1/f NOISE AS A NONSTATIONARY PROCESS

By

JAWAHAR LAL TANDON

Bachelor of Technology
Indian Institute of Technology
Kanpur, India
1970

Master of Science
Oklahoma State University
Stillwater, Oklahoma
1973

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
May, 1976

1/f NOISE AS A NONSTATIONARY PROCESS

Thesis Approved:

Hans R. Bilger
Thesis Adviser

M. A. Nicar

Bennett Basore

Robert J. Mulholland

Hugh L. Scott

D. D. Durham
Dean of the Graduate College

ACKNOWLEDGEMENTS

I wish to express my sincere thanks and gratitude to Professor Hans R. Bilger, my thesis adviser, for his interest, guidance and encouragement during the entire phase of my graduate study. His dedicated approach to scientific problems with insight and open mind was instrumental in writing this thesis. His family is also acknowledged for their interest and concern.

A special thanks is conveyed to Professor M-A. Nicolet for his advice, guidance and help. The year 1973-1974 spent at California Institute of Technology, Pasadena, Calfornia, was fruitful and rewarding.

The other members of my committee, namely, Dr. B. L. Basore, Dr. R. J. Mulholland and Dr. H. L. Scott are also thanked for their interest and help.

The discussions which I had periodically with Dr. C. S. Sims and Dr. T. J. Boehm are acknowledged.

Mr. P. S. Vijayakumar is thanked for excellent typing, and also his wife Geetha for her help.

Finally, I want to dedicate this thesis to my parents for making it possible.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page

CHAPTER I

INTRODUCTION

1.1 Fluctuations in the Measured Properties of Systems

In an endeavor to understand the nature of physical systems, measurements of certain defined properties are essential. These measurements not only provide empirical data to characterize the systems, but also give insight into the formalism of a set of hypotheses on which generalizations can be made for a better understanding of the system. The advantages of generalizations are multifold; they are employed to redefine the properties of the systems, they are used to predict the behavior of a similar class of systems, they point to the philosophy of future measurements, etc. This is the scientific method.

Any kind of measurement of a defined property of a physical system is uncertain in character. The uncertainty can be brought forth by several factors, e.g., the uncertainty could be inherent in the defined property itself; it could result through the influence of the environment; it could be a function of the equipment used for measurements, the techniques used for analysis, or even the experimenter himself; etc. In this perspective all measurements are probabilistic in nature and, therefore, any hypotheses derived from them should involve representations in terms of averages, e.g., means, variances, correlations, etc.

For our mode of analysis, we define fluctuations in a defined property of a physical system as a random uncertainty in the measurement of the property. In general, these fluctuations could exist in a class of identical systems (ensemble) at an instant of time; and also, as a function of time in a single system (time evolution of a system). The random nature of fluctuations in our definition should be emphasized. We exclude, as far as possible, any kind of biases or trends from our uncertain measurements to define fluctuations. In this sense, any kind of measurement of a defined property of a physical system has fluctuations ultimately.

1.2 Characterization and Identification of Different Kinds of Fluctuations

Over the years, in the study of fluctuations, the term "noise" has been used to represent "fluctuations". We shall use both terms interchangeably. Because of the random nature of noise we have to resort to averages of measured quantities (for convenience!) in order to characterize different kinds of fluctuations. In fluctuation theory, the concept of power spectral density (Papoulis, 1965, Chapter 10), $S_{XX}(f)$, of a fluctuating property $x(t)$ of a system as a function of frequency f, has received considerable attention as an average measured quantity to characterize and identify different kinds of fluctuations. We shall postpone the detailed discussion of $S_{XX}(f)$ till Chapter III where mathematical tools used in the fluctuation theory are discussed. However, for the present analysis, the various kinds of noise can be identified from the features of the measured $S_{XX}(f)$ as a function of f for a particular system. Some of the important kinds of noise are classified as follows:-

(a) White noise: In this case, $S_{XX}(f)$ = constant over the frequency range of interest (e.g., Nyquist noise, Shot noise (Van der Ziel, 1959)).

(b) Generation-Recombination (g-r) noise: In this case, $S_{XX}(f) \propto \frac{1}{1 + (2\pi f\tau)^2}$ typically, where τ is the time constant of a generation-recombination process occurring in the physical system. This kind of noise is of common occurrence in semiconductors where we have the generation and recombination of holes and electrons (Van der Ziel, 1959).

(c) Excess noise: In this case, $S_{XX}(f) \propto 1/f^{\alpha}$, where α is a constant close to 1.

(d) Burst noise: In this case, $S_{XX}(f)$ has a higher value over a certain short frequency range as compared to the value over most of the frequency range.

1.3 Low Frequency Fluctuations (1/f Noise)

Because excess noise has a spectral density $S_{XX}(f) \propto 1/f^{\alpha}$ ($\alpha \simeq 1$), it is generally referred to as 1/f noise. This kind of noise is the dominant form of noise at low frequencies because of its inverse frequency dependence and generally exists in systems which are in thermodynamic nonequilibrium. 1/f noise was first observed by Johnson (1925, 1971) around 1925 in electron tubes. Since then its presence has been found not only in different electronic systems but also in various other physical systems (see Chapter II).

1.4 1/f Noise in Electronic Systems

Over the last fifty years electronic systems have received considerable attention in the instrumentation technology. Most properties of the physical systems are now transduced into electronic signals because of their ease in processing. In other words, electronic systems constitute the heart of the measurement method. Now, since electronic systems also possess fluctuations, the noise in electronic systems imposes a lower limit to their applicability for low level processing of the transduced signals of the properties of physical systems. This is a very strong justification for why noise in electronic systems has been studied extensively over the last half century as compared to direct basic fluctuations of the defined properties of other physical systems. 1/f noise in electronic systems imposes a fundamental lower limit for low-level low-frequency signal processing and, therefore, has shared extensively its importance in the study of electronic noise.

Since the first observance of 1/f noise in electron tubes by Johnson (see Section 1.3), electronic 1/f noise has been found in almost all electronic systems (see Chapter II) in a thermodynamic nonequilibrium situation, and the frequency range of observance has been more than twelve decades (see Chapter II). In particular, to name a few, electron tubes, resistors, metal films, p-n junction diodes, bipolar transistors, Zener diodes, FET's, MOSFET's, superconductors have all been shown to possess 1/f noise. In an effort for the search of the origin of 1/f noise several noise mechanisms have also been proposed by several researchers (see Chapter IV). This has led to a variety of different names to describe the same 1/f noise depending upon its relationship to the other properties of the electronic system under investigation. "Flicker noise", "Contact noise", "Current noise", "Pink noise", "Excess noise", "Surface noise" etc., are the names used to describe the same 1/f noise. In retrospect, considering the modern state of research in 1/f noise, it would not be an exaggeration to make the statement that "All electronic devices/components/systems in thermodynamic nonequilibrium possess 1/f noise at low enough frequencies". It should be mentioned that, from the experimental standpoint, it is probably more difficult to refute this statement than to verify it.

1.5 Why Study 1/f Noise?

Figure 1 shows the electromagnetic spectrum (Heirtzler, 1962). It is drawn open-ended at the two ends to indicate the point that no limits to frequency can be assigned to the electromagnetic phenomena. The low frequency end (i.e. below 1 Hz), often referred to as the micropulsations region, is attaining more and more attention in modern investigations. Some of the phenomena that occur in this region are also listed in Figure 1. It should be mentioned in passing, that since the frequency bounds on the spectrum do not exist, if we choose f = 1 Hz as a reference, there are probably as many decades of useful phenomena occurring above 1 Hz as there are below 1 Hz. Now, since 1/f noise dominates at low frequencies its study proves important because it tends to mask several interesting phenomena that occur at low frequencies. Inherent in this statement is the idea that understanding of the 1/f noise mechanism could reveal methods whereby it could be reduced in

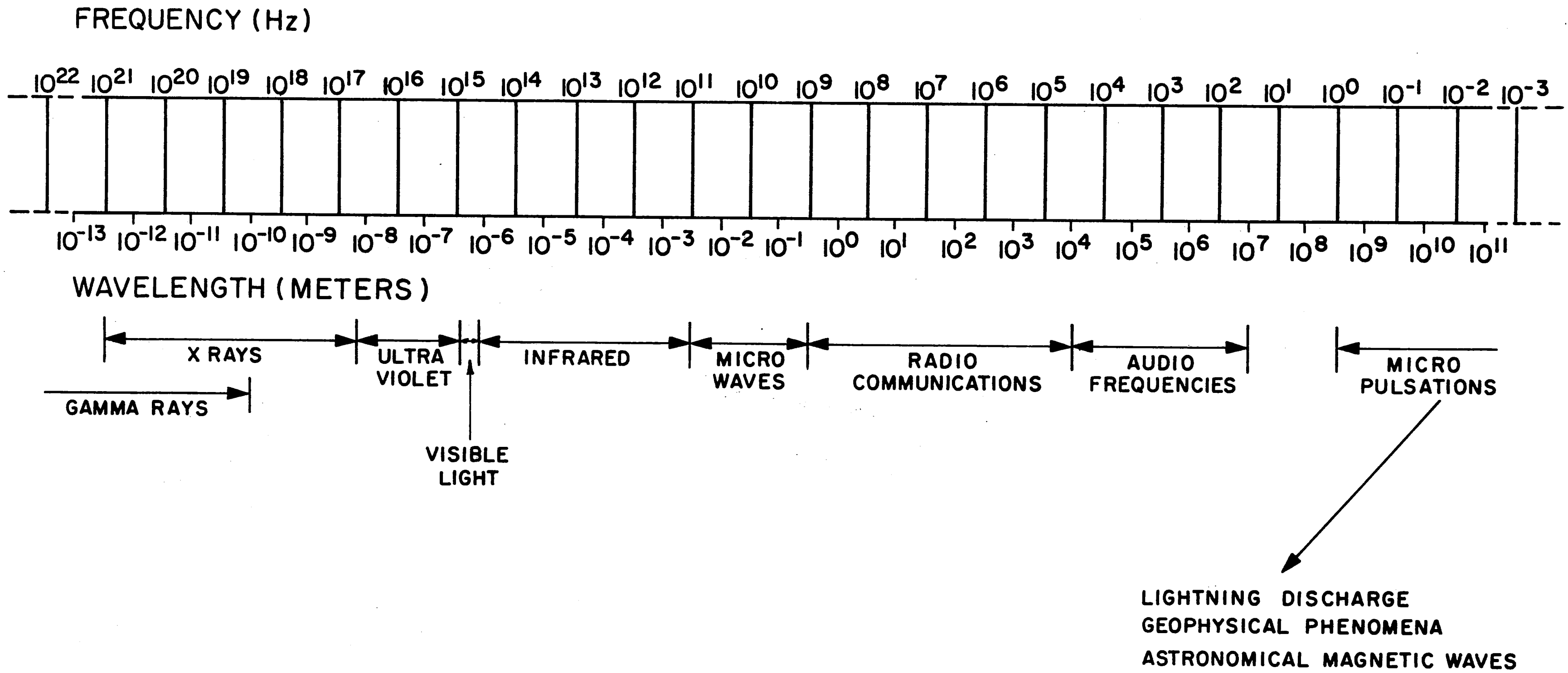


Figure 1. The Electromagnetic Spectrum (Heirtzler, 1962)

magnitude, thus making several interesting low frequency phenomena apparent. Investigation of 1/f noise in physical systems also proves worth while from several other standpoints:

(a) The reduction of 1/f noise in electronic systems helps in developing more sensitive low frequency instrumentation.

(b) The study of other kinds of noise (e.g. thermal noise) becomes more difficult in the presence of 1/f noise because of the dominance of 1/f noise at low frequencies. If, however, the 1/f noise mechanism is understood, its reduction makes the investigation of other kinds of noise easier. This aspect is illustrated by Figure 2, which shows the noise power density spectrum of a silicon single injection diode at high electric fields where the hole mobility is a function of the electric field (Tandon, 1975a). In this case, the interest is in determining the high frequency asymptote of the measured noise power spectral density in order to search "hot carrier" effects. Note that because of the presence of 1/f noise, the determination of the high frequency asymptote within the frequency range of measurement becomes difficult.

(c) The phenomena of so-called "drift", "running-away" of the properties of physical systems involve low frequency (long term) changes. A study of 1/f noise, because of its higher magnitude at low frequencies, could provide insight into such phenomena.

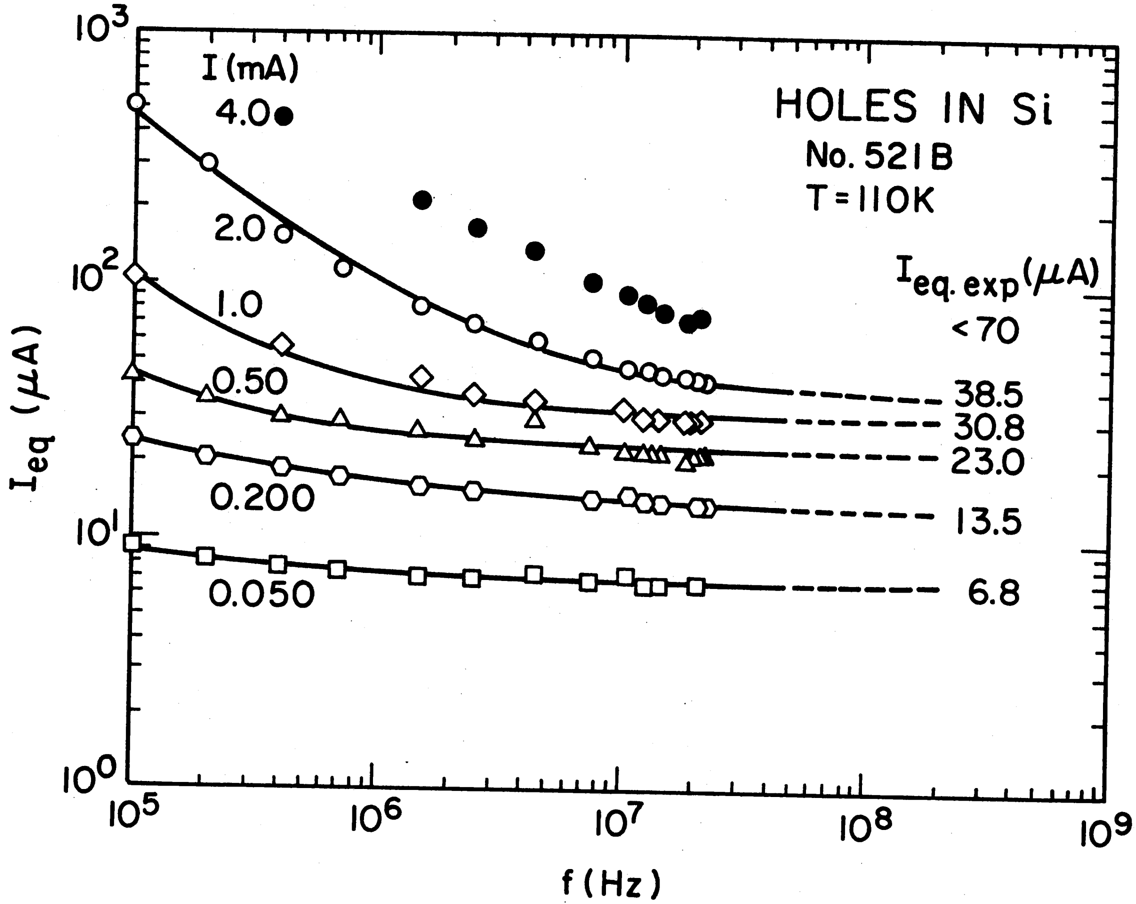


Figure 2. Noise Power Density Spectrum of a Silicon Single Injection Diode at High Electric Fields Where the Hole Mobility Is a Function of the Electric Field (Tandon, 1975). Here $I_{eq} \propto$ Noise Power Density. Note that the Presence of 1/f Noise Makes the Estimation of the High Frequency Asymptote Difficult

1.6 Purpose of This Thesis

In spite of the immense amount of effort, both experimental and theoretical in the understanding of 1/f noise, the mechanism generating it is still not well understood (Bell, 1968; Müller, 1971; Hooge, 1972; Malakhov, 1975).

In this thesis, the phenomenon of 1/f noise is viewed from a general standpoint as being existent in a wide variety of physical systems, not necessarily electronic, which are in thermodynamic non-equilibrium. The idea that 1/f noise results from a nonstationary stochastic process (see Chapter III for definitions) is proposed and evidence is provided to support it. This is a new approach to the problem of 1/f noise. The fundamental occurrence of 1/f noise ultimately at low enough frequencies in open thermodynamic physical systems is also emphasized.

In Chapter II an account of some of the major experiments on 1/f noise is given. Some of the more recent measurements on non-electronic systems are also presented. On the basis of the empirical information through measurements, certain hypotheses are made regarding the process generating 1/f noise. It is emphasized that the process should satisfy these hypotheses. In Chapter III the relevant mathematical definitions and concepts involved in the characterization of fluctuations are given. These are later used in the subsequent Chapters. Chapter IV contains a discussion of the important, different theories which purport to explain the 1/f noise phenomenon since 1925. Some of the mathematical difficulties associated with arriving at the 1/f type power density spectrum are given. A classification of the approaches, in an effort to understand 1/f noise, is attempted and an evaluation is made in terms of the stipulated hypotheses in Chapter II. Finally, the need for a general 1/f noise theory is recognized. The nonstationarity of the stochastic process responsible for 1/f noise is pointed out in Chapter V using analytical arguments. This is based on the hypotheses cited in Chapter II. In Chapter VI a possible class of nonstationary processes is analyzed, in terms of analytical conditions, which could be responsible for the generation of 1/f noise. A time-dependent autocorrelation function for such processes is proposed. Finally, in Chapter VII, an electronic system, namely an ion-implanted resistor, is picked as an example to demonstrate the nonstationary character of the 1/f noise mechanism. Measurements on the electronic 1/f noise and the long term variation of the resistance are reported, which suggest the existence of a nonstationary stochastic process responsible for the generation of 1/f noise.

CHAPTER II

EXPERIMENTS ON 1/f NOISE

In this Chapter some of the major experiments conducted in the study of 1/f noise are reported. The objective is to document the existence of the 1/f type fluctuations in various systems (both electronic and non-electronic) without dealing with the details of the relationship of 1/f noise to the properties of the system.

2.1 The Empirical Nature of the 1/f Noise Phenomenon

The support for the existence of fluctuations responsible for the generation of 1/f noise is based purely on experimental grounds. We say a physical system possesses 1/f noise if the measurements of the noise power spectral density (see Chapter III) of a property made on the system reveal the $1/f^{\alpha}$ (α = constant) dependence at low frequencies without significant departure over a wide range of frequencies. In other words, the characterization of the existing excess noise phenomenon in a physical system is made through the spectral (frequency) distribution of the noise power spectral density of a property.

2.2 1/f Noise Measurements in Electronic Systems

In electronic systems noise measurements normally involve estimating the spectral density of the fluctuating voltage or the current ($S_{VV}(f)$ in V^2/Hz or $S_{II}(f)$ in A^2/Hz respectively). These estimates can in almost all cases be related to the resistance fluctuations. Table I gives a brief summary of major electronic systems in which the presence of $1/f^{\alpha}$ type noise has been reported in the measured $S_{VV}(f)$ or $S_{II}(f)$ in a thermodynamic nonequilibrium situation. In constructing this Table out of the vast amount of experiments, an attempt is made to select only the ones which are considered representative of a given type of system and which indicate somewhat the limits of frequency over which the $1/f^{\alpha}$ type behavior is observed. An effort is also made to include recent information. It is evident from the Table that 1/f noise exists in a wide variety of electronic systems and, at this time, extends in frequency range from approximately 5×10^{-7} Hz to 10^{6} Hz (the low frequency end is the experimental limit, whereas the high frequency end is decided by other noise mechanisms, e.g. Nyquist noise, which tend to dominate at high enough frequencies). This is a huge frequency range for any phenomenon to be documented. As a counter example, one may state that the Nyquist theorem for thermal noise of a resistor ($S_{VV}(f) = 4kTR$, where k = Boltzmann's constant, T = absolute temperature and R = resistance) is firmly accepted, although its experimental support is outright scant, if compared to 1/f noise, e.g., there exist no measurements to demonstrate Nyquist noise down to even 10^{-3} Hz.

Figures 3, 4, 5, 6 and 7 are extracted from Table I as classic examples where the occurrence of 1/f noise at very low frequencies is evidenced. It should be mentioned that within measurement errors, no departure from the $1/f^{\alpha}$ behavior is observed, although some

TABLE I

EVIDENCE OF 1/f NOISE IN ELECTRONIC SYSTEMS

Electronic System	Reference	Particular Subsystem Investigated	Approximate Frequency Range (Hz) For Which $S_{XX}(f) \propto 1/f^{\alpha}$
Resistors	Rollin (1953) Figure 3	Pyrolytic Carbon Resistors	$2.5\times10^{-4} - 10^{1}$ $\alpha \simeq 1$
	Van Vliet (1956)	Cds-Ag Crystals	$5\times10^{0} - 10^{6}$ $\alpha \simeq 1$
	Montgomery (1952)	Ge Filaments	$2\times10^{1} - 10^{6}$ $\alpha \simeq 1$
	Bilger (1974) Boehm (1975a)	Ion-Implanted Resistors	$5\times10^{-4} - 10^{6}$ $\alpha \simeq 1$
P - N Junctions	Firle (1955) Figure 4	Si P-N Junction Diode	$6\times10^{-5} - 10^{-2}$ $\alpha = 1.2$
Zener Diodes	Ringo (1972)	Si Zener Diodes	$10^{-1} - 10^{1}$ $\alpha \simeq 1$
	Boehm (1975a, 1975b)	Si Zener Diodes	$10^{-4} - 5\times10^{-1}$ $\alpha \simeq 1$
Bipolar Transistors	Baldinger (1968) Figure 5	Si Bipolar Transistor	$10^{-4} - 5\times10^{-2}$ $\alpha = 0.86$
	Jaeger (1970)	Si Planar Transistors	$2\times10^{1} - 10^{3}$ $\alpha \simeq 1$
Metal Films	Hoppenbrouwers (1970)	Au, Cu, Ag, Al, Pt, Sn, Cr Films	$1.3\times10^{2} - 5\times10^{4}$ $\alpha \simeq 1$
	Clarke (1974)	Cu, Ag, Au, Sn, Bi, Manganin Films	$10^{-1} - 2.5\times10^{1}$ $\alpha \simeq 1$
FETS	Lauritzen (1965)	Si FETS	$10^{0} - 10^{4}$ $\alpha \simeq 1$

TABLE I (Continued)

Electronic System	Reference	Particular Subsystem Investigated	Approximate Frequency Range (Hz) For Which $S_{XX}(f) \propto 1/f^{\alpha}$
MOSFETS	Mansour (1968) Hawkins (1968) Figure 6	Si MOSFETS	$5x10^{-5}$ - $1x10^{0}$ $1 < \alpha < 1.2$
	Berz (1970)	Si MOSFETS	10^{1} - $4x10^{4}$ $\alpha \simeq 1.2$
Thermo and Concentration Cells	Hooge (1972)	Si Point Contact Thermo Cells and Ionic Solution Cells	10^{2} - 10^{4} $\alpha \simeq 1$
	Kleinpenning (1974)	Intrinsic and Extrinsic Ge and Si Thermo Cells	10^{2} - 10^{4} $\alpha \simeq 1$
Electron Tubes	Johnson (1925, 1971)	Filament Electron Tubes	$8x10^{0}$ - 10^{3} $\alpha \simeq 1$
	Graffunder (1939)	Temperature Limited Electron Tubes	$4x10^{1}$ - 10^{4} $\alpha \simeq 1$
Super-Conductors	Clarke (1975)	Josephson Junctions	$5x10^{-2}$ - 10^{2} $0.9 < \alpha < 1.15$
Integrated Circuits	Caloyannides (1974) Figure 7	Operational Amplifier	$5x10^{-7}$ - 10^{0} $\alpha = 1.23$

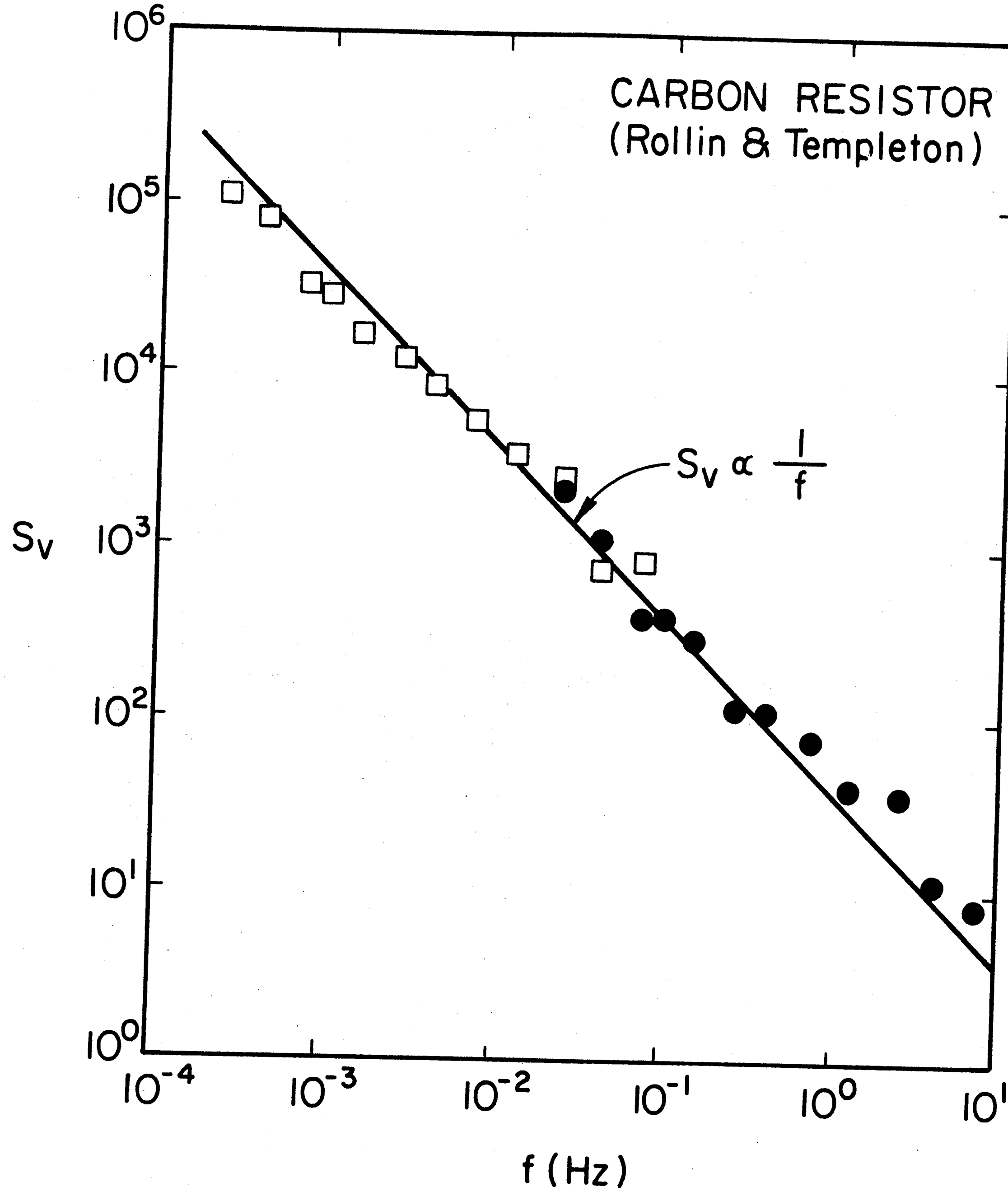


Figure 3. Excess Noise Voltage Spectral Density S_V in a Pyrolytic Carbon Resistor (Rollin, 1953). The Vertical Scale Has Arbitrary Units

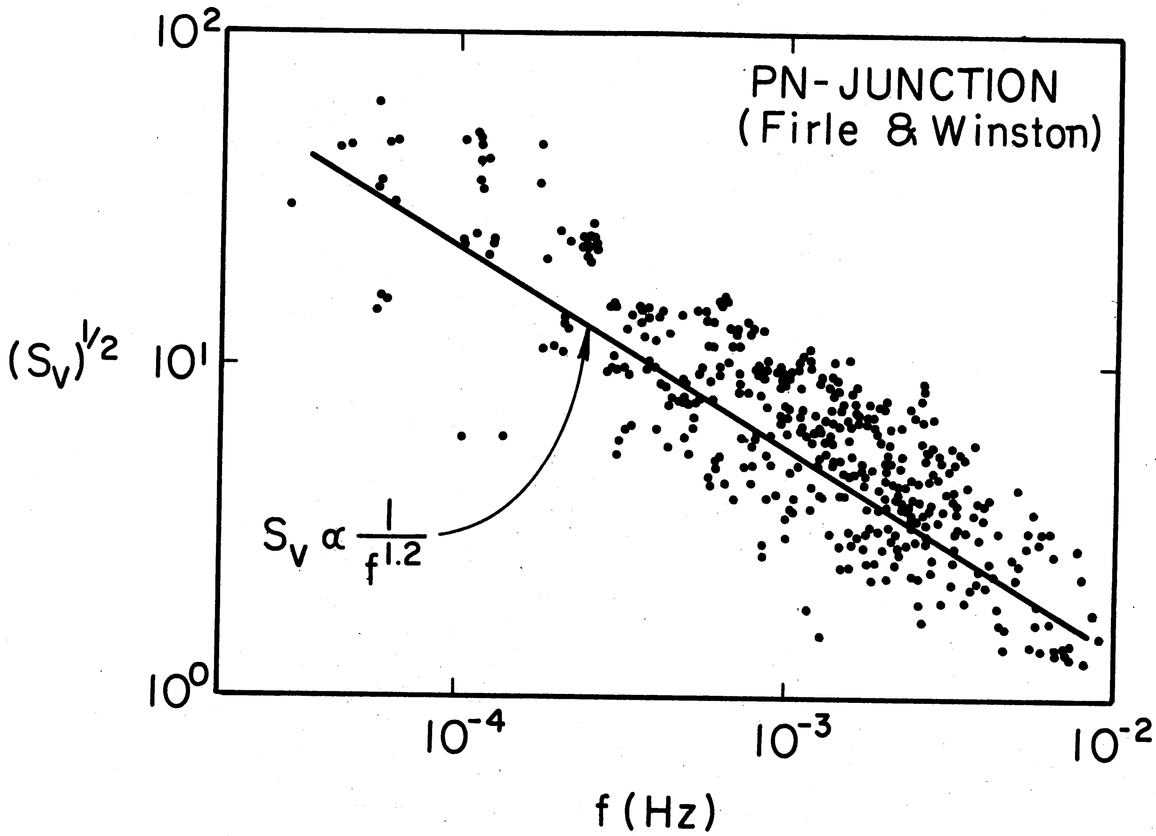


Figure 4. Noise Voltage Spectrum of a Silicon PN Junction Diode, a Composite of 11 measurements (Firle, 1955). The Vertical Scale Has Arbitrary Units

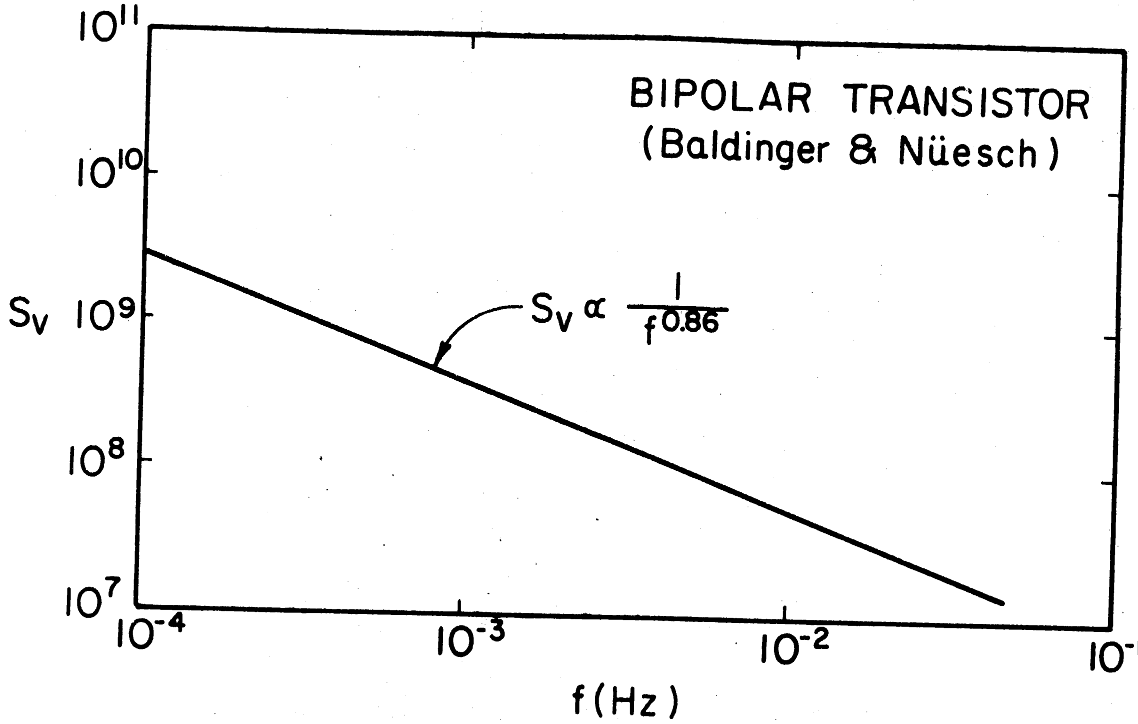


Figure 5. Representation of Noise Measurements on a Bipolar Transistor Pair (BCY 55, Selected With I_C = 100 μA) (Baldinger, 1968). The Vertical Scale Is in Arbitrary Units

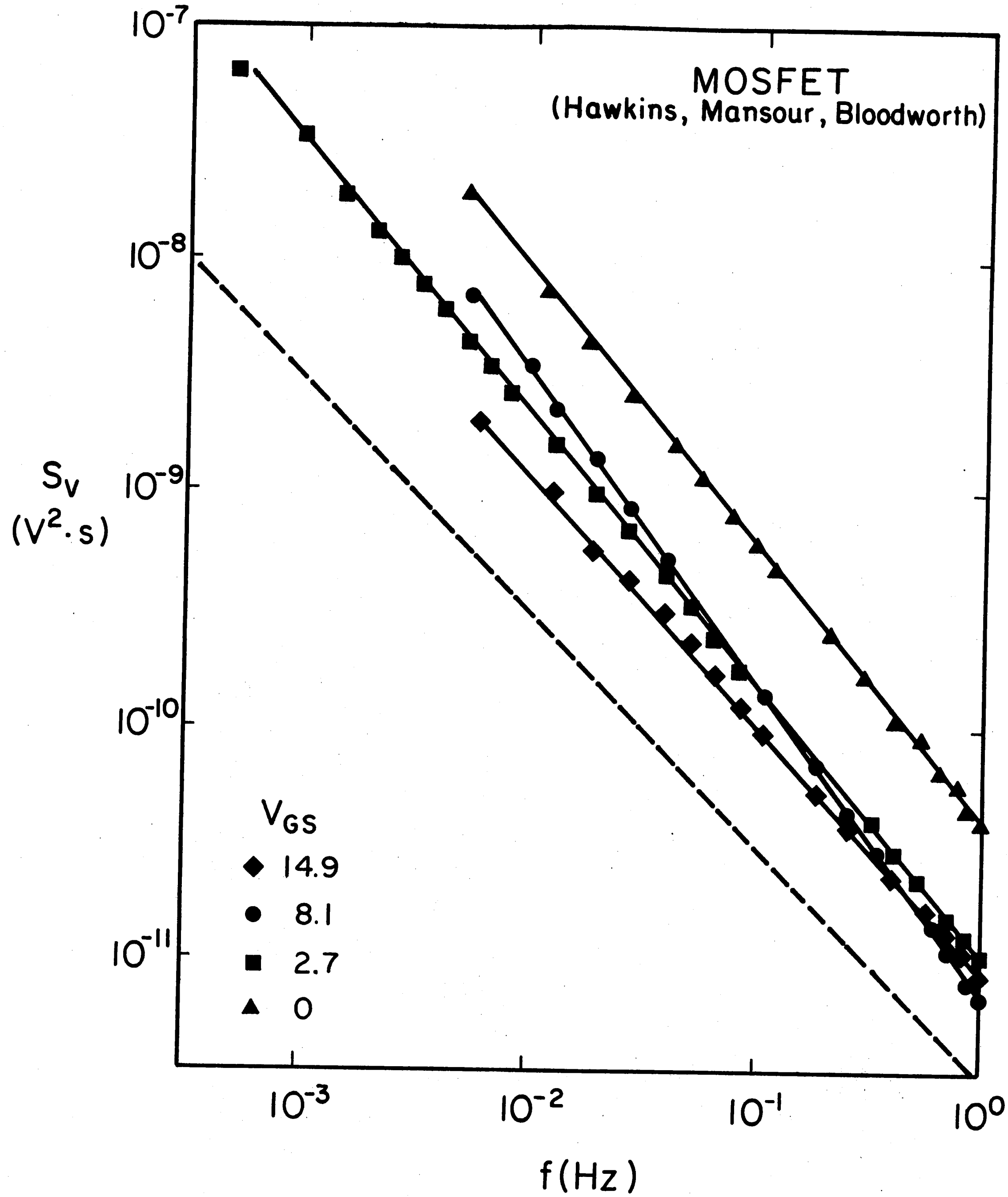


Figure 6. Noise Power Spectrum of an N Channel Depletion Type MOS Transistor (Hawkins, 1968)

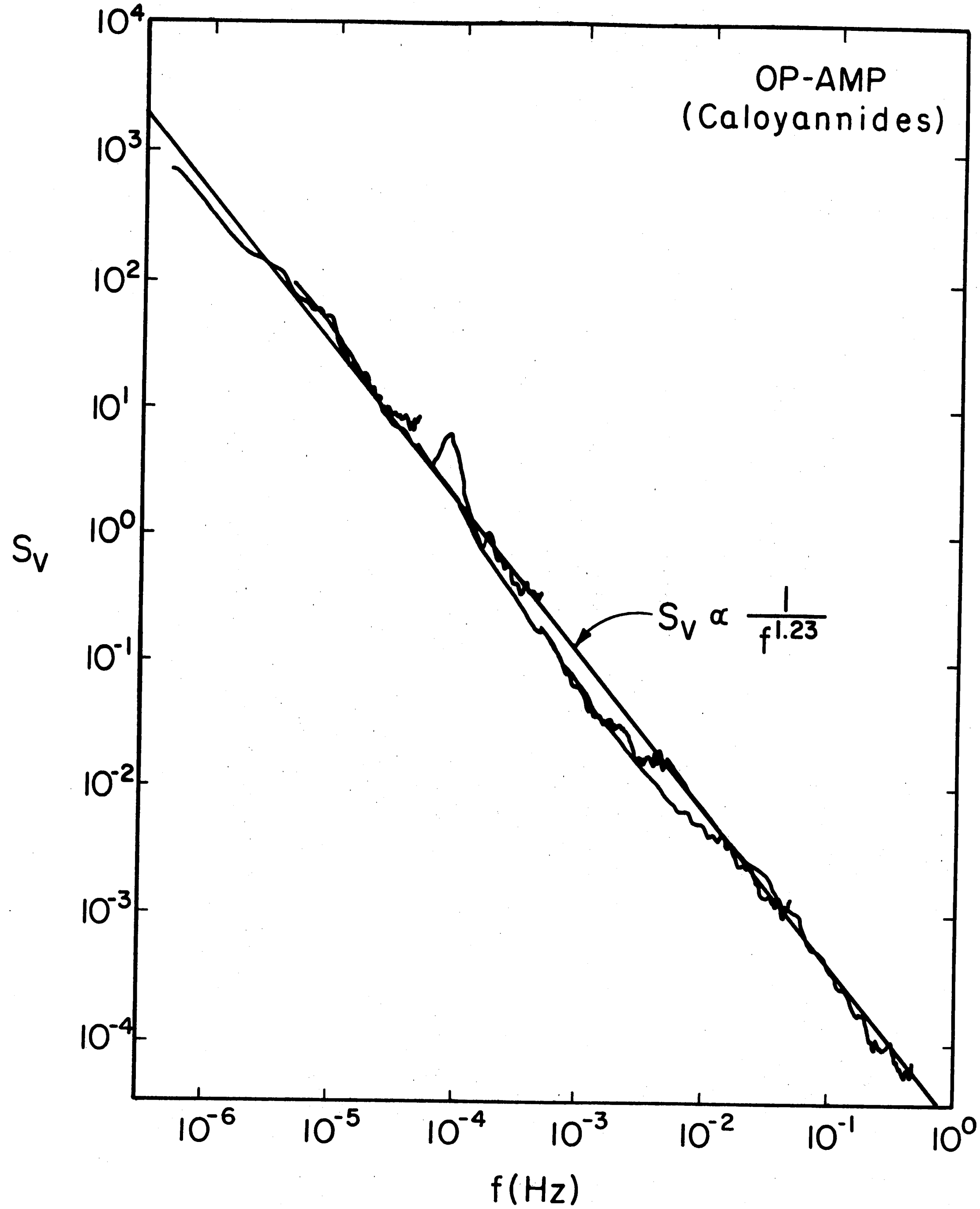


Figure 7. Noise Power Spectrum of a Commercial Operational Amplifier (Fairchild μA 709) (Caloyannides, 1974). The Vertical Scale is in arbitrary Units

of them were deliberately set up to search for any kind of levelling off at the low frequency end. Also, since vastly different techniques were used to estimate the power spectra in these measurements, viz. slow tape recording with speed-up for analysis, optical techniques, electronic sampling with digital processing, it is highly improbable that the method of analysis or measurement is responsible for the apparent 1/f noise.

A certain objection can be raised regarding the validity of the low frequency noise spectral estimates as depicted in Figures 3 through 7. Since the measurement of such fluctuations, which are typically in the microvolt range, extend over rather long periods of time ($1/10^{-6}$ Hz $\simeq$ 12 days), the stability of the noise spectral analyzer itself, or of any reference quantities, may become a problem, such that the fluctuations of the analyzer (or of the references) may not be separable any longer from the fluctuations of the system under investigation. This problem is, however, solved by stating that the electronic system combined with the analyzer can be treated as a generator of 1/f noise, again demonstrating that the presence of 1/f noise is universal.

2.3 Evidence of 1/f Noise in Non-Electronic Systems

Recent investigations show that 1/f noise is not restricted to electronic systems only. Table II lists situations where the measurements on a fluctuating property of a system in thermodynamic nonequilibrium reveal 1/f noise. Figures 8, 9, and 10, taken from Table II as examples, illustrate that the spectra, within errors of estimation, possess the $1/f^{\alpha}$ dependence at low frequencies.

TABLE II

EVIDENCE OF 1/f NOISE IN NON-ELECTRONIC SYSTEMS

System	Reference	Particular Subsystem Investi- gated	Property Analyzed	Approximate Frequency Range (Hz) For Which $S_{XX}(f) \propto 1/f^{\alpha}$
Bio- physical	Campbell (1972) Figure 8	Insulin Needs of an Unstable Diabetic	Fluctuations in the Rate of Insulin Intake	$10^{-8} - 10^{-5}$ $\alpha \simeq 1$
	Verveen (1968) Figure 9	Nerve Membrane	Voltage Fluctuations Across Nerve Fibres	$10^{-1} - 10^{2}$ $\alpha \simeq 1$
Astro- nomical	Munk (1960) Figure 10	Earth	Angular Fluctuations in the Earth's Rotation	$10^{-9} - 2x10^{-8}$ $\alpha = 2.8$
Physical	Perrone (1969)	Northwest Atlantic Ocean	Acoustic Ocean Ambient Noise	$10^{1} - 3x10^{3}$ $\alpha \simeq 1$
	Attkinson (1963)	Quartz Crystals	Frequency Fluctuations in Quartz Crystals	$5x10^{-7} - 10^{-6}$ $\alpha = 1.4$

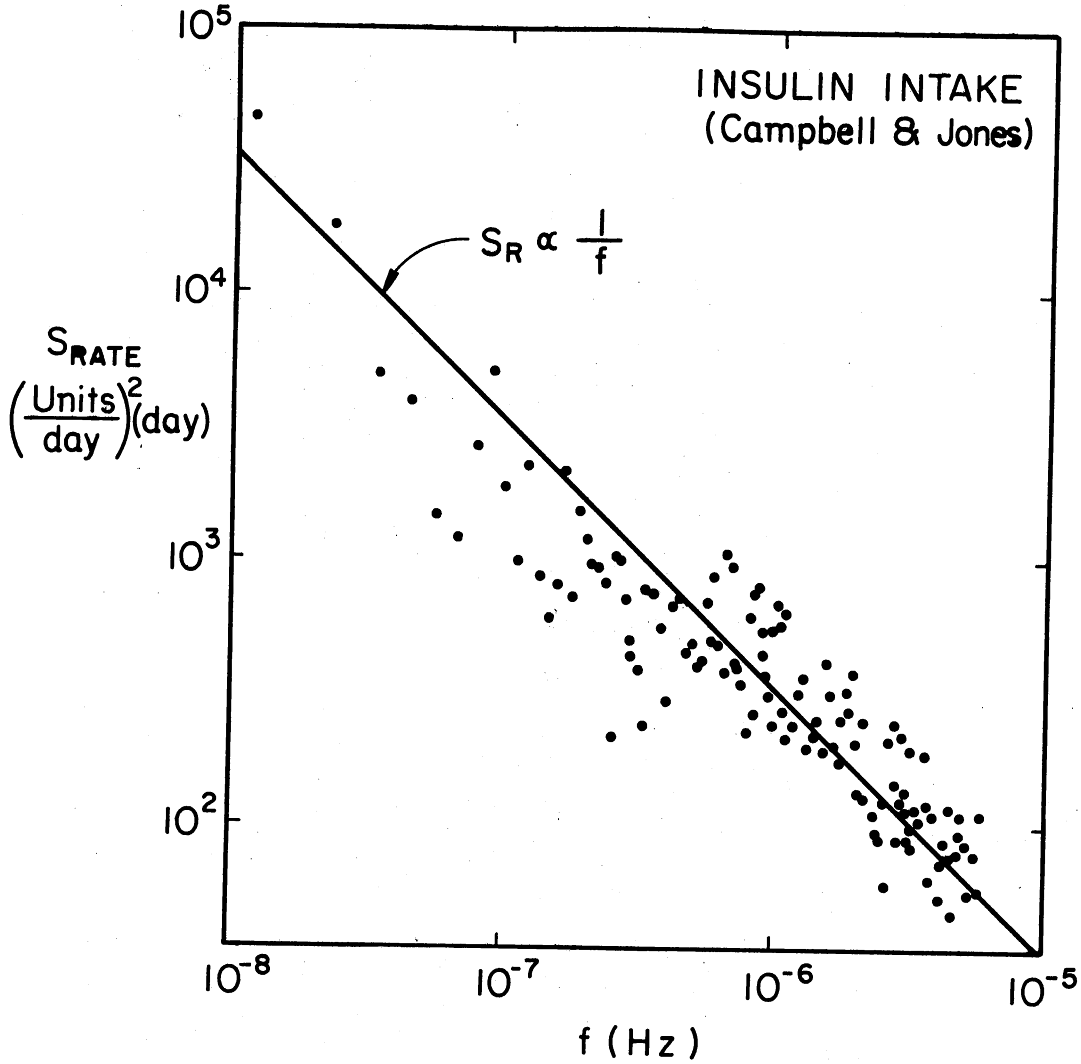


Figure 8. The Variance Spectrum of Insulin Intake of an Unstable Diabetic Patient Over an 8 Year Period (Campbell, 1972)

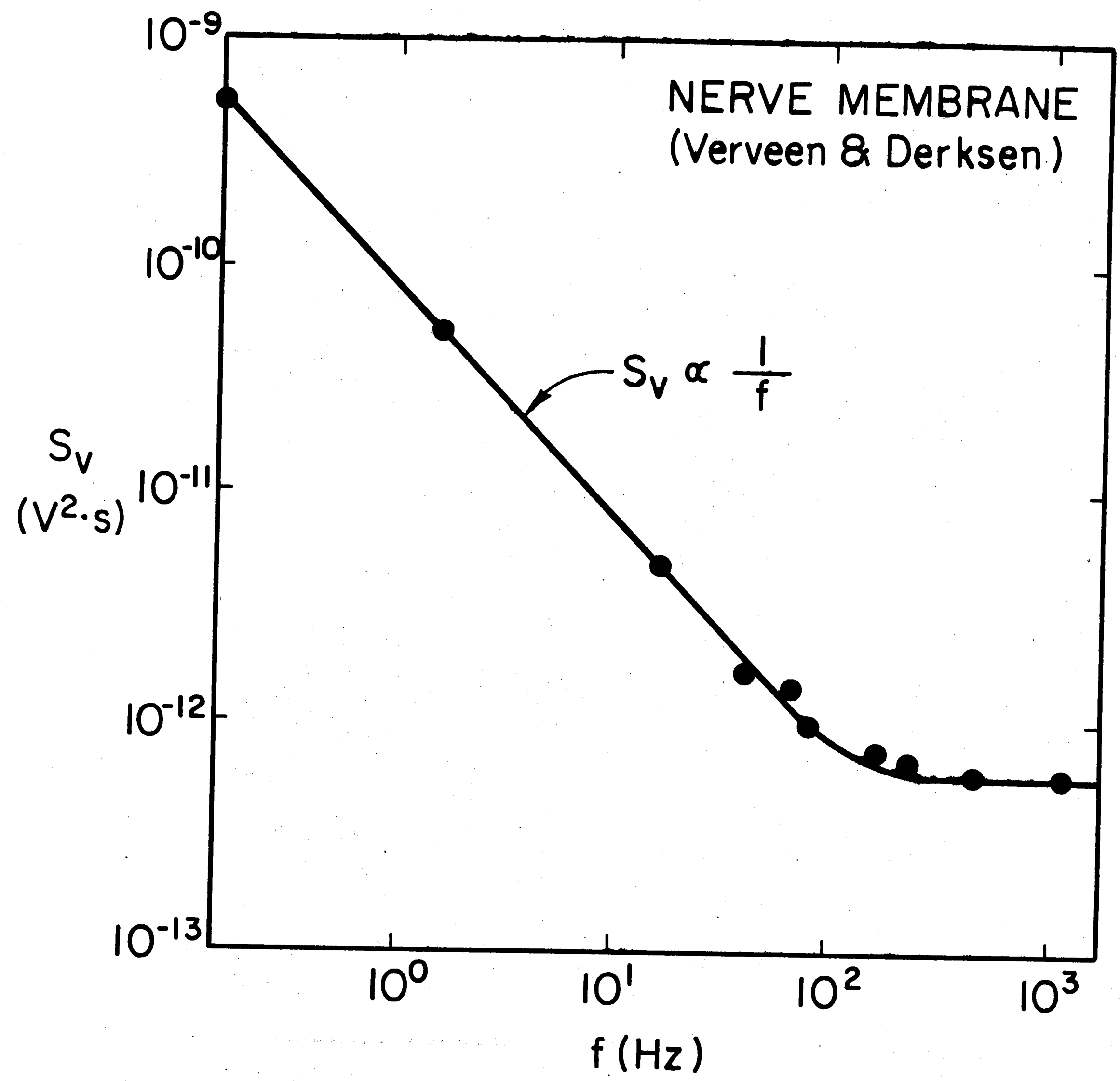


Figure 9. Voltage Fluctuation Spectrum Across the Resting Membrane of Myelinated Nerve Fibres (Verveen, 1968)

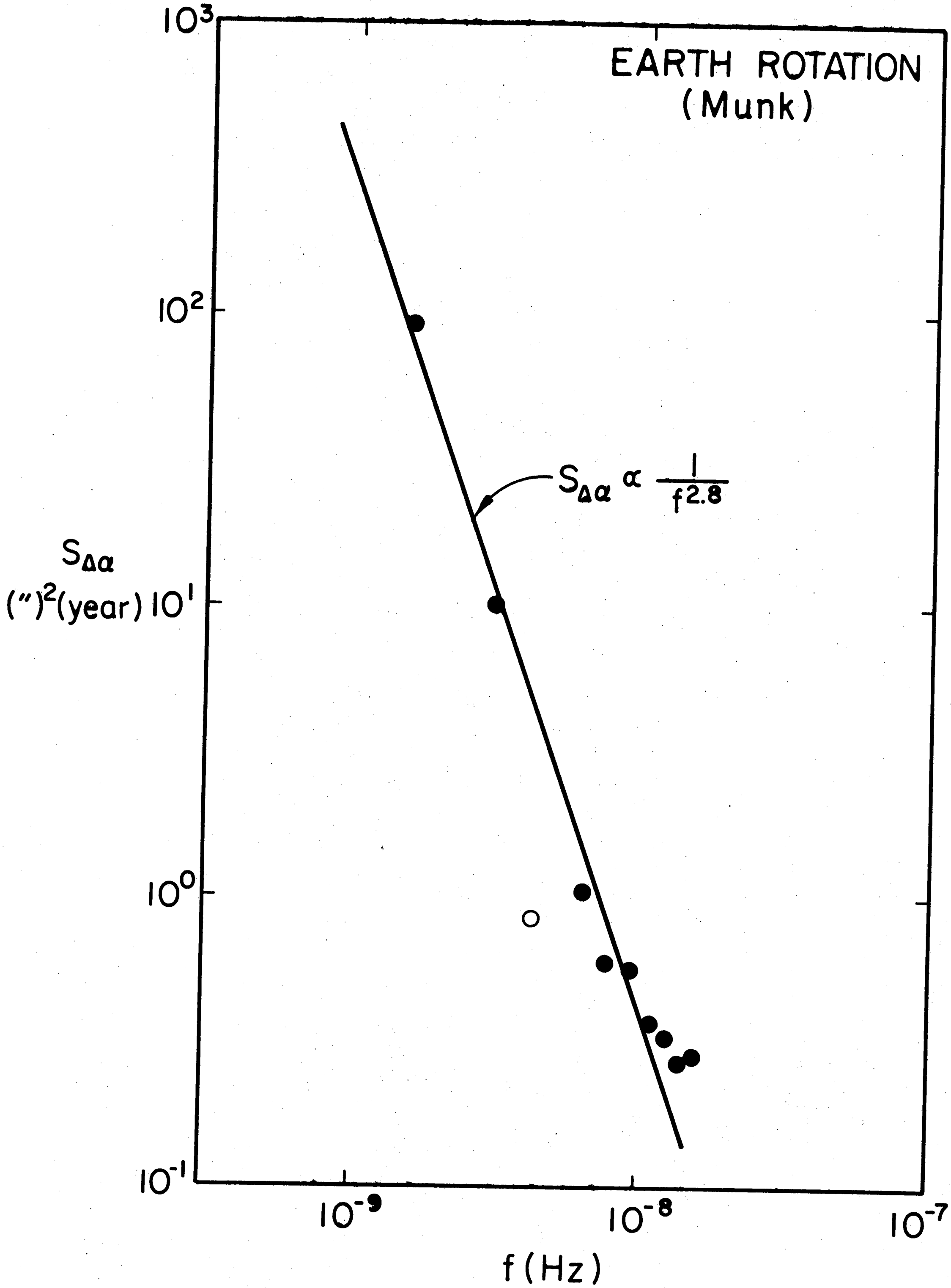


Figure 10. Angular Fluctuation Spectrum in the Earth's Rotation (Munk, 1960)

2.4 Basic Hypotheses About 1/f Noise

Considering the evidence provided in Sections 2.3 and 2.4, the following three hypotheses can be made about the 1/f noise phenomenon, with reasonable confidence:

A. The phenomenon of 1/f noise is general; in particular it is not restricted to electronic systems; it exists objectively, and is not affected by the type of measurement or mode of analysis. The fact that the study of 1/f noise has been associated with electronic systems is incidental because of their developing utility in the last fifty years.

B. Physical systems are capable of fluctuations in their properties whose spectral density is of the type $S_{XX}(f) \propto 1/f^{\alpha}$, over all observable frequencies. In this perspective, 1/f noise is "unbounded" as regards the frequency range of its existence is concerned. Practically, the high frequency end of $S_{XX}(f)$ could be masked by other noise mechanisms but the low frequency end always reveals the $1/f^{\alpha}$ dependence.

C. The observed spectra of the type $S_{XX}(f) \propto 1/f^{\alpha}$ are stable in the sense that repeated observations give rise to the same spectrum, within errors of observation. This aspect is demonstrated by measurements and quantitative evidence for a special case of an electronic system is given in Chapter VII.

As a consequence of the above hypotheses one may state that ultimately, any physical system produces fluctuations in its properties, which are of the $1/f^{\alpha}$ type when out of thermodynamic equilibrium. This statement is true in such a widespread class of systems, that it could be accepted as a general empirical fact.

CHAPTER III

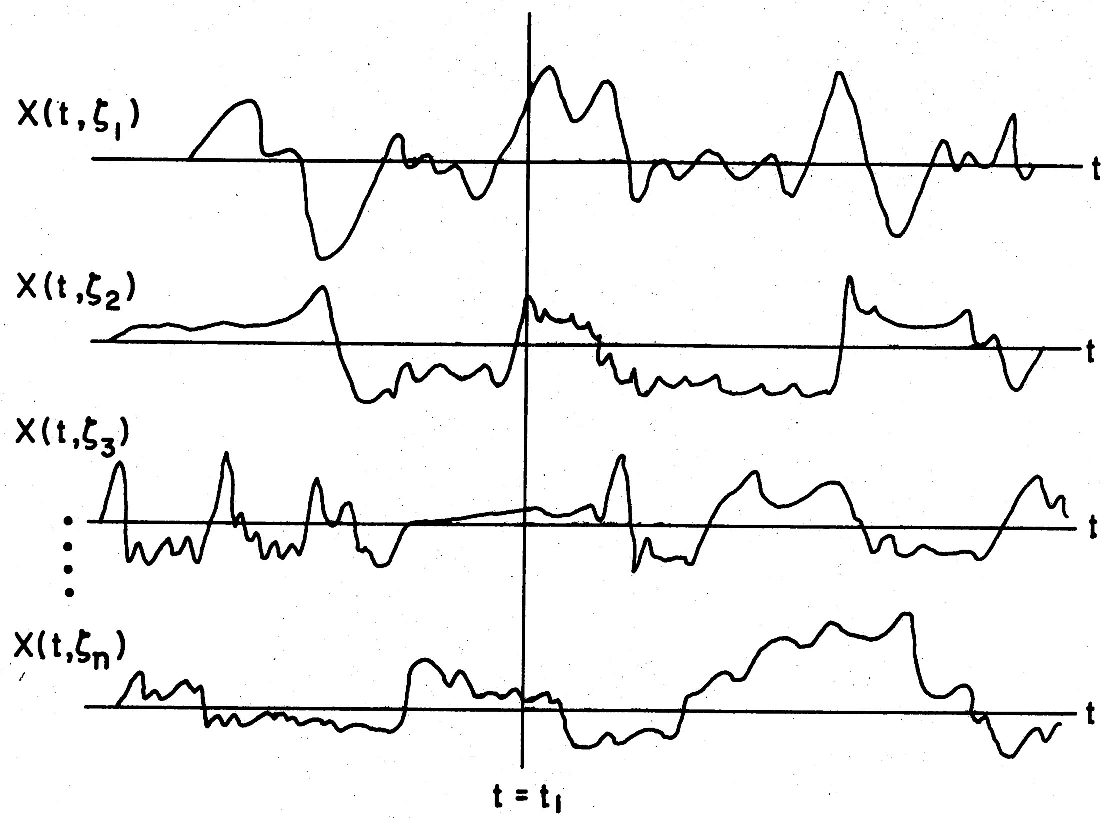
RELEVANT MATHEMATICAL CONCEPTS INVOLVED IN THE CHARACTERIZATION OF FLUCTUATIONS

Because of the random nature of fluctuations, various methods can be used to describe them. In general it is not possible to completely specify a random process. The description of a random process is statistical, i.e., the characterization is in terms of certain averages, and therefore, there exists an inherent limitation to the amount of knowledge one can obtain from such a study, due to the lack of complete information. In this Chapter, definitions and concepts which lead to the notion of power spectral density as an average quantity to characterize a random process are discussed and examined. Since the existence of the excess noise phenomenon is evidenced by the $1/f^{\alpha}$ dependence of its power spectral density (see Chapter II), an understanding of the concept of power spectral density is developed in this Chapter which is later used in the subsequent Chapters in an effort to investigate some of the properties of excess noise.

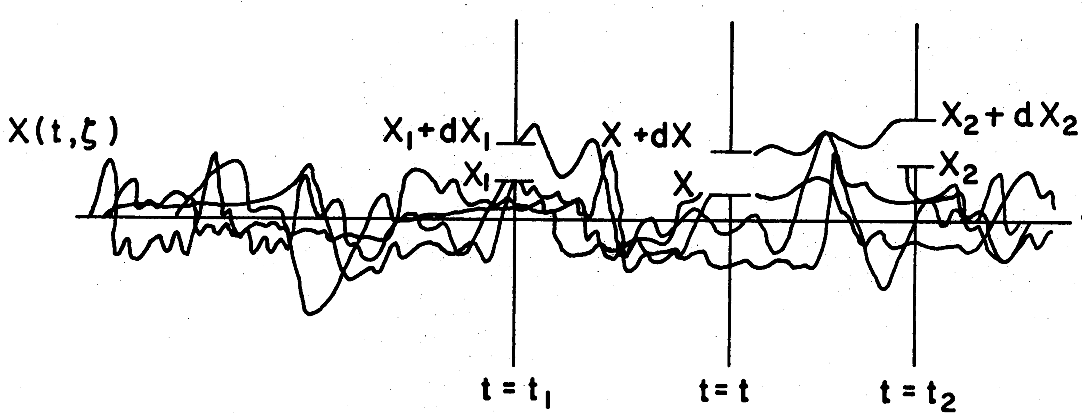
3.1 The Concept of a Stochastic Process

A stochastic process (also called a general random process) is an extension of the concept of a random variable (Lathi, 1968, Chapter 3; Papoulis, 1965, Chapter 9). In the case of random variables, one assigns a number $x(\zeta)$ to a random outcome ζ of a given experiment. In the case of stochastic processes, to each outcome ζ a waveform x(t) is assigned (which is a function of time), according to some rule $x(t,\zeta)$. Thus the space consisting of outcomes is a collection of waveforms, called an ensemble. The family of functions, one for each ζ, is referred to as a stochastic process.

A stochastic process can be viewed as a function of two variables t and ζ (Figure 11(a)). The domain of ζ is the set of all possible outcomes, and the domain of t is the set of real numbers (the time axis). For a specific outcome ζ_i, the expression $x(t,\zeta_i)$ signifies a single time function. For a specific time t_i, $x(t_i,\zeta)$ is a quantity depending on ζ, i.e., a random variable. Finally, $x(t_i,\zeta_i)$ is a mere number. It is common practice to represent a stochastic process by the notation x(t), for convenience, where the dependence on ζ is implied. The fluctuations, as defined in Chapter I, are stochastic in nature. In general, the family of functions of a stochastic process are complicated indeed. As an example, suppose x(t) represents the irregular motion of a particle due to its impact with the surrounding medium (Brownian motion - e.g. motion of electrons in a resistor). A specific outcome of this experiment is the selection of a particle, and the resulting x(t) is a curve. This curve is irregular and cannot be described by a formula. Now, if the motions of the different particles are considered, a family of time functions of the type shown in Figure 11(a) are obtained, constituting a stochastic process. It should be mentioned that if a specific x(t) is known for $t < t_1$ (see Figure 11(a)), one cannot predict its future values. Also the known value of x(t) at $t = t_1$ for a particle does not make it possible to estimate the exact value of x(t) of an another particle.



(a)



(b)

Figure 11. Graphical Representation of a Stochastic Process. (a) Ensemble Waveforms (b) Illustration for Interpreting Probability Density Functions

3.2 Probability Density, Mean, Variance, and Autocorrelation Function of a Stochastic Process

Any knowledge about a stochastic process can be obtained only by using a probabilistic description. Figure 11(b) is derived from Figure 11(a) where the amplitudes $x(t,\zeta_i)$ are drawn on a single time axis. The first-order probability density, $p(x;t)$, is defined as the probability density of the amplitudes of the stochastic variable $x(t)$ at time t. Thus, $p(x;t)dx$ represents the probability that the amplitude of $x(t)$ will lie in the interval $(x, x+dx)$ at the instant t (see Figure 11(b)). The interpretation of the second-order probability density function for the process $x(t)$ is made in terms of the probability, $p(x_1,x_2;t_1,t_2)dx_1dx_2$, of the joint event when the amplitudes of the variable $x(t)$ are in the range (x_1, x_1+dx_1) at $t = t_1$ and in the range (x_2, x_2+dx_2) at $t = t_2$ (see Figure 11(b)). Here $p(x_1,x_2;t_1,t_2)$ is the second-order probability density function for the process $x(t)$. Higher order probability density functions can be defined similarly, but they will not be considered here, as a description in terms of the second-order probability density function suffices for the computation of variance, autocorrelation and power spectral density for the process (Lathi, 1968, Chapter 3).

The computation of mean, variance and autocorrelation function is done using the first-order and the second-order probability density functions as follows:

$$\text{First-Order Mean, } \overline{x(t)} \equiv \int_{-\infty}^{+\infty} x\, p(x;t)\, dx \qquad (3.2.1)$$

$$\text{Second Order Mean, } \overline{x^2(t)} \equiv \int_{-\infty}^{+\infty} x^2 \, p(x;t) \, dx \qquad (3.2.2)$$

$$\text{Variance, } \sigma^2(t) = \overline{(x - \overline{x(t)})^2} \equiv \int_{-\infty}^{+\infty} (x - \overline{x(t)})^2 \, p(x;t) \, dx \qquad (3.2.3)$$

$$\text{Autocorrelation, } R_{XX}(t_1,t_2) = \overline{x_1 x_2} \equiv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \, x_2 \, p(x_1,x_2;t_1,t_2) \, dx_1 \, dx_2 \qquad (3.2.4)$$

3.3 Stationarity, Nonstationarity and Ergodicity

A stochastic process is defined to be strictly stationary if all orders of its probability density functions are independent of the shift in the time origin. Mathematically, for a strictly stationary process:

$$p(x;t) = p(x)$$

$$p(x_1,x_2;t_1,t_2) = p(x_1,x_2;t_2 - t_1)$$

$$\vdots$$

$$p(x_1,x_2,..,x_n;t_1,t_2,..,t_n) = p(x_1,x_2,..,x_n;\tau_1,\tau_2,..,\tau_{n-1})$$

$$\text{where } \tau_k = t_{k+1} - t_1 \text{ (k = integer} \geq 1) \qquad (3.3.1)$$

and in particular,

$$\overline{x(t)} = \overline{x} \qquad (3.3.2)$$

$$\overline{x^2(t)} = \overline{x^2} \tag{3.3.3}$$

$$\sigma^2(t) = \sigma^2 \tag{3.3.4}$$

$$\begin{aligned} R_{XX}(t_1,t_2) &= R_{XX}(t_2 - t_1) \\ &= R_{XX}(\tau) \text{ where } \tau \equiv t_2 - t_1 \ . \end{aligned} \tag{3.3.5}$$

In certain cases a stochastic process may have only the first and the second-order probability density functions independent of the shift in the time origin, whereby, Equations (3.3.2) to (3.3.5) are true. Such a process is called a wide-sense stationary process.

A nonstationary process is defined as a stochastic process when any order of its probability density function is dependent on the time origin. In this respect, stationarity is a special case of nonstationarity. Strictly speaking no physical process is stationary because every physical process must begin and end at some finite instants of time. Obviously, the statistics cannot be independent of the time origin in such a case. A stationary process is an idealized model.

An ergodic stochastic process is a special case of a stationary process. For an ergodic process the ensemble averages (over different members of the ensemble) are equal to the time averages (over one of the members of the ensemble). Thus,

$$\overline{x} \equiv \int_{-\infty}^{+\infty} x \ p(x) \ dx = \widetilde{x(t)} \equiv \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x(t) \ dt \tag{3.3.6}$$

$$\overline{x^2} \equiv \int_{-\infty}^{+\infty} x^2 \ p(x) \ dx = \widetilde{x^2(t)} \equiv \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t) \ dt \tag{3.3.7}$$

$$\overline{(x - \bar{x})^2} \equiv \int_{-\infty}^{+\infty} (x - \bar{x})^2 \, p(x) \, dx$$

$$= \widetilde{(x - \widetilde{x(t)})^2} \equiv \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} (\widetilde{x(t)} - x(t))^2 \, dt \quad (3.3.8)$$

$$R_{XX}(\tau) \equiv \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 \, x_2 \, p(x_1, x_2; \tau) \, dx_1 \, dx_2$$

$$\equiv \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x(t) \, x(t+\tau) \, dt \qquad (3.3.9)$$

Figure 12 (Lathi, 1968, Chapter 3) summarizes the concepts of non-stationarity, stationarity and ergodicity.

3.4 The Concept of Power Spectral Density

The power spectral density $S_{XX}(f)$ (f = frequency) of a stochastic process x(t) gives the power distribution per unit frequency of the various components present in the process as a function of frequency. Any stochastic process can be viewed as an energy signal and therefore can be represented in terms of $S_{XX}(f)$ by using Fourier Transform techniques (Lathi, 1968, Chapter 3).

3.4.1 Power Spectral Density of a Stationary Process

For a stationary stochastic process x(t), the power spectral density is defined as the Fourier Transform of its autocorrelation

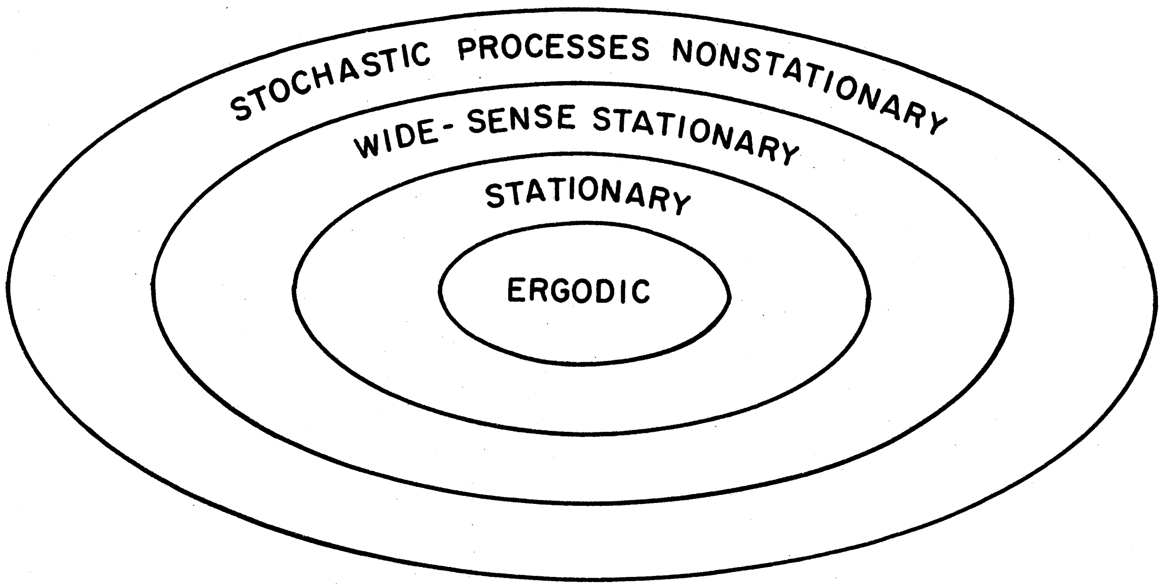


Figure 12. Classification of Stochastic Processes (Lathi, 1968, Chapter 3)

function as follows:

$$S_{XX}(f) \equiv \int_{-\infty}^{+\infty} R_{XX}(\tau) \exp(-j2\pi f\tau)\, d\tau \qquad (3.4.1)$$

3.4.2 Power Spectral Density of a Nonstationary Process

Unlike a stationary process, the representation of the signal power in terms of its frequency distribution becomes more difficult and ambiguous for a nonstationary process. For a nonstationary process y(t), the autocorrelation function $R_{YY}(t_1,t_2)$ (Equation (3.2.4)) is in general a function of two variables t_1 and t_2, hence the method used to obtain the power spectral density in the case of a stationary process cannot be applied directly. Several methods to obtain the power spectral density of a nonstationary process are discussed by Bendat (1966) in Chapter 9. If one transforms the variables t_1 and t_2 by writing $t = t_1$ and $\tau = (t_2 - t_1)$, $R_{YY}(t_1,t_2)$ can be expressed as $R_{YY}(t,\tau)$ for a nonstationary process y(t) (Lampard,1954). The notion of instantaneous power spectral density $S_{YY}(t,f)$, which is a function of t and f, can be introduced as the Fourier Transform of $R_{YY}(t,\tau)$ with respect to the variable τ (a generalized Wiener-Khintchine Theorem - Lampard, 1954; Turner, 1954; Levin, 1964) as follows:

$$S_{YY}(t,f) \equiv \int_{-\infty}^{+\infty} R_{YY}(t,\tau) \exp(-j2\pi f\tau)\, d\tau \qquad (3.4.2)$$

It should be mentioned that $S_{YY}(t,f)$ does not exist for all nonstationary processes y(t). Some of the conditions which have to be satisfied for $S_{YY}(t,f)$ to exist and to be unique are discussed by

Donati (1971). Practically, the quantity $S_{YY}(t,f)$ is not measurable (Bendat, 1966, Chapter 9). However, since the measurement of power spectral density involves time averaging (see Chapter VI), another quantity $S_{YY}(T,f)$ can be defined and realized experimentally, which is the time average of $S_{YY}(t,f)$ for a finite time T. Thus,

$$S_{YY}(T,f) = \frac{1}{T} \int_0^T S_{YY}(t,f)\, dt. \qquad (3.4.3)$$

In this respect any measurement of the power spectral density of a nonstationary process could be interpreted in terms of $S_{YY}(T,f)$.

CHAPTER IV

DISCUSSION OF 1/f NOISE THEORIES

Upto the present date almost all ideas proposed to explain and/or interpret the $1/f^{\alpha}$ type fluctuations have been catered towards electronic devices and systems. Much of the effort has been expended to relate 1/f noise with other physical properties of an electronic system. In this Chapter, several major different approaches which envisage to explain electronic 1/f noise are summarized and discussed. The restrictive domain of application of a particular approach is realized. Some of the mathematical and physical difficulties associated with the understanding of 1/f noise are mentioned. An evaluation of some of the major 1/f noise theories is made, wherever possible in terms of the stipulated hypotheses based on experimental evidence (see Chapter II). Finally, considering the general occurrence of 1/f noise, a need for a general 1/f noise theory is recognized.

4.1 Chronological Development in the Understanding of 1/f Noise

One of the earliest attempts to explain 1/f noise in vacuum tubes was made by Schottky (1926), who termed the phenomenon as "Flicker effect". He attributed the origin of 1/f noise in tubes to the fluctuations in the properties of the surface of the cathode as due to the interaction with foreign atoms present in the tube. Schottky's ideas were not only restricted to vacuum tubes, but also calculations made on the basis of his assumptions did not agree too well with Johnson's earlier experiments (Schottky, 1926). In the early 1930's, noise in carbon granules as used in microphones and other resistive structures revealed the $1/f^{\alpha}$ behavior at low frequencies (Frederick, 1931; Otto, 1935). It was demonstrated that 1/f noise was dependent on the current carrying contacts. This resulted in a new term for 1/f noise, the "Contact noise". However, further experiments (Van Vliet, 1956) where care was taken to make cleaner contacts, established that 1/f noise did not originate from the external current carrying contacts. A diffusion model for 1/f noise in resistors was proposed by Richardson (1950), where the calculation was made considering a resistor being linearly coupled to a diffusing medium undergoing thermally excited fluctuations. The inability of the diffusion mechanisms to explain 1/f noise was, however, pointed out by Van Vliet (1958). During the 1950's the fast development of solid state technology shifted the interest in the study of 1/f noise towards investigating semiconductor device structures. The surface model of 1/f noise was developed (McWhorter, 1957; Van der Ziel, 1959), whereby the origin of 1/f noise in a semiconductor was stipulated to arise from the interaction of charge carriers with the traps in the oxide near the conducting path through tunneling. Application of the surface model together with the better understanding of the oxide-semiconductor interface in the early 1960's resulted in the fabrication of semiconductor devices with less 1/f noise. A new name was given to 1/f noise, the "Surface noise". In spite of the ability to produce very clean oxide-semiconductor interfaces (Grove, 1967, Chapter 12), it was discovered in the 1960's that 1/f noise is still present at low enough frequencies in devices in which the surface controls the conduction process (Mansour, 1968; Hawkins, 1968; Berz, 1970). This led to suggesting an additional "bulk" source of 1/f noise, if not an alternate source in certain structures. As was pointed out in Chapter II, several structures, e.g. metal films, where the surface does not play a significant role in the conduction mechanism, strongly suggest the presence of a "bulk" source for 1/f noise. The question whether 1/f noise originates from the bulk or from the surface or from both in a device structure is still open and unanswered even up to the present. Thus, given a device structure, considering the present knowledge of 1/f noise, it is not possible to establish before measurement the magnitude of 1/f noise, let alone the respective contributions from the bulk and/or the surface.

One of the interesting characteristics of 1/f noise, apart from the $1/f^{\alpha}$ behavior, is its presence in a state when the system is in thermodynamic nonequilibrium. In the measurement of 1/f noise in an electronic system the state of thermodynamic nonequilibrium is typically achieved by a d.c. current flow. The dependence of electronic 1/f noise magnitude on the current (I), which is generally found as being proportional to I^2, gives further insight into the 1/f phenomenon. In the early days (Frederick, 1931; Otto, 1935) when 1/f noise was thought as being originating from the current carrying contacts, the proportionality of the 1/f noise magnitude to I^2 was argued as resulting due to the power dissipation at the contacts. In this sense, the current flow was responsible for generating 1/f noise, and 1/f noise was called the "Current noise". The modern approach to explain I^2 dependence is in terms of resistance fluctuations (Bell, 1968). In this perspective, the current flow is merely a probe to detect 1/f noise rather than a source for it. The resistance fluctuations are further interpreted as the fluctuations in the number of charge carriers (Bell, 1968) or as mobility fluctuations of the free charge carriers, as recent measurements on the fluctuations of thermo emf of semiconductors (Kleinpenning, 1974) have indicated. As the resistance is inversely proportional to the product of the number of charge carriers and the mobility, the interpretation of electronic 1/f noise in terms of the resistance fluctuations is generally accepted, although the dependence on the number of carriers and/or the mobility cannot be ascertained until more precise measurements on systems can be done, where a distinction can be made between mobility fluctuations and carrier fluctuations.

In recent years, especially in the last decade, the approach to explain 1/f noise has been more on general grounds, primarily because of its occurrence in a wide variety of electronic systems (see Chapter II). However, the scope is still restricted to electronic systems only. It is admitted that, in a loose sense, electronic 1/f noise is probably as fundamental in origin as thermal noise and, therefore, considerable effort is oriented towards understanding the 1/f behavior rather than investigating its dependence on physical properties of specific electronic systems. Some of the approaches are tabulated in Section 4.4.

4.2 Mathematical and Physical Difficulties In Stipulating Processes That Generate 1/f Noise

The diverging behavior of the power spectral density of 1/f noise at low frequencies imposes serious problems, both mathematically and physically. As was pointed out in Chapter III, the 1/f type fluctuations are stochastic. The conventional method of obtaining the power spectral density which involves the application of the Wiener-Khintchine cosine-transform theorem, and which results invariably in an even functional dependence of power spectral density with respect to frequency (Papoulis, 1965, Chapter10), cannot be applied to explain 1/f noise (see Chapter V). Another difficulty associated with stipulating a process that would generate a 1/f spectrum is the uniqueness problem. As was discussed in Chapter III, the power spectral density is an average quantity, therefore it does not contain all the information about a process as one would like to know. Several different processes could yield the same power spectral density (Papoulis, 1965, Chapter 10). Thus, given a 1/f spectrum, it is difficult to arrive at a unique process responsible for the generation of 1/f noise. A third problem, probably not so serious, deals with the finiteness of the total integrated power as obtained from a 1/f spectrum. A beautiful discussion of this problem is given by Bell (1968). A typical resistor which has a spectrum $S_{VV}(f) = 10^{-12}/f$ (1 μV r.m.s. of noise power per decade of frequency) when 1 volt d.c. bias is applied, is considered. If the upper frequency limit for 1/f noise is chosen to be at 10 GHz (10^{10}Hz), the period of the lowest frequency would then have to be of the order of 10^{10^9} s in order to make the integrated noise power equal to 1% of the d.c. power. This time limit far exceeds the age of the universe ($\simeq 10^{10^{1.2}}$ s). Thus, although the problem of finiteness of the total integrated 1/f noise power may seem difficult to circumvent mathematically, yet from the practical standpoint it is not so serious considering the incredibly small magnitude of the 1/f type fluctuations.

A process that attempts to explain the 1/f type fluctuations at very low frequencies also faces objections from the physical standpoint. Since the evidence of 1/f noise in a system involves long term measurements, the stability of the properties (both physical and chemical) of the system becomes a serious question. In this sense, a stipulation of a physical process for the generation of 1/f noise in a system should incorporate in some fashion the change in the properties of the system. If, however, a process envisages to explain the 1/f behavior independent of the change in the properties of the system, sufficient evidence should be established regarding the stability of the properties of the system at least for the period of duration for which 1/f noise is demonstrated to exist.

Because of the problems cited above, there is no satisfactory theory for 1/f noise although several attempts have been made.

4.3 Classification of Important 1/f Noise Theories Up To February, 1976

In this Section an attempt is made to classify several approaches to explain 1/f noise, especially electronic 1/f noise, in terms of four categories, namely, physical, statistical, phenomenological and empirical. Of course, there is quite a bit of overlap between the different categories, and in certain cases a classification is difficult. Nevertheless, such a classification demonstrates the immense amount of interest in the 1/f phenomenon over the years, for possibly all approaches one could think of to study a scientific phenomenon have been applied to investigate 1/f noise.

Table III is compiled from a survey of the vast literature on 1/f

TABLE III

CLASSIFICATION OF 1/f NOISE THEORIES

Approach	Reference	Basic Idea in Brief	Experimental Support
	Schottky (1926)	Flicker-effect: Fluctuations in the properties of the Cathode	Vacuum Tubes: Johnson (1925) Schottky (1926)
	Richardson (1950)	Diffusion Model: Coupling to a diffusing medium undergoing thermally excited fluctuations	-----
	Van Vliet (1958)	Inability of the diffusion model to explain 1/f noise	-----
	McWhorter (1957) Van der Ziel (1959, Ch. 5)	Surface Model: Interaction of the charge carriers with the traps in the oxide through tunneling	MOSFETS: Sah (1966) Berz (1970) Leuenberger (1971) Resistors: Leuenberger (1967)
Physical	Bakanov (1965) Ganefel'd (1969)	1/f noise as a manifestation of an anisotropic current instability in a semiconductor, for which non-potential waves propagating in a plane perpendicular to the current are responsible.	-----
	Müller (1970, 1971, 1974a, 1974b)	1/f noise generated due to the microscopic shot noise sources in a p-n junction diode through thermal feedback	P-N Junctions: Müller (1971, 1974a, 1974b)
	Clarke (1974)	Thermal Diffusion Mechanism: Frequency dependence of the spatial correlation of 1/f noise and its dependence on the temperature coefficient of resistance	Metal Films: Clarke (1974)

TABLE III (Continued)

Approach	Reference	Basic Idea in Brief	Experimental Support
Physical	Handel (1975a, 1975b, 1975c, 1975d)	Quantum Theory of 1/f Noise: Interaction of a current carrier with the quantized electromagnetic field	-----
	Tunaley (1974)	1/f spectrum can result if the time between collisions or traps of charge carriers in a resistor has an infinite variance	-----
	Stephany (1975)	Assumption of a complex autocorrelation function results in a 1/f spectrum using the idea that the electron velocity increases by the action of the applied electronic field on the lattice atoms of a solid	-----
	Weissman (1975)	Diffusion controlled thermodynamic fluctuations produce 1/f spectrum for resistors whose surfaces have sharp corners	-----
	Malakhov (1975)	1/f noise in radioelements due to the eroding of the potential barriers by the diffusion of carriers	-----
Statistical	Bernamont (1937)	Generation of 1/f noise by the superposition of the shot noise spectra with a time constant distribution	-----
	Barnes (1966)	Generation of 1/f noise from white noise by the method of fractional order of integration	-----

TABLE III (Continued)

Approach	Reference	Basic Idea in Brief	Experimental Support
Statistical	Halford (1968)	Constraints, necessary and sufficient conditions for the time dependent perturbations that involve time constants and which generate a $1/\|f\|^{\alpha}$ spectrum	-----
Phenomeno-logical	Bell (1955)	1/f noise due to the fluctuations in the number of charge carriers	-----
	Van der Ziel (1966)	Prediction of the life time of a transistor by 1/f noise measurements	-----
	Offner (1970)	1/f noise generated due to random walk plus drift of carriers	-----
	Teitler (1970)	1/f noise as a fluctuation of the local reference level	-----
	Bloodworth (1971)	Possible relation between 1/f noise and drift in MOS transistors	-----
	Ringo (1972)	Possible relation between 1/f noise and drift in Zener diodes	-----
Empirical	Hoppenbrouwers (1970)	1/f noise is inversely proportional to the total number of free carriers in a resistor	Metal Film Spreading Resistors: Hoppenbrouwers (1970) Epitaxial Silicon: Hooge (1970)
	Conti (1970)	Characterization of the "Surface" and the "Bulk" sources of 1/f noise in bipolar transistors	NPN Planar Transistors: Conti (1970)

noise. Almost all major approaches up to date are included. The central theme of each approach is given in brief. The electronic system where a particular approach in certain cases has been applied with some experimental support is mentioned, and a reference is given.

4.4 Evaluation of Major 1/f Noise Theories in Terms of the Stipulated Hypotheses

In this Section several approaches to explain electronic 1/f noise as tabulated in Table III are discussed briefly and their results evaluated, wherever possible, in terms of the hypotheses cited in Chapter II.

The "Flicker effect" as proposed by Schottky (1926) for vacuum tubes predicts the noise spectrum to be a constant at low frequencies, with a $1/f^2$ roll off at high frequencies. This behavior is calculated considering the small shot effect resulting due to the interaction of foreign atoms with the surface of the cathode emitting electrons responsible for conduction. 1/f noise is explained to be a region between the constant value and the $1/f^2$ dependence of the noise spectrum. The "Flicker effect" is probably restricted to vacuum tubes only. In addition, computations made by Schottky (1926) on the basis of the "Flicker effect" do not agree too well with the fine set of earlier experiments conducted by Johnson (1925).

The general linear theory of fluctuations arising from diffusional mechanisms as proposed by Richardson (1950) attempts to calculate the spectral density for the electrical resistance when it is linearly coupled to a diffusing medium (particles or heat) undergoing thermally excited fluctuations. Consideration of several types of coupling results in the general result for the power spectral density $S(f) \propto D^{-n-\nu/2} f^{-1+n+\nu/2}$, where D is the diffusion constant, ν is the dimension of the wave number $(\vec{k})$ space and n is an integer. Also $-1 < 2n + \nu + 1 < 3$. A $S(f) \propto 1/f$ - spectrum results if $\nu = 2$ and $n = -1$. Van Vliet (1957) discusses the inability of the diffusion mechanisms to produce 1/f noise. It is argued and shown through calculations, picking practical values, that the diffusion mechanisms must result in a flat noise spectrum at low frequencies which is not observed experimentally. The difficulty in obtaining a 1/f spectrum at higher frequencies is also pointed out.

The statistical model of Bernamont (1937) suggests that it is possible to obtain a 1/f spectrum for a range of frequencies if one considers the superposition of the shot noise spectra of the type $\dfrac{\tau}{1 + (2\pi f\tau)^2}$ (τ = time constant for a physical process), with the τ's distributed as $1/\tau$ in the range $\tau_1 < \tau < \tau_2$. Thus,

$$S(f) \propto \int_{\tau_1}^{\tau_2} \frac{\tau}{1 + (2\pi f\tau)^2} \frac{1}{\tau} d\tau$$

(see Van der Ziel, 1959, Chapter 5)

or $$S(f) \propto \frac{1}{f} (\tan^{-1}2\pi f\tau_2 - \tan^{-1}2\pi f\tau_1)$$

or $$S(f) \propto \begin{cases} \tau_2 & \text{for } 2\pi f\tau_2 \ll 1 \\ 1/f & \text{for } 2\pi f\tau_1 < 1 < 2\pi f\tau_2 \\ 1/f^2 & \text{for } 2\pi f\tau_1 \gg 1. \end{cases}$$

This indirect method of generating a 1/f spectrum is employed in McWhorter's surface model (McWhorter, 1957; Van der Ziel, 1959, Chapter 5) and also in Müller's thermal feedback model (Müller, 1970, 1971, 1974a, 1974b). In McWhorter's approach, the process of surface tunneling of the charge carriers to and from the traps located in the oxide is shown to result in a $1/\tau$-type trapping time constant distribution. In Müller's method, the thermal time constants associated with the mechanism of heat conduction from the device to the ambient on a microscopic level, and which modulate the properties of the device through thermal feedback, are argued to be responsible for the generation of 1/f noise. The ability of the thermal conduction mechanism to produce a spread in the thermal time constants, especially with a $1/\tau$ distribution, is rather doubtful, as recent discussions have revealed (Bilger, 1976). Nevertheless, both approaches as adopted by McWhorter and Müller, predict flattening of the noise spectrum at low frequencies (a property of Bernamont's statistical model) which is not observed experimentally.

Bakanov (1965) and Ganefel'd (1969) suggest low frequency solid state (electron-hole) plasma oscillations resulting in an anisotropic current instability as a possible cause for excess noise in semiconductors. The $1/f^{\alpha}$ character of excess noise is not evident from their analysis. Moreover, an experimental support is lacking.

The thermal diffusion mechanism employed by Clarke (1974) gives the noise voltage spectral density $S_V(f) \simeq \dfrac{V^2\beta^2T^2}{3N_A\left[3 + 2\ln\dfrac{\ell_1}{\ell_2}\right]f}$ for thin metal film structures, where β is the temperature coefficient of resistance R ($= \frac{1}{R}\frac{\partial R}{\partial T}$), T is the absolute temperature (K), V is the d.c. voltage bias on a sample with length ℓ_1 and width ℓ_2 and N_A is the number of atoms in the sample. This value of $S_V(f)$ is computed by approximating $S_T(f)$ (spectral density of the temperature fluctuations) to be a constant for $f \ll f_1$ and proportional to $f^{-3/2}$ for $f \gg f_2$, where f_1 and f_2 are the characteristic frequencies calculated by using the relation $f_i = D/2\pi\ell_i^2$ for a thermal diffusion process with the diffusion constant D. For the frequency range $f_1 < f < f_2$, $S_T(f)$ is merely postulated to be proportional to 1/f. $S_T(f)$ and $S_V(f)$ are related by the expression $S_V(f) = V^2\beta^2 S_T(f)$. The variance of temperature $\overline{(\Delta T)^2} = \int_0^\infty S_T(f)\, df$ is obtained from the thermodynamic result $\overline{(\Delta T)^2} = kT^2/C_V$ where C_V is the specific heat of the sample ($\simeq 3N_A k$, k = Boltzmann constant, Reif, 1965). It should be pointed out that no attempt has been made in this analysis to explain the 1/f region of $S_V(f)$. The assumption of $S_V(f)$ being a constant at low frequencies (as one would expect for the diffusion mechanisms - see Van Vliet, 1958) is rather uncomfortable from the experimental standpoint. Moreover, for a sample which has $\beta = 0$, the Clarke effect is zero ($S_V(f) = 0$). Such a sample may still show 1/f noise considering the universal occurrence of 1/f noise. Nevertheless, this thermal diffusion model has good agreement with measurements carried out by Clarke (1974) and Hoppenbrouwers (1970).

The Quantum theory of Handel (1975a, 1975b, 1975c, 1975d) is an interesting generalized approach to the problem of 1/f noise which applies to all systems involving transport of carriers. In this approach the interaction of a current carrier with the quantized electromagnetic field is considered. Scattering with bremsstrahlung which modulates the current is claimed to be a cause for 1/f noise. The noise current spectrum $S_I(f)$ as calculated by this model possesses a 1/f dependence down to an arbitrarily low value of frequency f_0, and is a constant for $f < f_0$. An estimation or interpretation of f_0 is missing. A numerical computation made by Handel (1975c) gives the possible minimal value of 1/f noise as 55% in excess of that measured by Hoppenbrouwers (1970). This difference is not explainable.

The treatment adopted by Tunaley (1974) considers the spectral density of the current fluctuations in a resistor under conditions when the time between collisions or traps has a finite variance. It is claimed that when the variance is infinite, a 1/f spectrum can result. The translation of this model into a physical situation where one could have infinite variance in the time between collisions or traps of carriers is rather difficult to comprehend. Moreover, as yet, there is no experimental comparison available for Tunaley's model.

An interpretation of 1/f noise in semiconductors and thin metal films is attempted by Stephany (1975) in terms of a bulk source considering a complex autocorrelation function for the current fluctuations which results if the units of the autocorrelation function are assumed to be those of power. Two noise generating currents are considered; the electron current (I_-) and the so-called "lattice current" (I_+). An assumption of a simple relaxation process with a carrier life time τ_0 describing the noise generating mechanism of both currents, results in the spectrum of the type $S(f) = \frac{4I_0^2R_0}{N_0}\left[\frac{\tau_0 + 4\pi f\tau_0^2\left(\frac{m_-}{m_+}\right)}{1 + (2\pi f\tau_0)^2}\right]^{1/2}$, where I_0 = d.c. current, R_0 = resistance of the sample, N_0 = total number of carriers, m_- = effective mass of the electrons and m_+ = effective mass of the lattice atoms. In the limit when $\tau_0 \rightarrow \infty$,

$S(f) \simeq \frac{8I_0^2R_0}{2\pi N_0}\left(\frac{m_-}{m_+}\right)^{1/2}\frac{1}{f}$ for $2\pi f >> \frac{1}{\tau_0}\left(\frac{m_+}{m_-}\right)^{1/2}$. It should be pointed out that the assumption of $\tau_0 \rightarrow \infty$ is not conceivable physically and, therefore, the 1/f spectrum as obtained by Stephany's model is not so clear.

Richardson's generalized theory for the calculation of noise power spectral densities for diffusion mechanisms (Richardson, 1950) is applied by Weissman (1975) for the singular case where the surfaces of a spreading resistor have sharp corners. It is shown that the noise produced by thermodynamic fluctuations (e.g. in the number of carriers) is of the type $S(f) \propto 1/f$ for $f > f_1$ and $S(f)$ = constant for $f < f_1$, where f_1 is a value of frequency dependent upon the geometry of the resistor and the diffusion constant (D). Again, using this model and considering the physical values of D, it is difficult to interpret the 1/f character at very low frequencies (see Van Vliet, 1958). Moreover, since D is strongly dependent on the temperature, Weissman's model predicts the strong variation of f_1 with the temperature, which is not observed experimentally.

The interesting recent approach outlined by Malakhov (1975) considers the presence of 1/f noise in thermodynamic nonequilibrium systems as natural as the occurrence of thermal or shot noise in equilibrium systems. It is proposed that 1/f noise in radioelements is generated during the eroding (aging) of the potential barriers. A calculation is made considering the aging of a sharp p-n junction caused by the diffusion of doping impurities. This approach, since it considers a long term change in a property of a system (the conductance in a p-n junction), shows promise for the interpretation of 1/f noise (see Section 4.2). However, by the method described by Malakhov the 1/f character of the fluctuations is not spelled out clearly.

The statistical model of 1/f noise proposed by Barnes (1966) is a mathematical exercise, where it is shown that by the method of the fractional order of integration one could generate a 1/f spectrum from the white noise spectrum. If x(t) represents a white noise process with $S_X(f) = c$ (c = constant), then the power spectral density of the m-fold integral of x(t) is given by $S_X^{(m)}(f) = c/|f|^{2m}$. A 1/f spectrum results if 2m = 1 (fractional order integration). It is difficult to see how this model could be used in a physical situation to provide an explanation for 1/f noise.

The criterion discussed by Halford (1968) stipulates the necessary and sufficient conditions for the generation of $1/|f|$ spectrum from the so called "reasonable time dependent perturbations" which occur at random with a distribution of time constants. It is shown that under these specific conditions a $1/|f|$ type spectrum can be obtained for an arbitrary but a finite range of frequencies. Physically, whether the perturbations are assumed to possess a time constant distribution is not known. Also, in order to explain 1/f noise spectrum for a large range of frequencies, one would require a huge range of time constants which are not realizable. Nevertheless, the models of Bernamont (1937), McWhorter (1957) and Müller (1970, 1971, 1974a, 1974b) satisfy the general criterion of Halford in the statistical sense.

The phenomenological approaches as tabulated in Table III discuss certain aspects of 1/f noise in relation to certain properties of specific systems with no attempt to explain the 1/f character of the noise spectrum. In the approach by Bell (1955) it is shown, through rough estimates for the noise in a resistor, that 1/f noise depends on the total number of carriers N more strongly than 1/N. A nonequilibrium process for the generation of 1/f noise is suggested. Van der Ziel (1966) suggests that the relative life expectancy of transistors can be estimated by 1/f noise measurements. It is shown that 1/f noise increases after a transistor is artificially aged at elevated temperatures. Offner (1970) provides a computer simulation of 1/f noise, whereby the mechanism for 1/f noise is claimed as a random walk plus drift. The 1/f character is, however, not clear from the numbers published by Offner. The approach outlined by Teitler (1970) considers two distinct aspects of the fluctuations, namely, the fluctuations around a reference level local in time; and the fluctuations of the local reference level. The fluctuations about the local reference level are just those that give rise to white noise. However, the fluctuations of the local reference level are claimed to result in 1/f noise in nonequilibrium systems if a specific nonlinear coupling between the different frequency regions of the noise power is assumed. The method is rather ambiguous. An attempt to answer the question of a possible relation between drift and 1/f noise is made by Bloodworth (1971) and Ringo (1972). Bloodworth suggests by the variance calculations of noise in MOS transistors, that sometimes 1/f noise may be the main cause of drift for periods between a day and a year.

Ringo compares the predicted mean square voltage drift, as obtained from 1/f noise measurements, with the measured mean square voltage drift for reference diodes. In certain cases a correlation between the predicted and the measured values is found to exist, although the measured values are approximately a factor 10^3 higher than the predicted values.

The beautiful set of measurements made by Hoppenbrouwers (1970) and Hooge (1970) in a wide variety of spreading resistors (metal films and epitaxial silicon) suggest a remarkable empirical relation for 1/f noise. It is shown that $\overline{\left(\frac{\Delta R}{R}\right)^2} = \frac{2x10^{-3}}{N_{tot}} \frac{\Delta f}{f}$, where $\overline{\left(\frac{\Delta R}{R}\right)^2}$ is the mean square relative fluctuations in the resistance R of a homogeneous sample with N_{tot} total number of carriers and Δf is the bandwidth of measurement. Several theoretical attempts (Handel, 1975c; Stephany, 1975) to explain this empirical relation have not yet been successful.

Finally, in the empirical approach by Conti (1970) extensive measurements are taken on NPN planar transistors to find out the spatial location of the 1/f noise sources. It is claimed that the planar transistors possess a 1/f noise source which mainly depends on the surface potential and the collector current. For the gated transistors and diodes, it is shown that the majority of 1/f noise comes from the bulk. The major source of 1/f noise is ascribed to a minority carrier recombination process in the emitter region.

4.5 Need for a General 1/f Noise Theory

From the discussion given in the previous Section, the following two shortcomings of the present state of 1/f noise theories are evident:-

(a) Almost all approaches deal with the transport of carriers in electronic systems and, in addition, are restricted in their application to only special kinds of systems.

(b) Most of the theories do not satisfy the hypothesis B (see Chapter II).

Considering the universal occurrence of 1/f noise in various kinds of systems, a general approach to the 1/f noise problem is needed, whereby a class of fluctuations, which yield the $1/f^{\alpha}$ type spectrum satisfying the hypotheses of Chapter II, should be investigated. Such an attempt is made in Chapters V and VI.

CHAPTER V

ON THE STATIONARITY OR THE NONSTATIONARITY OF 1/f NOISE

The basic hypotheses as developed in Chapter II are used in this Chapter as a building block to decide whether the process generating 1/f noise should be stationary or nonstationary. Using analytical arguments it is shown that, in order that the hypothesis B (Chapter II) be satisfied, the 1/f mechanism must be nonstationary. This is an interesting result in the sense that 1/f noise then provides probably a first experimental example of a nontransitory type nonstationary stochastic process. Moreover, the universal occurrence of 1/f noise in systems suggests the presence of nonstationary fluctuations as a fundamental property of a large class of systems.

5.1 1/f Noise and Stationarity

Following the definitions given in Chapter III, $R_{XX}(t_1,t_2) = R_{XX}(\tau)$ $(\tau \equiv t_2 - t_1)$ for a stationary stochastic process x(t). The relationship between $R_{XX}(\tau)$ and $S_{XX}(f)$ is given by the Fourier Transform pair (Papoulis, 1965, Chapter 10):

$$S_{XX}(f) = \int_{-\infty}^{+\infty} R_{XX}(\tau) \exp(-j2\pi f\tau)\, d\tau \text{ , and} \qquad (3.4.1)$$

$$R_{XX}(\tau) = \int_{-\infty}^{+\infty} S_{XX}(f) \exp(j2\pi f\tau)\, df \tag{5.1.1}$$

A necessary and sufficient condition for $R_{XX}(\tau)$ of the process x(t) (Bendat, 1966, p. 72) is

$$R_{XX}(-\tau) = R_{XX}(\tau) \quad , \tag{5.1.2}$$

i.e. $R_{XX}(\tau)$ is an even function of τ.

In order to investigate the capability of a stationary stochastic process x(t) to generate a 1/f spectrum satisfying the hypothesis B (Chapter II) we determine the asymptotic behavior of $S_{XX}(f)$ for $f \to 0$. Taking the derivative of Equation (3.4.1) using Leibniz' theorem and estimating the result for $f \to 0$ we have,

$$\lim_{f\to 0} \frac{dS_{XX}(f)}{df} = \lim_{f\to 0} \int_{-\infty}^{+\infty} \frac{\partial}{\partial f} \left[R_{XX}(\tau) \exp(-j2\pi f\tau) \right] d\tau$$

$$= \lim_{f\to 0} \int_{-\infty}^{+\infty} -j2\pi\tau\, R_{XX}(\tau) \exp(-j2\pi f\tau)\, d\tau$$

$$= -j2\pi \int_{-\infty}^{+\infty} \tau\, R_{XX}(\tau)\, d\tau \quad . \tag{5.1.3}$$

Now since $R_{XX}(\tau)$ is necessarily even (Equation (5.1.2)),

$\int_{-\infty}^{+\infty} \tau R_{XX}(\tau)\, d\tau = 0$, i.e. for the stationary stochastic process x(t),

the power spectral density $S_{XX}(f)$ must "flatten" at $f\to 0$ $\left(\frac{dS_{XX}(f)}{df} \right.$ must vanish). This violates the hypothesis B which is based on the measurements of 1/f noise in a variety of systems where the "flattening" is not observed even at very low frequencies. Of course, one could question the hypothesis B by arguing that the "flattening" could be observed if one pushed further down the lower frequency end of the noise power spectral density measurement. Such an argument has the difficulty of preconceiving the idea of stationarity and trying to explain the contradictory experimental evidence with conjectural results of further experimentation at the cost of the experimenter's patience. In conclusion, the present experimental documentation of 1/f noise suggests more strongly the acceptance of the hypothesis B, thus pointing to the result that 1/f noise cannot be a consequence of a stationary stochastic process.

5.2 1/f Noise Should Result From a Nonstationary Stochastic Process

With reference to Figure 12, and considering the arguments provided in Section 5.1, it is evident that an explanation of the observed phenomenon of 1/f noise falls beyond the domain of stationary (and also wide-sense stationary - see Chapter III) stochastic processes, or in other words, 1/f noise should result from a nonstationary stochastic process.

The nonstationary character of the process generating 1/f noise can now be employed as a new viewpoint to understand why most of the theories, as discussed in Chapter IV, have not been successful to explain 1/f noise. Since most of the approaches involve stationary kinetics, they invariably result in the noise power spectrum flattening at low frequencies which is not acceptable from the experimental standpoint (hypothesis B). It is interesting to mention that an exhaustive survey of the literature on the fluctuation phenomena reveals the study of stationary processes in almost all cases with exception of a few situations where the nonstationarity is dealt with only as a transient (asymptotic stationary processes, Donati (1971)). In this perspective, 1/f noise is probably a first example of a physically realizable non-transitory type nonstationary phenomenon which exists in a wide variety of systems.

One important aspect of the nonstationary stochastic process resulting in 1/f noise should be emphasized. The criterion set forth in the hypothesis C (Chapter II), namely, the measured noise spectra are stable, suggests that although the basic process for the generation of 1/f noise is nonstationary, the measured power spectral density is nevertheless independent of time. Some properties of such a process are investigated in Chapter VI.

CHAPTER VI

SOME FEATURES OF A NONSTATIONARY PROCESS YIELDING 1/f NOISE

In fluctuation theory the study of nonstationary stochastic processes is rather a new subject. As was mentioned in Chapter III, a nonstationary process has time dependent probability density functions and, therefore, in general the statistical properties, e.g. mean, variance, autocorrelation etc., are also time dependent. This makes the characterization of nonstationary processes cumbersome and difficult not only from the experimental standpoint but also from the mathematical and conceptual point of view. A neat elegant scheme to characterize nonstationary stochastic processes does not exist at present.

In this Chapter the measured noise power density spectrum of the type $1/f^{\alpha}$ in a general system is interpreted in terms of a nonstationary stochastic process. The concept of power spectral density is applied in the operational sense (see Chapter III) in terms of the methodology of measurement. Some of the properties of the nonstationary process which should satisfy the hypotheses A, B and C (Chapter II) are developed. Finally, an example of such a process is given.

6.1 Noise Power Spectral Density Measurement Methodology

The concept of power spectral density, which is so powerful in studying stationary stochastic processes, becomes shaky in its applicability and interpretation in relation to nonstationary processes. The spectral distribution of nonstationary fluctuations is not well understood in the analytical sense. Thus, given a nonstationary process is responsible for the generation of 1/f noise, the methodology of the noise power spectral density measurement should be carefully scrutinized in order to arrive at an operational interpretation of the concept of power spectral density. It should be mentioned that the only reason behind picking the idea of power spectral density for characterizing the nonstationary stochastic process generating 1/f noise is to explain an immense amount of experimental measurements which are stable and reproducible. However, one could adopt an alternative scheme for characterizing the same nonstationary process, whereby probably one could totally do away with the spectral description.

Figure 13 depicts the general methodology of the power spectral density measurement. We consider here a nonstationary stochastic source y(t) defined over the time interval between $-\infty$ and t and zero elsewhere. We assume that the blocks representing the measurement equipment perform ideal mathematical operations. The average energy contained in the component of frequency f of the process y(t) can be expressed (Lampard, 1954) as :

$$E_{YY}(t,f) = \overline{Y(t,f)\, Y^*(t,f)}$$

$$= \overline{\left(\int_{-\infty}^{t} y(t_1)\, \exp\,(-j2\pi f t_1)\, dt_1\right)\left(\int_{-\infty}^{t} y(t_2)\, \exp\,(j2\pi f t_2)\, dt_2\right)}$$

$$= \int_{-\infty}^{t}\int_{-\infty}^{t} \overline{y(t_1)\, y(t_2)}\, \exp\,(j2\pi f(t_2 - t_1))\, dt_1\, dt_2 \qquad (6.1.1)$$

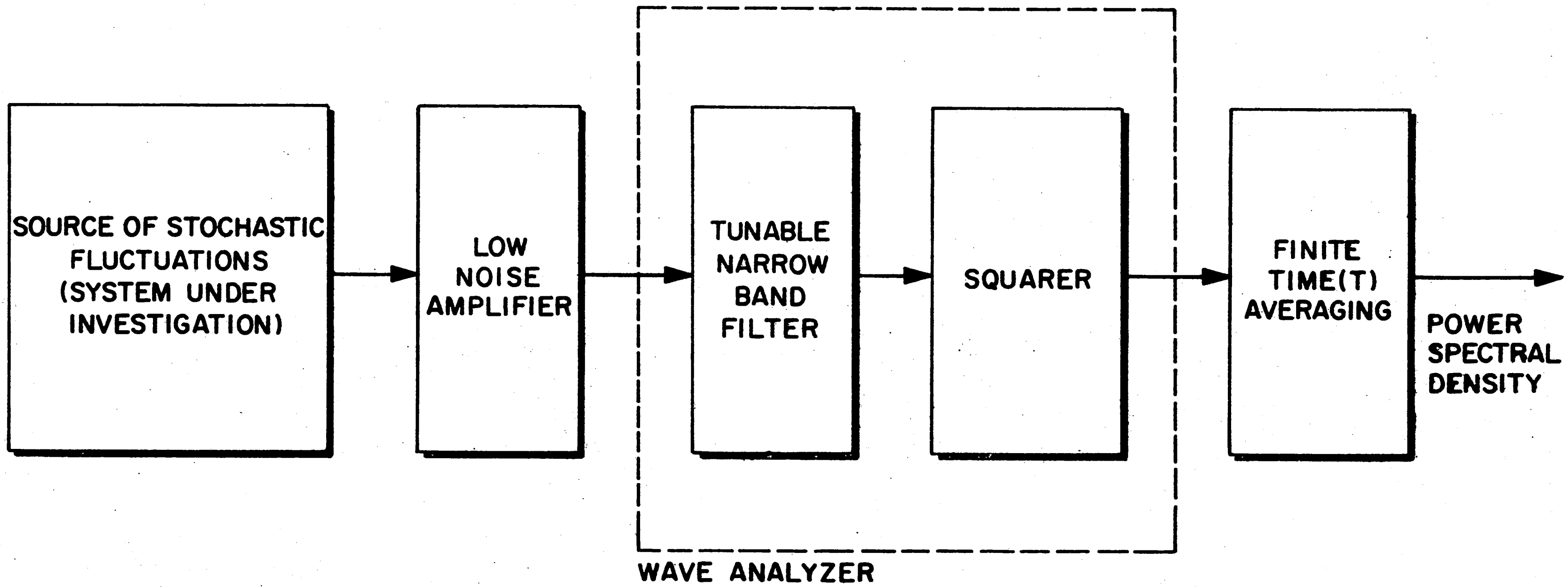


Figure 13. Block Diagram Representing the Noise Power Spectral Density Measurement Methodology

Here $Y(t,f)$ is the running Fourier Transform of $y(t)$ over the limits $-\infty$ and t. It is also assumed that the operations of time integration and ensemble averaging can be interchanged (Lathi, 1968, Chapter 3). The output of the Squarer (Figure 13) can now be interpreted in terms of the instantaneous power spectral density $S_{YY}(t,f)$ as:

$$S_{YY}(t,f) = \frac{\partial}{\partial t} E_{YY}(t,f)$$

$$= \frac{\partial}{\partial t}\left(\int_{-\infty}^{t} \int_{-\infty}^{t} \overline{y(t_1)\, y(t_2)} \exp\left(j2\pi f(t_2 - t_1)\right) dt_1\, dt_2 \right) . \tag{6.1.2}$$

If one transforms the variables t_1 and t_2 by writing $t_1 = t$ and $t_2 = t + \tau$, $S_{YY}(t,f)$ can be related to the autocorrelation function $R_{YY}(t,\tau)$ (Lampard, 1954) as:

$$S_{YY}(t,f) = \int_{-\infty}^{+\infty} R_{YY}(t,\tau) \exp(-j2\pi f\tau)\, d\tau . \tag{6.1.3}$$

The final power spectral density measurement is obtained by taking the finite time (T) average of $S_{YY}(t,f)$ (Figure 13), i.e.

$$S_{YY}(T,f) = \frac{1}{T} \int_{0}^{T} S_{YY}(t,f)\, dt . \tag{6.1.4}$$

It is thus seen that, considering the methodology of the power spectral density measurement, an operational interpretation of the measured power spectral density of a nonstationary process can be made in terms of $S_{YY}(T,f)$.

It is easy to show that the above interpretation of the power spectral density measurement gives the right value for a stationary stochastic process, x(t). For the process x(t), $R_{XX}(t,\tau) = R_{XX}(\tau)$. Hence, considering again ideal measurement equipment, Equations (6.1.3) and (6.1.4) reduce to

$$S_{XX}(t,f) = \int_{-\infty}^{+\infty} R_{XX}(\tau) \exp(-j2\pi f\tau)\, d\tau = S_{XX}(f) \quad (6.1.5)$$

and

$$S_{XX}(T,f) = \frac{1}{T} \int_{0}^{T} S_{XX}(t,f)\, dt$$

$$= \frac{1}{T} \int_{0}^{T} S_{XX}(f)\, dt = S_{XX}(f) \quad . \quad (6.1.6)$$

Since Equation (6.1.5) is the definition of power spectral density, Equation (6.1.6) shows that one indeed measures the power spectral density for the process x(t).

6.2 Some Properties of the Nonstationary Process Possibly Responsible for 1/f Noise

The hypotheses B and C developed on experimental evidence (Chapter II) can now be employed to predict certain properties of the nonstationary process responsible for the generation of 1/f noise in a general system. If one considers a nonstationary process, y(t), which exists for the time $t = T_0$ after the start of observation at t = 0, the stability criterion of the hypotheses C (Chapter II) demands

$$S_{YY}(T_0,f) = \frac{1}{T_0} \int_0^{T_0} S_{YY}(t,f)\, dt = S_{YY}(f) \quad . \qquad (6.2.1)$$

Also, the $1/f^{\alpha}$ character (hypothesis B - Chapter II) requires

$$S_{YY}(T_0,f) = S_{YY}(f) = c/f^{\alpha} \; .$$

$$(c \text{ and } \alpha \text{ are constants} > 0) \qquad (6.2.2)$$

Solutions of Equation (6.2.2) are in general not unique (see Chapter IV - Section 4.2) and are difficult to determine. In other words, there could be several processes y(t) which could satisfy Equation (6.2.2). Nevertheless, Equations (6.2.1) and (6.2.2) are strong conditions, thus suggesting that the nonstationarity of the process y(t) responsible for generating 1/f noise must be of a special kind.

Using Equations (6.1.3) and (6.2.2), a general expression for the autocorrelation function ($R_{YY}(t,\tau)$) can be written for the nonstationary process y(t) responsible for the generation of 1/f noise satisfying the hypotheses A, B and C (Chapter II) as follows:

$$S_{YY}(f) = \frac{1}{T_0} \int_0^{T_0} S_{YY}(t,f)\, dt$$

$$= \frac{1}{T_0} \int_0^{T_0} \left[\int_{-\infty}^{+\infty} R_{YY}(t,\tau) \exp(-j2\pi f\tau)\, d\tau \right] dt$$

$$= \frac{c}{f^{\alpha}} \qquad (6.2.3)$$

It is clear that the nonstationary process y(t) which results in a measured power spectral density of the type $1/f^{\alpha}$ must necessarily satisfy Equation (6.2.3).

6.3 A Possible Time Dependent Autocorrelation Function For the 1/f Noise Mechanism

In this Section a possible solution of Equation (6.2.3) is given and discussed. We consider again a nonstationary process y(t) which exists for the time $t = T_0$ beyond the start of observation at $t = 0$. If the autocorrelation function ($R_{YY}(t,\tau)$) for the process y(t) is given by

$$R_{YY}(t,\tau) = \frac{A}{(1 - t/T_0)^{1/\alpha}} \cos\left[\frac{2\pi f_0 \tau}{(1 - t/T_0)^{1/\alpha}}\right] \tag{6.3.1}$$

(A, f_0, α are constants > 0)

then we can show that Equation (6.2.3) is satisfied for $f_0 < f < \infty$, and $c = A\alpha(f_0)^{\alpha-1}/2$.

From Equations (6.1.3) and (6.3.1)

$$S_{YY}(t,f) = \int_{-\infty}^{+\infty} R_{YY}(t,\tau) \exp(-j2\pi f\tau)\, d\tau$$

$$= \frac{A}{(1 - t/T_0)^{1/\alpha}} \int_{-\infty}^{+\infty} \cos\left[\frac{2\pi f_0 \tau}{(1 - t/T_0)^{1/\alpha}}\right] \exp(-j2\pi f\tau)\, d\tau$$

$$= \frac{A}{2\,(1 - t/T_0)^{1/\alpha}} \left\{ \int_{-\infty}^{+\infty} \exp\left[\frac{j2\pi f_0 \tau}{(1 - t/T_0)^{1/\alpha}}\right] \exp(-j2\pi f \tau)\, d\tau \right.$$

$$\left. + \int_{-\infty}^{+\infty} \exp\left[\frac{-j2\pi f_0 \tau}{(1 - t/T_0)^{1/\alpha}}\right] \exp(-j2\pi f \tau)\, d\tau \right\}$$

$$= \frac{A\pi}{(1 - t/T_0)^{1/\alpha}} \left\{ \delta\left[-2\pi f + \frac{2\pi f_0}{(1 - t/T_0)^{1/\alpha}}\right] \right.$$

$$\left. + \delta\left[-2\pi f - \frac{2\pi f_0}{(1 - t/T_0)^{1/\alpha}}\right] \right\}$$

$$= \frac{A\pi}{(1 - t/T_0)^{1/\alpha}} \left\{ \delta\left[\frac{-2\pi f\,(1 - t/T_0)^{1/\alpha} + 2\pi f_0}{(1 - t/T_0)^{1/\alpha}}\right] \right.$$

$$\left. + \delta\left[\frac{-2\pi f\,(1 - t/T_0)^{1/\alpha} - 2\pi f_0}{(1 - t/T_0)^{1/\alpha}}\right] \right\}$$

$$= A\pi\delta\left[-2\pi f\,(1 - t/T_0)^{1/\alpha} + 2\pi f_0\right]$$

$$+ A\pi\delta\left[-2\pi f\,(1 - t/T_0)^{1/\alpha} - 2\pi f_0\right]$$

$$= A\pi T_0 \delta\left[2\pi f_0 T_0 - 2\pi f T_0\,(1 - t/T_0)^{1/\alpha}\right]$$

$$+ A\pi T_0 \delta\left[-2\pi f_0 T_0 - 2\pi f T_0\,(1 - t/T_0)^{1/\alpha}\right] . \tag{6.3.2}$$

In deriving Equation (6.3.2) some of the properties of the Dirac delta function (Schiff, 1968, p. 57.) are used. Putting the result of

Equation (6.3.2) in Equation (6.1.4), an expression for $S_{YY}(T_0,f)$ is obtained as follows:

$$S_{YY}(T_0,f) = \frac{1}{T_0} \int_0^{T_0} S_{YY}(t,f)\, dt$$

$$= A\pi \left\{ \int_0^{T_0} \delta \left[2\pi f_0 T_0 - 2\pi f T_0 \left(1 - t/T_0 \right)^{1/\alpha} \right] dt \right.$$

$$\left. + \int_0^{T_0} \delta \left[-2\pi f_0 T_0 - 2\pi f T_0 \left(1 - t/T_0 \right)^{1/\alpha} \right] dt \right\} \qquad (6.3.3)$$

Making a transformation of variables by writing

$$u = 2\pi f_0 T_0 - 2\pi f T_0 \left(1 - t/T_0 \right)^{1/\alpha}$$

and $v = -2\pi f_0 T_0 - 2\pi f T_0 \left(1 - t/T_0 \right)^{1/\alpha}$, Equation (6.3.3) can be expressed as:

$$S_{YY}(T_0,f) = A\pi\alpha \left[\int_{2\pi f_0 T_0 - 2\pi f T_0}^{2\pi f_0 T_0} \delta(u)\, (2\pi f)^{-\alpha} \left(2\pi f_0 - u/T_0 \right)^{\alpha-1} du \right.$$

$$\left. + \int_{-2\pi f_0 T_0 - 2\pi f T_0}^{-2\pi f_0 T_0} \delta(v)\, (2\pi f)^{-\alpha} \left(-2\pi f_0 - v/T_0 \right)^{\alpha-1} dv \right] . \qquad (6.3.4)$$

For realistic values of frequencies $f > 0$, the second integral in Equation (6.3.4) is zero and

$$S_{YY}(T_0,f) = A\pi\alpha\, (2\pi f)^{-\alpha}\, (2\pi f_0)^{\alpha-1}$$

$$= \begin{cases} \frac{A\alpha(f_0)^{\alpha-1}}{2} \frac{1}{f^\alpha} & \text{for } f_0 < f < \infty \\ 0 & \text{for } 0 < f < f_0 \end{cases} \quad (6.3.5)$$

Equation (6.3.5) is same as Equation (6.2.3) with $c = A\alpha(f_0)^{\alpha-1}/2$. Thus a nonstationary process y(t) with the autocorrelation function given by Equation (6.3.1) generates a measured spectrum of the type $1/f^\alpha$.

From the autocorrelation function (Equation (6.3.1)) the following interesting characteristics of the process y(t) possibly responsible for the generation of 1/f noise in a general system can be stated:

(a) Since the second order mean, $\overline{y^2(t)} = R(t,0) = \frac{A}{(1 - t/T_0)^{1/\alpha}}$, the process y(t) has a slow increase in its second order mean with time for large values of T_0. Such a result one would generally expect for a nonstationary process. Also, $\overline{y^2(t)}$ tends to become infinite at $t = T_0$, i.e., at a time beyond which the process ceases to exist.

(b) The process y(t) has the lowest frequency fluctuation component with the frequency value f_0. It is noted that there is no flattening of the spectrum for $f>f_0$. For $0<f<f_0$, there is a discontinuity in the spectrum and $S_{YY}(T_0,f)=0$. f_0 may be a function of certain properties of a particular system.

(c) The constants A and α may be considered to be functions of the system parameters and also of the forces which drive a system out of thermodynamic equilibrium (e.g. external bias on a resistor).

It must be emphasized that Equation (6.3.1) is only a solution of Equation (6.2.3). Of course, there could be other solutions also. Nevertheless, one important aspect is revealed from the example of Equation (6.3.1), namely, it is possible to conceive a nonstationary stochastic process which yields the measured $1/f^{\alpha}$ type spectrum in a general system satisfying the basic hypotheses of Chapter II.

CHAPTER VII

EXPERIMENTAL INVESTIGATION OF 1/f NOISE AND THE LONG TERM CHANGE IN THE RESISTANCE OF AN ION-IMPLANTED RESISTOR - EVIDENCE OF NONSTATIONARITY

Measurements on an electronic system, namely, an ion-implanted resistor, are reported in this Chapter as an example to demonstrate the general characteristics of 1/f noise discussed in the earlier Chapters. It is shown that, within experimental errors, the hypotheses B and C (Chapter II) are satisfied by the measured noise spectrum of the resistor. Under bias, the average of the resistance fluctuations in the resistor for a fixed short duration of time is found to be dependent on time. This aspect is accepted as an evidence for the existence of a nonstationary stochastic process responsible for the generation of 1/f noise.

7.1 Ion-Implanted Resistor Under Investigation

Exploratory measurements of 1/f noise in boron implanted silicon resistors were done previously (Tandon, 1973a, 1973b; Bilger, 1974). Details on the devices are given in the works by Tandon (1973a) and Dill (1971). Here the resistor R_{68} in Device No.27 is selected for experimental investigation. Device No.27 is implanted with 1.0×10^{13} boron ions $/cm^3$ at an energy of 80 keV, which are the intermediate values for the available range of devices. The resistor R_{68} has a box-type structure with the approximate dimensions 406 μm x 25.4 μm x 0.3 μm, and has linear current-voltage characteristics for d.c. voltage bias $\leq$ 8V (Tandon, 1973a).

7.2 1/f Noise Measurements

Figure 14 displays two measurements of the current noise spectrum $S_{II}(f)$ of R_{68} in Device No.27 taken approximately 2½ years apart at a d.c. current I_D = 50.0 μA and a temperature = 298 $\pm$ 0.5 K (Tandon, 1975b). No power was applied to the resistor during the 2½ years storage. As can be seen from Figure 14, within the accuracy of measurements, the noise power spectrum is reproducible. The remeasured value of resistance of R_{68} at a current bias of I_D = 50.0 μA was found to be 93.4 kΩ as compared to 93.2 kΩ 2½ years earlier. This difference is within ~ .3% arising due to the systematic errors in the measurement equipment. Since, the resistance does not change appreciably one would expect the thermal noise to be essentially constant. This is verified by Figure 14, where the white noise levels of the two spectra at high frequencies are seen to be closely equal and corresponding to the theoretical level given by the Nyquist theorem. In addition, the lower frequency ends of spectra down to 10 Hz in Figure 14, which reveal 1/f noise, are also equal within the error of measurement ~ 20%. This result establishes that the measured 1/f noise spectra are reproducible and stable (hypothesis C - Chapter II).

A further low frequency noise measurement on R_{68} in Device No.27, down to approximately 5 x 10^{-5} Hz, is shown in Figure 15 (Boehm,1975a). Here the data are represented in terms of the noise voltage spectrum

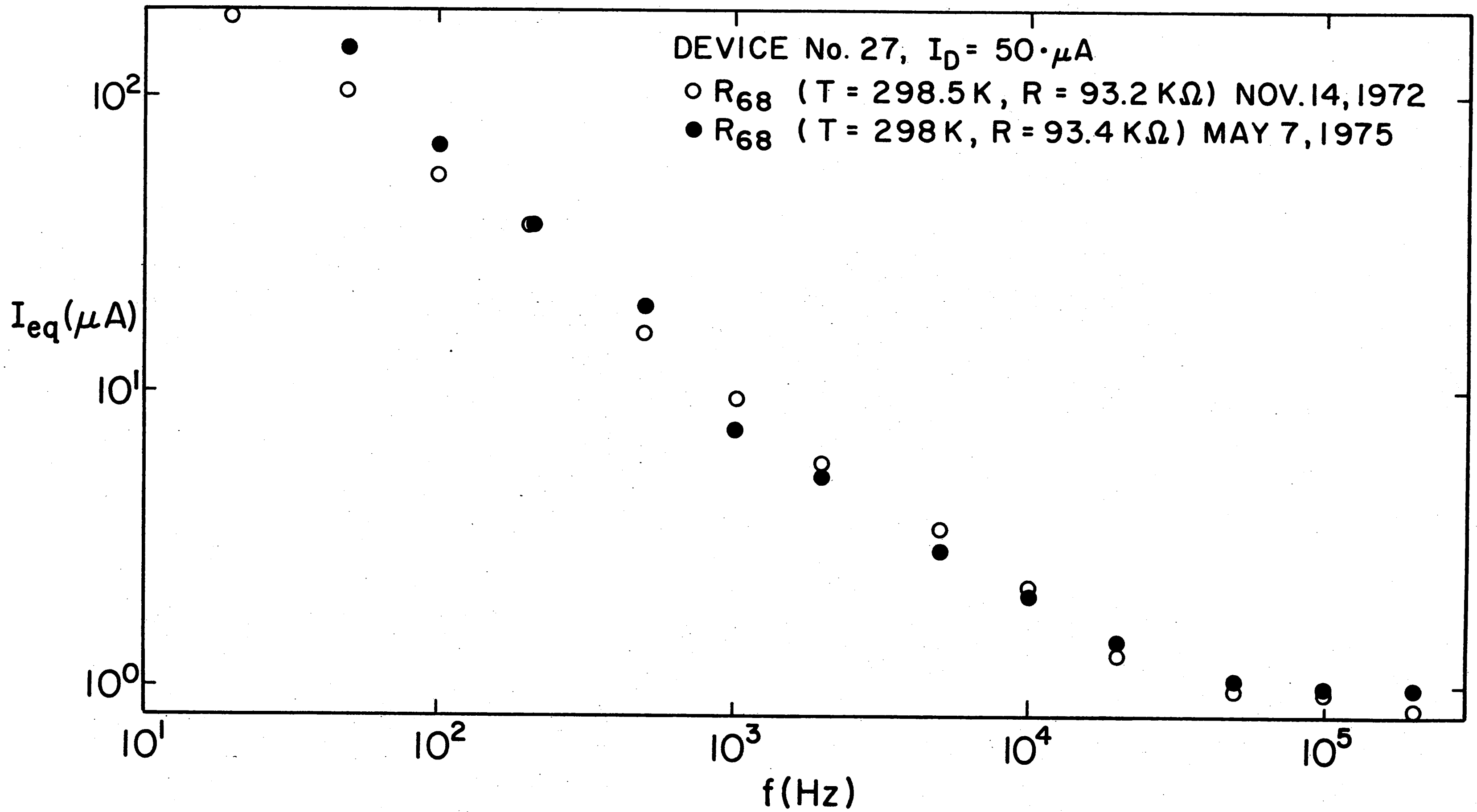


Figure 14. Two Noise Spectra Measured in an Interval of 2½ Years, on Resistor R_{68} in Device No. 27.

Note that $S_{VV}(f) = R^2\overline{I^2}/\Delta f = 2qI_{eq}R^2$

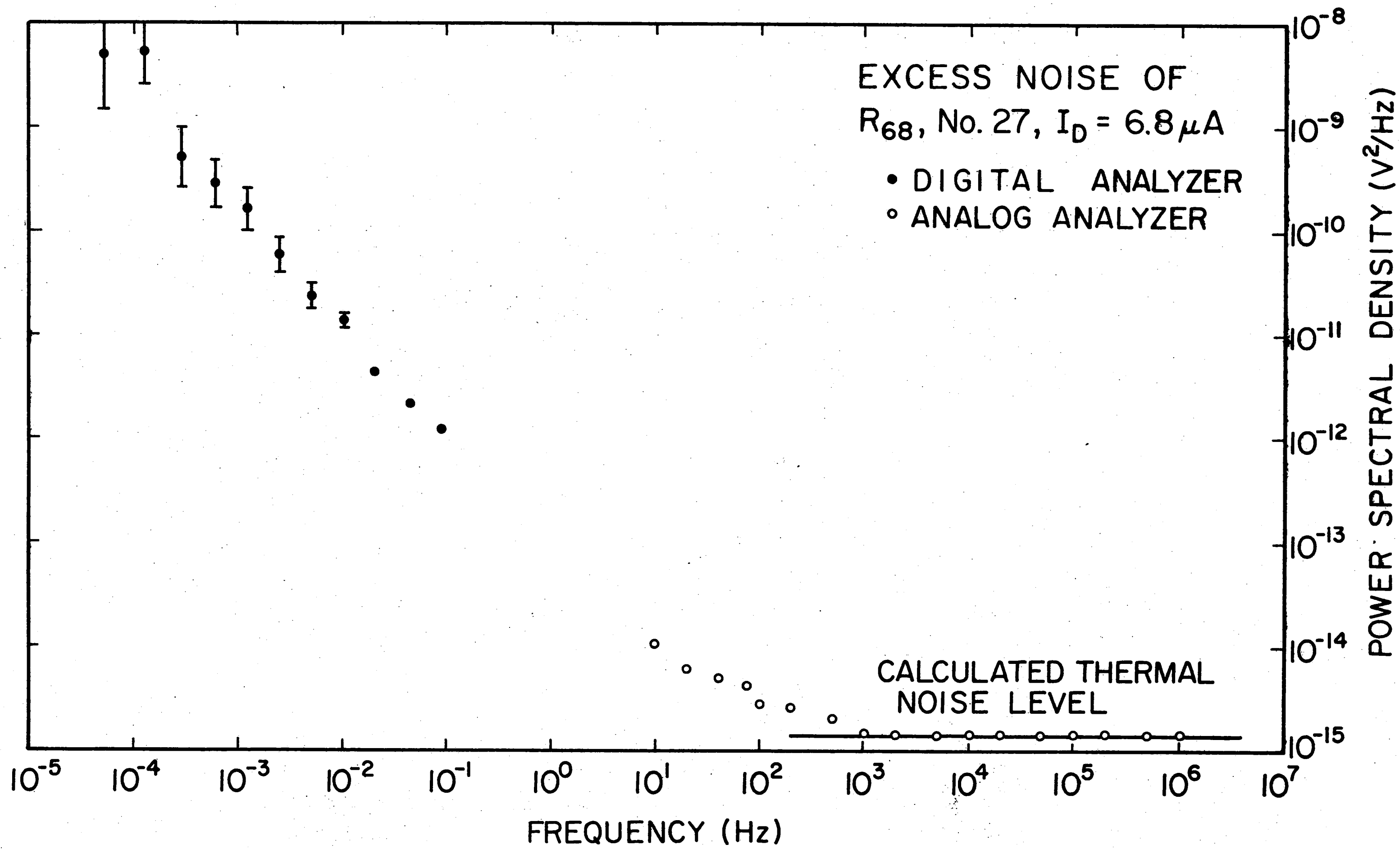


Figure 15. Noise Spectrum $S_{VV}(f)$ of R_{68} in Device No. 27 Measured By an Analog Technique (Open Circles) and a Digital Technique (Dots). $S_{VV}(f) = R^2 S_{II}(f)$

$S_{VV}(f)$ for a lower d.c. bias current $I_D = 6.8\ \mu A$. Important features are revealed from the analysis of Figure 15. Firstly, the noise spectrum of the resistor R_{68} has a high frequency asymptote given by the thermal noise verifiable by the Nyquist theorem, whereas the low frequency end has the $1/f^{\alpha}$ character down to at least 5×10^{-5} Hz. This supports the hypothesis B (Chapter II). Secondly, the high frequency portion of $S_{VV}(f)$ (open circles) evaluated by an analog technique (Tandon, 1973a) matches quantitatively with the low frequency portion (dots) computed by a digital technique (Boehm, 1975a). (One can reasonably assume that no drastic behavior occurs in the range of frequencies 10^{-1} - 10 Hz where unfortunately no data is available due to the shortcomings of the measurement techniques). This demonstrates that the noise spectral estimations (in particular, 1/f noise measurements) are independent of the type of techniques employed for measurement (see Chapter II - hypothesis A).

7.3 Direct Variation of Resistance Measurements

A high precision direct time averaged estimation of voltage fluctuations across the resistor R_{68} in Device No.27 was made employing the bridge arrangement as shown in Figure 16. The d.c. bias voltage across the device was 6.75 V. Since the resistance has a relatively high temperature coefficient ($\Delta R/(R\Delta T) = 3.6 \times 10^{-3}/^{\circ}C$), the resistor was inserted in a thermostat set at 32.25 $^{\circ}$C and whose peak-to-peak temperature fluctuation is less than 1 mK over periods of weeks (Boehm, 1975a). The power supply (with a temperature-stabilized Zener diode) for bridge and preamplifier showed negligible fluctuations over periods of the order of a day. The result of such a measurement, made over a period

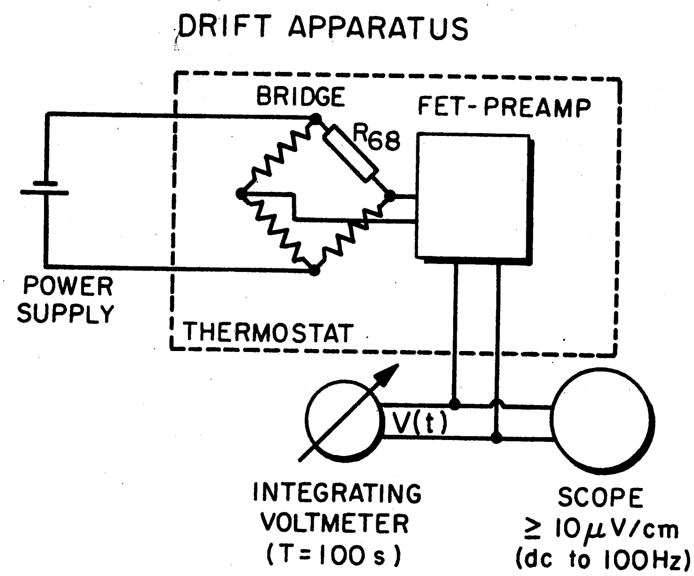


Figure 16. Bridge Arrangement Used to Measure "Drift" of the Resistor R_{68} in Device No. 27

of about 1 day, is shown in Figure 17, where ΔV = integrated value of $v(t)$ over a short time duration of 100 seconds is plotted as a function of time. As can be seen, there is no sign of any periodic variations due to the temperature of the power supply reference. Also, since identical test runs over equivalent lengths of time show a negligible value of ΔV for the system amplifier + power supply + bridge (with R_{68} replaced by an equivalent metal film resistor), the picture presented by Figure 17 can be considered as a "drift" effect existent in the resistor R_{68}. Thus, the resistor R_{68} is found to be unstable under bias. From Figure 17, the slope $(d/dt)\Delta V = 2\ \mu V/min \simeq 3\ mV/day$, which translates into the variation in resistance of R_{68} as $(1/R)(d/dt)R \simeq 5 \times 10^{-4}$ /day under bias. Such a magnitude is significant, for it would give rise to $(\Delta R)/R \simeq 0.5$ in 2½ years, under bias, as compared to $(0.2 \pm 3)\%$ observed variation without bias.

7.4 Relationship Between 1/f Noise and the Variation of Resistance Measurements

It was argued in Section 7.3 that $v(t)$, as measured by the bridge arrangement of Figure 16, represents the resistance fluctuations of R_{68}. It is noted that, because of the band limitation of the Oscillo-scope, $v(t)$ only has frequency components below 100 Hz and which are integrated by the Voltmeter (Figure 16). Since the noise spectrum of R_{68} possesses predominantly the $1/f^{\alpha}$ type noise below 100 Hz (Figures 14 and 15), it is evident that ΔV (Figure 17) is essentially representa-tive of the integrated value of the time record of $1/f^{\alpha}$ type resistance fluctuations inherent in the resistor R_{68}. If one considers an

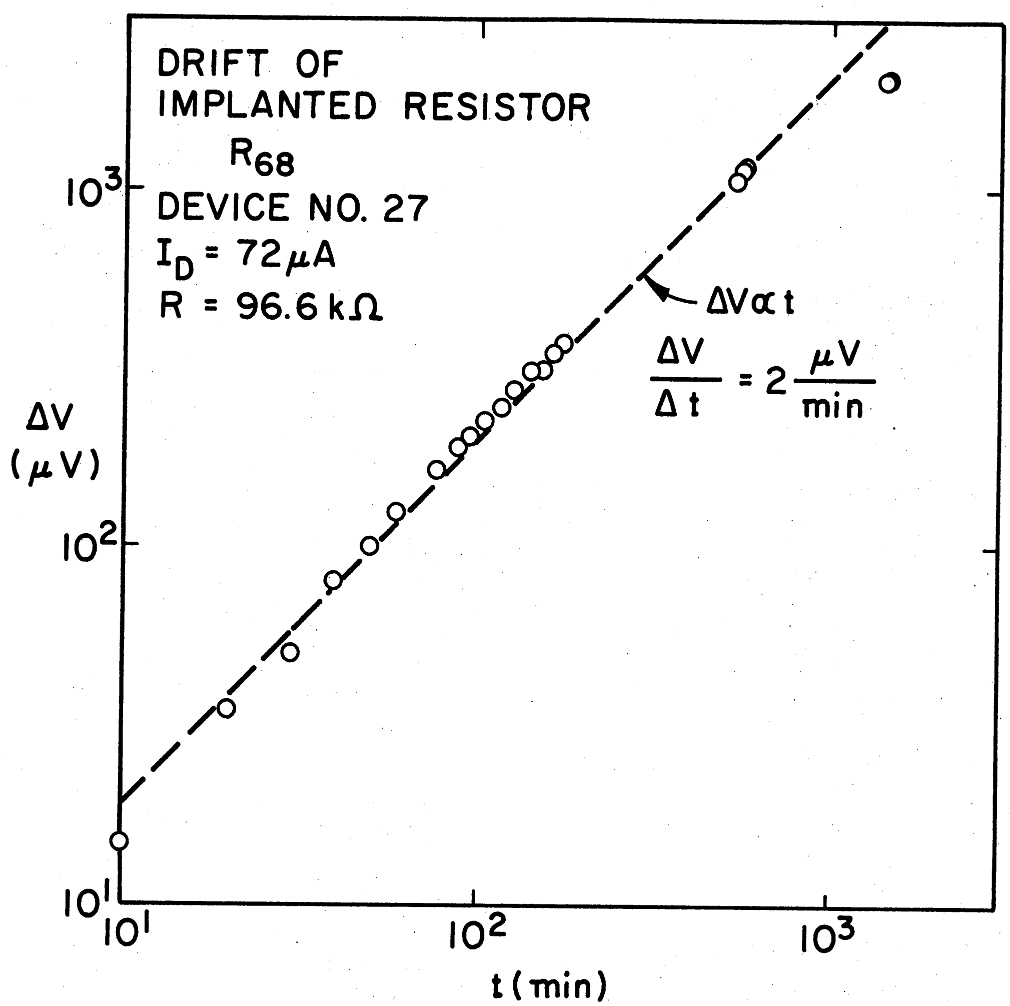


Figure 17. Unbalance of the Bridge Arrangement of Figure 16 As a Function of Time

elementary stochastic process for the resistance fluctuations in R_{68}, $\Delta V(t)$ can be interpreted as being equal to 100 x $\overline{V(t)}$ (V-sec), where $\overline{V(t)}$ is the ensemble average of the stochastic variable v(t). Thus, as $\Delta V(t)$ is a function of t (Figure 17), one finds that the stochastic process for the resistance fluctuations in R_{68} responsible for the $1/f^{\alpha}$ type measured power spectral density is nonstationary. (If $\overline{y(t)}$ is a function of time for a stochastic process y(t), then y(t) is nonstationary - see Chapter III). It should be realized, on the contrary, that if v(t) were stationary one would expect $\Delta V(t)$ to be a constant and independent of time.

The fact that $\Delta V(t)$ is proportional to t (Figure 17), can be considered as a sufficient proof for the existence of a nonstationary process in R_{68} which is responsible for the $1/f^{\alpha}$ type measured power spectral density. However, the kind of the nonstationary process cannot be realized by the mere information of the dependence of $\Delta V(t)$ on t. To establish the specific nonstationary process that exists in R_{68}, and which results in the measured $1/f^{\alpha}$ type spectrum, additional information e.g. the second order mean, autocorrelation etc. of v(t) is needed. Such measurements have not been done. Nevertheless, the conclusion that the process generating the $1/f^{\alpha}$ type measured noise spectrum is nonstationary, which was derived using general analytical ideas in Chapter V, is verified here experimentally at least for the case of an electronic system, namely, an ion-implanted resistor.

The "drift" effect, which involves genuine resistance change of R_{68} (Figure 17) under bias, can now be interpreted as the first order ensemble average of a more general nonstationary process existing in the resistor. In this perspective, if the nonstationary process is known, then the "drift" problem is solved. From the physical standpoint, a model for such a nonstationary process is not available and cannot be easily deduced from this study.

CHAPTER VIII

CONCLUSIONS

A general methodical approach to explain 1/f noise present in a wide variety of systems, not necessarily electronic, has been undertaken in this thesis. Recognizing that the evidence for the existence of 1/f noise rests purely on empirical grounds, experimental information, which reveals the $1/f^{\alpha}$ dependence in the measured noise power spectral density of various systems, is examined. On the basis of the observed features of the measured 1/f noise in a large class of systems, a general set of hypotheses for 1/f noise is derived (Chapter II). It is emphasized that the measured 1/f noise in systems and the possible approaches to explain it should satisfy these hypotheses.

Various possible attempts made in the past to explain 1/f noise are classified and discussed (Chapter IV). Effort is made to include the up-to-date information. It is recognized that almost all of the approaches have been catered towards electronic devices and, therefore, it is difficult to see how they can be employed to explain 1/f noise in other systems. Moreover, since most of the approaches up to the present do not satisfy one or more of the hypotheses postulated in Chapter II, a need for a general 1/f noise theory is realized.

In an effort to stipulate features of a future general 1/f noise theory, the question whether 1/f noise results from a stationary or a nonstationary stochastic process is answered (Chapter V). The question of possibly "flattening" of the 1/f noise spectrum at the lower frequency end is rejected as being academic in favor of the hypothesis B (Chapter II). An analytical treatment of the stationarity of stochastic processes together with the acceptance of the hypothesis B reveals that a true $1/f^{\alpha}$ type noise power spectrum cannot be obtained from a stationary stochastic process. It is, therefore, proposed that 1/f noise should result as a consequence of a nonstationary stochastic process.

Features of a general nonstationary stochastic process which could explain 1/f noise are explored (Chapter VI). Although the application of the concept of power spectral density to nonstationary processes is shaky, yet an operational interpretation of the measured power spectral density is proposed by considering the methodology of measurement and the concept of instantaneous power spectral density. A necessary condition (Equation (6.2.3)) for the autocorrelation function of a nonstationary process which would yield the $1/f^{\alpha}$ type measured power spectrum is developed. It is emphasized that a nonstationary process for 1/f noise should satisfy this condition. Since Equation (6.2.3) is not sufficient there could be several nonstationary processes that could satisfy it. A possible solution of Equation (6.2.3) in terms of a time dependent autocorrelation function of a nonstationary stochastic process is proposed and is shown to result in a $1/f^{\alpha}$ type measured power spectrum. Although a physical or mathematical interpretation, in terms of the time dependent probability density functions for such a solution is lacking, it nevertheless demonstrates that is possible to conceive a nonstationary stochastic process which generates the $1/f^{\alpha}$ type measured power spectrum in a general system satisfying the basic hypotheses of Chapter II.

As an experimental proof to the nonstationarity of the 1/f phenomenon, an electronic system (an ion-implanted resistor) is picked as an example (Chapter VII). The measurements on 1/f noise, when the resistor is biased, satisfy the basic hypotheses of Chapter II. The resistor is found to be unstable under bias and the measured "drift" is considered to be a sufficient proof for the existence of a nonstationary stochastic process in the resistor. Since both 1/f noise and the "drift" effect are measured on the same fluctuating quantity, one in the frequency domain and the other in the time domain, "drift" can be considered as the first order ensemble average of a more general nonstationary stochastic process which is responsible for 1/f noise.

8.1 Recommendations for Further Study

The nonstationarity of 1/f noise existing in a wide variety of systems out of thermodynamic equilibrium should be considered as a major result of this thesis. This opens a new interest in the study of nonstationary processes, not only from the mathematical standpoint, but also from the physical point of view. Because of the stability of the measured 1/f noise spectrum, 1/f noise can be considered to be probably a first example of a nonstationary process which exists in a stationary state in systems which are out of thermodynamic equilibrium. Several systems which possess 1/f noise can be treated as sources of nonstationary fluctuations which can be employed to study the response of other systems to nonstationary stochastic inputs.

In order to characterize the specific nonstationary process which is responsible for 1/f noise, direct studies of the statistical properties of 1/f noise sources seem important. In the recent years, some of these studies have been done (Brophy, 1970; Purcell, 1972; Dell, 1973; Bell, 1975; Stoisiek, 1976). In the works of Brophy (1970), Dell (1973) and Bell (1975), the notion of "variance noise", which is the fluctuations in the variance of a noise source, is employed and it is shown that the "variance noise" is significantly larger in the case of a 1/f noise source as compared to the case of a Nyquist noise source. Purcell (1972) computes the spectral distribution of the "variance noise" and it is shown that the power spectral density of "variance noise" is also of the $1/f^{\alpha}$ type. Although the nonstationary character of the 1/f noise source is hinted in the investigations by Brophy (1970), Dell (1973), Bell (1975) and Purcell (1972), it is difficult to see how the information provided by them can be easily translated into the determination of a nonstationary process responsible for the generation of 1/f noise. It should be mentioned that the measurements made by Stoisiek (1976) test only the stationarity of the power spectral density measurement of 1/f noise (stability of the measured 1/f noise spectra), and, therefore, cannot be employed to test the stationarity or the nonstationarity of the 1/f noise source. A major difficulty encountered in all these studies of the statistical properties of 1/f noise is the nonexistence of elegant schemes to investigate nonstationary stochastic processes. Thus it seems reasonable to develop better statistical methods for characterizing nonstationary processes before a nonstationary model for 1/f noise can be proposed and conclusively verified experimentally. Such a study should be interesting in retrospection of obtaining a general model for 1/f noise.

BIBLIOGRAPHY

Attkinson, W. R., L. Fey, and J. Newman.

1963 "Spectrum Analysis of Extremely Low Frequency Variations of Quartz Oscillators." Proc. IEEE, 51, 379.

Bakanov, S. P., and A. A. Rukhadze.

1965 "Excitation of Electromagnetic Waves in Plasmas Situated in External Electric Fields." Sov. Phys. - JETP, 21, 1113-1120.

Baldinger, E., and E. Nüesch.

1968 "Rauschen Von Transistoren bei sehr tiefen Frequenzen." Helv. Phys. Acta, 41, 313-322.

Barnes, J. A., and D. W. Allan.

1966 "A Statistical Model of Flicker Noise." Proc. IEEE, 54, 176-178.

Bell, D. A.

1955 "Phenomenological Approach to Current Noise." Br. J. Appl. Phys., 6, 284-287.

1968 "Present Knowledge of 1/f Noise." Conference on Physical Aspects of Noise in Electronic Devices, (Sept. 11-13), Nottingham, England, 106-115.

Bell, D. A., and S. P. B. Dissanayake.

1975 "Variance Fluctuations of 1/f Noise." Electron. Lett., 11, 274.

Bendat, J. S., and A. G. Piersol.

1966 Measurement and Analysis of Random Data. New York: John Wiley and Sons, Inc.

Bernamont, J.

1937 "Fluctuations in the Resistance of Thin Films." Proc. Phys. Soc. (London), 49, 138-139.

Berz, F., and C. G. Prior.

1970 "Test of McWhorter's Model of Low-Frequency Noise in Si M.O.S.T.S." Electron. Lett., 6, 595-597.

Bilger, H. R., J. L. Tandon, and M-A. Nicolet.

1974 "Excess Noise Measurements in Ion-Implanted Silicon Resistors." Solid-State Electron., 17, 599-605.

Bilger, H. R., and D. Halford.

1976 Private Communication.

Bloodworth, G. G., and R. J. Hawkins.

1971 "Drift and Low Frequency Noise." Radio and Electron. Eng., 41, 61-64.

Boehm, T. J.

1975a "A Noise Spectrum Analyzer for Frequencies Less Than One Hertz and Investigation of Low Frequency Zener Diode Noise." (Unpub. Ph. D. thesis, Oklahoma State University.)

Boehm, T. J., and H. R. Bilger.

1975b "Low Frequency Excess Noise in Zener Diodes." Fourth International Conference on Physical Aspects of Noise in Solid State Devices, (Sept. 9-11), Noordwijkerhout, The Netherlands, 61-64.

Brophy, J. J.

1970 "Low Frequency Variance Noise." J. App. Phys., 41, 2913-2916.

Caloyannides, M. A.

1974 "Microcycle Spectral Estimates of 1/f Noise in Semiconductors." J. App. Phys., 45, 307-316.

Campbell, M. C., and B. W. Jones.

1972 "Cyclic Changes in Insulin Needs of an Unstable Diabetic." Science, 177, 889-891.

Clarke, J., and R. F. Voss.

1974 "1/f Noise From Thermal Fluctuations in Metal Films." Phys. Rev. Lett., 33, 24-27.

Clarke, J., and G. Hawkins.

1975 "Low Frequency Noise in Josephson Junctions." Fourth International Conference on Physical Aspects of Noise in Solid State Devices, (Sept. 9-11), Noordwijkerhout, The Netherlands, 48-51.

Conti, Mario.

1970 "Surface and Bulk Effects in Low Frequency Noise in NPN Planar Transistors." Solid-State Electron., 13, 1461-1469.

Dell, R. A.

1973 "Experimental Study of 1/f Stationarity by Digital Techniques." J. App. Phys., 44, 472-476.

Dill, H. G., R. W. Brouwer, and J. W. Toombs.

1971 "Ion-Implanted MOS Technology." Radiat. Eff., 7, 45-57.

Donati, F.

1971 "Finite-Time Averaged Power Spectra." IEEE Trans. Inf. Theory., IT-17, 7-16.

Firle, T. E., and H. Winston

1955 "Noise Measurements in Semiconductors at Very Low Frequencies." J. App. Phys., 26, 716-717.

Frederick, H. A.

1931 "The Development of the Microphone." J. Acoust. Soc. Am., 3 (Supplement to No.1, July), 1-27.

Ganefel'd, R. V., and I. I. Repa.

1969 "Excess Noise in Semiconductors." Sov. Phys. - Semicond., 3, 22-24.

Graffunder, W.

1939 "Das Röhrenrauschen bei Niederfrequenz." Telefunken Röhre., 15, 41-63.

Grove, A. S.

1967 Physics and Technology of Semiconductor Devices. New York: John Wiley and Sons, Inc.

Halford, P. H.

1968 "A General Mechanical Model for $|f|^{\alpha}$ Spectral Density Random Noise With Special Reference to Flicker Noise $1/|f|$." Proc. IEEE., 56, 251-258.

Handel, P. H.

1975a "1/f noise - An Infrared Phenomenon." Phys. Rev. Lett., 34, 1492-1495.

1975b "Nature of 1/f Phase Noise." Phys. Rev. Lett., 34, 1495-1498.

1975c "Quantum Theory of 1/f Noise." Phys. Lett., 53A, 438-440.

1975d "Introduction to the Quantum Theory of 1/f and 1/Δf Noise." Fourth International Conference on Physical Aspects of Noise in Solid State Devices, (Sept. 9-11), Noordwijkerhout, The Netherlands, 69-72.

Hawkins, R. J., I. R. M. Mansour, and G. G. Bloodworth.

1968 "Current Noise in M.O.S. Transistors at Very Low Frequencies." Conference on Physical Aspects of Noise in Electronic Devices, (Sept. 11-13), Nottingham, England, 127-130.

Heirtzler, J. R.

1962 "The Longest Electromagnetic Waves." Sci. Am., (March), 128-137.

Hooge, F. N., H. J. A. van DIJK, and A. H. M. Hoppenbrouwers.

1970 "1/f Noise in Epitaxial Silicon." Philips Res. Rep., 25, 81-86.

Hooge, F. N.

1972 "Discussion of Recent Experiments on 1/f Noise." Physica, 60, 130-144.

Hoppenbrouwers, A. M. H., and F. N. Hooge.

1970 "1/f Noise of Spreading Resistances." Philips Res. Rep., 25, 69-80.

Jaeger, R. C., and A. J. Brodersen.

1970 "Low-Frequency Noise Sources in Bipolar Junction Transistors." IEEE Trans. Electron Devices, ED-17, 128-133.

Johnson, J. B.

1925 "The Schottky Effect in Low-Frequency Circuits." Phys. Rev., 26, 71-85.

1971 "Electronic Noise: the First Two Decades." IEEE Spectrum, (Feb.), 42-46.

Kleinpenning, Th. G. M.

1974 "1/f Noise in Thermo emf of Intrinsic and Extrinsic Semiconductors." Physica, 77, 78-98.

Lampard, D. G.

1954 "Generalization of the Wiener-Khintchine Theorem to Nonstationary Processes." J. App. Phys., 25, 802-803.

Lathi, B. P.

1968 An Introduction to Random Signals and Communication Theory. Pennsylvania: The International Textbook Company.

Lauritzen, P. O.

1965 "Low Frequency Generation Noise in Junction Field Effect Transistors." Solid-State Electron., 8, 41-58.

Leuenberger, F.

1967 "Audiofrequency Noise in Silicon." Electron. Lett., 3, 400.

1971 "Absence of 1/f Noise in M.O.S. Transistors Operated in Saturation." Electron. Lett., 7, 561.

Levin, M. J.

1964 "Instantaneous Spectra and Ambiguity Functions." IEEE Trans. Inf. Theory, IT-10, 95-97.

Malakhov, A. N., and A. V. Yakimov.

1975 "Non-Equilibrium Nature of the 1/f Noise." Fourth International Conference on Physical Aspects of Noise in Solid State Devices, (Sept. 9-11), Noordwijkerhout, The Netherlands, 81-84.

Mansour, I. R. M., R. J. Hawkins, and G. G. Bloodworth.

1968 "Measurement of Current Noise in M.O.S. Transistors from 5×10^{-5} to 1 Hz." Radio and Electron. Eng., (April), 212-216.

McWhorter, A. L.

1957 "1/f Noise and Germanium Surface Properties." Semiconductor Surface Physics. Ed. R.H. Kingston. Philadelphia: University of Pennsylvannia Press, 207-228.

Montgomery, H.C.

1952 "Electrical Noise in Semiconductors." Bell Syst. Tech. J., 31, 950-975.

Müller, O., and J. Pest.

1970 "Thermal Feedback in Power Semiconductor Devices." IEEE Trans. Electron Devices, ED-17, 770-782.

Müller, O.

1971 "Thermal Feedback: A Cause for 1/f Flicker Noise in Transistors." International Symposium on Noise of Active Semiconductor Components, (Sept.), Toulouse, France, 95-103.

1974a "A Formula for 1/f Flicker Noise in p-n Junctions." Arch. Elektron and Uebertragungstech., Band 28, 429-432.

1974b "Thermal Fluctuations and Flicker Noise in p-n Junction Diodes." IEEE Trans. Electron Devices, ED-21, 539-540.

Munk, W. H.

1960 The Rotation of the Earth. Cambridge: University Press, 192.

Offner, F. F.

1970 "1/f Noise in Semiconductors." J. App. Phys., 41, 5033-5034.

Otto, R.

1935 Hochfreq. Tech. Elektroakust., 45, 187-198.

Papoulis, A.

1965 Probability, Random Variables, and Stochastic Processes. New York: McGraw-Hill Book Company.

Perrone, A. J.

1969 "Deep-Ocean Ambient Noise Spectrum in the Northwest Atlantic." J. Acoust. Soc. Am., 46, 762-770.

Purcell, W. E.

1972 "Variance Noise Spectra of 1/f Noise." J. App. Phys., 43, 2890-2895.

Reif, F.

1965 Fundamentals of Statistical and Thermal Physics. New York: McGraw-Hill Book Company, 254.

Richardson, J. M.

1950 "The Linear Theory of Fluctuations Arising From Diffusional Mechanisms-An Attempt at a Theory of Contact Noise." Bell Syst. Tech. J., 29, 117-141.

Ringo, J. A., and P. O. Lauritzen.

1972 "Voltage Drift Related to Low Frequency Noise in Reference Diodes." Proc. IEEE, 60, 236-237.

Rollin, B. V., and I. M. Templeton.

1953 "Noise in Semiconductors at Very Low Frequencies." Proc. Phy. Soc. (London), B66, 259-261.

Sah, C. T., and F. F. Hielscher.

1966 "Evidence of the Surface Origin of the 1/f Noise." Phys. Rev. Lett., 17, 956-958.

Schiff, L. I.

1968 Quantum Mechanics. New York: McGraw-Hill Book Company, 57.

Schottky, W.

1926 "Small Shot Effect and Flicker Effect." Phys. Rev. 28, 74-103.

Stephany, J. R.

1975 "Origin of 1/f Noise." J. App. Phys., 46, 665-667.

Stoisiek, M., and D. Wolf.

1976 "Recent Investigations on the Stationarity of 1/f Noise." J. App. Phys., 47, 362-364.

Tandon, J. L.

1973a "Excess Noise Spectral Analysis of Boron Implanted Layers in Silicon." (Unpub. M.S. thesis, Oklahoma State University.)

Tandon, J. L., H. R. Bilger, and M-A. Nicolet.

1973b "Excess Noise Spectral Analysis of Boron Implanted Layers in Silicon." SWIEEECO Record, 561-568.

Tandon, J. L., H. R. Bilger, and M-A. Nicolet.

1975a "Thermal Equlibrium Noise of Space Charge Limited Current in Silicon for Holes with Field-Dependent Mobility." Solid-State Electron., 18, 113-118.

Tandon, J. L., H. R. Bilger, and M-A. Nicolet.

1975b "Excess Noise in Ion-Implanted Silicon Resistors." Fourth International Conference on Physical Aspects of Noise in Solid State Devices, (Sept. 9-11), Noordwijkerhout, The Netherlands, 85-89.

Teitler, S., and M. F. M. Osborne.

1970 "Phenomenological Approach to Low-Frequency Electrical Noise." J. App. Phys., 41, 3274-3276.

Tunaley, J. K. E.

1974 "Nyquist's Theorem and 1/f Noise." J. App. Phys., 45,482-483.

Turner, C. H. M.

1954 "On the Concept of an Instantaneous Power Spectrum, and its Relationship to the Auto-Correlation Function." J. App. Phys., 25, 1347-1351.

Van der Ziel, A.

1959 Fluctuation Phenomena in Semiconductors. London: Butterworths Scientific Publications

Van der Ziel, A., and Hu Tong.

1966 "Low-Frequency Noise Predicts When a Transistor Will Fail." Electronics, (Nov. 28), 95-97.

Van Vliet, K. M., J. Block, C. Ris, and J. Stekette.

1956 "Measurements of Noise and Response to Modulated Light of Cadmium Sulphide Single Crystals." Physica, 22, 723-740.

Van Vliet, K. M., and A. Van der Ziel.

1958 "On the Noise Generated by Diffusion Mechanisms." Physica, 24, 415-421.

Verveen, A. A., and H. E. Derksen.

1968 "Fluctuation Phenomena in Nerve Membrane." Proc. IEEE, 56, 906-916.

Weissman, M. B.

1975 "Simple Model for 1/f Noise." Phys. Rev. Lett., 35, 689-692.

VITA

Jawahar Lal Tandon

Candidate for the Degree of

Doctor of Philosophy

Thesis: 1/f NOISE AS A NONSTATIONARY PROCESS

Major Field: Electrical Engineering

Biographical:

Personal Data: Born on September 19, 1949, in Agra, India, the son of Mr. and Mrs. A. P. Tandon.

Education: Attended primary and secondary schools in New Delhi, India, and graduated from Air Force Central School, New Delhi, India in September, 1965; received the Bachelor of Technology degree in Electrical Engineering from Indian Institute of Technology, Kanpur, India, in October, 1970; received the Master of Science degree from Oklahoma State University, Stillwater, Oklahoma in May, 1973; attended California Institute of Technology, Pasadena, California from September, 1973 to August, 1974; completed requirements for the Doctor of Philosophy degree at Oklahoma State University, Stillwater, Oklahoma in May, 1976.

Professional Experience: Graduate Teaching/Research Assistant, School of Electrical Engineering, Oklahoma State University, Stillwater, Oklahoma, January, 1972 to May, 1973 and September, 1974 to present; Development Engineer, Hewlett Packard Laboratories, Palo Alto, California, Summer, 1973; Graduate Research Assistant, California Institute of Technology, Pasadena, California, September, 1973 to August, 1974.

Professional Organizations: Member of Eta Kappa Nu.