dc-field-induced enhancement and inhibition of spontaneous emission in a cavity

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We demonstrate how spontaneous emission in a cavity can be controlled by the application of a dc field. The method is especially suitable for Rydberg atoms. We present a simple argument based on Stark shifts for the control of emission.

The manipulation of spontaneous emission has been extensively studied. Methods involving either external fields [1–8] or appropriate cavities [9,10] have been suggested. The use of external fields enables one to control spontaneous emission via quantum interference effects [5,6]. Purcell [11] recognized how the emission rate in a single mode cavity can be much higher than in free space. Several demonstrations of the cavity-enhanced spontaneous rates exist in the literature [9,10]. Kleppner [12] discovered that the radiation rate in a cavity can be inhibited by choosing the transition frequency, such that the density of states at this frequency is insignificant. Quantum interference between various channels [3,4,6,7] could also result in the inhibition of emission. Further, very interesting, experiments [1,2,6–8] on the field-induced inhibition of emission in a cavity were reported. In these experiments the applied fields were resonant with atomic transitions. In this paper we show how a possible control of spontaneous emission can be obtained by using dc fields. We treat the case of atoms in a cavity and explain in rather simple terms the origin of the control produced by the dc fields. To be precise we are considering only the effect of dc field on the part of decay, which is due to the emission in the cavity mode. The decay of the atom depends on the detuning between the atomic frequency and the cavity frequency. The application of the dc field makes the detuning dependent on the field (Stark effect) and thus the dc field provides a control of the spontaneous emission. The dc-field-induced modification of spontaneous emission in free space is treated in Ref. [3].

We next describe how to calculate the dc-field-induced modification of the decay characteristics in a cavity. For our purpose we consider a two-level atom placed in a cavity, and a dc field (or low-frequency field) is injected inside the cavity. The Hamiltonian of the system can be written as

$$H = \hbar\omega_0 S^z + \hbar\omega_c a^\dagger a + \hbar g(aS^+ + S^- a^\dagger) + \hbar\mathcal{E}\cos\Omega t(S^+ + S^-), \quad (1)$$

where ω_0 is the atomic transition frequency, ω_c is the cavity mode frequency, and g is the atom-cavity coupling constant. The term $\mathcal{E}\cos\Omega t$ corresponds to a low-frequency field, if Ω is chosen to be very small. Note that $\mathcal{E}$ has dimensions of frequency. The cavity field has been expressed in terms of annihilation and creation operators a, $a^\dagger$ and S^+, S^-, S^z are usual atomic spin operators. We perform master equation calculations for the atom-cavity system. The density matrix of the system ρ will evolve as

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] - \kappa(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a), \quad (2)$$

where 2κ gives the leakage of photons. It is related to the cavity Q via $\kappa = \omega_c/2Q$. We will work in a frame rotating with atomic frequency ω_0. The density matrix in this frame is given by

$$\tilde{\rho} = e^{i\omega_0(S^z + a^\dagger a)t/\hbar}\rho e^{-i\omega_0(S^z + a^\dagger a)t/\hbar}. \quad (3)$$

Using Eqs. (2) and (3) we obtain the equation for $\tilde{\rho}$,

$$\dot{\tilde{\rho}} = -\frac{i}{\hbar}[H_a,\tilde{\rho}] - \kappa(a^\dagger a\tilde{\rho} - 2a\tilde{\rho}a^\dagger + \tilde{\rho}a^\dagger a) - \frac{i}{\hbar}[H_d,\tilde{\rho}], \quad (4)$$

where

$$H_a = -\hbar\Delta a^\dagger a + \hbar g(aS^+ + S^- a^\dagger),$$

$$H_d = \hbar\frac{\mathcal{E}}{2}\{S^+(e^{i(\omega_0+\Omega)t} + e^{i(\omega_0-\Omega)t}) + S^-(e^{-i(\omega_0+\Omega)t} + e^{-i(\omega_0-\Omega)t})\},$$

$$(5)$$

and $\Delta = \omega_0 - \omega_c$ is the detuning. We first note that the experiments of Lange and Walther correspond to using a microwave field, and thus $\Omega \sim \omega_0$. The results of Purcell and Kleppner also follow from the master equation (4). For $\mathcal{E}=0$ and $g \ll \kappa$, we can derive an equation for the atomic density matrix $\tilde{\rho}_a$,

$$\tilde{\rho}_a = \text{Tr}_c\tilde{\rho}, \quad (6)$$

where Tr_c is trace over cavity field, by adiabatically eliminating cavity variables. This leads to

$$\dot{\tilde{\rho}}_a = -i[\delta_0 S^z, \tilde{\rho}_a] - \Gamma_0(S^+ S^- \tilde{\rho}_a - 2S^- \tilde{\rho}_a S^+ + \tilde{\rho}_a S^+ S^-), \quad (7)$$

where

$$\Gamma_0 = \frac{g^2\kappa}{\kappa^2 + \Delta^2}, \quad \delta_0 = \frac{g^2\Delta}{\kappa^2 + \Delta^2}. \quad (8)$$

For resonant cavity $\omega_c = \omega_0$, $\delta_0 = 0$ and the decay rate $\Gamma_0 = g^2/\kappa$. There is cavity-induced enhancement if g^2/κ is greater than the free space decay rate. Note that as the cavity is detuned ($\Delta \neq 0$) Γ_0 decreases, which is Kleppner's result for a single mode cavity. Experimental observation of the Purcell effect was made by Goy *et al.* [9]. Next, we investigate the effect of the applied dc or low-frequency field. Note that the last term in the master equation (4) is highly oscil-
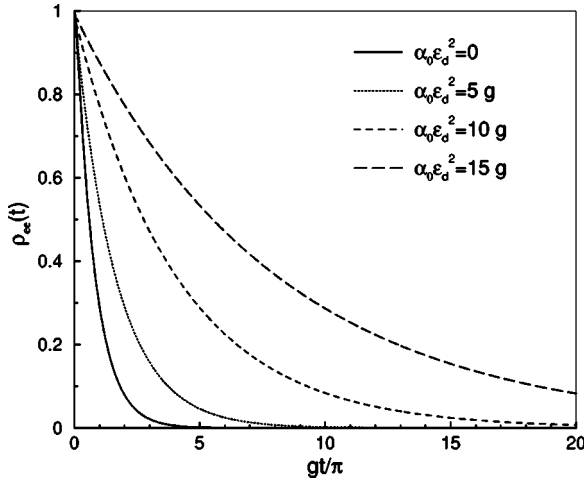
FIG. 1. The probability of the atom remaining in its excited state, $\rho_{ee} \equiv \langle e,0|\rho|e,0 \rangle$ vs time, for $\kappa = 5g$, $\Delta = 0$, $\Omega = 0$, and for the different values of the dc field $\mathcal{E}_d$.

lating. We do time averaging for this, as such terms oscillating at the cavity frequency would not be normally observed. The time averaging is well justified here, as all other relevant time scales g^{-1}, κ^{-1}, Δ^{-1} are much larger than $(\omega_0 \pm \Omega)^{-1}$. The inequality $\omega_0 \gg g$, κ, Δ enables us to do the time averaging in a much simpler fashion, i.e., we can essentially ignore the terms having H_a and κ in (4). We relegate the details of time averaging to the Appendix. The calculation leads to the following time-averaged master equation:

$$\dot{\widetilde{\rho}} = i[\Delta_e a^\dagger a, \widetilde{\rho}] - ig[(aS^+ + S^- a^\dagger), \widetilde{\rho}] - \kappa(a^\dagger a \widetilde{\rho} - 2a\widetilde{\rho}a^\dagger + \widetilde{\rho}a^\dagger a), \tag{9}$$

where

$$\Delta_e = \Delta + 2\omega_0 \mathcal{E}^2/(\omega_0^2 - \Omega^2). \tag{10}$$

We note that the dc field contributes to the Stark shift of the two levels in question. We further note that these two atomic levels can also be shifted because of the interaction of the dc field with other levels. These can be accounted for by introducing the polarizabilities α_e and α_g of the levels $|e\rangle$ and $|g\rangle$ [13,14]. We can rewrite Eq. (10) as

$$\Delta_e = \Delta + \alpha_0 \mathcal{E}_d^2, \ \alpha_0 = \alpha_e - \alpha_g, \tag{11}$$

where $\mathcal{E}_d$ is now the dc field in esu. The formulation of the Appendix can also be used to produce the well-known expressions for the $\alpha's$. The value of α_0 is known for many low-lying, as well as Rydberg, transitions. The values of α_0 have been calculated in the literature by converting infinite sums into the solution of differential equations.

Equation (9) can be solved, assuming that the atom is initially excited and the cavity field is in vacuum state. Equation (9) can be converted into a set of coupled equations in terms of the states $|e,0\rangle$, $|g,1\rangle$, and $|g,0\rangle$. The results of the numerical integration are shown in Fig. 1 for different values of the parameter Δ_e. Clearly there is inhibition as Δ_e increases. The effective detuning Δ_e changes due to the applied dc field. For a fixed cavity detuning Δ the dc field can make Δ_e larger or smaller depending on the sign of Δ. The results

can be understood by deriving analytical results in the bad cavity limit [15] $g \ll \kappa$ (and more precisely $g^2 \ll \kappa^2 + \Delta_e^2$). In this limit we can obtain a simpler equation for the atomic density matrix $\widetilde{\rho}_a$, defined by Eq. (6). The final result for the atomic system is

$$\dot{\widetilde{\rho}}_a = -i[\delta_e S^z, \widetilde{\rho}_a] - \Gamma_e(S^+ S^- \widetilde{\rho}_a - 2S^- \widetilde{\rho}_a S^+ + \widetilde{\rho}_a S^+ S^-), \tag{12}$$

where

$$\Gamma_e = \frac{g^2 \kappa}{\kappa^2 + \Delta_e^2}, \ \delta_e = \frac{g^2 \Delta_e}{\kappa^2 + \Delta_e^2}. \tag{13}$$

Here, Γ_e is the dc-field-modified decay parameter and δ_e is the net frequency shift. The ratio η of the decays in the presence and absence of dc field is given by

$$\eta = \frac{\Gamma_e}{\Gamma_0} = \frac{\kappa^2 + \Delta^2}{\kappa^2 + \Delta_e^2}. \tag{14}$$

Clearly the dc field modifies the decay rate, which depends on the detuning. For the cavity resonant to the atomic transition ($\Delta = 0$), using Eq. (10), η reduces to

$$\eta = \frac{\kappa^2}{\kappa^2 + \alpha_0^2 \mathcal{E}_d^4} \approx \left(1 + \frac{4\mathcal{E}^4}{\kappa^2 \omega_0^2}\right)^{-1}, \ \text{for } \Omega = 0. \tag{15}$$

It is clear from Eq. (15) that the dc field inhibits the decay rate. Note that the inhibition starts becoming significant for

$$\alpha_0 \mathcal{E}_d^2 \sim \kappa. \tag{16}$$

Let us estimate the condition (16) for the Na Rydberg transition $23S_{1/2} \rightarrow 22P_{3/2}$, whose frequency is 340 GHz. For the sake of argument, we also assume $\alpha_0 \mathcal{E}_d^2 \sim 2\mathcal{E}^2/\omega_0$. This transition has a dipole moment $d \sim 10^{-15}$ esu. The atom is placed in the cavity having one mode resonant to the atomic transition. Let us choose the cavity decay rate $\kappa = 1$ MHz. The condition (16) then leads to a Rabi frequency $\mathcal{E}$ of the order 400 MHz, which in turn, requires a dc field of the order 10^{-2} esu. We note that the required dc field is small enough, so that the perturbative results for the Stark shift hold. We further note that the scalar and tensor polarizabilities are available for some S and P levels of Na [13,14], although the absolute values for both the $23S_{1/2}$ and $22P_{3/2}$ level are not available in Fabre *et al.* [13]. However, the reported polarizabilities for say, the $23P$ level, are of the order of a few MHz/(Volt/cm)^2. Thus the condition (16) is realistic, and our finding that the dc field can be used to control spontaneous emission, can be implemented by the appropriate choice of the Rydberg transitions, [cf. the condition (16)]. We emphasize that we are discussing the inhibition or enhancement of spontaneous emission on a given transition, which is resonant with the cavity. This, for example, is the transition $23S \rightarrow 22P$ in the experiments of Goy *et al.* [9]. The authors of Ref. [9] emphasize this, as well, and it is in the spirit of the suggestion of Purcell [11]. It must be noted that the field-ionization techniques enable one to study transitions selectively [16].

In the case of cavities detuned from the atomic transition, spontaneous decay is smaller and the decay rate is given by
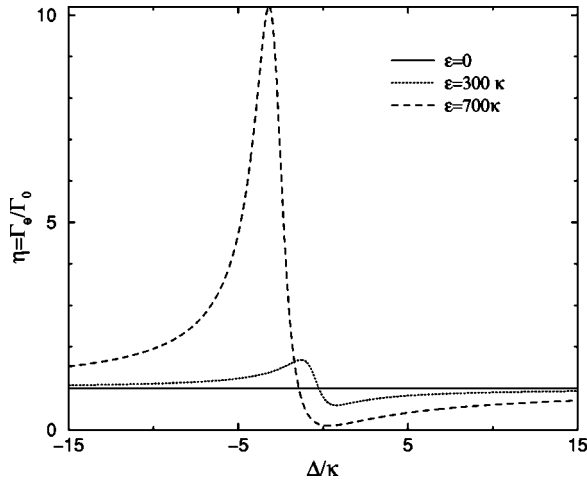
FIG. 2. The ratio (η) of the decays in the presence and the absence of a dc field vs Δ/κ. The parameters are $\omega_0 = 3.4 \times 10^5 \kappa$ and $\Omega = 0$.

$\Gamma = g^2 \kappa / (\kappa^2 + \Delta^2)$. Further inhibition of the decay rate is possible by applying a dc field. When a cavity is tuned below the atomic transition frequency (Δ is positive), then there is a significant inhibition of spontaneous decay, which increases further as the applied dc field is increased. On the other hand, when a cavity is tuned above the atomic frequency (Δ is negative), there is an enhancement in the atomic decay, i.e., on increasing the value of an applied dc field, the atom decays faster.

In Fig. 2 we show the behavior of the factor η as a function of Δ for different values of the dc field. The enhancement, as well as the inhibition of spontaneous decay, occurs, depending on whether the cavity is tuned above or below the atomic frequency. The results shown in Fig. 2 are consistent with the results obtained by direct solution of Eq. (9).

In conclusion, we find that in the presence of a dc field, spontaneous emission can be inhibited significantly in the case of cavities resonant to atomic transition. In the case of cavities having negligible mode density around atomic frequency, spontaneous emission itself is smaller, and the presence of a dc field shows significant inhibition or enhancement depending on whether the cavity is tuned below the atomic transition frequency or above the transition frequency.

APPENDIX

We outline how the time averaging is to be done. Let us consider the Schrödinger equation,

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\frac{i}{\hbar} V(t)|\psi(t)\rangle, \tag{A1}$$

where $V(t)$ consists of rapidly oscillating terms only, so that the time average of $V(t)$ is zero. Let $|\psi\rangle$ be written as

$$|\psi\rangle = |\bar{\psi}\rangle + |\phi\rangle, \tag{A2}$$

where $|\bar{\psi}\rangle$ is time-averaged part and $|\phi\rangle$ is the rapidly oscillating part. On substituting (A2) in (A1) we find that to the lowest order in $V(t)$,

$$|\phi\rangle = -\frac{i}{\hbar} \int_0^t V(\tau) d\tau |\bar{\psi}\rangle, \tag{A3}$$

and

$$\frac{\partial}{\partial t}|\bar{\psi}(t)\rangle = -\frac{i}{\hbar} \bar{V}(t)|\bar{\psi}\rangle, \tag{A4}$$

where

$$\bar{V}(t) = -\frac{i}{\hbar} \overline{V(t) \int_0^t V(\tau) d\tau}. \tag{A5}$$

The field-induced shift term in (9) is obtained by using Eq. (A5).

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