributed. While there is a lack of literature specifically addressing the normality issue, some authors ([Boer, 1984]; [Kaplan, 1975] and [Luh, 1968]) have questioned the assumption of normality. Their objections are discussed in the literature review.

1.4 Objectives of this Study

Excepting the materiality significance rule, all other CVID models described require knowledge of the distribution properties of the cost variances. Since most models have usually assumed a normal distribution, it would be useful to investigate the sensitivity of the decision effectiveness of these models to the distribution properties assumed. If this investigation reveals that different distribution assumptions lead to different optimal rules, or if it reveals that a decision rule optimal under one distribution assumption can perform very poorly under other distribution assumptions, then firms should begin to determine the actual distribution forms of their variances. If the investigation reveals otherwise, it would serve as an useful formal justification for using the convenient assumption of normally distributed cost variances in CVID problems.

Thus, the three primary objectives of the study are:

(1) To examine the distribution properties of actual cost variances collected from industry.

(2) To develop a practical approach for modeling nonnormal cost-variance distributions.

(3) To investigate how optimal decisions under various CVID models are affected by the nonnormality of cost variances.

1.5 Organization of the Thesis

Chapter II reviews the literature on CVID models, cost variances and distribution properties of costs. Actual cost variances obtained from a manufacturing firm are analyzed for normality in Chapter III. Alternative approaches for modeling nonnormal variances are discussed in the fourth chapter. Chapter V discusses the CVID models with which the effects of variances' nonnormality will be investigated. The simulation and computational methodologies used in this investigation are outlined in Chapter VI. Chapter VII presents the results of this thesis. Chapter VIII presents the summary and conclusions of this thesis.

CHAPTER II

REVIEW OF THE LITERATURE

Reviewed below are literatures on (1) CVID models and (2) distribution properties of costs and cost variances.

2.1 CVID Models

Kaplan [1975], in a review article, classified cost variance investigation models along two dimensions: (1) whether the decision was made on the basis of a single observation or multiple observations, and (2) whether or not both costs and benefits of investigation are incorporated in the model. For the first category, the single observation models are discussed first. One example of these models is the rule of investigating all unfavorable variances. Another example of these models is a decision rule which investigates all cost variances that exceed the standard by a fixed percentage. Statistical significance rules consider expected dispersion of the cost variances. The standard deviation is used as the measure of dispersion. Examples of such rules include models which investigate variances by a specified number of standard deviations, usually two or three. The advantage of these single observation models is that they are simple to use. Unfortunately, these models determine the desision rule subjectively.

The cumulative sum procedure is an example of a CVID model which uses multiple observations. This approach, introduced by Page [1954], attempts to detect a shift in the mean of a process. This approach sums the differences between the observations and the target mean for a series of observations. If the process is in-control, over time the sum should follow a random walk with a mean of zero. A negative or positive drift indicates the mean of the process has shifted. A benefit of the cumulative sum procedure is that, ideally, it will detect this "shift" in the mean of the process earlier than a mechanical statistical significance rule. However, none of these first category models consider the benefits and costs of investigation, nor do they include the costs of failing to correct an out-of-control process. These omissions are weaknesses of these models.

The second category of CVID models considers costs and benefits of investigation. Bierman, Fouraker, and Jaedicke [1961], can be considered to be the founder of this model. According to their model, the probability, p, that an observation came from the in-control distribution is computed. This probability can be determined using the past history of the firm. The cost of an investigation, C, and the benefit from correcting an out-of-control situation, L, must also be determined. An investigation is conducted if $C < (1 - p) L$. However, Dyckman [1969] criticized Bierman, Fouraker, and Jaedicke's [1961] model for ignoring all prior information. Experienced managers will have subjectively estimated the ratio of in-control observations to out-of-control observations and this information should be included in the decision model. Consequently, Dyckman [1969], Kaplan [1969], and Dittman and Prakash [1978 and 1979], improved Bierman, Fouraker, and Jaedicke's [1961] model by making use of prior observations and also including the costs and benefits of investigation in their decision model. These three models are discussed as follows.

Dyckman [1969] developed a single-period Bayesian model. This model determines the probability (q_i) that the process is in-control at the end of the period. The probability q_i is determined as follows [Dyckman, 1969]:

$$q_i = g \left[\frac{f_1(x) q_{i-1}}{f_1(x) q_{i-1} + f_2(x) \; (1 - q_{i-1})} \right] \tag{1}$$

where:

g = probability that the process remains in-control at the end of the period given that it entered the period in-control

f_1 = density function for observed cost variance x given x is from the in-control distribution

f_2 = density function for observed cost variance x given x is from the out-of-control distribution

q_{i-1} = probability the process is in-control at the end of period i-1

q_i = probability the process is in-control at the end of period i

x = cost variance for period i

If the probability the process is in-control (q_i) is less than the "trigger" value, q_n^*, an investigation should be made. The calculation of q_n^* is as follows:

$$q_n^* = 1 - \frac{C}{\left[g^n \cdot n \cdot (\mu_2-\mu_1) + \sum_{j=1}^{n-1} g^j \cdot (1-g) \cdot j \cdot (\mu_2-\mu_1) \right]} \qquad (2)$$

where:

C = investigation cost

g = probability that the process remains in-control at the end of the period given that it entered the period in-control

n = number of months left in the year

μ_1 = mean of the in-control distribution

μ_2 = mean of the out-of-control distribution

In an extension of the above approach, Kaplan [1969] developed a multi-period model. An advantage of Kaplan's [1969] model over Dyckman's [1969] model is that the costs and benefits of <u>future</u> investigation decisions are considered in Kaplan's model. The probability of the process being in-control (q_i) is determined the same as for Dyckman's [1969] model (equation 1 in section 2.1). The critical probability which triggers an investigation is found using a dynamic programming procedure. If the revised probability of the process being in-control (q_i) is less than the critical probability, the process should be investigated. A characteristic of this model is that it results in CVIDs which minimize discounted future costs.[1] There are two cost equations developed. One equation is the sum of discounted future costs assuming an investigation is made in the current period. The second cost equation is the sum of discounted future costs assuming an investigation is <u>not</u> made in the current period. The critical value, q_n^*, is determined by finding the value which makes the two aforementioned cost equations equal in the following minimization:

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[1] Future costs consist of two parts. The two parts are future investigation costs and future operating costs.

$$V_n[q] = \min \begin{bmatrix} C + \int (x + V_{n-1}(\tau_g x)) \cdot \\ \quad (g\, f_1(x) + (1 - g) f_2 \cdot (x))dx; \\ \int (x + V_{n-1}(\tau_q x)) \; (q f_1(x) \\ \quad + (1 - q) f_2(x))dx \end{bmatrix} \quad (3)$$

where:

- C = investigation cost
- x = observed cost
- g = probability that the process remains in-control at the end of the period given that it entered the period in-control
- f_1 = density function for observed cost x given x is from the in-control distribution
- f_2 = density function for observed cost x given x is from the out-of-control distribution
- τ = Bayesian revision operator
- p = prior probability of being in-control
- $V_n[q]$ = the value of having a probability q of being in-control next period with n periods left in the year

Two assumptions may limit the applicability of both Dyckman's [1969] model and Kaplan's [1969] model. One assumption is that the process can be represented as a two state system. That is, the process is either in-control or out-of-control. A second assumption is that the process can always be returned to the in-control state.

Dittman and Prakash [1978 and 1979] developed a Markovian approach to the CVID problem. This approach is similar to Kaplan's [1969] except that Dittman and Prakash determine a critical cost rather than a critical probability. When the reported cost exceeds the critical cost, the process is investigated. One advantage of the Markovian approach is that it does not require dynamic programming, which is a requirement of Kaplan's [1969] approach. Thus, the Markovian approach is much easier and less costly to implement than the dynamic programming approach.[2] A second advantage of the Markovian approach is that it does not require Bayesian updating of probabilities. Due to the two aforementioned advantages of the Markovian approach, it is a simpler model to operationalize than both the single-period and multiple-period Bayesian models. When the actual cost exceeds the critical cost, the process is investigated. While the Markovian approach was developed using costs rather than cost variances, this approach can be applied to cost variances since cost variances are a linear transformation of actual costs.

The critical cost is the value which minimizes the following expression [Dittman and Prakash, 1979]:

$$V^{*} = \min \left[f_1(x) - \frac{A}{B} \right] \cdot \left[\frac{1 - f_2(x)}{1 - g f_2(x)} \right] \quad (4)$$

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[2] The dynamic programming approach is very complex. Computer programming costs to set up and maintain this model would severely limit, if not eliminate, the use of this model.

where:

x = observed cost

g = probability that the process remains in-control at the end of the period given that it entered the period in-control

f_1 = density function for observed cost x given x is from the in-control distribution

f_2 = density function for observed cost x given x is from the out-of-control distribution

$A = C + (1 - g) \cdot K - g\ (\mu_2 - \mu_1)$

$B = g \cdot C$

C = investigation cost

K = cost to correct the process

μ_1 = mean of the in-control distribution

μ_2 = mean of the out-of-control distribution

V^* = critical cost

The expected cost saving per period equals μ_2 less the long run expected cost per period. Dittman and Prakash [1979] compared the cost savings[3] of Kaplan's [1969] dynamic programming approach with the cost savings of the Markovian approach. The results of Dittman and Prakash [1979] indicate the lost cost savings from using the Markovian approach rather than the dynamic programming

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[3] Cost savings are defined as μ_2 less long run expected cost per period. For example, assume μ_2 is equal to 20, and long-run expected expected costs are $5.50 and $5.75 for the dynamic programming and Markovian control approaches, respectively. Expected cost savings per period are $14.50 and $14.25 for the dynamic programming and Mrkovian control approaches, respectively. The lost cost savings from using the Markovian approach would be $0.25.

approach are very small. If the costs of implementing[4] and maintaining the models were to be included in the analysis, undoubtedly the Markovian approach would prove to be more cost efficient than the dynamic programming approach.

2.2 Evaluation of CVID Models

The performance of various CVID models described above has been evaluated by Magee [1976] and Jacobs [1978] to determine whether a particular model is superior. Simulation was used by Magee [1976] to evaluate the performance of seven cost variance investigation rules. Performance was measured in terms of `total cost,' which was defined as the sum of investigation costs and the `operating costs.' The investigation cost is defined as the cost necessary to investigate and, if necessary, bring the process back into the in-control state. Three investigation cost amounts ($10, $30, and $50) were used in the study. Operating costs were defined as the costs generated by the stochastic process. Six of the seven investigation rules ranged in degree of complexity from the simplest rule of investigating all unfavorable cost

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[4] This assertion is made primarily on the basis of personal observation. The are numerous complexities involved in the computer programming required to use the dynamic programming model. The dynamic programming model requires many "calls" to various function routines and the use of many IMSL subroutines. The dynamic programming model would require much more programming time than the Markovian control model.

variances to Kaplan's [1969] dynamic programming approach. The seventh rule assumed perfect knowledge concerning the state of the cost process. This rule was used as a benchmark for evaluation of the other rules. The results indicated the most effective rule was Kaplan's [1969] dynamic programming approach. However, the difference between Kaplan's rule and the simple rule of investigating all cost variances which exceed the standard cost by more than two standard deviations was not large.

Jacobs [1978] evaluated the performance of six decision models in a field experiment. Similar to Magee's [1976] study, the six decision models evaluated ranged in degree of complexity from a control chart to the dynamic programming approach. Effectiveness of the models was evaluated using two techniques. One evaluation technique was an analysis of the frequencies of type I and type II errors.[5] A second evaluation technique was an analysis of decision costs incurred by the various models. Included in decision costs were error costs (of type I and type II errors) and investigation costs.

The results did not indicate that one particular model is consistently superior. As a group, the

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[5] A type I error occurs when an in-control process is investigated and a type II error occurs when an out-of-control process is not investigated.

multi-observation models[6] performed somewhat better than the single-observation models. Within the group of multi-observation models, Dyckman's [1969] single-period Bayesian model and Kaplan's [1969] dynamic programming approach tended to give similar results.

Jacob's [1978] results may not be comparable to other studies since the analysis was based on physical usages rather than dollar amounts. The CVID models described in Section 2.1 suggested making decisions based on <u>cost</u> <u>dollar</u> variances rather than usage quantity variances. It is unknown if the results of an analysis based on quantities can be generalized to costs.

2.3 Distribution of Cost Variances

An important assumption underlying all the CVID models described above is that cost variances are normally distributed. Luh [1968, p. 124], in discussing the use of statistical control techniques in deciding when to investigate cost variances, states, "the assumption of normal distribution of cost . . . . would appear to lack sound theoretical basis." Consequently, Luh [1968] proposes an alternative procedure called "controlled cost." This approach consists of first estimating a probability distribution of "controlled" (or in-control) costs. Then,

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[6] Multi-observation models make use of prior observations. Single-period models do not make use of prior observations. Only the current observation is considered in making a decision.

a probability distribution of actual costs is obtained. To determine whether actual peformance is "in-control," the probability distribution of actual costs is compared with that of the controlled cost. Numerous theorems in mathematical statistics [Kendall and Stuart, 1969] may be used to test the hypothesis that the two samples are drawn from the same universe.

Boer [1984, p. 54], in the most recent review article of CVID models noted that "no studies of the distribution of actual costs have been published in the accounting literature." The only study which attempted to examine the distribution properties of variances is the study of Jacobs and Lorek [1980]. They examined actual manufacturing data for normality and independence. However, instead of using dollar cost variances, they examined the distribution properties of usage quantities. The data they used consisted of actual daily and weekly usages of materials and utilities for several processes in a large grain processing firm. Nine processes were analyzed using daily data. Both skewness and kurtosis for seven of these daily processes were significantly different than what would be expected for a normal distribution. For the other two processes, either skewness or kurtosis was significantly different than what would be expected for a normal distribution. However, when weekly data were analyzed, the normality hypothesis was accepted for all seven processes.

The results of this study indicate normality may be dependent upon the time between observations.

Boer [1984] provided two reasons why cost variances may not be normally distributed. The first reason is that the data are assumed to come from a constant system of chance causes. However, there is little likelihood that a plant or department remains stable from one period to the next. This instability may be due to different worker personalities, varying moods of supervisors, pressures from plant management and differences in material qualities.

The second reason relates to the problems in specifying a frequency distribution for making probabilistic statements about costs because accountants work with sample means of costs, which will be normally distributed according to the central limit theorem. However, if the underlying population is not normally distributed, the estimated parameters may not be an accurate estimate of the corresponding population parameters.

2.4 Summary and Conclusion

The preceding review reveals a void in two aspects. First, there is a lack of empirical research on the distribution properties of actual dollar cost variances. Second, there is a lack of understanding as to whether the optimal CVID rule is sensitive to the distribution properties of cost variances. In summary, the effect of the distribution properties of cost variances on the CVID is unknown. The remainder of this thesis will address these two issues.

CHAPTER III

ANALYSIS OF ACTUAL MANUFACTURING COST VARIANCES AND ESTIMATION OF PARAMETERS

3.1 Introduction

This chapter discusses methods for measuring and testing nonnormality and the use of these methods to evaluate the distribution properties of actual cost variances collected from a medium size manufacturing plant.

3.2 Tests of Normality and Descriptive Statistics

There are many statistical tests of normality (see [D'Agostino and Stephens, 1986 (ch. 9)] for an extensive review). These tests of normality can be grouped into five categories: chi-square test, empirical distribution function (EDF) tests, moment tests, regression tests, and miscellaneous tests. The first four of these normality tests will be discussed and the reasons for using certain tests and not considering other tests will be indicated. Due to the large number of tests in the fifth category of "miscellaneous" tests these will not be discussed in detail.

D'Agostino [1986] states the chi-square test should not be used in testing departures from normality when the full ungrouped sample of data is available because other tests are more powerful. In general, the chi-square test is not a powerful test of normality. Given the other four tests are more powerful than the chi-square, this study will not use the chi-square test to test for normality.

Two of the most prominent tests based on the empirical distribution function are the Kolmogorov [1933]-Smirnov [1939] test and the Anderson-Darling A^2 test [1954]. The Kolmogorov-Smirnov test has poor power in comparison to the other tests available [D'Agostino, 1986]. The Anderson-Darling A^2 test is considered to be the most powerful of all the EDF tests but it has not been studied as extensively as the moments tests or the regression tests. Thus, it is unknown how the power of the Anderson-Darling A^2 test compares with some of the other tests which have been studied and are considered to be the most powerful. Due to the reasons indicated above, neither the Kolmogorov-Smirnov test or the Anderson-Darling A^2 test will be used in the present study.

The third category of tests for normality is that of moment tests. Pearson [1895] observed that deviations from normality could be characterized by the standard third and fourth moments of a distribution. The third and fourth moments, respectively, of a normal distribution are determined as follows:

$$\sqrt{\beta_1} = \frac{E\ (X - \mu)^3}{\sigma^{3/2}} = 0 \tag{5}$$

and

$$\beta_2 = \frac{E\ (X - \mu)^4}{\sigma^4} = 3 \tag{6}$$

The third standardized moment $\sqrt{\beta_1}$ is a measure of the skewness of a distribution. If a distribution is symmetric about its mean μ, as is the normal distribution, $\sqrt{\beta_1} = 0$. Values of $\sqrt{\beta_1}$ not equal to 0 indicate skewness and nonnormality.

The fourth standardized moment β_2 is a measure of the kurtosis or peakedness of a distribution. If the distribution is normal, $\beta_2 = 3$. Values of β_2 not equal to 3 indicate nonnormality. β_2 also indicates tail thickness of a distribution. Values of $\beta_2 > 3$ indicate distributions with "thicker" than normal tails, and values of $\beta_2 < 3$ indicate distributions with "thinner" than normal tails.

Pearson [1895] suggested that the standardized third and fourth moments of the sample can be used to judge nonnormality. The third and fourth moments of the sample are determined as follows, respectively:

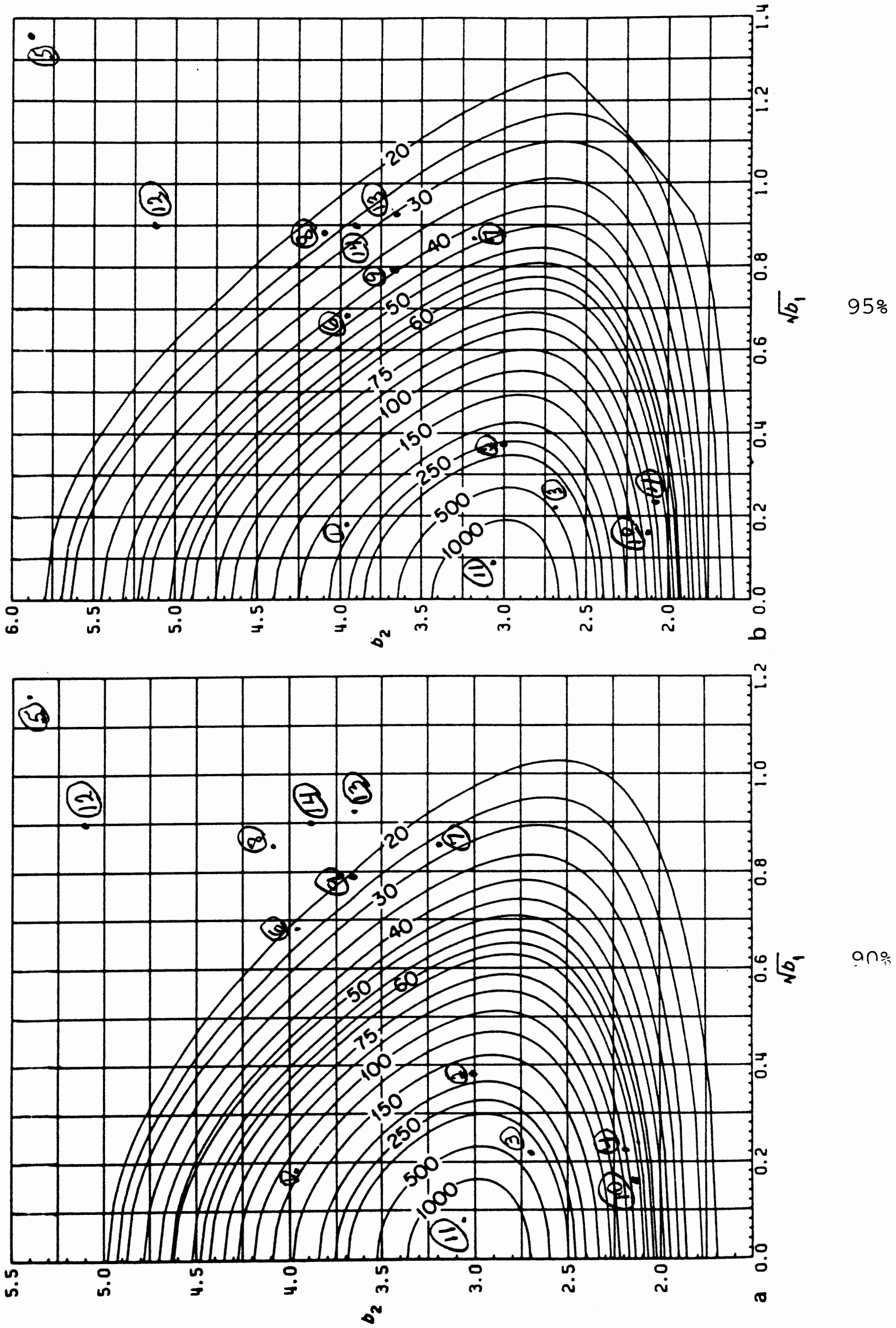
$$\sqrt{b_1} = \left[\sum_{i=1}^{n} (X_i - \bar{X})^3\right] / \left[\sum_{i=1}^{n} (X_i - \bar{X})^2\right]^{3/2} \quad (7)$$

and

$$b_2 = \left[\sum_{i=1}^{n} (X_i - \bar{X})^4\right] / \left[\sum_{i=1}^{n} (X_i - \bar{X})^2\right]^{2} \quad (8)$$

Among the many moment tests of normality, some attempt to detect nonnormality due to skewness while others attempt to detect nonnormality due to kurtosis. The more powerful "omnibus tests" of normality are those which consider both skewness and kurtosis. In a recent review, D'Agostino [1986] indicated that the Shapiro-Wilk W test [Shapiro and Wilk, 1965] and the K_s^2 test [Bowman and Shenton, 1986] are two of the best omnibus tests available. The K_s^2 is a moment test of normality and the Shapiro-Wilk W test is a regression test normality. The K_s^2 will be discussed first.

The K_s^2 test consists of calculating the sample skewness ($\sqrt{b_1}$) and kurtosis (b_2). The couplet ($\sqrt{b_1}, b_2$) is plotted on the 90% or 95% contour chart (Figure 1). If the plotted point is internal to the appropriate contour, the null hypothesis of normality is accepted. Using the data presented for Department 8 in Table II, an example of how to use the contour chart is presented. Department 8 had 42 observations, skewness ($\sqrt{b_1}$) of 0.87, and kurtosis (b_2) of 4.10. When this couplet is plotted on the 95% contour chart (Figure 1 (b)), we find the point is outside of the contour for n = 42. Thus, the appropriate K_s^2



Source: (Bowman and Shenton, 1986)

Figure 1. Contours for K_s^2 Test

decision is to reject the hypothesis of normality at the 5% significance level. The couplets ($\sqrt{b_1}, b_2$) for the direct labor efficiency variance amounts for each department are plotted on the 90% and 95% coutour charts (Figures 1(a) and 1(b)).

D'Agostino [1986] states the Bowman-Shenton K_s^2 test is sensitive to a wide range of nonnormal populations. Since the K_s^2 test is considered one of the best omnibus tests available [D'Agostino, 1986], it is one test which will be used in this study to evaluate the actual cost variance data for normality.

The fourth category of tests for normality is that of regression tests. The Shapiro-Wilk W test is a regression test of normality. It is considered by Bowman and Shenton [1986] as one of the best omnibus tests available.

The Shapiro-Wilk W test statistic is determined as follows:

$$W = \frac{\left[\sum_{i=1}^{n} a_i X_i \right]^2}{\sum_{i=1}^{n} \left[X_i - \bar{X} \right]^2} \qquad (9)$$

The values (X_i) are ordered from smallest to largest. The values are then multiplied by the weights a_i. The a_i values for n = 3 to 50 were given by Shapiro and Wilk [1965]. The W value can be treated like an R^2 value. Large values of W indicate normality and small values indicate nonnormality. The computed W test statistic is

TABLE I

CRITICAL VALUES FOR THE SHAPIRO-WILK TEST OF NORMALITY

	Significance level α								
	Lower tail								Upper tail
n	0.01	0.02	0.05	0.10	0.50	0.10	0.05	0.02	0.01
3	0.753	0.756	0.767	0.789	0.959	0.998	0.999	1.000	1.000
4	.687	.707	.748	.792	.935	.987	.992	.996	.997
5	.686	.715	.762	.806	.927	.979	.986	.991	.993
6	0.713	0.743	0.788	0.826	0.927	0.974	0.981	0.986	0.989
7	.730	.760	.803	.838	.928	.972	.979	.985	.988
8	.749	.778	.818	.851	.932	.972	.978	.984	.987
9	.764	.791	.829	.859	.935	.972	.978	.984	.986
10	.781	.806	.842	.869	.938	.972	.978	.983	.986
11	0.792	0.817	0.850	0.876	0.940	0.973	0.979	0.984	0.986
12	.805	.828	.859	.883	.943	.973	.979	.984	.986
13	.814	.837	.866	.889	.945	.974	.979	.984	.986
14	.825	.846	.874	.895	.947	.975	.980	.984	.986
15	.835	.855	.881	.901	.950	.975	.980	.984	.987
16	0.844	0.863	0.887	0.906	0.952	0.976	0.981	0.985	0.987
17	.851	.869	.892	.910	.954	.977	.981	.985	.987
18	.858	.874	.897	.914	.956	.978	.982	.986	.988
19	.863	.879	.901	.917	.957	.978	.982	.986	.988
20	.868	.884	.905	.920	.959	.979	.983	.986	.988
21	0.873	0.888	0.908	0.923	0.960	0.980	0.983	0.987	0.989
22	.878	.892	.911	.926	.961	.980	.984	.987	.989
23	.881	.895	.914	.928	.962	.981	.984	.987	.989
24	.884	.898	.916	.930	.963	.981	.984	.987	.989
25	.888	.901	.918	.931	.964	.981	.985	.988	.989
26	0.891	0.904	0.920	0.933	0.965	0.982	0.985	0.988	0.989
27	.894	.906	.923	.935	.965	.982	.985	.988	.990
28	.896	.908	.924	.936	.966	.982	.985	.988	.990
29	.898	.910	.926	.937	.966	.982	.985	.988	.990
30	.900	.912	.927	.939	.967	.983	.985	.988	.900
31	0.902	0.914	0.929	0.940	0.967	0.983	0.986	0.988	0.990
32	.904	.915	.930	.941	.968	.983	.986	.988	.990
33	.906	.917	.931	.942	.968	.983	.986	.989	.990
34	.908	.919	.933	.943	.969	.983	.986	.989	.990
35	.910	.920	.934	.944	.969	.984	.986	.989	.990
36	0.912	0.922	0.935	0.945	0.970	0.984	0.986	0.989	0.990
37	.914	.924	.936	.946	.970	.984	.987	.989	.990
38	.916	.925	.938	.947	.971	.984	.987	.989	.990
39	.917	.927	.939	.948	.971	.984	.987	.989	.991
40	.919	.928	.940	.949	.972	.985	.987	.989	.991
41	0.920	0.929	0.941	0.950	0.972	0.985	0.987	0.989	0.991
42	.922	.930	.942	.951	.972	.985	.987	.989	.991
43	.923	.932	.943	.951	.973	.985	.987	.990	.991
44	.924	.933	.944	.952	.973	.985	.987	.990	.991
45	.926	.934	.945	.953	.973	.985	.988	.990	.991
46	0.927	0.935	0.945	0.953	0.974	0.985	0.988	0.990	0.991
47	.928	.936	.946	.954	.974	.985	.988	.990	.991
48	.929	.937	.947	.954	.974	.985	.988	.990	.991
49	.929	.937	.947	.955	.974	.985	.988	.990	.991
50	.930	.938	.947	.955	.974	.985	.988	.990	.991

Source: (Shapiro-Wilk, 1965).

compared with the critical values of W (Table I), which were also given by Shapiro and Wilk [1965]. If the W test statistic (equation 9) is greater than or equal to the critical value from Table I, the null hypothesis of normality would be accepted. If the W test statistic is less than the critical value, we would conclude the data are not normally distributed.

Many studies have investigated the sensitivity of the various tests of normality to determine if there is a single test that is optimal for all possible deviations from normality. These studies have investigated a wide range of nonnormal populations for a variety of sample sizes. The results of these studies indicate no one test is optimal for all possible deviations from normality. However, D'Agostino [1986] states that the Shapiro-Wilk W test is a very sensitive omnibus test and for many skewed populations clearly the most powerful test. For these reasons, the Shapiro-Wilk W test and the Bowman-Shenton K_s^2 will be used to test the actual cost variance data for normality.

3.3 Actual Cost Variances: Test for Normality

Actual cost variances of a medium size manufacturing plant of a Fortune 500 company were collected from its fourteen production departments. The data consist of weekly standard direct labor costs and direct labor efficiency variances. To investigate the distribution properties of these variance data, the two tests of normality are applied on the two sets of data, namely, the departmental direct labor efficiency variance amounts and the departmental direct labor efficiency variance expressed as a percentage of the departmental standard direct labor cost.

Tables II and III present measures of skewness and kurtosis for each department's variances and the test statistics for testing the following hypotheses:

H_0: The population from which the sample of direct labor efficiency variances was drawn is normally distributed.

H_1: The population from which the sample of direct labor efficiency variances was drawn is not normally distributed.

TABLE II

SUMMARY STATISTICS AND RESULTS OF HYPOTHESES
(based on direct labor efficiency variance amounts)

Dept.	n	$\sqrt{b_1}$	b_2	W_{OSL}	K_s^2 Decision
1	42	0.17	3.93	0.058	Accept H_o
2	42	- 0.38	3.00	0.448	Accept H_o
3	42	0.23	2.68	0.350	Accept H_o
4	42	- 0.24	2.18	0.340	Accept H_o
5	42	3.45	15.64	0.010	Reject H_o
6	42	- 0.68	3.92	0.404	Accept H_o
7	42	- 0.86	3.20	0.016	Accept H_o
8	42	0.87	4.10	0.084	Reject H_o
9	42	0.79	3.62	0.038	Reject H_o
10	42	- 0.16	2.15	0.367	Accept H_o
11	41	- 0.09	3.10	0.731	Accept H_o
12	42	- 0.89	5.17	0.096	Reject H_o
13	33	- 0.93	3.66	0.019	Reject H_o
14	34	- 0.90	3.85	0.049	Reject H_o

n = number of direct labor efficiency variance observations for the department

X_i = direct labor efficiency variance observation

$$\bar{X} = \sum_{i=1}^{n} X_i \;/\; n$$

$$\sqrt{b_1} = \left[\sum_{i=1}^{n} (X_i - \bar{X})^3\right] \Big/ \left[\sum_{i=1}^{n} (X_i - \bar{X})^2\right]^{3/2}$$

$$b_2 = \left[\sum_{i=1}^{n} (X_i - \bar{X})^4\right] \Big/ \left[\sum_{i=1}^{n} (X_i - \bar{X})^2\right]^2$$

W_{OSL} : test of normality based on the Shapiro-Wilk W test (refer to Table I for critical values)

K_s^2 : decision based on $\alpha = 0.05$ (refer to Figure 1 for the contour charts)

NOTE: $\sqrt{b_1}$ is the sample estimate of skewness, it equals 0 for a normal distribution.

b_2 denotes kurtosis, it equals 3 for a normal distribution.

TABLE III

SUMMARY STATISTICS AND RESULTS OF HYPOTHESES
(based on direct labor efficiency variance percentages)

Dept.	n	$\sqrt{b_1}$	b_2	W_{OSL}	K_s^2 Decision
1	42	4.22	23.63	0.010	Reject H_o
2	42	0.73	2.84	0.039	Accept H_o
3	42	4.40	25.58	0.010	Reject H_o
4	42	1.09	4.97	0.013	Reject H_o
5	42	2.05	6.31	0.010	Reject H_o
6	42	0.27	3.22	0.224	Accept H_o
7	42	1.42	6.06	0.010	Reject H_o
8	42	1.06	3.93	0.010	Reject H_o
9	42	1.16	3.87	0.010	Reject H_o
10	42	0.99	3.38	0.010	Reject H_o
11	41	4.33	24.82	0.010	Reject H_o
12	42	2.63	9.66	0.010	Reject H_o
13	33	3.52	15.69	0.010	Reject H_o
14	34	2.26	7.21	0.010	Reject H_o

n = number of direct labor efficiency variance observations for the department

W_{OSL} : test of normality based on the Shapiro-Wilk W test (refer to Table I for critical values)

K_s^2 : decision based on $\alpha = 0.05$ (refer to Figure 1 for the contour charts)

While Tables II and III show that the skewness of all variance distributions deviates from 0 and their kurtosis deviates from 3, the purpose of the statistical normality tests is to find out whether these deviations are sufficiently large enough to imply that they are not due to random sampling errors from a normal distribution, but that the underlying population is indeed nonnormal.

Instead of providing the W test statistics, the W_{OSL} (i.e., the observed significance level of the W test statistic) values are given in Tables II and III. W_{OSL} can be determined by interpolating between the significance levels (α) given in Table I.

The cost variance amounts (Table II) for Department 13 are used to illustrate how the W_{OSL} values are determined. The W test statistic, calculated using equation 9, for the cost variance amounts of Department 13 is 0.916. The number of observations (n) for Department 13 is 33. In Table I, for n = 33, the significance level (OSL) for a W test statistic of 0.916 is between 0.01 and 0.02 (W test statistic values of 0.906 and 0.917, respectively). The test is made in the lower tail because studies by Shapiro and Wilk [1968] suggested that when the sample is not from a normal distribution, low values of W will usually result. Interpolating between the W_{OSL} values of 0.01 and 0.02 results in a W_{OSL} value of 0.019 for the W test statistic of 0.916.

For the direct labor efficiency variance amounts, the test statistics in Table II show that, at the 0.05 significance level, the W test rejects the normality hypothesis for variances for five departments (#5, 7, 9, 13, 14), and the K_s^2 test rejects this hypothesis for six departments (#5, 8, 9, 12, 13, 14). At the 0.1

significance level, the W test will reject the normality hypothesis for 8 of the 14 departments. For the labor efficiency variances expressed as a percentage of the standard direct labor cost, the test statistics in Table III indicate that, at the 0.05 significance level, both tests reject the normality hypothesis for all but two departments (#2 and 6). While nobody should doubt that some variance distributions are normally distributed, it is evident from Tables II and III that there is little justification to assume that all variance distributions are normally distributed.

CHAPTER IV

APPROACHES FOR MODELING NONNORMAL COST VARIANCE DISTRIBUTIONS

4.1 Introduction

CVID models and analyses require the representation of variances by statistical distribution functions. This section briefly reviews the selection of versatile distribution functions capable of representing the kind of general nonnormal variance distributions observed in the preceding section.

4.2 Systems of Distributions

Ideally, a distribution function should be chosen by the following three-step iterative process:

(1) Identify a family of distribution functions which appears appropriate.

(2) Determine the parameters of the distribution function that best fits the empirical distribution on hand.

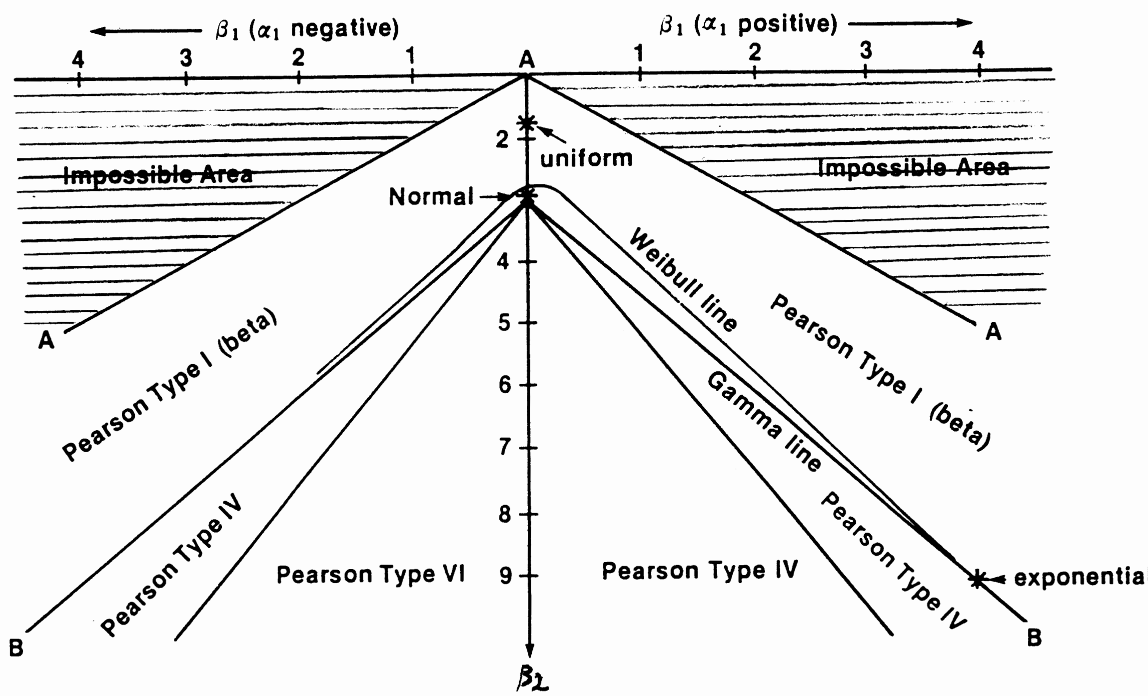
(3) Decide whether an adequate fit has been provided by the chosen family of distribution functions.

It is well recognized in the statistics literature that, in order to fit nonnormal empirical distributions of general shapes, a family of four-parameter distribution functions has to be used. This fitting process will be illustrated in Section 4.4 with an example. Many four-parameter families of distributions have been developed, one may refer to Kendall and Stuart [1969] for the theory of four-parameter distribution functions and to Schmeiser [1977] for a convenient compendium of available four-parameter families.

Among the many four-parameter families available, this study will use primarily the Pearson family to represent the nonnormal variance distributions. For some simulations, the Johnson family may also be used. The choice of these two distributions is justified in the following section and Section 4.4.

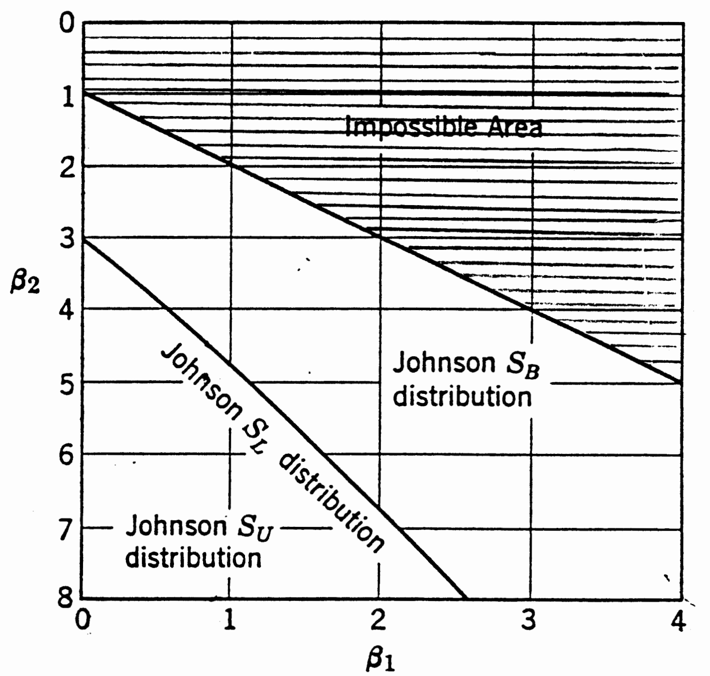
4.3 Merits of the Pearson and Johnson Systems

The Pearson and Johnson Systems are used in this study based on the following four criteria. First, these two systems can cover the entire possible area of skewness-kurtosis combinations. The skewness-kurtosis diagram in Figure 2 shows that, except for skewness-kurtosis combinations in the shaded "impossible area," an empirical distribution can have any skewness-kurtosis combination. The limited set of skewness-kurtosis combinations that can be represented by two and three-parameter distribution functions such as the normal, uniform, gamma and lognormal



Source: (Pearson and Hartley, 1970).

Figure 2. Skewness-Kurtosis Diagram



Source: (Johnson, 1949).

Figure 3. Skewness-Kurtosis Diagram

are depicted as points or lines in Figure 2. Only four-parameter distributions can possibly cover "areas" of skewness-kurtosis combinations, but not all four-parameter distributions can cover the entire possible area of skewness-kurtosis combinations depicted in Figure 2. The Pearson family consists of three "main" types (Types I, VI and IV) of distribution functions, and as shown in Figure 2, together these three functions cover the entire possible area of skewness-kurtosis combinations. Figure 3 shows that, between the S_B and S_U members of the Johnson family, the entire area of possible skewness-kurtosis combinations is also covered.

The second criterion used to select a distribution family is the ease of fitting. This criterion means it should be reasonably easy to determine the parameters of the distribution function that provides the best fit to the empirical distribution under consideration. Given the mean, variance, skewness and kurtosis of an empirical distribution, convenient closed-form formulas are available for determining the parameters of the fitting Pearson function [Elderton and Johnson, 1969]. Although similar closed-form formulas are unavailable for the Johnson family, tables [Pearson and Hartley, 1972] and subroutines are available to perform the same task. These tables and subroutines are not as convenient as the formulas available for the Pearson family.

The third criterion used is the ease of generating random variates from the distribution function. This criterion is important since simulation will be used in this study and values of cost variances have to be generated randomly by a computer. Random variates from Johnson distributions can be very easily and economically generated, so can variates from the type I Pearson distribution. Variates from types IV and VI Pearson distributions cannot be easily generated, therefore a Johnson function will be used for a variance distribution whose skewness-kurtosis combination falls in the types IV or VI area (see Figure 2).

The final selection criterion used is "popularity." The Pearson and Johnson families are the two oldest four-parameter systems of distribution functions, and have been widely used. In the accounting literature, applications of the Pearson distributions have been illustrated by Liao [1975] and Kottas, Lau and Lau [1978].

4.4 Selecting a Distribution

The direct labor efficiency variance amounts of Department 14 (see Table II) will be used to select a distribution for the study. A histogram of the cost variance data is presented in Figure 4. The skewness and kurtosis of the sample data were calculated using equations 7 and 8 of Section 3.2, respectively. Skewness was - 0.90 and kurtosis was 3.85. The sample mean was $439 and the

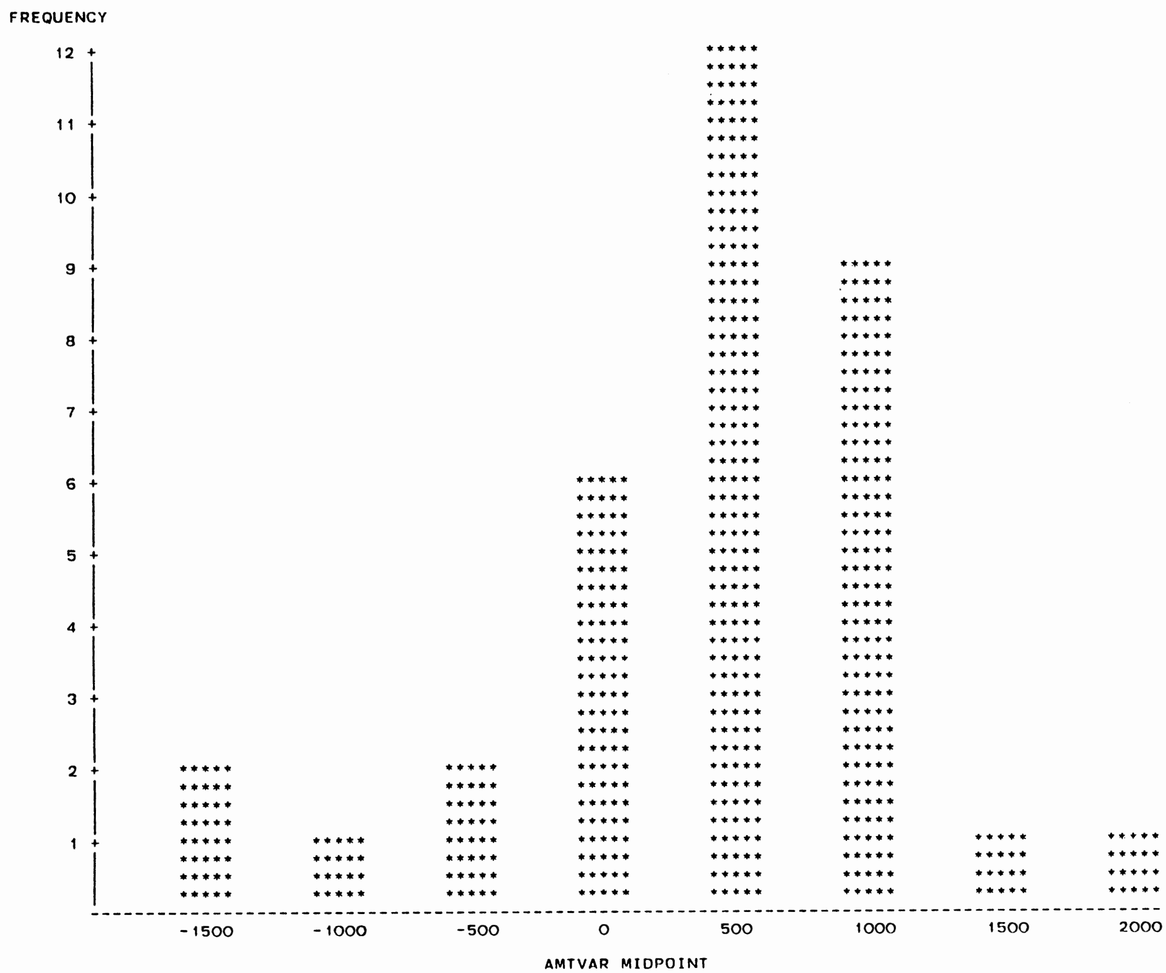


Figure 4. Histogram of Department 14 Cost Variance Amounts

standard deviation was $751. Both tests of normality (discussed in Section 3.2) indicate the cost variance amounts of Department 14 are not normally distributed.

The following three steps will be used to identify the appropriate distribution for this study. The first step is to identify a family of distribution functions which appears appropriate. Based on the histogram presented in Figure 4 and the discussion of Section 4.3, the Pearson family of distributions appears appropriate. The histogram shows the data are negatively skewed. We find this particular skewness-kurtosis combination (-0.90, 3.85) falls within the beta area of Figure 2. Thus, of all the distributions, the beta distribution is selected as the initial candidate.

The second step is to determine the parameters of the distribution function that best fits the empirical distribution on hand. There are two methods to use for determining the parameters of the distribution function. One method is called maximum likelihood. This procedure dictates one should examine the likelihood function of the sample values and take as estimates of the unknown parameters those values that maximize this likelihood function [Larson, 1982]. The maximum likelihood method is generally recognized as the better method for fitting a distribution but it is much more difficult to operationalize than the second method, the method of moments. Thus, for this study, the method of moments will be used to determine the parameters of the distribution function that best fits the empirical distribution on hand.

The method of moments uses the first four sample moments of the data set to estimate the parameters of the assumed population. The first four moments are determined as follows:

$$\mu_1 = \bar{X} = \sum_{i=1}^{n} X_i / n \tag{10}$$

$$\mu_2 = \left[\sum_{i=1}^{n} (X_i - \bar{X})^2\right] / n \tag{11}$$

$$\mu_3 = \left[\sum_{i=1}^{n} (X_i - \bar{X})^3\right] / n \tag{12}$$

$$\mu_4 = \left[\sum_{i=1}^{n} (X_i - \bar{X})^4\right] / n \tag{13}$$

where:

$\bar{X} = \sum_{i=1}^{n} X_i / n$

n = number of direct labor efficiency variance observations for the department

X_i = direct labor efficiency variance observation

Relative measures of skewness ($\sqrt{\beta_1}$) and kurtosis (β_2) can be determined using the first four moments. The measures are as follows:

$$\sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{3/2}} \tag{14}$$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} \tag{15}$$

The parameters of the beta distribution function, a, b, p, and q are determined as follows [Elderton and Johnson, 1969]:

Compute $r = 6(\beta_2-\beta 1-1)/(6+3\beta_1-2\beta_2)$ (16)

$w = \sqrt{(r+2)^2\beta_1+16(r+1)}$

$p,q = r/2[1\pm(r+2)\sqrt{\beta_1}/w]$ (17)

($q>p$ if $\beta_1>0$, $p\geq q$ otherwise)

$a = \mu_1-p\sqrt{\mu_2}w/[2(p+q)]$ (18)

$b = a+\sqrt{\mu_2}w/2$ (19)

Using the sample data of Department 14 in the above equations results in the following values for a, b, p, and q:

a = - 7212.40
b = 1748.06
p = 14.32
q = 2.45

The "a" and "b" values represent the lower and upper bounds of the beta distribution which have been fit to the sample data. The "p" and "q" values represent shape parameters of the beta distribution. Figure 5 includes a histogram of the sample data (from Figure 4) and the beta probability function which has been fit to the data.

The third step in choosing a distribution function is to decide whether an adequate fit has been provided by the chosen distribution family. The chi-square test will be used to determine whether the selected beta distribution adequately fits the sample data. The chi-square test is used to test the following hypotheses:

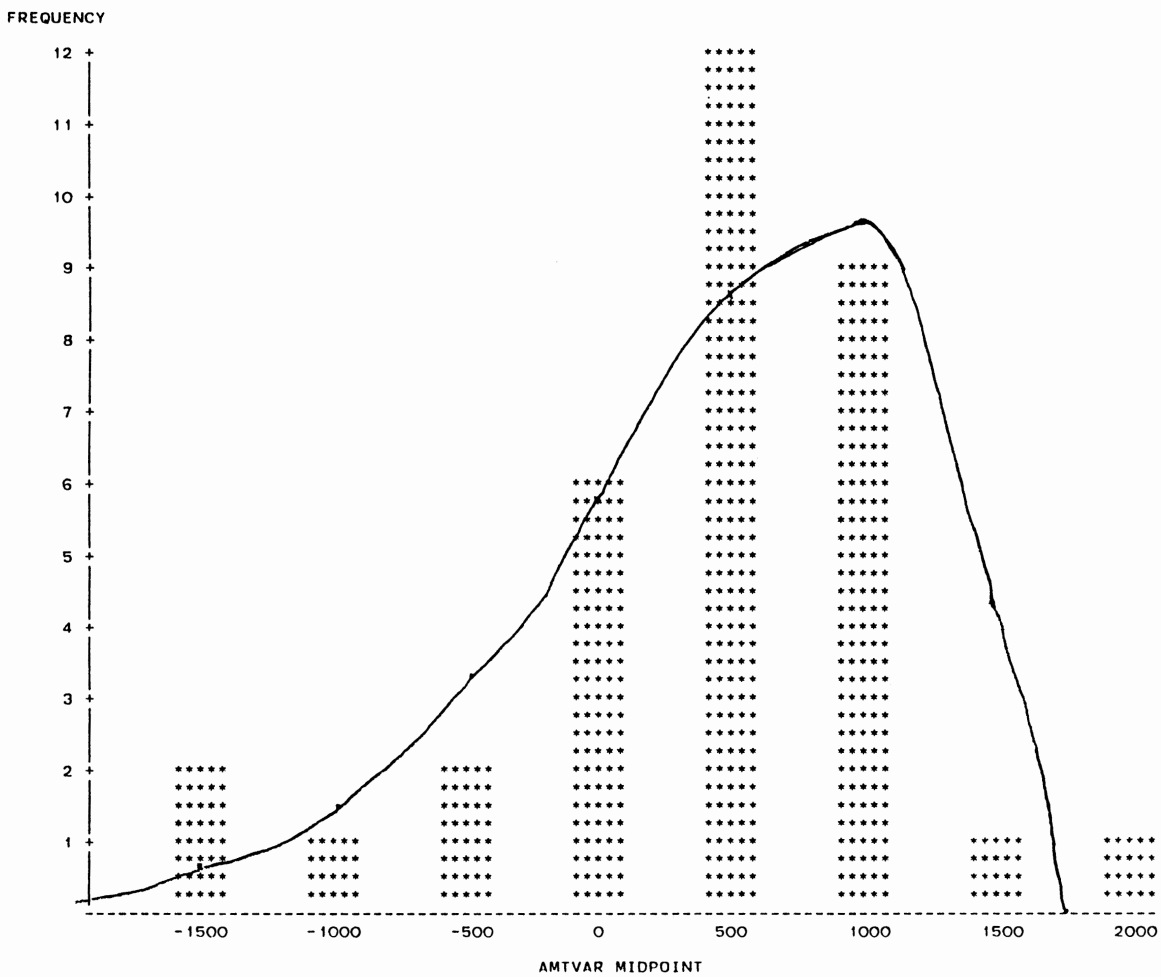


Figure 5. "Fitted" Beta Distribution Function

H_0: The beta distribution adequately fits the empirical data.

H_1: The beta distribution does not adequately fit the empirical data.

The chi-square test consists of dividing the range of the observed values into a number of intervals. The cost variance amounts ranged from $-7200 to $2000. Seven intervals were used. The actual frequency and expected frequency for each interval is determined. The actual frequency is determined by simply counting the number of observations in each interval. The expected frequency is determined by taking the probability of an observation being in the interval times the total number of observations. The value of the chi-square test statistic is computed as follows:

$$X^2 = \Sigma \frac{(\text{Actual Frequency} - \text{Expected Frequency})^2}{\text{Expected Frequency}} \quad (20)$$

Interval	Actual Freq.	Expected Freq.	$\frac{[\text{Actual Frequency} - \text{Expected Frequency}]^2}{\text{Expected Frequency}}$
1501 to 2000	1	0.97	0.0009
1001 to 1500	7	7.64	0.0536
501 to 1000	11	9.56	0.2169
1 to 500	7	7.37	0.0186
-499 to 0	4	4.49	0.0535
-999 to -500	1	2.32	0.7510
-7200 to -1000	3	1.65	1.1045
	34	34.00	2.1990

The chi-square test statistic for this illustration is 2.1990. The degrees of freedom for this test is the number of intervals minus the number of restrictions placed upon the data. Seven intervals were used in computing the chi-square test statistic and there were four restrictions as the first four moments were specified. Thus, the degrees of freedom for the chi-square test are three. The critical value for the chi-square test with α equal to 0.05 and three degrees of freedom is 7.815. Since the computed test statistic is less than the critical value, the chi-square test suggests the beta distribution adequately fits the sample data of Department 14.

The fitting procedure for the empirical data examined above indicates, of all the nonnormal distributions, the beta distribution seems to be the appropriate distribution to use for modeling the Department 14 direct labor efficiency variance amounts. If a similar fitting procedure is applied to other departments' direct labor efficiency variance data, other distribution functions may be selected.

One of the major questions of interest in this study is how distribution assumptions affect cost variance investigation decisions. The normal distribution is symmetrical around its mean. The density function for the normal distribution is as follows:

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2} \tag{21}$$

where:

x = observation

μ = mean of the distribution

σ = standard deviation of the distribution

The nonnormal distribuion used in this study is the beta distribution. The density function for the beta distribution is as follows:

$$f(x) = \left[K \cdot (x-A)^{\alpha-1} (B-x)^{\beta-1} \right] \tag{22}$$

where:

$$K = \left[\Gamma(\alpha+\beta)/\Gamma(\alpha)/\Gamma(\beta)/(B-A)^{(\alpha+\beta-1)} \right]$$

x = observation

α = parameter of the beta distribution

β = parameter of the beta distribution

A = lower bound of the beta distribution

B = upper bound of the beta distribution

As indicated above, the density functions for the two distributions are different. Thus, if a decision rule requires a density function calculation the distribution assumption becomes critical. Since the density functions are different the decision resulting from the use of a decision rule is dependent upon the distribution assumption. An example of different distribution assumptions affecting decisions is presented in Chapter 6.

4.5 Summary

This chapter discussed an approach which could be used to model nonnormal cost variance distributions. The three-step process for choosing a distribution function was illustrated using the sample data of Department 14. The initial data analysis indicated a Pearson family distribution function, the beta distribution, was an initial candidate which could be used to model the nonnormal cost variance data of Department 14. The required parameters for using the beta distribution were calculated. A chi-square test indicated the beta distribution adequately fit the empirical data of Deepartment 14. Since the modeling in the remaining sections of the thesis uses the parameters of the Department 14 data, the beta distribution appears to be the appropriate distribution to use in this study. Thus, Chapter V discusses the CVID models which will be modeled using the beta distribution.

CHAPTER V

CVID RULES AND MODELS TO BE INVESTIGATED

5.1 Introduction

Various CVID models to aid managers in this decision have been proposed in the literature (see Chapter II for a review of these models). This study will investigate the effect of cost variances' nonnormality on the performance of the seven CVID rules considered by Magee [1976]. In addition, the Markovian control model proposed by Dittman and Prakash [1978] will also be evaluated.

5.2 Rules Included in Magee's Study

Magee [1976] investigated the performance of the following seven CVID decision rules using simulation:

(1) Investigate all unfavorable cost variances.
(2) Investigate all cost variances that exceed the standard by 10 percent.
(3) Investigate all cost variances that exceed the standard by at least one standard deviation.
(4) Investigate all cost variances that exceed the standard by at least two standard deviations.
(5) Using the cost variance observation and Bayesian revision, find the probability the process is in control at the end of the period. If this probability is less than a predetermined critical value, an investigation is signalled. This is the model developed by Dyckman [1969].
(6) This rule is similar to rule 5 except the critical value is found using a dynamic programming procedure. This is the model developed by Kaplan [1969].
(7) This rule assumes perfect knowledge of the true state. It is used as a benchmark for evaluating the performance of the other rules.

In the simulation analyses, Magee [1976] used normally distributed cost variances. The criterion is to minimize `total costs,' defined as the sum of investigation costs and `operating costs' (the cost generated by the random process). Magee's [1976] results indicate that, of all the rules considered, the investigation rule based on perfect knowledge (number 7) had the lowest average total cost. Of the first six decision rules under uncertainty Kaplan's [1969] dynamic programming approach (number 6) resulted in the least total costs. However, the difference between this and the much simpler rule of investigating all cost variances exceeding the standard by two standard deviations is not large.

The results of Magee's [1976] study also indicate, for all decision rules, average total costs <u>decrease</u> as the transition probability increases. This result seems reasonable. When the transition probability increases, there is a larger probability the process will be in the in-control state at the end of the period. Thus, with more cost variances from the in-control distribution, we would expect lower decision costs.

Magee [1976] used three different investigation cost amounts in his study. The effect of the amount of the investigation cost on average total costs appears to depend upon the decision rule. For the first three decision rules (all unfavorable, 10%, and 1σ), average total costs always increase as the investigation cost increases. For decision rules four, five, and six, average total costs usually decrease as the investigation cost increases.[7]

5.3 Dittman-Prakash's Model

Subsequent to Magee's [1976] study, Dittman and Prakash [1978 and 1979] developed a Markovian approach to the CVID problem. This approach is similar to Kaplan's [1969] except that Dittman and Prakash determine a critical cost rather than a critical probability. When the reported cost exceeds the critical cost, the process is investigated. One advantage of the Markovian approach is that it does not require dynamic programming, which is a requirement of Kaplan's [1969] approach. A second

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[7] Magee [1976] did not provide an explanation for the cost relationships mentioned.

advantage of the Markovian approach is that it does not require Bayesian updating of probabilities. Due to the two aforementioned advantages of the Markovian approach, it is a simpler model to operationalize than both the single-period and multiple-period Bayesian models. When the actual cost exceeds the critical cost, the process is investigated. While the Markovian approach was developed using <u>costs</u> rather than <u>cost variances</u>, this approach can be applied to cost variances since cost variances are a linear transformation of actual costs.

The critical cost variance is the value which minimizes the following expression [Dittman and Prakash, 1979]:

$$V^* = \min \left[f_1(x) - \frac{A}{B} \right] \cdot \left[\frac{1 - f_2(x)}{1 - gf_2(x)} \right] \qquad (23)$$

where:

x = observed cost variance

g = probability that the process remains in-control at the end of the period given that it entered the period in-control

f_1 = density function for observed cost variance x given x is from the in-control distribution

f_2 = density function for observed cost variance x given x is from the out-of-control distribution

$A = C + (1 - g) \cdot K - g\ (\mu_2 - \mu_1)$

$B = g \cdot C$

C = investigation cost

K = cost to correct the process

μ_1 = mean of the in-control distribution

μ_2 = mean of the out-of-control distribution

V^* = critical cost

When the reported cost variance exceeds this critical cost variance, the process is investigated.

Since the Dittman and Prakash [1978 and 1979] Markovian approach has never been compared with the other CVID models, it would be worthwhile to examine its performance as compared to those CVID models used by Magee [1976]. Thus, in this study, eight CVID models will be first compared using normally distributed cost variances. Subsequently, the performance of these CVID models will be investigated under situations where the cost variances are nonnormal and have various skewness-kurtosis combinations.

The reason for examining this second aspect will be discussed in the next chapter.

CHAPTER VI

ANALYSES OF ALTERNATIVE COST VARIANCE INVESTIGATION MODELS WITH NONNORMALITY: AN ILLUSTRATION

6.1 Introduction

The purpose of this section is to illustrate the effects of assuming improper distribution properties on the cost variance investigation decision.

6.2 Single-Period Bayesian Model

The single-period Bayesian model [Dyckman, 1969] is a two state (in-control, out-of-control), two action (investigate, do not investigate), single-period model. There is Bayesian updating of the probability of being in either state. Given a cost variance observation, x, at the end of period i, the posterior state probability of x representing an out of control variance can be determined by applying Bayes' theorem.

The probability density functions for normal and nonnormal distributions are different. This difference could result in different posterior state probabilities (q_is) leading to different decisions and different decision costs. This is illustrated using normal and nonnormal distributions in the following example.

The formulas of Chapter II will be used in this example. Assume the following parameters:

$\mu_1 = 0$
$\mu_2 = 20$
$C = 30$
$g = .9$
$K = 0$

Assume at the end of period one (n=11) a cost variance of $3.21 is observed. Chapter IV indicated the cost variances were nonnormally distributed and could be fit to a beta distribution. To compute a posterior probability of the cost variance being from the in-control distribution, the following beta density functions would be used:

$$\frac{f_1(x)}{f_2(x)} = \frac{\left[K \cdot (x-A)^{\alpha-1} (B-x)^{\beta-1} \right]}{\left[L \cdot (x-C)^{\alpha-1} (D-x)^{\beta-1} \right]} \qquad (24)$$

where:

$$K = \left[\Gamma(\alpha+\beta)/\Gamma(\alpha)/\Gamma(\beta)/(B-A)^{\alpha+\beta-1)} \right]$$

$$L = \left[\Gamma(\alpha+\beta)/\Gamma(\alpha)/\Gamma(\beta)/(C-D)^{\alpha+\beta-1)} \right]$$

x = observed cost variance

α = parameter of the beta distribution

β = parameter of the beta distribution

A = lower bound of the in-control beta distribution

B = upper bound of the in-control beta distribution

C = lower bound of the out-of-control beta distribution

D = upper bound of the out-of-control beta distribution

Solving equation 24 results in a likelihood ratio of 0.454. Using this likelihood ratio ($f_1(x)/f_2(x)$) in the Bayesian model (equation 25) results in a posterior probability which is determined as follows:

$$q_i = g \left[\frac{f_1(x)q_{i-1}}{f_1(x)q_{i-1} + f_2(x)\ (1 - q_{i-1})} \right] \qquad (25)$$

where:

g = probability that the process remains in-control at the end of the period given that it entered the period in-control

f_1 = density function for observed cost variance x given x is from the in-control distribution

f_2 = density function for observed cost variance x given x is from the out-of-control distribution

q_{i-1} = probability the process is in-control at the end of period i-1

q_i = probability the process is in-control at the end of period i

x = cost variance for period i

(q_i) of 0.857. With n being equal to eleven, the following equation (24) results in a probability of 0.757 for q_n^*:

$$q_n^* = 1 - \frac{C}{\left[g^n \cdot n \cdot (\mu_2 - \mu_1) + \sum_{j=1}^{n-1} g^j \cdot (1-g) \cdot j \cdot (\mu_2 - \mu_1) \right]} \qquad (26)$$

where:

C = investigation cost

g = probability that the process remains in-control at the end of the period given that it entered the period in-control

n = number of months left in the year

μ_1 = mean of the in-control distribution

μ_2 = mean of the out-of-control distribution

Since q_n^* is less than q_i the cost variance would not be investigated.

Alternatively, instead of using the beta distribution, a normal distribution is used. The density functions $f_1(x)$ and $f_2(x)$ are determined using the following formulas:

$$\frac{f_1(x)}{f_2(x)} = \frac{\frac{1}{\sigma\sqrt{2\pi}}\ e^{-(x-\mu_{IN})^2/\ 2\sigma^2}}{\frac{1}{\sigma\sqrt{2\pi}}\ e^{-(x-\mu_{OUT})^2/\ 2\sigma^2}} \tag{27}$$

where:

x = observed cost variance

σ = standard deviation of the distribution

μ_{IN} = mean of the in-control distribution

μ_{OUT} = mean of the out-of-control distribution

Solving the preceding equation results in a likelihood ratio of 0.712. Using this likelihood ratio in the Bayesian model results in a posterior probability (q_i) value of 0.834. q_n^* is the same as for the beta distribution, 0.757. As with the beta distribution, q_n^* is less than q_i and the cost variance would not be investigated. For the first period, the distributional assumptions did not have an effect on the decision.

Assume the cost variance at the end of the second period (n = 10) is $14.00. If the beta distribution is assumed, the likelihood ratio equals 0.709, q_i equals 0.805, and q_n^* equals 0.744. Since q_n^* is less than q_i the cost variance would not be investigated. If it is assumed the cost variances are normally distributed the likelihood ratio equals 0.122 and q_i equals 0.724. Since q_n^* (0.744) is greater than q_i the cost variance would be investigated and investigation costs of $30 would be incurred. This example indicates distribution assumptions do affect decisions and decision costs. The reason is because the density functions are dependent upon the type of distribution assumed. The result was that the different distribution assumptions resulted in different decisions for the second period. The next section discusses the simulation analyses which will be used to investigate how the various CVID models are affected when an incorrect assumption of normality is made.

6.3 Simulation Analyses

To perform the simulation analyses, IMSL[8] will be used to generate random numbers from two (in-control and out-of-control) nonnormal distributions. The decisions and decision costs for each decision rule will be evaluated twice. For each decision rule the cost variances are first assumed to be normally distributed. Subsequently the correct nonnormal distributions of the cost variances are then used.

A distribution which can be fitted to the desired parameters must be selected. Using the criteria discussed

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[8] IMSL refers to International Mathematical and Statistical Library. This software consists of fortran subroutines for mathematical and statistical analysis.

in Chapter IV resulted in the beta distribution being used to generate random numbers for the nonnormal distributions.

Table IV presents one set of parameters used in the simulations. Since the present study is investigating the effect of distributional properties of cost variances, the parameters are for the in and out-of-control cost variance distributions, not the cost distributions. The mean and standard deviation of both (beta and normal) in-control distributions are zero and 20, respectively. The mean and standard deviation of both out-of-control distributions are 20. The set of nonnormal distributions has skewness of negative 0.90 and kurtosis of 4.00. The skewness and kurtosis of the normal distributions are zero and 3.00, respectively. In the studies of Magee [1976] and Dittman and Prakash [1979] the investigation costs used were equal to a fixed amount for each decision. This study will also assume investigation costs to be constant. As in Magee's [1976] study, three different levels of investigation costs will be examined. Magee [1976] used two means for the out-of-control distribution and three transition probabilities. Similarly, this study will also investigate two different means for the out-of-control distribution and three different transition probabilities.

TABLE IV

PARAMETERS USED IN SIMULATIONS

Beta Distributions

In Control		Out of Control	
μ =	0.00	μ =	20.00
σ =	20.00	σ =	20.00
$\sqrt{b_1}$ =	- 0.90	$\sqrt{b_1}$ =	- 0.90
b_2 =	4.00	b_2 =	4.00

Normal Distributions

In Control		Out of Control	
μ =	0.00	μ =	20.00
σ =	20.00	σ =	20.00
$\sqrt{b_1}$ =	0.00	$\sqrt{b_1}$ =	0.00
b_2 =	3.00	b_2 =	3.00

As illustrated in the numerical example, the distribution properties of cost variances affect the cost variance investigation decisions and the amount of decision costs incurred. This study intends to use simulation to examine how the seven[9] CVID models are affected by using incorrect distribution assumptions. Furthermore, this study will also examine the sensitivity of the CVID models to

[9] Preliminary analysis indicated there was not significant differences in decision costs between Kaplan's [1969] dynamic programming approach and the Markovian control model [Dittman and Prakash, 1979]. Thus, the dynamic programming approach will not be considered in any further analysis.

different measures of skewness and standard deviations of the distributions.

6.4 Simulation Results

Table V indicates the eight CVID rules and cost variance distribution assumptions which were examined for sensitivity to distribution properties. Tables VI and VII present the amount of the decision costs and the rank (based on decision costs) of each decision rule. The decision costs are defined as the sum of the cost variance amounts generated by the random process and costs of investigations. Similar to Magee's [1976] study, for each decision model, twelve monthly decision costs are accumulated. To consider the inherent variability of the simulation process, the 12-months' accumulated costs were then simulated 200 times. An average of these decision costs was then obtained. Thus, the results presented in Tables VI and VII are the average decision costs over 200 12-month years.

The parameters for Tables VI and VII are the same as those used in Magee's [1976] study for cases 17 and 18, respectively. The parameters are as follows:

Parameters for Case 17	Parameters for Case 18
μ_1 = 0.00	μ_1 = 0.00
μ_2 = 50.00	μ_2 = 50.00
C = 60.00	C = 60.00
g = 0.70	g = 0.90
K = 0.00	K = 0.00

Tables VI and VII present the decision costs for each decision rule under three different sets of assumptions. Included is the rank of each decision rule. The rank is based on average total cost, with the least costly decision rule being assigned a rank of one. Three conclusions are drawn from the results presented in Tables VI and VII. First, the more complex CVID models result in lower decision costs than the simpler models. The single-period Bayesian model and the Markovian control model consistently outperformed the other models. A second conclusion is that the assumption of distribution properties does affect decision costs. This result is easily seen by comparing the decision costs for the three different sets of assumptions for each decision rule. More specifically, when we compare the decision costs of the single-period Bayesian model and the Markovian control model, we find an incorrect assumption of normality results in slightly greater decision costs than the correct assumption of nonnormality. A third conclusion reached from this analysis is that the rank of the decision rule can be affected by the distribution properties.

Since the results of Tables VI and VII indicate distribution properties can affect decision costs of the various CVID models, a more extensive analysis is presented in Chapter VII.

TABLE V

DECISION RULES AND ASSUMPTIONS REGARDING DISTRIBUTIONS

	Magee Study	Gribbin Study	
Assumed Distribution	Normal	Nonnormal	Normal
Actual Distribution	Normal	Nonnormal	Nonnormal
Decision Rule			
1. Investigate all unfavorable variances.			
2. Investigate all cost variances that exceed the standard by 10 percent.			
3. Investigate all cost variances that exceed the standard by at least one standard deviation.			
4. Investigate all cost variances that exceed the standard by at least two standard deviations.			
5. Single-period Bayesian model.			
6. Markovian control model.			
7. Perfect knowledge (This rule will serve as a benchmark for the evaluation of the other decision rules).			

TABLE VI

AVERAGE TOTAL COST* AND RANK** FOR CASE 17 OVER 200 12-MONTH PERIODS (Using Simulation)

	Magee Study	Gribbin Study	
Assumed Distribution	Normal	Nonnormal	Normal
Actual Distribution	Normal	Nonnormal	Nonnormal
Decision Rule			
1. Investigate all unfavorable variances.	644 (7)	668 (7)	668 (7)
2. Investigate all cost variances that exceed the standard by 10 percent.	551 (6)	562 (6)	562 (6)
3. Investigate all cost variances that exceed the standard by at least one standard deviation.	475 (5)	461 (5)	461 (5)
4. Investigate all cost variances that exceed the standard by at least two standard deviations.	434 (4)	403 (4)	403 (4)
5. Single-period Bayesian model.	415 (2)	378 (1)	383 (1)
6. Markovian control model.	430 (3)	397 (3)	400 (3)
7. Perfect knowledge	407 (1)	388 (2)	388 (2)

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** Rank is based on average total cost with the decision rule having the minimum average total cost being assigned a rank of 1.

TABLE VII

AVERAGE TOTAL COST* AND RANK** FOR CASE 18 OVER 200 12-MONTH PERIODS (Using Simulation)

	Magee Study	Gribbin Study	
Assumed Distribution	Normal	Nonnormal	Normal
Actual Distribution	Normal	Nonnormal	Nonnormal
Decision Rule			
1. Investigate all unfavorable variances.	446 (7)	487 (7)	487 (7)
2. Investigate all cost variances that exceed the standard by 10 percent.	323 (6)	345 (6)	345 (6)
3. Investigate all cost variances that exceed the standard by at least one standard deviation.	229 (5)	219 (5)	219 (5)
4. Investigate all cost variances that exceed the standard by at least two standard deviations.	151 (2)	138 (4)	138 (3)
5. Single-period Bayesian model.	161 (3)	132 (2)	141 (4)
6. Markovian control model.	166 (4)	136 (3)	137 (2)
7. Perfect knowledge	127 (1)	125 (1)	125 (1)

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** Rank is based on average total cost with the decision rule having the minimum average total cost being assigned a rank of 1.

CHAPTER VII

RESULTS OF STUDY

7.1 Introduction

The simulation analysis was performed using IMSL. The parameters presented in Table IV of Section 6.3 were used for the simulation analysis. Tables VIII through IX present the results for each decision rule. Estimated for each of the eighteen cases were total costs (includes cost variances plus costs of investigations) and the standard deviation of total costs. This study used the same time interval and period as Magee [1976], two hundred twelve-month years.

One objective of this study is to investigate how decision costs of CVID models are affected by an incorrect assumption of distribution properties. Section 7.2 presents the decision costs of each assumption for each CVID rule investigated in this study. The decision costs resulting from making an incorrect assumption of the standard deviation of the cost variance distributions are presented in Section 7.3. Section 7.4 presents the decision costs resulting from making an incorrect assumption of the skewness of the cost variance distributions.

7.2 Sensitivity of CVID Models to Distribution Properites

One purpose of this study is to investigate how optimal decisions are affected by the nonnormality of cost variances. This objective is fulfilled by using the CVID models used in Magee's [1976] study and the Markovian control model [Dittman and Prakash (1978 and 1979)]. First, for each decision rule, nonnormal cost variance distributions are assumed in making the CVID. Secondly, an assumption of normal cost variance distributions are used in making the CVID. The results of this analysis are discussed in this section.

Of the seven CVID models examined, decision rules one through four are not dependent upon distributional properties since these decision rules make an investigation decision based on whether the observed cost variance is greater than a predetermined cutoff value. The decision costs for these four decision rules are presented in Tables VIII to XI.

Tables XII and XIII present the costs of the single-period Bayesian model and the Markovian control models, respectively. These models are sensitive to distributional properties. The Bayesian model requires the calculation of a critical probability. Using a Bayesian revision process, the probability that an observed cost variance is from the "out-of-control" cost variance distribution is determined. If this probability is less than the critical probability the cost variance is investigated. This revision process is repeated for each of the twelve months of the year. The revised probability is dependent upon the density functions of the distributions. The actual density functions of the distributions are nonnormally distributed. To examine the effect of distribution properties on decision costs, this study first treats each of the eighteen cases in Table XII as obtained from a nonnormal distribution. Thus, the revised probabilities were determined using the density function of a beta distribution. In the second situation, an assumption assumption of normality was used for all eighteen cases. Thus, the revised probabilities were determined using the density function of a normal distribution.

In comparing the decision costs of the two different assumptions, the following results are observed. As can be seen from Table XII, the beta assumption resulted in lower decision costs in seven cases out of eighteen, whereas the assumption of normality resulted in lower decision costs in

TABLE VIII

DECISION RULE 1: ALL UNFAVORABLE

AVERAGE TOTAL COST* AND STANDARD DEVIATION OVER 200 12-MONTH PERIODS (Using Simulation)

$\mu_1 = 0,\ \sigma_1 = \sigma_2 = 20$

Case	μ_2	C	g	Cost	Standard Deviation
1	20	10	0.5	213	82
2	20	10	0.7	153	85
3	20	10	0.9	94	81
4	20	30	0.5	385	104
5	20	30	0.7	311	112
6	20	30	0.9	237	109
7	20	60	0.5	643	140
8	20	60	0.7	547	154
9	20	60	0.9	451	116
10	50	10	0.5	398	116
11	50	10	0.7	259	111
12	50	10	0.9	127	93
13	50	30	0.5	582	139
14	50	30	0.7	423	135
15	50	30	0.9	271	121
16	50	60	0.5	859	176
17	50	60	0.7	668	176
18	50	60	0.9	487	165

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

TABLE IX

DECISION RULE 2: 10% RULE

AVERAGE TOTAL COST* AND STANDARD DEVIATION
OVER 200 12-MONTH PERIODS
(Using Simulation)

$\mu_1 = 0,\ \sigma_1 = \sigma_2 = 20$

Case	μ_2	C	g	Cost	Standard Deviation
1	20	10	0.5	203	83
2	20	10	0.7	139	79
3	20	10	0.9	72	84
4	20	30	0.5	341	107
5	20	30	0.7	256	104
6	20	30	0.9	168	111
7	20	60	0.5	549	146
8	20	60	0.7	430	145
9	20	60	0.9	310	154
10	50	10	0.5	391	118
11	50	10	0.7	244	115
12	50	10	0.9	104	93
13	50	30	0.5	547	144
14	50	30	0.7	371	142
15	50	30	0.9	200	119
16	50	60	0.5	786	186
17	50	60	0.7	562	184
18	50	60	0.9	345	161

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

TABLE X

DECISION RULE 3: 1 σ RULE

AVERAGE TOTAL COST* AND STANDARD DEVIATION OVER 200 12-MONTH PERIODS (Using Simulation)

$\mu_1 = 0,\ \sigma_1 = \sigma_2 = 20$

Case	μ_2	C	g	Cost	Standard Deviation
1	20	10	0.5	197	83
2	20	10	0.7	133	77
3	20	10	0.9	57	79
4	20	30	0.5	294	108
5	20	30	0.7	208	100
6	20	30	0.9	107	100
7	20	60	0.5	439	149
8	20	60	0.7	321	137
9	20	60	0.9	180	135
10	50	10	0.5	383	125
11	50	10	0.7	232	112
12	50	10	0.9	86	93
13	50	30	0.5	516	156
14	50	30	0.7	323	139
15	50	30	0.9	139	117
16	50	60	0.5	714	205
17	50	60	0.7	461	183
18	50	60	0.9	219	155

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

TABLE XI

DECISION RULE 4: 2 σ RULE

AVERAGE TOTAL COST* AND STANDARD DEVIATION OVER 200 12-MONTH PERIODS (Using Simulation)

$\mu_1 = 0,\ \sigma_1 = \sigma_2 = 20$

Case	μ_2	C	g	Cost	Standard Deviation
1	20	10	0.5	206	71
2	20	10	0.7	165	89
3	20	10	0.9	80	89
4	20	30	0.5	233	82
5	20	30	0.7	187	98
6	20	30	0.9	92	96
7	20	60	0.5	274	104
8	20	60	0.7	220	115
9	20	60	0.9	110	109
10	50	10	0.5	394	128
11	50	10	0.7	243	121
12	50	10	0.9	81	105
13	50	30	0.5	496	154
14	50	30	0.7	307	146
15	50	30	0.9	103	120
16	50	60	0.5	649	195
17	50	60	0.7	403	187
18	50	60	0.9	138	144

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

TABLE XII

DECISION RULE 5: SINGLE-PERIOD BAYESIAN

AVERAGE TOTAL COST* OVER 200 12-MONTH PERIODS
(Using Simulation)

				$\mu_1 = 0,\ \sigma_1 = \sigma_2 = 20$			
				AVERAGE TOTAL COST		STANDARD DEVIATION	
				Assumed Distribution			
Case	μ_2	C	g	Beta	Normal	Beta	Normal
1	20	10	0.5	221	221	70	69
2	20	10	0.7	168	168	73	73
3	20	10	0.9	100	99	72	72
4	20	30	0.5	**	**	**	**
5	20	30	0.7	176	182	96	99
6	20	30	0.9	82	95	91	97
7	20	60	0.5	**	**	**	**
8	20	60	0.7	**	**	**	**
9	20	60	0.9	93	101	95	102
10	50	10	0.5	414	414	110	110
11	50	10	0.7	284	284	105	105
12	50	10	0.9	150	152	83	79
13	50	30	0.5	491	497	154	143
14	50	30	0.7	431	430	115	119
15	50	30	0.9	170	168	107	108
16	50	60	0.5	**	**	**	**
17	50	60	0.7	378	383	184	177
18	50	60	0.9	132	141	139	140

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** In these cases, the values of the parameters were such that an investigation never was considered desirable.

TABLE XIII

DECISION RULE 6: MARKOVIAN CONTROL

AVERAGE TOTAL COST* OVER 200 12-MONTH PERIODS (Using Simulation)

				$\mu_1 = 0,\ \sigma_1 = \sigma_2 = 20$			
				AVERAGE TOTAL COST		STANDARD DEVIATION	
				Assumed Distribution			
Case	μ_2	C	g	Beta	Normal	Beta	Normal
1	20	10	0.5	196	198	82	78
2	20	10	0.7	132	135	82	85
3	20	10	0.9	57	58	77	77
4	20	30	0.5	**	**	**	**
5	20	30	0.7	186	184	104	98
6	20	30	0.9	79	80	90	94
7	20	60	0.5	**	**	**	**
8	20	60	0.7	**	**	**	**
9	20	60	0.9	108	106	112	109
10	50	10	0.5	380	382	125	123
11	50	10	0.7	228	229	125	112
12	50	10	0.9	76	74	94	89
13	50	30	0.5	493	492	146	142
14	50	30	0.7	298	299	148	148
15	50	30	0.9	101	102	118	117
16	50	60	0.5	**	**	**	**
17	50	60	0.7	397	400	197	194
18	50	60	0.9	136	137	146	144

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** In these cases, the values of the parameters were such that an investigation never was considered desirable.

TABLE XIV

DEICISION RULE 7: PERFECT KNOWLEDGE

AVERAGE TOTAL COST* AND STANDARD DEVIATION
OVER 200 12-MONTH PERIODS
(Using Simulation)

$\mu_1 = 0, \sigma_1 = \sigma_2 = 20$

Case	μ_2	C	g	Cost	Standard Deviation
1	20	10	0.5	180	84
2	20	10	0.7	102	81
3	20	10	0.9	30	70
4	20	30	0.5	304	110
5	20	30	0.7	174	105
6	20	30	0.9	54	80
7	20	60	0.5	489	154
8	20	60	0.7	281	149
9	20	60	0.9	89	103
10	50	10	0.5	365	124
11	50	10	0.7	209	119
12	50	10	0.9	66	87
13	50	30	0.5	489	154
14	50	30	0.7	281	149
15	50	30	0.9	89	103
16	50	60	0.5	674	202
17	50	60	0.7	388	196
18	50	60	0.9	125	129

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

TABLE XV

DECISION RULE 6: MARKOVIAN CONTROL

TOTAL COST* OVER A 12-MONTH PERIOD (Using Numerical Methods)

$\mu_1 = 0,\ \sigma_1 = \sigma_2 = 20$

Case	μ_2	C	g	Assumed Distribution: Beta	Assumed Distribution: Normal
1	20	10	0.5	203	204
2	20	10	0.7	143	145
3	20	10	0.9	64	66
4	20	30	0.5	**	**
5	20	30	0.7	204	205
6	20	30	0.9	93	93
7	20	60	0.5	**	**
8	20	60	0.7	**	**
9	20	60	0.9	127	131
10	50	10	0.5	378	379
11	50	10	0.7	236	237
12	50	10	0.9	84	84
13	50	30	0.5	493	493
14	50	30	0.7	309	309
15	50	30	0.9	109	110
16	50	60	0.5	**	**
17	50	60	0.7	411	414
18	50	60	0.9	145	149

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** In these cases, the values of the parameters were such that an investigation never was considered desirable.

only three cases.[10] For those seven cases the magnitude of difference in decision costs is slightly greater than for the three cases. This analysis seems to show that making a wrong assumption of distribution properties results in slightly higher decision costs than the right assumption.

Tables XIII and XV present the decision costs of the Markovian control model. The Markovian control determines an optimal cutoff value by minimizing a cost function (equation 4 of Section 2.1). The results of Table XIII are the results of simulation and the results of Table XV were determined using numerical methods. It is important to consider the nature of a Markovian proces in evaluating the results of these two methodologies. A Markovian process goes through a transition phase before the steady-state phase is attained. The simulation results are for a twelve month period. This twelve month period is the beginning of the transition phase. The numerical methodology considers long-term costs, those costs which will be incurred after the process reaches the steady-state phase. An advantage of the numerical methods approach is the elimination of sampling error associated with simulation. Thus, the numerical methodology approach produces results which are mathematically `exact.'

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[10] The results of the cases in which the normality assumption resulted in lower decision costs than the beta assumption may be exaplained by sampling error which is a disadvantage of simulation.

In comparing the decision costs of the two different assumptions, the following results are observed. Using simulation, Table XIII shows the two distributions do not have the same decision costs. Out of a total of eighteen cases, ten cases show the beta distribution assumption resulting in slightly lower decision costs than the normal distribution assumption, whereas only four cases show otherwise. The parameters of the remaining four cases were such that an investigation never was considered desirable.

Using numerical methods, Table XV shows that the two distributional assumptions do have equal decision costs for four cases. The beta distribution assumption again resulted in lower decision costs than the normal distribution assumption for nine cases. None of the cases under the assumption of normality has decision costs lower than that of the beta distribution.[11]

The results presented in Tables XII, XIII and XV indicate that an incorrect assumption of normality does affect decision costs when the single-period Bayesian and Markovian control models are used. In addition, the effect of the incorrect assumption of normality does not seem to be dependent upon the mean of the out of control distribution, μ_2. Differences between decision costs are observed when μ_2 = 20 (cases one through nine) and when

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[11] This result supports the aforementioned proposition that the inco distribution assumption of normality resulted in lower decision costs than correct distribution assumption due to sampling error associated with simu

$\mu_2 = 50$ (cases ten through eighteen). The above analysis is only applicable when using the measures of 20 for the standard deviation, -0.90 for skewness, and 4.00 for kurtosis. However, these three parameters of a probability distribution may have various measures (see Pearson [1895] for a more detailed discussion).

In order to appropriately model a cost variance process, these three parameters must be estimated. However, managers may not have accumulated the necessary data necessary to make an accurate estimate of the aforementioned parameters. Also, to accumulate the data necessary for these estimates is a costly undertaking. Thus, knowing which of the three parameters should be accurately estimated and which of the parameters one needs not be concerned with is useful information for managers. Thus, the following two sections examine the sensitivity of decision costs to incorrect assumptions of the standard deviation and skewness.[12]

7.3 Incorrect Assumption of the Standard Deviation

This section investigates the effect of an incorrect assumption of the standard deviation and the results are presented in Tables XVI and XVII. The only difference in

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[12] Two of the three parameters are examined in this study. Future research will examine the sensitivity of decision costs to an incorrect assumption of kurtosis.

the parameters of these two tables is the mean of the out of control cost variance distribution, μ_2. Table XVI uses a value of 20 for μ_2 and Table XVII uses 50 for μ_2.

Magee's [1976] results indicated the dynamic programming model of Kaplan [1969] was the optimum model in terms of minimizing average total costs (sum of investigation costs and operating costs). Dittman and Prakash [1979] compared the cost savings of the dynamic programming model with the cost savings of the Markovian control model. The results of Dittman and Prakash indicate the difference in cost savings of the dynamic programming approach and the Markovian control model are very small. If the labor costs of programming and implementing the two models were to be included in the analysis, undoubtedly the Markovian control model would prove to be more cost efficient than the dynamic programming approachis. Thus, the Markovian control model is used for the sensitivity analysis in this section and the Section 7.4.

Three standard deviations were used in the analysis. The standard deviations of the cost variance distributions were <u>assumed</u> to be either 10, 20, or 30. Also, the <u>actual</u> standard deviations were either 10, 20, or 30. Tables XVI and XVII indicate an incorrect assumption regarding the standard deviation of the cost variance distributions affects the decision costs. As would be expected, the minimum decision costs are incurred when the <u>assumed</u> and <u>actual</u> standard deviations are the same. Thus, for each scenario presented in Tables XVI and XVII the diagonal represents the minimum decision costs.

Decision costs not on the diagonal represent costs incurred when an incorrect assumption regarding the standard deviation is made. The off-diagonal decision costs are always equal to or more than the costs on the diagonal. Thus, it is costly to make an incorrect assumption regarding the standard deviation.

The various scenarios presented in Tables XVI and XVII suggest the standard deviation becomes more important as the investigation cost (C) and the transition probability (g)[13] increase. Referring to Table XVI, when investigation costs (C) are $10 and the transition probability is 0.5, and the actual standard deviation is 20, an incorrect assumption does not affect total costs. Total cost whether the standard deviation is assumed to be 10, 20, or 30. However, when investigation costs are $60 and the transition probability is 0.9, the assumption regarding the standard deviation becomes very important. If the actual standard deviation is 20 and the correct assumption is made, total cost is $152. But if an incorrect assumption of 10 is made, total cost is $192. Tables XVI and XVII indicate the assumption regarding the standard deviation

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[13] The transition probability (g) is the probability that the process remains in-control at the end of the period given that it entered the period in-control. Managers could estimate g using historical data.

TABLE XVI

DECISION RULE 6: MARKOVIAN CONTROL

TOTAL COST* OVER A 12-MONTH PERIOD INCORRECT ASSUMPTION OF σ (Using Numerical Methods)

$\mu_1 = 0$, $\mu_2 = 20$

C = 10

Actual σ	g = 0.5 Assumed 10	20	30	Actual σ	g = 0.7 Assumed 10	20	30	Actual σ	g = 0.9 Assumed 10	20	30
10	193	194	196	10	127	127	127	10	52	56	68
20	205	205	205	20	146	146	146	20	72	70	71
30	210	210	210	30	154	154	154	30	81	79	79

C = 30

Actual σ	g = 0.5 Assumed 10	20	30	Actual σ	g = 0.7 Assumed 10	20	30	Actual σ	g = 0.9 Assumed 10	20	30
10	**	**	**	10	198	219	238	10	83	117	200
20	**	**	**	20	227	214	222	20	126	111	125
30	**	**	**	30	245	226	220	30	154	131	124

C = 60

Actual σ	g = 0.5 Assumed 10	20	30	Actual σ	g = 0.7 Assumed 10	20	30	Actual σ	g = 0.9 Assumed 10	20	30
10	**	**	**	10	**	**	**	10	121	183	238
20	**	**	**	20	**	**	**	20	192	152	179
30	**	**	**	30	**	**	**	30	248	185	166

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** In these cases, the values of the parameters were such that an investigation never was considered desirable.

TABLE XVII

DECISION RULE 6: MARKOVIAN CONTROL

TOTAL COST* OVER A 12-MONTH PERIOD
INCORRECT ASSUMPTION OF σ
(Using Numerical Methods)

$\mu_1 = 0, \mu_2 = 50$

C = 10

Actual σ	g = 0.5 Assumed 10	20	30	Actual σ	g = 0.7 Assumed 10	20	30	Actual σ	g = 0.9 Assumed 10	20	30
10	361	362	368	10	217	218	220	10	73	73	73
20	379	377	380	20	237	237	239	20	89	89	89
30	397	394	393	30	259	257	256	30	106	106	106

C = 30

Actual σ	g = 0.5 Assumed 10	20	30	Actual σ	g = 0.7 Assumed 10	20	30	Actual σ	g = 0.9 Assumed 10	20	30
10	481	482	486	10	290	291	294	10	98	100	110
20	503	501	503	20	322	321	322	20	128	123	127
30	524	521	520	30	355	353	352	30	165	154	151

C = 60

Actual σ	g = 0.5 Assumed 10	20	30	Actual σ	g = 0.7 Assumed 10	20	30	Actual σ	g = 0.9 Assumed 10	20	30
10	**	**	**	10	398	404	430	10	134	142	176
20	**	**	**	20	441	430	440	20	182	166	179
30	**	**	**	30	490	468	461	30	246	213	203

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** In these cases, the values of the parameters were such that an investigation never was considered desirable.

becomes more important as the investigation cost (C) and the transition probability (g) increase.

This result may have implications for managers. If the transition probability is small (0.5) and the investigation cost is small, it may not be cost effective estimate. However, if the transition probability is large (0.7 or 0.9) and the investigation cost is large, it may be cost effective to collect enough data to accurately estimate the standard deviations of the cost variance distributions.

7.4 Incorrect Assumption of Skewness

This section investigates the effects of an incorrect assumption of skewness and the results are presented in Tables XVIII and XIX. Three measures of skewness were used to investigate the sensitivity of decision costs to an incorrect assumption. The measures of skewness used were 0.3, 0.6, and 0.9. The decision costs of the "Correct" columns of Tables XVIII and XIX were determined based on the correct assumption that the cost variances were beta distributed. As for the "Naive" columns, the decision costs were determined based on the incorrect assumption that the cost variances were normally distributed.

The results presented in Tables XVIII and XIX indicate an incorrect assumption of skewness may affect decision costs. The results suggest the more skewed the cost variance distributions, the more costly it would be when an

TABLE XVIII

DECISION RULE 6: MARKOVIAN CONTROL

TOTAL COST* OVER A 12-MONTH PERIOD
INCORRECT ASSUMPTION OF $\sqrt{\beta_1}$
(Using Numerical Methods)

$\mu_1 = 0$, $\mu_2 = 20$

$\sigma_1 = \sigma_2 = 20$

C = 10

g = 0.5			g = 0.7			g = 0.9		
$\sqrt{\beta_1}$	Correct	Naive	$\sqrt{\beta_1}$	Correct	Naive	$\sqrt{\beta_1}$	Correct	Naive
0.3	206	206	0.3	146	147	0.3	71	72
0.6	206	206	0.6	147	148	0.6	73	74
0.9	207	207	0.9	147	152	0.9	77	78

C = 30

g = 0.5			g = 0.7			g = 0.9		
$\sqrt{\beta_1}$	Correct	Naive	$\sqrt{\beta_1}$	Correct	Naive	$\sqrt{\beta_1}$	Correct	Naive
0.3	**	**	0.3	216	216	0.3	114	114
0.6	**	**	0.6	218	218	0.6	118	118
0.9	**	**	0.9	220	221	0.9	123	124.

C = 60

g = 0.5			g = 0.7			g = 0.9		
$\sqrt{\beta_1}$	Correct	Naive	$\sqrt{\beta_1}$	Correct	Naive	$\sqrt{\beta_1}$	Correct	Naive
0.3	**	**	0.3	**	**	0.3	156	156
0.6	**	**	0.6	**	**	0.6	160	161
0.9	**	**	0.9	**	**	0.9	164	168

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** In these cases, the values of the parameters were such that an investigation never was considered desirable.

TABLE XIX

DECISION RULE 6: MARKOVIAN CONTROL

TOTAL COST* OVER A 12-MONTH PERIOD
INCORRECT ASSUMPTION OF $\sqrt{\beta_1}$
(Using Numerical Methods)

$\mu_1 = 0,\ \mu_2 = 50$

$\sigma_1 = \sigma_2 = 20$

C = 10

$\sqrt{\beta_1}$	g = 0.5 Correct	g = 0.5 Naive	$\sqrt{\beta_1}$	g = 0.7 Correct	g = 0.7 Naive	$\sqrt{\beta_1}$	g = 0.9 Correct	g = 0.9 Naive
0.3	376	376	0.3	236	236	0.3	89	90
0.6	373	374	0.6	234	234	0.6	90	91
0.9	367	371	0.9	226	230	0.9	85	90

C = 30

$\sqrt{\beta_1}$	g = 0.5 Correct	g = 0.5 Naive	$\sqrt{\beta_1}$	g = 0.7 Correct	g = 0.7 Naive	$\sqrt{\beta_1}$	g = 0.9 Correct	g = 0.9 Naive
0.3	503	503	0.3	323	323	0.3	126	126
0.6	503	505	0.6	325	326	0.6	131	131
0.9	502	509	0.9	319	330	0.9	136	140

C = 60

$\sqrt{\beta_1}$	g = 0.5 Correct	g = 0.5 Naive	$\sqrt{\beta_1}$	g = 0.7 Correct	g = 0.7 Naive	$\sqrt{\beta_1}$	g = 0.9 Correct	g = 0.9 Naive
0.3	**	**	0.3	435	435	0.3	172	172
0.6	**	**	0.6	441	441	0.6	180	180
0.9	**	**	0.9	452	452	0.9	193	193

* Total cost includes cost variances plus costs of investigations. Figures are rounded to nearest dollar.

** In these cases, the values of the parameters were such that an investigation never was considered desirable.

incorrect assumption of normality is used. This conclusion is supported by observing that the magnitude of the differences between the "Correct" and "Naive" columns increases as the skewness of the distributions increases. For example, in examining the results of Table XIX, with a transition probability (g) is 0.5, we find the assumption of $\sqrt{\beta_1}$ does not affect decision costs when $\sqrt{\beta_1}$ is 0.3. However, when $\sqrt{\beta_1}$ is 0.9, the "Naive" assumption results in decision costs \$4 (\$371 - \$367) more than the "Correct" assumption when investigation costs are \$10. When investigation costs are \$30 and $\sqrt{\beta_1}$ is 0.9, the "Naive" assumption results in decision costs \$7 (\$509 - \$502) more than the "Correct" assumption. This aforementioned result does not appear to depend upon the the transition probability (g) as the differences between the columns usually increase with skewness, regardless of the transition probability. However, this result is only observed when investigation costs are either \$10 or \$30. The assumption of $\sqrt{\beta_1}$ does not usually affect decision costs when investigation costs are \$60. The results of this analysis indicate as the actual cost variance distribution becomes more skewed, the more serious the consequences of making an incorrect assumption normality.

7.5 Summary

Simulation and numerical methods were used to investigate the effects of distributional properties on decision costs of various CVID models. First, eight CVID models were used to investigate the sensitivity of decision costs to an incorrect assumption regarding distribution properties. Each of the eight CVID rules was used to determine decision costs. The following measures of the second, third, and fourth moments of the cost variance distributions were used in the analysis. The standard deviation was 20, skewness was -0.90, and kurtosis was 4.00.

First, an incorrect assumption of normality was used to estimating decision costs. Second, a correct assumption of nonnormality was used to estimate decision costs. The results indicate in some cases more decision costs are incurred when an incorrect assumption regarding distribution properties is made.

The Markovian control model was used to investigate the sensitivity of decision costs to an incorrect assumption regarding the standard deviation and skewness. The results indicate decision costs are sensitive to the standard deviation and skewness. The assumption regarding the standard deviation became important when investigation costs (C) and the transition probability (g) were large. The assumption regarding skewness became more important as the skewness of the actual distribution increased. Thus, when using a CVID model to aid in the decision making process, it may be cost effective for managers to collect enough data to accurately estimate the standard deviations and skewness of the cost variance distributions.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

8.1 Introduction

The CVID has been one of the most widely researched topics in managerial accounting. The rules examined for aiding managers in making the cost variance investigation decision range from a simple rule of investigating all unfavorable cost variances to the Markovian control approach. Most of the models require knowledge of the distribution properties of the cost variances. The models have assumed the cost variance distributions are normally distributed. The literature indicates this may not be a realistic assumption. Thus, the results and implications of this thesis described in the following sections may be of interest to managers who use a CVID rule in making the cost variance investigation.

8.2 Summary of Results

This thesis set out to fulfill three objectives:

(1) To examine the distributional properties of actual cost variances collected from industry.

(2) To develop a practical approach for modeling nonnormal cost-variance distributions.

(3) To investigate how optimal decisions under various CVID models are affected by the nonnormality of cost variances.

Actual cost variances of a medium size manufacturing plant of a Fortune 500 company were collected from its fourteen production departments. The data consisted of weekly direct labor efficiency cost variance amounts and direct labor efficiency variance percentages. Two tests of normality, the Shapiro-Wilk W test [Shapiro and Wilk, 1965] and the K_s^2 test [Bowman and Shenton, 1986], indicated that for some departments the actual cost variance amounts and direct labor efficiency variance percentages are not normally distributed.

The second objective was to develop a practical approach for modeling nonnormal actual cost variance distributions. To model nonnormal cost variance distributions, a family of four-parameter distribution functions has to be used. Ideally, a distribution function should be chosen by the following three-step iterative process:

(1) Identify a family of distribution functions which appear appropriate.

(2) Determine the parameters of the distribution function that best fits the empirical distribution on hand.

(3) Decide whether an adequate fit has been provided by the chosen family of distribution functions.

These three steps were illustrated with an example in Section 4.4. A histogram of cost variance data suggested the Pearson distribution family could be used and the beta distribution was selected as an initial choice. The parameters which best fit the distribution were determined using equations sixteen through nineteen in Section 4.4. A chi-square test indicated the beta distribution adequately fit the empirical data.

Simulation and numerical methods were used to investigate the effects of distributional properties on decision costs of various CVID models. First, seven CVID models were used to investigate the sensitivity of decision costs to an incorrect assumption regarding distribution properties. Each of the seven CVID rules was used to determine decision costs.

Given the actual cost variance distribution is a beta distribution, two sets of decision costs were estimated. First, an incorrect assumption of normality was used to estimate decision costs. Second, a correct assumption of nonnormality was used to estimate decision costs. The results (Tables VI and VII) support two conclusions. The first conclusion is the more sophisticated CVID models resulted in lower decision costs than the less sophisticated models. This result appears reasonable because the more sophisticated CVID models consider the costs and benefits of an investigation. The second conclusion is, for the more sophisticated CVID models, more decision costs are incurred when an incorrect assumption regarding distribution properties is made than when the correct assumption is made. For example, in examining the results of Table VI, when the cost variances were correctly assumed to be nonnormally distributed, the average total cost was $397. However, when the cost variances were incorrectly assumed to be normally distributed, the average total cost was $400. This same relationship of costs also occurred for the single-period Bayesian model. When the cost variances were correctly assumed to be nonnormally distributed, the average total cost was $378. When the cost variances were incorrectly assumed to be normally distributed, the average total cost was $383. These results illustrate an incorrect assumption of distribution properties results in greater decision costs than the correct assumption.

The implementation of a CVID model requires that managers estimate various parameters. Such parameters include the mean of the out-of-control cost variance distribution, investigation costs, and the transition probability. This estimation process is a costly activity (in terms of time and effort for acquiring the necessary information) for managers to undertake. Thus, in order to investigate which of the required parameters should be reasonably estimated and which parameters are not critical for the successful implementation of a CVID model, two CVID models were used to investigate the sensitivity of decision costs to different means (μ_2) of the out-of-control cost variance distribution, investigation costs (C), and transition probabilities (g). The results for the single-period Bayesian model and the Markovian control model are presented in Tables XII, XIII, and XV. The effect of each of these variables on decision costs is discussed next.

The results presented indicate that decision costs increase as the mean of the out-of-control cost variance distribution (μ_2) increases. For example, in examining the decision costs incurred by the Markovian control model using simulation (Table XIII), case 1 assuming the beta distribution resulted in average total cost of $196. This cost was for μ_2 equal to $20. With μ_2 equal to $50 (case 10), average total cost is $380. These results seem reasonable. One would expect decision costs to increase if the mean of the out of control cost variance distribution increases.

The effect of the investigation cost (C) is discussed next. When the Markovian control model is used (Table XIII), an increase in investigation cost always results in an increase in average total cost. For example, when the beta distribution is assumed case 11 (with C equal to $10) resulted in average total cost of $228. With a change of C to $30 (case 14), average total cost increases to $298. Likewise, with investigation cost equal to $60 (case 17), average total cost increases to $397. The aforementioned relationship holds for all of the cases when the Markovian control model is used.

When the single-period Bayesian model (Table XII) is used, changes in investigation costs result in both average total cost increases and decreases. For example, when the beta distribution is assumed, case 12 (with C equal to $10) resulted in average total cost of $150. With an increase of C to $30 (case 15), average total cost cost increases to $170. However, with an increase of C to $60 (case 18), average total cost decreases to $132. Thus, a change in investigation cost results in mixed effects on average total cost when a single-period Bayesian model is used.

The results presented in these tables indicate average total cost decreases as the transition probability increases. This result seems reasonable since a larger transition probability means there is a greater probability the process will be in control at the end of the period. Thus, with a larger transition probability, more cost variance observations are from the in control distribution than with a smaller transition probability.

The Markovian control model was used to investigate the sensitivity of decision costs to an incorrect assumption regarding the standard deviation and skewness. The results indicate decision costs are sensitive to the standard deviation and skewness. An example from Table XVI is used to illustrate the sensitivity of decision costs to an incorrect assumption of the standard deviation. For Table XVI $\mu_1 = 0$ and $\mu_2 = 20$. For C = 60 and g = 0.9, total costs were $121 when the actual standard deviation and the assumed standard deviation were both 10. However, when the actual standard deviation was 10 but was assumed to be 30, total costs were $238. This incorrect assumption of the standard deviation resulted in total costs being almost twice the amount of total costs incurred when the the correct assumption was made.

An example from Table XIX is used to illustrate the sensitivity of decision costs to an incorrect assumption of skewness. For Table XVI $\mu_1 = 0$ and $\mu_2 = 50$. For C = 30 and g = 0.7, total costs were $319 when the actual skewness and the assumed skewness were both 0.9. However, when the actual skewness was 0.9 but was assumed to be 0, total costs were $330. The two above examples indicate an incorrect assumption of the standard deviation aand skewness may result in greater total costs than the correct assumption. Thus, when using a Markovian control model to aid in the decision making process, it may be cost effective for managers to collect enough data to accurately estimate the standard deviations and skewness of the cost variance distributions.

From the results just discussed, it appears that the more sophisticated CVID models incurred lower decision costs and should be used. However, when using one of the sophisticated CVID models, managers should not ignore distributional properties. The results suggest it is beneficial for managers to compile enough cost data in order to accurately estimate the standard deviations and skewness of the cost variance distributions.

8.3 Implications and Suggestions

This study provides some evidence regarding the distribution properties of direct labor cost efficiency variances for a manufacturing firm. The results indicate one should not make unrealistic assumptions regarding distribution properties when using a CVID model. Whether the results of this study apply to other types of cost variances and other types of industries is an empirical question which can only be answered by further research. While there is a preponderance of recent literature [Howell and Soucy, 1988] which indicates labor costs as a percentage of total manufacturing costs have been decreasing, labor costs are still the <u>major</u> cost for many organizations. For example, labor is the most costly input in education, health care, and many other service type organizations such as law and acccounting [Dietemann, 1988]. Thus, for these industries, labor is a scarce resource which must be used in the most economically efficient manner. One tool which can be used to determine whether labor is being used in the most efficient manner are cost variances. The evidence presented in this study indicates that, when a cost variance investigation decision model is used as an aid to determine whether a cost variance is from the in or out-of-control distribution, distribution properties should not be ignored.

In conclusion, even though direct labor costs may be decreasing as a percentage of total manufacturing costs, service industries are becoming increasingly important to our economy. Labor is the most important cost to such industries. One performance evaluation measure within these industries is how well the managers control labor costs. Managers may use CVID rules as an aid in this control process. The results of this study suggest managers should not ignore distribution properties when such rules are used.

Criticisms of some of the more sophisticated CVID models include lack of knowledge regarding required information ([Magee, 1976] and [Boer, 1984]). In other words, how can a manager assess a distribution's mean and standard deviation to successfully implement a CVID model? The professional literature suggests the recent advances in computer technology has placed this data at the fingertips of accountants. Walker and Surdick [1988, p. 25] recently stated, "PC software packages priced under $1,000 place graphics portrayal of variances and comparative data at the fingertips of controllers." With such information readily available, data required to implement a CVID model as complex as the Markovian control model can easily be estimated.

The recent advances in computer technology also allow the cost variance investigation decision to be made more frequently than weekly. With the profilieration of computers, managers have the ability to monitor cost variances on a daily, or even hourly basis. An advantage of more frequent monitoring is that the process could be corrected more quickly.

When the Markovian control model is used for the CVID, there are many variables which must be estimated. As discussed in this study, the four moments of the cost variance distributions must be estimated. This study investigated the sensitivity of decision costs to an incorrect assumption of the second and third moments, the standard deviation and skewness, respectively. Future research could investigate the sensitivity of decision costs to an incorrect assumption of kurtosis. There are three other variables which must also be estimated for the Markovian control model. These variables are the investigation cost, the transition probability, and the mean of the out-of-control cost variance distribution. Future research could investigate the sensitivity of decision costs to an incorrect assumption regarding these three variables.

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